Switched System Model-Based Approaches for Detection and Isolation of Multiplicative Intermittent Faults in Dynamic Systems

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Mathematical notations

Notation	Description
$\mathcal{RH}_\infty$	the set of all stable transfer matrices
$\forall$	for all
$\in$	belong to
$\exists$	exist
$\Longrightarrow$	imply
$\Longleftrightarrow$	equivalent to
$:=$	defined as
$\|\| \cdot \|\|$	Euclidean norm of a vector
$\|\| \cdot \|\|_2$	$\mathcal{L}_2$ norm of a signal
$\|\| \cdot \|\|_{rms}$	root mean square norm of a signal
$\|\| \cdot \|\|_\infty$	$\mathcal{L}_\infty$ norm of a signal
$\hat{\mathbf{x}}$	estimate of the state vector $\mathbf{x}$
$\mathbf{X}^T$	transpose of $\mathbf{X}$
$\mathbf{X}^{-1}$	inverse of $\mathbf{X}$
$\mathbf{X} > \mathbf{0}$	$\mathbf{X}$ is positive definite matrix
$\mathcal{R}^n$	space of n-dimensional vectors
$\mathcal{R}^{n\times m}$	space of n by m matrices
$\mathtt{Sym}\{\mathbf{X}\}$	$\mathbf{X} + \mathbf{X}^T$
$\mathtt{det}\{\mathbf{X}\}$	determinant of $\mathbf{X}$
$eig(\mathbf{X})$	eigenvalues of $\mathbf{X}$
$max(min)$	maximum (minimum)
$\mathtt{sup}$	supremum
sgn	take the sign
arg	find the argument that gives the maximum value from a target function
$*$	an ellipsis for the symmetry terms in symmetric block matrices
$\mathcal{B}_q$	space of the vector $\mathbf{x} \in \mathcal{R}^n$ satisfying $\|\|\mathbf{x}\|\| \leq q$ for some $q > 0$
$\mathbf{I}_m$	m by m identity matrix
$\mathbf{0}_m$	m by m zero matrix

$\mathbf{0}_{m\times n}$	m by n zero matrix
Δ	model uncertainty
$\beta(\cdot) \in \mathcal{K}$	$\beta : \mathcal{R}_+ \to \mathcal{R}_+$ is continuous, strictly increasing, and satisfies $\beta(0) = 0$
$\beta(\cdot) \in \mathcal{K}_\infty$	$\beta \in \mathcal{K}$, and in addition, $\lim\limits_{s\to\infty} \beta(s) = \infty$
$\beta(\cdot) \in \mathcal{L}$	β is continuous, strictly decreasing, and satisfies $\lim_{s\to\infty} \beta(s) = 0$
$\phi(s,t) \in \mathcal{KL}$	for each fixed t the function is of class $\mathcal{K}$ and for each fixed s it is of class $\mathcal{L}$

1 Introduction

Multiplicative intermittent faults observed in industrial processes and systems imply serious threats to system reliability and decrease production safety. Therefore, this thesis aims at the investigation of intermittent fault detection methods in stochastic and deterministic systems. A detection scheme is developed for systems with multiplicative intermittent faults. These points of view will be presented successively in this thesis.

1.1 Background and motivation

A fault, which is defined as an unexpected change occurring in a process, is a special state that the process cannot properly perform its specified function under normal conditions. According to the occurrence characters, faults can be classified into the following categories:

- permanent faults (PFs) take effect on the process permanently until repairing. They are very common faults that occurring in industrial processes;
- transient faults (TFs) that appear and disappear instantaneously and their occurrence is almost completely random in nature. For instance, in electronic systems, they are usually caused by electromagnetic radiation, overheating and input power variations [11];
- intermittent faults (IFs) that often reoccur and last for a limited time interval, then disappear without any external correction. The cause of the IFs can be hardware failures in mechanic systems, digital circuits, the power industry, software problems, cyber-attacks, etc.

There have already been many research efforts made on the detection of PFs since the 1970s, while TFs have less impact on the system, and in some cases they are regarded as a part of noises. Compared with TFs, IFs may repeatedly occur at the same location, the influence of IFs, especially amplitude and duration, could increase because of the cumulative effect. It may lead to further deterioration of the entire system and cause severe accidents, for example:

- in digital circuits, IFs are the most frequent failures. According to statistics, the probability of the occurrence of an IF has reached nearly 90%, which is 30 times that of PFs [44, 65]. Most IFs are caused by hardware failures such as vibration, electro-migration, manufacturing defect, etc., and IFs caused by the software are memory leak, disk error, etc. [2], the cyber-attacks is a typical kind of IFs, which attracts a lot of attention;

- in the electric power transmission industry, arcing faults, which are typical IFs, may cause devastation damages such as electric fires and explosions [47, 89]. Moreover, due to the complex operating environment, there exists a large amount of undetected IFs in electric power distributed systems, which may lead to more severe hazards and decrease the reliability of power delivery [20];

- in the transport industry, the most important parts, such as bears, motors and gears, show repetitively characters and may fail intermittently. These problems will result in connection loosing, solder joint crack, corrosion, etc. [71] Such faults will significantly increase the maintenance costs and introduce addition financial burdens. According to American Trans Air members statistics, IFs lead to thousands of flight delays and cancellations, the total economic loss is annually 100 million dollars [78].

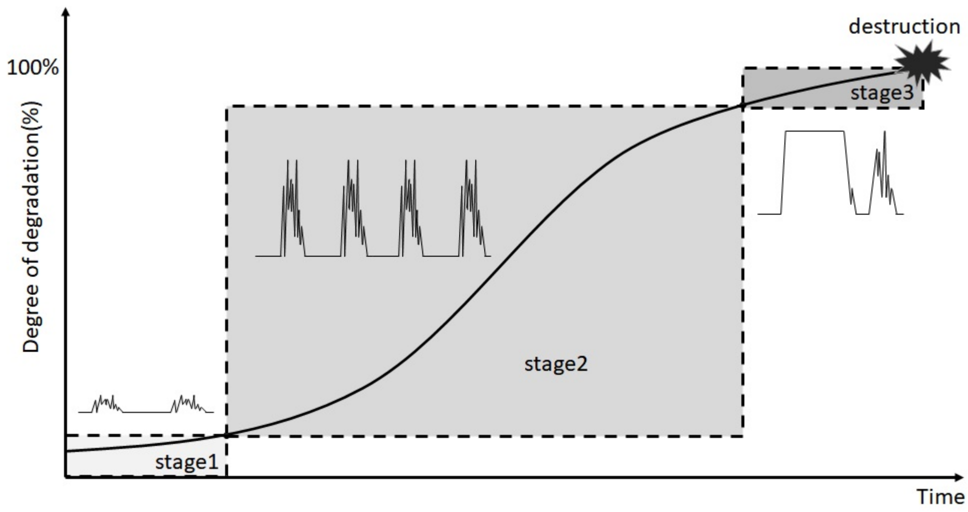

Figure 1.1: Evolution process of systems with IFs [10].

In the application domains, the "severity" of IFs will increase when the system is constantly running. As mentioned in [10], the frequency, amplitude and duration of IFs may increase over time, and the corresponding evolution process is shown in Fig. 1.1.

Faults in systems can also be classified into additive faults and multiplicative faults. Their characteristics and corresponding research results are summarized as follows:

- an additive fault is always modeled by additive term in state space representation (e.g. $E_f f$), notice that the occurrence of an additive fault cannot affect the system stability. There are also numerous investigations on dealing with additive faults. A recent useful method is developed by using the Plug-and-Play concept, which is interpreted as a reconfiguration of the controller after adding new sensors or subsystems of the process [6, 73], the additive faults could be compensated and the system could have optimal performance;

- the multiplicative fault may lead to the variation of the system structure, and cause changes in system eigen-dynamics. They widely exist in engineering systems, such as high-speed trains [14] and wind turbine systems [76].

In the past decades, investigations on fault detection (FD) for IFs have attracted considerable attention [1, 11, 66]. [96] gives a review on diagnosis techniques for IFs and discusses their strengths, limitations and future directions.

According to the above background knowledge, a multiplicative IF processes the following characteristics: occurring intermittently, changing the overall structure of the system, and even affecting the stability of the system. For the research on multiplicative IFs detection, we need to pay attention to three aspects: (1) modeling the system with IFs in a suitable way, (2) developing a proper FD scheme, and (3) studying the influence of multiple IFs' frequency and duration on system stability.

Since the IFs can be viewed as (frequent) switchings between two modes: inactive (fault-free) mode and active (faulty) mode, the overall system dynamics can be considered as a switched system. Roughly speaking, a switched system consists of a series of subsystems described by differential or difference equations and a logic rule that regulates the switchings between them. In this context, dynamic systems with multiplicative IFs are switched systems, and we could analyze their stability conditions by the well-developed switched system theory. Strongly motivated by these observations, the next subsections attempt to review some of the major developments in FD and switched systems.

1.1.1 Fault detection and fault classification schemes

With the increasing complexity of industrial processes and systems, early detection, classification and diagnosis of faults become more and more important. Due to this, related issues attract considerable attention both in academic and industrial fields.

According to the previous introduction, it can be seen that the faults could cause harmful results to the systems. Therefore, detecting the faults as early as possible is important. To this end, numerous FD approaches have been developed since the 1970s, including hardware redeundancy schemes, model-based FD, signal-based FD, data-driven FD methods, etc.

The development of model-based FD scheme begins by replacing hardware redundancy with analytical redundancy [4]. In this approach, the observer-based residual generator is developed, which is the fundamental of this thesis, and the schematic of the model-based FD scheme is shown in Fig. 1.2. In [34], an observer-based residual generator is established for FD schemes, based on this observer-based framework, many achievements have been made in different research domains, more details can be found in [7, 19, 25, 64].

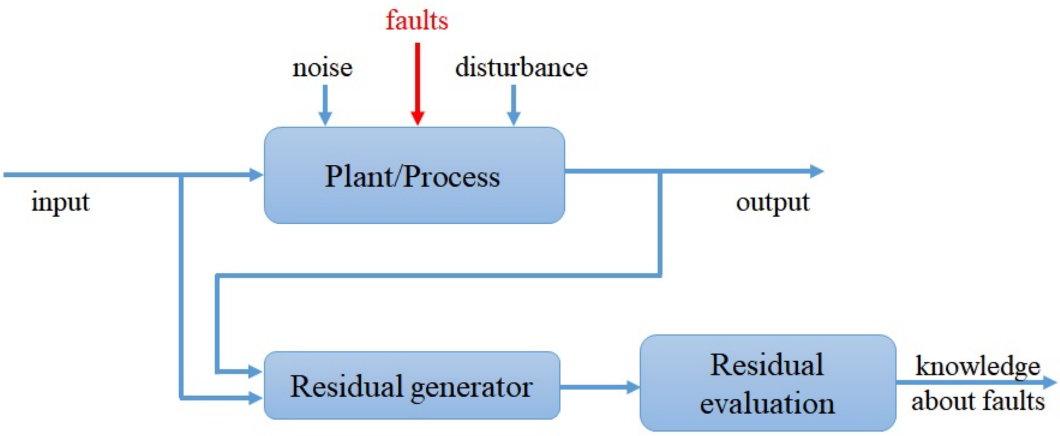

Figure 1.2: Schematic of the model-based FD schemes.

With the development of computer science and data network technologies, it is possible to propose process monitoring and FD schemes by analysing a large amount of data in an acceptable time interval. Motivated by this, the FD systems in data-driven framework have made great progress, many statistical methods have been developed, such as principal analysis [68], partial least squares [18], canonical correlation analysis [16], independent component analysis [88], etc. In [26], the data-driven FD approaches are systematically introduced.

For stochastic systems and deterministic systems, there have been considerable research results on FD issues. For the system with white Gaussian noise, an FD scheme consists of residual generation, evaluation (statistic test) and threshold setting. The FD methods have been addressed in different approaches, for instance, the residuals are generated by Kalman filter in [60], an FD strategy is achieved using generalized likelihood ratio test method in [83], etc. Parallel to the stochastic systems, FD problems for systems with norm-bounded disturbances and uncertainties are also discussed. Several methods are proposed, such as the decoupling approach [15], the linear matrix inequality technique-aid approach [25], $\mathcal{L}_\infty$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based method [86], and in this case, an adaptive threshold is adopted for robust FD systems. In conclusion, a standard procedure for model-based FD is investigated, including residual generation, residual evaluation and decision making.

1.1.2 Switched systems

In complex industrial processes, the continuous and discrete dynamics may co-exist and influence each other, and the parameters may vary with the changes of time or state. Hence, the hybrid system is proposed to describe the dynamic behaviours of such systems. As an important branch of hybrid systems, a switched system consists of a class of subsystems and a logical switching rule that governs the dynamics of the particular subsystems, and Fig. 1.3 shows the structure of a typical switched system. Investigations on switched systems have attracted considerable attention, and some scholars focus on some issues of switched system such as: stability conditions [58], control issues [94], FD schemes [67, 74], etc. In practice, there are numerous applications of switched systems, e.g., direct current power converters [23], wind turbine control [48], network control [77], etc.

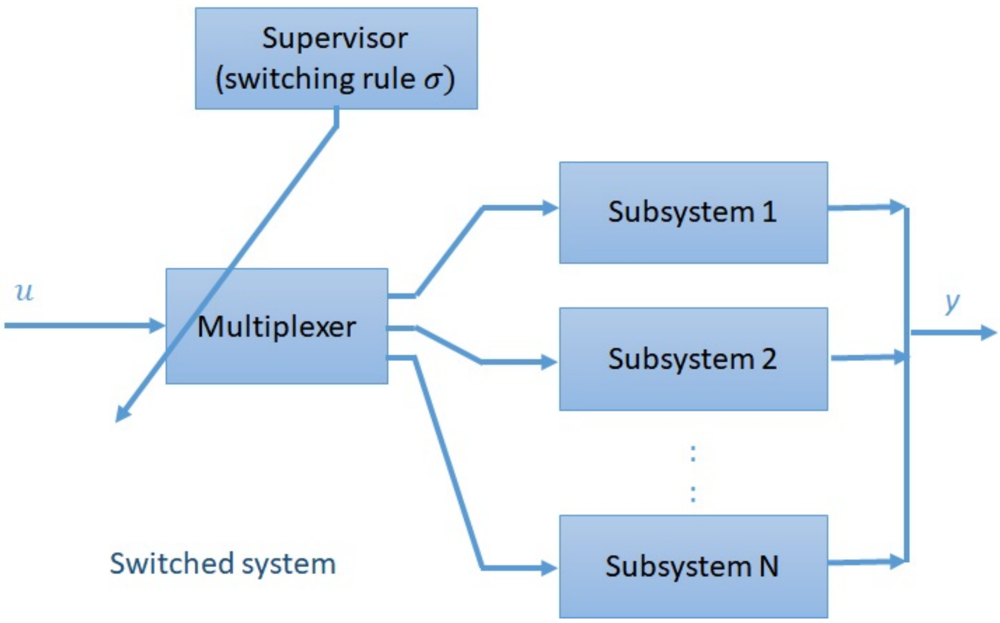

Figure 1.3: Schematic of switched systems.

One of the special characters of a switched system is its stability condition. For example, even though all the subsystems are exponentially stable, the switched systems could have divergent trajectories under a certain switching signal [24]. On the other hand, a switched system consisting of unstable sub-systems may be exponentially stable when choosing a proper switching rule. That means the stability of switched systems depends not only on the dynamics of each subsystem, but also on the switching rule. Therefore, it is necessary to analyze stability conditions in all studies related to switched systems.

Until now, many efforts have been dedicated to the stability analysis, and there exist some widely utilized methods: (1) find quadratic Lyapunov function for all its subsystems and ensure the stability in arbitrary case [21, 53]; (2) design a certain switching rule (see [80] and the references therein); (3) analyze the stability conditions under a time-constrain switching case, which is well-known as average dwell time (ADT) method [39, 62, 90, 91, 95]. In all the above attempts, the ADT method is mostly used in recent years. In this method, the switching frequency and the time interval between switchings play important roles in the stability analysis.

1.2 Objectives of the thesis

Inspired by the aforementioned motivations, the main objectives of this thesis are to investigate the approaches for detecting multiplicative IFs, to develop the FD schemes for stochastic and uncertain systems. More specifically, the tasks of this thesis are stated as follows:

1. Model linear time-invariant dynamic systems with multiplicative intermittent faults as switched systems.
2. Propose an FD approach with a mode estimation unit for switched systems subject to norm-bounded disturbance and uncertainties.
3. Develop a likelihood ratio (LR) test-based detection scheme for detecting IFs in stochastic systems.
4. Propose a detection scheme with an embedded mode estimation unit for multiplicative IFs and analyze the stability condition of the FD system.

1.3 Outline of the thesis

As shown in Fig. 1.4, this thesis consists of 7 chapters, and the structure as well as the contribution of each chapter are summarized as follows.

Chapter 1: Introduction

This chapter introduces the motivation, objectives, outline and contributions of this thesis.

Chapter 2: Basics of Fault Detection Systems

In Chapter 2, review the state-of-the-art of FD schemes for both stochastic and deterministic systems. To this end, knowledge of modeling systems and faults, the basics of FD systems and specific techniques are given. They provide fundamentals for developing FD approaches.

Chapter 3: Fundamentals of Switched Systems

In this chapter, the basics of switched systems and IFs are presented. A brief introduction of switched systems is given, including background, stability analysis under arbitrary and strict switching. Then the framework of using switched systems to describe the systems with intermittent faults is established.

Chapter 4: Design of observer-based Fault Detection schemes for uncertain switched systems

This chapter is dedicated to the design issues of observer-based FD for switched systems with norm-bounded uncertainties and disturbances. To this end, a so-called $\mathcal{L}_\infty/\mathcal{L}_2$ type of residual generator is investigated with the aid of the mode-dependent ADT method. To estimate the switching instant and improve the conservative threshold setting due to the asynchronized residual generation, a mode estimation unit is developed in the FD decision logic.

Chapter 5: A Fault Detection Approach for Stochastic Systems with Multiplicative Intermittent Faults

This chapter discusses a switched system theory-based framework for detecting multiplicative IFs in stochastic systems. Since the IFs can be viewed as switchings between inactive (fault-free) mode and active (faulty) mode, the overall system dynamics are governed by a switched system. The residual signals in both modes can be generated by the Kalman filter algorithm and the detection decision is proposed by LR test-based residual evaluation and threshold setting. The mode estimation unit is developed and in the last subsection, and the stability condition is analyzed based on the ADT method.

Chapter 6: Application to benchmark processes

In this chapter, the effectiveness and feasibility of the proposed FD schemes are verified by two examples.

Chapter 7: Conclusions and Future Work

This chapter provides a summary of the contributions in this thesis and gives the conclusion of this work. Then some future scopes are discussed in the end of this chapter.

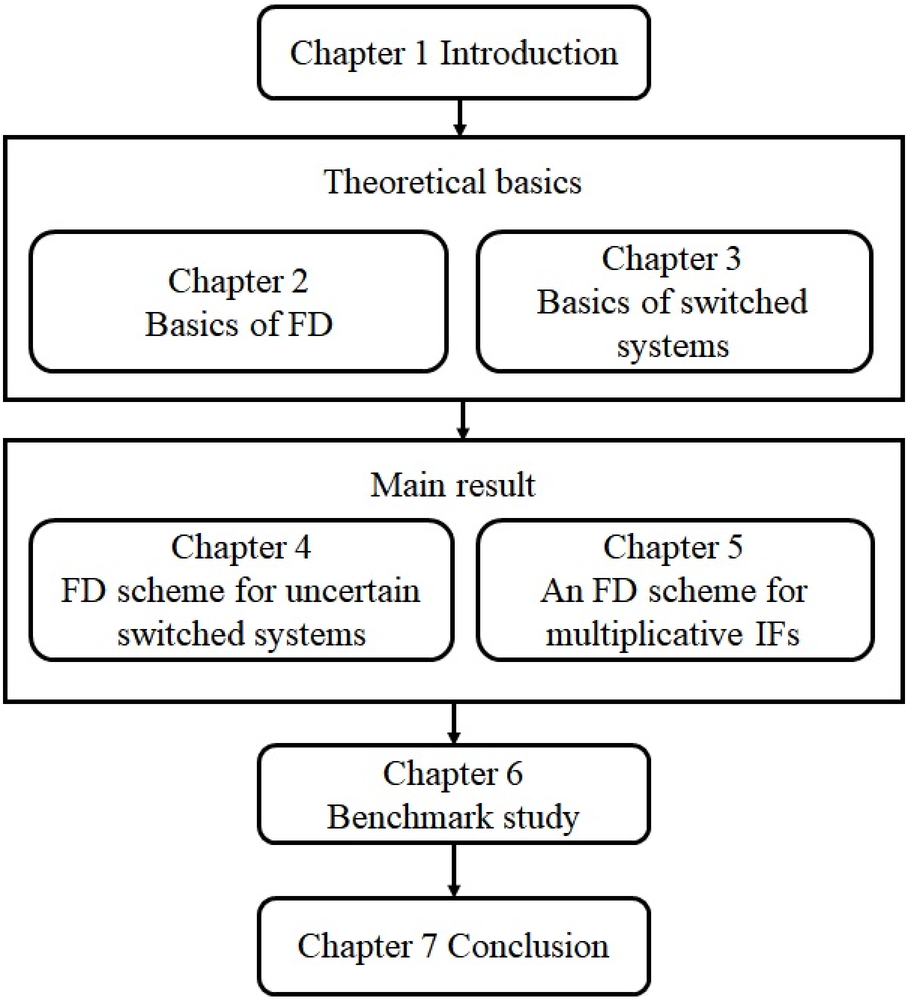

Figure 1.4: Organization of the thesis.

2 Basics of Fault Detection Systems

In this chapter, an overview of the basic knowledge of FD and the state-of-the-art of FD schemes is given. First present the mathematical description of systems and then review the basics of FD problems of stochastic and deterministic systems, which includes the basic concepts, residual generation and evaluation techniques, threshold setting and decision making.

2.1 Mathematical description of dynamic systems

This section introduces mathematical descriptions of dynamical systems. Lots of attention are paid on the systems in both faulty and fault-free cases, including two standard mathematical model forms, transfer function matrix and state space representation. The objective of this section is to build mathematical models for: (1) fault-free and disturbance-free systems, (2) systems with disturbances and uncertainties, and (3) systems with faults.

2.1.1 Description of nominal system behaviour

Linear time-invariant (LTI) systems are widely investigated in research and application domains due to their simple form. In order to describe an LTI system, two typical standard mathematical forms are introduced. One of them is the transfer function matrix, which shows the input-output dynamic behaviour in the frequency domain. A discrete-time system in nominal (fault-free and disturbance-free) as

$$y(z) = G_{yu}(z)u(z), \tag{2.1}$$

where $u \in \mathcal{R}^{n_u}$ and $y \in \mathcal{R}^{n_y}$ refer to the input and output signal, respectively; $G_{yu}(z) \in \mathcal{LH}_{\infty}^{n_y \times n_u}$ is the transfer function matrix of the LTI system which describes the transfer dynamics between the input and output signals. We use z to denote the variable of z-transform for discrete-time signals.

In addition, the system (2.1) can be mathematically modeled by the following state space form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k), x(0) = x_0, \\ y(k) = Cx(k) + Du(k), \end{cases} \tag{2.2}$$

where $x(k) \in \mathcal{R}^{n_x}$ represents the state variable of the system, $x_0 \in \mathcal{R}^{n_x}$ is the initial state. A, B, C and D are system matrices with appropriate dimensions. When $n_u = 1, n_y = 1$, the system is a Single-Input Single-Output (SISO) system; otherwise, the system is a Multiple-Input Multiple-Output (MIMO) system. We use k to denote the discrete-time samples.

The relationship of these two representations can be written as

$$G_{yu}(z) = C(zI - A)^{-1}B + D. \tag{2.3}$$

It is noteworthy that in this thesis the (A, B, C, D) is assumed to be the minimal realization of the transfer function $G_{yu}(z)$. Moreover, for simplicity of notation, state space minimal realization is denoted as follows:

$$\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] := C(zI - A)^{-1}B + D. \tag{2.4}$$

2.1.2 Coprime factorization technique

For LTI systems, a coprime factorization provides an alternative representation of the system transfer function matrix. Generally speaking, a coprime factorization over $\mathcal{RH}_\infty$ is the factorization of a transfer matrix by two stable and coprime transfer function matrices.

Definition 2.1. *[25] Two matrices $\hat{M}(z)$ and $\hat{N}(z)$ in $\mathcal{RH}_\infty$ are called left coprime over $\mathcal{RH}_\infty$ if there exist another two matrices $\hat{X}(z)$ and $\hat{Y}(z)$ in $\mathcal{RH}_\infty$ that satisfy*

$$\begin{bmatrix} \hat{M}(z) & \hat{N}(z) \end{bmatrix} \begin{bmatrix} \hat{X}(z) \\ \hat{Y}(z) \end{bmatrix} = I. \tag{2.5}$$

Similarly, two transfer matrices $M(z)$ and $N(z)$ in $\mathcal{RH}_\infty$ are called right coprime over $\mathcal{RH}_\infty$ when there exist two transfer matrices $X(z)$ and $Y(z)$ in $\mathcal{RH}_\infty$ satisfying

$$\begin{bmatrix} X(z) & Y(z) \end{bmatrix} \begin{bmatrix} M(z) \\ N(z) \end{bmatrix} = I. \tag{2.6}$$

Definition 2.2. *Let G_{yu} be a transfer function matrix of an LTI system, then the left coprime factorization (LCF) of $G_{yu}(z)$ is defined by $G_{yu}(z) = \hat{M}(z)^{-1}\hat{N}(z)$, where $\hat{M}(z)$ and $\hat{N}(z)$ are LCF pair over $\mathcal{RH}_\infty$. The right coprime factorization (RCF) of $G_{yu}(z)$ is given by $G_{yu}(z) = N(z)M(z)^{-1}$, where $M(z)$ and $N(z)$ are the RCF pair over $\mathcal{RH}_\infty$.*

The lemma given in [45] describes the relationship between coprime and the system state representation.

Lemma 1. *[45] Suppose that $G_{yu}(z)$ is a proper transfer function matrix of an LTI system with a minimal state space realization*

$$G_{yu}(z) = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right].$$

Choose matrices F and L, such that $(A+BF)$ and $(A-LC)$ both are Schur matrices, define

$$\begin{aligned} \begin{bmatrix} M(z) & -\hat{Y}(z) \\ N(z) & \hat{X}(z) \end{bmatrix} &= \left[\begin{array}{c|cc} A+BF & B & L \\ \hline F & I & 0 \\ C+DF & D & I \end{array}\right], \\ \begin{bmatrix} X(z) & Y(z) \\ -\hat{N}(z) & \hat{M}(z) \end{bmatrix} &= \left[\begin{array}{c|cc} A-LC & -(B-LD) & -L \\ \hline F & I & 0 \\ C & -D & I \end{array}\right]. \end{aligned} \tag{2.7}$$

Then

$$G_{yu}(z) = \hat{M}^{-1}(z)\hat{N}(z) = N(z)M^{-1}(z), \tag{2.8}$$

are the LCF and RCF of $G_{yu}(z)$. Moreover, the so-called Bezout identity holds

$$\begin{bmatrix} X(z) & Y(z) \\ -\hat{N}(z) & \hat{M}(z) \end{bmatrix} \begin{bmatrix} M(z) & -\hat{Y}(z) \\ N(z) & \hat{X}(z) \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}. \tag{2.9}$$

2.1.3 Description of systems with uncertainties

In real applications, there always exist some unknown inputs, for example, norm-bounded disturbances, measurement/process noises, structure uncertainties, and so on. In order to model the systems with different types of disturbances, it is needed to extend the nominal system model form that is previously introduced.

A. Noises

Considering a simple model with white Gaussian noises, the system model (2.2) is extended to:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + \omega(k), x(0) = x_0, \\ y(k) = Cx(k) + Du(k) + \upsilon(k). \end{cases} \tag{2.10}$$

Here noise signals $\omega(k), \upsilon(k)$ are statistically independent of $u(k)$ and $x(0)$, and they follow the next assumption.

Assumption 2.1. *The white Gaussian noises in (2.10) are zero mean Gaussian noises with covariance matrices Σ_ω and Σ_υ, respectively, i.e., the characters of the noise can be described as follows:*

$$\begin{aligned}
&E[\omega(k)] = 0, E[\upsilon(k)] = 0, \\
&E[\omega(k)\omega^T(k)] = \Sigma_\omega, E[\upsilon(k)\upsilon^T(k)] = \Sigma_\upsilon, \\
&\forall i \neq j, E[\omega(i)\omega^T(j)] = 0, E[\upsilon(i)\upsilon^T(j)] = 0.
\end{aligned}$$

B. Disturbances

Considering the system is disturbed by norm-bounded disturbances, the system model (2.2) can be extended to:

$$y(z) = G_{yu}(z)u(z) + G_{yd}(z)d(z), \tag{2.11}$$

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + E_d d(k), x(0) = x_0, \\ y(k) = Cx(k) + Du(k) + F_d d(k), \end{cases} \tag{2.12}$$

where $G_{yd}(z)$ is a proper real-ration matrix, E_d, F_d are matrices with proper dimensions, d is a norm-bounded unknown input. A precise description of norms is given in Section 2.3.1.

C. Model uncertainties

Consider an extended form of the transfer matrix model (2.1)

$$y(z) = G_{\Delta,yu}(z)u(z), \tag{2.13}$$

where Δ indicates structure model uncertainty. It can be represented by the following additive form (2.14) and multiplicative form (2.15), respectively,

$$G_{\Delta,yu}(z) = G_{yu}(z) + \Delta, \tag{2.14}$$

$$G_{\Delta,yu}(z) = (I + \Delta)G_{yu}(z), \tag{2.15}$$

where the structure model uncertainty Δ is a norm-bounded matrix defined by $\bar{\sigma}(\Delta) \leq \delta_\Delta, \bar{\sigma} = \max(\text{eig}(\sqrt{\Delta^T\Delta}))$.

For state space representation, the extended form is given by

$$\begin{cases} x(k+1) = (A + \Delta_A)x(k) + (B + \Delta_B)u(k), x(0) = x_0, \\ y(k) = (C + \Delta_C)x(k) + (D + \Delta_D)u(k), \end{cases} \tag{2.16}$$

whose uncertainties are norm-bounded.

2.1.4 Description of systems with faults

Faults are intolerable input signals occurring in systems. In this section, three fault classification definitions are introduced according to the fault occurrence places in the operation, time characters and the effect on system dynamics.

First, the system (2.1) with faults can be modeled by:

$$y(z) = G_{yu}(z)u(z) + G_{yf}(z)f(z), \tag{2.17}$$

where $f \in \mathcal{R}^{n_f}$ is an unknown vector and G_{yf} is proper real-time matrix. Suppose the minimal state space realization of (2.17) is:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + E_f f(k), x(0) = x_0, \\ y(k) = Cx(k) + Du(k) + F_f f(k), \end{cases} \tag{2.18}$$

where E_f, F_f are known matrices with proper dimensions. Obviously,

$$G_{yf}(z) = F_f + C(zI - A)^{-1}E_f.$$

As illustrated in Fig. 2.1, we can divide faults into three categories according to their places in the operation of the system:

1. actuator faults f_A: faults cause changes in actuators,
2. process faults f_P: faults exist in the process,
3. sensor faults f_S: faults act in measuring components.

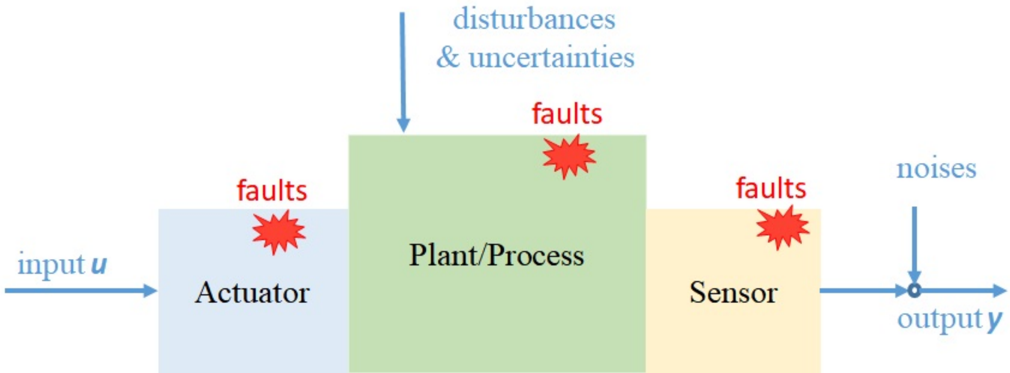

Figure 2.1: Faults in processes [25].

In addition, faults can also be classified into three types according to their occurrence characters (see Fig. 2.2): PFs, IFs, and TFs.

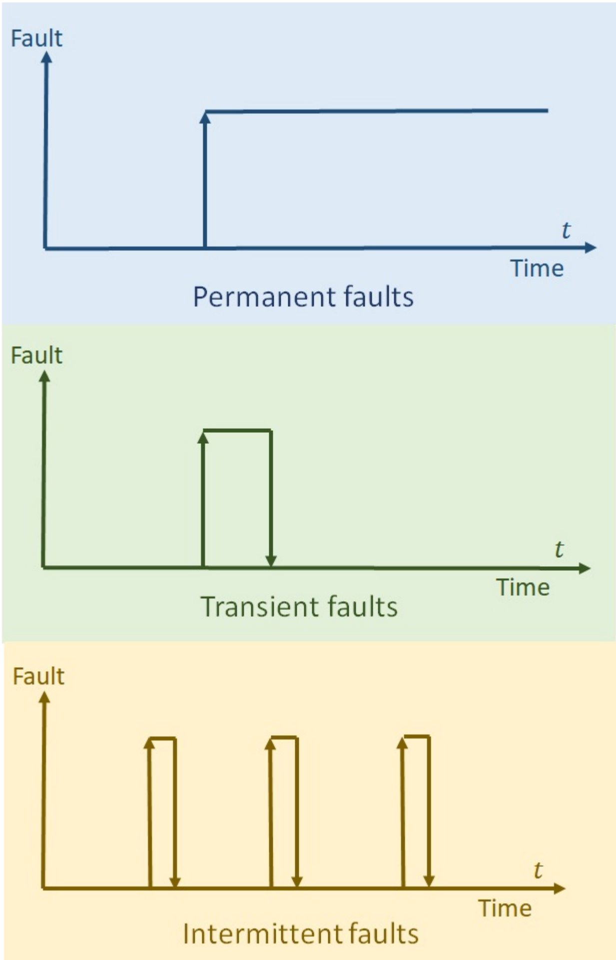

Figure 2.2: Faults categories via time characters.

These three types of faults may appear in systems of real applications. Notably, in some situations they can also transform from one to another, see Fig. 2.3.

Faults described by (2.17) are called additive faults due to the way of affecting the system dynamics linearly. There exists another type of faults called multiplicative faults. Different from the additive faults which do not influence the system stability, multiplicative faults can affect system stability by changing the eigen-dynamics of systems. Suppose that a system with multiplicative faults is expressed as follows:

$$\begin{cases} x(k+1) = (A + \Delta A_F)x(k) + (B + \Delta B_F)u(k), \\ y(k) = (C + \Delta C_F)x(k) + (D + \Delta D_F)u(k), \end{cases} \tag{2.19}$$

where $\Delta A_F, \Delta B_F, \Delta C_F, \Delta D_F$ represent multiplicative faults. Because of the variation of the system structure, the system stability can also be changed. The stability conditions of systems with multiplicative faults are determined by eigenvalues of the matrix $(A + \Delta A_F)$.

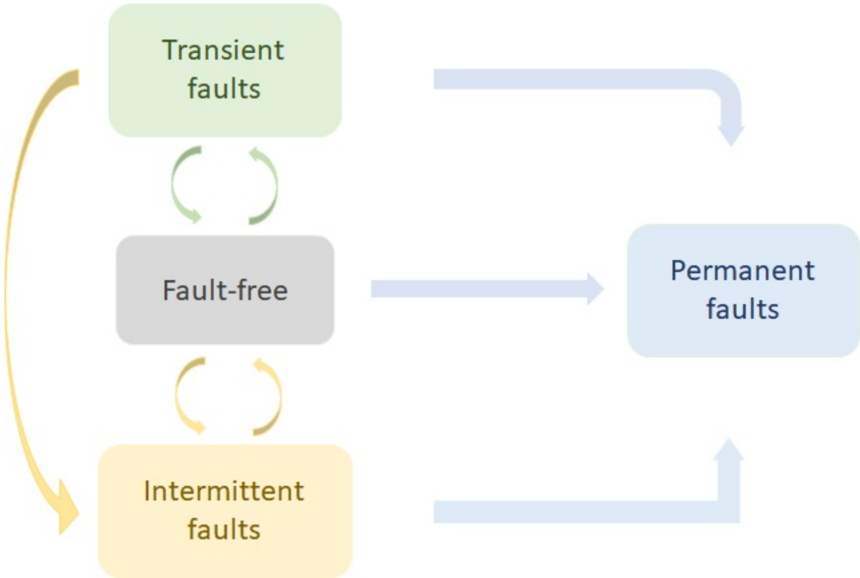

Figure 2.3: Transformation between TFs, IFs and PFs.

2.2 Basic FD problems

This section focuses on preliminaries of FD issues for systems in the statistical framework, including basic ideas, residual generation, residual evaluation and threshold setting. Some useful algorithms for FD system design are also summarized here.

2.2.1 The basics of FD systems in the statistical framework

Based on the measurement model, some essential FD concepts are introduced in this subsection. Measurement models of static and dynamic processes with faults can be expressed as

$$y(z) = f(z) + \epsilon(z) \tag{2.20}$$

$$y(z) = G_{yu}(z)u(z) + G_{yf}(z)f(z) + G_{y\epsilon}(z)\epsilon(z), \tag{2.21}$$

where $y \in \mathcal{R}^m$, and ϵ is subject to normal distribution.

An FD problem can be addressed by the hypothesis testing that consists of two cases: the null hypothesis H_0 and the alternative hypothesis H_1. Then, the FD problem can be re-formulated as follows:

$$\begin{aligned} H_0, \textit{null hypothesis} &: \quad f = 0, \textit{fault-free case}, \\ H_1, \textit{alternative hypothesis} &: \quad f \neq 0, \textit{faulty case}. \end{aligned} \tag{2.22}$$

By applying hypothesis testing, a process of detecting faults is to determine whether the statistic supports the alternative hypothesis and rejects the null hypothesis at the same time.

In the system model (2.20), y can be chosen as a test statistic. Then, test statistic conditions can be described by two hypotheses: (1) null hypothesis $H_0 : y = 0$ and (2) alternative hypothesis $H_1 : y \neq 0$.

However, in real world, there always exist different types of disturbances. In practice, a so-called limit monitoring analysis is widely utilized for FD purpose. For a given output signal y, test statistic conditions can be described as

$$\begin{aligned} &H_0, \textit{null hypothesis} &&: \; y_{\min} \leq y \leq y_{\max}, \\ &H_1, \textit{alternative hypothesis} &&: \; y < y_{\min} \text{ or } y > y_{\max}. \end{aligned} \tag{2.23}$$

Here y_{min} and y_{max} are thresholds that denote the minimum and maximum values of y, respectively.

For an FD scheme, two important probabilities are defined: false alarm rate (FAR) and fault detection rate (FDR). FAR indicates the probability that an FD system announcing a false faulty alert in fault-free case. FDR means the probability that a fault is detected in faulty case. These are defined as follows.

Definition 2.3. *The following probabilities are called FAR and FDR, respectively,*

$$\begin{aligned} FAR &= prob \; \{y < y_{min} \; or \; y > y_{max} \mid f = 0\}, \\ FDR &= prob \; \{y < y_{min} \; or \; y > y_{max} \mid f \neq 0\}. \end{aligned} \tag{2.24}$$

We tend to find an FD scheme that for a given FAR, the corresponding FDR is at largest; or for a given FDR, to find an FD approach that the corresponding FAR is at smallest.

In previous discussion, basic concepts about FD systems are given based on the simplest model. In real applications, dynamic systems are more complex than this. In order to increase the robustness to the model uncertainties, disturbances, we can set a proper observer running parallel to the process. In fact, the observer applied for FD purpose is an output observer whose existence conditions are less strict than a state observer, which is often used for control purposes. And an output observer can deliver an (optimal) estimate of the process output. This method was proposed in the 1970s. More details can be seen in [25, 34].

In an FD system, a comparison of the output signals with their estimates delivered by an observer is made. The difference between measured output signals and their estimates is called a residual, whose function can be used as a test statistic for the FD purpose.

An observer-based FD system is schematically shown in Fig. 2.4.

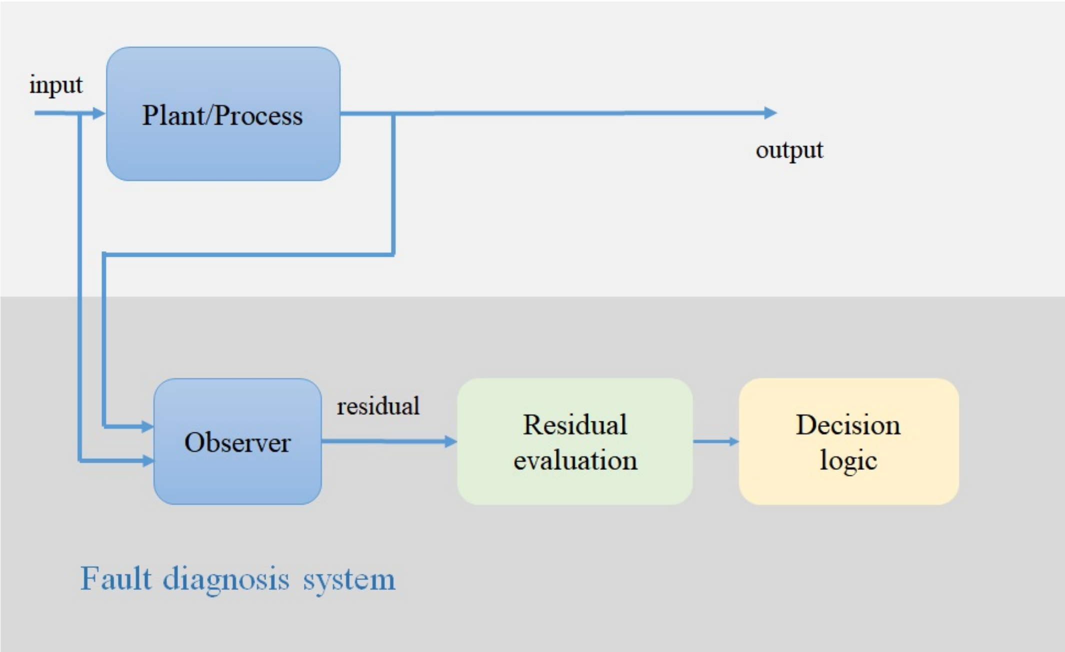

Figure 2.4: Diagram of fault diagnosis systems [25].

2.2.2 Residual generation

In this subsection, lots of attentions are paid on observer-based residual generation. First, an observer interpretation of the LCF is given. Then, a brief introduction of several residual generation methods is given, including fault detection filter, diagnostic observer and parity space-based residual generation approaches.

A. Fault detection filter

Fault detection filter (FDF) is a type of observer-based residual generator, which was first proposed in [4] in early 1970s. Suppose the system (2.2) has a full-order observer

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(y(k) - \hat{y}(k)), \\ \hat{y}(k) = C\hat{x}(k) + Du(k), \end{cases} \tag{2.25}$$

where $\hat{x}(k)$ is the observer state vector and it represents the estimate of $x(k)$, $\hat{y}(k)$ is the estimate of $y(k)$, and L denotes the observer gain matrix.

Define the residual signal as:

$$r(k) = y(k) - \hat{y}(k) = y(k) - C\hat{x}(k) - Du(k), \tag{2.26}$$

and suppose the input signals of the observer are $u(k)$ and $y(k)$, then we can rewrite the observer as:

$$\begin{cases} \hat{x}(k+1) = (A - LC)\hat{x}(k) + (B - LD)u(k) + Ly(k), \\ r(k) = -C\hat{x}(k) - Du(k) + y(k). \end{cases} \tag{2.27}$$

Introducing the error signal $e(k) = x(k) - \hat{x}(k)$, it yields

$$e(k+1) = (A - LC)e(k), r = Ce(k), \tag{2.28}$$

where $eig(A - LC) < 1$. In this case, $\hat{x}(k)$ provides an estimate for $x(k)$, and there exists

$$\lim_{k\to\infty} e(k) = \lim_{k\to\infty} (x(k) - \hat{x}(k)) = 0.$$

In order to (1) increase design freedoms, (2) enhance the sensitivity to faults, and (3) strengthen the robustness to disturbances, the residual generator is extended to the following form

$$r(k) = V^*(y(k) - \hat{y}(k)) \tag{2.29}$$

by introducing a post-filter V^*.

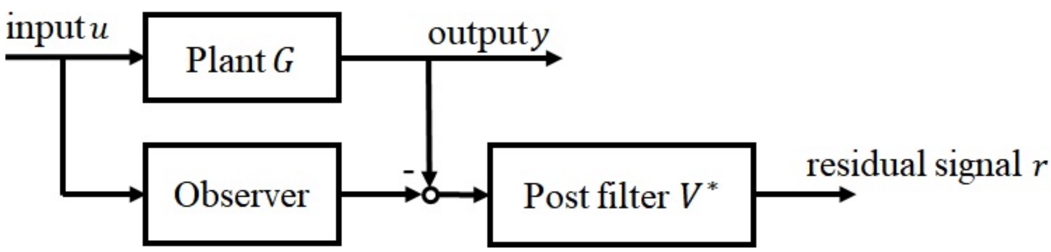

Figure 2.5: The diagnosis observer.

An observer is working online, and thus a full-order observer will cost more computation resources than a reduced observer. In this context, a reduced-order observer-based residual generator is more practical in the same performance. It motivates us to develop Luenberger type observer-based residual generators which is discussed in the sequel.

B. Diagnostic observer

Because of its similarity to the Luenberger type observer and flexible structure, the diagnostic observer (DO) becomes one of the most widely investigated model-based residual generator forms, related details can refer to [7, 15, 25]. A DO can be described by

$$\begin{cases} x_o(k+1) = Gx_o(k) + Hu(k) + Ly(k), \\ y_o(k) = Wx_o(k) + Vy(k) + Qu(k), \end{cases} \tag{2.30}$$

where $x_o(k) \in \mathcal{R}^{n_s}$ represents observer state, n_s denotes orders of this DO. Suppose that $G_{yu}(z) = C(zI - A)^{-1}B + D$, then matrices G, H, L, W, V, Q, together with matrix T have to satisfy the following Luenberger conditions:

$$\begin{aligned} &\textit{I.} \quad G \textit{ is stable}, \\ &\textit{II.} \quad TA - GT = LC, H = TB - LD, \\ &\textit{III.} \quad C = WT + VC, Q = D - VD. \end{aligned} \tag{2.31}$$

With the matrices in (2.31), the DO can generate unbiased estimation signal y_o of the output signal $y(k)$, which satisfies

$$\lim_{k\to\infty} (y(k) - y_o(k)) = 0.$$

To show its capacity in residual generation, we can check the error dynamic system with $e = Tx - x_o$. Considering an LTI system with noise signal and additive faults, the error dynamics becomes

$$\begin{cases} e(k+1) = Ge(k) + (TE_f - LF_f)f(k) + T\omega(k) - Lv(k), e(0) = e_0, \\ r(k) = We(k) + (F_f - VF_f)f(k) + (I - V)v(k). \end{cases} \tag{2.32}$$

Inspired by (2.29), we can set a post-filter V^* (see Fig. 2.5), then the observer dynamics turns to be

$$\begin{cases} x_o(k+1) = Gx_0(k) + Hu(k) + Ly(k), x_o(0) = x_0, \\ r(k) = \bar{V}y(k) - \bar{W}x_o(k) - \bar{Q}u(k), \end{cases} \tag{2.33}$$

where

$$\bar{V} = V^*(I - V), \bar{W} = V^*W, \bar{Q} = V^*Q.$$

With the novelly introduced post-filter V^*, Luenberger condition (III) turns to

$$\textit{III.}\ \bar{V}C = \bar{W}T = 0, \bar{Q} = \bar{V}D.$$

C. Kalman filter

It is well-known that the Kalman filter is the optimal linear filter when the noises in the process is white, and subject to Gaussian distribution, so it is widely applied in many researches and real applications. In this part, Kalman filter is applied to the residual generation.

Given a system with white Gaussian noises (2.10), the Kalman filter-based residual generator is given as follows:

$$\begin{cases} \hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(k)(y(k) - \hat{y}(k)), \\ \hat{y}(k) = C\hat{x}(k) + Du(k), \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{2.34}$$

where $\hat{x}(k)$ and $\hat{y}(k)$ represent estimates of state and output, $r(k)$ denotes the residual signal, $K(k)$ is called the time-varying Kalman filter gain matrix. The Kalman filter is a time-varying system given by following recursive functions:

(i) initial values:

$$\hat{x}(0) = E[x(0)] = \bar{x}, P(0) = E[(x(0) - \bar{x})(x(0) - \bar{x})^T]; \tag{2.35}$$

(ii) optimal state estimation:

$$\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + K(k)(y(k) - C\hat{x}(k) - Du(k)); \tag{2.36}$$

(iii) Kalman filter gain matrix:

$$P(k+1) = AP(k)A^T - K(k)\Sigma_r(k)K^T(k) + \Sigma_\omega \tag{2.37}$$

$$\Sigma_r(k) = \Sigma_v + CP(k)C^T \tag{2.38}$$

$$K(k) = (AP(k)C^T + S_{\omega v}^T)\Sigma_e^{-1}(k); \tag{2.39}$$

where $P(k)$ denotes the error covariance and $\Sigma_r(k)$ is the covariance of residual signal $r(k)$, expressed as

$$P(k) = E[(x(k) - \hat{x}(k))(x(k) - \hat{x}(k))^T], \tag{2.40}$$

$$\Sigma_r(k) = E[r(k)r^T(k)]. \tag{2.41}$$

The residual signal delivered by Kalman filter-based residual generator is of minimal covariance $P(k)$, which is an important character of Kalman filter. And the Kalman filter can be achieved by online implementation (2.36)-(2.39).

Suppose the process is stationary, and $k \to \infty$, the Kalman filter gain $K(k)$ is a constant matrix and expressed as

$$K = (APC^T + S_{\omega v}^T)\Sigma_r^{-1}, \tag{2.42}$$

$$P = APA^T - K\Sigma_r K^T + \Sigma_\omega, \tag{2.43}$$

$$\Sigma_r = \Sigma_v + CPC^T. \tag{2.44}$$

Equation (2.43) is the so-called Riccati equation, the steady-state Kalman filter gain K can be calculated by solving Riccati equation.

D. Parity space

Inspired by the work of Chow and Willsky in [19], the parity space method has been intensively investigated over the past several decades. In [25], it is proved that parity space residual generators is equal to certain types of observer structure. Therefore, we introduce it here as an observer-based method.

Consider an LTI system described by the state space model. First define some vectors that are used in party space approaches, such as state, input, output vectors, etc.,

$$\Lambda_s(k) = \begin{bmatrix} \lambda(k) \\ \lambda(k+1) \\ \vdots \\ \lambda(k+s) \end{bmatrix} \in \mathcal{R}^{(s+1)k_\lambda}, \lambda \in \mathcal{R}^{k_\lambda}. \tag{2.45}$$

To construct a residual generator, for a given order s of parity space, the system can be expressed as:

$$y_s(k) = \Gamma_s(k)x(k-s) + H_{u,s}u_s(k) + H_{\omega,s}\omega_s(k) + \upsilon(k) + H_{f,s}f_s(k), \tag{2.46}$$

where $y_s, u_s, x_s, \omega_s, \upsilon_s, f_s$ are constructed as $\Lambda_s(k)$ in (2.45), and

$$\Gamma_s = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^s \end{bmatrix}, H_{u,s} = \begin{bmatrix} D & 0 & \cdots & 0 \\ CB & D & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}B & \cdots & CB & D \end{bmatrix},$$

$$H_{d,s} = \begin{bmatrix} F_d & 0 & \cdots & 0 \\ CE_d & F_d & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}E_d & \cdots & CE_d & F_d \end{bmatrix}, H_{\omega,s} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1} & \cdots & C & 0 \end{bmatrix},$$

$$H_{f,s} = \begin{bmatrix} F_f & 0 & \cdots & 0 \\ CE_f & F_f & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{s-1}E_f & \cdots & CE_f & F_f \end{bmatrix}.$$

The basic idea of residual generation based on parity relationship is to take the advantage of the fact that for $s \geq n$, the following condition holds:

$$rank(\Gamma_s) \leq n < \textit{the row number of matrix } \Gamma_s = (s+1)m.$$

This condition ensures that there exists at least one non-null row vector $v_s \in \mathcal{R}^{1\times m(s+1)}$ satisfying $v_s\Gamma_s = 0$. The set of such kind of row vectors is called parity space vectors, noted as $P_s\{v_s | v_s\Gamma_s = 0\}$. With parity vectors, the residual generator can be constructed by

$$r(k) = v_s(y_s(k) - H_{u,s}u_s(k)). \tag{2.47}$$

The schematic description of the parity space scheme is shown in Fig. 2.6.

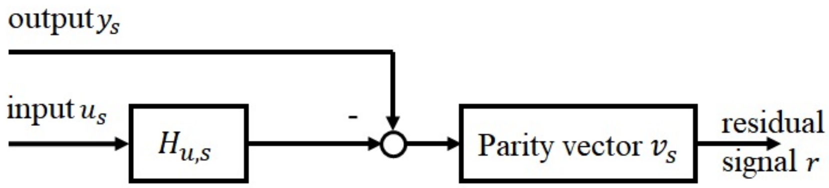

Figure 2.6: Schematic description of parity space scheme.

Therefore, the residual signal generated by (2.47) is constructed by a residual signal composed of the faults and a white Gaussian noise, which can be expressed as

$$r(k) = v_s(H_{f,s}f_s(k) + H_{\omega,s}\omega_s(k)), \tag{2.48}$$

in which we can see the design parameters of the parity relation-based residual generator is the parity vector, and the selection of it has a significant influence on the performance of the residual generator. One of the important advantages of the parity space-based residual generation method is that it can be designed in a straightforward manner.

E. SKR identification method

For complex industrial systems, high-precision mathematical models are difficult to build. With the development of the data processing technique, the data-driven residual generator design method becomes an alternative approach. In this regard, a novel data-driven design is proposed in [26]. The stable kernel representation (SKR) of systems is identified from the input/output data model with which the residual generator can be constructed directly. A brief introduction of this scheme is given in this subsection.

Recall (2.45) and introduce

$$\Lambda_{k,s} = [\Lambda_s(k) \quad \Lambda_s(k+1) \quad \cdots \quad \Lambda_s(k+N-1)], \tag{2.49}$$

where N is a sufficiently large integer.

Consider an I/O data model in (2.46) in the disturbance-free and fault-free case. It follows that $Y_{k,s} = \Gamma_s X_k + H_{u,s}U_{k,s} + H_{\omega,s}\Xi_{k,s} + V_{k,s}$ and can be written as

$$\begin{bmatrix} U_{k,s} \\ Y_{k,s} \end{bmatrix} = \begin{bmatrix} I & 0 \\ H_{u,s} & \Gamma_s \end{bmatrix} \begin{bmatrix} U_{k,s} \\ X_k \end{bmatrix} + \begin{bmatrix} 0 \\ H_{\omega,s}\Xi_{k,s} + V_{k,s} \end{bmatrix}. \tag{2.50}$$

Denote

$$Z_f = \begin{bmatrix} U_{k,s} \\ Y_{k,s} \end{bmatrix}, Z_p = \begin{bmatrix} U_{k-s_p-1,s_p} \\ Y_{k-s_p-1,s_p} \end{bmatrix}. \tag{2.51}$$

With the I/O model, the process to calculate SKR can be summarized as the Algorithm 2.1.

Algorithm 2.1 Identification of SKR based on data-driven method

1. Form the I/O data model and construct $Z_p, U_{k,s}, Y_{k,s}$.
2. Do QR-decomposition

$$\begin{bmatrix} Z_p \\ U_{k,s} \\ Y_{k,s} \end{bmatrix} = \begin{bmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix}.$$

3. Do singular value decomposition (SVD)

$$\begin{bmatrix} R_{21} & R_{22} \\ R_{31} & R_{32} \end{bmatrix} = [U_1 \quad U_2] \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$

4. The kernel representation is

$$\mathcal{K}_s = [\mathcal{K}_{s,u} \quad \mathcal{K}_{s,y}] = U_2^T \in \mathcal{R}^{((s+1)m-n)\times(s+1)(n_u+n_y)}.$$

5. The covariance matrix is

$$\Sigma_{res} = \frac{\mathcal{K}_{s,y} R_{33} (\mathcal{K}_{s,y} R_{33})^T}{N-1}.$$

Based on Algorithm 2.1, a data-driven design residual generator can be realized as Algorithm 2.2.

Algorithm 2.2 Data-driven design of observer-based residual generator

1. Run the Algorithm 2.1 and get $\mathcal{K}_s$.
2. Let $\phi_s^\perp$ be a row of $\mathcal{K}_s$ and it has the formulation $\phi_s^\perp = [\phi_{s,u}^\perp \quad \phi_{s,y}^\perp]$.
3. Construct observer-based residual generator:

$$\begin{cases} x_o(k+1) = A_o x_o(k) + B_o u(k) + L_o y(k), \\ r(k) = g y(k) - c_o x_o(k) - d_o u(k), \end{cases}$$

where

$$A_o = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix} \in \mathcal{R}^{s\times s}, c_o = [0 \quad \cdots \quad 0 \quad 1] \in \mathcal{R}^s,$$

$$L_0 = -\begin{bmatrix} \phi_{s,y}^\perp(1:n_y) \\ \vdots \\ \phi_{s,y}^\perp((s-1)n_y+1:sn_y) \end{bmatrix}, g = \phi_{s,y}^\perp((sn_y+1):(s+1)n_y),$$

$$B_o = -\begin{bmatrix} \phi_{s,u}^\perp(1:n_u) \\ \vdots \\ \phi_{s,u}^\perp((s-1)n_u+1:sn_u) \end{bmatrix}, d_o = \phi_{s,u}^\perp((sn_u+1):(s+1)n_u).$$

2.2.3 Residual evaluation and decision logic

As shown in Fig. 2.4, after residual generation, the generated residual signals should be evaluated. To this end, χ^2 test and GLR test are introduced in this subsection. And based on it, a threshold is established. The last step is to set a decision logic. In the subsequent sections, all these issues are studied.

A. Likelihood ratio test

The LR test method is very popular to detect changes in the binary case, and it is widely applied in the FD framework. The meaning of the LR method is to determine which hypothesis is more "likely" to happen and reject the other hypothesis. Because LR test statistic provides an optimal solution for detecting issues, it is widely applied in (optimal) detection problems.

Given the system model (2.20), the log-likelihood ratio for data y_i can be defined as

$$s(y_i) = \ln \frac{p_f(y_i)}{p_0(y_i)}, \tag{2.52}$$

with

$$p_0(y_i) = \frac{1}{\sqrt{(2\pi)^m \det(\Sigma_\epsilon)}} e^{-\frac{1}{2}y^T\Sigma^{-1}y}, p_f(y_i) = \frac{1}{\sqrt{(2\pi)^m \det(\Sigma_\epsilon)}} e^{-\frac{1}{2}(y-f)^T\Sigma^{-1}(y-f)}.$$

Since the noises in function (2.20) is white Gaussian noise, the likelihood function is chosen to be the PDF of y_i.

The test statistic $s(y_i)$ represents likelihood ratio of $p_f(y_i)$ and $p_0(y_i)$. The situation $s(y_i) > 0 \Leftrightarrow p_f(y_i) > p_0(y_i)$ means probability of the system in faulty case is larger than that in faulty-free case, and vice versa.

If there exist a series of data $y_i, i = 1, 2, \cdots, n$, available for detecting purpose, the log-LR test statistic can also be chosen as

$$S_n(y_1) = \sum_{i=1}^{n} s(y_i). \tag{2.53}$$

LR-based statistic testing is the optimal scheme when fault case f is known. In general, faults are unknown in industrial applications. In this case f can be substituted by its estimate. The statistic test with an LR tool and estimate of fault is called the GLR method, which is introduced in next part.

B. Generalized likelihood ratio test

For the purpose of detecting the unknown changes, a so-called GLR test method is developed. First, to estimate the fault,

$$\max_f S_n(y_1) = \max_f \textstyle\sum_{i=1}^{n} \ln \dfrac{p_f(y_i)}{p_0(y_i)} \Rightarrow \hat{f} = arg \max_f S_n(y_1). \tag{2.54}$$

Then with the estimate of f, the residual signal r can be used to construct the likelihood ratio $s(r_i), S_n(r_i)$, which can be used for GLR test-based FD schemes.

Notice that, the well-known χ^2 test statistic is the result of the GLR applied to the case with change in mean, and a χ^2 test is a statistical hypothesis test that is applied to perform when the test statistic is χ^2 distributed under the null hypothesis. It is very useful in dealing with the FD issues in the process with white Gaussian noises.

In the systems with white Gaussian noises, the residual signal r can be generated by a Kalman filter algorithm-based residual generator. In this case, r is subject to normal distribution $r \sim \mathcal{N}(0, \Sigma_r)$, n_r denotes the dimension of r.

Consider this residual signal r, the evaluation function can be set as

$$J_{\chi^2} = r^T \Sigma_r^{-1} r, \tag{2.55}$$

because $\Sigma_r^{-\frac{1}{2}} r \sim \mathcal{N}(0, I)$, there has $J_{\chi^2} \sim \chi^2_{n_r}$,where the $\chi^2(n_r)$ is χ^2 distribution with n_r degrees of freedom. For applying the χ^2 test statistic, the threshold computation can be done by:

1. choosing an FAR α, then determining χ_α with the χ^2 distribution table;

2. setting threshold as $J_{th} = \chi_\alpha$.

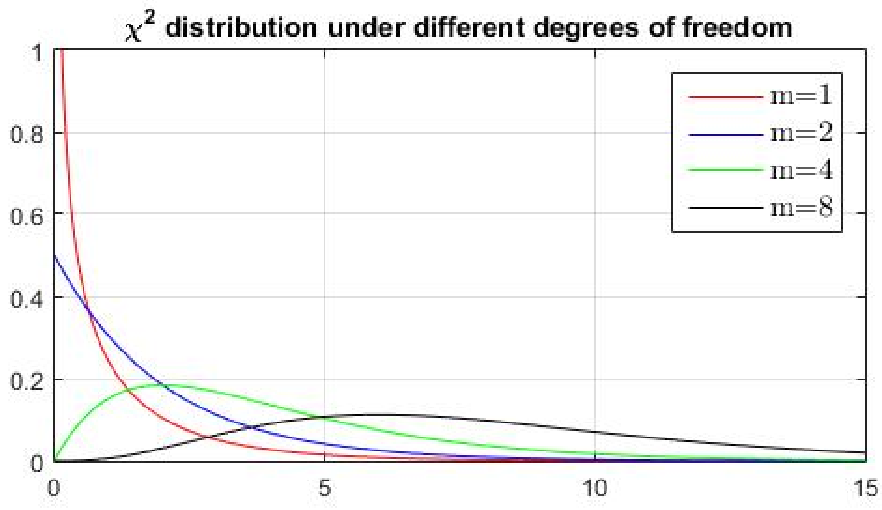

Figure 2.7: χ^2 distribution under different degrees of freedom.

According to Fig. 2.7, it is obvious that for a chosen FAR α, the corresponding threshold J_{th} can be found in χ^2 format with a certain degrees of freedom.

When a Kalman filter is used, notice that there exist the following conditions:

1. the maximum likelihood estimate of the fault f is $arg\max_f S_n(y_1)$;
2. the maximum LR is the χ^2 test statistic;
3. this χ^2 distribution form can be applied to determining the threshold J_{th}.

With the threshold J_{th}, the basic idea of LR-based fault detection scheme can be presented as

$$\begin{cases} s(y_i) < J_{th} \Rightarrow H_0 \\ s(y_i) \geq J_{th} \Rightarrow H_1. \end{cases} \tag{2.56}$$

Besides the FD scheme for the case with a changing mean, known as χ^2 method, the FD scheme for the case with changing covariance is also discussed in [25]. However, when both mean and covariance are changed, the probability computation based on the test statistic is challenging. To deal with this problem, a randomized algorithm can be applied to determine the threshold for a given FAR in subsequent chapters.

2.3 FD Preliminaries on systems with norm-bounded disturbances

In this section, the systems with norm-bounded disturbances and structure uncertainties are considered. To this end, a proper observer to generate residual signal is proposed, and the corresponding techniques are given here.

2.3.1 Basic concepts of FD for deterministic systems

In the FD framework for systems with norm-bounded disturbances, the norm is a powerful tool to measure vectors. In deterministic systems, for FD purposes, an evaluation function is denoted by J, and J can be computed by the norm of residual signals. To be precise, $\mathcal{L}_2$, $\mathcal{L}_\infty$ and root-mean-square (RMS) norms are widely used for residual evaluation and threshold computation. Here we introduce the definition of different types of norms and corresponding thresholds.

1. $\mathcal{L}_2$-norm: The physical meaning of $\mathcal{L}_2$ is the energy level of the considered signal. The $\mathcal{L}_2$-norm of a vector-valued signal $u(k)$ and evaluation function J_2 are defined by

$$\|u\|_2 = (\textstyle\sum_{k=0}^{\infty} u^T(k)u(k))^{\frac{1}{2}}, \tag{2.57}$$

$$J_2 = \|r(k)\|_2 = (\textstyle\sum_{k=0}^{\infty} r^T(k)r(k))^{\frac{1}{2}}, \tag{2.58}$$

$$J_{th,2} = \sup_{f=0} J_2. \tag{2.59}$$

In (2.57), $u^T(k)u(k)$ represents the instantaneous power, and $\|u\|_2$ stands for the total energy of the signal. In practice, it is impossible to measure the total energy in the time interval $[0, \infty)$, therefore, one of the solutions is to choose a time interval $[k_1, k_2]$. The $\mathcal{L}_2-$norm in time interval $[k_1, k_2]$ is denoted by $J_{2,[k_1,k_2]}$ and defined by

$$J_{2,[k_1,k_2]} = \|r(k)\|_2 = (\textstyle\sum_{k=k_1}^{k_2} r^T(k)r(k))^{\frac{1}{2}}, \tag{2.60}$$

$$J_{th,2,[k_1,k_2]} = \sup_{f=0} J_{2,[k_1,k_2]}. \tag{2.61}$$

2. RMS-norm:

To calculate the RMS value instead of $\mathcal{L}_2-$norm is another useful method. The RMS value stands for the average energy of a signal over a certain time interval. The norm and corresponding evaluation function are correspondingly defined by

$$\|u\|_{RMS} = (\frac{1}{N} \textstyle\sum_{k=0}^{N-1} u^T(k)u(k))^{\frac{1}{2}}, \tag{2.62}$$

$$J_{RMS} = \|r(k)\|_{RMS} = (\frac{1}{N} \textstyle\sum_{j=1}^{N} \|r(k+j)\|^2)^{\frac{1}{2}}, \tag{2.63}$$

$$J_{th,RMS} = \sup_{f=0} J_{RMS}. \tag{2.64}$$

2.3.2 Residual dynamics for systems with norm-bounded disturbances

As the FDF mentioned in Section 2.2.2, a standard observer can be proposed as (2.25), the dynamics of estimation error $e(k) = x(k) - \hat{x}(k)$ can be described by

$$\begin{cases} e(k+1) = (A - LC)e(k) + (E_f - LF_f)f(k) + (E_d - LF_d)d(k), \\ r(k) = Ce(k) + F_f f(k) + F_d d(k), \end{cases} \tag{2.65}$$

which delivers a residual signal described by

$$r(z) = \hat{N}_d(z)d(z) + \hat{N}_f(z)f(z), \tag{2.66}$$

$$\hat{N}_d(z) = C(zI - A + LC)^{-1}(E_d - LF_d) + F_d, \tag{2.67}$$

$$\hat{N}_f(z) = C(zI - A + LC)^{-1}(E_f - LF_f) + F_f. \tag{2.68}$$

In this case, the main task turns into determining an observer gain L, which makes the influence of disturbances under a given bound, and make the influence of faults as large as possible. To this end, the linear matrix inequality (LMI) technique can be applied as a mathematical tool to achieve solutions, more results can be seen in [25, 46, 59].

Notice that, in disturbance-free case, the occurrence of a fault results in $\lim_{k\to\infty} r(k) \neq 0$. In real applications there always exist some tolerable disturbances, such that it is not proper to set the decision rule as $\lim_{k\to\infty} r(k) = 0$ directly. To deal with this problem, an FD scheme for a deterministic system is investigated by

(1) decoupling the residual signal from disturbances;

(2) evaluating residual signal by a proper norm;

(3) determining the corresponding threshold and setting decision logic.

2.3.3 Residual evaluation and decision logic

For systems with norm-bounded disturbances, strategies of residual evaluation are applied by using the well-established robust control theory. In real applications, methods of residual evaluation should be chosen according to the actual requirement.

Generally, in systems with deterministic disturbances, a threshold is said to be the tolerant limit of disturbances and uncertainties in fault-free case. For a chosen residual evaluation function, the standard threshold can be computed by:

$$J_{th,e} = \sup_{f=0} J_e = \sup_{f=0} \|r\|_e, \tag{2.69}$$

where J_e represents the evaluation function of residual signal, which can be $\mathcal{L}_2$-norm, RMS-norm of residual signals. With (1) test statistic J in statistical framework or evaluation function J_e generated by observer-based residual generator, and (2) the corresponding threshold $J_{th,e}$, then decision rule is set as (see Fig. 2.8):

$$\begin{cases} J_e < J_{th}, & H_0, \textit{null hypothesis, fault-free,} \\ J_e \geq J_{th}, & H_1, \textit{alternative hypothesis, faulty.} \end{cases} \tag{2.70}$$

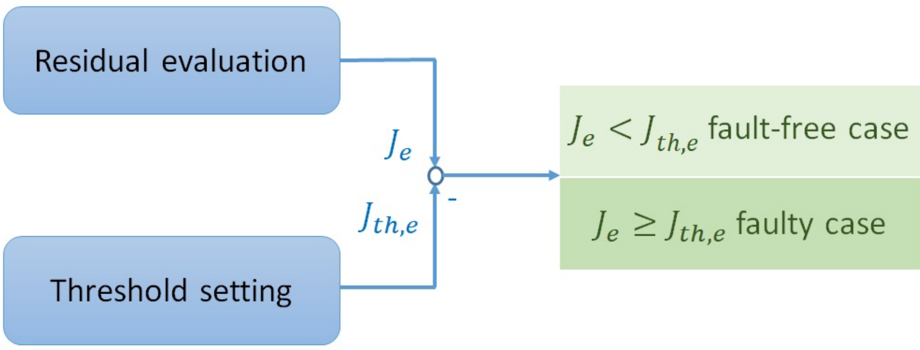

Figure 2.8: An intuitive explanation of decision logic.

2.4 Concluding remarks

This chapter presents a brief introduction to FD systems, including the preliminaries for both stochastic and deterministic systems.

The first part focuses on modeling nominal systems, disturbances and faults. A widely applied mathematical tool, the coprime factorization technique is also introduced here. FD systems for both stochastic and deterministic systems are given. In stochastic systems, basic concepts are given first, it is followed by residual generation methods, including fault detection filter, Kalman filter, diagnosis observer, parity space approach and data-driven methods. Then, LR test and GLR test are introduced, and corresponding thresholds computation is addressed. As the result of the GLR test which is applied to the case with only change in mean, χ^2 test statistic is also given in this section. Based on them, the decision logic is given. Parallel to stochastic systems, systems with norm-bounded disturbances is discussed.

3 Fundamentals of Switched Systems

Parallel to preliminaries given in Chapter 2, this chapter introduces basic knowledge about switched systems and IFs. Firstly, we focus on the background and recent research processes of switched systems, then it is followed by a further introduction to IFs, and a system with multiplicative IFs modeled by switched systems is given.

3.1 Motivations and problem formulation

From the practical application of view, many industrial systems show different system dynamics under different working conditions, and some of them possess a switching feature among different dynamics, such as chemical processes, mechanical systems, etc. In order to study these complex systems better, researchers have proposed the concepts of hybrid systems and switched systems to model and analyze such systems. The main feature of hybrid systems and switched systems is that they can model different system dynamics by means of different subsystems, which meets the demands of the description of complex systems.

It is apparent that systems with IFs present strong switching characteristics. As mentioned in Chapter 2, IFs widely exist in industrial systems, especially in systems that contain periodic and repetitive components and actions, for instance, a plant equipped with gears, motors, and so on. The main feature of an IF is that it disappears after period of time. Because of these characteristics, a system with IFs can be regarded as a switched system that switches between a normal subsystem and a faulty subsystem, and treat the fault occurrence and disappearance as the switching rule. In this context, faults can be described as a switching from a normal subsystem to a faulty subsystem.

Moreover, in switched systems, the frequency of switchings and the duration of each subsystem are two important that influence factors the stability conditions. Hence, the stability analysis methods for switched systems can be used to study the stability conditions of systems with multiplicative IFs. To this end, switched systems are introduced to model the systems with IFs and utilize the well-developed theory on stability analysis of switched systems to analyze the stability conditions.

Motivated by the above discussion, related knowledge on switched systems and IFs is given in this chapter, which also serves as essential parts of subsequent FD studies. To be specific, the background of switched systems, stability analysis methods, and systems with multiplicative IFs modeled by switched systems are successively presented in detail.

3.2 Background of switched systems

With the increasing complexity of modern industrial control systems, simple LTI systems are insufficient to comprehensively describe complicated industrial systems and processes. To model complex systems, hybrid systems were proposed in the early 1990s [36]. In recent decades, different branches of hybrid systems, especially the so-called switched systems, have attracted considerable attention. In this section, mathematical preliminaries on hybrid and switched systems are given.

A classic switched system consists of a class of subsystems and a switching rule governing the dynamics of particular subsystems. Here, subsystems are utilized to describe different system dynamics, and a switching law gives the rule for switchings between different subsystems. As an important branch of hybrid systems, switched systems drew the attention of researchers in both theoretical and application domains. For systems with complex trajectories of processes and time-varying systems, the switched system is an efficient tool to model them.

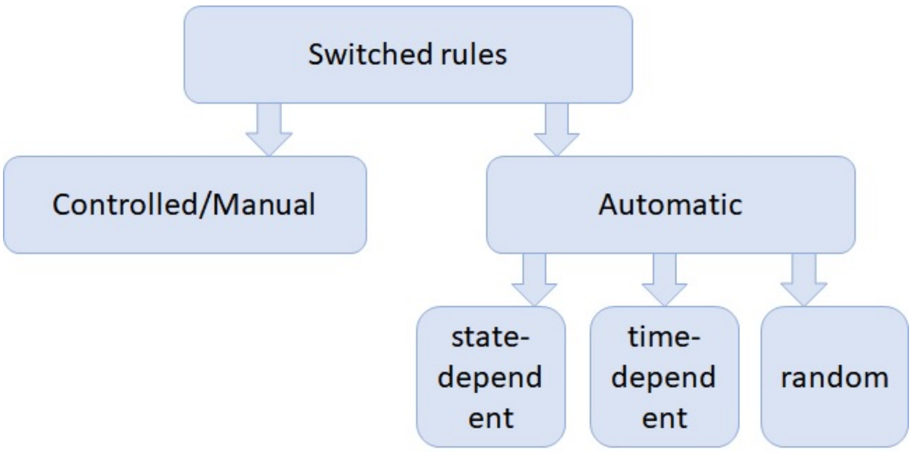

Figure 3.1: Schematic of switching rules.

Subsystems are used to describe different system dynamics, and the switching law gives the rules for switching between different subsystems. Switching rules are roughly divided into four types: autonomous, controlled, time-dependent, and state-dependent rules, which are shown in Fig. 3.1.

In the research field of switched systems, recent efforts focus on many issues, such as stability analysis, filter design, robust control and fault detection. Among these studies, stability analysis is the basis of all investigations. [58] has summarized many well-known stability analysis methods, such as the methods base on common Lyapunov function, multiple Lyapunov function and piecewise Lyapunov function. Moreover, stability conditions under arbitrary switching were also provided. As a typical time-constraint (or restrict switching) method, a so-called ADT method has been intensively studied in recent decades [84, 95]. In [95], a mode-dependent average dwell time (MDADT) method was proposed, and extended the application of the ADT method.

Based on stability researches of switched systems, filtering or state estimation [75] and FD [37] have emerged as significant research areas. Considerable achievements on observer-based FD for switched systems have been made, and FD issues for discrete-time and continuous-time switched systems have been discussed in [52] and [81], respectively. However, most studies were proposed for the case of models and observers operating in a synchronous manner. In some specific applications, because of the time lag between sub-models and observers, there is a phenomenon that subsystems and corresponding observers would switch asynchronously. In [82], an asynchronous switching FD problem has been investigated, and a maximum dwell-time approach was applied to deal with asynchronous switching control problems.

Sub-models in switched systems are usually described by different equations. For continuous-time subsystems, differential equations are utilized, while discrete-time subsystems are generally described by difference equations. The state space representation of a discrete-time switched system is expressed as follows:

$$x(k+1) = f_{\sigma(k)}(x(k), u(k)), \tag{3.1}$$

where $x(k) \in \mathcal{R}^{n_x}$ denotes the system state, $u(k) \in \mathcal{R}^{n_u}$ represents the input signal, σ denotes the switching rule that governs the switchings between sub-systems in a finite set $\mathcal{S} = \{1, 2, \cdots, N\}$, with $\sigma \in \mathcal{S}$ and N is the number of subsystems.

In this thesis, only time-dependent switching and autonomous switching are studied. A linear discrete-time switched system in state space formulation is given by:

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k), \\ y(k) = C_{\sigma(k)}x(k) + D_{\sigma(k)}u(k), \end{cases} \tag{3.2}$$

where $A_{\sigma(k)}, B_{\sigma(k)}, C_{\sigma(k)}, D_{\sigma(k)}$ are matrices with appropriate dimensions, $y(k) \in \mathcal{R}^{n_y}$ denotes the output signal. Switched system (3.2) can also be described by subsystems in the following form with $i \in \mathcal{S}$:

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k), \\ y(k) = C_i x(k) + D_i u(k). \end{cases} \tag{3.3}$$

Fig. 3.2 gives the structure of switched systems in form of (3.3).

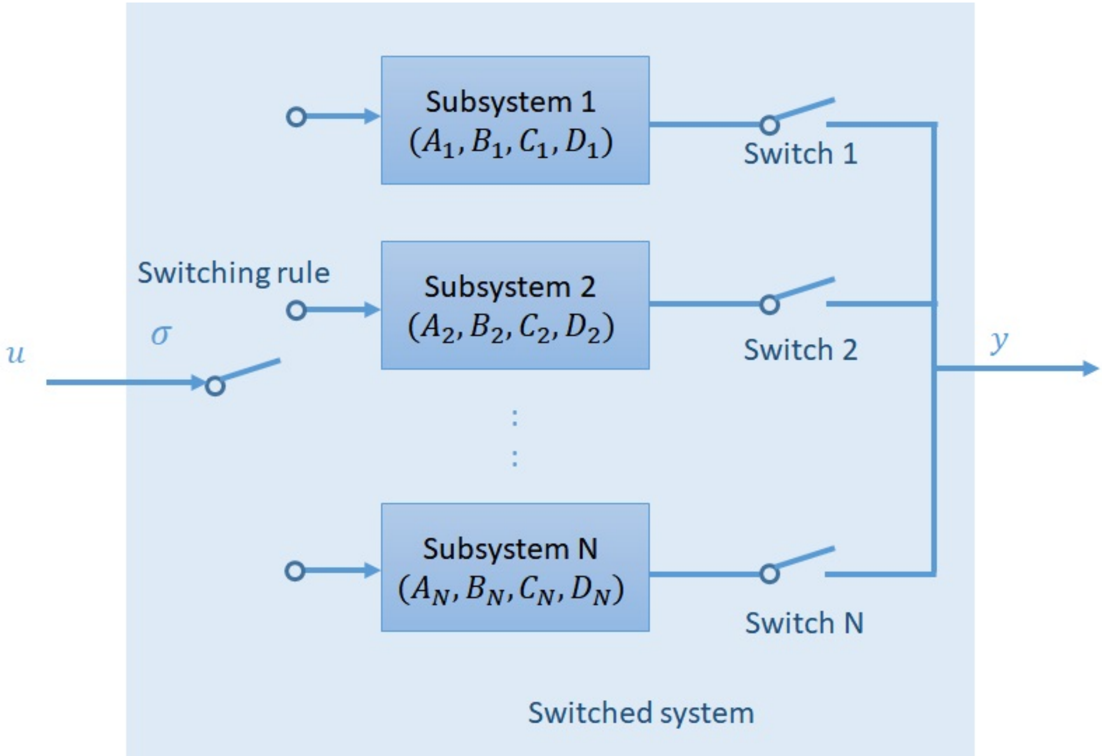

Figure 3.2: Structure of switched systems.

Due to the effects of modeling uncertainties on complex systems, switched systems with uncertainties have also attracted more and more attentions. There exist many related studies, such as stability analysis [12], H_∞ state-feedback control [13, 17], H_∞ filtering design [22], etc. Moreover, studies on the FD and fault tolerant control of the switched systems have become hot topics in recent years. Numerous achievements have been obtained on robust FD scheme [24], LMI-based FD scheme [9], FD for switched systems with time-delay [29, 31, 33], and fault tolerant control issues for switched systems [32].

3.3 Stability analysis

There exist several interesting phenomena concerning switched system stability. For instance, when all subsystems are stable, the whole system trajectories may diverge after several switchings. There exists a situation that even some subsystems are unstable, a system can still have convergent trajectories by selecting a proper switching rule. Therefore, over the past decades many scholars paid attention to the stability problems, and stability analysis for switched systems is a prerequisite in the study of many problems. Furthermore, due to these characteristics, some restricted switching rules are designed to ensure the stability of the whole system, and constraint conditions of switching rules should be given in some integrated designs.

Therefore, research results on stability analysis of switched systems have two important branches:

(1) for a given switched system with a certain switching rule, analyze stability conditions;

(2) for a switched system with known subsystems, design a stabilizing switching rule.

Methods based on Lyapunov functions are most commonly applied in switched systems with no restricted switchings. In the case under restricted switching, the ADT method is usually applied to analyze the stability conditions.

In the following subsections, the stability analysis of both switched systems with or without restricted switching rules is introduced.

3.3.1 Stability analysis under arbitrary switching

This section is dedicated to the case that when there is no restriction on switching rules of the switched systems. This operating mode is also called switched systems under arbitrary switchings. Here we investigate the stability conditions of the whole system. Stability issues are commonly studied using Lyapunov methods, and two most useful tools are introduced here: (1) common quadratic Lyapunov function (CQLF) and (2) switched quadratic Lyapunov function (SQLF).

CQLF Method

The existence of a CQLF for all subsystems accesses the quadratic stability of switched systems. Quadratic stability is a special case of exponential stability that ensures the asymptotic stability of the whole system. It is defined in Definition 3.1.

Definition 3.1. *A discrete-time system $x(k+1) = Ax(k)$ is said to be quadratically stable if there exists a positive definite symmetric matrix $P = P^T > 0, P \in \mathcal{R}^{n_x \times n_x}$, such that:*

$$A^T PA - P < 0. \tag{3.4}$$

According to the Definition 3.1, it is easy to see that if there exists a symmetric positive definite matrix P for each subsystem satisfying:

$$A_i^T PA_i - P < 0, \forall i \in \mathcal{S}, \tag{3.5}$$

then the global stability of the system under arbitrary switching is guaranteed by CQLF.

The CQLF method has a simple form and it is suitable for arbitrary switching between each subsystem. If there exists a matrix P satisfying (3.5), switched systems are ensured to be stable. It is worth noticing that the conservativeness of CQLF becomes higher as the number of subsystems increases, at the same time, LMIs in (3.5) are more difficult to be solved. In the case that an analytical solution is difficult to achieve, a numerical method is an effective alternative. For example, in [57], a gradient descent algorithm was proposed to solve solutions to a finite number of steps. However, the CQLF method leads to higher conservativeness and it is difficult to find a CQLF for a switched system. To this end, many methods aiming at reducing the conservativeness have been proposed.

SQLF Method

To reduce the conservativeness, many researchers have focused on a less conservative class of Lyapunov functions, known as SQLF [30, 58]. Consider a series of matrices $P_i, \forall i \in \mathcal{S}$, a global Lyapunov function is constructed as

$$V(k, x(k)) = x^T(k) P_{\sigma(k)} x(k). \tag{3.6}$$

Stability conditions of switched systems with arbitrary switching rules can be guaranteed by solving certain LMIs as given in the following lemma [21, 58].

Lemma 2. *If there exist positive definite symmetric matrices $P_i \in \mathcal{R}^{n_x \times n_x}$ and matrices $F_i, G_i \in \mathcal{R}^{n_x \times n_x}$ satisfying*

$$\begin{bmatrix} A_i F_i^T + F_i A_i^T - P_i & A_i G_i - F_i \\ * & P_j - G_i - G_i^T \end{bmatrix} < 0, \tag{3.7}$$

for all $i, j \in \mathcal{S}$, the system (3.3) is asymptotically stable.

It is obvious that when $P_i = P_j, \forall i, j \in \mathcal{S}$, SQLF becomes CQLF, which means CQLF is a special case of SQLF. Although the conservativeness of SQLF is lower than that of CQLF, the conditions are still too strict. Equation (3.7) should be satisfied between every two i-th and j-th subsystems. In real applications, it is very difficult to find feasible solutions as the number of subsystems increases. In other words, with the increasing number of subsystems, the conservativeness of the SQLF method is getting larger.

In order to reduce the difficulty of finding a feasible solution to (3.7), or, to reduce the conservativeness of the stability analysis method, much attention has been paid to analysing stability conditions under restrict switching, which is introduced in next subsection.

3.3.2 Stability analysis under restrict switching

According to the examples given in [24, 55], divergent trajectories have occurred after switchings between two stable subsystems. This phenomenon indicates that divergence can be introduced by switchings, or switchings may lead to system unstable behaviors. In some cases, switched systems can not preserve their stability under arbitrary switching, but can be stable under restricted switching rules. In real applications, some physical constraints in systems and processes may cause adverse limitations in switchings. Therefore, analyzing and designing restricted switching rules has become one of the main tasks in switched system studies.

In this subsection, the stability conditions of switched systems under restricted switching rules are introduced. Restrictions can be time-domain restrictions (dwell time of each subsystem, frequency of switching) and state space restrictions (Lyapunov function's value, which is also called generalized energy). These two kinds of restrictions have been studied through the piecewise quadratic Lyapunov function method and ADT method, respectively.

Piecewise Quadric Lyapunov Function Method

For stability analysis under restricted switching, a basic idea is that multiple Lyapunov functions for subsystems are concatenated to produce a non-traditional Lyapunov function. This non-traditional Lyapunov function may not monotonically decrease along trajectories, which means the generalized energy of systems may not decrease during the working time.

In general, there exist two different conditions: (1) all subsystems are stable, conditions of Lyapunov function's values after switching instants can be larger than before; (2) some subsystems are unstable, in other words, part of trajectories are divergent, in which case, switched systems also can be stable by restricting conditions of Lyapunov function's values at switching instants.

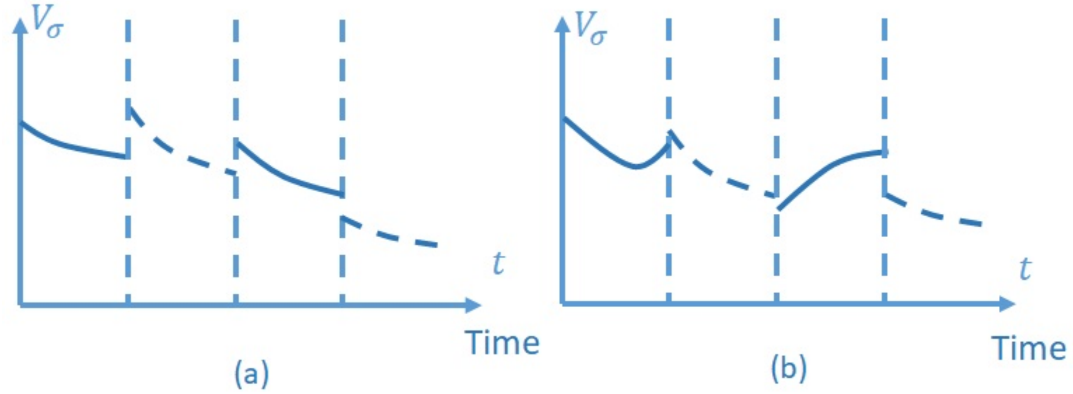

Figure 3.3: Piecewise Lyapunov function results.

There exist several piecewise Lyapunov function results, a simple result is that each Lyapunov-like function is decreasing, and at each switching instant the value of Lyapunov-like function does not increase [24]. To obtain less conservative results, another situation is illustrated in Fig. 3.3 (a) [8]. The value of the Lyapunov-like function before a switching is not larger than the value after the switching, such that $V_i(x(k_i)) \geq V_{i+1}(x(k_{i+1})), i \in \mathcal{S}$, then the switched system is asymptotically stable.

Furthermore, the switched system even can ensure its stability when the Lyapunov-like function value increases during a time interval if the increment is bounded by a certain kind of function. The situation is illustrated in Fig. 3.3 (b), detailed introduce and examples can refer to [5].

As introduced in [58], a piecewise quadric Lyapunov function method has been proposed based on a non-traditional Lyapunov function. Denote $\Omega_i \subset \mathcal{R}^n$ as a partition of the state space $\mathcal{R}^n$. The piecewise Lyapunov functions $V_i(x(k)) = x^T(k)P_i x(k)$ satisfy:

1. for $x(k) \in \Omega_i$, $\exists P_i$, such that $A_i^T P_i A_i - P_i < 0$;
2. there exist constants $\beta_i \geq \alpha_i > 0$ such that $\alpha_i \|x(k)\|^2 \leq V_i(x(k)) \leq \beta_i \|x(k)\|^2$ holds for all $x \in \Omega_i$;
3. at switching instants, Lyapunov function's values at switching instant are non-increasing, and can be expressed as $x^T(k)P_j x(k) \leq x^T(k)P_i x(k)$, where the system is switching from the i-th subsystem to the j-th subsystem.

Average Dwell-time Method

Under some working conditions, the frequency of switchings between different subsystems may cause diverged trajectories even if all subsystems are stable. And sometimes there also exist unstable subsystems caused by abnormal working conditions, e.g. components failures. If a system stays in unstable subsystems too long or switchings are too frequent, the switched system may be unstable. It indicates that if setting an appropriate constraint condition for switching frequency or dwell time of unstable subsystems, the stability of the whole system can be ensured. Based on this fact, a widely used method, the so-called ADT method has been investigated to analyse stability conditions of switched systems. Examples and more related studies can be found in [38, 39, 41].

According to references [39, 61, 95], we introduce several useful definitions and lemma.

Definition 3.2. *[95] For a switching signal $\sigma(k)$ and $k \geq k_0 \geq 0$, let $N_\sigma(k, k_0)$ denote the number of $\sigma(k)$ in the time interval (k_0, k). A positive constant τ_a is called the average dwell time for the switching signal $\sigma(k)$, if*

$$N_\sigma(k, k_0) \leq N_0 + \frac{k - k_0}{\tau_a} \tag{3.8}$$

holds for all $k \geq \tau \geq 0$, and there exists positive number N_0 (called the chatter bound).

Definition 3.3. *[53] Given model (3.1), the equilibrium $x = 0$ is globally uniform exponentially stable (GUES) under certain switching signal σ, if for $u(k) = 0$ and initial condition x_{k_0}, there exist constants $K > 0, \lambda > 0$ such that the solution of systems satisfies $\|x_k\| \leq Ke^{-\lambda(k-k_0)}\|x_{k_0}\|$.*

It has been reported in [56, 58] that the following lemma holds.

Lemma 3. *[95] Consider a switched system $x(k+1) = f_{\sigma(k)}(x(k)), \sigma \in \Gamma$ and there exist positive constants $0 < \alpha < 1, \mu > 1$. Suppose that there exist $\mathcal{C}^1$ functions $V_\sigma : R^n \to R$ and two class $\mathcal{K}_\infty$ functions β_1, β_2 such that $\forall\sigma$,*

$$\beta_1(\|x(k)\|) \leq V_\sigma(x(k)) \leq \beta_2(\|x(k)\|), \tag{3.9}$$

$$V_\sigma(x(k+1)) - V_\sigma(x(k)) \leq -\alpha V_\sigma(x(k)), \tag{3.10}$$

and $\forall(\sigma, \sigma') \in \Gamma \times \Gamma, \sigma \neq \sigma'$,

$$V_\sigma(x(k_l)) \leq \mu V_{\sigma'}(x(k_l)). \tag{3.11}$$

Then, a system is GUES for any switching signal with ADT

$$\tau_a > \tau_a^* = -\ln\mu / \ln(1-\alpha). \tag{3.12}$$

It is clear that the average value of the "dwell time" between any two switchings should be no smaller than τ_a. If all subsystems are GUES, the switched system can remain stable with a sufficiently large τ_a. The specific proof process can be found in [56].

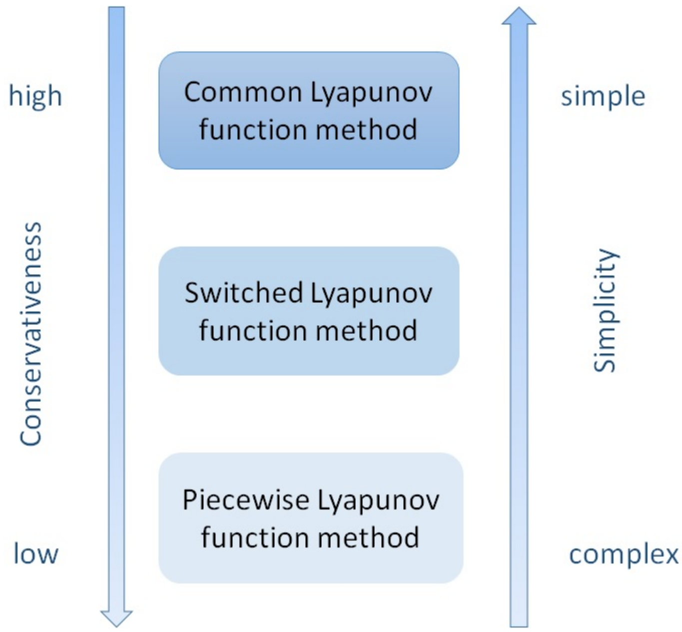

Figure 3.4: Conservativeness and methods.

According to the above discussions of stability analysis methods, we can see that the CQLF method has the simplest form and highest conservativeness. SQLF method has a more complex mathematical form, but it is easier to get a feasible solution. Obviously, conditions of arbitrary switching are less strict than restricted switching and the conservativeness is higher than that under restricted switching.

Fig. 3.4 expresses the relationship between conservativeness and the possibility to get a solution to stability conditions.

3.4 Intermittent faults

According to Chapter 2, IFs widely exist in industrial processes and some PFs can be transformed from IFs. Therefore, accurate and early detecting of IFs can help to prevent serious failures and increase the reliability of the whole system. Several methods have been developed to describe and detect IFs. One of the most popular techniques is to model systems with IFs by means of the Markov jump system theory, more details can be found in [87, 93]. However, a known Markov chain is needed for modeling the system with IFs, and the Markov chain represents the probability of switching in each subsystem. Because the probability of switching is difficult to measure and estimate, methods based on the Markov jump system have limitations in applications. Besides, there are other studies on detection of IFs, such as standard detection scheme [11], FD for hybrid systems [85], FD for time-delay systems [35], FD for embedded systems [43], etc.

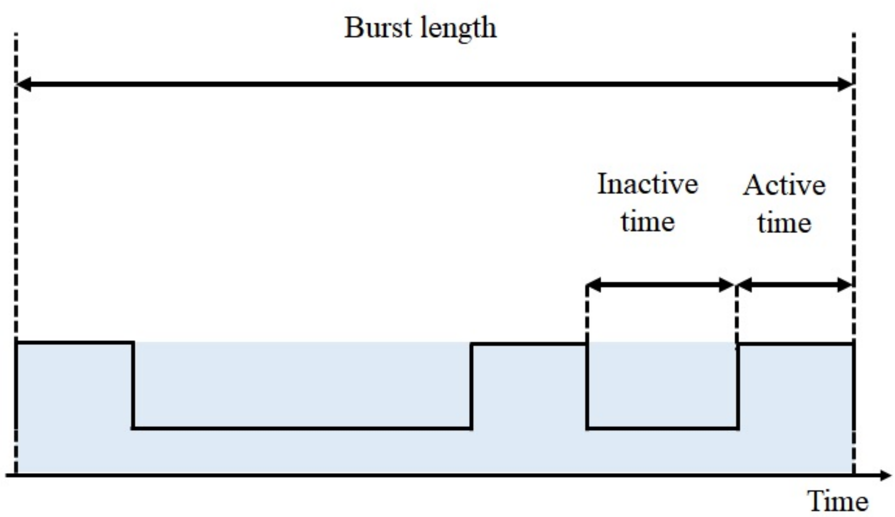

Figure 3.5: Three characters of IFs.

In real applications, there exist both additive IFs and multiplicative IFs. Multiplicative

IFs are more common in industrial systems and processes, however, they can change the system eigen-dynamics and even make the system unstable. However, a Markov jump system is always utilized for additive IFs. In order to facilitate the study of multiplicative IFs, in this thesis we choose the switched system as a new mathematical tool to fit the whole complex system composed of different subsystems and switching signals. Then, we further propose a framework for switched systems on detection and isolation issues with multiplicative IFs.

In practice, some types of IFs show periodicity character, and there also exist some IFs that occur irregularly. One can use three key parameters to describe one type of IFs, burst length, active time, and inactive time [63]. As shown in Fig. 3.5, the burst length is the time-interval of IFs occurring, the active time is the positive pulse width of one IF activation, while inactive time is the time interval of no IFs occurring. These three parameters determine the time characteristics of one type of IF. It is obvious that for IFs with periodic occurrence properties, the length of inactive and active time is a constant.

For the purpose to study the systems with multiplicative IFs, the switched system is applied to describe them. According to this, a noise-free system with multiplicative IFs is expressed as follows:

$$\begin{cases} x(k+1) = A_\sigma x(k) + B_\sigma u(k), \\ y(k) = C_\sigma x(k) + D_\sigma u(k), \end{cases} \tag{3.13}$$

where $A_\sigma, B_\sigma, C_\sigma, D_\sigma$ are matrices with appropriate dimensions. The case $\sigma = 0$ represents the fault-free case, and $\sigma = 1, 2, \cdots,$ represents the system trajectory being in faulty cases.

Remark 3.1. *Systems with additive IFs can also be described by switched systems. However, parameter matrices A, B, C, D are the same as fault-free subsystems, which means the eigen-dynamics and stability conditions are invariant. The systems with additive faults can be expressed as follows*

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + E_f f_\sigma(k), \\ y(k) = Cx(k) + Du(k) + F_f f_\sigma(k), \end{cases}$$

where $f_\sigma(k)$ represents the different additive faults.

3.5 Concluding remarks

In this chapter, related knowledge of switched systems and IFs has been presented, which provides the theoretical basis of this thesis.

First, a brief introduction of switched systems has been provided. Due to the characteristics of the switched systems that subsystems and switching rules can well represent complex systems in real applications, it is important to study switched systems. Switched systems with norm-bounded disturbance and uncertainties will be studied in Chapter 4. The systems with IFs can also be described by switched systems with normal subsystems and faulty subsystems. Hence, multiplicative IFs based on switched system model will be discussed in Chapter 5.

Then, stability conditions of switched systems under arbitrary switching and restrict switching have been introduced, respectively. CQLF, SQLF, piecewise Lyapunov functions and ADT methods are given. It is worth mentioning that we will mainly focus on the ADT method to address the studies on stability analysis of switched systems in Chapter 4 and Chapter 5.

Considering that the early detection of IFs is important and there have been few results on multiplicative IFs detection schemes, the characters and modeling of multiplicative IFs have been discussed in the end. Motivated by this, the investigation on IF detection schemes based on switched systems will be addressed in detail.

4 Design of observer-based Fault Detection schemes for uncertain switched systems

To meet demand for early FD in switched systems with norm-bounded uncertainties, a so-called $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme is investigated in this chapter.

Although there have already been numerous studies on FD schemes for switched systems under the condition that the systems and observers are in synchronized manner, the case in asynchronized manner is still not sufficiently studied, which attracts our interest to study this topic in this chapter. To this end, the existence conditions of $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD systems are studied first. It is followed by an $\mathcal{L}_\infty/\mathcal{L}_2$ FD scheme based on ADT stability situations with a mode estimation unit. Finally, the LMI-aided technique is applied to threshold computation.

4.1 Problem Formulation

For uncertain systems and systems with norm-bounded disturbance, some FD schemes have been investigated for the analysis and integrated design in many different types of systems, such as Markov jump systems [70], T-S fuzzy systems [51], and linear parameter varying systems [72]. Based on $\mathcal{L}_2$ stability theory [86], a dynamic threshold is proposed by considering the influence of the input variables on residual signals. The so-called $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ FD schemes are proposed [86, 50]. To optimize the parameters related to dynamical thresholds, LMI tools can be applied to obtain sufficient conditions for the dynamical thresholds computing [49, 51].

It has been mentioned in Chapter 2 that white Gaussian noise and norm-bounded uncertainties can be used to model unknown disturbances in processes. In addition, for systems with norm-bounded unknown input and uncertainties, the $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ FD schemes can also be investigated for the analysis and design of observers. Moreover, the asynchronous phenomena between systems and observers widely exist in real applications and since there are few studies on switched systems with asynchronous manners, an FD approach for such kinds of systems is proposed here. In this chapter, switched systems with asynchronous manners between sub-models and observers are considered. A detection scheme is proposed and dynamical thresholds are calculated by means of LMI tools.

The main objective of this chapter is to address the analysis and integrated design issues of the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD for switched systems with uncertainties. Existence conditions of $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD systems for switched systems are studied. For the real-time FD scheme, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based residual generator is developed. By means of the LMI technique, dynamical thresholds are computed. In addition, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme and a procedure for estimating switching modes are developed in this chapter.

4.2 $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ types of observer-based FD systems

In this section,we mainly focus on the existence conditions of $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD systems for uncertain switched systems. Consider a discrete-time switched system

$$\begin{cases} x(k+1) = A_{\sigma(k)}x(k) + B_{\sigma(k)}u(k) + \Delta_A x(k) + \Delta_B u(k) + M_d d(k) + E_{f,\sigma(k)}f(k), \\ y(k) = C_{\sigma(k)}x(k) + \Delta_C x(k) + N_d d(k) + F_{f,\sigma(k)}f(k), \end{cases} \tag{4.1}$$

where $x(k) \in \mathcal{R}^{n_x}$ is the system state with initial condition $x(0) = x_0$, and $y(k) \in \mathcal{R}^{n_y}$ is a measurable output. $u(k) \in \mathcal{R}^{n_u}$ is the input signal, $d(k)$ is an unknown input vector with bounded norm. $A_{\sigma(k)}, B_{\sigma(k)}, C_{\sigma(k)}, M_d, N_d, E_{f,\sigma(k)}$ and $F_{f,\sigma(k)}$ are known system matrices with appropriate dimensions. $\sigma(k) \in \mathcal{S}$ is a switching signal with $\mathcal{S} = \{1, 2, 3, \ldots, M\}$. Uncertainties are norm-bounded, that is, for given $\epsilon_{a,\sigma}, \epsilon_{b,\sigma}, \epsilon_{c,\sigma} > 0$ it holds that $\|\Delta_A\| \leq \epsilon_{a,\sigma}$, $\|\Delta_B\| \leq \epsilon_{b,\sigma}$, $\|\Delta_C\| \leq \epsilon_{c,\sigma}$.

The observer-based residual generator dynamics can be formulated as follows:

$$\begin{cases} \hat{x}(k+1) = A_{\hat{\sigma}(k)}\hat{x}(k) + B_{\hat{\sigma}(k)}u(k) + L_{\hat{\sigma}(k)}(y(k) - \hat{y}(k)), \\ \hat{y}(k) = C_{\hat{\sigma}(k)}\hat{x}(k), \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{4.2}$$

where $\hat{x}(k) \in \mathcal{R}^{n_x}$ denotes the estimate of state $x(k)$, $\hat{y}(k) \in R^{n_y}$ is the estimate of output, $r(k) \in R^{n_y}$ represents the residual signal, for the purpose that avoiding loss of information about the faults, the residual vector has the same dimension as the output vector. $\hat{\sigma}(k)$ is the switching rule of the observer.

In the following subsections, the existence conditions of $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD systems are discussed, which are the basis of the FD schemes discussed in this chapter.

4.2.1 An $\mathcal{L}_2$ observer-based FD system

According to [27, 51, 86], the input-output stability theory and weakly output reconstructibility can be applied to investigate an $\mathcal{L}_2$ observer-based FD scheme for nonlinear systems. Then the $\mathcal{L}_2$ observer-based FD approach can be studied for switched systems. To this end, a useful definition is introduced first, then a theorem revealing the existence conditions of FD systems for system (4.1) is presented.

Definition 4.1. *System (4.1) is said to be $\mathcal{L}_2$ re-constructible if there exist (i) an observer-based residual generator (4.2), (ii) functions $\varphi_1(\cdot) \in \mathcal{K}, \varphi_2(\cdot) \in \mathcal{K}_\infty$, and $\gamma_0(\cdot) \geq 0$, such that $\forall x, \hat{x} \in \mathcal{B}_q$, $\mathcal{B}_q := \{x \in \mathcal{R}^{k_x} : \|x\| \leq q, q > 0\}$.*

$$\textstyle\sum_{m=0}^{k} \varphi_1(\|r(m)\|) \leq \sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + \gamma_0(x(0), \hat{x}(0)), \tag{4.3}$$

where $\tilde{u}(k) = [u^T(k) \quad d^T(k)]^T$, $\mathcal{B}_q := \{x \in \mathcal{R}^n : \|x\| \leq q$ for some $q > 0\}$.

In this section, we focus on the case that the system is $\mathcal{L}_2$ re-constructible. To this end, a sufficient condition for it is given in the following theorem.

Theorem 4.1. *For the given switched system (4.1), if there exist (i) an observer-based residual generator (4.2), (ii) a Lyapunov function $V(k, x(k), \hat{x}(k)) \geq 0$, functions $\varphi_1(\cdot) \in \mathcal{K}$, and $\varphi_2(\cdot) \in \mathcal{K}_\infty$, such that $\forall x, \hat{x} \in \mathcal{B}_q$*

$$V(k+1, x(k+1), \hat{x}(k+1)) - V(k, x(k), \hat{x}(k+1)) \leq -\varphi_1(\|r(k)\|) + \varphi_2(\|\tilde{u}(k)\|), \tag{4.4}$$

then the switched system (4.1) is $\mathcal{L}_2$ re-constructible.

Proof. It is evident from (4.4) that

$$\begin{array}{rcl} V\left(k+1, x(k+1), \hat{x}(k+1)\right) - V\left(k, x(k), \hat{x}(k)\right) & \leq & -\varphi_1(\|r(k)\|) + \varphi_2(\|\tilde{u}(k)\|), \\ V\left(k, x(k), \hat{x}(k)\right) - V\left(k-1, x(k-1), \hat{x}(k-1)\right) & \leq & -\varphi_1(\|r(k-1)\|) + \varphi_2(\|\tilde{u}(k-1)\|), \\ & \vdots & \\ V\left(1, x(1), \hat{x}(1)\right) - V\left(0, x(0), \hat{x}(0)\right) & \leq & -\varphi_1(\|r(0)\|) + \varphi_2(\|\tilde{u}(0)\|). \end{array} \tag{4.5}$$

It is obvious that

$$V(k+1, x(k+1), \hat{x}(k+1)) \geq 0,$$

the sum of the both sides of the above inequalities yields

$$\begin{array}{r} V\left(k+1, x(k+1), \hat{x}(k+1)\right) - V\left(0, x(0), \hat{x}(0)\right) \leq \\ -\sum_{m=0}^{k} \varphi_1(\|r(m)\|) + \sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|). \end{array} \tag{4.6}$$

Setting $\gamma_0(x(0), \hat{x}(0)) = V(0, x(0), \hat{x}(0))$, the inequality

$$\begin{aligned} \textstyle\sum_{m=0}^{k} \varphi_1(\|r(m)\|) &\leq -V(k+1, x(k+1), \hat{x}(k+1)) \\ &+ \textstyle\sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + \gamma_0(x(0), \hat{x}(0)), \end{aligned} \tag{4.7}$$

holds, which leads to (4.3). The proof is completed. ■

By adopting

$$\varphi_1(\|r(k)\|) = r^T(k)r(k), \tag{4.8}$$

$$\varphi_2(\|\tilde{u}(k)\|) = \gamma^2 \tilde{u}^T(k)\tilde{u}(k), \tag{4.9}$$

equation (4.4) can be formulated as:

$$V(k+1, x(k+1), \hat{x}(k+1)) - \gamma_0(x(0), \hat{x}(0)) \leq -\sum_{m=0}^{k} r^T(m)r(m) + \gamma^2 \sum_{m=0}^{k} \tilde{u}^T(m)\tilde{u}(m).$$

For an $\mathcal{L}_2$ observer-based FD system, the evaluation function can be defined as

$$J(k) = \sum_{m=0}^{k} r^T(m)r(m), \tag{4.10}$$

correspondingly, the threshold can be set as

$$J_{th}(k) = \textstyle\sum_{m=0}^{k} \tilde{u}^T(m)\tilde{u}(m)) + \bar{\gamma}_0, \tag{4.11}$$

$$\bar{\gamma}_0 = \sup_{x(0), \hat{x}(0)} V(x(0), \hat{x}(0)). \tag{4.12}$$

The occurrence of a fault can be detected by using the decision logic

$$\begin{cases} J(k) \leq J_{th}(k) \Rightarrow \textit{fault-free}, \\ J(k) > J_{th}(k) \Rightarrow \textit{faulty}. \end{cases} \tag{4.13}$$

In this $\mathcal{L}_2$ observer-based FD scheme, the threshold is related to the input signal, disturbance and the initial value. The threshold is dynamic and shows lower conservativeness than a fixed threshold.

As the evaluation function (4.10) shows, after running the $\mathcal{L}_2$ observer-based FD scheme for a time interval $[0, k]$, we can determine whether a fault occurs or not. It indicates that the real-time performance of $\mathcal{L}_2$ observer-based FD scheme is poor. To this end, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme is investigated to meet the requirement of real-time FD.

Remark 4.1. *In the theorem of [86], the evaluation window $\mathcal{L}_2$ observer-based FD system are assumed to be infinitively large. In practice, a large evaluation window will cause a considerably time-delay in FD schemes. And in complex systems, a large evaluation window also means a high threshold, which will increase the conservativeness of the FD scheme. As a result, a reasonable time interval should be chosen in practice. In this section, k is the length of the evaluation window.*

4.2.2 An $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system

To attain a real-time detection scheme for potential faults, an $\mathcal{L}_\infty/\mathcal{L}_2$ type of observer has been investigated [27, 49, 50]. In this subsection, the existence conditions of $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD approach is studied for switched system (4.1). For this purpose, the following definition is introduced.

Definition 4.2. *Switched system (4.1) is said to be $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructible if there exist (i) an observer-based residual generator (4.2), and (ii) functions $\varphi_1(\cdot) \in \mathcal{K}, \varphi_2(\cdot), \varphi_3(\cdot) \in \mathcal{K}_\infty$, and $\gamma_0(\cdot) \geq 0$, such that $\forall x, \hat{x} \in \mathcal{B}_q$*

$$\varphi_1(\|r(k)\|) \leq \sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + \gamma_0(x(0), \hat{x}(0)). \tag{4.14}$$

Based on Definition 4.2, we can get the following theorem.

Theorem 4.2. *Given switched system (4.1), if there exist (i) an observer-based residual generator (4.2), (ii) a Lyapunov function $V(k, x(k), \hat{x}(k)) \geq 0$ and functions $\varphi_1(\cdot) \in \mathcal{K}, \varphi_2(\cdot) \in \mathcal{K}_\infty$, such that $\forall x, \hat{x} \in \mathcal{B}_q$*

$$\varphi_1(\|r(k)\|) \leq V(k, x(k), \hat{x}(k)), \tag{4.15}$$

$$V\,(k+1, x(k+1), \hat{x}(k+1)) - V(k, x(k), \hat{x}(k)) \leq \varphi_2(\|\tilde{u}(k)\|), \tag{4.16}$$

then the switched system (4.1) is $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructible.

Proof: It is evident from (4.16) that

$$\begin{gathered}
V(k+1, x(k+1), \hat{x}(k+1)) - V(k, x(k), \hat{x}(k)) \leq \varphi_2(\|\tilde{u}(k)\|) \\
V(k, x(k), \hat{x}(k)) - V(k-1, x(k-1), \hat{x}(k-1)) \leq \varphi_2(\|\tilde{u}(k-1)\|) \\
\vdots \\
V(1, x(1), \hat{x}(1)) - V(x(0), \hat{x}(0)) \leq \varphi_2(\|\tilde{u}(0)\|) \\
V(k+1, x(k+1), \hat{x}(k+1)) \leq \textstyle\sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + V(x(0), \hat{x}(0)).
\end{gathered} \tag{4.17}$$

Together with (4.15), we have

$$\varphi_1(\|r(k)\|) \leq \sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + V(x(0), \hat{x}(0)). \tag{4.18}$$

Since $\gamma_0(x(0), \hat{x}(0)) = \sup_{x(0), \hat{x}(0)} V(x(0), \hat{x}(0))$, it is obvious that (4.14) holds. The proof is completed. ∎

As a result, if the switched system (4.1) is $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructible, an observer-based FD system can be realized by defining the evaluation function and threshold as follows:

$$J(k) = \varphi_1(\|r(k)\|), \tag{4.19}$$

$$J_{th}(k) = \textstyle\sum_{m=0}^{k} \varphi_2(\|\tilde{u}(m)\|) + \sup_{x(0),\hat{x}(0)} \gamma_0(x(0), \hat{x}(0)). \tag{4.20}$$

Then FD can be achieved by using the decision logic

$$\begin{cases} J(k) \leq J_{th}(k) \Rightarrow \ \textit{fault-free}, \\ J(k) > J_{th}(k) \Rightarrow \ \textit{faulty}. \end{cases} \tag{4.21}$$

In this subsection, unknown input signals and uncertainties are considered to be norm bounded, and it is noteworthy that the FD approach given promises zero FAR. Both $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD schemes can be applied to switched systems with known switching signals. However, when the switching rule is not known in prior, situations may become more complex.

To settle these situations, switched systems in synchronized and asynchronized manners are established in the next section, and an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD approach under ADT stability condition is proposed.

4.3 An $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme for switched systems in asynchronized manner

In this section, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD design by means of ADT theory is investigated for switched systems in both synchronized and asynchronized manners. A mode estimation unit is embedded which can determine whether a fault or a switching occurs.

In the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD design, the residuals are generated by $\mathcal{L}_\infty/\mathcal{L}_2$ observer first, then the evaluation function is $\mathcal{L}$-norm, which indicates the residual is measured at each time instant. The threshold is $\mathcal{L}_2$-norm of the extended input signal, including disturbances and modal uncertainties. On this basis, the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system is proposed.

It should be emphasized that the target of mode estimation embedded in the detection logic is to (i) reduce the conservativeness of the threshold setting, and (ii) increase the fault detection rate.

4.3.1 Model description

The i-th subsystem of the switched system and the j-th residual generator can be expressed as

$$\begin{cases} x(k+1) = (A_i + \Delta_A)x(k) + (B_i + \Delta_B)u(k) + M_d d(k) + E_{f,i}f(k), \\ y(k) = (C_i + \Delta_C)x(k) + N_d d(k) + F_{f,i}f(k), \end{cases} \tag{4.22}$$

and

$$\begin{cases} \hat{x}(k+1) = A_j\hat{x}(k) + B_j u(k) + L_j(y(k) - \hat{y}(k)), \\ \hat{y}(k) = C_j\hat{x}(k), \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{4.23}$$

respectively, where $i, j \in \mathcal{S}$.

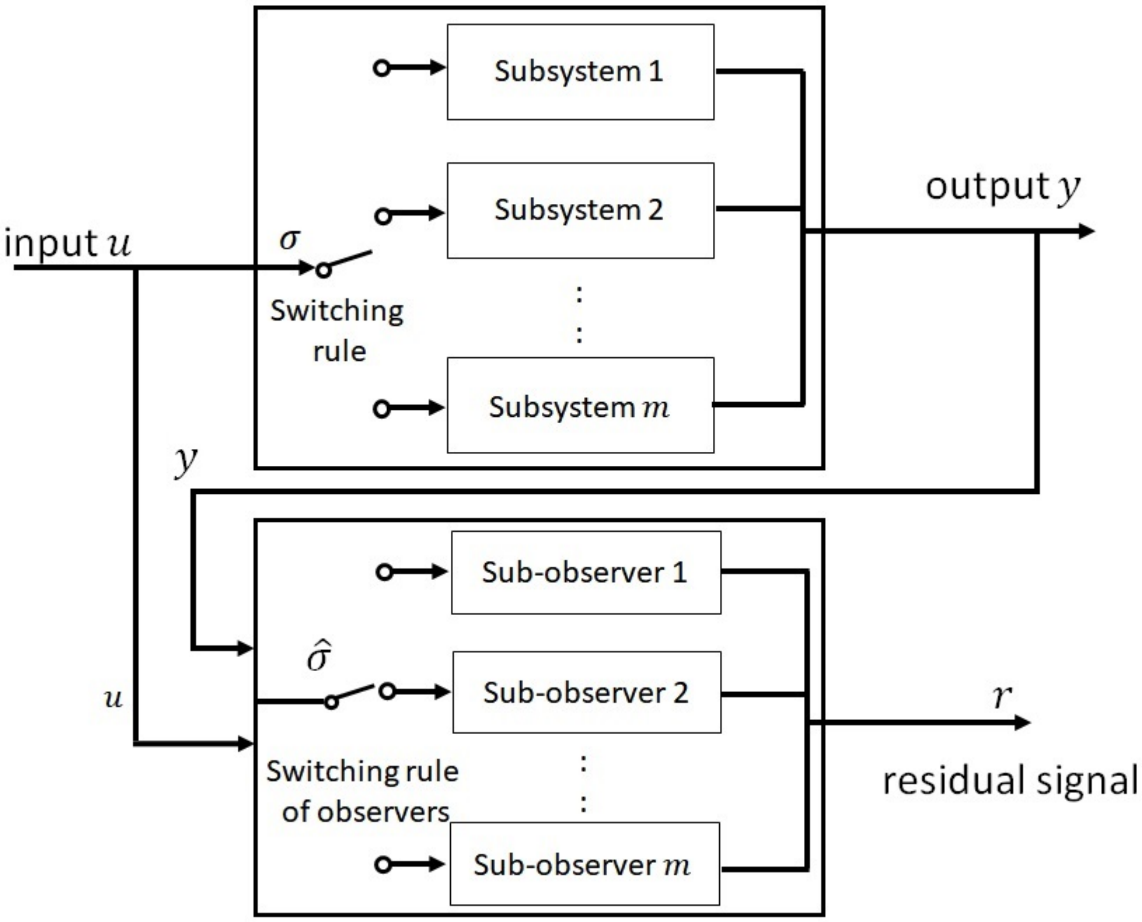

Figure 4.1: Switched systems and asynchronously switching observers.

The switchings between different observers depend on the switching signal, and each subsystem has a corresponding sub-observer, see Fig. 4.1. Here, the switching rule of subsystem and sub-observer are expressed by σ and $\hat{\sigma}$, respectively.

In this section, the switching rule of given switched system (4.1) is unknown, and it is obvious that there exist two switching modes:

1. $\sigma(k) = \hat{\sigma}(k)$, both the system under consideration and the observer are running with the "dynamics" corresponding to the same sub-model before the switching.

2. $\sigma(k) \neq \hat{\sigma}(k)$, the subsystem and the observer are running with different dynamics corresponding to different sub-models before the switching.

The asynchronous manners between subsystems and observers is illustrated in Fig. 4.2. Here, the switched system and switched observers are running in a time interval $[k_0, k_N]$, and it is obvious that there exists a time-delay between the switching of the subsystem and the switching observer. The system switches from the i-th subsystem to the $(i+1)$-th subsystem at k_i, while the corresponding observer switches at $k_i + \Delta_i$. The time-delay Δ_i can be different at different switchings. The system and the corresponding observer are synchronous in matched periods, while the asynchronous manner appears in unmatched periods.

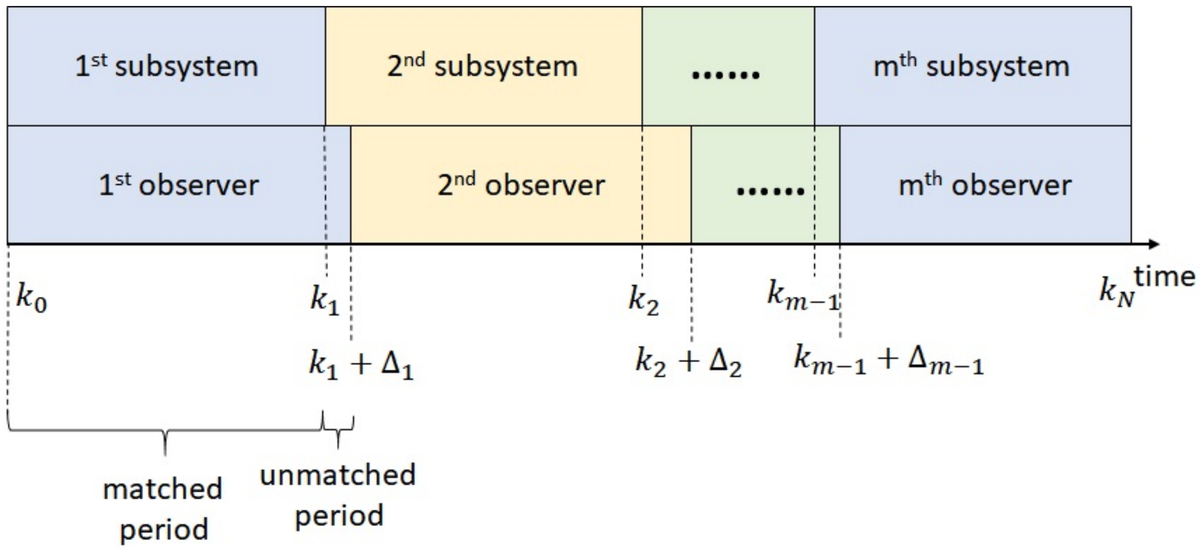

Figure 4.2: Asynchronous switchings between subsystems and observers.

In the following part, the ADT stability theory discussed in Chapter 3 is applied to ensure the stability of FD schemes for switched systems. For the switched systems with both synchronized and asynchronized observers, an MDADT [95] approach is proposed as follows.

Definition 4.3. *(MDADT) For any $T > t \geq 0$, over the time interval (t, T), let $N_{\sigma,i}$ be the switching times that the i-th subsystem is activated and $T_i(t, T)$ be the total operating time for the i-th subsystem. For any $T_i > 0$ and positive number $N^*_{0,i}$, there exists:*

$$N_{\sigma,i} \leq \frac{T_i(t,T)}{\tau_{\sigma,i}} + N^*_{0,i}, i \in \mathcal{S}, \tag{4.24}$$

then we can say that σ has an MDADT $\tau_{\sigma,i}$.

Remark 4.2. *The ADT switching property is that the average time intervals between any two consecutive subsystems are at least τ, which is independent to all the subsystems. Therefore, the ADT calculated by all subsystems may increase the Lyapunov function value at the switching instants, it may lead to a certain rise of the conservativeness. To this end, MDADT method is investigated to reduce the conservativeness.*

Existence conditions of $\mathcal{L}_2$ observer-based FD systems for switched systems cannot be proved under ADT stability theory. So that an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system is investigated in the next subsection.

4.3.2 An $\mathcal{L}_\infty/\mathcal{L}_2$ FD scheme with mode estimation

In this section, an LMI-aided integrated design of FD approach for switched systems with asynchronized switching manners is investigated. To this end, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based residual generator is studied, and thresholds in synchronized and asynchronized manners are calculated. A decision rule with a mode estimation unit is set to distinguish different faulty modes and estimate switching instants.

In our study, the switching rule of the switched system is unknown a prior, and thereby a detection scheme for distinguishing mode switching is necessary. Obviously, there exists a time-delay for the switching of the residual generator, which delivers asynchronized switching between the system and the residual generator. That indicates the residual signal contains the information for both the fault and switching, which motivates us to propose an FD scheme with mode estimation.

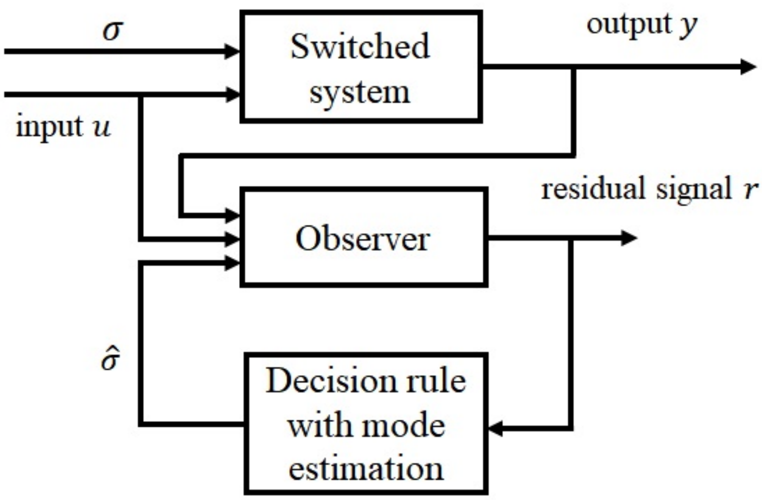

Figure 4.3: Schematic description of the FD scheme in asynchronous manner.

As a result, the main tasks of the FD scheme design are: (1) investigate an $\mathcal{L}_\infty/\mathcal{L}_2$ type of observer-based residual generator; (2) propose a detection rule with an embedded mode estimation unit, see Fig. 4.3.

Define

$$e(k) = x(k) - \hat{x}(k), \theta(k) = \begin{bmatrix} e(k) \\ x(k) \end{bmatrix}, \tilde{u}(k) = \begin{bmatrix} u(k) \\ d(k) \end{bmatrix}. \tag{4.25}$$

The overall dynamics of the residual generator can be formulated as follows:

1. in synchronous periods $k \in [k_l + \kappa_l, k_{l+1}], l = 0, 1, \ldots, \kappa_l$ denotes the switching instant in $[k_l, k_{l+1}]$ as shown in Fig. 4.4,

$$\begin{cases} \theta(k+1) = (\bar{A}_i + \bar{\Delta}_{A_i})\theta(k) + (\bar{B}_i + \bar{\Delta}_{B_i})\tilde{u}(k), \\ r(k) = (\bar{C}_i + \bar{\Delta}_{C_i})\theta(k), \end{cases} \tag{4.26}$$

   where

$$\bar{A}_i = \begin{bmatrix} A_i - L_i C_i & 0 \\ 0 & A_i \end{bmatrix}, \bar{B}_i = \begin{bmatrix} 0 & M_d \\ B_i & 0 \end{bmatrix}, \bar{C}_i = \begin{bmatrix} -C_i & 0 \end{bmatrix}, \tag{4.27}$$

$$\bar{\Delta}_{A_i} = \begin{bmatrix} 0 & -L_i\Delta_C + \Delta_A \\ 0 & \Delta_A \end{bmatrix}, \bar{\Delta}_{B_i} = \begin{bmatrix} \Delta_B & 0 \\ \Delta_B & 0 \end{bmatrix}, \bar{\Delta}_{C_i} = \begin{bmatrix} 0 & \Delta_C \end{bmatrix}.$$

2. in asynchronous periods $k \in [k_l, k_l + \kappa_l], l = 0, 1, \ldots,$ where $\sigma(k) = i, \hat{\sigma}(k) = j,$

$$\begin{cases} \theta(k+1) = (\bar{A}_{ij} + \bar{\Delta}_{A_{ij}})\theta(k) + (\bar{B}_{ij} + \bar{\Delta}_{B_{ij}})\tilde{u}(k), \\ r(k) = (\bar{C}_{ij} + \bar{\Delta}_{C_{ij}})\theta(k). \end{cases} \tag{4.28}$$

where

$$\begin{aligned} \bar{A}_{ij} &= \begin{bmatrix} A_j - L_jC_j & (A_i - A_j) - L_j(C_{ij} - C_j) \\ 0 & A_i \end{bmatrix}, \\ \bar{B}_{ij} &= \begin{bmatrix} B_i - B_j & M_d \\ B_i & 0 \end{bmatrix}, \bar{C}_{ij} = \begin{bmatrix} -C_j & C_i - C_j \end{bmatrix}, \end{aligned} \tag{4.29}$$

$$\bar{\Delta}_{A_{ij}} = \begin{bmatrix} 0 & -L_j\Delta_C + \Delta_A \\ 0 & \Delta_A \end{bmatrix}, \bar{\Delta}_{B_{ij}} = \begin{bmatrix} \Delta_B & 0 \\ \Delta_B & 0 \end{bmatrix}, \bar{\Delta}_{C_{ij}} = \begin{bmatrix} 0 & \Delta_C \end{bmatrix}. \tag{4.30}$$

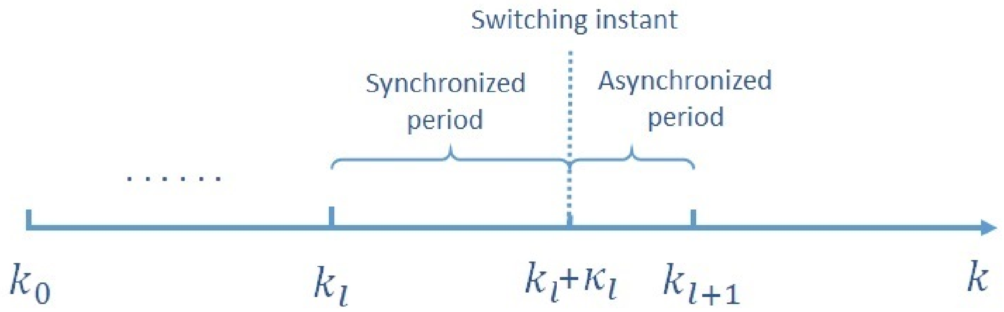

Figure 4.4: Timing diagram.

For our purpose, we first study existence conditions for $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructability.

Theorem 4.3. *Consider system (4.28), if there exist positive constants α_i, η_i, $\bar{\beta}_i$, $\mu_{1,i}$, $\mu_{2,i}$, $i \in \mathcal{S}$, $\exists 0 < \lambda < 1$, $0 < \gamma < 1$ and piecewise Lyapunov function*

$$\bar{V}(\theta(k)) = \begin{cases} \bar{V}_{\sigma(k)}(\theta(k)), k \in [k_l + \kappa_l, k_{l+1}], \\ \bar{V}_{\sigma(k)\hat{\sigma}(k)}(\theta(k)), k \in [k_l, k_l + \kappa_l], \end{cases} \tag{4.31}$$

such that the following inequalities hold

$$\bar{V}_{ij}(\theta(k+1)) \leq (1+\eta_j)\bar{V}_{ij}(\theta(k)) + \bar{\beta}_j\tilde{u}^T\tilde{u}, \tag{4.32}$$

$$\bar{V}_i(\theta(k+1)) \leq (1-\alpha_i)\bar{V}_i(\theta(k)) + \bar{\beta}_i\tilde{u}^T\tilde{u}, \tag{4.33}$$

$$\theta^T(k)(\bar{C}_i + \delta_C)^T(\bar{C}_i + \delta_C)\theta(k) \leq \bar{V}_{ij}(\theta(k)), \tag{4.34}$$

$$\theta^T(k)(\bar{C}_{ij} + \delta_C)^T(\bar{C}_{ij} + \delta_C)\theta(k) \leq \bar{V}_i(\theta(k)), \tag{4.35}$$

$$\bar{V}_j(\theta(k)) \leq \mu_{1,j}\bar{V}_{ij}(\theta(k)), \bar{V}_{ij}(\theta(k)) \leq \mu_{2,i}\bar{V}_i(\theta(k)), \forall i,j \in \mathcal{S}, \tag{4.36}$$

and the MDADT for any switching signal satisfies

$$\tau_{\sigma_i} > \tau^\star_{\sigma_i} := \frac{\ln \mu_{1,i}\mu_{2,i}}{\xi}, \frac{T_\eta(k_0,k)}{T_\alpha(k_0,k)} < -\frac{\ln(1-\alpha_i)+\xi}{\ln(1+\eta_i)+\xi}, \tag{4.37}$$

where $l = 0, 1, \cdots$, the system (4.28) is $\mathcal{L}_\infty/\mathcal{L}_2$ reconstructible and it holds

$$\begin{array}{rcl} r^T(k)r(k) & \leq & \prod_{i=0}^{l}(\mu_{1,i}\mu_{2,i})^{N_{0,i}} \sum_{m=k_0}^{k} \bar{\beta}_{\sigma(m)}\tilde{u}^T(m)\tilde{u}(m) \\ & + & \lambda^{k-k_0}\prod_{i=0}^{l}(\mu_{1,i}\mu_{2,i})^{N_{0,i}}\bar{V}_{\sigma_{k_0}}(\theta(k_0)), \end{array} \tag{4.38}$$

where $N_\sigma(k_0, k), N_{\hat{\sigma}}(k_0, k)$ are counters of the switching number of a residual generator in a time interval $[k_0, k]$, $T_\alpha(k_0, k), T_\eta(k_0, k)$ represent the running time interval of both synchronous and asynchronous periods, respectively.

Proof. Suppose that $k_l \leq k \leq k_l + \kappa_l$ and $\bar{k}_l = k_l + \kappa_l$. It follows from (4.32)-(4.36) that

$$\begin{array}{rcl} \bar{V}(\theta(k)) & \leq & \bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{V}_{\sigma_l}(\theta(\bar{k}_l)) + \sum_{m=\bar{k}_l}^{k}\bar{\beta}_{\sigma_l}\bar{\alpha}_{\sigma_l}^{k-m}\tilde{u}^T(m)\tilde{u}(m) \\ & \leq & \mu_{1,\sigma_l}\bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{V}_{\sigma_{l-1}\hat{\sigma}_l}(\theta(\bar{k}_l)) + \sum_{m=\bar{k}_l}^{k}\bar{\beta}_{\sigma_l}\bar{\alpha}_{\sigma_l}^{k-m}\tilde{u}^T(m)\tilde{u}(m) \\ & \leq & \bar{\eta}_{\sigma_l}^{\bar{k}_l-k_l}\mu_{1,\sigma_l}\bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{V}_{\sigma_{l-1}\hat{\sigma}_l}(\theta(k_l)) \\ & & +\sum_{m=k_l}^{\bar{k}_l}\mu_{1,\sigma_l}\bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{\beta}_{\hat{\sigma}_l}\bar{\eta}_{\sigma_l}^{\bar{k}_l-m}\tilde{u}^T(m)\tilde{u}(m) \\ & & +\sum_{m=\bar{k}_l}^{k}\bar{\beta}_{\sigma_l}\bar{\alpha}_{\sigma_l}^{k-m}\tilde{u}^T(m)\tilde{u}(m) \\ & \leq & \mu_{1,\sigma_l}\mu_{2,\sigma_{l-1}}\bar{\eta}_{\sigma_l}^{\bar{k}_l-k_l}\mu_{1,\sigma_l}\bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{V}_{\sigma_{l-1}}(\theta(k_l)) \\ & & +\sum_{m=k_l}^{\bar{k}_l}\mu_{1,\sigma_l}\bar{\alpha}_{\sigma_l}^{k-\bar{k}_l}\bar{\beta}_{\hat{\sigma}_l}\bar{\eta}_{\sigma_l}^{\bar{k}_l-m}\tilde{u}^T(m)\tilde{u}(m) \\ & & +\sum_{m=\bar{k}_l}^{k}\bar{\beta}_{\sigma_l}\bar{\alpha}_{\sigma_l}^{k-m}\tilde{u}^T(m)\tilde{u}(m) \\ & \leq & \prod_{i=0}^{l}\mu_{2,i}^{N_{\hat{\sigma}_i}(k_0,k)}\mu_{1,i}^{N_{\sigma_i}(k_0,k)}\bar{\eta}_i^{T_{\eta_i}}\bar{\alpha}_i^{T_{\alpha_i}}\bar{V}_{\sigma_{k_0}}(\theta(0)) \\ & & +\sum_{m=k_0}^{k}\prod_{i=0}^{l}\mu_{1,i}^{N_{\sigma_i}(m,k)}\mu_{2,i}^{N_{\hat{\sigma}_i}(m,k)}\bar{\beta}_{\sigma(i)}\bar{\eta}_i^{T_{\eta_i}(m,k)}\bar{\alpha}_i^{T_{\alpha_i}(k_0,k)}\tilde{u}^T(m)\tilde{u}(m), \end{array} \tag{4.39}$$

where $T_{\eta_i}(t, T), T_{\alpha_i}(t, T)$ denote the running time of asynchronous and synchronous period in $[t, T]$ in the i-th subsystem. We also suppose that

$$\bar{\alpha}_i = 1 - \alpha_i, \bar{\eta}_i = 1 + \eta_i.$$

As introduced in Theorem 3.3, the process is GUES over time interval $[k_0, k]$, if $\exists 0 < \lambda < 1, 0 < \gamma < 1$ and $\tilde{u} = 0$, there exists

$$\bar{V}(\theta(k)) \quad \leq \quad \lambda^{k-k_0}\bar{V}(\theta(0)), \tag{4.40}$$

such that (4.39) implies

$$\prod_{i=0}^{l}\mu_{2,i}^{N_{\hat{\sigma}_i}(k_0,k)}\mu_{1,i}^{N_{\sigma_i}(k_0,k)}(1+\eta_i)^{T_{\eta_i}(k_0,k)}(1-\alpha_i)^{T_{\alpha_i}(k_0,k)} \leq \lambda^{k-k_0}. \tag{4.41}$$

Taking the log of both sides of (4.41), it gives

$$\sum_{i=0}^{l}(N_{\hat{\sigma},i}\ln\mu_{2,i} + N_{\sigma_i}\ln\mu_{1,i}) + T_{\eta_1}(k_0,k)\ln(1+\eta_i) + T_{\alpha_1}(k_0,k)\ln(1-\alpha_i) \leq (k-k_0)\ln\lambda.$$

Combining with (4.37) and $\sum_{i=0}^{l}(T_{\eta_i}(k_0,k)+T_{\alpha_i}(k_0,k))=k-k_0$, we have

$$T_{\eta_i}(k_0,k)\ln(1+\eta_i)+T_{\alpha_i}(k_0,k)\ln(1-\alpha_i)\le(\ln\lambda+\xi)(k-k_0), \tag{4.42}$$

then the following inequality

$$r^T(k)r(k)\le\bar{\rho}\sum_{m=k_0}^{k}\bar{\beta}_{\sigma(m)}\tilde{u}^T(m)\tilde{u}(m)+\lambda^{k-k_0}\bar{\rho}\bar{V}_{\sigma_{k_0}}(\theta(k_0)) \tag{4.43}$$

holds and there exists

$$\bar{\rho}=\prod_{i=0}^{l}(\mu_{1,i}\mu_{2,i})^{N_{0,i}}. \tag{4.44}$$

Together with $r^T(k)r(k)\le\bar{V}_{\sigma_k}(\theta(k))$, it is obvious that (4.38) holds. The proof is thus completed. ■

If the conditions given in Theorem 4.3 are satisfied, the Lyapunov functions can be expressed by

$$\bar{V}(\theta(k))=\begin{cases}\theta^T(k)P_{\sigma(k)}\theta(k),k\in[k_l+\kappa_l,k_{l+1}],\\ \theta^T(k)P_{\sigma(k)\hat{\sigma}(k)}\theta(k),k\in[k_l,k_l+\kappa_l],\end{cases} \tag{4.45}$$

where $l=0,1,\dots,$ the design scheme of the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based residual generation can be realized in form of LMIs, as expressed in the following theorem.

Theorem 4.4. *Consider system (4.22) and observer-based residual generator (4.28), if there exist matrices $L_i,\bar{P}_i>0,\bar{P}_{ij}>0$ and positive constants α_i,η_i, $\bar{\beta}_i$, ϵ_a, ϵ_b, ϵ_c, $i,j\in\mathcal{S}$ such that*

$$\begin{bmatrix}-(1-\alpha_i)\bar{P}_i & 0 & \bar{A}_i^T\bar{P}_i & 0 & \epsilon_a\\ * & -\bar{\beta}_i & \bar{B}_i^T\bar{P}_i & 0 & \epsilon_b\\ * & * & -\bar{P}_i & \bar{P}_i & 0\\ * & * & * & -I & 0\\ * & * & * & * & -1/\Delta^2\end{bmatrix}\le 0, \tag{4.46}$$

$$\begin{bmatrix}-(1+\eta_j)\bar{P}_ij & 0 & \bar{A}_{ij}^T\bar{P}_{ij} & 0 & \epsilon_a\\ * & -\bar{\beta}_j & \bar{B}_{ij}^T\bar{P}_{ij} & 0 & \epsilon_b\\ * & * & -\bar{P}_{ij} & \bar{P}_{ij} & 0\\ * & * & * & -I & 0\\ * & * & * & * & -1/\Delta^2\end{bmatrix}\le 0, \tag{4.47}$$

$$\begin{bmatrix}-\bar{P}_i & \bar{C}_i^T+\epsilon_c\\ * & -I\end{bmatrix}\le 0, \tag{4.48}$$

$$\begin{bmatrix} -\bar{P}_{ij} & \bar{C}_{ij}^T + \epsilon_c \\ * & -I \end{bmatrix} \leq 0, \tag{4.49}$$

then (4.37) holds for any switching signal with MDADT.

Proof. It is evident from (4.32) and (4.33) that

$$\theta^T(k+1)\bar{P}_i\theta(k+1) \leq (1-\alpha_i)\theta^T(k)\bar{P}_i\theta(k) + \bar{\beta}_i\tilde{u}^T(k)\tilde{u}(k), \tag{4.50}$$

$$\theta^T(k+1)\bar{P}_{ij}\theta(k+1) \leq (1+\eta_j)\theta^T(k)\bar{P}_{ij}\theta(k) + \bar{\beta}_j\tilde{u}^T(k)\tilde{u}(k). \tag{4.51}$$

Applying Schur complement lemma [21], it can be easily proven that (4.46) and (4.47) yield for any switching signal with MDADT, such that (4.48) and (4.49) holds. The proof is done. ■

Same as conditions of $\mathcal{L}_2$ observer-based FD scheme, $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructibility serves as a sufficient condition for the existence of an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system and threshold setting.

With the minimized $\bar{\beta}_i, i \in \mathcal{S}$ subject to (4.46)-(4.49) and

$$\bar{\rho} = \prod_{i=0}^{l}(\mu_{1,i}\mu_{2,i})^{N_{0,i}}, \tag{4.52}$$

the inequality

$$r^T(k)r(k) \leq \bar{\rho}\sum_{m=k_0}^{k}\bar{\beta}_{\sigma(m)}\tilde{u}^T(m)\tilde{u}(m) + \lambda^{k-k_0}\bar{\rho}\bar{V}_{\sigma_{k_0}}(\theta(k_0)) \tag{4.53}$$

holds. An $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system can be realized by adopting the residual generator (4.28) and setting the evaluation function in asynchronized manner as

$$J_{\hat{\sigma}}(k) = r^T(k)r(k), \tag{4.54}$$

with the threshold

$$\bar{J}_{\hat{\sigma},th}(k) = \bar{\rho}\sum_{m=k_0}^{k}\bar{\beta}_{\sigma(m)}(u^T(m)u(m) + \delta_d^2) + \lambda^{k-k_0}\bar{\rho}\bar{V}_{\sigma_{k_0}}(\theta(k_0)). \tag{4.55}$$

The evaluation function and threshold in asynchronized manner can also be presented as recursive formulation. With initial condition $\bar{J}_{th,i}(k_0) = \lambda\bar{\rho}\bar{V}_{\sigma_{k_0}}(\theta(k_0))$, we can set that

$$J_{\hat{\sigma}}(k) = r^T(k)r(k), \tag{4.56}$$

$$\bar{J}_{\hat{\sigma},th}(k+1) = \lambda\bar{J}_{\hat{\sigma},th}(k) + \bar{\rho}\bar{\beta}_{\hat{\sigma}}(u^T(k)u(k) + \delta_d^2). \tag{4.57}$$

It can be seen that (4.57) is a dynamic threshold with a mode-dependent parameter $\bar{\beta}_{\hat{\sigma}}$, which is generally less conservative than a constant threshold. However, such a threshold is still conservative because it should satisfy both the synchronized and asynchronized situations between the system and residual generator modes. If a small fault occurs in synchronous periods, the threshold (4.57) may not be appropriate which leads to the reduction of fault detectability. Furthermore, it is impossible to distinguish the switchings between different faulty modes. To this end, a less conservative FD approach is proposed for synchronous periods as follows.

When in synchronous periods, $\sigma(k) = \hat{\sigma}(k)$, with observer-based residual generator (4.26), the existence conditions of $\mathcal{L}_\infty/\mathcal{L}_2$ re-constructability for synchronized periods can be realized by Theorem 4.5.

Theorem 4.5. *Consider system (4.22) and observer-based residual generator (4.26) in synchronized manner, if there exist positive constants $\alpha_i, \beta_i, \gamma$ and piecewise Lyapunov function $\bar{V}_i, i \in \mathcal{S}$, such that following inequalities*

$$\bar{V}_i(\theta(k+1)) \leq (1-\alpha_i-\gamma)\bar{V}_i(\theta(k)) + \beta_i \tilde{u}^T \tilde{u}, \tag{4.58}$$

$$\theta^T(k)(\bar{C}_i+\delta_C)^T(\bar{C}_i+\delta_C)\theta(k) \leq \bar{V}_i(\theta(k)), \tag{4.59}$$

$$\bar{V}_j(\theta(k)) \leq \mu_i \bar{V}_i(\theta(k)), \forall i,j \in \mathcal{S} \tag{4.60}$$

hold and if any switching signal with MDADT satisfies

$$\tau_{\sigma_i} > \tau^{\star}_{\sigma_i} := \frac{\ln \mu_i}{\xi} = \frac{\ln \mu_{1,i}\mu_{2,i}}{\xi}, \tag{4.61}$$

then the system (4.22) is $\mathcal{L}_\infty/\mathcal{L}_2$ reconstructible and there exist

$$\begin{array}{l} r^T(k)r(k) \leq \rho \sum_{m=k_0}^{k} \beta_i \tilde{u}^T(m)\tilde{u}(m) + \rho \lambda^{k-k_0} \bar{V}_{\sigma_{k_0}}(\theta(0)), \\ \rho = \prod_{i=0}^{l} \mu_i^{N_{0,i}}. \end{array} \tag{4.62}$$

Proof. It follows from (4.58)-(4.60) that

$$\begin{array}{rcl} \bar{V}(\theta(k)) & \leq & \prod_{i=0}^{l} \mu_i^{N_{\sigma_i}(k_0,k)} (1-\alpha_i-\gamma)_i^T(k_0,k) \bar{V}_{\sigma_{k_0}}(\theta(k_0)) \\ & + & \sum_{m=k_0}^{k} \prod_{i=0}^{l} \mu_i^{N_{\sigma_i}(m,k)} \beta_i (1-\alpha_i-\gamma)^{T_{\alpha_i}(m,k)} \tilde{u}^T(m)\tilde{u}(m), \end{array} \tag{4.63}$$

where $T_{\alpha_i}(k_0,k)$ denotes the total duration time in time interval $[k_0,k]$ for mode $i, i \in \mathcal{S}$. Recalling that MDADT conditions (4.61), we can get

$$\ln(1-\alpha_i-\gamma)T_{\alpha_i}(k_0,k) + N_{\sigma_i}(k_0,k)\ln \mu_i < \lambda(k-k_0). \tag{4.64}$$

The proof is done. ■

In synchronous periods, the design scheme of $\mathcal{L}_\infty/\mathcal{L}_2$ residual generator can be realized in a form of LMIs as presented in the following corollary.

Corollary 4.1. *Consider system (4.22) and residual generator (4.26), if there exist matrices $L_i, P_i > 0$ obtained from (4.46) and (4.47), and positive constants $\alpha_i, \beta_i, \epsilon_a, \epsilon_b \epsilon_c, i, j \in \mathcal{S}$ such that*

$$\begin{bmatrix} -(1-\alpha_i-\gamma)P_i & 0 & \bar{A}_i^T P_i & 0 & \epsilon_a \\ * & -\beta_i & \bar{B}_i^T P_i & 0 & \epsilon_b \\ * & * & -P_i & P_i & 0 \\ * & * & * & -1 & 0 \\ * & * & * & * & -1/\Delta^2 \end{bmatrix} \leq 0, \tag{4.65}$$

$$\begin{bmatrix} -P_i & \bar{C}_i^T + \epsilon_c \\ * & -I \end{bmatrix} \leq 0, \tag{4.66}$$

then (4.61) holds for for any switching signal with MDADT.

With the minimized $\beta_i, i \in \mathcal{S}$, it is evident that the inequality

$$r^T(k)r(k) < \rho \sum_{m=k_0}^{k} \beta_i \lambda^{k-m}(u^T(m)u(m) + \delta_d^2) + \lambda^{k-k_0}\rho \bar{V}_{\sigma_{k_0}}(\theta(k_0)) \tag{4.67}$$

holds. An $\mathcal{L}_\infty/\mathcal{L}_2$ type of FD approach for switched systems in synchronized manner can be realized by setting the evaluation function as

$$J_{\hat{\sigma}} = r^T(k)r(k), \tag{4.68}$$

and the thresholds is set as

$$J_{th,i}(k) = \rho \sum_{m=k_0}^{k} \beta_i \lambda^{k-m}(u^T(m)u(m) + \delta_d^2) + \lambda^{k-k_0}\rho \bar{V}_{\sigma_{k_0}}(\theta(k_0)). \tag{4.69}$$

As shown in (4.56) and (4.57), the evaluation function and threshold in synchronized manner can also be presented as recursive formulation, with initial situation $J_{th,i}(0) = \lambda\rho\bar{V}_{\sigma_{k_0}}(\theta(0))$, there exist:

$$J_{\hat{\sigma}}(k) = r^T(k)r(k), \tag{4.70}$$

$$J_{th,\hat{\sigma}}(k+1) = \lambda J_{th,\hat{\sigma}}(k) + \rho\beta_i\lambda(u^T(k)u(k) + \delta_d^2). \tag{4.71}$$

It is evident that $\forall i \in \mathcal{S}, \beta_i \leq \bar{\beta}_i$, and a threshold in synchronized manner (4.71) is generally lower than that in asynchronized manner (4.57), which indicates the increase of the fault detectability.

As a result, a threshold in the synchronized manner is used to detect faults in normal working conditions, and the threshold in asynchronized manner is utilized to distinguish whether there occur faults or switchings between different subsystems. The precise decision logic is given in the following subsection.

4.3.3 The decision rule for mode estimation embedded FD scheme

In this subsection, we discuss the decision rule on an FD approach with a mode estimation unit.

In the previous subsection, two thresholds in (4.71) and (4.57) for synchronized and asynchronized manners are investigated, and both thresholds are running in parallel. For an FD system, there exist three different cases:

1. $J_{\hat{\sigma}(k)}(k) < J_{th,\hat{\sigma}(k)}$, fault-free case.
2. $J_{\hat{\sigma}(k)}(k) \geq \bar{J}_{th,\hat{\sigma}(k)}(k)$, faulty case.
3. $J_{th,\hat{\sigma}(k)}(k) \leq J_{\hat{\sigma}(k)}(k) < \bar{J}_{th,\hat{\sigma}(k)}(k)$, it is necessary to distinguish whether a fault occurs or a switching between different subsystems occurs.

For the case $J_{th,\hat{\sigma}(k)}(k) \leq J_{\hat{\sigma}(k)}(k) < \bar{J}_{th,\hat{\sigma}(k)}(k)$, it is necessary to propose a mode estimation unit to determine whether a fault occurs or a switching is performed. To this end, after $J_{\hat{\sigma}(k)}(k) \geq J_{th,\hat{\sigma}(k)}$ is detected, all residual generators for different subsystems are activated in parallel in a restrict time interval $[k, k+\tau], \tau = \min \tau_{\sigma_i}, i \in \mathcal{S}$. Two types of thresholds are presented in Fig. 4.5.

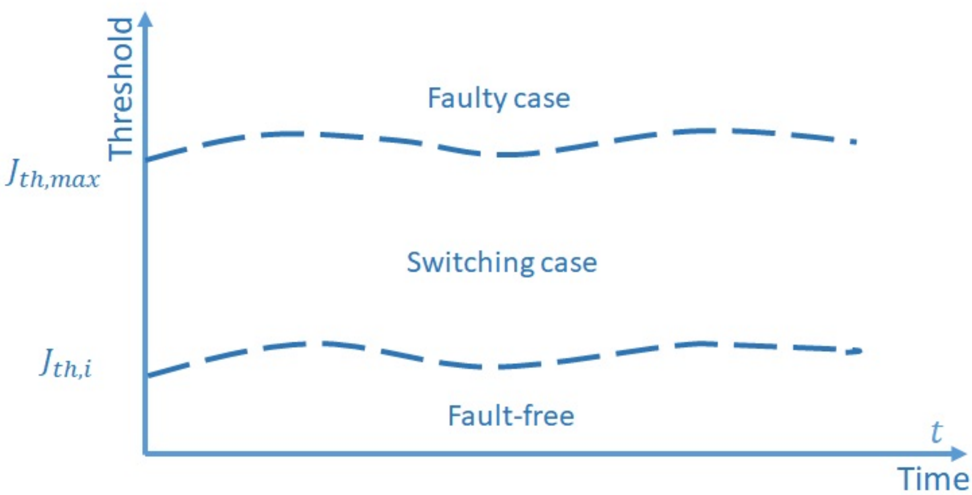

Figure 4.5: Thresholds for proposed FD systems.

The initial value of each residual generator is set as the current value of the adopted residual generator. Based on the evaluation function $\forall j \in \mathcal{S}, J_j$ given in (4.56) and thresholds $J_{th,j}, \bar{J}_{th,j}$ given in (4.71) and (4.57), the process mode should be estimated in an FD system. All sub-observers of all subsystems should be activated in parallel in a time interval $[k_1, k_2]$. A time interval $[k_1, k_2]$ is so-called asynchronous periods $T_{\eta,i}$ for each subsystem i. Consider MDADT stability, time intervals should satisfy conditions given in (4.37). After activating all observers, the initial value of each residual generator is set as the value of current state in the current residual generator. Then, we can get residual signals from all residual generators and the corresponding evaluation function $J_j(k), j \neq i, i, j \in \mathcal{S}$. Thresholds in the synchronized manner for each observer $J_{th,j}$ can also be calculated. From all these available data, a detection logic for FD scheme with mode estimation is:

1. $\exists j, J_j(k) \geq \bar{J}_{th,j}$, faulty case, the evaluation function is above the threshold in asynchronous manner.

2. $\forall j, J_j(k) \geq J_{th,j}$, faulty case, no residual generator matching the system.

3. $\exists j, J_j(k) < J_{th,j}$, switching case, there exist several residual generators matching the system, while a mode estimation expression can be expressed as

$$\hat{\sigma}(k) = arg \min_{j \in \Theta} \frac{J_j(k)}{J_{th,j}(k)}, \Theta = \{j \mid J_j < J_{th,j}\}. \tag{4.72}$$

In conclusion, an FD scheme with mode estimation can be summarized as Algorithm 4.1. It is noteworthy that τ is chosen such that the ratio of the match periods $T_\alpha(k_0, k)$ and asynchronous periods $T_\eta(k_0, k)$ satisfy (4.37). The mode estimation unit in the detection rule is to reduce the conservative threshold setting and increase the fault detectability.

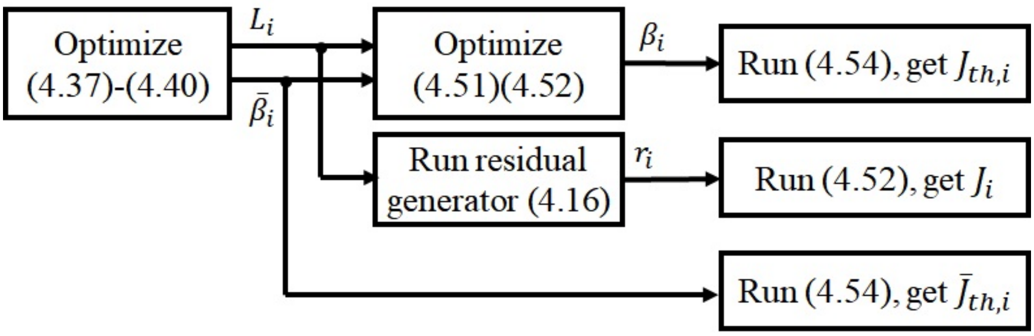

Figure 4.6: Calculation steps of evaluation function and thresholds.

The specific calculation of parameters β_i and $\bar{\beta}_i$ can be seen in Fig. 4.6, the decision rule including mode estimation is shown in Fig. 4.7.

Algorithm 4.1 An FD scheme with mode estimation for switched systems

1. Run the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based residual generator (4.26) and (4.28), and optimize parameters $\bar{\beta}_i$ in function (4.46)-(4.49) by using LMI tools.
2. With matrices L_i calculated in (4.46)-(4.49), optimize parameters β_i in (4.65)-(4.66) by means of LMI.
3. Set the evaluation function and thresholds with (4.56) and (4.71), the decision rule is set as

$$\begin{cases} J_i(k) < J_{th,i}(k), \textit{fault-free case, go to step 3,} \\ J_i(k) \geq \bar{J}_{th,i}(k), \textit{faulty case, alarm,} \\ J_{th,i}(k) \leq J_i(k) < \bar{J}_{th,i}(k), \textit{go to step 4.} \end{cases}$$

4. Active all residual generators $j, j \neq i, j \in \mathcal{S}$, set evaluation functions and thresholds with (4.56)-(4.57), and run decision logic as

$$\begin{cases} \exists j, J_j(k) \geq \bar{J}_{th,j}, \textit{faulty case, alarm,} \\ \forall j, J_j(k) \geq J_{th,j}, \textit{faulty case, alarm,} \\ \exists j, J_j(k) < J_{th,j}, \textit{switching case, go to step 5.} \end{cases}$$

5. Set the mode estimation logic as

$$\hat{\sigma}(k) = arg \min_{j \in \Theta} \frac{J_j(k)}{J_{th,j}(k)}, \Theta = \{j \mid J_j < J_{th,j}\}.$$

It is mentioned that in most of the work about asynchrnoized FD schemes for hybrid systems, a constant threshold is applied which is chosen as the maximum of the evaluation function after considering all cases. In this chapter, we improve the threshold in two aspects: 1) a mode-dependent $\mathcal{L}_2$ gain is calculated for each sub-mode; 2) dynamical thresholds are introduced, which reduces the conservativeness of the thresholds.

Remark 4.3. *For the switched system in asynchronized manner whose stability is ensured by ADT stability conditions, only the $\mathcal{L}_\infty/\mathcal{L}_2$ type of observer-based FD systems exist, therefore, we apply such type of FD scheme in this case. For the systems satisfying the existing conditions of both types of observer-based FD systems, the FD scheme with $\mathcal{L}_2$ observer-based FD scheme is always utilized for detecting persistent faults, while the FD scheme based on $\mathcal{L}_\infty/\mathcal{L}_2$ type of observer copes with instant faults.*

Remark 4.4. *In this chapter, an FD scheme for discrete-time switched systems in asynchronous manners is introduced, the FD system for continuous-time switched systems is studied in [49].*

Remark 4.5. *In industrial applications, the time-delay generally exists in dynamic systems, which may deteriorate the FD performance significantly. In this thesis, we use asynchronized* *period to describe the time delay between the system and the observer and settle the problem. This method also can be extend to switched time-delay systems, which motivated us to study further in the future.*

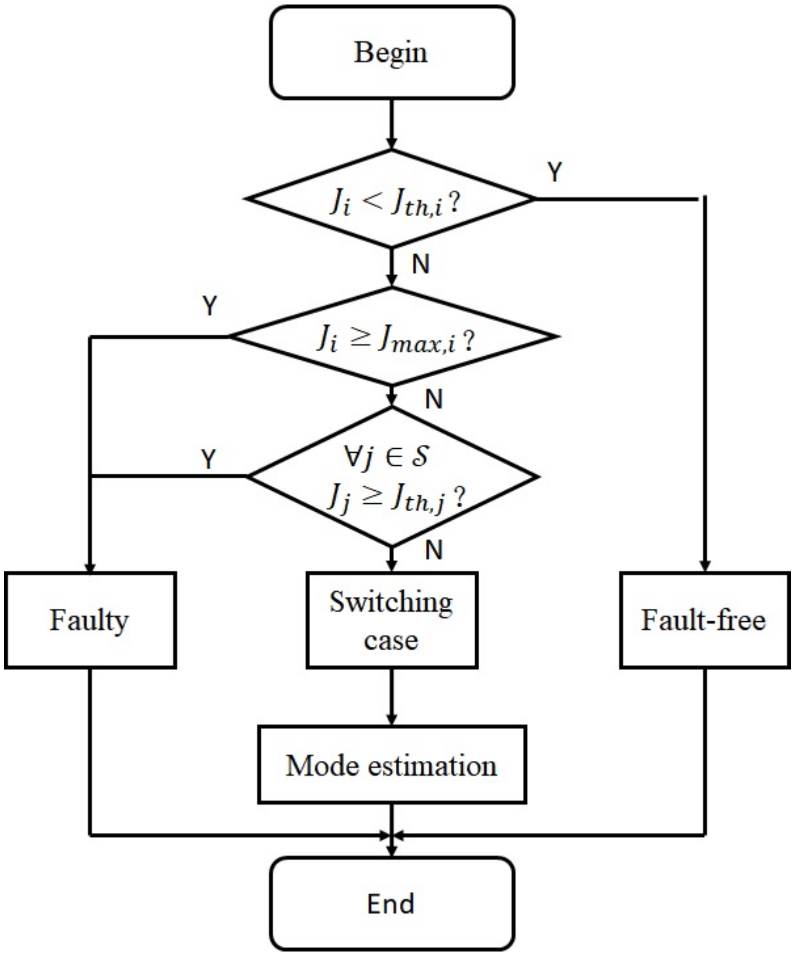

Figure 4.7: Decision rule.

4.4 Concluding remarks

This chapter mainly focuses on an integrated design of $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD schemes for switched systems, while an embedded mode estimation unit is addressed to distinguish faulty modes and switchings.

To this end, existence conditions of $\mathcal{L}_2$ observer-based FD schemes for switched systems are studied at first. Furthermore, to improve the real-time performance of the FD scheme, an $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD system is proposed. Then, matched and unmatched periods are introduced to describe asynchronized manners in switched systems, which is the main contribution of this thesis for FD schemes on switched systems. Furthermore, an integrated design of $\mathcal{L}_\infty/\mathcal{L}_2$ type of residual generator is proposed for switched systems under ADT stability conditions. Parameters corresponding to dynamical thresholds are optimized by means of LMIs. With two thresholds for synchronized and asynchronized periods, a decision rule is set with mode estimation unit.

5 A Fault Detection Scheme for Stochastic Systems with Multiplicative Intermittent Faults

In Chapter 4, the FD scheme for switched systems with deterministic disturbance has been investigated. In this chapter, we focus on the FD schemes for stochastic switched systems. As introduced in Chapter 2, IFs appear for a period of time and then disappear, so the dynamic systems with multiplicative IFs can be treated as switched systems, and the detection of switchings can be regarded as an FD scheme. Motivated by this observation, we propose an FD approach for stochastic dynamic systems with multiplicative IFs. To this end, linear time-varying (LTV) systems and time-varying Kalman filter-based residual generation are firstly introduced as preliminaries. After that, we model stochastic dynamic systems with multiplicative IFs by means of switched systems, which is the core idea of this chapter. Then, an LR test-based FD scheme is presented, in which a randomized algorithm-aided (RA-aided) technique is used for threshold computation. Finally, the stability margin for the FD scheme is studied.

5.1 Problem Formulation

Based on the knowledge that the dynamical behaviors of a system with IFs can be described by IF active modes (faulty subsystems) and IF inactive mode (fault-free subsystem), the system dynamics with multiplicative IFs can be modeled as different subsystems. Therefore, one idea to model systems with multiplicative IFs is to regard them as switched systems, and the detection of faults is reformulated into the detection of switchings between different subsystems.

For systems with white Gaussian noises, the well-developed Kalman filter algorithm is widely used for generating residual signals, more detailed information can be found in [40, 42]. Considering the switched system which is a special type of LTV system, the time-varying Kalman filter is applied in our work for residual generation.

It is well known that the LR test is widely used to detect potential faults, and for some types of complex systems, e.g., the switched systems, the LR technique [69] can be used to distinguish different subsystems. It indicates that the LR test can be applied to detect and isolate IF active modes (faulty subsystems) in the context of detecting multiplicative IFs.

Considering that the distribution of the likelihood function cannot be described in an analytic form, we further apply the so-called RA-technique [28] for threshold setting that optimally satisfies the required FD performance.

Moreover, since the overall system dynamics is governed by a switched system, it may consequently become unstable. In order to ensure the stability of the whole system, stability conditions with dwell time in each faulty subsystem are discussed by means of MDADT stability theory. Because the MDADT method has a strict minimal average dwell time limit in each subsystem, estimating the switching instant is necessary for the FD scheme to be developed.

To this end, an LR test-based detection scheme for stochastic switched systems is investigated in this chapter with a mode estimation unit. The main tasks of this chapter are summarized as follows:

1. introducing the time-varying Kalman filter algorithm-based residual generator for LTV systems;

2. modeling stochastic dynamic systems with multiplicative IFs by switched systems;

3. investigating FD issues for stochastic dynamic systems with multiplicative IFs, including the Kalman filter-based residual generation, the LR test-based residual evaluation and RA-aided threshold setting;

4. analyzing the system stability conditions in terms of switching frequency and the time between switchings.

5.2 Kalman filter-based residual generator for LTV systems

In this section, issues of modeling of the linear time-varying (LTV) systems and the time-varying Kalman filter-based residual generator for LTV systems are addressed, which provides preliminaries for this chapter.

5.2.1 The LTV system

Consider an LTV system with white Gaussian noises as follows:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) + \omega(k), \\ y(k) = C(k)x(k) + D(k)u(k) + \upsilon(k), \end{cases} \tag{5.1}$$

where $x(k) \in R^{n_x}, u(k) \in R^{n_u}, y(k) \in R^{n_y}$ are state, input and output vector, respectively, $A(k), B(k), C(k), D(k)$ are time-varying system matrices with appropriate dimensions. It is assumed that $(C(k), A(k))$ is uniformly detectable. $\omega(k)$ and $\upsilon(k)$ correspondingly represent process and measurement noise vectors, which are the white Gaussian noise subject to Assumption 2.1, i.e.,

$$\omega(k) \sim \mathcal{N}(0, \Sigma_\omega(k)), \upsilon(k) \sim \mathcal{N}(0, \Sigma_\upsilon(k)), \tag{5.2}$$

and they are uncorrelated with the state, input vector and initial values

$$\begin{gathered} \mathcal{E}\left(\begin{bmatrix} \omega(i) \\ \upsilon(i) \\ x(0) \end{bmatrix} \begin{bmatrix} \omega(j) \\ \upsilon(j) \\ x(0) \end{bmatrix}^T \right) = \begin{bmatrix} \begin{bmatrix} \Sigma_\omega(i) & S_{\omega\upsilon}(i) \\ S_{\upsilon\omega}(i) & \Sigma_\upsilon(i) \end{bmatrix} \delta_{i,0} & 0 \\ 0 & \Pi_0 \end{bmatrix}, \\ \delta_{i,0} = \begin{cases} 1, i = j \\ 0, i \neq j \end{cases}. \end{gathered} \tag{5.3}$$

5.2.2 A time-varying Kalman filter-based residual generator

As described in [4], the time-varying Kalman filter-based residual generator for LTV systems (5.1) and (5.2) are given as follows:

$$\begin{cases} \hat{x}(k+1) = A(k)\hat{x}(k) + B(k)u(k) + K(k)(y(k) - \hat{y}(k)), \\ \hat{y}(k) = C(k)\hat{x}(k) + D(k)u(k), \\ r(k) = y(k) - \hat{y}(k), \end{cases} \tag{5.4}$$

where $\hat{x}(k)$ and $\hat{y}(k)$ represent the estimates of state and output, $r(k)$ denotes the residual signal, $K(k)$ is the Kalman filter gain matrix.

Let $P(k)$ be the co-variance matrix of estimation error, i.e., $P(k) = E[(x(k) - \hat{x}(k))(x(k) - \hat{x}(k))^T]$. The Kalman filter-based algorithm is given in a recursive form, see Algorithm 5.1.

Since $\omega(k)$ and $\upsilon(k)$ are white Gaussian noises, it is well known that the residual signal $r(k)$ delivered by Kalman filter is white and is of minimum covariance matrix. With the covariance matrix $\Sigma_r(k)$ calculated in Algorithm 5.1, the residual signal $r(k)$ is denoted by

$$r(k) = f(k) + \varepsilon(k), \varepsilon(k) \sim \mathcal{N}(0, \Sigma_r(k)), \tag{5.5}$$

where $f(k)$ represents any possible faults.

Algorithm 5.1 Recursive computation of time-varying Kalman filter

Initial value:
$\hat{x}(0) = 0, P(0) = P_0$.
Predicted state estimate:
$\hat{x}(k+1 \mid k) = A(k)\hat{x}(k \mid k-1) + B(k)u(k) + K(k)r(k)$.
Residual signal:
$r(k) = y(k) - \hat{y}(k \mid k-1)$.
Predicted output estimate:
$\hat{y}(k \mid k-1) = C(k)\hat{x}(k \mid k-1) + D(k)u(k)$.
Innovation of co-variance and Kalman filter gain:
1 : $K(k) = (A(k)P(k \mid k-1)C(k)^T + S_{\omega\upsilon})\Sigma_r^{-1}(k)$,
2 : $\Sigma_r(k) = C(k)P(k \mid k-1)C^T(k) + \Sigma_\upsilon = \mathcal{E}(r(k)r^T(k))$,
3 : $P(k+1 \mid k) = A(k)P(k \mid k-1)A^T(k) + \Sigma_\omega - K(k)\Sigma_r(k)K^T(k)$.

Remark 5.1. *It is mentioned that the LTV system is of a general form of linear dynamic systems, there are many well-known systems that can be modeled as LTV systems, such as linear parameter varying systems and networked systems. The main objective studied in this thesis, i.e., the switched system, is a typical LTV system.*

5.3 Residual generation for stochastic dynamic systems with multiplicative IFs

Modeling LTI stochastic systems with multiplicative IFs by switched systems is the core idea of the FD framework, in which the detection of switchings from the fault-free subsystem to faulty subsystems implies a successful detection of the IFs. To build this framework, switched systems and the modeling of stochastic dynamic systems with multiplicative IFs are introduced, which is followed by the residual generator design.

5.3.1 Modelling of stochastic dynamic systems with IFs

Consider a switched system

$$\mathcal{G}_\sigma : \begin{cases} x(k+1) = A_\sigma x(k) + B_\sigma u(k) + \omega_\sigma(k), \\ y(k) = C_\sigma x(k) + D_\sigma u(k) + \upsilon_\sigma(k), \end{cases} \tag{5.6}$$

where $x(k) \in R^{n_x}, u(k) \in R^{n_u}, y(k) \in R^{n_y}$ are state, input and output vector, respectively. $A_\sigma, B_\sigma, C_\sigma, D_\sigma$ are system matrices with appropriate dimensions, where $\sigma \in \mathcal{S}, \mathcal{S} = \{0, 1, 2, \cdots, m\}$ represents the switching rule.

For $\sigma = i$, the system (5.6) is an LTI system and is called the i-th subsystem. $\omega_\sigma(k)$ and $\upsilon_\sigma(k)$ represent the process and measurement noise vectors in the σ-th subsystems, respectively. The white Gaussian noises are subject to $\omega_\sigma \sim \mathcal{N}(0, \Sigma_{\omega,\sigma}), \upsilon_\sigma \sim \mathcal{N}(0, \Sigma_{\upsilon,\sigma})$.

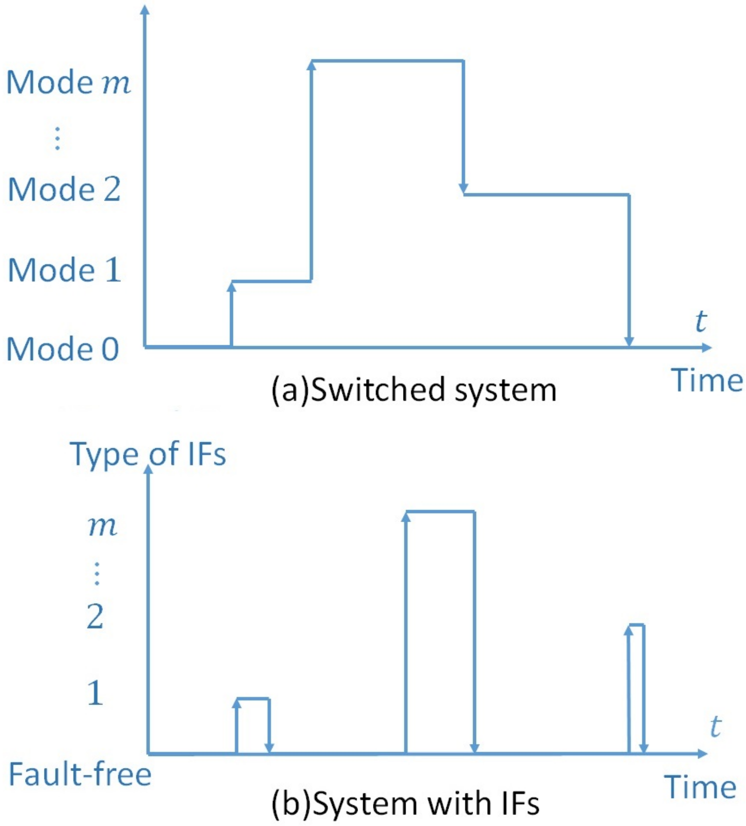

Figure 5.1: (a) Switched system and (b) a stochastic system with multiplicative IFs.

Modeling stochastic systems with multiplicative IFs by means of switched systems is the core idea of our work, different subsystems are utilized to describe both inactive mode (fault-free subsystem) and active modes (faulty subsystems). The stochastic system with multiplicative IFs can be modeled by switched system (5.6), $\sigma = 0$ represents the inactive mode (fault-free subsystem) and $\sigma = 1, 2, \cdots, m \in \mathcal{S}$, represent the modes of different types of multiplicative IFs.

Different from the switched system described in Chapter 4 (see Fig. 5.1(a)), the system with multiplicative IFs we considered in this chapter, for instance, as shown in Fig. 5.1(b), is firstly running in fault-free subsystem, then the 1-st, m-th and 2-nd IFs are successively performed. From the figure we also can see that different types of IFs have different lasting time, so the detection scheme for IFs should include the detection of both IFs' appearance and disappearance.

5.3.2 Kalman filter-based residual generator

Because the stochastic system with multiplicative IFs is viewed as a special LTV system, a bank of time-varying Kalman filters are applied as residual generators for stochastic dynamic systems with multiplicative IFs.

In this subsection, we adopt the following Kalman filter-based residual generators

$$\begin{cases} \hat{x}_{\hat{\sigma}}(k+1) = A_{\hat{\sigma}}\hat{x}_{\hat{\sigma}}(k) + B_{\hat{\sigma}}u(k) + K_{\hat{\sigma}}(k)(y(k) - \hat{y}_{\hat{\sigma}}(k)), \\ \hat{y}_{\hat{\sigma}}(k) = C_{\hat{\sigma}}\hat{x}_{\hat{\sigma}}(k) + D_{\hat{\sigma}}u(k), \\ r_{\hat{\sigma}}(k) = y(k) - \hat{y}_{\hat{\sigma}}(k), \end{cases} \tag{5.7}$$

where $\hat{\sigma} \in \mathcal{S}$ represents the switching rule of the Kalman filter-based residual generator, and $\hat{x}_{\hat{\sigma}}(k) \in \mathcal{R}^{n_x}$ denotes the estimation of state $x(k)$ for the $\hat{\sigma}$-th residual generator. $\hat{y}_{\hat{\sigma}}(k) \in R^{n_y}$ is the estimation of output $y(k)$, $r_{\hat{\sigma}}(k) \in R^{n_y}$ represents the residual signal. $K_{\hat{\sigma}}$ is the Kalman gain.

In the switched system (5.6) and Kalman filter (5.7), when $\sigma = \hat{\sigma}$, the system and Kalman filter are in synchronous mode, while $\sigma \neq \hat{\sigma}$ represents the asynchronous mode.

To calculate the Kalman filter gain matrices $K_{\hat{\sigma}}$, we can apply Algorithm 5.1 with $A(k) = A_{\hat{\sigma}}, B(k) = B_{\hat{\sigma}}, C(k) = C_{\hat{\sigma}}, D(k) = D_{\hat{\sigma}}, K(k) = K_{\hat{\sigma}}$. For each subsystem, Kalman filter-based residual generator delivers a residual signal $r_{\hat{\sigma}}$, which is of minimum covariance matrix $\Sigma_{\hat{\sigma}}$. The whiteness of the residual signals allows them to be applicable for FD purpose, which will be discussed in the following section.

5.4 LR test-based FD approach for stochastic dynamic systems with multiplicative IFs

LR methods are very popular methods in dealing with the change detection problems, and they are also widely applied in FD schemes. From the Neyman-Pearson Lemma, we know that, if the likelihood function of the fault f is known, the use of LR gives an optimal FD solution at the significant level α for a given threshold, which has with the best fault detectability. Motivated by this observation, in this section, we will study LR test-based FD schemes for stochastic dynamic systems with multiplicative IFs.

5.4.1 Problem formulation

It is known that if the process and measurement noises are white Gaussian noises, likelihood functions can be chosen as probability density functions (PDFs) of concerned residuals.

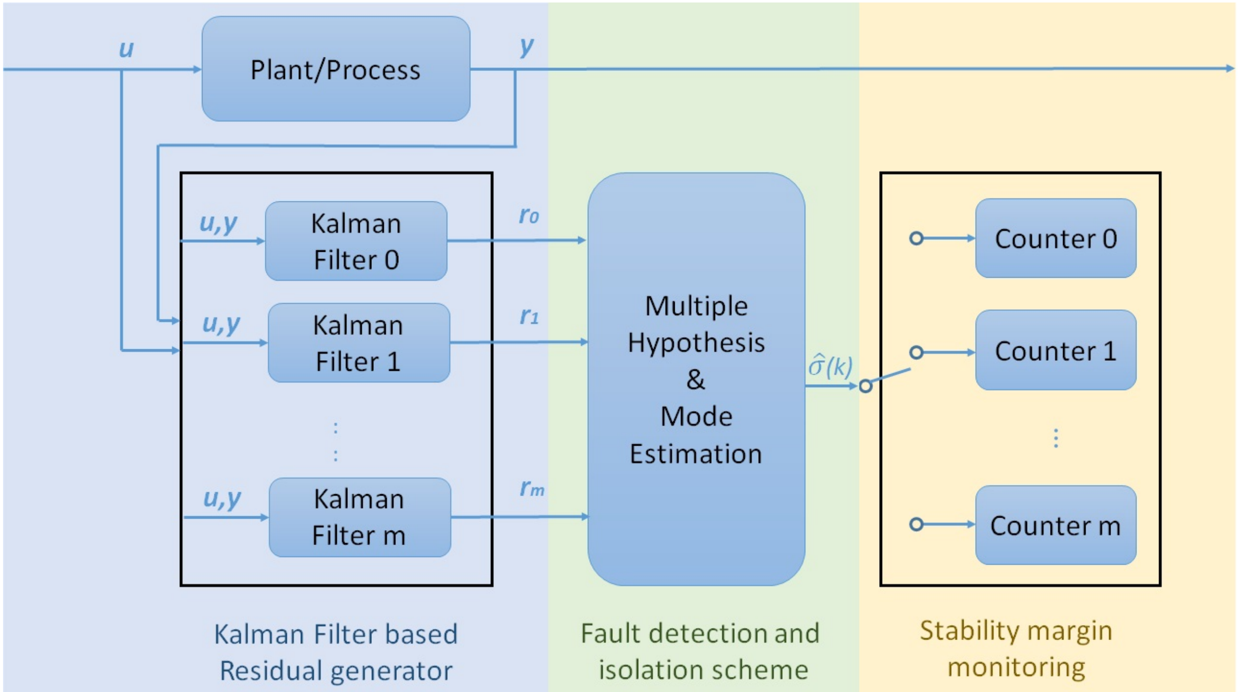

Figure 5.2: FD scheme for stochastic dynamic systems with multiplicative IFs.

Therefore, for stochastic dynamic systems with multiplicative IFs (5.6), with the residuals generated by a bank of time-varying Kalman filters (5.7), LRs for the null hypothesis (fault-free subsystem) and alternative hypothesis (faulty subsystems) are defined.

In order to express the FD scheme more clearly, a binary case in which there exist only two subsystems is studied first, and it is followed by RA-aided thresholds computation. Then, the FD scheme for systems with multiple multiplicative IFs is addressed.

Since the system dynamics are governed by a switched system, whose stability can be affected by frequent switchings, stability analysis for the FD system is necessary. Considering the fact that the MDADT method indicates that switching frequency and dwell time in each subsystem can influence the stability of the switched system, a counter for each subsystem is set for recording switching instants.

As a result, the FD scheme includes three parts:

1. residual generation: by means of a bank of Kalman filter-based residual generators;
2. detection and isolation scheme: by investigating an FD approach for switched systems

embedded with a mode estimation unit;

3. stability condition monitoring unit: by recording switchings.

The FD scheme is illustrated in Fig. 5.2.

5.4.2 LR test-based FD scheme in binary case

In this subsection, an LR test-based FD scheme is investigated. To present the LR test-based FD scheme clearly, we first simplify the discussion and consider the situation that there exist two known subsystems with $\sigma \in \{0, 1\}$. This detection of the binary case is viewed as the basis of the FD scheme for switched systems with multiple subsystems. In the binary case, we suppose that the system is running in one subsystem, and our task is to propose an LR test-based FD approach for a system consisting of the 0-th mode (fault-free subsystem) and the 1-st mode (IF subsystem).

Likelihood functions

In the time interval $[k_0, k_N]$, suppose that the switched system is running in the 0-th mode, and switches to the 1-st mode at $k_s \in [k_0, k_N]$. Define the binary hypothesis as

$$\begin{aligned} &H_0 : \mathcal{G} = \mathcal{G}_0, k \in [k_0, k_N], \\ &H_1 : \mathcal{G} = \begin{cases} \mathcal{G}_0, k \in [k_0, k_s - 1), \\ \mathcal{G}_1, k \in [k_s, k_N]. \end{cases} \end{aligned} \tag{5.8}$$

The null hypothesis means there is no switching in the time interval $[k_0, k_N]$, while the alternative hypothesis represents the case that the system switches from the 0-th subsystem to the 1-st at k_s.

Based on the switched system (5.6) and Kalman filter-based residual generator (5.7), the residual signal r_σ is generated. In binary case $\sigma = 0, 1$, the white Gaussian noises are $\omega_\sigma \sim \mathcal{N}(0, \Sigma_{\omega,\sigma}), \upsilon_\sigma \sim \mathcal{N}(0, \Sigma_{\upsilon,\sigma})$. When the process under consideration is stationary, the Kalman filter gain K_σ and covariance Σ_σ of residual signal r_σ are given by solving the following Riccati equation

$$K_\sigma = A_\sigma P_\sigma C_\sigma^T (\Sigma_{\upsilon,\sigma} + C_\sigma P_\sigma C_\sigma^T)^{-1}, \tag{5.9}$$

$$P_\sigma = A_\sigma P_\sigma A_\sigma^T - L(\Sigma_{\upsilon,\sigma} + C_\sigma P_\sigma C_\sigma^T) K_\sigma^T + \Sigma_{\omega,\sigma}, \tag{5.10}$$

$$\Sigma_\sigma = \Sigma_{\upsilon,\sigma} + C_\sigma P_\sigma C_\sigma^T. \tag{5.11}$$

With residuals r_σ delivered by Kalman filter-based residual generators and their covariance matrices Σ_σ, the hypotheses H_0 and H_1 are expressed as

$$\begin{aligned} &H_0 : r = r_0, k \in [k_0, k_N], \\ &H_1 : r = \begin{cases} r_0, k \in [k_0, k_s - 1), \\ r_1, k \in [k_s, k_N). \end{cases} \end{aligned} \tag{5.12}$$

It is clear from (5.12) that under the H_1 hypothesis, both of the mean and covariance matrices of the residual signal have changed after k_s. Therefore, our tasks is to design a detection scheme which can: (1) detect the switching instant; (2) detect changes both in mean and covariance matrix. Based on the preliminaries given in Chapter 2, the LR-based method is applied.

For switched systems with white Gaussian noises, the PDF of the residual signal $p_{\theta_i}(r_i(k))$ is given by

$$p_{\theta_i}(r_i(k)) = \frac{1}{\sqrt{(2\pi)^{n_r}\det(\Sigma_i)}} e^{-\frac{1}{2}r_i^T(k)\Sigma_i^{-1}r_i(k)}. \tag{5.13}$$

According to the log-LR calculated by PDF (5.13), in the time interval $[k_0, k_N]$, we can define LR as test statistic $J_{0,1}(k)$ by

$$\begin{aligned} J_{0,1}(k) &:= \ln \frac{\prod_{k=k_0}^{k_s-1} p_{\theta_0}(r_0(k)) \prod_{k=k_s}^{k_N} p_{\theta_1}(r_1(k))}{\prod_{k=k_0}^{k_N} p_{\theta_0}(r_0(k))} \\ &= \ln \frac{\prod_{k=k_s}^{k_N} p_{\theta_1}(r_1(k))}{\prod_{k=k_s}^{k_N} p_{\theta_0}(r_0(k))} \\ &= \ln \frac{\prod_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_1)}} e^{-\frac{1}{2}r_1^T(k)\Sigma_1^{-1}r_1(k)}}{\prod_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_0)}} e^{-\frac{1}{2}r_0^T(k)\Sigma_0^{-1}r_0(k)}}. \end{aligned} \tag{5.14}$$

An RA-based threshold computing

From (5.14) we see that the switching instant $\hat{k}_s$ should be estimated first, and it can be estimated by the maximum value of the product of likelihood functions in both subsystems as follows:

$$\hat{k}_s = arg\max_{k_s} \ln \prod_{k=k_0}^{k_s-1} p_{\theta_0}(r_0(k)) \prod_{k=k_s}^{k_N} p_{\theta_1}(r_1(k)). \tag{5.15}$$

In addition to computing the LR for FD design, we need to determine the corresponding threshold afterwards. With the $\hat{k}_s$ solved from (5.15), LR $J_{0,1}$ in binary case is expressed as

$$J_{0,1}(k) = \frac{1}{2}\sum_{k=\hat{k}_s}^{k_N} \left[\ln \frac{\det(\Sigma_0)}{\det(\Sigma_1)} + r_0^T(k)\Sigma_0^{-1}r_0(k) - r_1^T(k)\Sigma_1^{-1}r_1(k) \right]. \tag{5.16}$$

It is observed from (5.16) that the evaluation $J_{0,1}$ consists of two parts which follow different probability distribution:

1. $\ln \dfrac{\det(\Sigma_0)}{\det(\Sigma_1)}$, which is a constant that can be calculated;

2. $r_0^T(k)\Sigma_0^{-1}r_0(k) - r_1^T(k)\Sigma_1^{-1}r_1(k)$, in which r_0, r_1 are subject to two different norm distribution.

In general, there exists no analytical form to compute FAR for the probability distribution of $J_{0,1}$. To solve this problem, the RA technique is applied to evaluate $J_{0,1}$ in this thesis.

In [28], the so-called RA technique has been proposed for threshold setting with FAR requirement being satisfied. And here we propose to apply the RA method for threshold setting, which can be briefly summarized as: (1) generating $\bar{N}$ independent identically distributed samples $x(1), x(2), ...x(\bar{N})$ for $s(x)$ with $\theta = \theta_0$, (2) determining s_{max} and the threshold J_{th} according to

$$s_{max} = \max\left\{s(x(1)), \cdots, s(x(\bar{N}))\right\}, J_{th} = s_{max}. \tag{5.17}$$

As introduced in [79], it is known that for any given $\delta \in (0,1), \varepsilon \in (0,1)$, if there exists

$$\bar{N} \geq \frac{\log \dfrac{1}{\delta}}{\log \dfrac{1}{1-\varepsilon}}, \tag{5.18}$$

then with a probability greater than $1-\delta$, it holds

$$\text{Prob}\{s(x) \leq J_{th} = s_{\max} \mid \theta = \theta_0\} \geq 1 - \varepsilon, \tag{5.19}$$

$$\Longleftrightarrow \text{Prob}\{s(x) > J_{th} \mid \theta = \theta_0\} \leq \varepsilon, \tag{5.20}$$

the probability $1-\delta$ is the so-called confident level.

Therefore, given the estimate of switching instant $\hat{k}_s$, the RA-aided threshold is calculated by generating $\bar{N}$ independent data $\{r_0\}_i = \{r_0(\hat{k}_s), \cdots, r_0(k_N)\}_i$ and $\{r_1\}_i = \{r_1(\hat{k}_s), \cdots, r_1(k_N)\}_i, i = 1, \cdots, \bar{N}$, when $\mathcal{G} = \mathcal{G}_0$. Notice that the data used for computation is in the time interval $[\hat{k}_s, k_N]$. The threshold can be chosen as the maximum of the $J_{0,1}|_i$, which can be expressed as

$$J_{th,0,1}|_{\hat{k}_s} = \max_{i=1,\cdots,\bar{N}} J_{0,1}|_{i,\hat{k}_s} \tag{5.21}$$

For defined hypothesis (5.12), the detection scheme can be achieved with the LR by a moving $k = k_0+1, k_0+2, ..., k_N$. Furthermore, corresponding thresholds can be determined with a bank of thresholds corresponding to every possible switching instants $\hat{k}_s \in [k_0+1, k_N]$. In this regard, each threshold value is computed by applying the RA-aided technique.

For each possible $\hat{k}_s$, the RA-aided threshold of (5.16) are determined by generating $\bar{N}$ independent data sets, and with these $\bar{N}$ sets of data $\{r_0\}_i, \{r_1\}_i$, the LR $J_{0,1}|_{i,\hat{k}_s}$ is computed by (5.16). Here, the data used for computing thresholds $J_{th,0,1}|_n, n = k_0+1, k_0+2, \ldots, \hat{k}_s$ are in the time intervals $[k_0+1, k_N], [k_0+2, k_N], \ldots, [\hat{k}_s, k_N]$, respectively.

As a result, the threshold $J_{th,0,1}(k)$ is set dynamical form, which is expressed as

$$J_{th,0,1}(k) = \max_{i=1,\cdots,\bar{N}} J_{0,1}|_{i,k}. \tag{5.22}$$

With a probability (confident level) larger than $1-\delta$, there is

$$\text{Prob}\{J_{0,1}(k) > J_{th,0,1}(k) \mid H_0\} \leq \varepsilon. \tag{5.23}$$

In conclusion, the RA-aided threshold values for every possible $\hat{k}_s$ is set offline, and it is described as the following Algorithm.

Algorithm 5.2 RA-aided threshold compution

Give probabilities $\varepsilon \in (0,1), \delta \in (0,1)$.

1. Compute $\bar{N}$ by (5.18).
2. Do loop statement:

   for $n = k_0 + 1 : 1 : k_N$

   (1) generate $\bar{N}$ sets of data $\{r_0(n), \cdots, r_0(k_N)\}_i, \{r_1(n), \cdots, r_1(k_N)\}_i$, $i = 1, \cdots, \bar{N}$, by (5.7) with $\mathcal{G} = \mathcal{G}_0$;

   (2) calculate LRs $J_{0,1}|_{i,n}$ by (5.16);

   (3) calculate threshold values at every possible $k \in [k_0, k_N]$ $J_{th,0,1}|_k$ by (5.21)

   end.
3. Set the threshold as $J_{th,0,1}(k) = \max_{i=1,\cdots,\bar{N}} J_{0,1}|_{i,k}$.

Based on the threshold values determined by Algorithm 5.2, LR test-based FD scheme with a mode estimation unit for stochastic switched systems is given in Algorithm 5.3.

Algorithm 5.3 An LR test-based FD scheme for binary case

1. Generate residuals $r_0(k), r_1(k)$ by Kalman filter-based residual generator (5.7).
2. Compute LR $J_{0,1}(k)$ (5.14) with residuals $r_0(k), r_1(k)$.
3. Estimate switching instant $\hat{k}_s$ by (5.15).
4. With the threshold $J_{th,0,1}(k)$ obtained by Algorithm 5.2, set the decision rule as
$$\begin{cases} J_{0,1}(k) < J_{th,0,1}(k) \Rightarrow \text{fault-free case}, \\ J_{0,1}(k) \geq J_{th,0,1}(k) \Rightarrow \text{faulty case}. \end{cases}$$

Remark 5.2. *The FD scheme for the case from the* 1*-st mode (IF subsystem) to the* 0*-th mode (fault-free subsystem) has the similar process as that given in Algorithm 5.2. In this case, to compute the RA-aided threshold values, the residuals r_0, r_1 should be generated when $\mathcal{G} = \mathcal{G}_1$. Then, the corresponding $J_{th,1,0}(k)$ is obtained with LR $J_{1,0}$.*

5.4.3 LR test-based FD scheme in multiple case

Based on the FD scheme in the binary case, in this subsection, the LR test-based FD scheme for stochastic dynamic systems with multiple multiplicative IFs (faulty subsystems) is studied. To this end, by means of Kalman filter-based residual generators, the residual signals of each subsystem are generated, and likelihood functions for each subsystem are computed. Then, the LR test is utilized and the mode estimation and the switchings instant estimation processes are given in the end.

Notice that, in this thesis, we only consider the case that the switchings are between the fault-free mode (the 0-th subsystem) and an IF mode (the i-th subsystem), $i \in \mathcal{S}$. The situation that the system switches from an IF mode to another type of IF is not discussed.

Kalman filter-based likelihood functions

For the switched system with multiple subsystems, suppose that the system is running over the time interval $[k_0, k_N]$, and it is in the 0-th subsystem (fault-free mode) at $k = k_0$. All Kalman filter-based residual generators for each subsystem is running in parallel, and all Kalman filter-based residual generators are activated.

In the IF active mode, changes happen in both expectation and covariance of the initial values, so the common FD scheme based on the χ^2 test cannot be applied in this situation. Considering the switched system (5.6), for $\sigma(k) \in \mathcal{S}$, the Kalman filter gain matrices are calculated by Algorithm 5.1.

Then, with the residual signal calculated for each subsystems $r_{\sigma(k)}(k)$ and the covariance matrix Σ_{r_σ}, the PDF of the residual signal for each subsystem is represented as

$$\mathcal{L}(\theta | r_{\sigma(k)}) = p_\theta(r_\sigma(k)) = \frac{1}{\sqrt{(2\pi)^{n_{r_\sigma}} det(\Sigma_\sigma)}} e^{-\frac{1}{2} r_\sigma(k)^T \Sigma_\sigma^{-1} r_\sigma(k)}. \tag{5.24}$$

An LR test-based FD scheme for stochastic dynamic systems with multiplicative IFs

With the likelihood functions for each subsystem (5.24), an LR test-based FD scheme is investigated. First, we introduce the multiple hypothesis put forward by Basseville and Nikiforov [3] to describe different subsystems in a stochastic switched system.

Suppose that the switching is from the 0-th to the i-th subsystem at $k = k_s, i \in \mathcal{S}$, the switching instant is presented as

$$\sigma(k_s - 1) = 0, \sigma(k_s) = i \neq 0. \tag{5.25}$$

Similar to the hypothesis condition in binary case (5.8), for given system $\mathcal{G}$, the multiple hypothesis is defined as

$$\begin{aligned}&H_0 : \mathcal{G} = \mathcal{G}_i, k = k_0, \cdots, k_N,\\ &H_i : \begin{cases} \mathcal{G} = \mathcal{G}_0, k = k_0, \cdots, k_s - 1,\\ \mathcal{G} = \mathcal{G}_i, k = k_s, \cdots, k_N.\end{cases}\end{aligned} \tag{5.26}$$

Here the null hypothesis suggests that no switchings occur in time interval $[k_0, k_N]$, while the alternative hypothesis is that the system switches from the 0-th subsystem to the i-th subsystem at $k = k_s$.

According to the hypothesis (5.26), the evaluation functions $J_{0,i}, i \in \mathcal{S}$ for each subsystem can be defined as

$$\begin{aligned} J_{0,i}(k) &:= \ln \frac{\prod_{k=k_0}^{k_s-1} \mathcal{L}(\theta_0 \mid r_0) \prod_{k=k_s}^{k_N} \mathcal{L}(\theta_i \mid r_i)}{\prod_{k=k_0}^{k_N} \mathcal{L}(\theta_0 \mid r_0)}\\ &= \ln \frac{\prod_{k=k_s}^{k_N} \mathcal{L}(\theta_i \mid r_i)}{\prod_{k=k_s}^{k_N} \mathcal{L}(\theta_0 \mid r_0)}. \end{aligned} \tag{5.27}$$

We consider the case that the system switches from the 0-th subsystem into the i-th subsystem, and residual signals $r_0(k), r_i(k)$ generated by Kalman filter based-residual generators (5.7). With log-LRs, evaluation functions are described as:

$$\begin{aligned} J_{0,i}(k) &= \ln \frac{\prod_{k=k_0}^{k_s-1} \frac{1}{\sqrt{\det(\Sigma_0)}} e^{-\frac{1}{2} r_0(k) \Sigma_0^{-1} r_0(k)} \prod_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_i)}} e^{-\frac{1}{2} r_i(k) \Sigma_i^{-1} r_i(k)}}{\prod_{k=k_0}^{k_N} \frac{1}{\sqrt{\det(\Sigma_0)}} e^{-\frac{1}{2} r_0(k) \Sigma_0^{-1} r_0(k)}}\\ &= \ln \frac{\prod_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_i)}} e^{-\frac{1}{2} r_i(k) \Sigma_i^{-1} r_i(k)}}{\prod_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_0)}} e^{-\frac{1}{2} r_0(k) \Sigma_0^{-1} r_0(k)}}, i \in \mathcal{S}. \end{aligned} \tag{5.28}$$

For FD purposes, all Kalman filter-based residual generators are running and evaluation functions $J_{0,i}$ are calculated in parallel. We should determine which mode the system switches into, and whether an unknown fault occurs or not.

Motivated by this, we need to estimate switching instants $k_{s,m}, m \in \mathcal{S}$ for all subsystems satisfying $J_{i,m} \geq J_{th,i,m}$, and decide the switching mode $\hat{\sigma}_{es}$ with $k_{s,m}$. Then, set the estimate of switching instant as $\hat{k}_s = k_{s,\hat{\sigma}_{es}}$. With $k_{s,m}$ and the switching mode $\hat{\sigma}_{es}$, the evaluation function $J_{i,\hat{\sigma}_{es}}$ is calculated. By applying RA-aided threshold $J_{th,i,\hat{\sigma}_{es}}$, we can determine that whether it is a switching case or not.

Firstly, we should estimate the switching instant $k_{s,m}$ for all possible modes, here we use the m-th subsystem to present this process. With the evaluation function (5.27), the estimation of switching instant $k_{s,m}$ can be calculated as

$$\hat{k}_{s,m} = arg \max_{k_s} \ln \prod_{k=k_0}^{k_s-1} \mathcal{L}(\theta_0 \mid r_0) \prod_{k=k_s}^{k_N} \mathcal{L}(\theta_m \mid r_m). \tag{5.29}$$

Based on the residual signals generated by Kalman filters, the estimate of $\hat{k}_{s,m}$ is given by

$$\begin{aligned}\hat{k}_{s,m} &= arg\max\nolimits_{k_s} \ln \prod\nolimits_{k=k_0}^{k_s-1} \frac{1}{\sqrt{\det(\Sigma_0)}} e^{-\frac{1}{2}r_0^T(k)\Sigma_0^{-1}r_0(k)} \prod\nolimits_{k=k_s}^{k_N} \frac{1}{\sqrt{\det(\Sigma_m)}} e^{-\frac{1}{2}r_m^T(k)\Sigma_m^{-1}r_m(k)} \\ &= arg\max\nolimits_{k_s} J_{0,m}(k) \\ &= arg\max\nolimits_{k_s} \sum\nolimits_{k=k_s}^{k_N} \left(\ln \frac{\det(\Sigma_0)}{\det(\Sigma_m)} + r_0^T(k)\Sigma_0^{-1}r_0(k) - r_m^T(k)\Sigma_m^{-1}r_m(k) \right).\end{aligned} \tag{5.30}$$

If there exist more than one subsystems satisfying $J_{0,m} < J_{th,0,m}$ and suppose that two of them are $p, q \in \mathcal{S}$, the inequality

$$J_{0,p}(k) > J_{0,q}(k) \Rightarrow \ln \frac{\prod_{k=\hat{k}_{s,q}}^{k_N} \frac{1}{\sqrt{\det(\Sigma_q)}} e^{-\frac{1}{2}r_q(k)\Sigma_q^{-1}r_q(k)}}{\prod_{k=\hat{k}_{s,p}}^{k_N} \frac{1}{\sqrt{\det(\Sigma_p)}} e^{-\frac{1}{2}r_p(k)\Sigma_p^{-1}r_p(k)}} > 0, \tag{5.31}$$

indicates that if $J_{0,p}(k) > J_{0,q}(k)$, the probability of the system running in the p-th subsystem is higher than that in the q-th subsystem. To calculate the estimate of mode $\hat{\sigma}_{es}$, we need to find the mode $m \in \mathcal{S}$ with largest probability, which means to find the mode with largest $J_{0,m}(k)$. The process to compute $\hat{\sigma}_{es}$ can be expressed by

$$\begin{aligned}\hat{\sigma}_{es} &= \arg\max\nolimits_m J_{0,m} \\ &= \arg\max\nolimits_m \sum\nolimits_{k=\hat{k}_{s,m}}^{k_N} \left[\ln \frac{\det(\Sigma_0)}{\det(\Sigma_m)} + r_0(k)^T\Sigma_0^{-1}r_0(k) - r_m(k)^T\Sigma_m^{-1}r_m(k) \right],\end{aligned} \tag{5.32}$$

with which we can determine the system is in the $\hat{\sigma}_{es}$-th subsystem and set $\hat{k}_s = \hat{k}_{s,\hat{\sigma}_{es}}$.

The evaluation function $J_{0,\hat{\sigma}_{es}}$ is

$$J_{0,\hat{\sigma}_{es}} = \max_{\hat{k}_s} \sum_{k=\hat{k}_s}^{k_N} \left(\ln \frac{\det(\Sigma_0)}{\det(\Sigma_{\hat{\sigma}_{es}})} + r_0^T(k)\Sigma_0^{-1}r_0(k) - r_{\hat{\sigma}_{es}}^T(k)\Sigma_{\hat{\sigma}_{es}}^{-1}r_{\hat{\sigma}_{es}}(k) \right). \tag{5.33}$$

As given in (5.22), for a given accuracy ε and confident level $1-\delta$, thresholds $J_{th,i,\hat{\sigma}_{es}}$ for stochastic dynamic systems are calculated by RA technique, see Algorithm 5.2 and Algorithm 5.3.

The FD scheme for stochastic switched systems with a mode estimation unit is summarized in Algorithm 5.4.

Algorithm 5.4 FD scheme for stochastic switched systems with multiple subsystems

Kalman filter-based residual generation.

1: For $\sigma = 0$, run Kalman filter (5.7) and generate residual $r_0(k)$.

2: For $\sigma = i, i \in \mathcal{S}$, active all Kalman filters (5.7) with initial values $x(0), \hat{x}(0)$, then generate $r_i(k)$.

The LR test-based FD scheme.

3: Construct likelihood functions $\mathcal{L}(\theta_\sigma | r_\sigma)$ for all $\sigma \in \mathcal{S}$.

4: Set evaluation functions $J_{0,i}$ and compute all switching instants $\hat{k}_{s,m}, m \neq i, m \in \mathcal{S}$ as (5.30).

5: Estimate the mode estimation as (5.32).

6: With $\hat{k}_s = \hat{k}_{s,\hat{\sigma}_{es}}$, set evaluation function $J_{0,\hat{\sigma}_{es}}(k)$ as (5.33)

7: Run Algorithm 5.2 and obtain thresholds $J_{th,0,\hat{\sigma}_{es}}(k)$.

8: Set the decision rule

$$\begin{cases} J_{0,\hat{\sigma}_{es}}(k) < J_{th,0,\hat{\sigma}_{es}}(k) \Rightarrow \text{fault-free case}, \\ J_{0,\hat{\sigma}_{es}}(k) \geq J_{th,0,\hat{\sigma}_{es}}(k) \Rightarrow \text{switching to the } \hat{\sigma}_{es}\text{-th subsystem.} \end{cases}$$

Under the circumstance, the evaluation function $J_{0,i}$ provides the maximal faulty detectability for a threshold $J_{th,0,i}$, and with a probability (confident level) greater than $1 - \delta$, there is

$$p = Prob\{J_{0,i} > J_{th,0,i} \mid \mathcal{G} = \mathcal{G}_0\} = \varepsilon.$$

Remark 5.3. *In this thesis, we only consider the situation that the switchings happen between fault-free mode (the* 0*-th subsystem) and one IF mode (the* i*-th subsystem). Therefore, when the system is running in the* i*-th subsystem, the task of the FD scheme is to detect the switching happening to a certain mode (*0*-th subsystem) and estimate the switching instant. The detection scheme is simplified, and can be regarded as the binary case, so it has a similar process as that shown in Algorithm 5.2.*

In this case, to compute the RA-aided threshold values, the residuals r_0, r_i *should be generated when* $\mathcal{G} = \mathcal{G}_i$*. Then, the corresponding* $J_{th,i,0}(k)$ *is obtained by calculating LR* $J_{i,0}$*.*

5.5 Stability analysis of the LR test-based FD system

As mentioned in Chapter 3, the time interval of the detection scheme may influence the stability condition of the switched system. Therefore, it is necessary to analyze the stability condition of the FD approach proposed in Section 5.4, and the time limit conditions for the

FD scheme should be discussed. To this end, the stability conditions of the FD approach for stochastic switched systems in multiple case are studied by means of the MDADT method. The MDADT-based stability condition can be determined by

1. counting switchings and recording the dwell time for each subsystem;
2. comparing the dwell time in the i-th subsystem with the i-th minimum MDADT.

5.5.1 Error dynamic system

With $k_s \in [k_0, k_N]$, synchronized mode and asynchronized mode are defined as

$$\sigma = \begin{cases} \sigma(k) = \sigma(k_0), k_0 \leq k \leq k_s, \text{synchronized mode}, \\ \sigma(k_s - 1) = \sigma(k_0) \neq \sigma(k), k_s \leq k \leq k_N, \text{asynchronized mode}. \end{cases} \tag{5.34}$$

The error signal is

$$e(k) := x(k) - \hat{x}_{\hat{\sigma}(k)}(k), k \in [k_0, k_N]. \tag{5.35}$$

With (5.6) and (5.7), in asynchronized mode the estimation error is governed by

$$\begin{aligned} \begin{bmatrix} e(k+1) \\ \hat{x}(k+1) \end{bmatrix} &= A_{\sigma(k),\hat{\sigma}(k)}(k) \begin{bmatrix} e(k) \\ \hat{x}(k) \end{bmatrix} \\ &+ B_{\sigma(k),\hat{\sigma}(k)}(k) u(k) + \bar{\omega}_{\sigma(k),\hat{\sigma}(k)}(k), \end{aligned} \tag{5.36}$$

where

$$A_{\sigma(k),\hat{\sigma}(k)}(k) = \begin{bmatrix} A_{\sigma(k)} - K_{\hat{\sigma}(k)} C_{\sigma(k)} & (A_{\sigma(k)} - A_{\hat{\sigma}(k)}) - K_{\hat{\sigma}(k)}(C_{\sigma(k)} - C_{\hat{\sigma}(k)}) \\ 0 & A_{\hat{\sigma}(k)} \end{bmatrix},$$
$$B_{\sigma(k),\hat{\sigma}(k)}(k) = \begin{bmatrix} B_{\sigma(k)} - B_{\hat{\sigma}(k)} \\ B_{\hat{\sigma}(k)} \end{bmatrix}, \bar{\omega}_{\sigma(k),\sigma(k_0)}(k) = \begin{bmatrix} \omega_{\sigma(k)} - K_{\hat{\sigma}(k)} \upsilon_{\sigma(k)} \\ \omega_{\hat{\sigma}(k)} \end{bmatrix}.$$

Note that the extended form of switched system (5.36) and error $e(k)$ in (5.35), the switched system may be in asynchronized operations. Therefore, our target is to analyze stability conditions for a system with (1) time-varying characters, (2) both synchronized and asynchronized manners.

To study the stability conditions of LTV systems, we assess a condition for GUES (defined in Chapter 4) of a switched system, then the switched system is exponentially stable for any switching signal. More information about stability analysis for switched systems can be found in [54].

The following lemma given in [92] ensures the stability of the Kalman filter applied to LTV systems, which is particular useful for switched systems.

Lemma 4. *[92] Let*

$$e(k+1) = (A(k) - K(k)C(k))e(k), \tag{5.37}$$

be the estimation error dynamics and assume that $A(k)$ is invertible, $(A(k), C(k))$ is observable. Then, the Kalman filter algorithm results in

$$V(e(k+1)) - V(e(k)) \le -\gamma(k)\|e(k)\|^2, \gamma(k) > 0, \tag{5.38}$$

$$V(e(k)) = e^T(k)\Sigma(k)e(k), \Sigma(k) = P^{-1}(k \mid k-1) > 0, \tag{5.39}$$

where $P(k \mid k-1)$ is the covariance matrix calculated by Kalman algorithm, see Algorithm 5.1.

In Lemma 4, there always exists a positive parameter $\gamma(k)$ to ensure the difference of Lyapunov functions are bounded, as in (5.38). According to this, we can extend the result to the error system in asynchronized mode (5.36), which is summarized in the following theorem.

Theorem 5.1. *For the given system (5.36), if the Kalman filter is asymptotically stable, then there exist a set of functions*

$$V_{\sigma(k)}(e(k)) = e^T(k)\Sigma_{\sigma(k)}(k)e(k), \tag{5.40}$$

where

$$\Sigma_{\sigma(k)} = P^{-1}_{\sigma(k)}(k),$$
$$\lambda_{\sigma(k),min} = \max_k \lambda_{min}(P_{\sigma(k)}(k)) = \min_k \lambda^{-1}_{max}(\Sigma_{\sigma(k)}(k)),$$

the matrix $P_{\sigma(k)}$ represents the covariance matrix for the σ-th subsystem which is calculated by Algorithm 5.1, such that

$$V_{\sigma(k)}(e(k+1)) - V_{\sigma(k)}(e(k)) \le -\gamma_{\sigma(k)}\|e(k)\|^2, \forall\sigma(k) \in \mathcal{S}, \tag{5.41}$$

where $\gamma_{\sigma(k)}$ is the parameter for the σ-th subsystem calculated by (5.38), and $\gamma_{\sigma(k)} := \min_k \gamma_{\sigma(k)}(k) > 0$.

Proof: Consider the function (5.37) with

$$A(k) = A_{\sigma(k)}, C(k) = C_{\sigma(k)}, \tag{5.42}$$

and suppose the Lyapunov function (5.40), then we have

$$V_{\sigma(k)}(e(k+1)) - V_{\sigma(k)}(e(k)) \le -\gamma_{\sigma(k)}(k)\|e(k)\|^2, \forall\sigma(k) \in \mathcal{S}. \tag{5.43}$$

For all possible k, choose the minimum of $\gamma_{\sigma(k)}(k)$ as $\gamma_{\sigma(k)} := \min_k$, it holds for (5.41). The proof is completed. ∎

Since additive disturbances have no influence on the system stability, only the stability conditions of the eigen-dynamics are analyzed, while influences of input variables are not considered in this situation.

Therefore, for stability analysis, the stochastic switched system in asynchronized manners is described by

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = A_{\sigma(k),\hat{\sigma}(k)} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \tag{5.44}$$

where $A_{\sigma(k),\hat{\sigma}(k)}$ is expressed in (5.36).

Furthermore, to study the stability conditions for (5.44), we define the Lypapunov function

$$\begin{aligned} V_{\sigma(k),\hat{\sigma}(k)} &= [x^T(k) \quad e^T(k)] \begin{bmatrix} P_{\sigma(k)} & 0 \\ 0 & \Sigma_{\hat{\sigma}(k)} \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \\ &= V_{\sigma(k)}(x(k)) + V_{\hat{\sigma}(k)}(e(k)), \end{aligned} \tag{5.45}$$

where

$$\begin{aligned} V_{\sigma(k)}(x(k)) &= x^T(k) P_{\sigma(k)} x(k), \\ V_{\hat{\sigma}(k)}(e(k)) &= e^T \Sigma_{\hat{\sigma}(k)} e(k), \\ P_{\sigma(k)} > 0, \Sigma_{\hat{\sigma}(k)} &= P^{-1}_{\sigma(k)}(k). \end{aligned} \tag{5.46}$$

5.5.2 MDADT method based stability analysis

Suppose that $T_{s,\sigma}, T_{t,\sigma}$ are the total time that the system runs in the σ-th synchronized and asynchronized mode over $[k_m, k_{m+1}]$. For $q \in [k_0, k]$ the Lyapunov function $V_{\sigma(q),\hat{\sigma}(q)}(x(q), e(q))$ is expressed as

$$\begin{aligned} V_{\sigma(q),\hat{\sigma}(q)}(x(q), e(q)) &= V_{\sigma(q)}(x(q)) + V_{\hat{\sigma}(q)}(e(q)), \\ V_{\sigma(q)}(x(q)) &= x^T(q) P_{\sigma(q)} x(q), \\ V_{\hat{\sigma}(q)}(e(q)) &= e^T(q) \Sigma_{\hat{\sigma}(q)} e(q), \Sigma_{\hat{\sigma}(q)} = P^{-1}_{\hat{\sigma}(q)}(k|k-1). \end{aligned} \tag{5.47}$$

With the Lyapunov functions (5.46) and (5.47), the stability conditions are addressed for statistic switched systems in asynchronized manners as follows.

Theorem 5.2. *Considering the switched system (5.44), if there exist (1) positive constants* $0 < \alpha_i < 1, \kappa_i > 0, \mu_{i,1} > 1, \mu_{i,2} > 1, \lambda > 0, i \in \mathcal{S}$, *(2) piece-wise Lyapunov function*

$$V(k) = \begin{cases} V_{\sigma(k)}(x(k), e(k)), k \in [k_m, k_{s,m}], \\ V_{\sigma(k),\hat{\sigma}(k)}(x(k), e(k)), k \in (k_{s,m}, k_{m+1}], \end{cases} \tag{5.48}$$

and (3) β_i satisfying

$$\begin{bmatrix} P_i & 0 \\ 0 & \left(\frac{1-\kappa_i}{1-\alpha_i}\right)^{k_{s,m}-k_m} \Sigma_i(k_m) \end{bmatrix} \leq \beta_i \begin{bmatrix} P_i & 0 \\ 0 & \Sigma_i(k_m) \end{bmatrix}, \tag{5.49}$$

such that $\forall i \in \mathcal{S}$

$$\begin{aligned} &V_i(x(k+1)) \leq (1-\alpha_i)V_i(x(k)), V_i(e(k+1)) \leq (1-\kappa_i)V_i(e(k)), \\ &V_0 \leq \mu_{i,1}V_{i,0}((x(k),e(k)), V_{i,0} \leq \mu_{i,2}V_i((x(k),e(k)), \end{aligned} \tag{5.50}$$

then for any switching signal with MDADT satisfying

$$T_{s,i}^* \geq T_{s,i} = \frac{\ln \beta_i \mu_{i,1}\mu_{i,2} + \lambda T_{t,i}}{\ln(1-\alpha_i)+\lambda}, \tag{5.51}$$

the stochastic switched system (5.44) is GUES over time interval $[k_0, k]$.

Proof: If the stochastic switched system (5.44) is subject to alternative hypothesis in (5.26), and it can be regarded as in synchronized mode in the time interval $[k_m, k_{s,m}]$. For $\sigma(l) = \hat{\sigma}(l) = i, i \in \mathcal{S}$, it follows from (5.50) that

$$\begin{aligned} & V_i(x(k_{s,m}), e(k_{s,m})) = V_i(x(k_{s,m})) + V_i(k_{s,m})(e(k_{s,m})) \\ \leq\ & (1-\alpha_i)^{k_{s,m}-k_m}V_i(x(k_m)) + (1-\kappa_i)^{k_{s,m}-k_m}V_i(e(k_m)), \end{aligned} \tag{5.52}$$

with $\kappa_i = 1 - \lambda_{i,min}\gamma_i < 1$, where $\lambda_{i,min}\gamma_i$ is calculated in Theorem 5.1.

While in time interval $[k_{s,m}, k_{m+1}]$, which is also called asynchronized mode, with $\mu_{i,2} > 1$ and (5.50), we have

$$V_{i,0}(x(k_{s,m}), e(k_{s,m})) \leq \mu_{i,2}V_i(x(k_{s,m}), e(k_{s,m})). \tag{5.53}$$

Then, in asynchronized mode, with $\mu_{i,2} > 1$, the Lyapunov function for asynchronized manners is expressed as

$$V_{i,0}(x(k_{s,m}), e(k_{s,m})) \leq \mu_{i,2}[(1-\alpha_i)^{k_{s,m}-k_m}V_i(x(k_m)) + (1-\kappa_i)^{k_{s,m}-k_m}V_i(e(k_m))].$$

For switched system (5.44), for $\sigma(l) = i, \hat{\sigma}(l) = 0, i \neq j, i \in \mathcal{S}, l \in [k_{s,m}, k_{m+1}]$, suppose that

$$V_{i,0}(x(l), e(l)) = [x^T(l) \quad e^T(l)]\bar{P}_{i,0}(l) \begin{bmatrix} x(l) \\ e(l) \end{bmatrix}, \tag{5.54}$$

$$V_{i,0}(x(l+1), e(l+1)) = [x^T(l) \quad e^T(l)]\bar{A}_{i,0}^T\bar{P}_{i,0}(l+1)\bar{A}_{i,0} \begin{bmatrix} x(l) \\ e(l) \end{bmatrix}, \tag{5.55}$$

with β_i in (5.49) satisfying

$$\begin{bmatrix} P_i & 0 \\ 0 & \left(\dfrac{1-\kappa_i}{1-\alpha_i}\right)^{k_{s,m}-k_m} \Sigma_i(k_m) \end{bmatrix} \leq \beta_i \begin{bmatrix} P_i & 0 \\ 0 & \Sigma_i(k_m) \end{bmatrix}, \tag{5.56}$$

there is

$$V_{i,0}(x(k_{m+1}), e(k_{m+1})) \leq \beta_i \mu_{i,2}(1-\alpha_i)^{k_{s,m}-k_m} V_i(x(k_m, e(k_m)). \tag{5.57}$$

Since $\mu_{i,1} > 1$, it implies that

$$V_i(x(k_{m+1}), e(k_{m+1})) \leq \mu_{i,1} V_{i,0}(x(k_{m+1}), e(k_{m+1})),$$

then the above inequality yields

$$V_i(x(k_{m+1}), e(k_{m+1})) \leq \beta_i \mu_{i,1}\mu_{i,2}(1-\alpha_i)^{k_{s,m}-k_m} V_i(x(k_m, e(k_m)).$$

Consider that over the time interval $[k_0, k]$, if the system is GUES, the following inequality holds

$$\begin{aligned} &\textstyle\prod_{i=0}^{l} \beta_i \mu_{i,1}^{N_{\hat{\sigma}(k_0,k)}} \mu_{i,2}^{N_{\sigma(k_0,k)}} (1-\alpha_i)^{T_{t,m}} V_{\sigma k_0}(x(k_0), e(k_0)) \\ \leq\ & e^{-\lambda(k-k_0)} V_{\sigma(k_0)}(x(k_0), e(k_0)), \end{aligned} \tag{5.58}$$

where $N_{\sigma(k_0,k)}$ represents the switchings from synchronized modes to asynchronized modes in the time interval (k_0, k), and $N_{\hat{\sigma}(k_0,k)}$ denotes the switching from asynchronized modes to synchronized modes. Taking log of both sides of (5.58), it gives

$$\sum_{i=0}^{l} \ln \beta_i + N_{\hat{\sigma}(k_0,k)} \ln \mu_{i,1} + N_{\sigma(k_0,k)} \ln \mu_{i,2} + T_{s,i} \ln(1-\alpha_i) \leq -\lambda(k-k_0). \tag{5.59}$$

For the whole time interval $[k_0, k]$ that (5.59) holds and $k - k_0 = \sum_{i=0}^{l} T_{s,i} + T_{t,i}$, in each time period $[k_m, k_{m+1}]$ with $\ln(1-\alpha_i) < 0$, time interval dwelling in each mode $T_{s,i}$ is represented as

$$T^*_{s,i} \geq T_{s,i} = \frac{\ln \beta_i \mu_{i,1}\mu_{i,2} + \lambda T_{t,i}}{\ln(1-\alpha_i) + \lambda}. \tag{5.60}$$

The proof is completed. ■

5.6 Concluding Remarks

In this chapter, the main focus is to propose an FD scheme for stochastic dynamic systems with multiplicative IFs, in which a mode estimation unit is embedded to determine faulty modes. To this end, modeling the stochastic dynamic systems by switched systems is given first, and in this chapter, only the case of switching between fault-free mode and one faulty mode is considered, the case that switching between two different multiplicative IFs is not discussed. Then, Kalman filter-based residual generator is introduced, with the generated residuals the likelihood functions are formulated directly. It is followed by an LR test-based FD scheme for stochastic switched systems, with which the threshold is determined by the RA-aided technique. To distinguish faulty subsystems, a mode estimation unit is designed. In the end, with the aid of ADT stability theory, stability conditions for the proposed FD scheme are provided.

6 Application to benchmark processes

In this chapter, we will show the effectiveness of the FD schemes proposed in the previous chapters by applications in the two-mass-spring system and a numerical example. To be specific, a two-mass-spring system is firstly used to present the effectiveness of $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD approaches proposed in Chapter 4 for uncertain switched systems.

6.1 Case studies on a two-mass-spring system

In this section, the two-mass-spring system is introduced. Then, the schemes discussed in Chapter 4, namely $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD schemes with embedded mode estimation unit are applied in such uncertain switched systems.

6.1.1 Process description

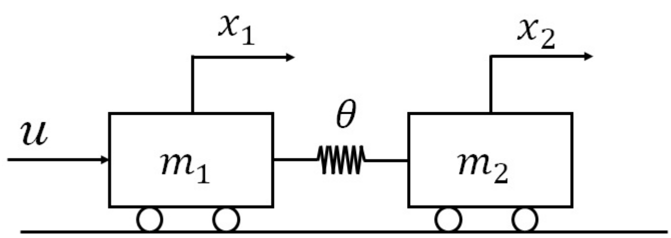

Figure 6.1: The schematic diagram of two-mass-spring system.

The two-mass-spring system (see Fig. 6.1) is a typical nonlinear system concerning a dynamical spring constant θ. To describe the real situation of the spring, the dynamical spring constant is given within $\theta \in [7.5, 15]$, which can be estimated by three spring constants in different lengths: $\theta_1 = 7.5, \theta_2 = 12, \theta_3 = 15$. If the spring elasticity coefficient changes drastically, the system output may have a large deviation. Therefore, it is necessary to identify the spring coefficient timely, then switch the observer to a certain sub-observer according to the chosen switching indicator $\hat{\theta}$.

Suppose the system state is $x = [\dot{x}_1^T \quad \dot{x}_2^T \quad x_1^T \quad x_2^T]^T$, where x_1, x_2 represent the positions of object 1 and 2, and $\dot{x}_1, \dot{x}_2$ refer to the velocity of the two objects. The output signal is the situations of two objectives which can be measured by situation sensors, denoted by $y = [x_1^T \quad x_2^T]^T$. The input signal u is the external force added on objective 1.

Based on the physical relation we have

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & 0 & \frac{\theta_i}{m_1} & -\frac{\theta_i}{m_1} \\ 0 & 0 & -\frac{\theta_i}{m_2} & \frac{\theta_i}{m_2} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} \frac{1}{m_1} \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t), \\ y(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} x(t), \end{cases} \tag{6.1}$$

where $i = 1, 2, 3$, and we set $m_1 = m_2 = 10$ in this example.

In this study, the output feedback controller (see Fig. 6.2) is implemented to stabilize the whole system. The signals are transformed into discrete-time signals through zero-order-holder (ZOH).

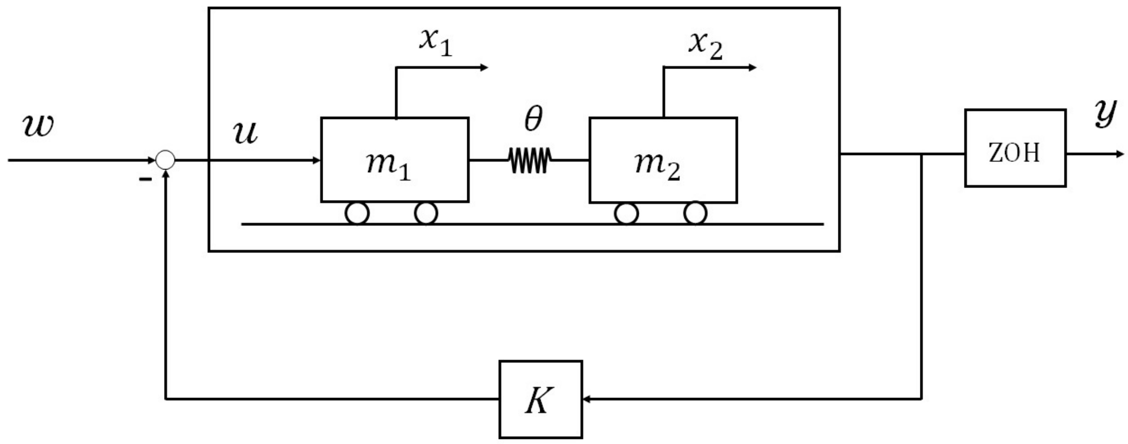

Figure 6.2: The closed-loop two-mass-spring system.

In order to monitor the two-mass-spring system, we need to connect the input and output data to a computer with typical hardware and software working platform for hardware-in-the-loop simulation. The data can be transformed into discrete-time signal. Based on the continuous-to-discrete transformation, we can get the discrete-time system matrices of the closed-loop system (see Fig. 6.2). With the spring constants θ_i $(i = 1, 2, 3)$ as the switching indicator, the two-mass-spring system can be approximated by the switched system

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k), \\ y(k) = C_i x(k), \end{cases} \tag{6.2}$$

with the following system matrices

$$A_1 = \begin{bmatrix} -10 & 96.4 & -37.2 & 86.89 \\ 0 & 0 & -0.75 & -0.75 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -5.9 & 16.4 & -14 & 17.8 \\ 0 & 0 & -1.2 & -1.2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -8.3 & 33.3 & -27.14 & 39.00 \\ 0 & 0 & -1.5 & -1.5 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, B = [0.1 \quad 0 \quad 0 \quad 0]^T,$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

To stabilize the switched system, the feedback controller gain for each subsystem can be chosen as

$$K_1 = [-100 \quad 96.44 \quad -379.5 \quad 876.4],$$
$$K_2 = [-59 \quad 166.4 \quad -153.0 \quad 190.0],$$
$$K_3 = [-83 \quad 333.0 \quad -286.4 \quad 404.9].$$

With the feedback controllers K_i $(i = 1, 2, 3)$, we can calculate the system matrices for closed-loop subsystems as

$$A_1 = \begin{bmatrix} 0.0464 & -4.7418 & 0.7705 & -4.2207 \\ 0.0003 & -0.4552 & 0.0393 & -1.0337 \\ -0.0402 & 0.9677 & -0.3559 & -0.8643 \\ -0.0086 & 0.3765 & -0.0862 & 0.3783 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.1743 & -0.9882 & -0.4920 & -2.2976 \\ -0.0377 & -0.3289 & -0.1092 & -1.3039 \\ -0.0568 & 1.0724 & -0.5093 & -0.0570 \\ -0.0376 & 0.5293 & -0.2594 & 0.2872 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.0145 & -2.6530 & 0.6764 & -3.0640 \\ 0.0007 & -0.6539 & 0.1243 & -1.3697 \\ -0.0566 & 0.5723 & -0.4549 & -0.7698 \\ -0.0226 & 0.3258 & -0.1868 & 0.0983 \end{bmatrix},$$

$$B_1 = [-0.004 \quad 0 \quad 0 \quad 0]^T,$$
$$B_2 = [-0.0057 \quad -0.0038 \quad 0.0046 \quad -0.0014]^T,$$
$$B_3 = [-0.0057 \quad -0.0023 \quad 0.001 \quad -0.0011]^T,$$
$$C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Notice that the spring constant is not measurable in practice. The spring deformation is assumed to be system uncertainties, and the unknown friction between objects is described by norm-bounded disturbance $d(k)$, denoted by $\|d\| \leq \delta_d, \delta_d > 0$. The switched system can be expressed as

$$\begin{cases} x(k+1) = (A_1 + \Delta_{A,1})x(k) + (B_1 + \Delta_{B,1})u(k) + M_d d(k) + E_f f(k), \\ y(k) = (C_1 + \Delta_{C,1})x(k) + N_d d(k) + F_f f(k), \end{cases} \tag{6.3}$$

where

$$M_d = [0.1 \quad 0.1 \quad 0.1 \quad 0.1]^T, N_d = 0.1,$$

and the uncertainties of each subsystems satisfy

$$\|\Delta_{A,i}\| \leq \epsilon_{1,i}, \|\Delta_{B,i}\| \leq \epsilon_{2,i}, \|\Delta_{C,i}\| \leq \epsilon_{3,i}.$$

6.1.2 $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme with mode estimation unit

In this subsection, the $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD scheme for uncertain switched systems is demonstrated by the two-mass-spring system. Consider the switched system (6.3) with $\epsilon_{1,i} = 10^{-7}, \epsilon_{2,i} = \epsilon_{3,i} = 10^{-6}, i = 1, 2, 3$. Set that

$$\alpha_1 = 0.3, \alpha_2 = 0.25, \alpha_3 = 0.35, \eta_1 = 1, \eta_2 = 1.1, \eta_3 = 0.9;$$
$$\mu_{1,1} = 1.5, \mu_{1,2} = 1.4, \mu_{1,3} = 1.6, \mu_{2,1} = 2, \mu_{2,2} = 2.1, \mu_{2,3} = 1.9,$$

and $\gamma = \gamma^* = 0.1, \xi = 0.1, N_0 = 1$. Applying the Algorithm 4.1, we obtain

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} 8.69 \times 10^{-6} \\ 5.36 \times 10^{-6} \\ 4.74 \times 10^{-6} \end{bmatrix}, \begin{bmatrix} \bar{\beta}_1 \\ \bar{\beta}_2 \\ \bar{\beta}_3 \end{bmatrix} = \begin{bmatrix} 5.13 \times 10^{-5} \\ 5.76 \times 10^{-5} \\ 5.95 \times 10^{-5} \end{bmatrix},$$

$$L_1 = \begin{bmatrix} 2561.0216 & -1077.4628 \\ 81.1023 & -26.9376 \\ 148.7674 & 71.2702 \\ 82.5884 & 209.4318 \end{bmatrix}, L_2 = \begin{bmatrix} 2323.5804 & 911.4842 \\ 92.4100 & -36.5026 \\ 140.0953 & 70.5025 \\ 94.2363 & 231.4496 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} 2987.5007 & 1019.1604 \\ 109.6171 & -30.5160 \\ 144.9732 & 69.6407 \\ 88.8201 & 218.5472 \end{bmatrix}.$$

The working process of the proposed FD scheme with mode estimation unit is shown in the Fig. 6.3 and Fig. 6.4.

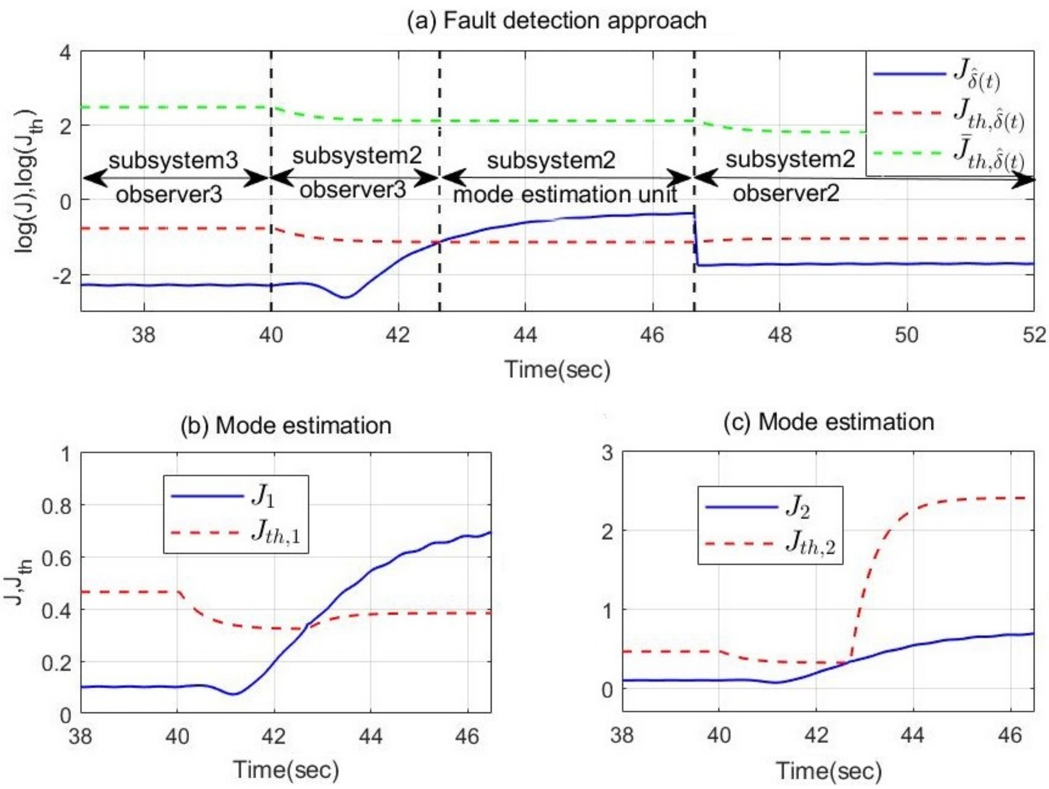

Figure 6.3: The FD scheme embedded with mode estimation unit: switching case.

For demonstration purposes, suppose the process is operating in mode 3 during $k \in [0, 40)$ sec, and the whole system switches from mode 3 into mode 2 at 40 sec. As shown in the Fig. 6.3 (b) and (c), the switching can be detected. Applying the mode estimation approach given in Chapter 4 with $\delta = 3$, it is obvious to see from the Fig. 6.3 (b) and (c), that the system is running in mode 2 after the switching. The mode estimation interval is chosen as 4 sec according to ADT theory.

Moreover, it can be seen in the time interval $k \in (40, 50)$ sec the FD scheme is verified to be effective after switching and both the whole system and the observer-based residual generator are running in a safe situation.

Furthermore, the same detection process is applied in the time interval $[68, 73)$ sec, the corresponding FD performance is shown in Fig. 6.4 (b) and (c). While activating the other two sub-observers, it is evident a fault is detected (see Fig. 6.4).

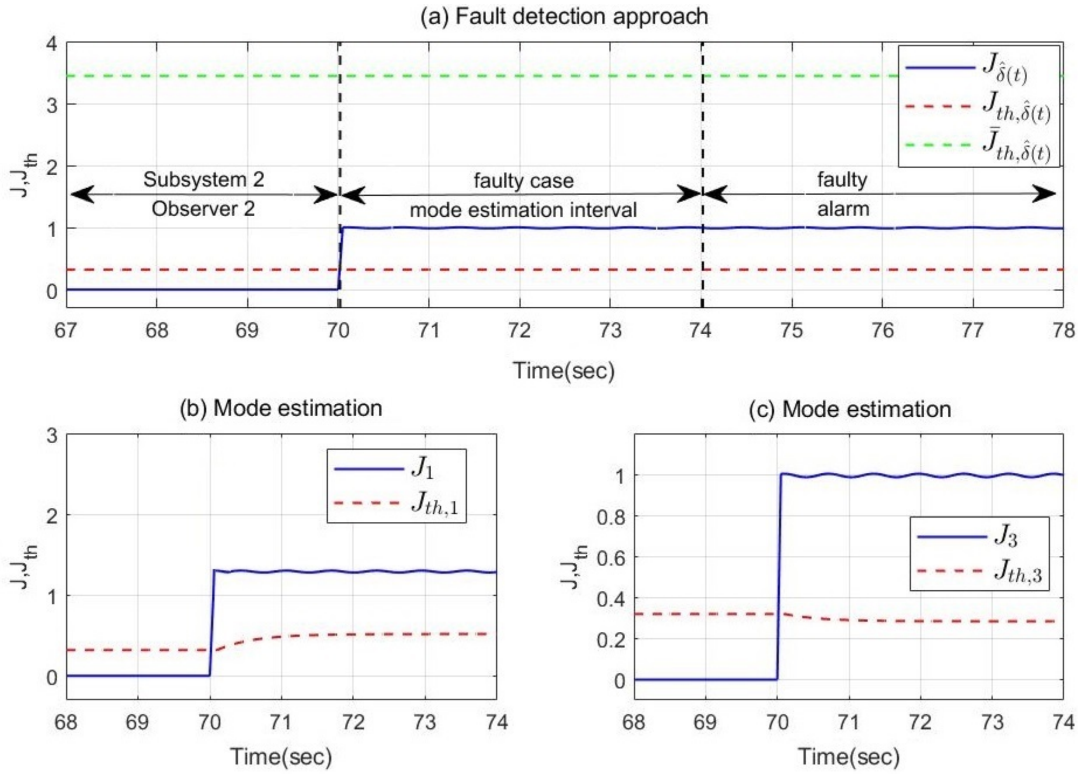

Figure 6.4: The FD scheme embedded with mode estimation unit: faulty case.

6.2 A numerical example for LR test-based FD scheme

In this subsection, a numerical example is given to demonstrate the log LR test-based detection scheme for multiplicative IFs proposed in Chapter 5.

Consider a discrete-time switched system with white Gaussian noises as follows

$$\begin{cases} x(k+1) = A_i x(k) + B_i u(k) + \omega_i(k), \\ y(k) = C_i x(k) + D_i u(k) + \upsilon_i(k), \end{cases} \tag{6.4}$$

where $x(k), u(k), y(k)$ are state, input and output vector, respectively. Notation $i, i = 0, 1, 2, 3$ is the mode of the system, the switching rule is unknown as a prior. In this example, the 0-th mode represents the fault-free subsystem, and other i-th modes ($i = 1, 2, 3$) denote three different multiplicative IFs. The detecting window is 5 samples.

The $\omega_i(k)$ and $\upsilon_i(k)$ are white Gaussian noises subject to $\omega_i \sim \mathcal{N}(0, \Sigma_{\omega,i})$, $\upsilon_i \sim \mathcal{N}(0, \Sigma_{\upsilon,i})$, respectively. The mode $i = 0$ denotes the fault-free subsystem, while $i = 1, 2, 3$ represents the i-th multiplicative IF mode, respectively. The system (6.4) is with following matrices

$$A_0 = \begin{bmatrix} -0.201 & 0.356 & -0.675 & 0.357 \\ -0.073 & -0.532 & 0.380 & 0.205 \\ 0.085 & 1.651 & 0.621 & -1.784 \\ -0.039 & 0.756 & 0.220 & 1.153 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -0.170 & 0.173 & 0.453 & 0.293 \\ 0.054 & -0.411 & 0.214 & 0.111 \\ -0.081 & 1.377 & -0.381 & -1.475 \\ -0.042 & 0.629 & -0.128 & 1.016 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.272 & 1.313 & -1.468 & 1.397 \\ 0.079 & -0.660 & 0.506 & 0.380 \\ -0.067 & 1.860 & 0.853 & 2.014 \\ 0.053 & 1.125 & -0.555 & 1.522 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.241 & -0.528 & -0.984 & -0.676 \\ -0.078 & -0.578 & 0.454 & 0.299 \\ 0.072 & 1.568 & -0.617 & 1.773 \\ 0.040 & -0.695 & 0.237 & -1.149 \end{bmatrix},$$

$$B_0 = B_1 = B_2 = B_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \\ 0 & -0.1 \\ 1 & 0 \end{bmatrix}, C_0 = C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

and the covariance of white Gaussian noises are

$$\begin{aligned} \omega_0 &= 1 \times 10^{-4} I_4, \omega_1 = 1.5 \times 10^{-4} I_4, \omega_2 = 2 \times 10^{-4} I_4, \omega_3 = 4 \times 10^{-4} I_4, \\ \upsilon_0 &= 2 \times 10^{-4} I_4, \upsilon_1 = 1.5 \times 10^{-4} I_4, \upsilon_2 = 1 \times 10^{-4} I_4, \upsilon_3 = 4 \times 10^{-4} I_4. \end{aligned}$$

The Kalman filter gain matrices are

$$K_0 = \begin{bmatrix} 0.0093 & -0.0165 \\ 0.3801 & 0.2255 \\ 0.7722 & -0.0899 \\ 0.0482 & 0.7523 \end{bmatrix}, K_1 = \begin{bmatrix} 0.1407 & 0.0740 \\ 0.3954 & 0.2157 \\ 0.6707 & -0.0734 \\ 0.0143 & 0.7334 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 0.5181 & 3.9590 \\ 0.4096 & 0.3648 \\ 0.8290 & 0.2438 \\ 0.0540 & 0.8474 \end{bmatrix}, K_3 = \begin{bmatrix} -0.7045 & 4.5529 \\ 0.4149 & -0.4309 \\ 0.6592 & 0.0703 \\ -0.0150 & 0.6026 \end{bmatrix}.$$

In the binary case.

In this study, the system is working in the 0-th mode during $[0, 500)$ samples, and in the 500 samples, the process switches into the 1-st mode. In the time interval $[500, 600)$, the system is running in the 1-st mode then switches back to the 0-th mode at 600 samples.

The change is detected with the log LR test-based FD scheme at $\hat{k}_{0,1} = 500$ samples, as shown in subfigure 1 in Fig. 6.5. while the disappearance of 1-st multiplicative IF is detected at $\hat{k}_{1,0} = 600$ samples, see subfigure 2 in Fig. 6.5.

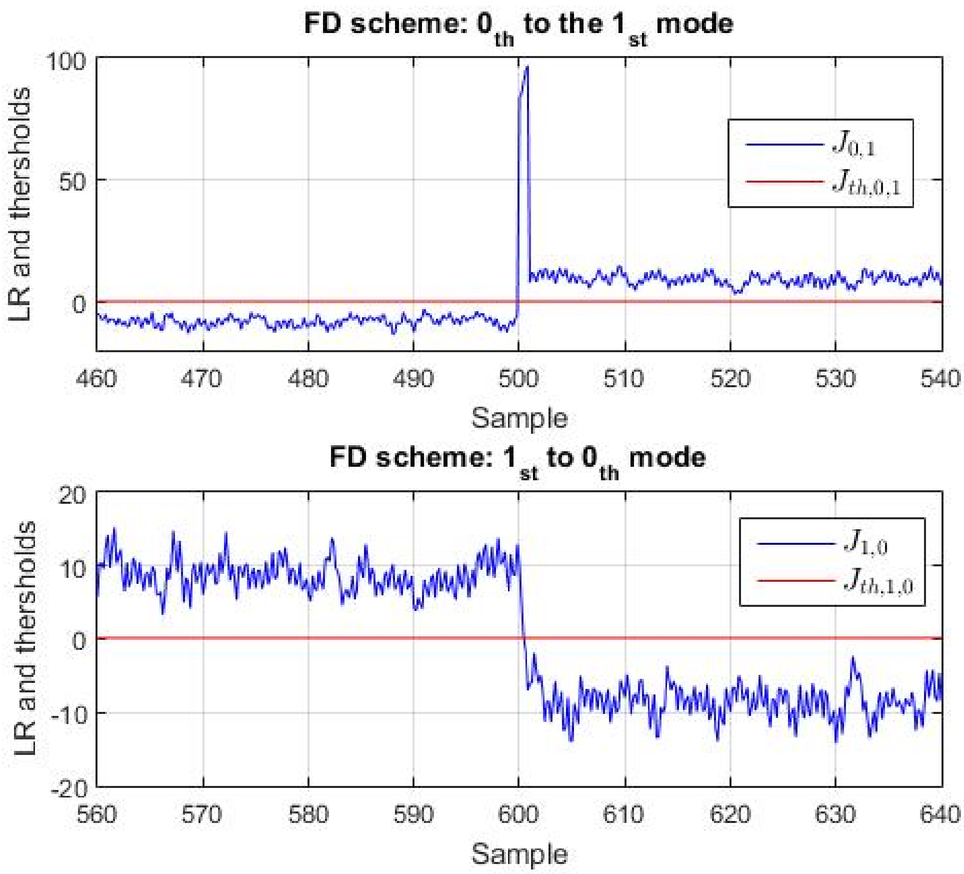

Figure 6.5: Detection of the 1-st multiplicative IF mode and the disappearance of the IF.

For calculating the threshold, choosing the accuracy $\varepsilon = 5\%$ and the confidence level $1 - \delta, \delta = 10^{-6}$, according to the Algorithm 5.3, the training samples should be $N_1 \geq 270$.

Two thresholds are calculated by $N_1 = 271$ training data are dynamic (varying in a small range). Here a typical value is given $J_{th,0,1} = 0.15, J_{th,1,0} = 0.56$.

The ADT parameters are with $\alpha_0 = 0.10, \alpha_1 = 0.15$, $\kappa_0 = 0.50, \kappa_1 = 0.55$, $\mu_{0,1} = 1.1, \mu_{0,2} = 1.9$ and $\mu_{1,1} = 1.05, \mu_{1,2} = 1.95$. By related calculation, the acceptable ratio of running time intervals in both modes is $0.11 < T_0/T_i < 4.9$, $i = 1, 2, 3$.

In the multiple case.

In this study, the system is working in normal mode during $[0, 700)$ samples, and in the 700 samples switches into the 3-rd mode. The tasks of the detection scheme are estimating the switching instant and distinguishing the multiplicative IF mode.

The ADT parameter are chosen as $\alpha_0 = 0.10$, $\alpha_1 = \alpha_2 = \alpha_3 = 0.15$, $\kappa_0 = 0.50$, $\kappa_1 = \kappa_2 = \kappa_3 = 0.55$, $\mu_{0,1} = 1.1$, $\mu_{0,2} = 1.9$ and $\mu_{1,1} = \mu_{2,1} = \mu_{3,1} = 1.05$, $\mu_{1,2} = \mu_{2,2} = \mu_{3,2} = 1.95$. After ADT theoretical calculation, the acceptable ratio of running time intervals in both modes is $0.11 < T_0/T_1 < 4.9$. The threshold $J_{th,0,3}$ is also a dynamic series, a typical value is given here $J_{th,0,3} = 2.39$.

The estimation of switching instants in every modes are $\hat{k}_{0,1} = 701$, $\hat{k}_{0,2} = 700$, $\hat{k}_{0,3} = 700$. Because $J_{0,3} = \max(J_{0,1}, J_{0,2}, J_{0,3})$, which means the system is most "likely" switching in the 3-rd mode. The performance of LR test-based FD scheme is demonstrated in Fig 6.6.

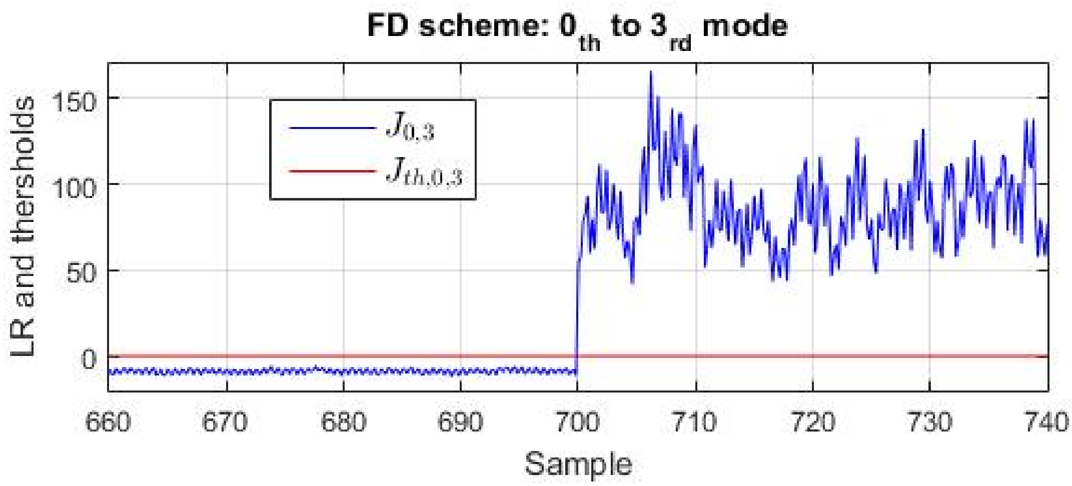

Figure 6.6: Detection of the 3-rd multiplicative IF mode.

6.3 Concluding remarks

In this chapter, two benchmark processes have been utilized to demonstrate the effectiveness of the proposed FD approaches. A two-mass-spring system is used for $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based FD schemes of uncertain switched systems in Chapter 4. A numerical example is demonstrated the proposed FD scheme in the binary case, which is discussed in Chapter 5. Furthermore, it has also been applied to show the feasibility of the log LR test-based FD scheme in multiple cases. The results in this chapter meet the requirement for reliable detection of potential faults.

7 Conclusions and Future Work

The main contribution of this thesis is to study the FD scheme with the IFs system modeled by the switched system. Based on this framework, FD schemes with a mode estimation unit are proposed for both stochastic and deterministic dynamic systems. This chapter first summarizes the main points of this thesis, and then points out several related topics that can be further investigated.

7.1 Conclusions

This thesis is dedicated to applying switched systems technique to the detection of systems with multiplicative IFs, and the specific content is arranged as follows.

In Chapter 1, a brief introduction to the developments and basic concepts of FD schemes, and switched systems are given at first. In order to model and detect the potential faults that occur and last for a limited time interval then disappear without any external correction, a new FD framework based on switched system is developed, in which a fault mode estimation unit is embedded and a proper decision rule is set up.

Mathematical and control theoretical preliminaries of this thesis are presented in Chapters 2 and 3.

In Chapter 2, a brief introduction of FD schemes designed for systems both in deterministic and statistical frameworks is presented. Including basic ideas, hypothesis testing, modeling of dynamic processes, the model-based and data-driven methods of FD systems design are addressed in this chapter.

In Chapter 3, the basic knowledge of switched systems is addressed. Switched systems received considerable attention in recent years and numerous research results have been reported, such as filtering, control and stability analysis. Among them, we focus on the stability analysis issues in this chapter. To be precise, situations under both arbitrary switching and restrict switching are discussed. For restricted switching conditions, a so-called ADT method has been applied, and it is further utilized in the following chapters to analyze stability conditions in both deterministic and stochastic systems. The further introduction of multiplicative IFs provides the objective of this thesis.

Considering the FD approach for switched systems with norm-bounded uncertainties and unknown input signals, an FD scheme is investigated with $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ observer-based residual generators in Chapter 4. Moreover, the system and observer running in asynchronized manner attract our interest to study such a topic in this chapter. To this end, the $\mathcal{L}_\infty/\mathcal{L}_2$ FD scheme for uncertain switched systems is addressed with a mode estimation unit. An LMI aided technique is applied to calculate dynamical thresholds.

In Chapter 5, our purpose is to build a switched system-based framework to detect IFs in stochastic dynamic systems. In this framework, the detection of such faults can be equivalently treated as the detection of the switchings between IFs inactive (fault-free) mode and active (faulty) mode. Based on this framework, an FD scheme with mode and switching instant estimation is investigated. To this end, LTV systems and Kalman filter-based residual generation are firstly introduced. After that, we construct the framework of applying switched systems to model systems with IFs properly. Then, with an RA-aided threshold, an LR test-based FD scheme for stochastic dynamic systems with IFs is presented. In the end, the stability margin for the FD scheme is studied according to ADT stability theory.

In Chapter 6, a two-spring-mass system is firstly considered and used to show the effectiveness of FD schemes proposed in Chapter 4, in which $\mathcal{L}_2$ and $\mathcal{L}_\infty/\mathcal{L}_2$ types of FD schemes are investigated for switched systems with norm-bounded uncertainties, since the elasticity coefficient of the spring may vary in practice. Then, a numerical example is utilized to demonstrate the feasibility of the LR test-based FD scheme in Chapter 5 in the binary case, and in multiple working cases, it is also put forward to demonstrate the feasibility of the LR test-based FD approaches.

In conclusion, in this thesis, the main contributions are:

1. construct a framework that applies switched systems to model systems with IFs;
2. propose an $\mathcal{L}_\infty/\mathcal{L}_2$ type of FD system for switched systems in asynchronous manners;
3. investigate LR test-based FD scheme for a dynamic systems with multiplicative IFs in the stochastic case and apply RA-aided technique to set corresponding thresholds;
4. propose an FD schemes for dynamic systems with multiplicative IFs with mode and switching instant estimation unit.

7.2 Future work

Besides this thesis made contribution to FD schemes for systems with multiplicative IFs, there are some topics worth being considered and explored.

FD design for systems with unknown IFs. For some dynamic processes and systems, there always exist some unknown IFs, so it is necessary to construct residual generators and propose corresponding FD schemes for these unknown IFs (subsystems). To this end, the construction of residual generators for systems with unknown IFs (subsystems) can employ data-driven methods.

FD design for systems with time-delay. In real industrial applications, the time-delay always exists, and a time-delay between dynamic system and observer is considered in Chapter 4. Furthermore, there exist other types of time-delay situations which motivates us to study further on FD topics.

FD design for other kinds of hybrid systems. The FD system for switched systems is proposed in this thesis, and there are other types of hybrid systems to be investigated, such as impulsive systems and Markov Jumping systems.

RA-aided FD design for systems with non-Gaussian noise. For systems with non-Gaussian noise and unknown disturbance, it is difficult to set a proper threshold for a certain evaluation function. In this case, the RA-aided technique can be applied to determine the corresponding threshold under a given accuracy and confident level.

The ideas, preliminaries, methods and techniques discussed in this thesis can also be the fundamental of fault-tolerant control systems design. Therefore, there exist several relevant topics to be studied in future. Some of the future work are outlined as follows:

1. Fault tolerant control systems for switched systems will be investigated, in which the Plug-and-Play (PnP) control strategies can be applied and ADT stability condition will be given;
2. The FD and fault tolerant control design for distributed systems will be considered in the framework of switched systems.

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List of publications

Journal Papers

1. **Y. Na**, L. Li, S. X. Ding, Y. Na and J. Qiu, An asynchronized observer based fault detection approach for uncertain switching systems with mode estimation, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 2, pp. 514-518, 2021.

Conference Papers

1. **Y. Na**, Y. Na, S. X. Ding, L. Li, A. Abdo, A fault detection scheme for uncertain switched systems under asynchronous switching, *IFAC-PapersOnLine*, vol. 51, no. 24, pp. 117-122, Warsaw, 2018.

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3. **Y. Na**, Y. Na and S. Peng, A Data-driven Approach for Identification and Detection of Intermittent Faults, *2021 4th IEEE International Conference on Industrial Cyber-Physical Systems (ICPS)*, pp. 455-460, 2021.

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