Econometric Modeling and Forecasting with Application in Electricity Markets

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Preface

The present thesis is a cumulative dissertation composed of the following three academic papers:

1. Kulakov S., X-model: Further Development and Possible Modifications, published in Forecasting 2020, 2, 20-35, accessible via `https://doi.org/10.3390/forecast2010002`

2. Kulakov S. and Ziel, F., The Impact of Renewable Energy Forecasts on Intraday Electricity Prices, published in Volume 10, Number 1 of The Quarterly Journal of the IAEE's Energy Economics Education Foundation, accessible via `https://doi.org/10.5547/2160-5890.10.1.skul`

3. Kulakov S. and Ziel, F., Determining Fundamental Supply and Demand Curves in a Wholesale Electricity Market, published in an open-access e-Print archive arXiv.org, accessible via `https://arxiv.org/pdf/1903.11383.pdf`

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Introduction and Conclusion

X-model: Further Development and Possible Modifications

The Impact of Renewable Energy Forecasts on Intraday Electricity Prices

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List of Variables

Introduction and Conclusion

d	Daily time index
$W^F_{d,h}$	Day-ahead forecast of wind infeed
$S^F_{d,h}$	Day-ahead forecast of solar infeed
$load^F_{d,h}$	Day-ahead load forecast
$P^{DA}_{d,h}$	Day-ahead price
$P_{d,h}$	Electricity price
$\varepsilon_{d,h}$	Error term
$Sup(x)$	Function denoting the supply curve as a function of volume x
h	Hourly time index
n, k, j, i, l, m	Indices
$P^{ID}_{d,h}$	Intraday price
$\boldsymbol{X}_{d,h}$	Matrix of regressors
r	Number of to-be-averaged wholesale supply curves
$avgsupregr$	Regression-based transformation
β	Regression coefficient
$\kappa_{S,d,h}$	Shift coefficient for the supply curve
$\kappa_{D,d,h}$	Shift coefficient for the demand curve
t	Time index
$sup1$	Transformation based on the inverse wholesale supply curve
$avgsupr$	Transformation based on r averaged wholesale supply curves
$Y_{d,h}$	Transformed price $P_{d,h}$
v	Utility's price tolerance level
$\boldsymbol{\beta}_{d,h}$	Vector of β-coefficients
$\boldsymbol{r}$	Vector of r variables
$X^c_{S,d,h}$	Volume in price class c on the supply curve
$X^c_{D,d,h}$	Volume in price class c on the demand curve
χ	Weight coefficient

X-model: Further Development and Possible Modifications

$P^{DA}_{d,h}$	Actual day-ahead price
D	Amount of days in a year
M_D	Amount of time series models for the demand curve
M_S	Amount of time series models for the supply curve
$\bar{Sup}(x)$	Average transformed supply curve as a function of volume x
$R(P)$	Binary function
k	Day index in the weekday dummy function
DoW	Day-of-the-week dummy
d	Daily time index
$Y_{m,d,h}$	Detrended version of $X_{m,d,h}$
$X^{volume}_{d,h}$	Difference between volumes in inelastic and wholesale equilibrium settings
$\varepsilon_{m,d,h}$	Error term
$xmod^{combined}$	Equally weighted combination of the original and modified X-models
$X^{price}_{d,h}$	Equilibrium price

$X_{d,h}^{Dem_{d,h}^{inelastic}}$	Equilibrium volume in the inelastic setting
$X_{d,h}^{volume,WS}$	Equilibrium volume in the wholesale setting
$\widehat{P}_{d,h}^{DA}$	Forecasted day-ahead price
$\check{S}_{d,h}$	Forecast for the entire supply curve after reconstruction
$Dem_t^{inelastic}$	Function denoting the inelastic demand curve
$Sup_t(x)$	Function denoting the transformed supply curve as a function of volume x
$WSDem_t(x)$	Function which defines the wholesale demand curve as a function of volume x
$WSSup_t(x)$	Function which defines the wholesale supply curve as a function of volume x
$\mathcal{W}(d)$	function which yields a number corresponding to the day of the week
$\mathbb{P}$	Grid of all possible prices between $P_{\min}$ and $P_{\max}$
V	Volumes on the curve $Sup_t^{-1}(P)$
$\mathcal{P}$	Grid of prices P which have at least one positive bid in the in-sample period
h	Hourly time index
$\phi = (1, 2)$	Index
l, j	Indices
$X_{D,d,h}$	Inelastic demand volume
$X_{d+1,h}^{generation}$	Load forecast
$\delta_{d,h,\mathbb{A},\mathbb{B}}$	Loss differential between models $\mathbb{A}$ and $\mathbb{B}$
$L_{d,h,\mathbb{A}}, L_{d,h,\mathbb{B}}$	Loss functions of models $\mathbb{A}$ and $\mathbb{B}$
$\boldsymbol{Z}_{d,h}$	Matrix of regressors (outlier processing model)
$P_{\max}$	Maximal bidding price allowed by the EPEX SPOT SE
$P_{\min}$	Minimal bidding price allowed by the EPEX SPOT SE
m	Model index
$\mathbb{A}$, $\mathbb{B}$	Model abbreviations
$xmod^{modified}$	Modified X-model with inelastic demand
T	Number of in-sample observations
$xmod^{orignal}$	Original X-model
$\lambda_{m,h}$	Penalization parameter
$P_{d,h}$	Price variable
$\check{P}_{d,h}$	Prices in $\mathcal{P}$ if $R(P) = 1$
β	Regression coefficient
$\tilde{Y}_{m,d,h}$	Scaled version of $Y_{m,d,h}$
$\phi_{m,h,l,j,k}, \mathcal{I}_{m,h}(l, j)$	Sets of lags
$X_{d+1,h}^{solar}$	Solar forecast
$\sigma_{d,h,\mathbb{A},\mathbb{B}}$	Standard deviation of $\bar{\delta}_{d,h,\mathbb{A},\mathbb{B}}$
V_*	Step in the volume grid
N	Supplementary variable
t	Time index
$\mathcal{C}$	Total amount of price classes on the supply curve
M	Total amount of time series models
t_{DM}	t-statistic for the DM-test
$C = (S, D)$	Variable denoting supply or demand curve
$\sigma_{m,h}^2$	Variance term
$\boldsymbol{\beta}$	Vector of β-coefficients
$\boldsymbol{\varepsilon}_{d,h}$	Vector of $\varepsilon_{m,d,h}$
$\boldsymbol{X}_{m,d,h}$	Vector of $X_{m,d,h}$
$\boldsymbol{Y}_{d,h}$	Vector of $Y_{m,d,h}$

$\boldsymbol{\mu}_h$	Vector of mean values of $X_{d,h}$
$\boldsymbol{X}_{m,d,h}$	Vector of regressors
$Y_{d,h}^{S,1}$	Volume at price $P_{\min}$ on the supply curve (outlier processing model)
$Y_{d,h}^{S,\gamma}$	Volume at price $P_{\min} + 5$ on the supply curve (outlier processing model)
$Y_{d,h}^{D,1}$	Volume at price $P_{\max}$ on the demand curve (outlier processing model)
$Y_{d,h}^{D,\gamma}$	Volume at price $P_{\max} - 5$ on the demand curve (outlier processing model)
$X_{S,d,h}^{c}$	Volume in price class c in the supply curve
$\check{V}_{d,h}$	Volume reconstruction function
$W_k(d)$	Weekday dummy function
$X_{d+1,h}^{wind}$	Wind forecast

The Impact of Renewable Energy Forecasts on Intraday Electricity Prices

nlm	Abbreviation for the first auction-curves-based model
$lnlm$	Abbreviation for the second auction-curves-based model
lm_1	Abbreviation for the first benchmark linear model
lm_2	Abbreviation for the second benchmark linear model
qlm	Abbreviation for the benchmark quadratic model
clq	Abbreviation for the first combined model
$clnq$	Abbreviation for the second combined model
$naive$	Abbreviation for the naive model
W_t^A	Actual amount of wind supply
S_t^A	Actual amount of solar supply
D	Amount of days in a year
W_t	Amount of energy harvested by a wind power plant
$\bar{\delta}^{\mathbb{A},\mathbb{B}}$	Average loss differential between models $\mathbb{A}$ and $\mathbb{B}$
ρ	Correlation coefficient
W_t^F	Day-ahead forecast of wind supply
S_t^F	Day-ahead forecast of solar supply
P_t^{DA}	Day-ahead price
d	Daily time index
ε_t	Error term
S_t^{Δ}	Forecast error as a difference between actual and forecasted solar supply
W_t^{Δ}	Forecast error as a difference between actual and forecasted wind supply
$Dem_t^{inelastic}$	Function which defines the inelastic demand curve
$Sup_t(x)$	Function which defines the transformed supply curve as a function of volume x
$WSDem_t(x)$	Function which defines the wholesale demand curve as a function of volume x
$WSSup_t(x)$	Function which defines the wholesale supply curve as a function of volume x
$\bar{\delta}_h^{\mathbb{A},\mathbb{B}}$	Hourly average loss differential between models $\mathbb{A}$ and $\mathbb{B}$
$\delta_h^{\mathbb{A},\mathbb{B},\phi}$	Hourly loss differential between models $\mathbb{A}$ and $\mathbb{B}$
$L_h^{\mathbb{A},\phi}, L_h^{\mathbb{B},\phi}$	Hourly loss functions of models $\mathbb{A}$ and $\mathbb{B}$
$\sigma_h^{\bar{\delta},\mathbb{A},\mathbb{B}}$	Hourly standard deviation of $\bar{\delta}_h^{\mathbb{A},\mathbb{B}}$
h	Hourly time index
$t_h^{\mathbb{A},\mathbb{B},\phi}$	Hourly t-statistic for the DM-test
$\widetilde{W_t}$	Incremental wind supply from additional wind power capacities

P_t^{ID}	Intraday price
$\phi = (1, 2)$	Linear/quadratic index
$\delta^{\mathbb{A},\mathbb{B},\phi}$	Loss differential between models $\mathbb{A}$ and $\mathbb{B}$
$L^{\mathbb{A},\phi}, L^{\mathbb{B},\phi}$	Loss functions of models $\mathbb{A}$ and $\mathbb{B}$
$\boldsymbol{Z}_t$	Main regression component in the considered models
$P_{\max}$	Maximal bidding price allowed by the EPEX SPOT SE
$P_{\min}$	Minimal bidding price allowed by the EPEX SPOT SE
$\omega = \mathbb{A}, \mathbb{B}$	Model abbreviations
$W_t^{\Delta-}$	Negative part in the wind forecast errors
$S_t^{\Delta-}$	Negative part in the solar forecast error
β	Regression coefficient
γ	Scale factor
$\sigma^{\bar{\delta},\mathbb{A},\mathbb{B}}$	Standard deviation of $\bar{\delta}^{\mathbb{A},\mathbb{B}}$
σ	Standard deviation term
t	Time index
$t^{\mathbb{A},\mathbb{B},\phi}$	t-statistic for the DM-test
$\boldsymbol{\beta}$	Vector of regression coefficients

Determining Fundamental Supply and Demand Curves in a Wholesale Electricity Market

$load_i$	Actual load value
$FDem$	Demand curve in the FM case
$E_{F,i}^p$	Elasticity coefficient
p^F	Equilibrium price in the FM case
p^W	Equilibrium price in the WS case
v^F	Equilibrium volume in the FM case
v^W	Equilibrium volume in the WS case
v^C	Equilibrium volumes suggested by the model in Coulon et al., 2014
$FDem_1(x)$	Final Utility's demand schedule as a function of volume x
$FSup_1(x)$	Final Utility's supply schedule as a function of volume x
i	Index
p^U	Internal price of the Utility
$p_{\max}$	Maximal bidding price allowed by the EPEX SPOT SE
$p_{\min}$	Minimal bidding price allowed by the EPEX SPOT SE
$pmin$	Minimally allowed bidding price in the model
$pmax$	Maximally allowed bidding price in the model
$p_{\max,2017}$	Maximal bidding price recorded in 2017
$p_{\min,2017}$	Minimal bidding price recorded in 2017
$\widetilde{FDem}_1(x)$	Non-shifted Utility's demand schedule as a function of volume x
$\widetilde{FSup}_1(x)$	Non-shifted Utility's supply schedule as a function of volume x
θ	Regression coefficient
a	Regression coefficient
$Dem_0(x)$	Retailer's demand schedule as a function of volume x
τ_1	Shift coefficient
h	Shift distance for determining the factor $ls(p, h)$

$ls(p, h)$	Slope of the demand curve in point p in EUR/MWh^2
α_1	Split coefficient between buy and sell orders for the Utility's demand
β_1	Split coefficient between buy and sell orders for the Utility's supply
$FSup$	Supply curve in the FM case
$Sup_0(x)$	Supplier's supply schedule as a function of volume x
ϕ_1	Utility's proportion in the lower segment of the wholesale supply curve
γ_1	Utility's proportion in the upper segment of the wholesale supply curve
WSup (x)	Wholesale supply curve as a function of volume x
WDem (x)	Wholesale demand curve as a function of volume x

List of Abbreviations

ASD	Aggregated Supply and Demand case
asinh	Areasinus hyperbolicus
AR(X)	Autoregressive model of order X
ARMA	Autoregressive moving average
ARIMA	Autoregressive integrated moving average
BIC	Bayesian information criterion
COVID-19	Coronavirus disease 2019
DM-test	Diebold-Mariano test
EXAA	Energy Exchange Austria
EPF	Electricity Price Forecasting
ENTSO-E	European Network of Transmission System Operators for Electricity
EUR	Euro
EPEX SPOT SE	European Power Exchange
FM	Fundamental Market equilibrium
GARCH	Generalized autoregressive conditional heteroscedasticity
GEFCom2014	Energy forecasting competition 2014
IPEX	Italian Power Exchange
LASSO	Least absolute shrinkage and selection operator
MAE	Mean absolute error
MW	Megawatt
MWh	Megawatt hour
OTC	Over-the-counter
RMSE	Root mean square error
WM	Wholesale Market case

1 Introduction

In the modern, increasingly interconnected and digitized world, the practice of forecasting is becoming a commonplace approach in a variety of areas of application (see e.g. the work in [Hyndman and Athanasopoulos, 2018] for a general review). Special attention of researchers and practitioners to forecasting does not appear surprising. Vast amounts of available data and massive computational capabilities of contemporary computers enable the employment of diverse techniques with the purpose of making predictions of the future. These techniques range from relatively subjective qualitative methods to highly sophisticated mathematical and statistical models. Forecasting horizons, too, vary, with some predictions aiming to precisely assess the future decades ahead. As a result, forecasts allow the degree of riskiness and uncertainty of a variety of projects to be evaluated and, if possible, minimized.

One of areas which fully embraced benefits of forecasting is the field of electricity markets. Particular interest to forecasting in this field arises, among other factors, from the very nature of electricity. As opposed to conventional commodities, electricity is economically non-storable, meaning that supply and demand of electricity must be continuously equilibrated to maintain balance in a power system (see e.g. the work in [Kaminski, 2012]). Besides unique properties of electricity, another factor that facilitated the deployment of forecasts in the field is rapid development and decentralization of electricity markets across the globe. Moreover, the share of weather-dependent and intermittent renewable energies in the worldwide power mix is rapidly increasing (see statistics provided in e.g. [Gielen et al., 2019]), meaning that both short- and long-term management of electricity markets will become more and more reliant on accurate predictions (see e.g. the work in [Hong et al., 2016] or [Hong et al., 2014] for a general overview of the topic). The coronavirus pandemic (COVID-19) has, of course, strongly affected the energy sector, further emphasizing the importance and utility of forecasting (see e.g. the work in [Obst et al., 2020] or [Scarabaggio et al., 2020])

From the above paragraph it follows that benefits of forecasting in electricity markets can be fully embraced to reduce uncertainty stemming from a large amount of variables. Moreover, the above paragraph has an implication that short-term forecasting horizon is given a special attention in electricity markets due to short-term optimization needs (see e.g. the work of [Ziel and Steinert, 2018] for a comprehensive review of medium- to long-term price forecasting). In turn, most relevant forecasting domains in the electricity field include, as follows from [Hong et al., 2020], wind and solar supply (see a review in [Sweeney et al., 2020] or e.g. the work in [Pinson et al., 2007] and [Lin et al., 2018] for the case of wind power; a see a review in [Yang et al., 2018] or e.g. the work in [Sharma et al., 2011] and [Rodríguez et al., 2018] for the case of solar power); total load amount (see e.g. the work in [Hong et al., 2016], [Amjady et al., 2010] or [Liu et al., 2017]) and electricity prices (see e.g. recent reviews provided in [Ziel and Weron, 2018] or [Nowotarski and Weron, 2018]). The amount of publications in these domains, as reported by [Hong et al., 2020], has increased from a little more than 500 papers in 2010 to almost 2000 papers in 2020, further emphasizing a growing interest in the topic.

The focus of the present thesis will be placed on one of these domains, namely on electricity price forecasting (commonly referred to simply as EPF). More specifically, the main part of the thesis will be dedicated to two types of relatively short-term electricity price forecasting: day-ahead and intraday. In Europe, the former one is commonly referred to as spot price forecasting and is understood as making a prediction of 24 electricity prices for 24 hours of the next day. The latter one usually denotes price predictions within one day, or within a specified period of time of the next day.

1.1 Day-Ahead and Intraday Markets in Germany

As the work in [Huisman et al., 2007] suggests, fundamentals and behavior of day-ahead and intraday prices differ from country to country. To keep further discussion consistent, day-ahead and intraday prices are defined on the grounds of the German electricity market. These definitions are chosen because econometric models developed in the present thesis are estimated on the German electricity market data.

In Germany, only around 50% of electricity is traded on wholesale markets, while the remaining is being processed over various over-the-counter (OTC) contracts. OTC contracts are typically opaque and are thus out of interest for the present discussion. On the contrary, as outlined in e.g. [Hagemann and Weber, 2013] or [Graf von Luckner and Kiesel, 2020], wholesale electricity market in Germany is relatively liquid, competitive and transparent. Whenever electricity is traded on a wholesale market in Germany, it will likely be conducted on one of the auctions organized by the European Power Exchange (EPEX SPOT SE) in German-Austrian bidding zone (see [EPEX, 2020] for a more thorough description). EPEX auctions remain most important electricity marketplaces in terms of volume in Germany. Moreover, they serve as a data basis for several regulatory calculations, including e.g. determination of feed-in tariffs in Germany.

A comprehensive overview of the German electricity market structure is provided in the paper by [Kiesel and Paraschiv, 2017], while a simplified temporary structure of the German electricity market with its two main auctions is demonstrated in Figure 1. These two auctions are most important for the present thesis and are referred to as day-ahead and 15-minutes intraday. Furthermore, besides the two auctions, there is so-called a continuous intraday market depicted in the Figure. For the sake of simplicity some auctions offered in Germany are neglected in the Figure, i.e. forward or day-after ones.

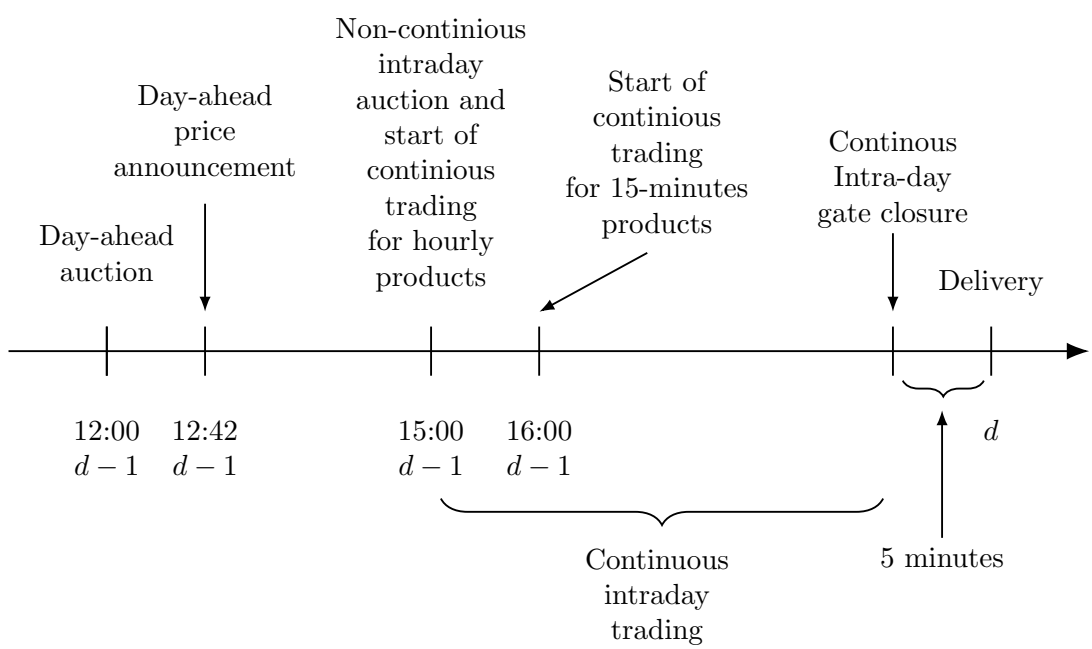


Figure 1: A simplified temporary structure of the German EPEX SPOT SE.

Following Figure 1, day-ahead auction is a non-continuous limit order book auction conducted at 12:00 a day prior to d. 24 hourly prices for respectively 24 hours of day d are established at

this market and published at approximately 12:42 of day $d-1$. The non-continuous intraday auction is the second auction. It is conducted at 15:00 a day prior to d. 96 prices in 15-minutes resolution for respectively 24 hours of day d (4 prices per hour) are published at approximately 16:00 of day $d-1$. Finally, a continuous intraday market opens at 15:00 for hourly products and at 16:00 for quarter-hourly products of day $d-1$ and runs until 5 minutes prior to the point of physical delivery of electricity.[1] Continuous intraday market involves the immediate execution of orders upon their receipt by market makers and specialists. As a result, following [Graf von Luckner and Kiesel, 2020], events such as the placement of a buy market order occur randomly in time in this market.

The importance of having several markets for electricity trading have been emphasized multiple times (see e.g. the work [Weber, 2010] besides the above mentioned papers in this section). While day-ahead market is primarily used to enable operational planning for the next day and offers market participants a certain amount of security, intraday markets (especially the continuous one) offer more flexibility to market participants and enable short-term balancing of electricity volumes, with this balancing being particularly important in times of increasing share of renewable energies and uncertainty induced by COVID-19 and the resulting demand volatility. Therefore, market participants can utilize advantages of three markets for their own benefits and build various trading strategies to optimize their risk exposure.

1.2 An Example of Electricity Price Time Series

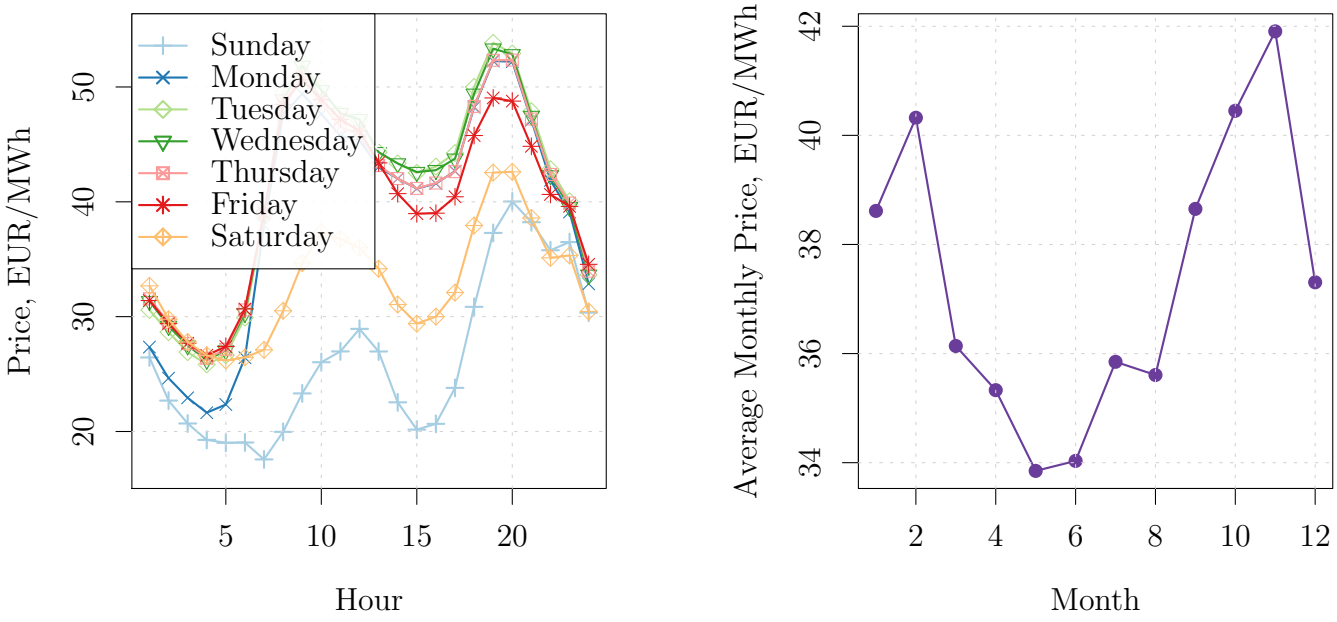
EPF has seen an upswing in interest not least due to the fact that electricity price time series follow certain patterns and can be forecasted efficiently (see e.g. the work in [Ziel et al., 2015a]). An exemplary time series with German day-ahead electricity prices is depicted in Figure 2.

As can be seen from Figure 2(a), daily electricity prices are cyclical and mainly depend on several factors. One such factor is routine human behavior. For example, prices are typically lower during the weekend due to lower levels of industrial activity. Less intensive industrial activity also leads to patterns commonly referred to as Monday and Friday effects. These effects denote the tendency of prices to be typically lower on Monday morning and Friday evening than on other days of the working week. Moreover, the Figure shows two price spikes, at approximately 11:00 and 18:00. These spikes can be explained by peaking industrial and households activities, respectively. Of course, prices go down during the night as a result of muted human activity.

Besides behavioral patterns, feed-in of renewable energies, especially wind and solar, exerts a crucial impact on electricity prices. Electricity obtained from wind and solar resources practically has zero marginal costs, meaning that higher supply of such electricity leads to a so-called merit order effect. Following [Sensfuß et al., 2008], the merit-order effect occurs when a greater amount of electricity harvested from renewable recourses is fed into a system, thus leading to lower electricity prices (assuming constant demand). Therefore, prices are typically lower in times of high wind and solar supply.

Over a longer time horizon, as depicted in Figure 2(b), typically lower prices can be seen in summer due to absent heating needs and relatively high supply of solar energy. Furthermore, other factors potentially affecting prices are e.g. holidays, big social events or structural changes in demand of electricity, as has happened recently due to the onset of COVID-19 (see the work

[1] Following [EPEX, 2020], it is a relatively recent market novelty introduced in 2017 that the continuous intraday market closes only 5 minutes prior to physical delivery. It used to be 45 minutes at the launch of the market and reduced to 30 minutes in 2015.



(a) Average hourly day-ahead prices

(b) Average monthly day-ahead prices

Figure 2: Average hourly (left) and monthly (right) day-ahead prices recorded in the German EPEX SPOT SE over the period 01.01.2016 to 22.02.2018

in [Narajewski and Ziel, 2020a] for a comprehensive overview of the impact of COVID-19 on the German electricity market).

1.3 Literature overview

A very comprehensive and thorough review of state-of-the-art electricity price forecasting techniques is provided in the papers by [Weron, 2014] and [Hong et al., 2020]. Following the former paper, EPF models can be subdivided into 5 groups: multi-agent (which simulate supply and demand by creating a heterogeneous system of agents); fundamental (which determine prices by modeling the impact of structural physical and economic factors); reduced-form (which evaluate endogenous variables in terms of observable exogenous variables and serve to detect relationship between these variables); computational intelligence (which take advantage of modern artificial intelligence techniques) and statistical (which are rather direct applications of statistical and econometric modeling tools). Models elaborated in the present thesis can be placed at the intersection of two of these groups: statistical and fundamental. There are two main reasons which explain such position of the models relative to conventionally defined groups of EPF.

On the one hand, the developed models take full advantage of properties, information and dependencies present in electricity price time series as well as other auxiliary data. The models are, in fact, based on historical time series and are fully utilizing contemporary statistical and econometric techniques to analyze the data and carry out price forecasts. The models are estimated using advanced computational methods, including LASSO-estimators and optimization-based algorithms. The comparison of the models is conducted on the grounds of typical EPF measures, including MAE, RMSE and Diebold-Mariano tests.

On the other hand, the models are utilizing recent developments in the area of energy economics

and structural modeling approaches. The models are not carrying out time-series-based electricity price forecasts in conventional sense. Instead, the models are predicting electricity prices on the grounds of historical wholesale supply and demand curves (also known as sale and purchase curve) recorded in an electricity market.

As a result, the models elaborated within the framework of the present thesis combine insights of market structure models with extensive contemporary econometric analysis. This combination has several advantages (they will be discussed in what follows) and has not been studied extensively in the academic literature. Therefore, the present thesis fills an existing gap and presents an extensive study of econometric modeling of electricity wholesale supply and demand curves as well as the application of these curves for the purposes of electricity price forecasting and electricity markets modeling. Hence, the forthcoming literature review will be focused on two main bodies of academic literature relevant for the present thesis: methods from the field of statistical forecasting and recent developments in the area of structural modeling of wholesale supply and demand curves observed in electricity markets.

1.3.1 Statistical Methods in Electricity Price Forecasting

One of the simplest statistical models used in the field of electricity price forecasting is a so-called naive model. This model was introduced by [Nogales et al., 2002] and assumes that the price on hour h of day d is equal to the price on hour h of day $d-1$. Other variations of the model consider the same hour h of the previous week, i.e. of day $d-7$, assuming that prices on hour h on e.g. Monday of one week are similar to prices on hour h on Monday of the previous week. Naive model is typically used as the most basic benchmark in a variety of EPF papers. Furthermore, as e.g. research in [Narajewski and Ziel, 2020b] shows, simplest naive models can be used successfully for predicting very short-term intraday prices (referred to as ID3 in the German electricity market).

Second group of considerably more advanced EPF models is so-called expert models. These models are typically AR(X)-based models and can incorporate several auxiliary external regressors such as e.g. wind and solar infeed.[2] Expert models in conventional sense are producing point forecasts and can be written in the following form

$$P_{d,h} = \beta_0 + \boldsymbol{\beta}_{d,h}\boldsymbol{X}_{d,h} + \varepsilon_{d,h} \tag{1}$$

where $P_{d,h}$ denotes a price on day d and hour h, β_0 is an intercept, $\boldsymbol{\beta}_{d,h}$ is a vector of regression coefficients, $\boldsymbol{X}$ is a matrix of regressors and $\varepsilon_{d,h}$ is an error term. Of course, there is a wide variety of different extensions to basic regression-based models. In one of the earlier studies, the work in [Cuaresma et al., 2004] shows that separating the entire price time series into 24 different ones for each hour of the day produces significantly better results. [Misiorek et al., 2006] argue that extending simple AR-typed model with external regressors yields significantly better results. More advanced weekly dependence structures have been identified in e.g. [Kristiansen, 2012], while [Ziel, 2018] provides a special study of holidays' impact on German electricity prices and argues that including holiday dummies improves forecasting quality. The work in [Ziel et al., 2015b] or [Weron and Misiorek, 2008] propose using data from spatially neighboring electricity markets (such as e.g. Austrian EXAA for the case of Germany) for improving forecasting accuracy. Another interesting approach was selected in [Karakatsani and Bunn, 2008] who show that models with time-varying coefficients are preforming better than those with fixed coefficients. To capture

[2] Of course, plain regression models exist too but, as is mentioned in [Weron, 2014], classical regression models are often mixed with AR(X) models.

yearly seasonal structure, [Knittel and Roberts, 2005] suggest to include temperature data and e.g. [Xu and Niimura, 2004] and [Ziel et al., 2015a] prove the effectiveness of wavelets such as periodic B-splines.

Besides rather classical AR(X) models, a lot of effort has been made in the direction of applying GARCH-typed models. The work in e.g. [Tan et al., 2010] or [Liu and Shi, 2013] provides a comparative study of various econometric models and their modified forms and shows that ARMA–GARCH-M models are in general an effective method for modeling and predicting the mean and volatility of electricity prices. Furthermore, jump diffusion models have been proposed in e.g. [Cartea and Figueroa, 2005] and more deeply analyzed in [Benth et al., 2012], showing effectiveness of such models. In a more recent study, [Muniain and Ziel, 2020] estimated mean and residuals separately and included a bivariate jump component in a mean reverting jump diffusion model in the residuals.

More sophisticated modeling techniques applied in EPF have also been focused on taking advantage of more advanced estimation techniques. One of them is the so-called LASSO (least absolute shrinkage and selection operator) estimation. Following [Tibshirani, 1996], LASSO minimizes the sum of a special penalization parameter and the conventional residual sum of squares. As a result, it is possible to plug a very large amount of regressors into a model and leave the selection task to the LASSO-estimator. [Ludwig et al., 2015] were among the first to use LASSO in the field of EPF, while [Ziel et al., 2015a] and [Ziel, 2016] conducted several studies on the German electricity price data and showed that LASSO is indeed a power and straightforward estimator that can be used in electricity price forecasting. [Ziel and Weron, 2018] showed that LASSO-estimated models in both univariate and multivariate settings outperform benchmark autoregressive models significantly. Furthermore, similar results were found in comparative studies carried out by [Uniejewski and Weron, 2018] and [Uniejewski et al., 2016] for spot prices. [Uniejewski et al., 2019b] and [Marcjasz et al., 2020] proved that LASSO can also be used for efficient modeling of short-term intraday prices.

Another area that has developed rather recently − and, more importantly, which provides opportunities for further developing research presented in this thesis − is transformation of underlying time series data prior to rolling out a forecast. Such transformation is usually used to tackle the issue of price spikes, as outlined in e.g. [Weron and Ziel, 2018]. The idea is to first transform the data into a less volatile and more stable time series, then perform modeling and forecasting on the transformed data and then apply the inverse transformation to the results. Among main papers which analyzed the benefits of data transformation are e.g. [Ziel and Weron, 2018] and [Uniejewski et al., 2017]. The latter paper provides a comprehensive comparison study of most common transformation techniques and shows that applying transformations leads to better forecasting performance. Furthermore, the topic of transformation can become especially important in the aftermath of COVID-19 crisis due to a large amount of negative price spikes present in electricity markets (see e.g. the work in [Narajewski and Ziel, 2020a] or [Zhong et al., 2020]).

Moreover, probabilistic forecasting with application in electricity markets has also been gaining in popularity over recent years. Probabilistic forecasts became especially promising after they showed best performance at several energy-related forecasting competitions, including e.g. GEFCom2014 (see an overview of GEFCom2014 in [Hong et al., 2016]). Following the paper by [Gneiting and Katzfuss, 2014], as opposed to point forecasts, probabilistic forecasts take the form of a predictive probability distribution over future quantities or events of interest. Recent review papers on the topic of probabilistic forecasting are e.g. [Nowotarski and Weron, 2018] , with the work in e.g. [Maciejowska et al., 2016], [Nowotarski and Weron, 2015] or [Uniejewski et al., 2019a] being most prominent. Among recent developments in probabilistic forecasting with application

in electricity price data are e.g. or [Narajewski and Ziel, 2020c] who take a novel approach to the problem of probabilistic forecasting and performed it by simulating trajectories in every trading window.

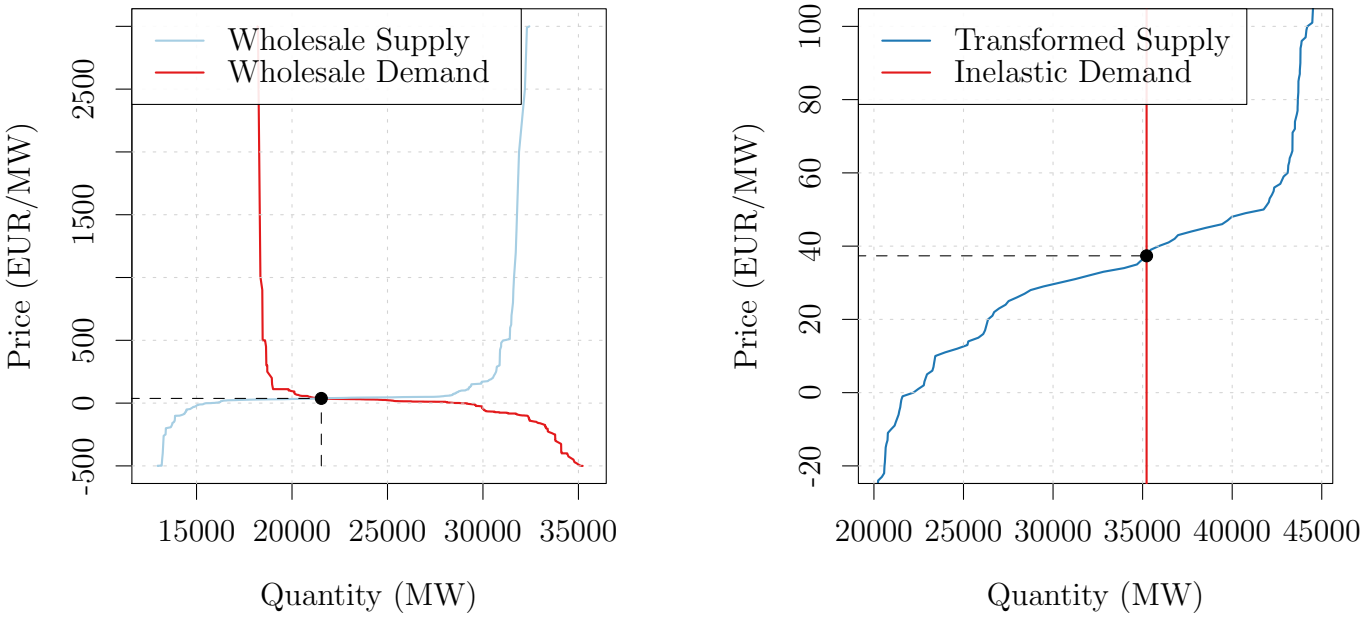
Finally, among areas of EPF specifically relevant for the present research is that of ensemble forecasting. The idea of this concept is to combine several forecasting models (or their results) into one. Following [Hong et al., 2020], combining forecasts has been widely acknowledged as one of the best practices in forecasting. A general review can be found in e.g. [Wallis, 2011], while particular applications in the energy field are presented in e.g. [Zheng et al., 2020] besides above mentioned [Ziel and Weron, 2018] or [Marcjasz et al., 2020]. Slightly different approach to combining forecasts was taken in [Marcjasz et al., 2019] and [Serafin et al., 2019] who combine probabilistic forecasts evaluated using different calibration windows. The work in the area of ensemble forecasting shows explicitly that that performance of combined model is superior to that of non-combined ones. The present thesis also supports this conclusion with empirical evidence.

1.3.2 Empirical Wholesale Auction Curves in Electricity Markets

Market clearing prices in competitive markets are typically determined and announced by a market maker and technically are intersections of supply and demand curves. From this perspective, day-ahead and intraday electricity prices are nothing else than intersections of the corresponding day-ahead and intraday wholesale electricity supply and demand curves (also known as sale and purchase curves, respectively). In turn, these curves are constructed by respectively aggregating supply and demand bids collected by an electricity exchange. The German EPEX SPOT, a classical market maker in this context, not only publishes market clearing prices but also reveals the entire wholesale supply and demand curves (see [EPEX, 2020]). An example of these curves is presented in Figure 3(a). Therefore, the data made available by the EPEX SPOT SE allows a more structural approach to be utilized when analyzing electricity prices. Moreover, following [Weron, 2014], there exists a wide variety of structural approaches to modeling in electricity markets. To keep the discussion concise, the focus of the present subsection will be placed solely on academic literature related to modeling electricity wholesale supply and demand curves.

In the earlier studies, auction curves as such were not publicly revealed. Therefore, there were several attempts to model these curves using different approaches. For example, [Barlow, 2002] create a diffusion model for electricity prices. The basis of this pricing model is a stochastic model of electricity supply and demand curves derived for the case of the Alberta and California markets. [Buzoianu et al., 2005] use latent supply and demand curve estimated on the grounds of the Californian electricity market data. The authors of this paper suppose that actual supply and demand curves are not observable. Therefore, these curves are approximated and estimated using particle filtering approach and information regarding daily market prices, traded volumes, temperatures and natural gas prices and supply. Several papers are dedicated to using more fundamental approaches and bid stacks for wholesale auction curves' estimation. Among these papers are e.g. [Howison and Coulon, 2009] or [Carmona et al., 2013].

Among more recent studies relevant for wholesale auction curves' forecasting are several papers which attempt to predict price and demand curves from a functional perspective. [Liebl et al., 2013] derive electricity price forecasts by interpreting hourly spot prices as noisy discretization points of smooth price-demand. The authors of this paper motivate their study by the merit-order model. [Míguez et al., 2015] make an attempt to use functional approach to predict day-ahead electricity price and demand curves on the grounds of the corresponding historical curves available in the Spanish markets. [Chen and Li, 2017] forecasts Californian daily electricity price curves by using



(a) Initial wholesale auction curves

(b) Transformed wholeasle auction curves using the method described in [Coulon et al., 2014]

Figure 3: Initial wholesale day-ahead supply and demand curves (left) and their transformed version (right) recorded in the German EPEX SPOT SE on 16-02-2018 23:00:00.

a time-varying functional autoregressive model and smoothing the 24 discrete hourly observations over a continuous time interval.

One of the biggest steps in the direction of econometric modeling and forecasting of wholesale supply and demand curves in electricity markets was done by [Ziel and Steinert, 2016]. The so-called X-model developed in this paper makes electricity price and volume forecasts, albeit in a unique manner. Instead of using electricity price time series and fitting an econometric model to this data, the X-model is based on historical data of the entire wholesale supply and demand curves. More specifically, the X-model selects several points on the curves, with each of these points corresponding to a certain price level. Then, historical volume data recorded in wholesale supply and demand curves at these price points is retrieved. Forecasting models are then applied to each of these volumes' time series. As a result, the X-model yields forecasts for two auctions curves for the next time period, meaning that the equilibrium between these two curves exactly constitutes price and volume predictions.

Another core finding, relevant in particular for the research elaborated within the present thesis, was described in the paper by [Coulon et al., 2014]. The authors of this paper developed a methodology that allowed the demand curve in a wholesale electricity market to be transformed into a perfectly inelastic curve. Important is that this transformation does not affect equilibrium price, while pushes up equilibrium volume. This happens because all elasticities present in the demand are shifted onto the supply side. An example of transformed day-ahead auction curves is presented in Figure 3(b), where curves depicted in Figure 3(a) serve as a basis for transformation.

In other research relevant for the topic, there is a paper by [Knaut and Paulus, 2016]. The authors of this paper argue that wholesale auction curves in an electricity market can be considered from two different perspectives, with one of them excluding arbitrage opportunities of some market

participants. Analogously to the method proposed in [Coulon et al., 2014], equilibrium prices stay the same independently on the chosen perspective on the market, while volumes are higher in the case when arbitrage opportunities are prohibited. [Shah and Lisi, 2020] use the data from the Italian market and assume that each auction curve is a single object in a functional space. Then, using autocorrelation structures, forecasts for the entire auction curves are rolled out. A similar approach, though a parametric one, is followed in [Canale and Vantini, 2016] to model the prices in the Italian gas market.

1.4 Contribution of the Present Thesis

The present thesis consists of the following three academic papers:

1. **"X-model: further development and possible modifications"**

   As its name suggests, the paper provides an improvement to the X-model developed by [Ziel and Steinert, 2016]. In fact, the paper is the first attempt to improve the X-model. The key idea of the paper is to take advantage of the wholesale auction curves' transformation technique developed by [Coulon et al., 2014] and apply this technique to the initial wholesale auction curves' data. Then, the X-model is to be used with the purpose of carrying out price and volume forecasts on the grounds of the transformed auction curves. As a result, the improved X-model has several critical advantages relative to the original one: (a) its computational requirements are significantly lower and execution speed is several times quicker, (b) it is less dependent on outliers present in the data and (c) it produces more accurate forecasts.

2. **"The Impact of Renewable Energy Forecasts on Intraday Electricity Prices"**

   A novel perspective on econometric modeling of electricity prices is elaborated in this paper. The developed model, besides being suitable for intraday price forecasting, proves that the impact of errors in wind and solar power forecasts on intraday electricity prices is non-linear. Intuition of the developed model can be explained as follows. The model approximates intraday supply curves by horizontally shifting transformed day-ahead wholesale supply curves. Magnitudes and directions of the shifts are determined by means of an optimization-based algorithm and are assumed to depend solely on sizes of errors in wind and solar power forecasts. The intersections of the shifted day-ahead supply curves with intraday demand curves coincide with forecasts for intraday prices.

   This approach has two main advantages. On the one hand, the model outperforms linear and quadratic expert-model-based benchmarks. On the other hand, the model allows for a straightforward interpretation of results. This holds because a contribution of each modeled parameter to a shift size can be traced clearly, which can not be done that easily in conventional models.

   Furthermore, this paper also features an auxiliary study proving that additional wind and solar power capacities induce non-linear changes in intraday price volatility. Finally, the paper is concluded with six real-world economic and policy implications of the obtained results.

3. **"Determining Fundamental Supply and Demand Curves in a Wholesale Electricity Market"**

This paper presents another novel approach to econometric modeling and forecasting with application in electricity markets. On the grounds of several assumptions, the model developed in this paper decomposes wholesale supply and demand curves into individual buy and sell orders of market participants. In doing so, the model eliminates arbitrage orders, thus enabling a theoretical arbitrage-free fundamental curves to be constructed. As a result, the model has two main advantages. First, the derived demand curves lies "in between" perfectly inelastic demand curve as in [Coulon et al., 2014] and the original demand curve. Second, the model assigns bid and ask orders to three types of market participants (Utility, Retailer and Supplier), therefore allowing the structure of the market to be better understood. Hence, the model, besides giving multiple insights about the German electricity market, yields a more precise approximation of the actual load profile than a model with perfectly inelastic demand. Furthermore, the model allows the actual demand elasticity in the market to be determined and, despite its novelty, produces results consistent with academic literature.

Given the above mentioned, contribution of the present thesis to academic literature can be described as follows. First, the thesis much extends the section of academic literature related to econometric modeling and forecasting of wholesale auction curves recorded in electricity markets. As has been outlined in the literature overview earlier, the amount of academic papers in this area is scarce, meaning that wholesale curves, despite the amount of structural information they provide, haven't been given enough academic attention. The present thesis fills this gap. Moreover, each model, although estimated on the grounds of the German electricity data, can (with some minor modifications) be applied to other competitive electricity markets, provided that regulation of these markets does not differ dramatically to that of EPEX SPOT SE.

Second, the thesis bridges the gap between statistical and structural approaches used in the field of EPF. Each of the models, although being based on wholesale auction curves and being underpinned by sophisticated economic reasoning, uses state-of-the-art econometric tools for time series analysis and day-ahead and intraday price forecasting. Therefore, the thesis relies upon literature from both econometric and economic fields, thus linking the two fields and allowing benefits of statistical and more structural approaches to be combined.

Third, in addition to conclusions drawn on the grounds of the developed models, the thesis opens possibilities for further research. Two of the elaborated models are novel and thus can be easily extended further. Given that these models are a combination of statistical and structural approaches, advancements in both of these approaches can be used to further develop the models. Furthermore, the proposed improvement of the X-model has also been only the first attempt to work with the X-model after the original paper. Further extensions of the X-model are possible too. Moreover, the models may potentially be applied in areas other than electricity markets, e.g. in gas or oil markets.

Fourth, on the grounds of the obtained results, the thesis comments on potential real-world economic and policy implications of continuously increasing share of renewable energies. Furthermore, the impact of arbitrage orders on electricity markets has been studied within the present thesis, thus allowing market makers to better understand the structure of electricity markets. Therefore, the findings of the thesis can be of interest to researchers and practitioners (including e.g. policymakers, grid and power plant operators or market speculators) alike.

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X-model: further development and possible modifications

Sergei Kulakov[1]

Abstract

The main goal of the present paper is to improve the X-model used for day-ahead electricity price and volume forecasting. The key feature of the X-model is that it makes a day-ahead forecast for the entire wholesale supply and demand curves. The intersection of the predicted curves yields the forecast for equilibrium day-ahead prices and volumes. We take advantage of a technique for auction curves' transformation to improve the original X-model. Instead of using actual wholesale supply and demand curves, we rely on transformed versions of these curves with perfectly inelastic demand. As a result, the computational requirements of our X-model are reduced and its forecasting power increases. Moreover, our X-model is more robust towards outliers present in the initial auction curves' data.

Keywords: Energy Economics; Energy Forecasting; Econometric Modeling; Electricity Supply; Electricity Demand

JEL: C5, D4, Q41, Q47

1 Introduction

Due to the fact that electricity markets are becoming increasingly more competitive, many participants of these markets seek to optimize their trading strategies and minimize their risk exposure. As a result, the field of electricity price forecasting has developed considerably over the past decades. Following [Weron, 2014], a large amount of methods has been created to solve a variety of forecasting problems. These methods are based on a wide range of possible modeling approaches and thus have their particular advantages and disadvantages.

Predicting price spikes has proven to be one of especially important issues in the field of electricity price forecasting. These low-probability and high-impact events not only influence bidding strategies of market participants, but also affect electricity production and consumption schedules. As a result, many forecasting models were designed with the aim to capture these price spikes well. In fact, relevant for the present paper are two groups of price forecasting models: time series-based and structural ones.

The former group can be represented by ARX-type models standard in electricity price forecasting. The latest iterations of these models were described in, e.g., [Uniejewski et al., 2016] or [Narajewski and Ziel, 2019] for day-ahead and intraday electricity markets. Moreover, GARCH-type models as in, e.g., [Tan et al., 2010], [Gianfreda and Grossi, 2012] or [Liu and Shi, 2013] have proven to be suitable for the forecasting of price spikes in the Spanish, Italian, and U.K. markets, respectively. Jump diffusion models (as in, e.g., [Cartea and Figueroa, 2005], [Benth et al., 2012], [Ioannou et al., 2018] or [Muniain and Ziel, 2018]) and regime switching models (see, e.g., the work in [Karakatsani and Bunn, 2008], [Weron, 2009], [Bordignon et al., 2013], [Eichler and Türk, 2013] or [Janczura and Weron, 2012]) have also been applied successfully to predict electricity prices in a variety of settings.

On the other hand, the family of models in the latter group is extensive and more diverse. To keep the literature review concise, the focus will be placed on models that study supply and demand curves in an electricity market. The work in [Barlow, 2002] described the price process in the Alberta and California markets using the real-world auction data. The work in [Buzoianu et al., 2012]

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conducted a study of Californian electricity prices by taking advantage of latent supply and demand curves. To estimate the curves, the authors of the paper relied on temperature data, seasonality factors, and gas availability. Models by, e.g., [Coulon et al., 2014] and [Aneiros et al., 2013] followed a more functional approach and were driven by real auction curves' data. Moreover, there exists a field of more structural approaches to modeling the auction curves. For example, the work in [Carmona et al., 2013] and [Howison and Coulon, 2009] use bid stacks to determine electricity spot prices on the basis of power demand and prices of generating fuels.

However, time series-based models typically neglect fundamental dependencies present in the electricity market. In turn, structural models usually do not take substantial advantage of the underlying historical data. A model that bridges the gap is the so-called X-model developed by [Ziel and Steinert, 2016]. More specifically, the X-model uses a time series-based approach to make a prediction for entire day-ahead wholesale supply and demand curves. The intersection of the predicted auction curves yields a forecast for equilibrium prices and volumes. It follows that the X-model includes the properties of both time series and structural analyses.

More importantly, there have been many advances in the field of modeling of wholesale supply and demand curves in electricity markets. For example, the work in [Shah and Lisi, 2019] proposed another functional approach to model the wholesale supply and demand curves. Using the example of the Italian electricity market, the authors of the paper suggested to treat each curve as a single structured object in a functional space and use the autocorrelation between the curves to conduct a forecast for the entire curves. A similar functional model (albeit only a parametric one) was developed in [Canale and Vantini, 2016] and was applied to the Italian gas market. The work in [Dillig et al., 2016] measured the influence of clean energies on electricity prices in the German market. In doing so, the model in [Dillig et al., 2016] added or subtracted amounts of renewable supply from the initial auction curves' data. As a result, the model shifted the wholesale auction curves to produce results. A similar approach was followed by [Kulakov and Ziel, 2019b] who studied the impact of errors in renewable energy forecasts on intraday electricity prices. The work in [Kulakov and Ziel, 2019b] obtained the results by shifting the day-ahead auction curves to approximate the intraday auction curves. The work in [Kulakov and Ziel, 2019a] relied on the findings of [Knaut and Paulus, 2016] to manipulate the observed wholesale auction curves and derive a fundamental model of the German electricity market.

The present paper extends the field of modeling of the wholesale auction curves and provides presumably the first improvement of the X-model. More specifically, the paper is based on the concept elaborated by [Coulon et al., 2014]. Following this concept, it is possible to transform initial wholesale auction curves into their analogues with perfectly inelastic demand. Important is the fact that the equilibrium price is the same before and after the transformation. We show that using the X-model on the transformed auction curves, i.e., predicting the transformed day-ahead wholesale supply and demand curves instead of the original ones, allows the forecasting accuracy of the X-model to be improved and its computational burden to be decreased. Moreover, the improved X-model becomes more robust towards outliers present in the initial auction curves' data.

The paper has the following structure. The remainder of the present section discusses the paper by [Coulon et al., 2014] and comments on the main idea of the present paper. Section 2 is divided into three subsections. The first one provides institutional details of the German day-ahead wholesale market; the second one comments on data sources; and the third one discusses data filtering. Section 3 is devoted to the methodology. This section first provides a detailed general description of the modified X-model and then elaborates on steps for developing the model. Section 4 presents the obtained results. Section 5 concludes the paper and discusses possibilities for further research.

1.1 Main Idea

As was mentioned earlier, the concept of the wholesale auction curves' transformation described in [Coulon et al., 2014] lies in the core of the present paper. An application of the concept to day-ahead wholesale supply and demand curves observed in the German electricity market is presented in Figure 1.

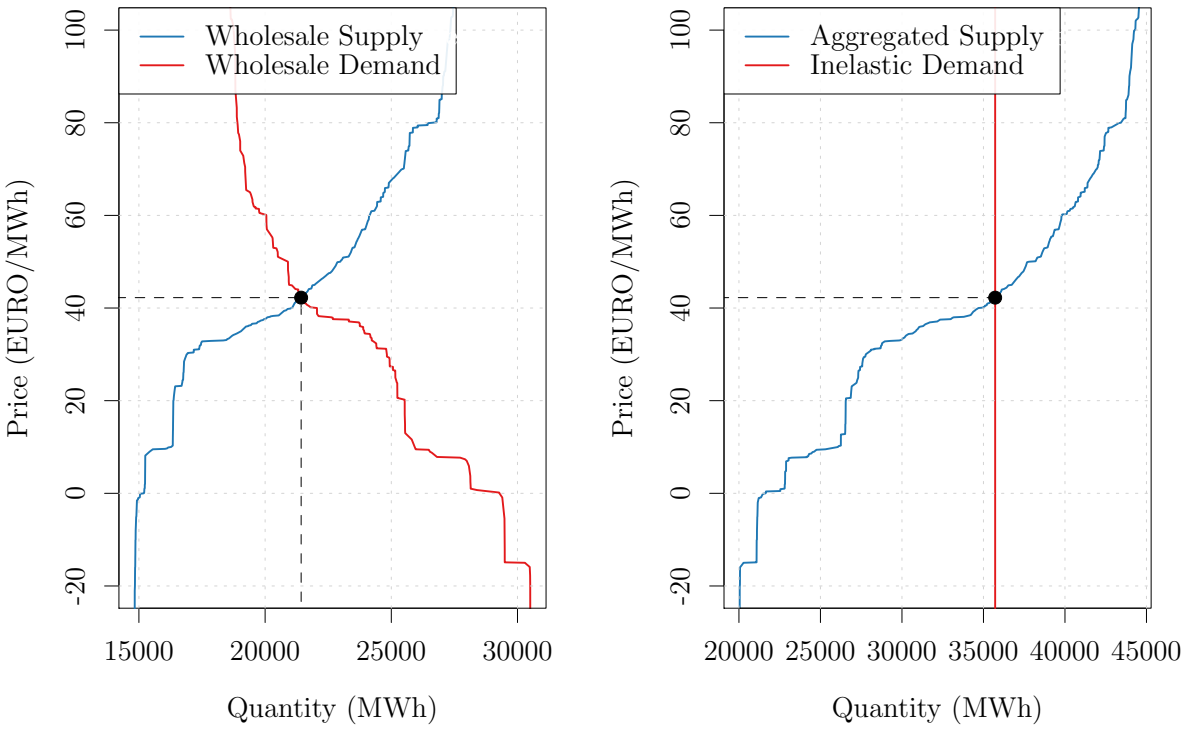

Figure 1: A wholesale market equilibrium in the EPEX SPOT SE on 2017-02-01 at 00:00:00 (left plot) vs. its manipulated form with an inelastic demand curve (right plot)

The intuition behind the concept is based on the fact that, besides the wholesale market, there exists a bilateral market for electricity trading. Electricity producers and consumers are directly connected with each other in this market. Therefore, if prices in the bilateral market are higher than prices in the wholesale market, some electricity suppliers decide to abstain from the production of electricity. Instead, they try to buy electricity in the wholesale market. Hence, the wholesale demand curve is elastic because some market participants are sensitive to the wholesale market price. However, if we assume that electricity suppliers cannot use wholesale arbitrage opportunities, only a few market participants actually are price sensitive.

Thus, as [Coulon et al., 2014] suggested, the existence of two parallel markets complicates the process of electricity markets' modeling. To avoid complications, the work in [Coulon et al., 2014] proposed to assume that all orders in the wholesale demand curve were of an arbitrage nature. Under this assumption, it is possible to shift all wholesale demand elasticities to the supply side. As a result, the demand curve turns inelastic. Moreover, equilibrium volume increases because the transformed supply curve incorporates the shifted demand orders. The equilibrium price, however, remains unchanged. Therefore, the transformed auction curves better reflect the fundamental market equilibrium because they take into account the existence of arbitrage orders.

There are two key benefits of the modified X-model with the transformed auction curves relative to the original X-model. First, predicting only one point instead of the entire demand curve requires a much smaller amount of computational time. Second, the modified X-model is less dependent on outliers present in the original auction curves' data. This holds because outliers can occur in each segment of the wholesale demand curve. The forecast of the wholesale demand can thus accumulate these outliers and lose its accuracy. On the contrary, the forecast of the inelastic demand cannot be affected by such a cumulative effect. Therefore, outliers in the initial auction curves influence the original X-model more than the modified one. As a result, the modified X-model yields more precise results.

2 Data

2.1 Institutional Details of the German Electricity Market

The EPEX SPOR SE runs several electricity trading auctions in the German bidding zone. The timing of these auctions and their detailed description is provided in, e.g., [Viehmann, 2017] or [Kiesel and Paraschiv, 2017]. The focus of the present paper will be placed on the day-ahead auction for hourly products. This auction takes place on a daily basis at 12:00 a day prior to the day of delivery. Market participants submit their bids to a system that acts as an auctioneer and establishes 24 equilibrium prices for the following day. To determine these prices, the system draws 24 combinations of day-ahead wholesale supply and demand curves on the grounds of the submitted bids. The intersections of the curves coincide with the announced prices. In turn, the prices are revealed as soon as possible starting at 12:42 on the day the auction clears. The maximal bid price in the German day-ahead market is equal to $P_{\max} = 3000$, and the lowest bidding price amounts to $P_{\min} = -500$.

Besides the day-ahead auction, several other trading venues exist in the German electricity market. Among them are continuous and non-continuous intraday markets for hourly and quarter-hourly products. The balancing market exists to account for discrepancies between final electricity supply and demand volumes. Furthermore, in addition to the wholesale market, electricity is also traded via OTC contracts.

2.2 Data Description

The present study was conducted on the day-ahead wholesale auction curves' data from the German EPEX SPOT SE (see [EPEX, 2019b] and [EPEX, 2019a]). An example of the observed curves is illustrated on the left-hand side of 1. It is important that the auction curves are constructed by aggregating price-volume combinations submitted by market participants to the electricity exchange. To ease further notation, we denote the bid volume at price $P_{\min}$ in the supply curve as $Y^{S,1}_{d,h}$ and the ask volume at price $P_{\max}$ in the demand curve as $Y^{D,1}_{d,h}$ where d and h are time indices. Thus, to build the supply curve at time point (d, h), volume $Y^{S,1}_{d,h}$ is first taken as a starting point. The bid volumes at prices $(P_{\min}, P_{\max}]$ are then one-by-one added as increments to $Y^{S,1}_{d,h}$ to draw the supply curve. The demand curve at (d, h) is then constructed analogously. Volume $Y^{D,1}_{d,h}$ is taken as a starting point, and ask volumes at prices $(P_{\max}, P_{\min}]$ are incremented one-by-one to $Y^{D,1}_{d,h}$ to finalize the demand curve.

Additional datasets were obtained from the ENTSOE (see [ENTSO-E, 2019]). These datasets included wind and solar power forecasts and total load forecast. The in-sample period extended

from 2016-01-01 to 2017-01-01. The out-of-sample period was the year 2017. The data were clock-change adjusted. The missing hours in March were calculated using the two values before and after them. The average value of the two double hours in October was taken to account for the clock-change.

2.3 Data Filtering

There were two clusters of outliers present in the original data. The first cluster was detected in $Y^{S,1}$ time series, the second one in $Y^{D,1}$. The fact that these clusters are outliers becomes apparent given that, e.g., observations $Y^{S,\gamma}_{d,h}$ and $Y^{D,\gamma}_{d,h}$ at prices $P^1_S = P_{\min} + 5$ and $P^1_D = P_{\max} - 5$, respectively, did not have any peculiarities. For instance, the volume size $Y^{S,1}_{d,h}$ in the starting point of the supply curve at time point 2016-04-01 00:00:00 equals 316 (MWh), and the corresponding volume size $Y^{S,\gamma}_{d,h}$ at P^1_S amounts to 16904.77 (MWh). Naturally, only the latter number corresponds to a realistic order size posted by a market participant. In total, 252 outlier observations were clustered between 2016-03-31 22:00:00 and 2016-05-18 00:00:00 in the values of $Y^{S,1}$, and 845 outlier observations are clustered in between 2016-04-01 07:00:00 and 2016-05-20 20:00:00 in the values of $Y^{D,1}$. No observations outside of these clusters exhibited outlier behavior. Hence, from an economic standpoint, it is possible that market participants tried to bid unrealistic volumes at the extremes of the auction curves in the hope to get very profitable deals.

Please note that these outliers influence the functioning of the X-model. This happens because they lie in the starting points $Y^{S,1}_{d,h}$ and $Y^{D,1}_{d,h}$ of the auctions curves. As was mentioned in the previous subsection, the auction curves are constructed by incrementing volumes over the starting points $Y^{S,1}_{d,h}$ and $Y^{D,1}_{d,h}$ of the curves, i.e., by adding volumes at prices $(P_{\min}, P_{\max}]$ to the volume sizes at $P_{\min}$ and $P_{\max}$ for the supply and demand curves, respectively. Therefore, if the volume forecasts for the starting points are influenced by outliers, the predictions for the entire curves, too, will be affected. In other words, if the outliers are not processed correctly, the forecasted supply and demand curves are shifted away from the true equilibrium because volumes at prices $(P_{\min}, P_{\max}]$ are incremented over wrongly predicted starting points. Furthermore, it is technically possible to directly use volumes $Y^{S,\gamma}_{d,h}$ and $Y^{D,\gamma}_{d,h}$ as starting points to construct the auction curves. The performance of the model will not be greatly reduced in this case because equilibrium prices almost never reach their extremes at $P_{\min}$ or $P_{\max}$.

To clean the outliers, a method proposed by [Weron, 2007] was used. A typical expert-type regression model similar to [Weron and Misiorek, 2008] or [Ziel, 2016] was constructed. The model has the following specification:

$$Y^{C,1}_{d,h} = \left(\boldsymbol{\beta}^C_{d,h}\right)' \boldsymbol{Z}^C_{d,h} + \varepsilon^C_{d,h} \tag{1}$$

where d and h are time indices; $C = (S, D)$ denotes supply and demand curves, respectively; $Y^{C,1}_{d,h}$ stands for an electricity volume in the starting point of a curve; $\boldsymbol{\beta}^C_{d,h}$ is a vector of regression coefficients and $\boldsymbol{Z}^C_{d,h} = (1, Y^{C,1}_{d-1,h}, Y^{C,1}_{d-2,h}, Y^{C,1}_{d-7,h}, \mathrm{DoW}^1_d, \mathrm{DoW}^6_d, \mathrm{DoW}^7_d, Y^{C,\gamma}_{d,h})$, where DoW denotes a weekday dummy. Thus, the outliers' processing model includes lags 1,2, and 7 and Monday, Saturday, and Sunday dummies. Moreover, the model is extended with additional regressors represented by the values of $Y^{S,\gamma}_{d,h}$ and $Y^{D,\gamma}_{d,h}$ at points $P^1_S = P_{\min} + 5$ and $P^1_D = P_{\max} - 5$ for the supply and demand curves, respectively. Note that in a normal (non-outlier) case, the values of $Y^{S,\gamma}_{d,h}$ for the supply curve and $Y^{D,\gamma}_{d,h}$ for the demand curve are usually very close to $Y^{S,1}_{d,h}$ and $Y^{D,1}_{d,h}$, respectively. Therefore, including $Y^{S,\gamma}_{d,h}$ and $Y^{D,\gamma}_{d,h}$ into the outliers' processing model allows the precision of the model to be improved substantially.

3 Methodology

3.1 General Description of the Modified X-Model

The X-model does not rely on equilibrium price and volume time series to carry out a day-ahead forecast of electricity prices or volumes. Instead, the X-model forecasts the entire day-ahead wholesale supply and demand curves. Since these curves are used to settle wholesale market prices, the intersection of the predicted curves yields a price or volume forecast. As was mentioned earlier, the modified version of the X-model is based on the transformed wholesale auction curves with perfectly inelastic demand. Therefore, the auction curves' transformation as described in [Coulon et al., 2014] is the first step to be undertaken.

Second, to make a prediction of the entire day-ahead supply curve at time period $t+1$, the model first selects several points on this curve. These points correspond to predefined price levels and in the original paper by [Ziel and Steinert, 2016] were referred to as price classes. The price classes are selected as follows. First, an average supply curve over T in-sample observations is computed. Then, M_S prices are selected on this curve. The goal when choosing these prices is to ensure that the horizontal distances between the selected price-volume combinations are identical. In other words, an equidistant volume grid is applied to the average supply curve to derive the price classes. Of course, the greater the number M_S of the price classes is, the more accurate the forecast becomes, but the higher the computational burden of the model is. Furthermore, given that the demand curve is perfectly inelastic, there is only one price class on the demand side. Hence, $M_D = 1$.

Third, historical volume sizes are recalled for each of the $M = M_S + M_D$ price classes. As a result, M time series with wholesale volumes at the corresponding price levels are drawn. A day-ahead forecasting model is then applied to each of the time series separately. As a result, the forecasts deliver M_S price-volume combinations on the supply curve and $M_D = 1$ price-volume combination for the demand at $t+1$.

Then, the obtained volume forecasts for $t+1$ in each of the M_S price classes are combined together (or, loosely speaking, connected with one another into one curve) to create a forecast for the entire supply curve at time period $t+1$. It is important that M_S price-volume combinations are not connected with each other with $M_S - 1$ straight lines. A more sophisticated method called supply curve reconstruction is used to retain the form and structure of the supply curve. Finally, the intersection between the predicted supply and demand curves is determined to produce the day-ahead price and volume forecasts at $t+1$.

3.2 Transformation of the Auction Curves

The first step in deriving the modified X-model is to transform the wholesale supply and demand curves. The formulas for the transformation are taken directly from [Coulon et al., 2014]. To simplify our notation, we consider both auction curves as functions of the market price. The inelastic demand curve can thus be represented as:

$$Dem_t^{inelastic} = WSDem_t^{-1}(P_{\max}) \tag{2}$$

where $WSDem$ is a function which denotes the demand curve in the wholesale market. The equation for the transformed supply curve reads

$$Sup_t^{-1}(z) = WSSup_t^{-1}(z) + WSDem_t^{-1}(P_{\min}) - WSDem_t^{-1}(z) \tag{3}$$

where $WSSup$ is a function that denotes the supply curve in the wholesale market.

3.3 Defining the Price Classes

aving transformed the auction curves, it is possible to apply the X-model and carry out price and volume forecasts. Please note that the formulas below are almost identical to those in the original paper by [Ziel and Steinert, 2016] to ensure the comparability of the original and modified X-models. However, the applied transformation of the auction curves requires us to focus only on the supply side because the demand curve is represented by a single point. As a result, we can omit many indices in the formulas below. Hence, the less sophisticated appearance of the mathematical part of the present paper is another simplification of the original X-model.

To construct the X-model, we need to determine price classes on the transformed supply curve. The price classes, as has already been mentioned, are points on the supply curve that correspond to certain prices and to which volume forecasting models are applied. The price classes are selected by first computing an average supply curve over T in-sample observations. To obtain this curve, we need to define a grid of prices $\mathcal{P}$, which have at least one positive bid volume during the in-sample period. Thus, it holds that $\mathcal{P} = \{P \in \mathbb{P} | V_t(P) > 0\}$ where $\mathbb{P}$ denotes a grid of all possible possible prices in the interval from $P_{\min}$ to $P_{\max}$ and V stands for volumes on curve Sup_t^{-1}. We then compute average volumes over T in-sample observations at prices $\mathcal{P}$ as:

$$\overline{V}(P) = \frac{1}{T} \sum_{t=1}^{T} V_t(P) \quad \text{where} \quad P \in \mathcal{P}. \tag{4}$$

Given the above expression, writing the equation for average curve $\overline{Sup_t}^{-1}$ over the in-sample period yields:

$$\overline{Sup}^{-1}(P) = \sum_{\substack{p \in \mathcal{P} \\ p \leq P}} \overline{V}(p) \quad \text{where} \quad P \in \mathcal{P} \tag{5}$$

Finally, we apply an equidistant volume grid with a step of $V_* = 500$ mW to the inverse $\overline{Sup}(V)$ of curve $\overline{Sup}^{-1}(P)$. This allows us to define the price classes as: $\mathcal{C} = \{\overline{Sup}(iV_*) | i \in \mathbb{N}\}$. As a result, the application of the equidistant volume grid with the selected size of V_* to the derived average supply curve $\overline{Sup}(V)$ yields $M_S = 19$ price classes.

Of course, the method of applying an equidistant volume grid to the average supply curve may appear too simple for determining the price classes. However, despite its simplicity, the method proves to be relatively efficient. Following an example illustrated in Figure 2, it is clear that the number of price classes is greater in the flatter segments of the transformed supply curve and is smaller in the steeper segments. However, note that the equilibrium price is typically realized in the flatter segments of the supply curve. Therefore, using the equidistant volume grid allows us to focus on more important sectors of the supply curve without using any sophisticated techniques. Furthermore, there is no certain mathematical reason for choosing the size of V_*. The selected value of V_* must allow us to obtain such an amount of price classes that their number is (a) sufficient enough for approximating the supply curve relatively precisely and (b) is not too large and is thus not computationally inefficient.

There are $M_S = 16$ price classes in the original paper by [Ziel and Steinert, 2016]. Given that there is only one price class on the demand side in our case (as compared to $M_D = 16$ in the original paper), we can use the spared computational time to select more price classes on the supply side and thus improve the quality of our supply forecast. The defined $M_S = 19$ price classes $\mathcal{C}$ are provided in Table 1.

Table 1: The defined $M_S = 19$ price classes $\mathcal{C}$

-500.0	-250.0	-100.1	-76.1	-15.8	3.6	10.0	13.6	19.2	22.3
25.9	30.0	34.0	39.1	49.4	81.0	200.0	1871.9	3000.0	

Furthermore, we define volume size $X_{S,t}^{(c)}$ in price class $c \in \mathcal{C}$ as:

$$X_{S,t}^{(c)} = \sum_{P \in \mathcal{P}(c)} V_t(P) \quad \text{where} \quad c \in \mathcal{C} \tag{6}$$

and the inelastic demand volume $X_{D,t}$ is given by:

$$X_{D,t} = Dem_t^{inelastic}. \tag{7}$$

Thus, the total amount of variables which we need to forecast equals to $M = M_S + M_D = 20$.

Equation 6 has a critical implication for the functioning of the X-model. Note that volume $X_{S,t}^{(c)}$ incorporates the sum of the volumes present in between price classes c and $c-1$. Hence, as subsection 2.2 suggests, the way the X-model builds the forecasted supply curve is similar to the way the actual wholesale supply curve is constructed. More specifically, volume $X_{S,t}^{1}$ in price class $c = 1$ is taken as the starting point. The volumes in the following price classes $(2, ..., M_S)$ are added to the value of $X_{S,t}^{1}$ as increments to draw the entire supply curve.

The above description implies that outliers present in the initial supply curves' data are accumulated in the forecast for the entire supply curve. If an outlier occurs in the price class c, not only the volume forecast for price class c is going to be affected. This outlier also affects the part of the supply curve in the segment from $c+1$ to M_S. This holds because the volumes in price classes $\{c+1, ..., M_S\}$are incremented over an influenced-by-an-outlier volume forecast for $X_{S,t}^{(c)}$. Hence, an entire part of the supply curve is going to be shifted due to the presence of an outlier in one price class c. Moreover, if outliers occur in several price classes (e.g., c and $c+N$ where $N < M_S$), then the impact of these outliers is accumulated in the segment from $c+N+1$ to M_S of the supply curve.

Of course, both the supply and demand curves in the original X-model suffer from the above described problem. On the contrary, the demand curve in the modified X-model is represented by only one point. Therefore, outliers cannot be accumulated in the demand forecast of the modified X-model. As a result, our model is more robust towards outliers and is thus more accurate than the original X-model.

3.4 Time Series Model

The forecasting model described below will be applied to carry out volume forecasts for each of $M = 20$ price classes. The model is analogous to the one in the original paper by [Ziel and Steinert, 2016] and is a simple ARX-type process with 5 external regressors. The external regressors are wholesale day-ahead market prices and volumes, as well as forecasts for electricity generation, wind, and solar power supply. Please note that in our case, the equilibrium volume coincides with the value of the inelastic demand function. Therefore, to account for the equilibrium volume separately, we consider the difference between the equilibrium volume in the setting with the transformed curves and the initial wholesale equilibrium volume, i.e., $X_{d,h}^{volume} = X_{d,h}^{Dem_{d,h}^{inelastic}} - X_{d,h}^{volume,WS}$. The time series for the modified X-model with inelastic demand is then:

$$X_{d,h} = \left(\left(X_{S,d,h}^{(c)} \right)_{c \in \mathcal{C}}, X_{D,d,h}, X_{d,h}^{price}, X_{d,h}^{volume}, X_{d+1,h}^{generation}, X_{d+1,h}^{wind}, X_{d+1,h}^{solar} \right) \tag{8}$$

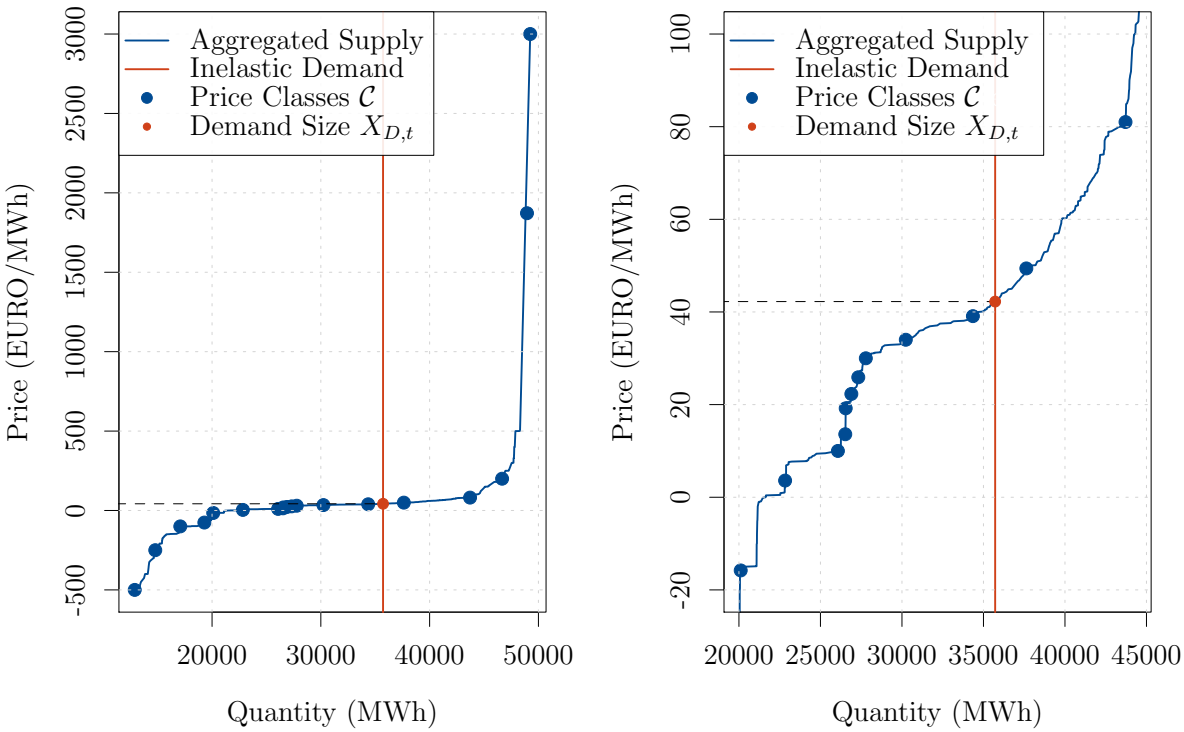

Figure 2: A wholesale market equilibrium in the EPEX SPOT SE on 2017-02-01 at 00:00:00 with transformed auction curves and highlighted price classes. The left hand side of the Figure shows the entire auction curves recorded at the time point. The right hand side of the Figure plots the same curves but focuses on the equilibrium between them.

which in this case includes $M + 5 = 25$ variables.

To capture the seasonal structure, a weekday dummy $W_k(d)$ is introduced. We assume that $\mathcal{W}(d)$ is a function which yields a number corresponding to the day of the week d and k is a day index with, e.g., $k = 1$ for Monday. Therefore, $W_k(d)$ equals to 1 if $\mathcal{W}(d) < k$ and $W_k(d) = 0$ if $\mathcal{W}(d) \geq k$. More specifically,

$$W_k(d) \begin{cases} 1, & \mathcal{W}(d) < k \\ 0, & \mathcal{W}(d) \geq k \end{cases} \tag{9}$$

Since we estimate the time series model by a BIC-based LASSO technique (for more see [Tibshirani, 1996] and [Schwarz et al., 1978]), the underlying data should be standardized. Therefore, we have to subtract the means from the original process, i.e., $\boldsymbol{Y}_{d,h} = \boldsymbol{X}^{(c)}_{d,h} - \boldsymbol{\mu}_h$ where $\boldsymbol{\mu}_h = \mathbb{E}(\boldsymbol{X}_{d,h})$. Therefore, the model under consideration can be written as follows:

$$Y_{m,d,h} = \sum_{l=1}^{M} \sum_{j=1}^{24} \sum_{k \in \mathcal{I}_{m,h}(l,j)} \phi_{m,h,l,j,k} Y_{l,d-k,j} + \sum_{k=2}^{7} \varphi_{m,h,k} W_k(d) + \varepsilon_{m,d,h} \tag{10}$$

where $\phi_{m,h,l,j,k}$, $\varphi_{m,h,k}$ and $\mathcal{I}_{m,h}(l,j)$ are sets of lags, m is a model index, and $\varepsilon_{m,d,h}$ is an error term. As in the original paper, the latter term is supposed to be i.i.d. with constant variance $\sigma^2_{m,h}$.

Please note that $\mathcal{I}_{m,h}(l,j)$ is defined as:

$$\mathcal{I}_{m,h}(l,j) = \begin{cases} \{1,2,3,...,36\}, \;\; m = l \;\text{ and }\; h = j \\ \{1,2,3,...,8\}, \;\; (m = l \;\text{ and }\; h \neq j) \;\text{ or }\; (m \neq l \;\text{ and }\; h = j) \\ \{1\}, \;\; m \neq l \;\text{ and }\; h \neq j \end{cases} \tag{11}$$

where the choice of lags and the corresponding motivation is elaborated at length in the original paper by [Ziel and Steinert, 2016]. More specifically, the process $Y_{m,d,h}$ for price class m at hour h includes: (a) 36 days of autoregressive lags, (b) values on 8 previous days at hours other than h, (c) values on the previous day at all hours in classes other than m and (d) load, wind and solar forecasts at all hours of the same day and at hour h on up to 7 days back.

To estimate $\boldsymbol{\beta}$-coefficients, we used the R-package `glmnet` which is described in, e.g., the paper by [Friedman et al., 2010]. The corresponding mathematical representation of scaled and estimated $\widehat{\widetilde{\boldsymbol{\beta}}}$-coefficients can be written as follows:

$$\widehat{\widetilde{\boldsymbol{\beta}}}_{m,h} = \underset{\boldsymbol{\beta} \in \mathbb{R}^{p_{m,h}}}{\arg\min} \sum_{d=1}^{n} \left(\widetilde{Y}_{m,d,h} - \boldsymbol{\beta} \widetilde{\mathbb{X}}_{m,d,h} \right)^2 + \lambda_{m,h} \sum_{j=1}^{p_{m,h}} |\boldsymbol{\beta}_j| \tag{12}$$

where $\mathbb{X}_{m,d,h} = (\mathbb{X}_{m,d,h,1}, ..., \mathbb{X}_{m,d,h,p_{m,h}})'$ is a $p_{m,h}$-dimensional vector of regressors, $\boldsymbol{\beta}_{m,h}$ is a corresponding vector of coefficients, $\widetilde{Y}_{m,d,h}$ denotes response variables, $\lambda_{m,h}$ stands for a penalization parameter, and tilde denotes a standardized version of a variable with its variance being scaled to one. Moreover, please note that non-standardized versions of the coefficients can be obtained easily by rescaling.

The volume forecast for the next day is thus given by:

$$\widehat{Y}_{m,n+1,h} = \sum_{l=1}^{M} \sum_{j=1}^{24} \sum_{k \in \mathcal{I}_{m,h}(l,j)} \widehat{\phi}_{m,h,l,j,k} Y_{l,n+1-k,j} + \sum_{k=2}^{7} \widehat{\varphi}_{m,h,k} W_k(n+1). \tag{13}$$

Then, we need to add sample means to the obtained values of $\widehat{Y}_{1,n+1,h}, ..., \widehat{Y}_{M,n+1,h}$ to compute the final day-ahead volume forecast $\widehat{X}_{1,n+1,h}, ..., \widehat{X}_{M,n+1,h}$. However, to calculate a precise forecast for the next day, simply adding mean values to the above defined process is not sufficient. We thus followed the procedure used in the original paper and ran a residual-based bootstrap simulation with $B = 10000$ bootstrap samples. Hence, we sampled from residual vector $\widehat{\boldsymbol{\varepsilon}}_{d,h} = (\varepsilon_{1,d,h}, ..., \varepsilon_{M,d,h})'$ only over the days d. We then used the mean of the simulated results to finalize the computation of our point forecasts.

3.5 Supply Curve Reconstruction

Application of the above described model to each of $M = 20$ time series yields day-ahead forecasts for the inelastic demand and for $M_S = 19$ points, which lie on the forecasted supply curve. Therefore, to complete the forecast for the entire supply curve, we need to connect the predicted M_S price-volume combinations with each other, i.e., draw a curve out of the predicted points. However, we want to retain and replicate the shape of the actual supply curve in our forecast. As was argued in the original paper or in, e.g., [Kulakov and Ziel, 2019b], the shape of the supply curve may influence electricity prices significantly. Hence, we did not simply draw a line over the

predicted points. Instead, we relied on a more sophisticated technique. This technique was referred to as curve reconstruction in the original paper and has the following definition:

$$\check{V}_{d,h}(P) = \frac{R(P)\overline{V}(P)}{\sum_{Q \in \mathcal{P}(c)} R(Q)\overline{V}(Q)} X^{(c)}_{S,d,h} \tag{14}$$

where $R(P) = 1$ if a price occurs at least two times a day and $R(P) = 0$ otherwise. Therefore, equation 14 allows us to divide prices in price grid $\mathcal{P}$ into two categories. The first category includes prices at which market participants did not bid often (i.e., at least two times a day). The formula neglects these prices in the further calculation. On the contrary, the volume predicted within a given price class is distributed over the prices in the second category. The more often market participants placed their bids at a particular price, the greater the share of volume assigned to this price would be. As a result, the formula retains the non-linear structure and composition of the original supply curve. Please note that reconstructing the demand curve is not necessary since this curve is perfectly inelastic. Moreover, Figure 3 provides a graphical representation of the predicted vs. actual curves after the curve reconstruction was carried out. From the figure (especially between price classes 49.4 and 81.0) it can be seen clearly that price classes are connected with curved lines.

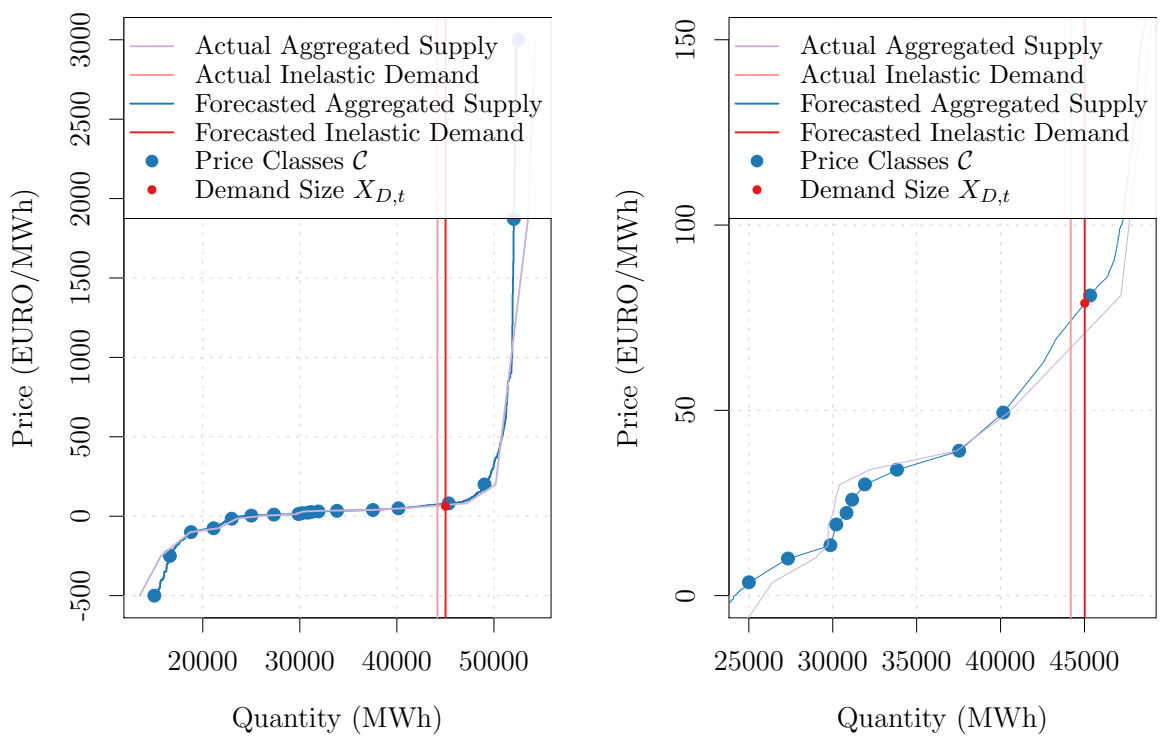

Figure 3: Market equilibrium forecast on 2017-02-01 at 10-00-00. The left hand side of the Figure shows the entire auction curves recorded at the time point. The right hand side of the Figure plots the same curves but focuses on the equilibrium between them.

Finally, the forecast for the entire supply curve can be defined as:

$$\check{S}_{d,h}(P) = \sum_{\substack{p \in \check{\mathcal{P}}_{d,h} \\ p \leq P}} \check{V}_{d,h}(p) \text{ for } \check{\mathcal{P}}_{d,h} \tag{15}$$

where $\check{\mathcal{P}}_{d,h} = \{P \in \mathcal{P} | R(P) = 1\}$. The price forecast is then an intersection between the above defined supply curve and the inelastic demand curve, i.e., $\widehat{P}^{DA}_{d,h} = \check{S}^{-1}_{d,h}(\widehat{X}_{D,d,h})$.

4 Results

To ease the notation, we will denote the original X-model by $xmod^{original}$ and the modified X-model by $xmod^{modified}$. Furthermore, the study was extended with a simple naive benchmark (as in, e.g., [Ziel et al., 2015]) denoted by *naive*. The *naive* model assumed that the value at $t+1$ was equal to the value at t. Moreover, following the findings of, e.g., [Granger and Ramanathan, 1984] or [Nowotarski et al., 2016], we considered an equally weighted mixture of the original and modified X-models. This mixture was referred as a combined X-model and had the specification $xmod^{combined} = 0.5P^{xmod^{original}}_{d,h} + 0.5P^{xmod^{modified}}_{d,h}$.

To test the performance of the models, a rolling window study was conducted. The size of the window was equal to 24 hours, and the out-of-sample period was the year 2017. The yearly comparison of the models is provided in Table 2. Please note that our definitions of the MAE and RMSE values were analogous to those in the original paper or in, e.g., [Uniejewski et al., 2017] and can be written as:

$$\text{MAE} = \frac{1}{24D}\sum_{d=1}^{D}\sum_{d=h}^{24}|P^{DA}_{d,h} - \widehat{P}^{DA}_{d,h}| \quad \text{and} \quad \text{RMSE} = \sqrt{\frac{1}{24D}\sum_{d=1}^{D}\sum_{h=1}^{24}\left(P^{DA}_{d,h} - \widehat{P}^{DA}_{d,h}\right)^2} \tag{16}$$

where $P^{DA}_{d,h}$ is an actual day-ahead price, $\widehat{P}^{DA}_{d,h}$ is a price forecast of a model, and $D = 364$. Thus, Table 2 shows explicitly that *naive* model has the worst performance in the comparison. Moreover, the modified X-model had the lowest MAE and RMSE values, even lower than those of the combined X-model. The better accuracy of the modified X-model can be explained by the model's ability to better process outliers. This ability was especially important given that the X-model was designed to better capture price spikes. Besides the superior performance, the modified X-model delivered results ca. three times quicker than the original X-model. Naturally, the improvement was present because the amount of forecasted variables was almost twice smaller in the modified version of the X-model. Please note that execution speed may vary depending on the specification of the LASSO model and its parameters.

Table 2: Comparison of the yearly MAE and RMSE values of the naive benchmark, the original X-model ([Ziel and Steinert, 2016]), the modified X-model with an inelastic demand curve, and an equally weighted mixture of the original and the modified X-models.

	MAE	**RMSE**	**Average execution time (min)**
naive	9.97	11.90	-
$xmod^{original}$	6.21	7.54	4.34
$xmod^{modified}$	**5.12**	**6.45**	1.40
$xmod^{combined}$	5.28	6.47	-

Furthermore, the MAE and RMSE values of the three X-models for each of the 24 hours of the day are provided in Figure 4. The naive model was neglected in this Figure due to the fact that its MAE and RMSE values were much higher than those of the X-models. As can be seen clearly from Figure 4, the modified X-model outperformed the original X-model during each hour of the

day. In turn, the combined X-model had lower MAE values than the modified X-model during the hours 9, 20, 21, and 22. Moreover, the hourly RMSE values of the combined model were lower than those of the modified X-model only during the hours 9 and 20.

To determine best model, we used the DM-test formula as defined in [Diebold, 2015]. The loss differential between models $\mathbb{A}$ and $\mathbb{B}$ for hour h was set to $\delta_{d,h,\mathbb{A},\mathbb{B}} = L_{d,h,\mathbb{A}} - L_{d,h,\mathbb{B}}$ where $L_{d,h}$ is the loss function of a model at hour h of day d. The respective loss functions of models $\mathbb{A}$ and $\mathbb{B}$ are $L_{\mathbb{A},d,h} = |\widehat{\varepsilon}_{\mathbb{A},d+i,h}|^{\varphi}$ and $L_{\mathbb{B},d,h} = |\widehat{\varepsilon}_{\mathbb{B},d+i,h}|^{\varphi}$, where $\varphi = 1, 2$ to compare the models with respect to both absolute and quadratic errors. The t-statistics of the DM-test is defined as $t_{DM} = \bar{\delta}_{h,\mathbb{A},\mathbb{B}}/\sigma_{\bar{\delta},h,\mathbb{A},\mathbb{B}}$ where $\bar{\delta}_{h,\mathbb{A},\mathbb{B}} = \frac{1}{D}\sum_{d=1}^{D}\delta_{d,h,\mathbb{A},\mathbb{B}}$ and $\sigma_{\bar{\delta},h,\mathbb{A}\mathbb{B}}$ denotes the standard deviation of $\bar{\delta}_{d,h,\mathbb{A},\mathbb{B}}$. The conducted DM-test proved that the modified X-model outperformed the original X-model over the course of the year 2017 (the corresponding p-value was equal to 2×10^{-9}). In fact, the results of hourly DM-tests indicated that the modified X-model was significantly better than the original X-model during each of the 24 hours of the day.

Furthermore, hourly DM-tests were conducted to compare the modified X-model $xmod^{modified}$ with the combined X-model $xmod^{combined}$ The obtained results are presented in Figure 5. The figure demonstrates that the latter model was significantly better than the former one during the hours 9 and 20 with respect to absolute errors. However, the combined X-model did not significantly outperform the modified X-model with respect to quadratic errors. A possible reason for this result is that the modified X-model was more robust towards outliers than the original X-model. Moreover, the fact that even the mixture of the two models failed to outperform the modified X-model significantly during most of the hours was yet another indication of the superiority of the modified X-model over the original one.

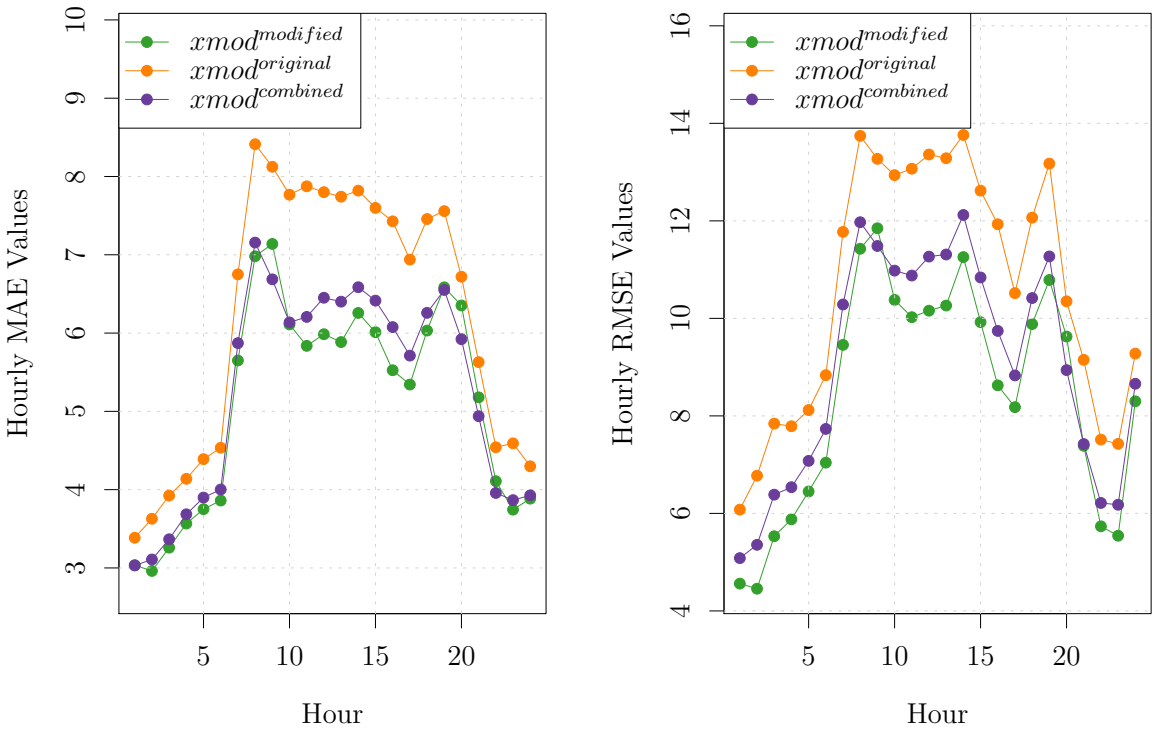

Figure 4: Hourly MAE (left hand side) and RMSE (right hand side) values of modified X-model ($xmod^{modifed}$), original X-model ($xmod^{original}$) and combined X-model ($xmod^{combined}$).

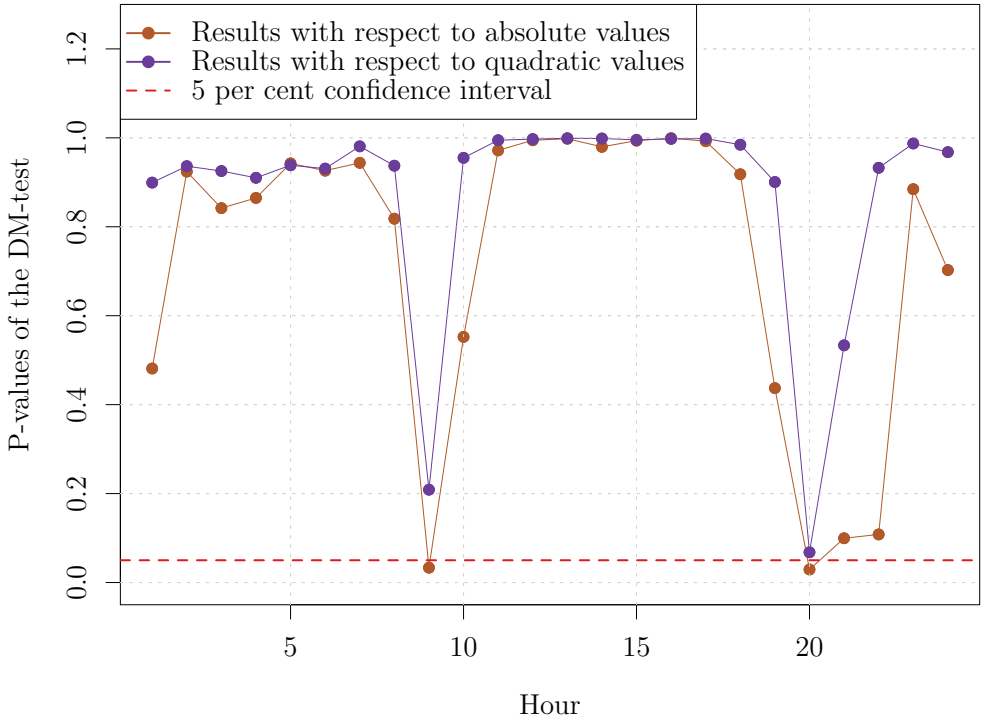

Figure 5: P-valeus for each hour of the day of the comparison of the models $xmod^{modified}$ vs. $xmod^{combined}$ according to the DM-test. As can be seen from the Figure, the modified X-model outperforms the combined X-model during most hours of the day.

5 Conclusions

The goal of the present paper was to improve the original X-model derived by by [Ziel and Steinert, 2016]. The improvement was based on the technique developed by [Coulon et al., 2014]. This technique allowed us to transform the wholesale supply and demand curves into their analogues with a perfectly inelastic demand curve. We showed that modifying the X-model by means of this technique led to a significant improvement of the final results.

More specifically, the modified X-model was shown to work faster. A decrease of the execution speed was achieved because the demand curve after the transformation was represented by only one point instead of several price classes. Moreover, the modified version of the X-model was shown to be more robust towards outliers present in the initial auction curves' data. Due to the way the auction curves were constructed by the electricity exchange, outliers were accumulated in the forecast for both the supply and demand curves in the original X-model. However, the cumulative effect was not present in the demand curve of the modified X-model. Therefore, the modified X-model yielded more accurate forecasts.

There are two possible ways to improve the X-model further. The first one is related to the issue of outliers. As was elaborated earlier, outliers are accumulated in the forecast for the entire supply curve in the modified X-model. This happens because the volumes in price classes $(2, ..., M_S)$ are added as increments to the volume $Y_{d,h}^{S,1}$ in the first price class. As a result, the entire segment

$(c, ..., M_S)$ with $c > 1$ of the supply curve is going to be shifted if there are outliers in any of price classes $(1, ..., c)$. To mitigate the problem, the following can be done. First, the last price class M_S (i.e., the one at $P_{\max}$) should be taken as the starting point for building the supply curve. In this case, volumes in price classes $(M_S - 1, ..., 1)$ are going to be incremented over the volume in price class M_S. Then, an average of the two supply curves with starting points $Y_{d,h}^{S,1}$ and $Y_{d,h}^{S,M_S}$, respectively, should be calculated. Hence, computing an average of two curves with opposite starting points allows the cumulative effect of outliers to be reduced. Analogously, a point in the middle of the supply curve can be taken as a starting point.

The second way concerns the selection of the price classes. The currently implemented method is rather simplistic and can be replaced with a more sophisticated technique. More specifically, applying an equidistant volume grid to an average supply curve does not provide any technical justification for selecting particular price classes. Instead, conventional clustering methods such as, e.g., the K-means algorithm or support vector machines can be used to select price classes.

Appendix

- Conflicts of interests: There are no conflicts of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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- Sample availability: Samples of the compounds as well as the source code are available from the authors for review purposes.

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The Impact of Renewable Energy Forecasts on Intraday Electricity Prices

Abstract

In this paper we study the impact of errors in wind and solar power forecasts on intraday electricity prices. We develop a novel econometric model which is based on day-ahead wholesale auction curves data and errors in wind and solar power forecasts. The model shifts day-ahead supply curves to calculate intraday prices. We apply our model to the German EPEX SPOT data. Our model outperforms both linear and non-linear benchmarks. Our study allows us to conclude that errors in renewable energy forecasts exert a non-linear impact on intraday prices. We demonstrate that additional wind and solar power capacities induce non-linear changes in the intraday price volatility. Finally, we comment on economical and policy implications of our findings.
Keywords: Energy economics, Energy forecasting, Energy Policy, Forecasting and Prediction Methods, Renewable Resources
JEL: C53, Q21, Q41, Q47, Q48

1 Introduction

1.1 Literature review

In an effort to curb climate change, contemporary government policies actively promote, support and even force the increased use of clean power. As a result, structural changes to energy systems are occurring. Moreover, wind and solar power is main driver of the energy transition. Multiple indicators predict their booming future due to their continuously falling costs, widespread availability and low global warming potential. Yet, energy harnessed by wind turbines or photovoltaic panels is intermittent. This signifies the importance of load and price forecasting.

The variability of the sun and wind energy is critical in the German EPEX SPOT. A simplified temporal trading scheme of this energy exchange is depicted in Figure 1. Two markets are of particular interest to us: day-ahead and continuous intraday. They differ in their temporal proximity to point t of physical electricity delivery and in their microstructures. The former market is a non-continuous limit order book auction conducted at 12:00 a day prior to t. The latter one is a continuous trading market which closes 30 minutes prior to t.

Note that prices in both markets are announced before the physical delivery of electricity occurs. Therefore, market prices are based on wind and solar power supply forecasts. Furthermore, forecasts are usually more precise in the intraday market. Hence, prices in the intraday market are closer to the actual fundamental equilibrium if market participants actively trade in both markets (see e.g. [Weber, 2010] or [Pape, 2017]). As a consequence, the influence of forecast errors on intraday prices drops the closer the trading occurs to the point of actual electricity delivery. In fact, the decrease of forecast errors can be non-linear as e.g. the work in [Kühnert, 2016], [Larson et al., 2016] or [Ahlstrom et al., 2013] suggest. [1] Moreover, note that the impact of forecast errors on intraday prices may depend on the size of the error. Therefore, this paper attempts to prove that the impact of forecast errors on intraday prices is non-linear.

The influence of forecast errors on electricity market prices has already been given a thorough attention in the academic literature. The work in [Von Roon and Wagner, 2009] demonstrated the

[1] Please note that in this paper we always mean forecast errors in wind and solar power forecasts when we refer to forecast errors.

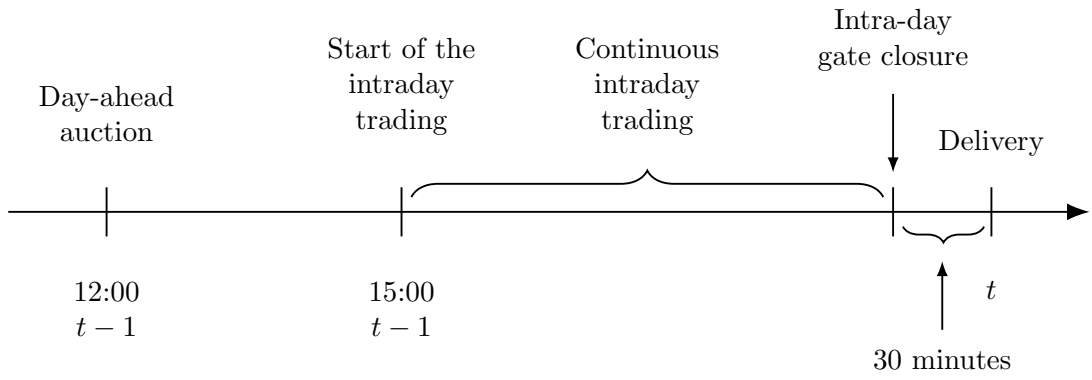


Figure 1: A simplified trading scheme of the German EPEX SPOT.

importance of errors in wind forecasts and attempted to measure the impact of these errors on intraday prices. In a more recent study, [Kiesel and Paraschiv, 2017] show that intraday prices are indeed affected by errors in renewable energy forecasts and are even sensible to the signs of forecast errors. The work in [Pape, 2017] shows that errors in wind and solar power forecasts affect not only intraday prices, but also controllable electricity producers. The work in [Pape, 2018] demonstrates that balancing day-ahead forecast errors by trading in the intraday market is preferred to using imbalancing mechanisms. [Garnier and Madlener, 2015] take the perspective of an operator who tries to compensate forecast errors in a continuous intraday market. In doing so, the authors of the paper attempt to determine optimal timing and volume decisions of the operator. The study of [Gürtler and Paulsen, 2018] uses panel data analysis and supports the conclusions drawn by [Kiesel and Paraschiv, 2017]. The key findings of the latter paper regarding asymmetries are disputed by [Ziel, 2017] who agrees that forecast errors influence intraday market prices but shows that the asymmetric effects are insignificant.

Non-linearities in the impact of forecast errors have also been analyzed in the academic literature. By investigating the Nord Pool data, [Jónsson et al., 2010] show that electricity prices react on adjustments in wind power predictions more strongly during the day than during the night. Furthermore, the study claims that the price reaction becomes smaller when the level of wind penetration rises. [Hagemann, 2013] relies on the German electricity data and demonstrates that the impact of wind forecasts on electricity prices is more significant from midnight to 8 a.m. due to office hours and forecast updates arriving. Moreover, many fundamental models which were applied to study the influence of forecast errors (e.g. [Goodarzi et al., 2019] or [Zareipour et al., 2009] besides the above mentioned) are non-linear by nature.

Nevertheless, the non-linear impact of errors in wind and solar power forecasts particularly on intraday electricity prices has not yet been studied. The present paper attempts to solve this problem and demonstrates that the impact of forecast errors on intraday prices depends on e.g. the sector of a merit-order curve in which an intraday price is realized. More importantly, one of key innovations of the present paper is an auction-curves-based non-linear econometric model which we develop. The core of the model is built around manipulations with empirical supply and demand curves (also known as sale and purchase curves) recorded in a day-ahead wholesale electricity market. Furthermore, we show that forecast errors influence the volatility of intraday prices in a non-linear manner.

The paper is organized as follows. In the second part of the present section we comment on our idea and provide an intuitive description of our auction-curves-based model. The first part of

section 2 is devoted to the data description. The second part of section 2 comments on a method to transform wholesale supply and demand curves into an equilibrium with a perfectly inelastic demand curve. Section 3 is dedicated to the description of our models. Section 4 presents the obtained results. More specifically, we discuss the results and features of our model, show the out-of-sample evaluation of the models and construct a numerical example to demonstrate the impact of forecast errors on the volatility of intraday prices. Furthermore, subsection 4.4 elaborates on economic and policy implications of our research. Section 5 concludes the paper.

1.2 Main idea

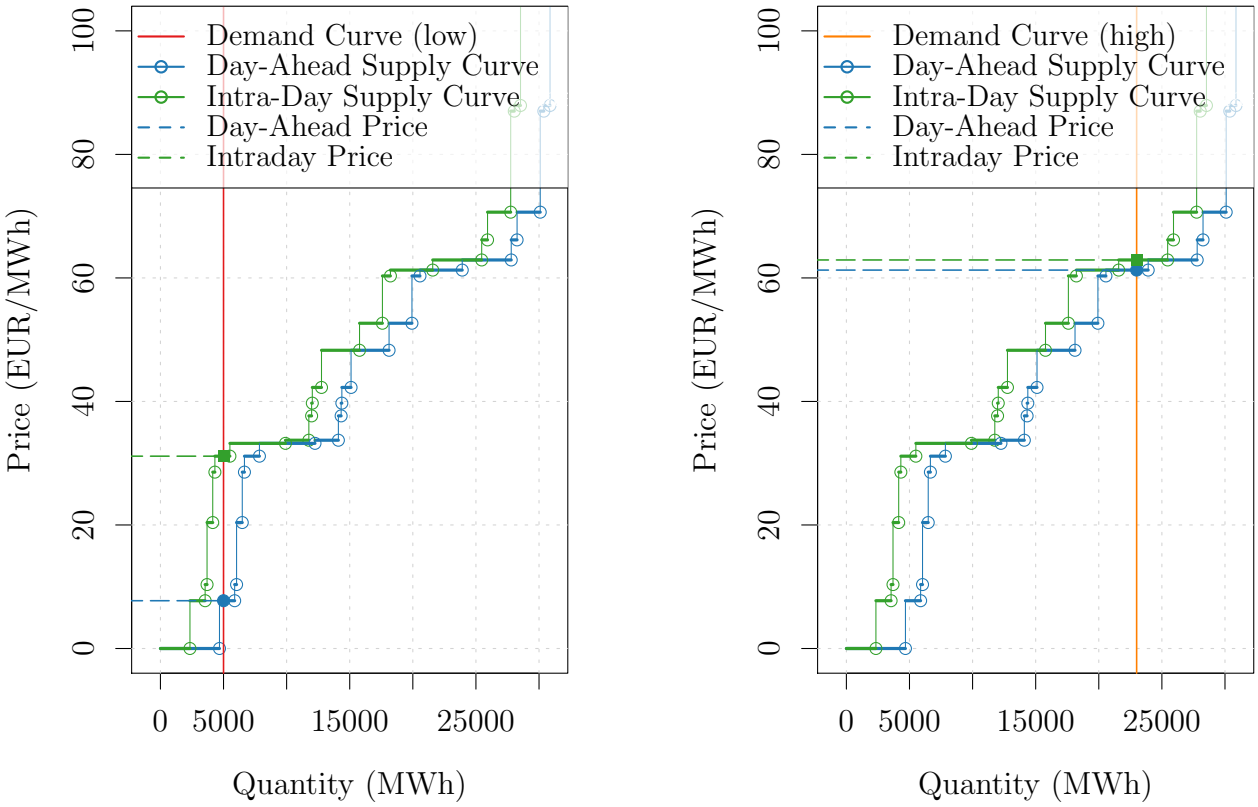
To elaborate on the main intuition behind our idea, we assume a toy example of an imaginary electricity market. This example is depicted in Figure 2. We suppose that the blue curves denote supply curves recorded in a wholesale day-ahead market. The green curves are the corresponding hypothetical intraday supply curves. For the matter of simplicity we assume that the distance between the blue and the green curves depends only on a forecast error of 2500 MW. Note that the blue curves are located to the right of the green curves. It follows that the actual amount of electricity was overestimated in the day-ahead market. Naturally, the blue curves would be shifted towards the green ones and the day-ahead price would be closer to the intraday price if the forecast in the day-ahead market would be more precise.

Note that the two curves, their shapes and the distances between them are identical on both sides of Figure 2. The only difference between the two sides of Figure 2 is the realized demand size. The intraday price is more different to the day-ahead price in Figure 2(a) (low demand case) than in Figure 2(b) (high demand case), even though the size of the forecast error is the same. Figure 2 thus demonstrates that forecast errors may influence intraday prices in a non-linear manner. As the Figure suggests, the sector of the supply curve in which the price is realized or the non-linear shape of the merit-order curve are factors which may determine the impact of forecast errors on intraday prices.

Figure 2 can also be used to discuss the functioning of our novel auction-curves-based econometric model. The model is optimization-based and is not built on the analysis of the day-ahead and intraday price time series. Instead, the model is based on manipulations with empirical wholesale supply and demand curves. More specifically, our model shifts the wholesale day-ahead supply curves to approximate the corresponding intraday supply curves. The magnitudes and the directions of the shifts are determined by (a) errors in wind and solar power forecasts and (b) absolute amounts of wind and solar power generated at the moment of delivery.[2] To optimally adjust the shift size, a non-linear optimization technique is applied. In other words, we add or subtract the adjusted forecast errors from the day-ahead supply. As a result, the day-ahead wholesale supply curves are shifted horizontally. The shifted day-ahead supply curves are thus our approximations of the intraday supply curves. Naturally, the intersections of the shifted day-ahead supply curves with the demand curves coincide with the intraday prices.[3] Therefore, our auction-curves-based

[2]We do not have the data regarding the wind and solar output at time points shortly before the delivery. Thus, to carry out our study, we focus on the actually realized data. Hence, we can not use our model for intraday price forecasting. However, our model still allows us to trace the non-linear impact of forecast errors on intraday prices.

[3]The fact that intersections of the auction curves yield equilibrium prices is the core of fundamental models (also known as structural models) elaborated by e.g. [Howison and Coulon, 2009] or [Carmona et al., 2013]. Moreover, the work in [Ziel and Steinert, 2016] provided a lengthy analysis of the intersections of the day-ahead wholesale auction curves. They showed that in 64 % of the cases the intersections between the auction curves are identical to the reported prices, in 89 % the error is less than 0.1 EUR and in 99.8 % the error is less than 1 EUR. The reason for the errors is e.g. the presence of block or other complex orders which are neglected in the auction curves.



(a) Low demand scenario with demand $D_{llow,t} = 5000$ MW

(b) High demand scenario with demand $D_{high,t} = 23000$ MW

Figure 2: A toy example of an electricity market with the distance between day-ahead and intraday supply curves being dependent only on a forecast error of 2500 MW.

model provides us with an innovative modeling approach of electricity prices. As opposed to conventional quadratic models, our model allows us to interpret the impact of each of the model's parameters. Hence, to draw further conclusions, we will compare our auction-curves-based model and its modifications with similarly parametrized linear, quadratic and combined benchmarks.

2 Inputs of the Models

2.1 Data description

Following the introductory section of the present paper, we focus on the German-Austrian EPEX SPOT market and study the period between 01.01.2016 and 31.12.2017 (see [EPEX, 2019a] and [EPEX, 2019b]). We denote day-ahead prices by P^{DA} and hourly weighted average intraday prices (usually referred to as VWAP by the EPEX SPOT) by P^{ID}.[4] As the regulation of the exchange

[4]Our choice of the weighted average intraday prices is motivated by the findings of [von Luckner et al., 2017]. Following the conclusions of this paper, the majority of orders in the continuous intraday market arrive shortly before the gate closure.

suggests, prices in the day-ahead market are bound with $P_{\max} = 3000$ EUR from above and with $P_{\min} = -500$ EUR from below. In turn, the price range in the intraday market comprises -9.999 EUR to 9.999 EUR. Moreover, besides the price data, from the EPEX SPOT we also obtain the data regarding the day-ahead wholesale supply and demand curves.

Furthermore, from ENTSO-E Transparency (see [ENTSO-E, 2019]) we have the data regarding the forecasted and actual wind and solar power supply. We index day-ahead forecasts of wind and solar generation by F and the corresponding realized values by A. Hence, there are two pairs of parameters: W^F and W^A, S^F and S^A, where W and S stand for wind and solar power, respectively. We compute forecast errors as $W^{\Delta} = W^A - W^F$ and $S^{\Delta} = S^A - S^F$.[5]

Figures 3 and 4 were constructed to present the data. The former Figure demonstrates an example of the wholesale supply and demand curves recorded in the German day-ahead market. The latter Figure shows the dynamics of day-ahead and hourly weighted average intraday prices, the total amount of wind and solar power supply and the forecast errors in wind and solar power output. From Figure 4 it can be seen that day-ahead prices may deviate from intra-day prices if the amount of wind or solar power was wrongly predicted. More specifically, the segment bounded by two red lines demonstrates that the intra-day prices can be smaller than the day-ahead prices (upper plot) when the actual amount of wind power (lower plot) was underestimated in the day-ahead market.

2.2 Transformation of empirical supply and demand curves

The toy example illustrated in Figure 2 plots a market equilibrium with a perfectly inelastic demand curve. Such setting allows us to shift the supply curve back and forward because market participants remain insensitive to the equilibrium price. However, Figure 3 demonstrates that the actual wholesale supply and demand curves are elastic. Shifting elastic curves may lead to ambiguous results because market participants may act differently depending on the equilibrium price.

To avoid this problem, we will use a method developed by [Coulon et al., 2014]. This method allows us to transform actual wholesale auction curves into a market equilibrium with perfectly inelastic demand. The equilibrium prices remain unchanged before and after the transformation, while the corresponding volume sizes increase.

The economic reasoning behind the transformation was elaborated at length in the original paper by [Coulon et al., 2014] or in e.g. [Kulakov and Ziel, 2019]. Moreover, the paper by [Knaut and Paulus, 2016] can help understand the reasoning because this paper describes trading strategies of market participants in a wholesale energy market. Generally speaking , the idea is to transfer all elasticities from the demand to the supply side. Following [Coulon et al., 2014], there exists two bilateral markets for electricity trading: an OTC market and a wholesale market. Since prices in the latter market can be lower than costs of electricity generation, arbitrage opportunities exist in the wholesale market. Therefore, market participants can try to buy electricity in the wholesale market instead of generating it. As a result, some orders in the wholesale demand curve can be of arbitrage nature. Therefore, even though being based on a somewhat relaxed assumption, transferring all demand elasticities to the supply side allows us to obtain a more stable and

[5]Please note that the data as to the renewable energy supply was provided in the quarter-hourly resolution. We used simple arithmetic averages to adjust the granularity of the data to the hourly resolution. We did not extrapolate missing data and omitted the time points in which a price-volume observation was not available in at least one of the datasets. We neglected daylight saving times within the current research and did not make any clock-change adjustments. Furthermore, we rounded the prices to two decimal places to spare computational time.

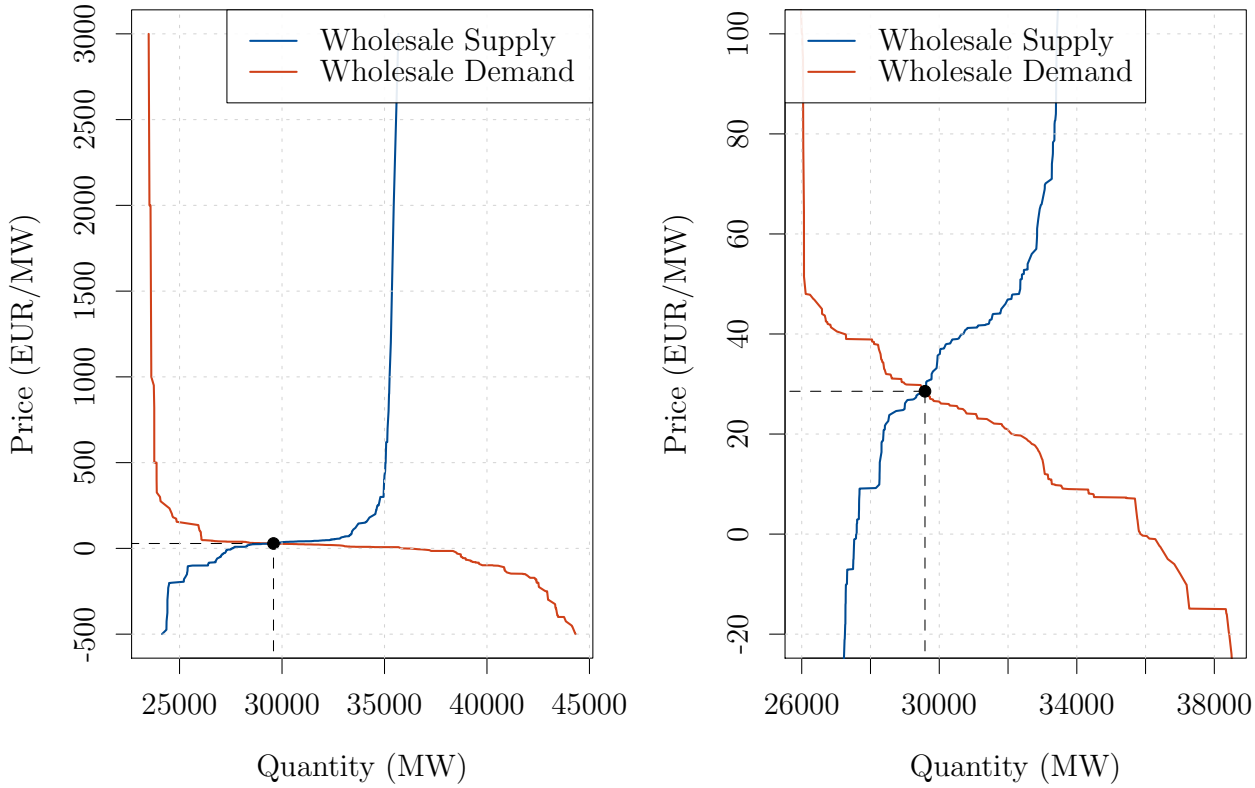


Figure 3: Wholesale supply and demand curves in the German electricity market on 2017-04-02 at 09:00:00. The Figure shows the entire auction curves (left) and the same two curves with a particular focus on their intersection (right) .

predictable market equilibrium with a perfectly inelastic demand curve.

Following the original paper, the expression for the inelastic demand curve can be written as

$$Dem_t^{inelastic} = WSDem_t^{-1}(P_{\min}) \tag{1}$$

where the demand curve in the wholesale market at time point t is denoted by $WSDem$ and $P_{\min} = -500$ as follows from the regulation of the EPEX SPOT. In turn, the equation for the transformed inverse supply curve can be written as

$$Sup_t^{-1}(z) = \underbrace{WSSup_t^{-1}(z)}_{\text{Contribution of the wholesale supply curve}} + \underbrace{WSDem_t^{-1}(P_{\min}) - WSDem_t^{-1}(z)}_{\text{Contribution of the wholesale demand cruve}} \tag{2}$$

where the supply curve in the wholesale market a time point t is denoted by $WSSup$. More specifically, the transformed version of the supply curve Sup_t at time point t consists of two parts: the initial wholesale supply curve $WSSup_t^{-1}(z)$ and an adjustment in the form of the horizontally mirrored wholesale demand curve $WSDem_t^{-1}(P_{\min}) - WSDem_t^{-1}(z)$. Therefore, following the equation and the above described intuition, the transformed supply curve Sup_t incorporates all demand elasticities additionally to the initial supply volumes. Furthermore, the supply and demand curves as defined in equations 1 and 2 allow the equilibrium price to remain unchanged before and

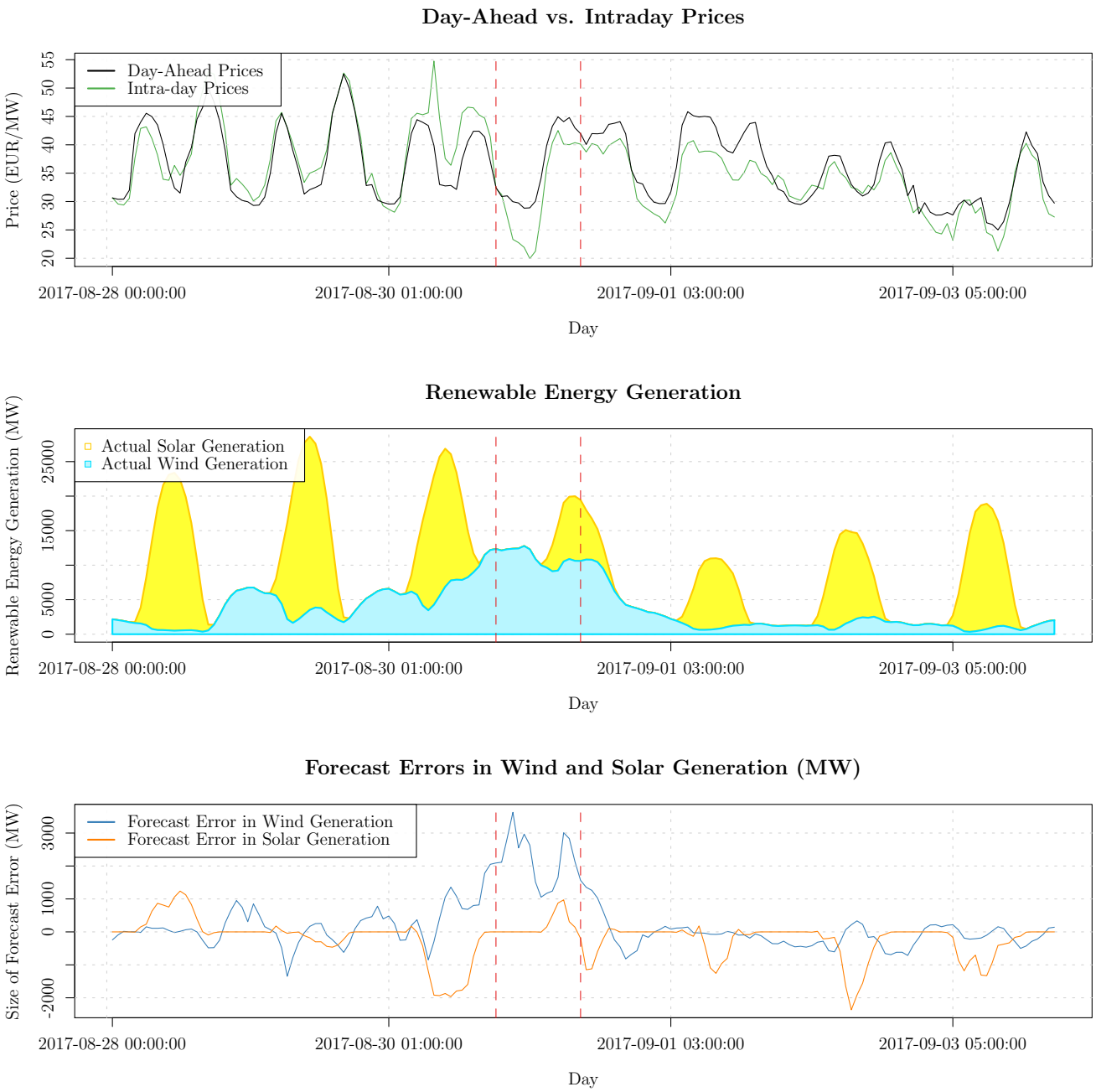


Figure 4: The dynamics of day-ahead and intra-day prices (upper plot), total generation of wind and solar energy (middle plot) and differences between actual and predicted wind and solar generation loads (lower plot) for a one-week sample from August, 28 to September, 03, 2017.

after the transformation. Furthermore, please note that equation 2 defines $Sup_t(z)$ automatically since the function is monotonic. Finally, an example of the transformed wholesale auction curves is illustrated in Figure 5.

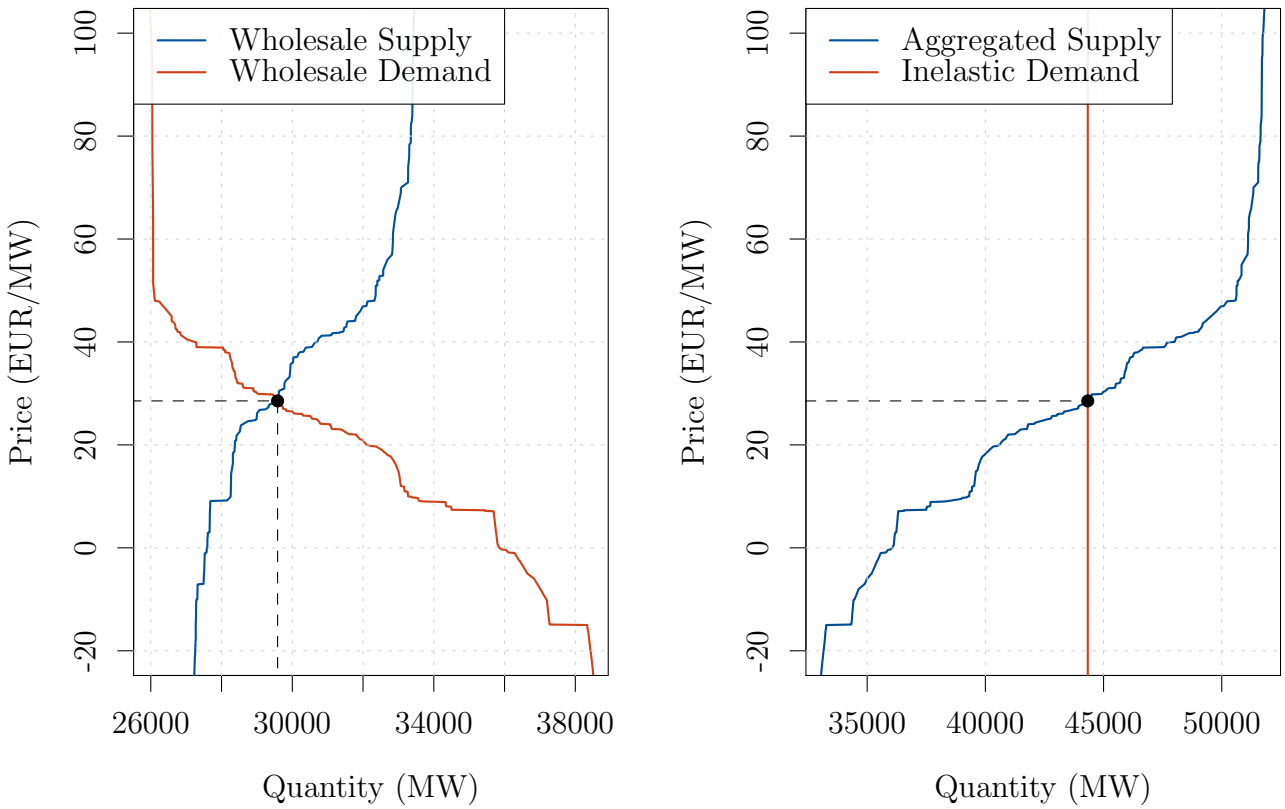


Figure 5: A wholesale market equilibrium on 2017-04-02 08-00-00 CET (left) vs. its manipulated form with the inelastic demand curve (right).

3 Methodology

3.1 Benchmark models

The benchmark models we use in the present study are state-of-the-art approaches in the field of intraday electricity price forecasting (see e.g. [Narajewski and Ziel, 2019] or [Uniejewski et al., 2019]). However, we neglect autoregressive components in our models to measure the impact of forecast errors on intraday prices.

The first benchmark model is a typical naive model $P_t^{naive} = P_t^{DA} + \varepsilon_t$, where P^{DA} stands for the day-ahead price at time point t and ε_t is an error term. The other models in our paper are based on a component denoted by $\boldsymbol{Z}_t$. This component is written in the following vector form and includes 6 elements

$$\boldsymbol{Z}_t = \left(W_t^{\Delta-}, W_t^{\Delta}, S_t^{\Delta-}, S_t^{\Delta}, W_t^{A}, S_t^{A}\right)' \tag{3}$$

where W_t^{Δ} and S_t^{Δ} are errors in wind and solar supply forecasts, respectively; $W_t^{\Delta-} = \max(-W_t^{\Delta}, 0)$ and $S_t^{\Delta-} = \max(-S_t^{\Delta}, 0)$ stand for the negative errors in wind and solar supply forecasts at t, re-

spectively; W_t^A and S_t^A denote the absolute volumes of the harvested wind and solar energy at t, respectively. Note that we model negative forecast errors separately analogously to what was done in e.g. the papers by [Soysal et al., 2017], [Kiesel and Paraschiv, 2017] or [Ziel, 2017]. Vector $\boldsymbol{Z}_t$ will thus be incorporated into several models and different estimation techniques will be used to determine the corresponding vectors of parameters $\boldsymbol{\beta} = (\beta_0, ..., \beta_i)'$.

The first linear benchmark model lm_1 can be characterized as follows

$$P_t^{lm_1} = \beta_0 + \boldsymbol{\beta}'_{1:6}\boldsymbol{Z}_t + P_t^{DA} + \varepsilon_t. \tag{4}$$

The second linear model lm_2 is analogous to the first one, save for the fact that term P_t^{DA} is assigned with its own β coefficient.

$$P_t^{lm_2} = \beta_0 + \boldsymbol{\beta}'_{1:6}\boldsymbol{Z}_t + \beta_7 P_t^{DA} + \varepsilon_t. \tag{5}$$

It follows that the model in equation 4 is a special case of the model in equation 5 with $\beta_7 = 1$. Moreover, the former model assumes a perfect cointegration of intraday and day-ahead prices. In fact the use of two similar models is justified because there is no clear academic consensus about which of them usually performs better (see e.g. [Soysal et al., 2017] or [Narajewski and Ziel, 2019]).

The last benchmark model is an extension of model lm_2 with quadratic terms. We will denote this model by qlm and make the following statement

$$P_t^{qlm} = \beta_0 + \boldsymbol{\beta}'_{1:6}\boldsymbol{Z}_t + \beta_7 P_t^{DA} + \boldsymbol{\beta}'_{8:13}\left(\boldsymbol{Z}_t \circ \boldsymbol{Z}_t\right) + \beta_{14}(P_t^{DA})^2 + \varepsilon_t, \tag{6}$$

where $\circ$ denotes the Hadamard or entrywise product.

3.2 Auction-curves-based models

Following the intuition elaborated in section 1.2, our auction-curves-based models do not focus on the day-ahead and intraday price time series. Instead, our models are based on manipulations with the wholesale auction curves. From this perspective, our models follow a novel approach, but extend the family of econometric models developed in e.g. [Ziel and Steinert, 2016], [Dillig et al., 2016], [Shah and Lisi, 2018] or [Kulakov and Ziel, 2019]. Moreover, we keep the same parameter specification in auction-curves-based and benchmark models to allow for the comparison of the models.

We will denote the first auction-curves-based model by nlm and define this model as follows

$$P_t^{nlm}(\boldsymbol{\beta}_{15:21}) = Sup_t(Dem_t^{inelastic} - \beta_{15} - \boldsymbol{\beta}'_{16:21}\boldsymbol{Z}_t)) + \varepsilon_t \tag{7}$$

where the price at t is an intersection of the shifted day-ahead supply curve with the inelastic demand curve. Furthermore, to estimate the optimal vector of coefficients $\boldsymbol{\beta}_{15:21}$, we solve a non-linear least squares problem in the form $\widehat{\boldsymbol{\beta}}_{nlm} = \arg\min_{\boldsymbol{\beta}\in\mathbb{R}^7}\left(P_t^{ID} - P_t^{nlm}(\boldsymbol{\beta}_{15:21})\right)^2$. In doing so, we use built-in `R` optimizer `optim` with default settings.

Our second model $lnlm$ incorporates linear model lm_2 and auction-curves-based model nlm. The price equation of the model can be written as follows

$$P_t^{lnlm}(\beta_{0:7,15:22}) = \beta_0 + \underbrace{\boldsymbol{\beta}'_{1:6}\boldsymbol{Z}_t + \beta_7 P_t^{DA}}_{\text{linear component}} + \beta_{22}\underbrace{Sup_t\big(Dem_t^{inelastic} - \beta_{15} - \boldsymbol{\beta}'_{16:21}\boldsymbol{Z}_t\big)}_{\text{non-linear component}} + \varepsilon_t \tag{8}$$

where the linear component coincides with the price produced by linear model lm_2 and the non-linear component is the price P^{nlm}. Writing the corresponding non-linear least squares problem yields $\widehat{\boldsymbol{\beta}}_{lnlm} = \arg\min_{\beta\in\mathbb{R}^{16}}\left(P_t^{ID} - P_t^{lnlm}(\beta_{0:7,15:22})\right)^2$.

3.3 Combined models

Following e.g. [Ziel and Weron, 2018], simply combining price models may yield further improvements of their performance. Therefore, additionally to the benchmark and auction-curves-based models, we also consider some of their equally weighted linear combinations. Please note that we report only on those models which were better compared to the non-combined models. First, model *clq* is a combination of linear model lm_2 and quadratic model *qlm* with $P_t^{clq} = 0.5P_t^{lm_2} + 0.5P_t^{qlm}$. We use model lm_2 and not lm_1 since the former one shows better performance with respect to its MAE and RMSE value. Second model *cnq* has equation $P_t^{cnq} = 0.5P_t^{nlm} + 0.5P_t^{qlm}$, third model *clnq* can be represented as $P_t^{clnq} = 0.5P_t^{lnlm} + 0.5P_t^{qlm}$.

4 Results

4.1 Model analysis

The obtained β-coefficients for the year 2017 are summarized in Table 2. Please note that β-coefficients of linear models lm_1 and lm_2 and quadratic model *qlm* are primarily negative. These findings are consistent with those of e.g. [Ziel, 2017], [Kiesel and Paraschiv, 2017], [Clò et al., 2015], [Ketterer, 2014] or [Gürtler and Paulsen, 2018] where the signs of the coefficients are similar to the ones we obtained. Hence, when the sizes of e.g. negative forecast errors grow, market participants expect a lower electricity supply. As a result, market prices rise and thus β-coefficients are negative.

Yet, the coefficients of the auction-curves-based models are primarily positive. The higher the magnitude of e.g. a positive forecast error is, the more will the merit-order be shifted to the right, and thus the lower the prices are (see e.g. [Neubarth et al., 2006], [Cludius et al., 2014], [Ketterer, 2014], [Kiesel and Paraschiv, 2017], [Fürsch et al., 2012] or [Roldan-Fernandez et al., 2016]). From this perspective, the intuition behind the functioning of both linear and auction-curves-based models is the same.

The application of model *nlm* to the real data is illustrated in Figure 6. The areas highlighted in various colors demonstrate the shift sizes induced by the components of the model. The presence of the shaded areas to the left of the intraday supply curve indicates that the shift was partially negative. Furthermore, the intersection of the shifted supply curve with the inelastic demand curve yields price P_t^{nlm}.

Moreover, we constructed Figure 7 as an example to show the differences in the impacts of positive and negative forecast errors on intraday prices. Both sides of the Figure plot the day-ahead supply curve (blue) recorded on 2017-04-19 at 10:00:00 and simulated shifts of this curve. The green curves show the shifts induced by negative forecast errors, the purple curves by positive. We used the framework of model *nlm* to build up the Figure. Both sides of the Figure incorporate a low demand scenario with $D_{low,t} = 37440$ MW and a high demand scenario with $D_{high,t} = 56740$ MW. We assume that the absolute amounts of wind and solar power output were equal to $W_t^A = 15000$ MW and $S_t^A = 15000$ MW. Figure 7(a) shows a scenario with forecast errors only in wind forecasts, i.e. $W^{\Delta-}{}_t = -5000$ MW, $W_t^{\Delta} = 5000$ MW and $S_t^{\Delta} = 0$ MW. On the contrary, Figure 7(b) shows a scenario with $S^{\Delta-}{}_t = -5000$ MW, $S_t^{\Delta} = 5000$ MW and $W_t^{\Delta} = 0$ MW.

From Figure 7 it appears clear that the horizontal distance between the green and blue curves is greater than the corresponding distance between the blue and purple curves. Following Table 2, coefficients $W_t^{\Delta-}$ and $S_t^{\Delta-}$ are positive. Therefore, the impact of negative forecast errors on the shift of the day-ahead curves is greater than the impact of positive errors. As a result, the green curves are moved further from the initial day-ahead supply curves. From the economic standpoint,

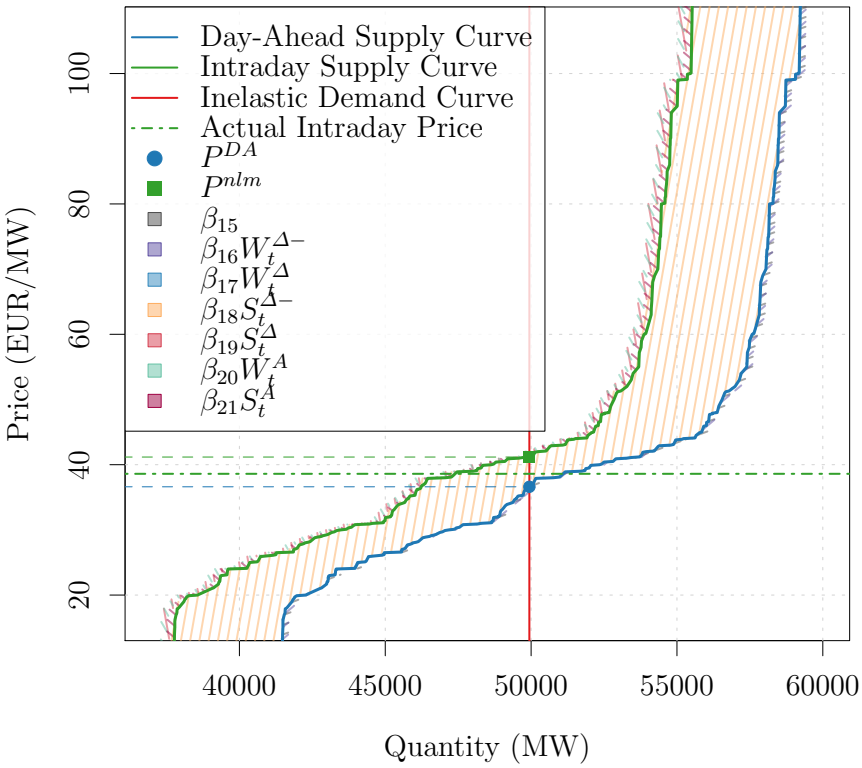
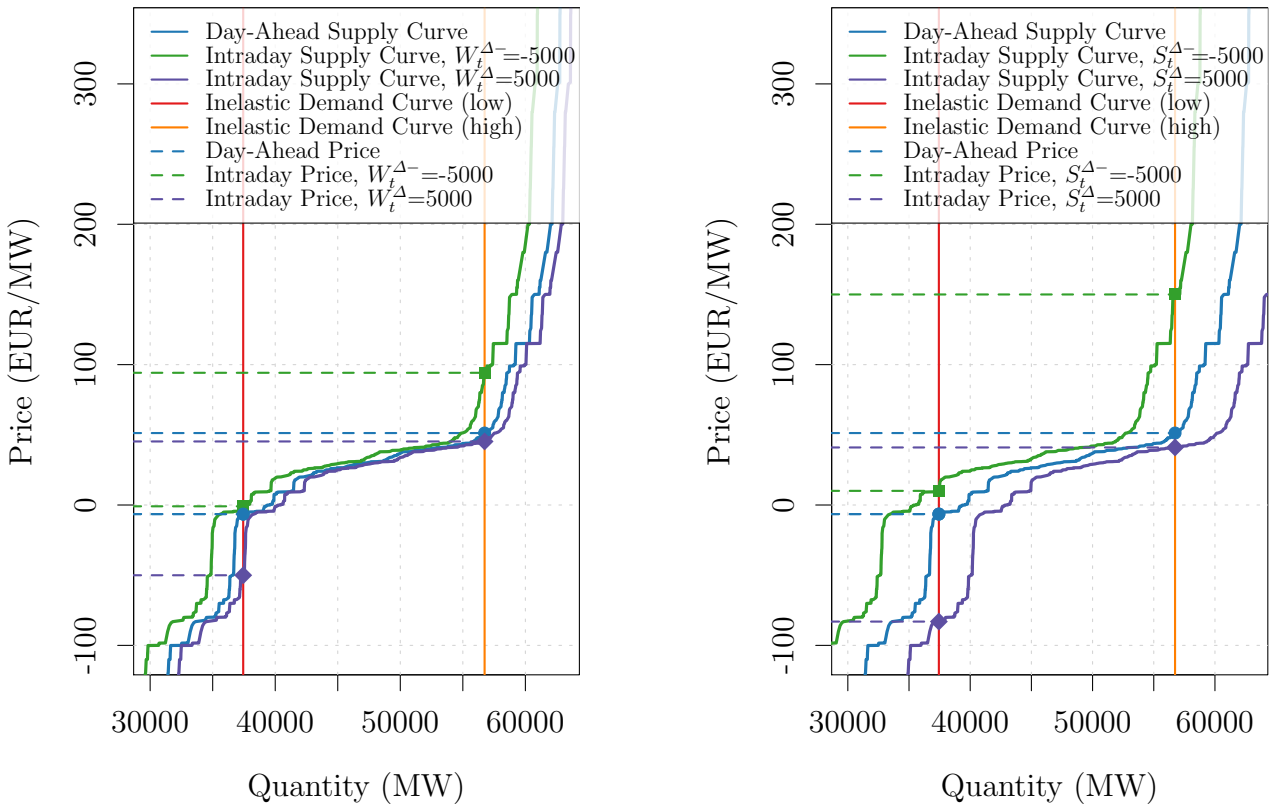


Figure 6: Price P_t^{nlm} on 2017-04-19 at 10:00:00 as the result of shifting the transformed day-ahead supply curve to the left.

we can argue that negative forecast errors can lead to the use of positive reserve power capacities . Running additional power plants is costly, which is why negative errors can exert a greater impact on intraday prices.

Moreover, it appears clear that the horizontal distances between the green, blue and purple curves are greater in Figure 7(b) than in Figure 7(a). In other words, an error in a solar power forecast causes a potentially greater impact on intraday prices than an equally sized impact in a wind power forecast due to solar occurring on hours with usually higher load (i.e. on peak hours). However, large forecast errors (as e.g. of size $S^{\Delta} = 5000$ MW or $S^{\Delta-} = -5000$ MW in our example) are relatively rare compared to similarly sized errors in wind forecasts due to a smaller amount of the generated solar output. For example, in our sample (for both years 2016 and 2017) the absolute mean error (MAE) in wind forecasts was about 1000 MW, for solar 330 MW. Thus, the magnitude of an absolute average error in a solar forecast was roughly a third of that of a wind forecast. However, the corresponding mean absolute percentage error (MAPE) values were about 10% for the wind power and about 8% for the solar power.

Furthermore, we also analyzed asymmetries in wind and solar power forecasts. Figure 8 plots the corresponding β-coefficients for the negative parts of the errors. Please note that the coefficients of linear model lm_2 (left side of the Figure) are negative, the coefficients of auction-curves-based model nlm (right side of the Figure) are positive. More importantly, both sides of Figure 8 show that the influence of negative forecast errors tends to drop over the year 2017. Similar findings are



(a) $W_t^{\Delta-} = -5000$ MW (green curve), $W_t^{\Delta} = 5000$ MW (purple curve) with $S_t^{\Delta} = 0$ MW

(b) $S_t^{\Delta-} = -5000$ MW (green curve), $S_t^{\Delta} = 5000$ MW (purple curve) with $W_t^{\Delta} = 0$ MW

Figure 7: The day-ahead curve (blue) recorded on 2017-04-19 10:00:00 and its simulated shifts as results of positive and negative forecast errors for a low demand case with $D_{low,t} = 37440$ MW and $P_t^{DA} = -6.48$ and a high demand case with $D_{high,t} = 56740$ MW and $P_t^{DA} = 51.25$.

in e.g. [Gürtler and Paulsen, 2018].

4.2 Out-of-sample evaluation

To evaluate the performance of the models, we first used the MAE and RMSE measures with the following specifications

$$\text{MAE} = \frac{1}{24D} \sum_{d=1}^{D} \sum_{d=h}^{24} |P_{d,h}^{ID} - \widehat{P}_{d,h}^{ID}|$$

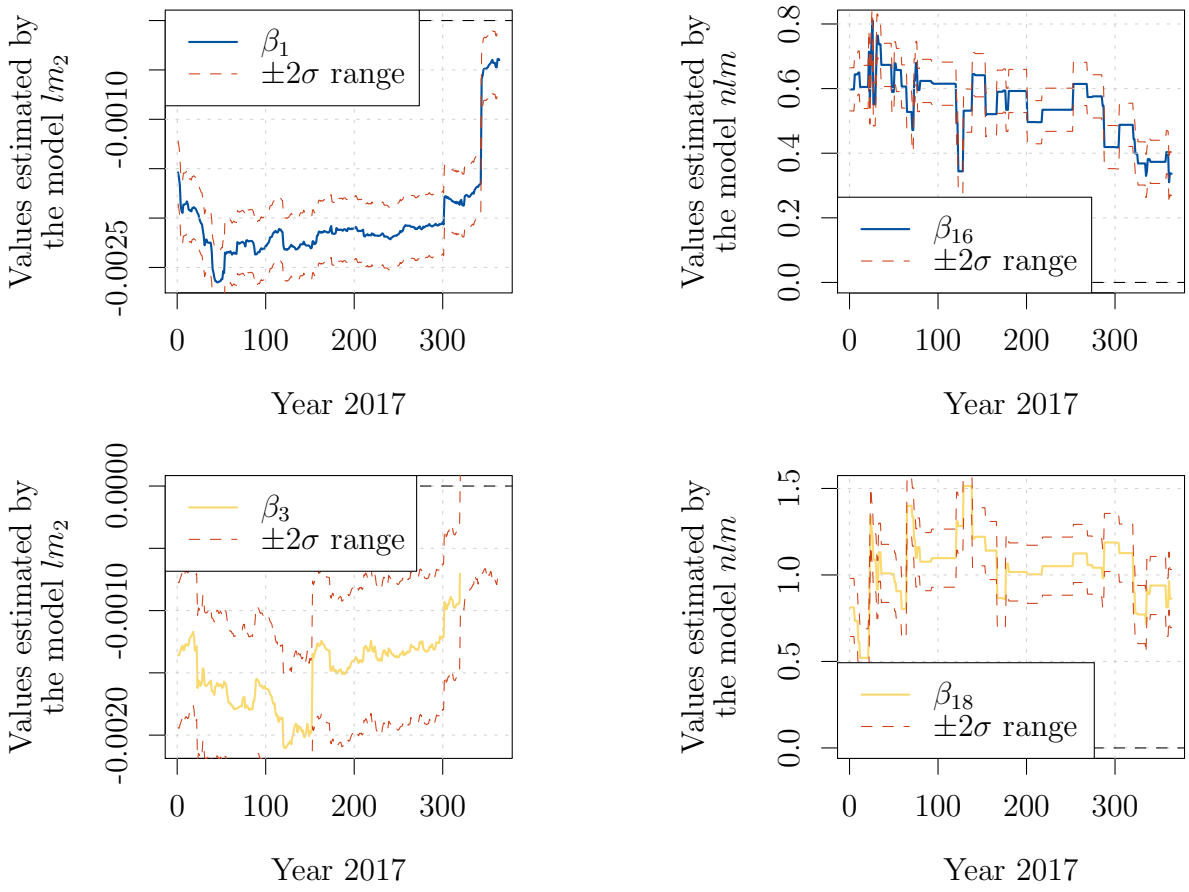


Figure 8: β-coefficients for the negative parts of forecast errors for models lm_2 and nlm

and

$$\text{RMSE} = \sqrt{\frac{1}{24D}\sum_{d=1}^{D}\sum_{h=1}^{24}\left(P_{d,h}^{ID} - \widehat{P}_{d,h}^{ID}\right)^2}$$

where $D = 364$ and h is a hour index. Hence, we used a rolling window study with 365 in-sample observations (year 2016) and 364 out-of-sample observations (year 2017). The window size was equal to 24 hours.

The results of the MAE and RMSE tests are summarized in Table 1. The Table allows us to see that model lm_2 has lower MAE and RMSE values than model lm_1. Model lm_2 was thus used in model $lnlm$. Furthermore, model nlm does not surpass the linear models and quadratic model qlm. Our model $lnlm$ performs better than model qlm with respect to the RMSE measure and has a very similar MAE value. In turn, combined model clq fails to beat model qlm. Moreover, the MAE value of combined model cnq and the RMSE value of model $clnq$ are best compared to those of the other models .

To determine best model among the ones considered, we used two types of the DM-tests. The first one is a multivariate version of the DM-test that provides one statistics on the grounds of 24-dimensional vector of errors. This approach was described in [Ziel et al., 2016]. The second one is the standard univariate DM-test that evaluates models in each of the 24 hours as defined in [Diebold, 2015]. To specify the parameters of the multivariate DM-test, we define the loss

	Naive	lm_1	lm_2	qlm	nlm	$lnlm$	clq	cnq	$clnq$
MAE	4.884	4.294	4.277	4.242	4.336	4.247	4.244	4.198	**4.185**
RMSE	8.048	7.352	7.286	7.103	7.123	7.097	7.171	**6.984**	7.010

Table 1: MAE and RMSE values of the models

differential between models $\mathbb{A}$ and $\mathbb{B}$ on day d as $\delta_d^{\mathbb{A},\mathbb{B},\varphi} = L_d^{\mathbb{A},\varphi} - L_d^{\mathbb{B},\varphi}$ where $L_d^{\omega,\varphi}$ is the loss function of model $\omega = \mathbb{A}, \mathbb{B}$ on day d with $\varphi = 1, 2$ to compare the models with respect to $\|\cdot\|_1$-norm ($\varphi = 1$) and Euclidian norm $\|\cdot\|_2$ ($\varphi = 2$). For the univariate DM-test, we define the loss differential between models $\mathbb{A}$ and $\mathbb{B}$ at hour h on day d as $\delta_{d,h}^{\mathbb{A},\mathbb{B},\varphi} = L_{d,h}^{\mathbb{A},\varphi} - L_{d,h}^{\mathbb{B},\varphi}$ where $L_{d,h}^{\omega,\varphi}$ is the loss function of model $\omega = \mathbb{A}, \mathbb{B}$ with $\varphi = 1, 2$ to compare the models with respect to absolute errors ($\varphi = 1$) and quadratic errors ($\varphi = 2$). The loss functions of models $\mathbb{A}$ and $\mathbb{B}$ in the multivariate and in the univariate settings can be written respectively as

$$L_d^{\omega,\varphi} = \left(\sum_{h=1}^{24} |\widehat{\varepsilon}_{d,h}^{\omega}|^{\varphi} \right)^{1/\varphi} \quad \text{and} \quad L_{d,h}^{\omega,\varphi} = |\widehat{\varepsilon}_{d,h}^{\omega}|^{\varphi},$$

where $\omega = \mathbb{A}, \mathbb{B}$ and $\varphi = 1, 2$. The test statistics of the multivariate and univariate DM-tests are respectively given by

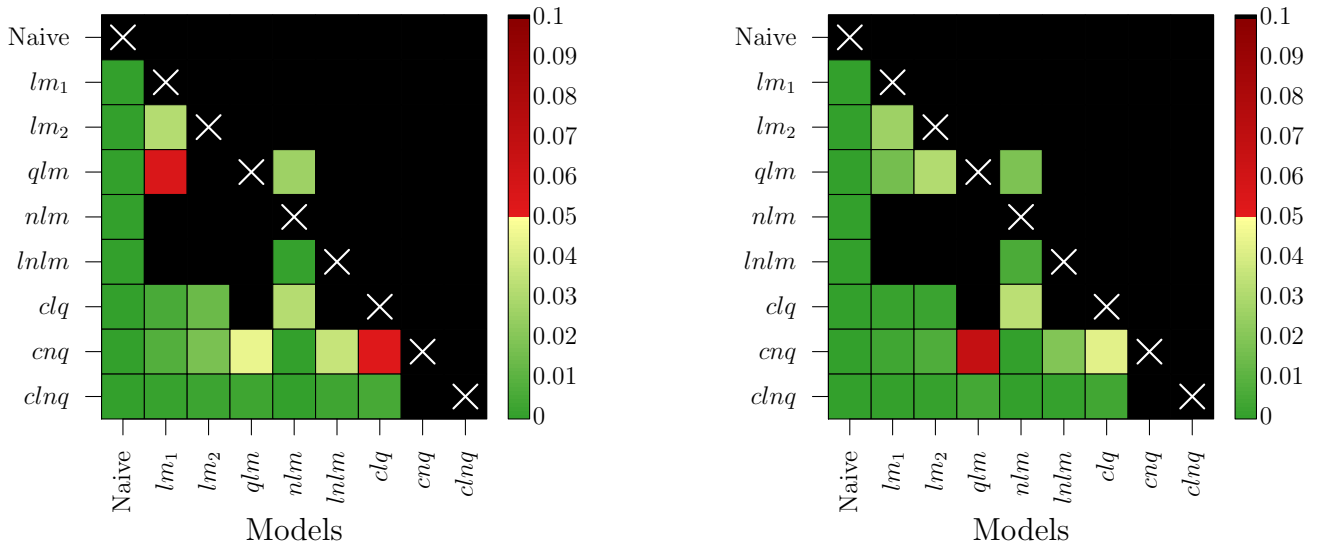
$$t^{\mathbb{A},\mathbb{B},\varphi} = \frac{\bar{\delta}^{\mathbb{A},\mathbb{B}}}{\sigma^{\bar{\delta},\mathbb{A},\mathbb{B}}} \quad \text{and} \quad t_h^{\mathbb{A},\mathbb{B},\varphi} = \frac{\bar{\delta}_h^{\mathbb{A},\mathbb{B}}}{\sigma_h^{\bar{\delta},\mathbb{A},\mathbb{B}}}$$

where $\bar{\delta}^{\mathbb{A},\mathbb{B}} = \frac{1}{D}\sum_{d=1}^{D} \delta_d^{\mathbb{A},\mathbb{B}}$ and $\bar{\delta}_h^{\mathbb{A},\mathbb{B}} = \frac{1}{D}\sum_{d=1}^{D} \delta_{d,h}^{\mathbb{A},\mathbb{B}}$. Moreover, $\sigma^{\bar{\delta},\mathbb{A},\mathbb{B}}$ and $\sigma_h^{\bar{\delta},\mathbb{A},\mathbb{B}}$ denote sample standard deviations of $\bar{\delta}_d^{\mathbb{A},\mathbb{B}}$ and $\bar{\delta}_{d,h}^{\mathbb{A},\mathbb{B}}$, respectively.

The comparison of all models with each other according to the multivariate DM-test is illustrated in Figure 9. More specifically, Figure 9(a) shows the comparison with $\varphi = 1$, Figure 9(b) with $\varphi = 2$. As Figure 9 suggests, linear model lm_2 performs better than model lm_1. Furthermore, neither quadratic benchmark qlm nor models nml and $lnlm$ are significantly better than model lm_2 when $\varphi = 1$. However, model qlm shows better performance than the linear models and than model nlm when $\varphi = 2$. In turn, the combined models tend to show overall best performance. More specifically, model clq outperforms models lm_2 and $lnlm$. Models cnq and $clnq$ outperform the other models in the comparison, however, none of the two can significantly outperform another. Hence, Figure 9 allows us to conclude that the combinations of linear and non-linear models show overall best performance among our models.

Figure 10 illustrates the hourly DM-test comparisons of models $clnq$ (Figure 10(a)) and $lnlm$ (Figure 10(b)) against our best benchmark model qlm. Given the 5% confidence interval, we see that model $clnq$ outperforms model qlm during several hours of the day. More importantly, as Figure 9 shows, model $clnq$ shows overall better performance than model qlm. Therefore, the overall performance improvement was achieved due to the fact that model $clnq$ performs better during several hours of the day and is not significantly worse during the other hours. Hence, we show that combining our auction-curves-based model $lnlm$ with the quadratic benchmark allows us to achieve a significantly better result. Moreover, Figure 10(b) demonstrates that model $lnlm$ beats model qlm during two peak hours of the day when $\varphi = 1$. However, model qlm is almost better than model $lnlm$ at hour 5 when $\varphi = 1$ and is better at hour 21 when $\varphi = 2$. Hence, given the conclusion drawn from Figure 9, model $lnlm$ is not significantly better than model qlm.

The above findings have the following implications. First, our model $clnq$ can be applied successfully to model intraday prices. Despite being unconventional, the model yields similar (and



(a) Multivariate DM-test with $\varphi = 1$

(b) Multivariate DM-test with $\varphi = 2$

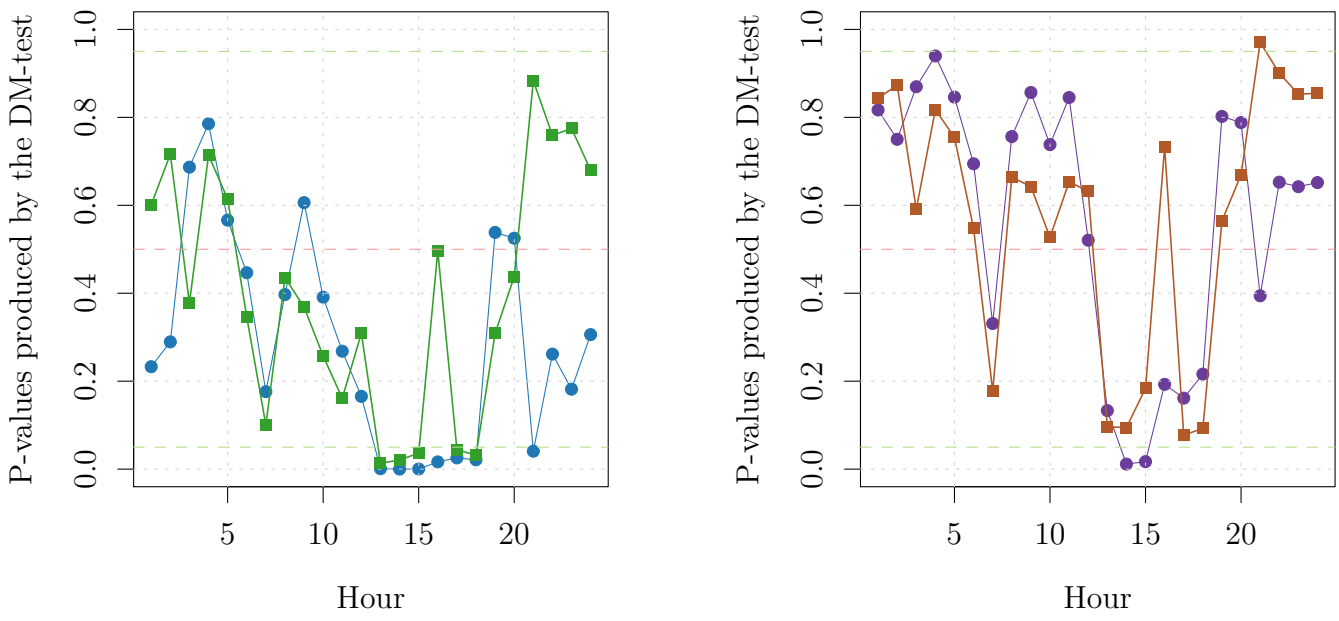
Figure 9: Results of the multivariate DM-test with $\varphi = 1$ (left) and with $\varphi = 2$ (right)

even superior) performance relative to quadratic model *qlm*. Furthermore, our model allows for a straightforward interpretation of results . As opposed to quadratic model *qlm*, we can easily interpret the influence of each of the considered parameters by studying the contributions of each parameter to the shift size. Second, given the fact that the main components of our models are forecast errors in wind and solar power supply and absolute amounts of wind and solar power, we can conclude that the impact of forecast errors on intraday prices is non-linear. This holds because model *qlm* outperforms the linear models and because our combined auction-curves-based models *cnq* and *clnq* shows a better performance even compared to quadratic model *qlm*. Hence, as was mentioned earlier, the non-linear shape of the merit order curve and the sector of this curve in which the equilibrium price is realized are possible reasons for the non-linear impact of forecast errors on intraday electricity prices. In fact, the exact shape of the actual merit order curves remains unknown when both day-ahead and intraday prices are established. Therefore, wind and solar forecast errors cause a non-linear impact on intraday market prices.[6]

4.3 Forecast errors and volatility of intraday prices

Following e.g. [Clò et al., 2015], additional wind and solar power capacities not only induced a merit-order effect in Italy, but also increased the volatility of electricity prices. These findings are consistent with the work in [Woo et al., 2011] where a similar study is conducted for the electricity market in Texas. The authors of the latter paper show that the rising wind generation induces an increase in the variance of 15-minutes electricity spot prices. The corresponding analysis of the German electricity market is provided in [Ketterer, 2014]. The conclusions of this paper show that a rise in the volatility of electricity prices may stem from the growing penetration of renewable resources.

[6]Following e.g. [Kyle, 1985] or [de Frutos and Manzano, 2014], higher market liquidity implies lower risk premia. Therefore, the impact of a higher forecast error on intraday prices may be lower in a deeper market. Yet, deriving a scientific proof of the statement is a subject of another study.



(a) Models *clnq* vs. *qlm*

(b) Models *lnlm* vs. *qlm*

Figure 10: Results of the DM-test comparison of models *clnq* vs. *qlm* (left) and *lnlm* vs. *qlm* (right) for each hour of the day for the out-of-sample year 2017. Circles denote p-values with $\varphi = 1$, squares denote p-values with $\varphi = 2$.

In the present section, we will develop a numerical example to show that the rising amount of wind or solar power capacities in fact increases the volatility of intraday prices. More importantly, our example demonstrates that the growth in the price volatility is driven by rising forecast errors and is non-linear.

We assume an operating onshore wind power plant and suppose that this power plant is extended with additional capacities. Let W_t be the amount of energy currently harvested by the plant, $\widetilde{W}_t$ an incremental wind supply from additional wind power capacities and $\gamma \geq 0$ a scale factor. As the conventional portfolio theory suggests (see e.g. [Berk and DeMarzo, 2007]), the generation variance of the extended wind power plant can be computed as follows

$$\mathrm{Var}[W_t + \gamma\widetilde{W}_t] = \mathrm{Var}[W_t] + 2\gamma\rho_{W_t,\widetilde{W}_t}\sqrt{\mathrm{Var}(W_t)\,\mathrm{Var}(\widetilde{W}_t)} + \gamma^2\,\mathrm{Var}[\widetilde{W}_t]$$

where ρ denotes the correlation coefficient between the outputs of the old and new capacities. Assuming that variances $\mathrm{Var}[W_t]$ and $\mathrm{Var}[\widetilde{W}_t]$ are equal allows the above expression to be represented as follows

$$\mathrm{SD}[W_t + \gamma\widetilde{W}_t] = \mathrm{SD}[W_t] \underbrace{\sqrt{1 + 2\gamma\rho_{W_t,\widetilde{W}_t} + \gamma^2}}_{\text{greater than 1 if } \rho_{W_t,\widetilde{W}_t} > -\gamma/2} \tag{9}$$

Hence, the standard deviation of the electricity output of the combined power plant increases when $\rho > -\gamma/2$ (which is especially the case for $\rho > 0$). Moreover, the unpredictability of the

volatile energy output implies that forecast errors, too, are high. In fact, [Weber, 2010] suggests that the magnitude and amount of forecast errors rises together with expanding wind and solar power capacities.

To show that growing forecast errors may induce a non-linear increase in the volatility of electricity prices, we consider a numerical example. As [Weber, 2010] suggests, we can assume that forecast errors are proportional to the standard deviation of the power generation. Given this assumption, we can modify component $\boldsymbol{Z}_t$ to test the sensitivity of our models to changes in the amounts of wind and solar power capacities. In line with equation 9, we suppose that our models are applied to a market with both old and additional wind and solar power capacities. We assume that modified component $\boldsymbol{Z}_t$ is denoted by $\boldsymbol{Z}_{t,\gamma}$ and can be represented as follows

$$\begin{aligned}\boldsymbol{Z}_{t,\gamma} = \Big(& W_t^{\Delta-}\sqrt{1+2\gamma_W\rho_{W_t,\widetilde{W}_t}+\gamma_W^2}\ ,\ W_t^{\Delta}\sqrt{1+2\gamma_W\rho_{W_t,\widetilde{W}_t}+\gamma_W^2}\ , \\ & S_t^{\Delta-}\sqrt{1+2\gamma_S\rho_{S_t,\widetilde{S}_t}+\gamma_S^2}\ ,\ S_t^{\Delta}\sqrt{1+2\gamma_S\rho_{S_t,\widetilde{S}_t}+\gamma_S^2}\ , W_t^A\big(1+\gamma_W\big)\ ,\ S_t^A\big(1+\gamma_S\big)\Big)\end{aligned} \tag{10}$$

To test our models under the new assumption, component $\boldsymbol{Z}_t$ was replaced with $\boldsymbol{Z}_{t,\gamma}$ in two of our models. We decided to compare our best linear model lm_2 with auction-curves-based model nlm to demonstrate the differences between linear and non-linear settings. We have chosen $\gamma = 0.1, 1, 5$ for the amounts of additional capacities and $\rho = 0, 0.5, 0.8$ as possible correlation coefficients to conduct the study. Table 3 summarizes the standard deviations of prices P^{lm_2} and P^{nlm} relative to the respective baseline models with $\gamma = 0$ and $\rho = 0$. Table 4 shows relative differences between the true and the modeled intraday prices for the 0.1% quantile relative to the respective baseline models with $\gamma = 0$ and $\rho = 0$. Table 5 shows the same as Table 4 but for the 99.9% quantile. Hence, selecting different values of scaling coefficients γ and ρ allows us to test and re-evaluate the model presented in this section under various assumptions. As a result, we show that e.g. a linear increase in the values of γ (i.e. a linear increase in additional wind and solar power capacities) induces a non-linear increase in the standard deviations of electricity prices. Therefore, given that the amount of forecast errors and the sizes of wind and solar power capacities are proportional, the model elaborated in this section is yet another confirmation of the fact that the impact of forecast errors in wind and solar power forecasts on electricity prices is non-linear.

Note that the values in all three tables tend to increase the greater the additional power capacities are (i.e. the bigger γ is) or the stronger the correlation between the old and the new capacities is (i.e. the higher ρ is). More importantly, the values grow much quicker in model nlm. This indicates that the increase in the values is non-linear. For example, when considering Table 3, non-linear effects can be seen by comparing the difference in the values produced by models lm_2 and nlm for $\gamma = 1, \rho = 0.8$ and $\gamma = 5, \rho = 0.8$, especially for W+S case. Therefore, given the assumption that forecast errors are proportional to the standard deviations, Table 3 shows that the volatility of intraday prices increases when forecast errors grow. Hence, the numerical example allows us to conclude that forecast errors not only have a non-linear impact on intraday prices, but also influence the intraday price volatility in a non-linear manner.

Furthermore, as the work in e.g. [Ziel, 2017] and [Monforti et al., 2014] shows, wind and solar forecast errors are nearly uncorrelated and the power output from wind and solar resources is negatively correlated. The model described in this section allows us to account for the effect of these correlations. More specifically, we achieve this effect in the 0.5(W+S) case, where, following equation 9, a part of the variation induced by additional wind and solar capacities is eliminated. Of course, this effect is not present in the other cases and thus the volatility is highest in the W+S case. As a result, Tables 3-5 show that simultaneous build up of wind and solar capacities in

the 0.5(W+S) case induces a relatively small change (or even decreases) the volatility of intraday prices, especially at the 0.1% and 99.9% quantiles.

Tables 4 and 5 offer another interesting conclusion. Table 4 shows that both in linear and non-linear models in the case of solar power the values can drop when γ increases from 0.1 to 1. This observation means that a moderate increase in the solar power capacities lowers the volatility of intraday prices at the 0.1% quantile. The drop in the volatility happens because e.g. negative price spikes occur less often due to the increased solar output during the peak hours. A similar effect can be seen in Table 5 which focuses on the 99.9% quantile. More specifically, the values in Table 5 can be lower than 1 in all four cases. However, the observed effect is stronger in linear model lm_2. It follows that moderate increase in wind and solar capacities can be beneficial for lowering the influence of positive price spikes.

The model elaborated in section 4.3 shows that locations of new wind and solar power plants must be selected to minimize the correlations between the outputs of the old and new capacities. This issue has already been investigated on the grounds of empirical data in e.g. the papers by [Palmintier et al., 2008], [Jónsson et al., 2010] and [Grams et al., 2017]. Moreover, our numerical example supports the findings of these papers. In fact, there are several factors which may influence the strength of the correlation. For example, a new wind power plant can be built spatially close to the old one, or a new plant can be built in a place with similar weather conditions to the old one. As a result, fluctuations in wind output will influence the power supply of both old and new power plants simultaneously. Therefore, e.g. an unexpectedly small amount of wind in the system will almost equally affect the power supply of both old and new plants. Hence, the volatility of the power supply (and thus of electricity prices) increases. On the other hand, if the outputs of the old and new plants are negatively correlated, a drop in the output of one power plant will be offset by an increase of the output of another. The impact of extreme events on intraday prices and their volatility, too, is lower the lower the correlations between the old and new plants are. As a result, the overall volatility of electricity supply remains lower when the correlations are minimized. Hence, basic conclusions of conventional portfolio theory regarding portfolio diversification (see e.g. [Cochrane, 2009]) hold in this context too.

4.4 Policy implications

The above described models demonstrate the need to decrease the non-linear impact of forecast errors on electricity prices and their volatility. The problem seems especially important given that the shares of clean power in the worldwide energy mix are expected to grow steadily. Given the setting of our models, the policy implications that we draw in this subsection are more applicable to the case of Europe, especially to the countries participating in the XBID (cross-border intraday initiative). Thus, the following can be done to achieve the reduction of the impact of forecast errors on electricity prices:

a. **Regional deployment**

The model elaborated in section 4.3 and Tables 3-5 show that locations of new wind and solar power plants must be selected to minimize the correlations between the outputs of the old and new capacities. This holds because lower variability of renewable energy supply will lead to lower forecast errors and thus lower variability of electricity prices. Therefore, our findings are consistent with academic research. More specifically, e.g. [Engeland et al., 2017] argue that renewable energies should not be concentrated in several parts of a country and should be spread more equally across the country's area because spatial diversification of renewable

resources on regional and local levels decreases the variability of energy output. Furthermore, as the work in e.g. [Drake and Hubacek, 2007], [Hasche, 2010], [Handschy et al., 2017], [Novacheck and Johnson, 2017] or [Eising et al., 2020] shows, the variability of wind supply drops when wind farms are spread geographically. Similar conclusions for the case of solar power are drawn in e.g. [Mills, 2010], [Perez et al., 2012] or [Perez et al., 2016]. Therefore, policy makers can employ a more intensive and directed subsidization of renewable resources in particular regions, or direct the support only on certain regions to trigger regional deployment.

b. **Diversity of renewable power supply**

As was mentioned earlier, the model developed in section 4.3 allows us to account for the effect of negative correlation between outputs of wind and solar power. Therefore, we can conclude that maintaining a balance between additional wind and solar capacities is important for offsetting the impact of forecast errors and keeping the prices more stable. Similar conclusions regarding the benefits of simultaneous integration of negatively correlated wind and solar power (as well as diverse wind and solar technologies) can be found in e.g. [Widén, 2011], [Sioshansi and Denholm, 2013], [Zheng et al., 2019] or [Eising et al., 2020] . Besides, expanding the energy mix with further renewable energy sources (e.g. geothermal or tidal) will induce positive portfolio smoothing effects. These effects lead to a further decrease in the volatility of energy output as described on the grounds of empirical data in e.g. [Awerbuch, 2006], [Engeland et al., 2017], [Monforti et al., 2014] or [Zipf and Möst, 2013].

c. **Cross-border interconnections and intraday trading**

The work in e.g. [Association, 2009], [Scholz, 2012], [Paternò et al., 2016] or [Brown et al., 2016] shows that building the cross-border transmission infrastructure to avoid country-specific bottlenecks is a financially efficient decision. In fact, [Schlachtberger et al., 2017] demonstrates that restricting continental transmission expansion in Europe leads to a non-linear cost increase of up to 30%. The work in [Grams et al., 2017] suggests that collaboration on the European level will decrease the variance of wind power output, whereas the absence of collaboration will imply a further increase in overall wind output variability. Therefore, a greater support (especially a greater centralized support on e.g. the EU-level) should be given to developing the cross-border electricity infrastructure (see e.g. [Jacottet, 2012] or [Puka and Szulecki, 2014]). In addition to the cross-border interconnections and for the case of Europe, it is also important to include more countries into the cross-border intraday trading system (XBID). As e.g. [Kath, 2019] or [TGE, 2019] argue, expanding the XBID system will allow for an increased European market market liquidity and thus improve overall market and transmission efficiency.

d. **Better quality of renewable energy forecasts**

Given that the impact of forecast errors on intraday prices is non-linear, the quality of renewable energy forecasts should be improved to minimize forecast errors. Of course, general improvements of methodology and computational capacities (see e.g. [Chang et al., 2014], [Antonanzas et al., 2016] or [Voyant et al., 2017]) should be considered in this context. Moreover, a measure suggested in [Joos and Staffell, 2018] or [Pinson, 2016] is to establish a centralized platform for information sharing between participants of energy markets, especially between power plant operators and transmission system operators. Furthermore, an improve-

ment of meteorological models can be beneficial for increasing the overall performance of electricity forecasting models (see e.g. [Andrade and Bessa, 2017] or [De Giorgi et al., 2011]).

e. **Flexible generation and demand side management**

Technically speaking and following e.g. [Pape et al., 2016], flexibility measures allow the merit-order to stay wider and more elastic for longer periods of time. Thus, as the intuition behind our auction-curves-based models (section 3.2) suggests, the impact of shifting the supply curve on intraday prices will be lower because a greater part of the shift will take place inside of more elastic segments of the merit order. In fact, measures on the supply side include greater investments into flexible power plants and energy storage capacities (see e.g. [Child et al., 2019], [Eyer and Corey, 2010] or [Lund et al., 2015]). Demand side management are discussed at length in a review paper by [Palensky and Dietrich, 2011] and include e.g. reducing demand during peak hours or load shifting. Examples of the latter are e.g. the use of excess electricity for domestic or industrial purposes, i.e. by replacing CHP units with boilers and electrical heating or by increasing the reliance on electric vehicles.

f. **Efficient and transparent renewable energy curtailment Management**

The practice of electricity curtailment suggests that electricity supply from renewable resources, especially from wind and solar power plants, should be cut off if further electricity infeed from these power plants threatens the stability of the energy system (a review of international practices is presented in [Bird et al., 2016]). More specifically for the case of Germany, guidelines for the curtailment are described in a regulation called EinsMan (Einspeisemanagement, see [Jacobsen and Schröder, 2012] or [Bundesnetzagentur, 2018]).However, researchers (see e.g. [Ostermann et al., 2019], [BDEW, 2017], [Ketterer, 2014] or [Joos and Staffell, 2018]) argue that centralized public disclosure of the extent and duration of the curtailment (e.g. on the grounds of a prognosis) will further improve the effectiveness of the curtailment. In fact, information should be made public as soon as possible, especially prior to moments of important decision making (e.g. for prior to the European coupling of the region day-ahead auction at 12:00 CE(S)T the case of Europe), because the announcement will allow traders to adjust their strategies for a potential curtailment. As a result, a timely announcement of the information about the curtailment will further decrease the impact of forecast errors on intraday market prices.

5 Conclusion

In this paper we studied the impact of errors in wind and solar power forecasts on wholesale intraday electricity prices. To derive our conclusions, we elaborated a novel econometric model. Our model is based on manipulations with empirical supply and demand curves recorded in a wholesale electricity market. To compute the intraday price at a given time point, we horizontally shift the corresponding day-ahead supply curve. The magnitude and the direction of the shift depend on errors in wind and solar power forecasts and absolute amounts of wind and solar power. The shifted day-ahead supply curve is our approximation of intraday supply curve. The intersection of the approximated intraday supply curve with the demand curve coincides with the intraday price. Given that we can see the contribution of each of the model's parameters to the shift size, the main advantage of our model is the ease of interpretation of the results.

Our results indicated that our auction-curves-based model outperforms other models in the study during several hours of the day. The quadratic benchmark performs better than the linear benchmarks and, since the parameters in all of our models included only forecast errors and absolute amounts of wind and solar power, we could conclude that that the impact of forecast errors on intraday prices is non-linear. Based on a numerical example, we show that the impact of forecast errors on the volatility of intraday prices is non-linear too.

Given the results of our study, we argue that it can be efficient to build additional wind and solar power capacities such that the correlations between the old and new plants is minimized. This can be achieved by improving regional deployment and cross-border interconnections. Moreover, we argue that it is efficient to minimize forecast errors themselves, introduce better flexibility measures (including flexible generation and storage capacities) and improve the demand side management. Finally, a central and timely announcement of potential curtailment measures (e.g. prior to day-ahead trading.) can further benefit the stability of energy system.

Further improvements of our auction-curves-based model are possible. For example, we only employed the shifts of the day-ahead supply curves to estimate the intraday supply curves. Additionally, we could also shift the demand curve. Moreover, using intraday data shortly before the delivery (and not the actually realized data) will allow the model to be used for intraday price forecasting. Improving the model described in Subsection 4.3 is possible by implementing a more sophisticated correlation structure between the generations of wind and solar power plants.

6 Appendix

	Multiplier	lm_1	lm_2	qlm	nlm	$lnlm$
β_0	1	$-0.19777_{\star\star\star}$	1.24052^{***}	$2.46489^{***}_{\star\star\star}$	–	0.10064
β_1	$W_t^{\Delta-}$	$-0.00039^{*}_{\star\star\star}$	$-0.00040^{*}_{\star\star\star}$	$0.00129^{***}_{\star\star\star}$	–	$0.00000_{\star\star\star}$
β_2	W_t^{Δ}	$-0.00214^{***}_{\star\star\star}$	$-0.00209^{***}_{\star\star\star}$	$-0.00410^{***}_{\star\star\star}$	–	$-0.00002_{\star\star\star}$
β_3	$S_t^{\Delta-}$	$-0.00043_{\star\star\star}$	$-0.00015_{\star\star\star}$	$-0.00173^{*}_{\star\star\star}$	–	$0.00000_{\star\star\star}$
β_4	S_t^{Δ}	$-0.00258^{***}_{\star\star\star}$	$-0.00273^{***}_{\star\star\star}$	$-0.00267^{***}_{\star\star\star}$	–	$-0.00002_{\star\star\star}$
β_5	W_t^{A}	$0.00009^{***}_{\star\star\star}$	$0.00005^{***}_{\star\star\star}$	$0.00014^{***}_{\star\star\star}$	–	$-0.00000_{\star\star\star}$
β_6	S_t^{A}	$0.00000_{\star\star\star}$	$-0.00002_{\star\star\star}$	$0.00010^{*}_{\star\star\star}$	–	$-0.00000_{\star\star\star}$
β_7	P_t^{DA}	–	$0.97019^{***}_{\star\star\star}$	$0.86481^{***}_{\star\star\star}$	–	$0.39731^{*}_{\star\star\star}$
β_8	$(W_t^{\Delta-})^2$	–	–	$0.00000^{***}_{\star\star\star}$	–	–
β_9	$(W_t^{\Delta})^2$	–	–	$0.00000^{***}_{\star\star\star}$	–	–
β_{10}	$(S_t^{\Delta-})^2$	–	–	$0.00000^{***}_{\star\star\star}$	–	–
β_{11}	$(S_t^{\Delta})^2$	–	–	$0.00000_{\star\star\star}$	–	–
β_{12}	$(W_t^{A})^2$	–	–	$0.00000^{***}_{\star\star\star}$	–	–
β_{13}	$(S_t^{A})^2$	–	–	$0.00000^{*}_{\star\star\star}$	–	–
β_{14}	$(P_t^{DA})^2$	–	–	$0.00125^{***}_{\star\star\star}$	–	–
β_{15}	1	–	–	–	$0.00004_{\star\star\star}$	$-0.00061_{\star\star\star}$
β_{16}	$W_t^{\Delta-}$	–	–	–	$0.33663^{***}_{\star\star\star}$	$0.90624_{\star\star\star}$
β_{17}	W_t^{Δ}	–	–	–	$0.39478^{***}_{\star\star\star}$	$0.24175^{***}_{\star\star\star}$
β_{18}	$S_t^{\Delta-}$	–	–	–	$0.86325^{***}_{\star\star\star}$	$1.48092^{***}_{\star\star\star}$
β_{19}	S_t^{Δ}	–	–	–	$0.37144^{***}_{\star\star\star}$	$0.25544^{***}_{\star\star\star}$
β_{20}	W_t^{A}	–	–	–	$-0.02659^{***}_{\star\star\star}$	$-0.06149_{\star\star\star}$
β_{21}	S_t^{A}	–	–	–	$-0.02590^{***}_{\star\star\star}$	$-0.01337_{\star\star\star}$
β_{22}	P_t^{nlm}	–	–	–	–	$0.55152^{***}_{\star\star\star}$

Table 2: The obtained β-coefficients, significance levels are: $\bullet$=10% *=5%, **=1%, ***=0.1% with respect to zero, $\circ$=10%, $\star$=5%, $\star\star$=1%, $\star\star\star$=0.1% with respect to one.

		Model lm_2				Model nlm			
γ	ρ	W	S	W+S	0.5(W+S)	W	S	W+S	0.5(W+S)
0.1	0.000	1.000	1.000	1.000	1.000	1.001	1.000	1.001	1.000
0.1	0.500	1.000	1.000	1.000	1.000	1.001	1.000	1.001	1.001
0.1	0.800	1.000	1.000	1.001	1.000	1.002	1.001	1.002	1.001
1	0.000	1.016	1.004	1.020	1.005	1.026	1.006	1.029	1.009
1	0.500	1.036	1.012	1.048	1.012	1.062	1.025	1.081	1.023
1	0.800	1.051	1.018	1.069	1.018	1.089	1.034	1.123	1.033
5	0.000	1.799	1.330	2.000	1.323	3.797	4.455	11.245	1.850
5	0.500	1.931	1.397	2.161	1.385	4.245	5.031	13.273	2.193
5	0.800	2.008	1.437	2.254	1.421	4.775	5.424	14.100	2.310
0	0	4.980			4.974				

Table 3: Standard deviations of prices P^{lm_2} and P^{nlm} relative to the respective baseline models with $\gamma = 0$ and $\rho = 0$. W abbreviates Wind, S abbreviates Solar.

		Model lm_2				Model nlm			
γ	ρ	W	S	W+S	0.5(W+S)	W	S	W+S	0.5(W+S)
0.1	0.000	1.005	1.000	1.005	1.002	1.047	1.000	1.047	1.023
0.1	0.500	1.000	0.997	0.997	0.999	1.042	0.992	1.042	1.009
0.1	0.800	0.997	0.996	0.993	0.996	1.036	0.986	1.036	1.009
1	0.000	1.009	0.981	0.990	0.995	1.124	0.986	1.124	1.095
1	0.500	0.977	0.963	0.940	0.970	1.059	0.986	1.068	1.094
1	0.800	0.960	0.951	0.915	0.957	1.060	0.986	1.127	1.040
5	0.000	1.076	1.043	1.232	0.965	1.851	2.166	3.158	2.105
5	0.500	1.101	1.103	1.340	0.939	1.956	2.195	3.705	2.106
5	0.800	1.129	1.136	1.401	0.945	2.438	2.200	4.009	2.107
0	0	-39.190			-29.605				

Table 4: Relative differences between true and modeled intraday prices at the 0.1% quantile relative to the respective baseline models with $\gamma = 0$ and $\rho = 0$. W abbreviates Wind, S abbreviates Solar.

		Model lm_2				Model nlm			
γ	ρ	W	S	W+S	0.5(W+S)	W	S	W+S	0.5(W+S)
0.1	0.000	0.993	1.000	0.993	0.997	0.962	0.973	0.961	0.963
0.1	0.500	0.991	1.000	0.991	0.996	0.991	0.963	0.991	0.990
0.1	0.800	0.990	1.000	0.990	0.995	1.022	0.962	1.022	0.991
1	0.000	0.966	1.000	0.966	0.979	1.060	1.046	1.061	1.025
1	0.500	0.962	1.000	0.962	0.977	1.335	1.095	1.605	1.105
1	0.800	0.967	1.000	0.967	0.976	1.468	1.095	1.851	1.109
5	0.000	1.680	1.435	1.911	1.178	7.273	19.345	19.572	3.349
5	0.500	1.870	1.558	2.133	1.188	13.708	19.587	19.723	3.648
5	0.800	1.963	1.607	2.265	1.242	19.242	19.739	19.947	3.985
0	0	25.504			26.135				

Table 5: Relative differences between true and modeled intraday prices at the 99.9% quantile relative to the respective baseline models with $\gamma = 0$ and $\rho = 0$. W abbreviates Wind, S abbreviates Solar.

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Determining Fundamental Supply and Demand Curves in a Wholesale Electricity Market

Sergei Kulakov[1] & Florian Ziel[2]

Abstract

In this paper we develop a novel method of wholesale electricity market modeling. Our optimization-based model decomposes wholesale supply and demand curves into buy and sell orders of individual market participants. In doing so, the model detects and removes arbitrage orders. As a result, we construct an innovative fundamental model of a wholesale electricity market. First, our fundamental demand curve has a unique composition. The demand curve lies in between the wholesale demand curve and a perfectly inelastic demand curve. Second, our fundamental supply and demand curves contain only actual (i.e. non-arbitrage) transactions with physical assets on buy and sell sides. Third, these transactions are designated to one of the three groups of wholesale electricity market participants: retailers, suppliers, or utility companies. To evaluate the performance of our model, we use the German wholesale market data. Our fundamental model yields a more precise approximation of the actual load values than a model with perfectly inelastic demand. Moreover, we conduct a study of wholesale demand elasticities. The obtained conclusions regarding wholesale demand elasticity are consistent with the existing academic literature.

Keywords: Energy economics, Demand Elasticity, Energy Demand, Wholesale Electricity Markets, Econometric Modeling

JEL: C5, D4, Q41, Q47

1 Introduction

The shift to renewable power is accelerating at a growing pace. Worldwide energy mix becomes increasingly reliant on wind and solar resources. Their apparent economic efficiency lures investments, while their environmental benefits attract vocal public support. Coping with their variability, however, is one of the major challenges of modern policymakers and electricity grid operators.

Even before the onset of green technologies it was clear that successful integration of renewables requires a thorough revision and redesign of current energy infrastructure. As e.g. [Turner, 1999] or [Boyle, 2004] suggest, the issue of variability of green power can be tackled in multiple ways. Among those ways are, of course, a greater reliance on energy storage technologies (see e.g. [Carrasco et al., 2006], [Ibrahim et al., 2008], or [Dunn et al., 2011]), development of decentralized and smart grids (see e.g. [Heier, 2014], [Liserre et al., 2010], [McDaniel and McLaughlin, 2009] or [Kempton and Tomić, 2005]) and, more importantly for the present paper, a more advanced demand side management (see e.g. [Palensky and Dietrich, 2011], [Mohsenian-Rad et al., 2010], [Siano, 2014]).

Modeling and forecasting in energy markets thus became an increasingly important and complex task (see e.g. [Weron, 2007] or [Burger et al., 2008]). Models vary in their degree of sophistication, and focus on a whole multitude of aspects. In turn, in this paper we elaborate an optimization-based fundamental model of a wholesale electricity market. More specifically, our model is based on econometric manipulations with wholesale electricity supply and demand curves. Important is the

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fact that these manipulations are rooted into determining a unique composition of the fundamental demand curve.

Our demand curve lies "in between" the wholesale demand curve (which incorporates many arbitrage orders) and a perfectly inelastic demand curve (which is an extreme assumption). In other words, our demand retains some elasticities, but eliminates arbitrage orders. More specifically, our demand incorporates only actual transaction with physical assess on both buy and sell sides. As a result, the demand curve in our fundamental model constitutes a better approximation of the true demand curve in a wholesale electricity market, i.e. of a curve which corresponds to the actual electricity load.

The paper is organized as follows. The remainder of the present section first provides a literature review. Then, subsections 1.1.1 and 1.1.2 elaborate on details of the papers by [Coulon et al., 2014] and [Knaut and Paulus, 2016], respectively. These papers are worthy particular attention because they lay the foundation for our model. Finally, the introduction is concluded with a general overview of our contribution. Section 2 is the description of our model. Subsection 2.1 comments on general intuition behind our idea. Subsection 2.2 is devoted to the technical summary of the model. Subsection 2.3 shows an application of our model to imaginary data. Institutional framework of the German electricity market as well as specifications of the data are outlined in section 3. Section 4 discusses the obtained results. Section 5 is the conclusion.

1.1 Literature review

As was mentioned before, the kernel of our model is built on determining the composition of the fundamental demand curve. Therefore, to keep the literature review relatively concise, its focus will be placed on three main topics: approaches to modeling residential and industrial demand, discussion of literature on wholesale demand elasticities, and econometric modeling of wholesale supply and demand curves. Special attention will be paid to works by [Coulon et al., 2014] and [Knaut and Paulus, 2016] because they lay the foundation for our research.

Residential and industrial demand for energy has been analyzed extensively in academic literature. Therefore, multiple approaches to modeling demand in electricity markets exist. Following [Labandeira et al., 2012], many of these approaches payed particular attention to estimating demand elasticity. Not surprisingly so, as [Kirschen, 2003] suggest. Knowing price sensitivity of private households and industry players can be of great utility to energy producers and grid operators. Moreover, [Albadi and El-Saadany, 2008] show that the success of different demand response programs and the corresponding cost-cuttings can be highly dependent on the accuracy of the demand elasticity estimation.

[Labandeira et al., 2012] and [Bigerna and Bollino, 2014] argue that there are two main ways to model electricity demand. The former one relies on macroeconomic data to construct aggregate econometric models. The data may include electricity prices, income levels and climatic conditions. This approach was followed by e.g. [Narayan and Smyth, 2005], [Narayan et al., 2007], [Bernstein and Griffin, 2006], or [Holtedahl and Joutz, 2004] to study residential demand, while e.g. [Kamerschen and Porter, 2004] and [Paul et al., 2009], and [Taylor et al., 2005] analyzed industrial and aggregated demand besides residential one.

The second way is to focus on microeconomic data or consumer surveys. A model may thus be based on characteristics, sizes, types, and preferences of households and companies. Among papers which restricted to this approach are e.g. [Alberini et al., 2011], [Fuks and Salazar, 2008], [Leth-Petersen, 2002], [Labandeira et al., 2006], [Fell et al., 2014], [Krishnamurthy and Kriström, 2015] or [Schulte and Heindl, 2017] who studied residential elasticities. Industrial demand is investigated

in, for example, [Woodland, 1993] and [Bardazzi et al., 2015].

On the contrary, the body of academic literature on modeling wholesale demand is rather scarce. Moreover, papers in this field may follow slightly different modeling approaches. A more traditional method is used by e.g. [Lijesen, 2007] who build up their model based on lagged day-ahead prices and temperature data. On the other hand, [Bönte et al., 2015] use wind speed as a main proxy for demand elasticity. So do [Knaut and Paulus, 2016]. [Bigerna and Bollino, 2014] pursue yet another novel approach. They critique a common practice of determining residual demand and estimate demand elasticity from the available wholesale bid data.

The present paper will be similar to the latter one in a sense that our modeling approach is also rooted into wholesale market data. However, we, too, develop a new and rather unconventional approach. We will focus solely on manipulations with wholesale supply and demand curves.

Theoretical background of our paper thus stems from the field of econometric modeling of auction curves in a wholesale electricity market. [Barlow, 2002] were among the first researchers who attempted to construct a model based on real-world electricity auction data. [Buzoianu et al., 2012] study Californian electricity prices on the grounds of latent supply and demand curves. To estimate the curves, they exploit temperature data, seasonality factors and gas availability. Furthermore, structural approaches have also be undertaken. The papers by [Carmona et al., 2013] and [Howison and Coulon, 2009] use bid stack to derive electricity spot prices on the grounds of power demand and prices of generating fuels.

Wholesale auction curves were analyzed and manipulated in the following papers. The work in [Ziel and Steinert, 2016] uses day-ahead EEX auction curves to forecast German day-ahead electricity pries. Instead of analyzing the price time series, the authors of the paper suggest to predict auction curves in their entirety. The intersection of the predicted curves, of course, coincides with the price forecast. [Dillig et al., 2016] try to quantify the impact of renewable energies on electricity prices in the German market. Their model is also based on the EEX curves. To obtain the results, the authors of the paper add or subtract amounts of renewable power generation from the initial auction curves data. In other words, their model shifts one of the curves depending on the supply of renewable energy. A similar technique is employed in [Kulakov and Ziel, 2019]. By shifting either the wholesale supply or demand curve, a non-linear impact of renewable energy generation on intra-day electricity prices is shown. The higher the renewable generation is, the stronger the prices in an intra-day market will react to the changes.

Finally, the papers written by [Coulon et al., 2014] and [Knaut and Paulus, 2016] require our particular attention because they build up the foundation for our research. These papers will thus be discussed at length in the forthcoming two sections.

1.1.1 Transformation of the wholesale auction curves

Of pivotal importance for the present study is a concept elaborated by [Coulon et al., 2014]. The concept suggests that it is possible to transform wholesale supply and demand curves such that the demand curve becomes inelastic. Noteworthy is the fact that the equilibrium price remains the same before and after the transformation. On the contrary, the equilibrium volume increases. Figure 1 provides a graphical representation of the concept's functioning. The left hand side of the Figure shows the actual wholesale market auction curves observed in the German EPEX SPOT SE on 2017-01-01 at 00:00:00. The right hand side of the Figure depicts a transformed version of the two curves with perfectly inelastic demand.

[Coulon et al., 2014] elaborate on the intuition behind their idea at length. Following their reasoning, there are only few price-sensitive consumers in power markets. On the other hand, the

demand curve in the wholesale market is elastic. At first, this contradiction seems counterintutive. However, besides the wholesale market, there exist another big bilateral venue where power suppliers are connected directly with their consumers. Therefore, whenever the prices in the wholesale market are lower than those in the bilateral one, multiple market participants discover arbitrage opportunities and decide to engage into speculation. They try to purchase electricity instead of producing it. Naturally, speculation leads to a surge in price sensitivity.

Hence, having two parallel markets may lead to considerable complications for energy sector modeling. To avoid them, [Coulon et al., 2014] suggest shifting the entire elasticity from the demand to the supply side. As a result, a perfectly inelastic demand curve will be obtained. Moreover, both the transformed supply and the demand curves are more stable and predictable because many orders in a wholesale market are of speculative nature.

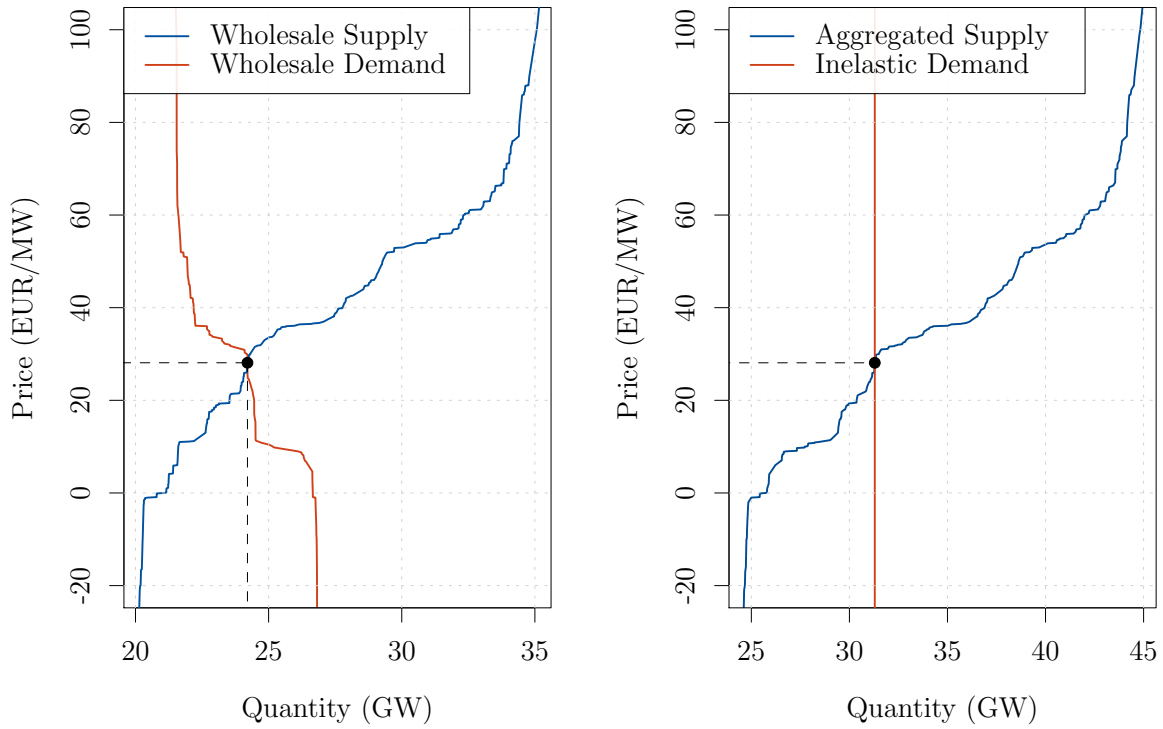


Figure 1: A wholesale market equilibrium in the EPEX SPOT SE on 2017-01-02 01:00:00 (left plot) vs. its manipulated form with an inelastic demand curve (right plot)

1.1.2 Aggregated Supply and Demand case and the Wholesale Market

The kernel of our model stems from a market transformation carried out by [Knaut and Paulus, 2016]. However, intuition behind the transformation has not been elaborated in the original paper. Therefore, prior to explaining the model itself, we have to comment on functioning of its underpinnings.

First of all, we introduce a toy example of a fictional electricity market. Describing the model on the grounds of the toy example appears to be more straightforward and much simpler. We assume an economy populated by three market participants: a Utility company, a Retailer, and a Supplier. The agents differ in their abilities to interact with the market. Utility can post both bid and ask orders, Retailer can only demand electricity from the market, and the choice of Supplier is

restricted only to bid orders. Of course, the number of market participants can easily be expanded. In our case, however, each of them can be treated as the respective generalized representation of a country's power utility companies, electricity retailers, and electricity suppliers.

Individual supply and demand schedules of the market agents in our toy example are represented in Figures 2(a), 2(b), and 2(c), respectively. Please note that the internal equilibrium price of Utility equals to zero. This price was selected because it allows us to demonstrate all features of our model more explicitly. Moreover, to ease further notation, each single step in the supply curves of the market participants will be referred to as a sell order. In turn, by a buy order we will denote a single step in the demand curves.

As [Knaut and Paulus, 2016] suggest, traders' orders can be aggregated by means of two different techniques. The two resulting equilibria will be referred to as an Aggregated Supply and Demand case (ASD) and a Wholesale Market (WM). Even though the choice of a technique does non influence equilibrium prices, it does affect equilibrium volumes and the compositions of the market supply and demand curves. The focus of the present section will thus be placed on the discussion of these two techniques.

Graphical depictions of both final equilibria are provided in Figures 2(d) and 2(e), respectively. Explaining how both solutions were obtained is another challenge we face at this stage. To simplify the forthcoming description, we create two auxiliary variables: a supply pool and a demand pool. Following their names, the variables will be used to collect buy and sell orders of the market participants. Separation into two pools will become especially useful for the discussion of the more cumbersome WM case.

Let us first concentrate on the ASD equilibrium and Figure 2(d). The demand curve in this case was built up as follows. First, we collected Utility's and Retailer's buy orders in the demand pool. Second, we sorted the orders in the demand pool according to their prices. Third, we plotted the obtained data. The supply curve in this case was then constructed analogously. First, we collected Utility's and Supplier's sell orders in the supply pool. Second, we sorted the orders in the supply pool according to their prices. Third, we plotted the obtained data. The upper part Table 1 summarizes components of both the supply and demand pools in the ASD case.

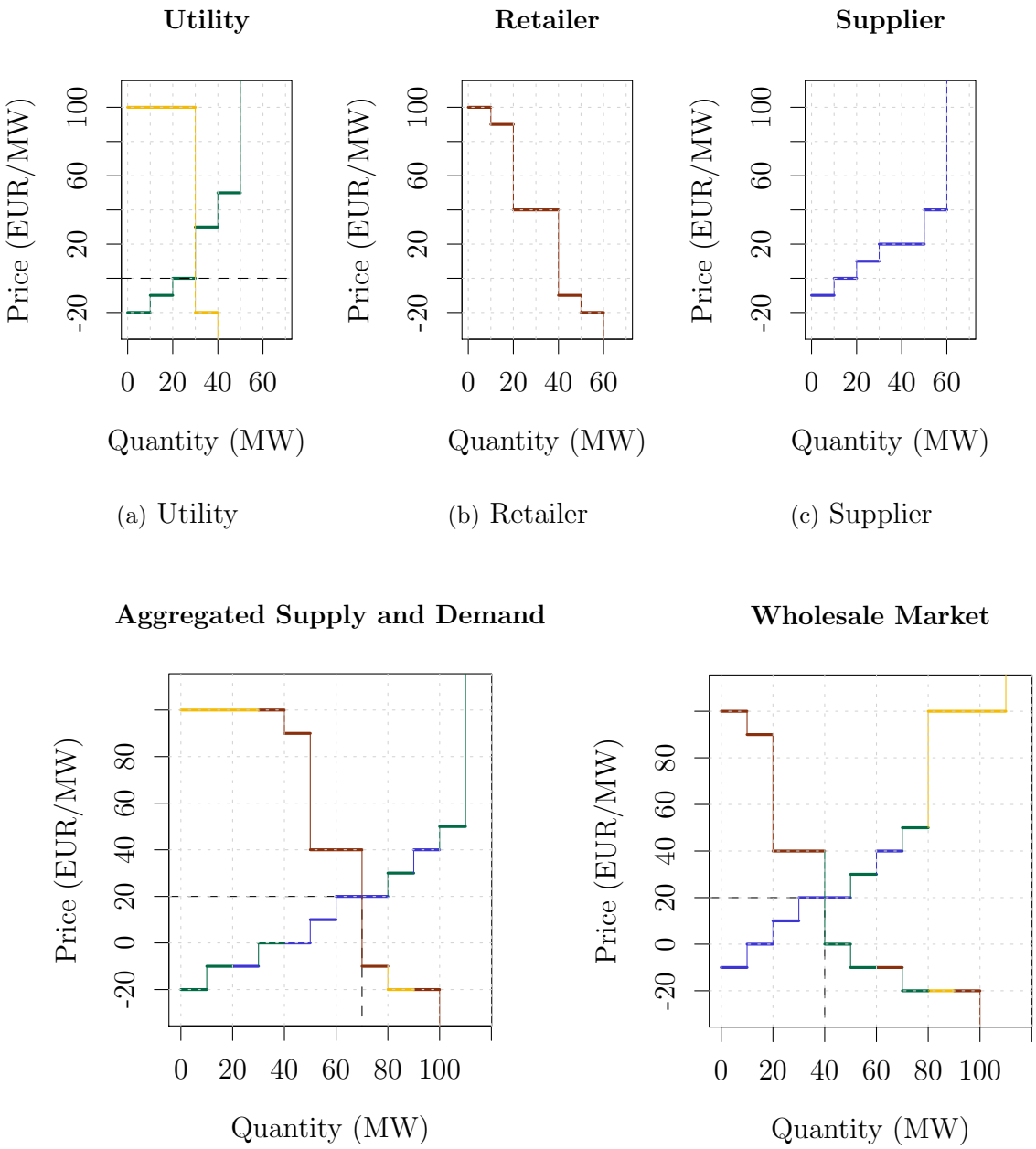
Much less intuitive is the functioning of the WM equilibrium. The main characteristic of the wholesale market is an assumption that Utility is allowed to engage into speculation.[3] As a result, the bidding strategy of Utility changes. On the other hand, the behavior of both Retailer and Supplier remains the same. This case is illustrated in Figure 2(e) and is supported by the lower part of Table 1. Moreover, to ease the forthcoming explanation, we divided the Utility's supply and demand curves into 4 sectors. These sectors are illustrated in Figure 3(a).

	Supply Pool	Demand Pool
ASD	Sell orders of Supplier Sell orders of Utility	Buy orders of Retailer Buy orders of Utility
WM	Sell orders of Supplier Sell orders of Utility from Sector 1 Buy orders of Utility from Sector 2	Buy orders of Retailer Buy orders of Utility from Sector 4 Sell orders of Utility from Sector 3

Table 1: Supply and Demand pools in the ASD and WM cases

Let us now consider the demand curve in the WM scenario. Given that Retailer can not act as

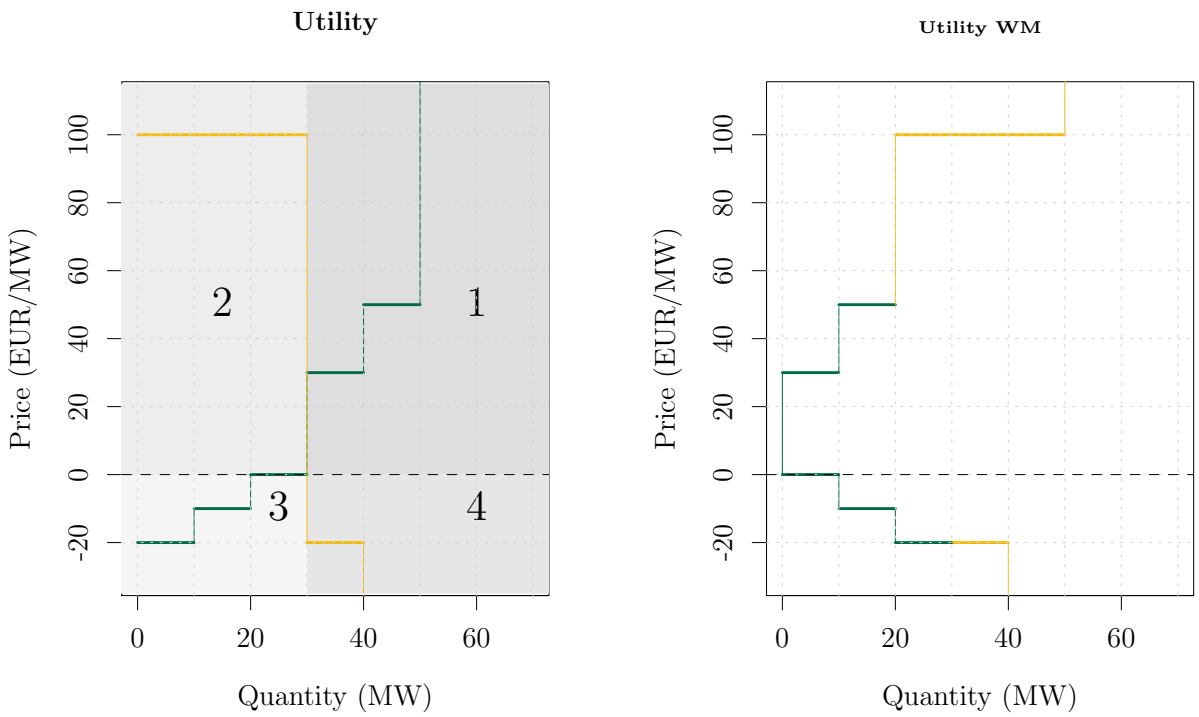
[3]Please note that by speculation we only assume that Utility tries to purchase electricity instead of producing it. The forthcoming description will show that this speculation is risk-free.



(a) Utility (b) Retailer (c) Supplier

(d) Aggregated supply and demand (e) Wholesale market

Figure 2: A toy example of an electricity market



(a) Utility Sectors

(b) Utility WM

Figure 3: Auxiliary plots for explanation of the WM case: (a) breakdown of the space of Utility into four sectors and (b) buy and sell orders of Utility in the WM case

a speculator, we place all its buy orders into the demand pool. Then, we shift our focus to Utility. We include orders located in sectors 3 and 4 in the demand pool. The following rationale can be used to justify the choice of these orders. First, the buy orders in sector 4 are located to the right of the internal equilibrium of Utility. Therefore, these orders are out-of-the-money and Utility can not capitalize on them. Utility thus hopes that they may be interesting to other market players. Hence, the buy orders from sector 4 end up in the demand pool. Second, placing the sell orders from sector 3 in the demand pool at first seems counterintuitive. However, we suppose that Utility decides to halt its own electricity production. Instead, Utility tries to purchase these orders at the same prices in the wholesale market. Purchasing rather than generating electricity may allow Utility to e.g. spare some maintenance costs.[4] Hence, Utility no longer tries to produce the orders located in sector 3 itself. On the contrary, Utility attempts to buy these orders in the market. Therefore, they are placed in the demand pool in the WM case.

At this stage there are three groups of orders left unassigned. They will thus become components of the supply pool. More specifically, the three groups are: the sell orders of Supplier, the sell orders of Utility from sector 1, and the buy orders of Utility from sector 2. Let us now discuss why the orders from these groups are placed into the supply pool. Given the above discussion, the treatment of the former group appears straightforward. The latter two groups, in turn, require a special attention. Analogously to the Utility's buy orders from sector 4, the sell orders in sector 1 are out-of-the-money. Therefore, Utility attempts to find new customers among other market

[4]Other possible reasons for speculation were mentioned in subsection 1.1.1

participants for these orders. Finally, intuition behind manipulations with the buy orders from sector 2 is connected to the sell orders from sector 3. From the previous paragraph we know that Utility intends to purchase the sell orders from sector 3 instead of producing them. To be able to carry out this transaction, Utility needs money. Thus, Utility places the buy orders from sector 2 in the supply pool. In doing so, Utility hopes that these orders would be acquired by other market participants. If other market agents indeed buy these orders, Utility obtains necessary funds to engage into the speculation and purchase the sell orders from sector 3.

Figure 3(b) may be useful to amend the above explanation. This Figure shows us supply and demand schedules of Utility in the WM case. Let us now examine these schedules in greater detail. First, we can see explicitly that the downward-sloping demand curve incorporates the sell orders from sector 3. Second, the buy orders from sector 2 were included into the upward-sloping supply curve. Third, there are no orders in the downward-sloping demand curve which are more expensive than the internal equilibrium price of Utility. Fourth, there are no orders in the upward-sloping supply curve which are cheaper than the internal equilibrium price of Utility. Finally, the internal equilibrium price of Utility remains the same in both ASD and WM cases.

Having elaborated on the fundamentals of both ASD and WM equilibria, we can now study their features. First of all, note that the equilibrium prices are the same in both cases. On the contrary, the equilibrium volumes are higher in the latter case. The discrepancy in the volume sizes can be explained by the fact that Utility tries to abstain from production of electricity in the WM scenario. Naturally, this assumption goes in line with subsection 1.1.1. Therefore, market clears under an assumption that Utility does not supply some electricity to the market. Second, elasticity of the demand curve is typically higher in the WM case. This fact seems obvious given that Utility tries to speculate in the wholesale market and is thus more sensitive to price fluctuations.

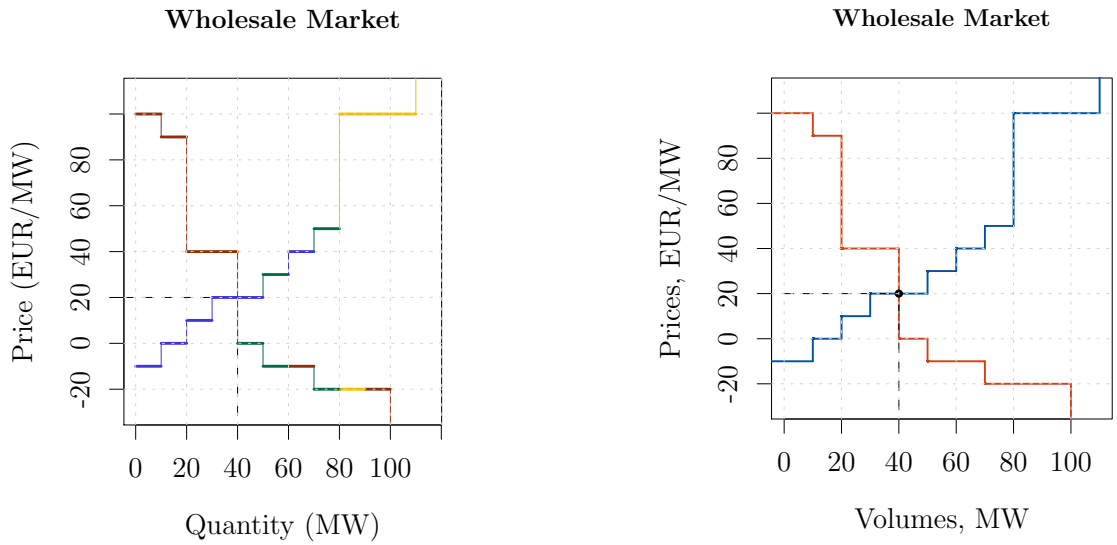
Moreover, there is a final commentary regarding the two equilibria. Note that Utility may realize that speculating is no longer profitable after the equilibrium price has been established. In other words, Utility will chose to refrain from speculation if its internal costs for electricity generation are lower than the market prices. In this case final load values in both ASD and WM scenarios will coincide. From this perspective, Utility does not face any risks when trying to speculate in a wholesale market. Therefore, Utility's orders in the wholesale supply and demand curves are arbitrage orders.

1.2 Main idea and motivation

Following [Coulon et al., 2014], a model with perfectly inelastic demand is an extreme representation of an electricity market. Of course, transferring all demand elasticities to the supply side allows us to simplify computations substantially. However, sensitivity of some market agents to the price should not be neglected. Thus, the model with perfectly inelastic demand remains an imperfect solution. On the other hand, the wholesale market equilibrium, too, is not fully informative. The presence of arbitrage orders in this equilibrium distorts true intentions of market participants. Therefore, as the paper by [Knaut and Paulus, 2016] suggests, the ASD equilibrium can provide us with a more precise approximation of a wholesale electricity market.

In fact, the ASD solution lies "in between" the solution with inelastic demand and the WM case. Therefore, the equilibrium volumes obtained in these two cases are respectively greater and lower than those in the ASD equilibrium. Moreover, the ASD market volumes, as we will later demonstrate, exhibit a stronger correlation with true total load values in an electricity market. Better suitability of the ASD solution for load approximation can be easily explained by commentaries made in the previous sections.

The model described in section 1.1.2 may suggest that obtaining the ASD solution is relatively simple. In reality, however, it not necessarily is. Many electricity exchanges, including e.g. the German EPEX SPOT SE, do not disclose individual buy and sell orders of market participants. Therefore, the data available to scientists only constitutes auction curves drawn in a wholesale market. Precise compositions of these curves and contributions of each market player to these curves is not announced publicly. Hence, if we remain in the world of our toy example, the auction curves which are typically disclosed to market participants look like those in Figure 4(b) and not like those in Figure 4(a).



(a) WM equilibrium in our toy example

(b) Actually disclosed WM auction curves from our toy example

Figure 4: Auxiliary plots for explanation of the WM case: (a) wholesale market equilibrium in our toy example (b) auction curves in this equilibrium as are typically disclosed by an electricity exchange

Nevertheless, we have a certain knowledge about how exactly both ASD and WM solutions can be assembled. Given this knowledge, it is possible to derive the supply and demand schedules of market participants from the wholesale market data. More specifically, our optimization-based model will back-decompose the wholesale auction curves into individual buy and sell orders of market agents. The ASD equilibrium can be constructed easily once the decomposition is complete.

2 Model

2.1 Intuition behind the functioning of the model

The to-be-undertaken decomposition of the wholesale auction curves is relatively sophisticated. Prior to explaining its details, two comments are to be made. First, in what follows we will use the name Fundamental Model (FM) to refer to the ASD case. The choice of the name appears rational because we will no longer apply our model to an imaginary data set. Therefore, our ASD solution will correspond to a fundamental model of a wholesale electricity market. Second, the

decomposition will be formulated as an optimization problem. To obtain our FM model, we will minimize the distance between the FM equilibrium volumes and the true load values. In fact, the decomposition will be carried out along the five steps listed below. Referring to these steps may ease the understanding of the model's description.

1. Define the basic mathematical framework of the model
2. Conjecture the internal equilibrium price of Utility. Since individual supply and demand schedules of market participants remain undisclosed, there is no information regarding the internal equilibrium of Utility. Hence, this step is critical because of two main reasons. First, the only difference between the ASD and WM cases lies in the Utility's bidding strategy. Second, the Utility's behavior is solely predetermined by its internal equilibrium price. Therefore, making an assumption regarding this price is necessary to perform the decomposition.
3. Break down the wholesale supply and demand curves into individual supply and demand schedules of market participants. To carry out the decomposition, we will rely on the conjectured internal price of Utility and on our knowledge about the functioning of the wholesale market. Moreover, important is that at this stage we will obtain supply and demand curves of Utility in the WM setting.[5] These curves have to be transformed from WM to FM case in the next step.
4. Convert the Utility's curves from WM to FM setting.
5. Combine the obtained supply and demand schedules of the market participants in an FM equilibrium. Ensure that the distance between the FM market volumes and true load values is minimized.

2.2 Model description

2.2.1 Defining the basic mathematical framework of the model

To explain the model, we will still remain in the world of the toy example introduced in section 1.1.2. Before we proceed further, let $p_{\min} = -20$ and $p_{\max} = 100$. These two borders correspond to the ones given in the toy example. We thus can define the supply and demand curves in a wholesale market as follows

$$\text{WSup} : \underbrace{(0, \infty)}_{\text{Volumes}} \mapsto [p_{\min}, p_{\max}] \tag{1}$$

$$\text{WDem} : (0, \infty) \mapsto [p_{\min}, p_{\max}] \tag{2}$$

with the respective inverses being given by

$$\text{WSup}^{-1} : [p_{\min}, p_{\max}] \mapsto (0, \infty) \tag{3}$$

$$\text{WDem}^{-1} : [p_{\min}, p_{\max}] \mapsto (0, \infty). \tag{4}$$

[5] The schedules of Utility in this setting were depicted in Figure 3(b)

Please note that the above defined curves correspond to those depicted in Figure 4(b). This implies that all manipulations described below will be performed on auction curves with unknown compositions, i.e. on curves without a clear breakdown into individual orders of market participants.

The market equilibrium can be clearly determined as follows

$$(v^W, p^W) = \{v, \text{WSup}(v) | \text{WSup}(v) = \text{WDem}(v), v > 0\} \tag{5}$$

where v denotes the volume and W stands for the wholesale market.

2.2.2 Conjecturing the internal equilibrium price of Utility

As was mentioned earlier, the internal equilibrium price of Utility predetermines the differences between the ASD and WM cases. Since this price is typically not announced publicly, we need to conjecture it.

Let us denote the internal equilibrium price of Utility by p^U. We suppose that price p^U is a linear function of the wholesale market price. This assumption, despite being simple, appears reasonable. Utility would not be able to operate in a market successfully if its prices would substantially deviate from market prices. The following statement can hence be made

$$p^U = a_0 + a_1 p^W \tag{6}$$

where a_0 and a_1 denote the intercept and the slope, respectively. Finally, to ease the forthcoming description of the model, we suppose that the Utility's internal equilibrium price is greater than the respective equilibrium market price.

2.2.3 Breaking down the wholesale supply and demand curves into individual supply and demand schedules of market participants

In this section we will determine supply and demand schedules of Supplier, Retailer, and Utility in the WM setting. As has already been said, the strategies of Supplier and Retailer do not vary between the FM and WM equilibria. The strategy of Utility, however, does. Hence, the two obtained supply and demand curves of Utility will be modified further in subsection 2.2.4. To be able to distinguish market participants from one another, will use index 0 to denote Supplier or Retailer, and will use index 1 for Utility.

Step 1: determining the supply and demand schedules of Supplier and Retailer

First of all, let us return to Figure 2(e) and focus on the wholesale supply curve. As can be seen from the Figure and as was mentioned in section 1.1.2, Utility will not place an order on the supply side if this order is priced below the Utility's internal equilibrium price p^U. Therefore, all orders in the wholesale supply curve which are cheaper than p^U belong to Supplier. In turn, orders more expensive than the price p^U can either be posted by Utility or by Supplier. In other words, the wholesale supply curve above the price p^U is split in some proportion between Utility and Supplier.[6]

The supply schedule of Supplier is thus built up of two components. The first component is the segment of the wholesale supply curve below price p^U. The second component is a part of the

[6] We will calculate this proportion by using an optimization tool later.

upper segment of the wholesale supply curve (above price p^U). The corresponding mathematical representation can be written as follows

$$\begin{aligned}\mathrm{Sup}_0^{-1}(p) = {} & \mathbb{1}_{[p_{\min},p^U)}\mathrm{WSup}^{-1}(p) \\ & + \mathbb{1}_{(p^U,p_{\max}]}\left(\mathrm{WSup}^{-1}(p^U) + (1-\gamma_1)\left(\mathrm{WSup}^{-1}(p) - \mathrm{WSup}^{-1}(p^U)\right)\right)\end{aligned} \tag{7}$$

where coefficient γ_1 denotes the Utility's proportion in the upper segment of the wholesale supply curve.

Of course, the demand schedules of Retailer can be derived analogously. Following Figure 2(e), the upper segment of the wholesale demand curve (above the price p^U) belongs solely to Retailer. The segment below price p^U is split in some proportion between Retailer and Utility. Hence, it holds that

$$\begin{aligned}\mathrm{Dem}_0^{-1}(p) = {} & \mathbb{1}_{(p^U,p_{\max}]}\mathrm{WDem}^{-1}(p) \\ & + \mathbb{1}_{[p_{\min},p^U)}\left(\mathrm{WDem}^{-1}(p^U) + (1-\phi_1)\left(\mathrm{WDem}^{-1}(p) - \mathrm{WDem}^{-1}(p^U)\right)\right)\end{aligned} \tag{8}$$

where coefficient ϕ_1 denotes the Utility's proportion in the lower segment of the wholesale demand curve.

Step 2: determining supply and demand schedules of Utility in the WM setting

From equation 7 and Figure 3(b) we know that the upper segment of the wholesale supply curve is split between Supplier and Utility. Therefore, the upward-sloping supply curve of Utility in the WM case can be written as

$$\mathrm{WSup}_1^{-1}(p) = \mathbb{1}_{(p^U,p_{\max}]}\gamma_1\left(\mathrm{WSup}^{-1}(p) - \mathrm{WSup}^{-1}(p^U)\right). \tag{9}$$

In turn, following equation 8, the downward-sloping curve can be represented as

$$\mathrm{WDem}_1^{-1}(p) = \mathbb{1}_{[p_{\min},p^U)}\phi_1\left(\mathrm{WDem}^{-1}(p) - \mathrm{WDem}^{-1}(p^U)\right). \tag{10}$$

Hence, equations 9 and 10 define Utility's auction curves in the WM case as depicted in Figure 5(a). The following section will be devoted to transforming these curves from WM to FM setting.

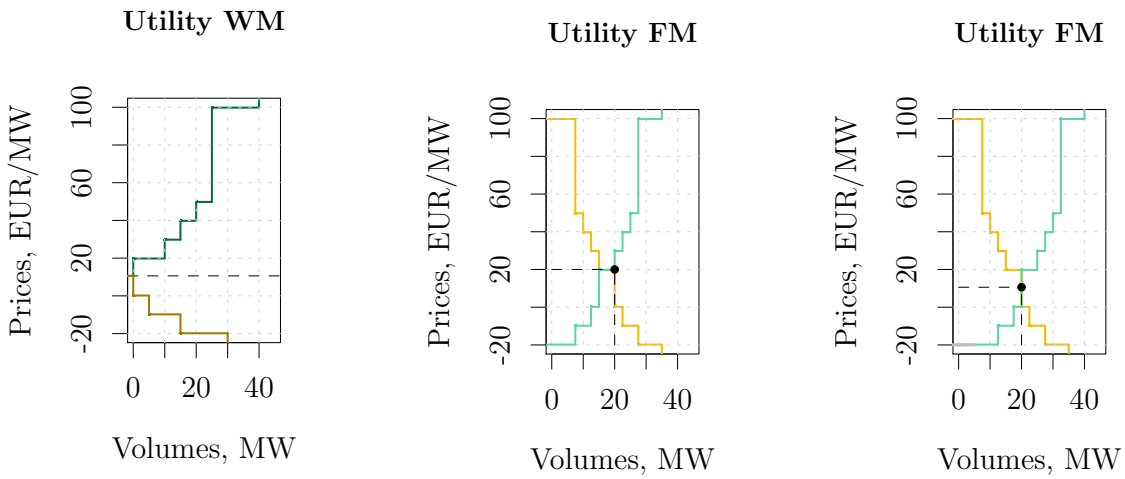
2.2.4 Converting the Utility's schedules from wholesale market to fundamental model setting

As section 1.1.2 suggests, Figure 5(a) corresponds to Figure 3(b). However, to be able to construct the FM equilibrium, Utility's auction curves in the initial setting (as depicted in e.g. Figure 3(a)) are required. Transforming the supply and demand curves of Utility from WM to FM case is thus the main aim of the present subsection. This transformation will be carried out in two steps.

Step 1: separating the Utility's WM curves into buy and sell orders

Remember that the supply and demand curves depicted in Figure 3(b) incorporate both buy and sell orders. However, following section 1.2 and Figure 5(a), we do not know the exact compositions of the Utility's curves in our case. Therefore, we have to divide the Utility's WM curves into buy and sell orders. Following the general structure of the model, an optimization-based technique will be used to determine the proportions in which orders are to be divided.

Let us now consider the downward-sloping demand curve in Figure 5(a). Suppose that orders in this curve are split equally between the buy and sell sides. To quantify this assumption, we



(a) Wholesale auction curves of Utility with $\gamma_1 = 0.5$ and $\phi_1 = 0.5$

(b) Unadjusted curves of Utility in FM setting with $\alpha_1 = 0.5$ and $\beta_1 = 0.5$

(c) Adjusted curves of Utility in FM setting with $\alpha_1 = 0.5$ and $\beta_1 = 0.5$

Figure 5: Auxiliary plot for the explanation of the WM case: transformation of the Utility's curves from WM to FM setting

introduce a new coefficient $\alpha_1 = 0.5$. Then, following Figure 3, half of the downward-sloping demand curve remains on the buy side. On the contrary, the other half of this curve is flipped onto the supply side. In other words, as we can see from Figure 5(b), the lower part (below price p^U) of the supply curve of Utility FM is a flipped half of the downward-sloping demand curve of Utility WM. Naturally, the supply and demand curves below the price p^U in the Utility FM case look like mirrored versions of one another because $\alpha_1 = 0.5$.

We can now use the same method to decompose the supply curve of Utility WM. Let us thus focus on the upward-sloping curve in Figure 5(a). Assume that the orders in this curve, too, are split equally between the buy and sell sides. Therefore, let $\beta_1 = 0.5$. As the description provided in the previous paragraph suggests, one half of the upward-sloping curve is flipped onto the demand side. Hence, as can be seen from Figure 5(b), the upper part of the demand curve (above the price p^U) in the FM case is a flipped half of the upward-sloping supply curve of Utility WM.

Hence, the supply curve depicted in Figure 5(b) can be represented mathematically as

$$\begin{aligned}\widetilde{\mathrm{FSup}_1}^{-1}(p) = {} & \mathbb{1}_{[p_{\min},p^U)}\alpha_1\left(\mathrm{WDem}_1^{-1}(p_{\min}) - \mathrm{WDem}_1^{-1}(p)\right) \\ & + \mathbb{1}_{(p^U,p_{\max}]}\left(\alpha_1\mathrm{WDem}_1^{-1}(p_{\min}) + (1-\beta_1)\mathrm{WSup}_1^{-1}(p)\right)\end{aligned} \tag{11}$$

where α_1 is a portion of sell orders in the downward-sloping curve of Utility WM and β_1 denotes a portion of buy orders in the upward-sloping curve of Utility WM. The first line of the above equation is thus a horizontally flipped part of the demand curve of Utility WM.[7]

[7]Note that the assumptions $\alpha_1 = 0.5$ and β_1 were only made to simplify the description of the model. The actual values of α_1 and β_1 have to be determined by means of an optimization tool.

In turn, the definition of the demand curve illustrated in Figure 5(b) reads

$$\widetilde{\mathrm{FDem}_1}^{-1}(p) = \mathbb{1}_{(p^U, p_{\max}]}\beta_1 \left(\mathrm{WSup}_1^{-1}(p_{\max}) - \mathrm{WSup}_1^{-1}(p)\right) + \mathbb{1}_{[p_{\min}, p^U)} \left(\beta_1 \mathrm{WSup}_1^{-1}(p_{\max}) + (1-\alpha_1)\mathrm{WDem}_1^{-1}(p)\right). \tag{12}$$

Equations 11 and 12, however, do not describe the final versions of Utility's curves in the FM case. These equations will be modified further in the next step.

Step 2: reconciling equilibrium prices of Utility WM and Utility FM

From section 1.1.2 and Figure 3 we know that the internal equilibrium price of Utility is the same in WM and FM settings. On the other hand, Figures 5(a) and 5(b) indicate explicitly that prices may differ after the decomposition of the wholesale auction curves. Naturally, this discrepancy violates our assumption. Therefore, we have to further manipulate the curves of Utility FM for the equilibrium prices of Utility in both WM and FM settings to be reconciled.

Our solution to solve the problem is rather simple. We merely decide to shift one of the Utility's curves to the right unless equilibrium prices in FM and WM cases are reconciled.[8] Despite being counterintuitive at first, this approach proves to be extremely efficient and does not contradict to the main aim of our model. Figures 5(b) and 5(c) provide us with the corresponding graphical representations.

Of course, shifting one of the curves to the right may imply that we "create" additional volumes. One would thus expect that we should subtract the added volumes from other parts of the curves. However, when the model is applied to real data, subtracting these volumes will inevitably distort either the internal price of Utility, or the equivalence of the prices in final WM and FM equilibria. Therefore, a question arises: where do these additional volumes come from?

Let us now suppose that we shift the Utility's supply curve.[9] Note that the need to maintain the stability of an energy system forces energy producers to offer must-run supply at minimal prices. Therefore, sell orders at the lowest price are typically much more voluminous than other sell orders. Following section 1.1.2, these must-run sell orders are located in the segment $p_{\min}$ of the downward-sloping demand curve of Utility WM. Hence, when converting the schedules of Utility form WM to FM setting, we, theoretically, should not only shift the supply curve of Utility FM to the right. We should also subtract the shift size from the segment $p_{\min}$ of the demand curve of Utility FM.

However, important is the fact that this segment is represented graphically only in the toy example. It is not displayed in real-world examples. Actually observed wholesale auction curves almost abruptly end at the prices $p_{\min}$ and $p_{\max}$.[10] In other words, we indeed can subtract the volumes in question, but only if we remain in the world of the toy example. Moreover, it is worthy to note that manipulations with the far-right segments of the curves influence neither Utility's price nor the prices in WM and FM cases. The equilibrium remains unchanged even if no volumes are subtracted. The shape of the demand curve, too, remains unaffected.

Figure 9 will provide an even greater motivation for using the second approach. As we will later demonstrate, equilibrium volumes we obtain approximate the actual load values better than the model with perfectly inelastic demand and the wholesale market volumes. Therefore, after several adjustments our model can be used successfully for load forecasting.

[8] Intuition behind which curve needs to be shifted will be elaborated in what follows.

[9] Of course, the reasoning is analogous for the demand side.

[10] Figure 7 plots two auction curves observed in the German day-ahead wholesale electricity market.

Let us now explain how the shift of Utility's curves is implemented into the model. First, we will denote the magnitude of the shift by τ_1 and assume that

$$\tau_1 = \widetilde{FSup_1}^{-1}(p^U) - \widetilde{FDem_1}^{-1}(p^U) \tag{13}$$

Hence, we determine the shift size by calculating the difference between volumes on supply and demand curves at point p^U. From equation 13 it appears clear that τ_1 is negative if the demand curve is located to the right of the supply curve at the point p^U. Hence, we will shift the Utility's supply curve if $\tau_1 < 0$. Otherwise, if $\tau_1 > 0$, then the demand curve has to be shifted.

Then, the equations for the adjusted supply and demand curves of Utility FM can be respectively represented as

$$\begin{aligned}\mathrm{FSup}^{-1}{}_1(p) = {} & \mathbb{1}_{[p_{\min},p^U)}\alpha_1\left(\mathrm{WDem}_1^{-1}(p_{\min}) - \mathrm{WDem}_1^{-1}(p)\right) \\ & + \mathbb{1}_{(p^U,p_{\max}]}\left(\alpha_1\mathrm{WDem}_1^{-1}(p_{\min}) + (1-\beta_1)\mathrm{WSup}_1^{-1}(p)\right) - \min(\tau_1, 0)\end{aligned} \tag{14}$$

and

$$\begin{aligned}\mathrm{FDem}^{-1}{}_1(p) = {} & \mathbb{1}_{(p^U,p_{\max}]}\beta_1\left(\mathrm{WSup}_1^{-1}(p_{\max}) - \mathrm{WSup}_1^{-1}(p)\right) \\ & + \mathbb{1}_{[p_{\min},p^U)}\left(\beta_1\mathrm{WSup}_1^{-1}(p_{\max}) + (1-\alpha_1)\mathrm{WDem}_1^{-1}(p)\right) + \max(\tau_1, 0).\end{aligned} \tag{15}$$

Figure 5(c) plots the final curves of Utility in the FM case. As can be seen from the Figure, the equilibrium price of Utility FM equals to that of Utility WM once the adjustment was made. Moreover, in our case factor τ_1 was positive since the initial supply curve of Utility FM is located to the left of the demand curve at the point p^U. Therefore, the supply curve was shifted to the right. The shift magnitude is highlighted in gray in Figure 5(c).

2.2.5 Assembling the Fundamental Model and formulating the optimization problem

The above defined supply and demand schedules of the market participants allow us to build up the FM equilibrium. Hence, the market demand curve in the FM case can be obtained by aggregating the sell orders of Retailer and Utility. The following statement can thus be made

$$\mathrm{FDem}^{-1} = \mathrm{Dem}_0^{-1} + \mathrm{FDem}_1^{-1}. \tag{16}$$

The market supply curve in the FM case can then be defined analogously as

$$\mathrm{FSup}^{-1} = \mathrm{Sup}_0^{-1} + \mathrm{FSup}_1^{-1} \tag{17}$$

It follows that the intersection between the FM supply and demand curves can be defined as

$$(v^F, p^F) = \{(v, \mathrm{FSup}(v) | \mathrm{FSup}(v) = \mathrm{FDem}(v)), v > 0\}, \tag{18}$$

where $p^W = p^F$ and $v^W < v^F$ as follows from section 1.1.2 and equation 5.

As the previous sections suggest, values of v^W in the wholesale market are much smaller than the actual load values. Note that not only arbitrage orders influence the difference. Final load reflects volumes from intra-day trading (both the intra-day auctions and continuous trading), OTC-deals, and potentially other markets.[11] Therefore, volumes v^F in our FM equilibrium, too,

[11] For example, the EXAA can be considered as an "other market" for the case of Germany.

must be smaller than the actual load values. Moreover, volumes v^F must be lower than those suggested by the model with perfectly inelastic demand. This holds because the latter model leaves no ealsticities at the demand side.

Finally, the core of our optimization model can be explained as follows. We will minimize the distance between the actual load values and the equilibrium volumes in the FM case. Speaking technically, a linear fit optimization problem can thus be written as follows

$$\arg\min_{\theta_0,\theta_1} Q(\theta_0,\theta_1) \quad \text{where} \quad Q(\theta_0,\theta_1) = \sum_{i=1}^{n} (load_i - \theta_0 - \theta_1 v^F)^2 \tag{19}$$

2.3 Applying the model to the toy example

To test the functioning of our model, we first remain in the world of our toy example. Naturally, we use Figure 4(b) as the starting point. For illustration purposes we suppose the following set of coefficients: $a_0 = 0.5$, $a_1 = 0.5$, $\gamma_1 = 0.5$, $\phi_1 = 0.5$, $\alpha_1 = 0.5$, $\beta_1 = 0.5$. The obtained model is presented in Figure 6. There are two properties of our model to be noted.

First, our knowledge of the bidding strategies allows us to compute the difference $v^F - v^W$ on the grounds of our model. From Figures 2(d) and 2(e) we know that orders to the left of the equilibrium in the WM case can belong only to Retailer or Supplier. On the contrary, orders to the left of the equilibrium in the FM case can be submitted by Retailer, Supplier, and Utility. Therefore, the difference $v^F - v^W$ stems from the Utility's orders. The following statement can thus be made

$$v^F - v^W = \begin{cases} \mathrm{FSup}_1^{-1}(p^U) - (\mathrm{FDem}_1^{-1}(p^F) - \mathrm{FDem}_1^{-1}(p^U)) & \text{if} \quad p^U \leq p^F \\ \mathrm{FDem}_1^{-1}(p^U) - (\mathrm{FSup}_1^{-1}(p^F) - \mathrm{FSup}_1^{-1}(p^U)) & \text{if} \quad p^U \geq p^F \end{cases} \tag{20}$$

Therefore, the relation between the prices p^U and p^F determines which of the auction curves has to be shifted to the right. Moreover, to compute the difference $v^F - v^W$ we have to focus on a curve that has not been adjusted.

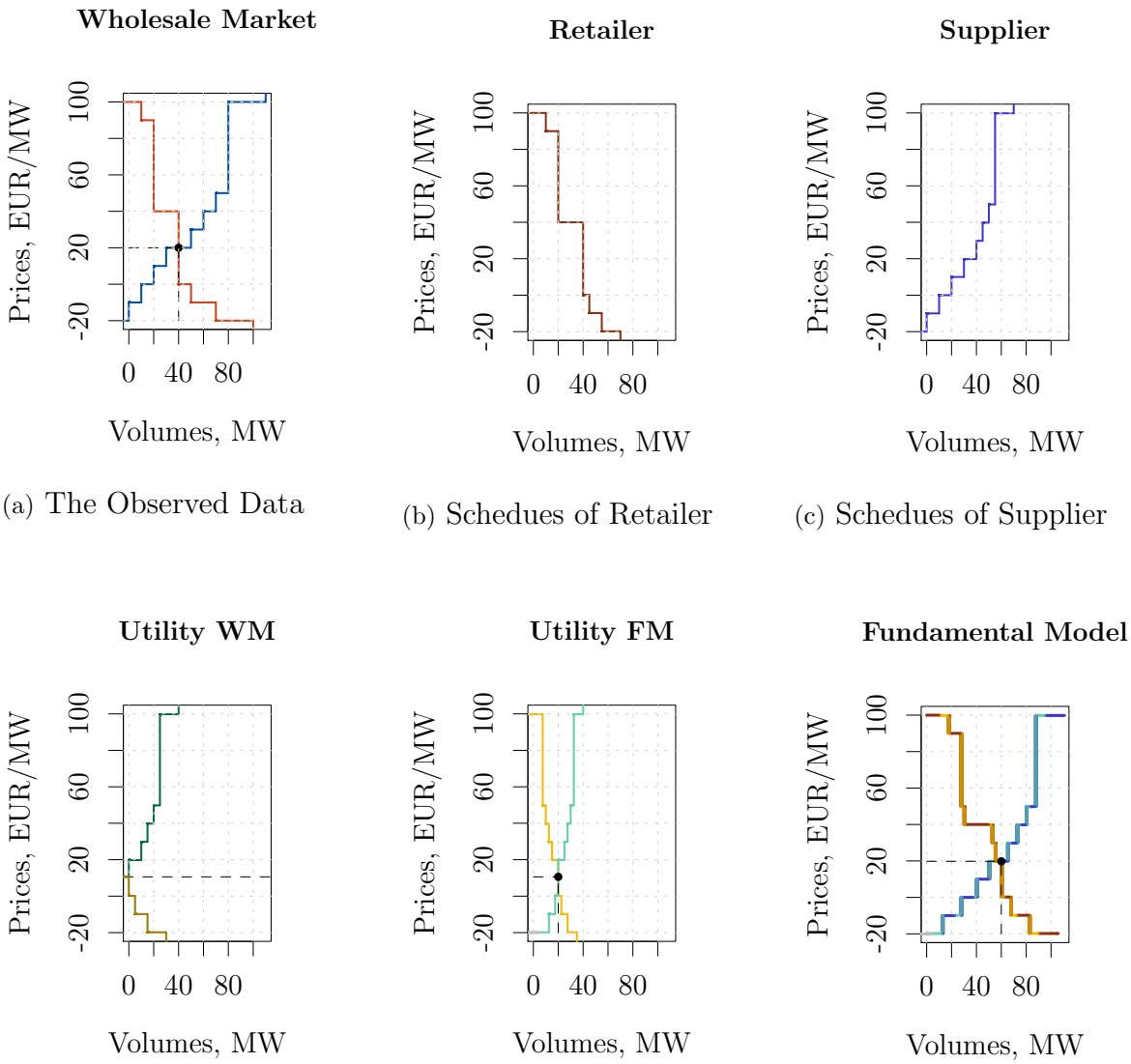
Second, the model with perfectly inelastic demand is an extreme form of our model. Under the parameter setting $a_0 = 0$, $a_1 = 1$, $\gamma_1 = 1$, $\phi_1 = 1$, $\alpha_1 = 0$, $\beta_1 = 0$ all demand elasticities are transferred to the supply side.[12] More specifically, this setting first assumes that Utility's internal price equals to the wholesale market price. Second, Utility attains all possible orders from the wholesale auction curves. Third, when converting Utility's schedules from WM to FM case, all orders are transferred from the demand to the supply side. Therefore, the demand curve is represented only by one point, all volumes are accumulated on the supply side, and the final equilibrium volume reaches its maximum.

3 Empirical Data

3.1 Institutional framework of the German electricity market

We chose to test our model on the German data. Wholesale electricity trading in Germany takes place on several different marketplaces. Their timing is illustrated in [Kiesel and Paraschiv, 2017]. Their detailed overview is given on the EPEX SPOT website or in e.g. [Hagemann and Weber, 2013]. Additionally to wholesale markets, electricity is actively traded via various OTC-contracts.

[12] Implications of each of the coefficients are be elaborated at length in section 4.1



(a) The Observed Data

(b) Schedues of Retailer

(c) Schedues of Supplier

(d) Utility before transformation

(e) Utility after transformation

(f) Fundamental Model

Figure 6: Obtaining the Fundamental Model using the wholesale market data from the toy example with $a_0 = 0.5$, $a_1 = 0.5$, $\gamma_1 = 0.5$, $\phi_1 = 0.5$, $\alpha_1 = 0.5$, $\beta_1 = 0.5$

First of all, a day-ahead auction for hourly products is conducted at 12:00 on a daily basis. Following the rules of the exchange, minimal price in the day-ahead market equals to $p_{\min} = -500$ EUR and maximal price is limited to $p_{\max} = 3000$ EUR. Market participants are expected to submit their bids to an auctioneer prior to the start of the auction. Once the bidding window closes, the EPEX system matches the orders, constructs wholesale supply and demand auction curves, and establishes 24 hourly prices for the next day. These prices are announced as soon as possible from 12:42. Besides the prices, also the auction curves are disclosed publicly. However, these curves are revealed only in their aggregated form, i.e. individual orders of market participants still remain concealed. Moreover, there exists another day-ahead auction for 15-minutes contracts. This auction clears at 15:00 every day.

In addition to the day-ahead market, a continuous intra-day trading for hourly contracts begins at 15:00 the day before physical electricity delivery and closes 30 minutes prior to the delivery. Continuous auction for 15-minutes contracts begins at 16:00 daily. Details of the German continuous intra-day market are discussed at length in e.g. [von Luckner et al., 2017].

Finally, energy can be traded in a balancing market to eliminate discrepancies between electricity supply and demand. Imbalances are traded ex-post. This market is studied at length in e.g. [Just and Weber, 2015].

3.2 Data Set

Our sample period extends from 31.12.2016 to 31.12.2017. To decrease computational burden of our model, we will set the upper and lower price bounds to $p_{\min,2017} = -83.05$ EUR and $p_{\max,2017} = 163.50$ EUR. The selected values correspond to the minimum and maximum observation, respectively, present in the in-sample period.

An example of the collected auction curves data is provided in Figure 7.

In fact, to define wholesale supply and demand curves (equations 3 and 4), we used a non-equidistant price grid with the following specification

$$pg = \{-500, -450, -400, ..., -83.05, -82.85, -82.65, ..., 163.50, 213.50, 263.50, ..., 3000\},$$

where pg stands for price grid. The use of such price grid allows us to incorporate all segments of the supply and demand curves while retaining a particular focus on their most important parts, i.e. on the interval from $p_{\min,2017}$ to $p_{\max,2017}$. Therefore, the computational burden of our model lessens, though the quality of the results remains unchanged.

Additionally to the auction curves data, we used total load data obtained from the ENTSOE. This data was though provided in a quarter-hourly format. Given that the auction curves data had a hourly resolution, we used simple arithmetic averages to manipulate the load data. Moreover, all data sets were clock-change adjusted. To replace missing hours in March, we used two values before and after these hours. In turn, simple arithmetic averages of two double hours in October were calculated to solve the problem.

4 Results

4.1 The obtained coefficients

The functioning of our model on real data is depicted in Figure 8 below. This Figure is based on 365 in-sample observations with 31.12.2016 being the starting point. The obtained coefficients

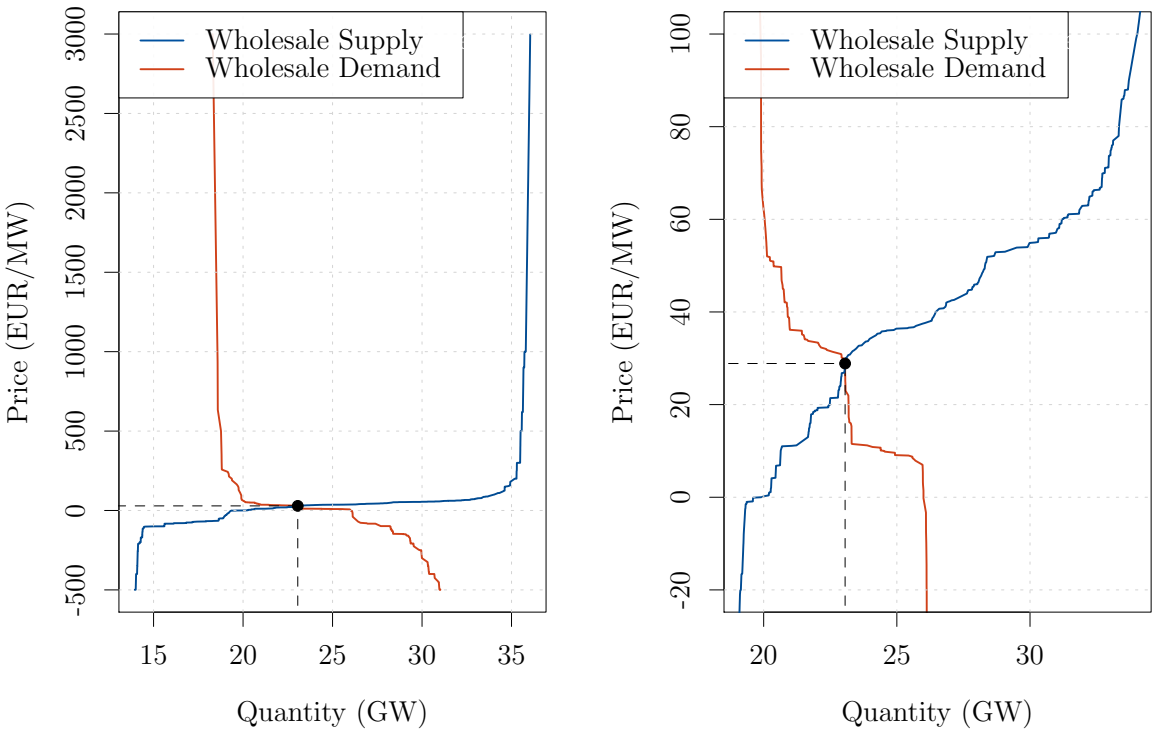


Figure 7: Wholesale market equilibrium on 2017-01-01 23:00:00. The left plot shows the entire auction curves observed at the time point. The right plot focuses on the equilibrium between these two curves.

for the year 2017 are summarized in Table 2 below. The conducted robustness checks show that the coefficients are significantly different from zero. A weak significance level is exhibited only by coefficient $\beta_1 = 0.019$. Moreover, all but coefficient $\phi_1 = 0.984$ are significantly different from one. These results allow us to conclude that the assumptions on which our model is built are valid and reasonable.

a_0	a_1	γ_1	ϕ_1	α_1	β_1
$5.890^{***}_{\star\star\star}$ (0.076)	$0.963^{***}_{\star\star\star}$ (0.006)	$0.510^{***}_{\star\star\star}$ (0.015)	$0.984^{***}_{\circ}$ (0.008)	$0.287^{***}_{\star\star\star}$ (0.008)	$0.019^{*}_{\star\star\star}$ (0.011)

Table 2: The obtained coefficients for the year 2017 with the following significance levels: $\bullet = 10\%$, $* = 5\%$, $** = 1$, $*** = 0.1\%$ with respect to 0; $\circ = 10\%$, $\star = 5\%$, $\star\star = 1\%$, $\star\star\star = 0.1\%$ with respect to 1.

Let us now analyze the above listed values more thoroughly. First, recall that coefficients a_0 and a_1 stand for the slope and intercept, respectively, in the linear function of the internal equilibrium price of Utility (equation 6). The obtained values indicate explicitly that the internal price of Utility does not deviate substantially from the market price. Therefore, an earlier assumption can be confirmed, i.e. Utility would struggle to make profits if its strategy would be much different to that of other market players.

The second pair of coefficients is γ_1 and ϕ_1. They describe how the wholesale auction curves are split between the three market participants. Following equations 7 and 9, the lower part of the

wholesale supply curve (below the Utility's internal price p^U) belongs solely to Supplier. In turn, the upper part (above the price p^U) includes orders of both Utility and Supplier. As the value of γ_1 suggests, orders in the upper part of the wholesale supply curve are split almost equally between Utility and Supplier. Therefore, inelastic parts are present neither in the schedules of Supplier (Figure 8(c)) nor in the upward-sloping curve of Utility WM (Figure 8(d)).

On the contrary, the split of the wholesale demand curve between Utility and Retailers is not equal. As was defined in equations 8 and 10, the upper part of the wholesale demand curve (above the price p^U) contains only Retailer's orders, whereas orders of both Utility and Retailer can be incorporated in its lower part. The value of ϕ_1 thus shows that Retailer retains only 0.016% of the orders from the lower part of the wholesale demand curve. The remaining 0.984% belongs to Utility. As a result, the lower part of the Retailer's demand curve is almost perfectly inelastic (Figure 8(b)).

Let us now shift our focus to coefficients α_1 and β_1. These coefficients, as is given in equations 14 and 15, determine the transformation of the Utility's schedules from WM to FM setting. The value of α_1 implies that roughly 30% of the upward-sloping supply curve of Utility WM is flipped onto the demand side. On the other hand, the value of β_1 indicates that more than 98% of the downward-sloping demand curve is transferred onto the supply side. Therefore, the lower part of the demand curve of Utility FM is almost perfectly inelastic. Moreover, the size of the lower part of the supply curve of Utility FM is almost maximal. Not only does Utility accumulate many orders from the wholesale demand curve (since ϕ_1 is very small), but also Utility flips almost the entirety of those orders onto the supply side.

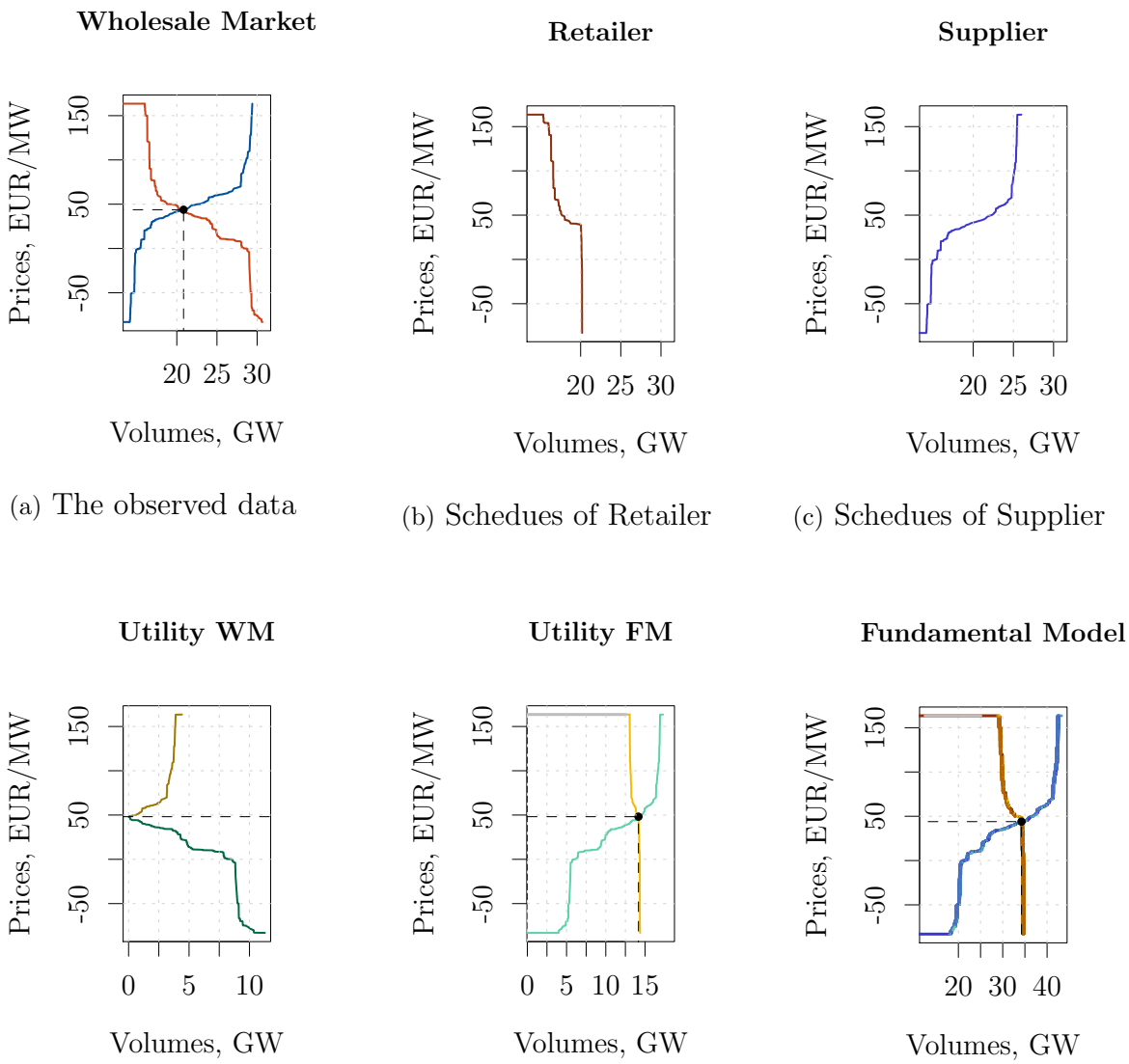
Finally, the obtained supply and demand schedules of the market participants allow us to assemble the Fundamental Model. Note that the lower part of the demand curve of Utility FM is almost fully inelastic. The lower part of the Retailer's curve, too, is inelastic. Therefore, the demand curve in our Fundamental Model consists of two parts: a relatively elastic upper part and almost fully inelastic lower part. Thus, following our initial hypothesis, the demand curve in our Fundamental Model lies "in between" a perfectly inelastic one (derived in [Coulon et al., 2014]) and the initial one in the wholesale market.

Moreover, as has been mentioned earlier, the coefficient setting $a_0 = 0$, $a_1 = 1$, $\gamma_1 = 1$, $\phi_1 = 1$, $\alpha_1 = 0$ and $\beta_1 = 0$ yields the model with perfectly inelastic demand. Let us scrutinize this setting more thoroughly. The internal equilibrium price of Utility is equal to the wholesale market price because $a_0 = 0$ and $a_1 = 1$. Then, Utility accumulates all possible sale and purchase orders from the wholesale supply and demand curves since $\gamma_1 = 1$ and $\phi_1 = 1$. $\alpha_1 = 0$ shows that the upward-sloping curve of Utility WM remains unmodified, whereas $\beta_1 = 0$ shows that the entire downward-sloping curve is flipped onto the supply side. As a result, equilibrium volumes are equal to those suggested by the model in [Coulon et al., 2014].

4.2 Actual electricity load and the values of v^F

Besides the developed market decomposition model, the utility of our research can be well demonstrated by Figure 9. This Figure illustrates actual load values, wholesale market equilibrium volumes, Fundamental Model equilibrium volumes and volumes suggested by the model with perfectly inelastic demand for a two week sample 08-21 June 2017. To ease further notation, let us denote the volumes suggested by the [Coulon et al., 2014] model by v^C.

From Figure 9 it can be seen clearly that the values produced by our Fundamental Model are often simply replicating the values of v^C. This observation seems intuitive given the fact that we often shift one of the Utility's curves as described in section 2.2.4. Volumes we add to reconcile



(a) The observed data

(b) Schedues of Retailer

(c) Schedues of Supplier

(d) Utility before transformation

(e) Utility after transformation

(f) Fundamental Model

Figure 8: Obtaining the Fundamental Model using the German wholesale market data on 2017-01-06 02:00:00 with $a_0 = 5.890$, $a_1 = 0.963$, $\gamma_1 = 0.510$, $\phi_1 = 0.984$, $\alpha_1 = 0.287$ and $\beta_1 = 0.019$

Utility's prices push final values v^F up. However, $v^F \leq v^C$ always holds since equilibrium volumes reach their maximum in the model with perfectly inelastic demand. Second, values v^F have higher correlation with the actual load values than those of the model by [Coulon et al., 2014]. To be more precise, the correlation between the load and v^W equals to 0.36, between the load and v^C to 0.63, and between the load and v^F to 0.75. Closer correlation between v^F and the load values can be seen explicitly during e.g. night hours on 11th, 12th, 16th and 17th of June or during day hours on 11th and 16th of June. For instance, during the early morning on 11th of June the actual load values drop as is suggested by our Fundamental model, yet a contrary tendency is predicted by the[Coulon et al., 2014] model.

Furthermore, note that the red curve in Figure 9 can be easily shifted upwards to match the actual load values. In fact, shifting the red curve upwards eliminates the limitations imposed by earlier manipulations with the Utility's curves (section 2.2.4). As has already been mentioned, volumes we add to reconcile Utility's prices do not influence the shape of the demand curve in our Fundamental Model. Therefore, these volumes only affect the position of the red curve relative to the blue curve, but not the correlation between these two curves.

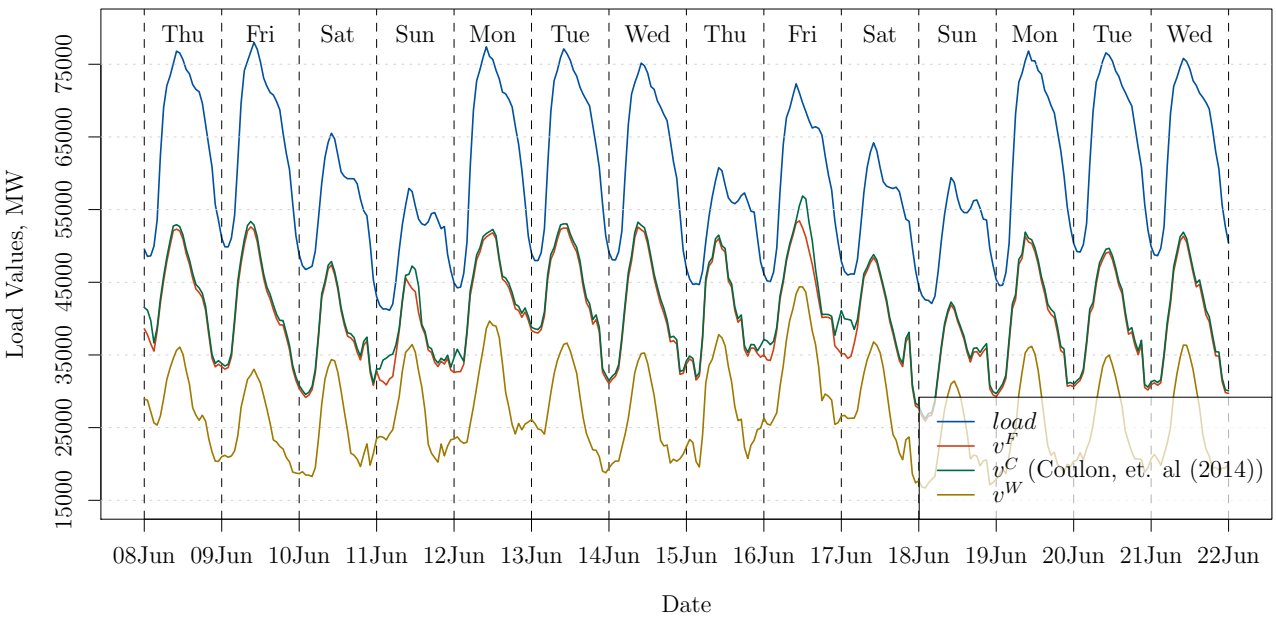


Figure 9: Fundamental Model volumes vs. values suggested by the [Coulon et al., 2014] model vs. actual load values vs. wholesale market volumes for a two weeks sample from 08 June 2017 to 21 June 2017

4.3 Determining the demand elasticity

Ultimately, an auxiliary study of demand elasticity was conducted. Given that our optimization model is non-linear, our demand elasticities, too, should be non-linear. Estimating slope coefficients by means of a linear regression does not allow non-elastic components to be traced. Therefore, we will apply a finite central difference method to determine demand elasticities. To calculate the slope, we will rely on a function which depends on two parameters p and h. The former parameter is a point on the demand curve, the latter one is a shift distance form the point p and is equal to

100 MW. Then,

$$ls(p,h) = \frac{\text{FDem}\left(\text{FDem}^{-1}(p)+h\right) - \text{FDem}\left(\text{FDem}^{-1}(p)-h\right)}{2h}, \tag{21}$$

where $ls(p,h)$ approximates the slope of the demand curve in point p and is measured in EUR/MWh^2. An elasticity coefficient can thus be determined as follows

$$E^p_{F,i} = \frac{p}{\text{FDem}^{-1}(p)} \cdot \frac{1}{ls(p,h)}. \tag{22}$$

We have then conducted the study at points $p \in \{0, 20, 25, 30, 35, 40, 50, 60\}$ and summarized the obtained results in Figures 10(a) and 10(b). The former one shows average values of $ls(p,h)$, the latter one depicts the corresponding average elasticities E^p_F.

Let us focus on both Figures simultaneously because the values of $ls(p,h)$ and E^p_F follow similar patterns. First, note that $ls(p,h)$ at points $p \in \{0, 20, 25\}$ tend to be further away from zero. In turn, the absolute values of E^p_F at points $p \in \{0, 20, 25\}$ tend to be relatively low. These two observations are consistent with the fact that the lower part of our fundamental demand curve is almost perfectly inelastic. Second, absolute values of $ls(p,h)$ and E^p_F at points $p \in \{30, 35, 40, 45, 50\}$ tend to be lower and higher, respectively, because these points are located in the most elastic parts of the fundamental demand curve. Moreover, the majority of equilibrium prices occurs within this price range.

More importantly, we see two spikes in the values of $ls(p,h)$ and E^p_F at points $p \in \{30, 35, 40, 45, 50\}$. These spikes occur during morning and evening hours. Similar tendencies have also been observed by [Knaut and Paulus, 2016] and [Bigerna and Bollino, 2014]. However, contrary to these papers, absolute values of our elasticity coefficients drop. This tendency, too, is not surprising, given the composition of our fundamental demand curve. Points which split our fundamental demand curve into elastic and inelastic parts are typically located within the range $p \in \{30, 50\}$. Therefore, following equation 21, the difference $\text{FDem}\left(\text{FDem}^{-1}(p)+h\right) - \text{FDem}\left(\text{FDem}^{-1}(p)-h\right)$ may be large if point p is located in the very elastic part of our fundamental demand. On the other hand, the values of E^p_F are small if p is located in its inelastic part. Hence, following section 2.2, we may conclude that many demand elasticities were transferred to the supply side at the spike points. From this perspective, our results are similar to those of [Knaut and Paulus, 2016] and [Bigerna and Bollino, 2014].

In fact, we can compare our results with only two studies on demand elasticity in the German wholesale electricity market. Our values (range from -0.207 to -0.01) are higher (in absolute terms) than those of [Knaut and Paulus, 2016] (range from -0.006 to -0.001), yet follow a similar pattern. On the other hand, our values are lower (in absolute terms) than the value -0.43 computed by [Bönte et al., 2015]. Naturally, the discrepancy between the values stems from the method we used to determine demand elasticity. [Knaut and Paulus, 2016] and [Bönte et al., 2015] used wind speed as the key parameter in their analyses. On the contrary, our values are non-linear and are obtained from an innovative fundamental model. Furthermore, the considered time frames, too, are different.

To carry out a deeper sensitivity analysis, Figures 10(c) and 10(d) were constructed. The former one summarizes average monthly elasticities for selected prices p, the latter one demonstrates day-of-the-week elasticities. To the best of our knowledge, seasonal and day-of-the-week breakdown of demand elasticities has never been conducted for the German wholesale electricity market.

As can be seen from Figure 10(c), there is a spike in demand elasticity in summer. Following the reasoning in the previous paragraph, the observed tendency is consistent with the findings of

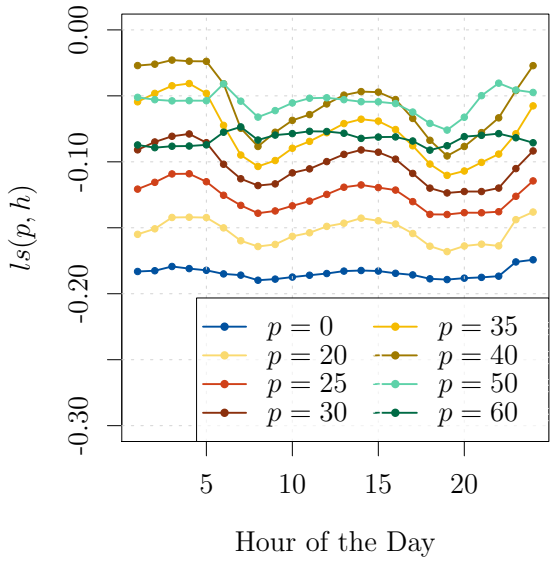
[Bigerna and Bollino, 2014] and does not seem counterintuitive. Finally, Figure 10(d) shows that elasticities tend to be higher during weekdays and lower over the weekend for higher values of p. The contrary can be said for lower values of p. The observed behavior can be well explained by routine electricity consumption patterns as described in e.g. [Weron, 2007].

5 Conclusion

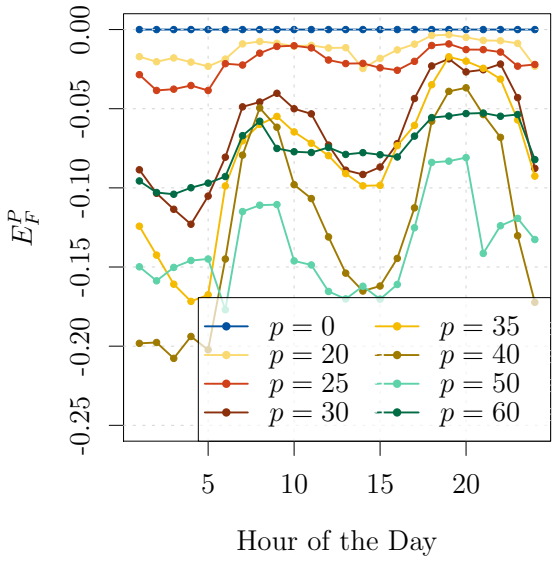
Auction curves recorded in a wholesale electricity market often contain a variety of arbitrage orders. The model developed in this paper attempts to detect and process these arbitrage orders. As a result, we can determine a true shape of the wholesale demand curve, i.e. the curve which is "cleaned" from those arbitrage orders. Our demand curve thus lies "in between" the elastic wholesale demand curve and a perfectly inelastic demand curve, the latter being obtained by transferring all elasticities to the supply side.

Our model allowed us to obtain a more profound understanding of the German wholesale electricity market. In particular, we scrutinized the compositions of the wholesale demand and supply curves. The knowledge of these compositions may be useful for further modeling of the auction curves. Moreover, we showed that our model can approximate actual load values observed in an electricity market better than the model with perfectly inelastic demand curve. Therefore, the solution we elaborated can be well suitable for load forecasting. Furthermore, our paper provides presumably the second (after [Knaut and Paulus, 2016]) in-depth analysis of demand elasticity in the German wholesale electricity market.

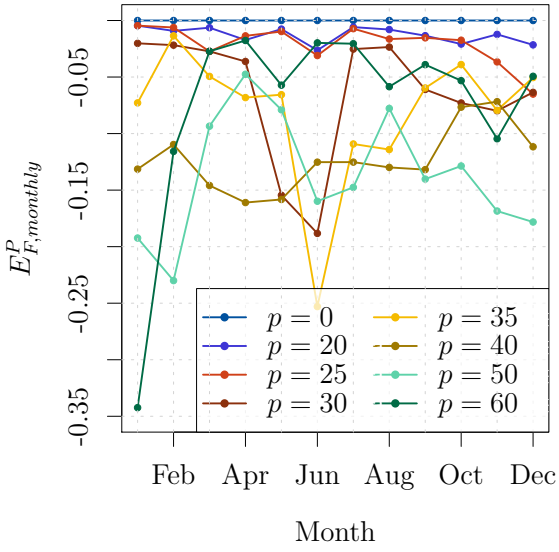
There exist numerous possibilities for further research. First, our model can be extended from the case of a market with single Utility, Retailer, and Supplier. One can easily incorporate e.g. multiple Utilities, several types of Retailers, or differently acting Suppliers into the model. Second, one can smoothen the transition from elastic to inelastic parts in our fundamental demand curve. Currently there is a single point which splits this curve into two parts. Third, transaction costs can be considered. Their presence may substantially alter the behavior of market participants. Finally, the model can be extended with multiple coefficients, especially for determining the optimal amount of must-run supply and demand. As was mentioned in section 2.2.4, volumes of sell orders at price $p_{\min}$ and of buy orders at price $p_{\max}$ are much larger than those of other orders. Therefore, must-run orders can be treated differently. In fact, knowing the size of must-run buy and sell orders would allow us to omit the manipulations described in the second step of section 2.2.4. Shifting one of the Utility's curves to reconcile equilibrium prices will no longer be necessary because the optimal sizes of must-run supply and demand would be determined by the model.



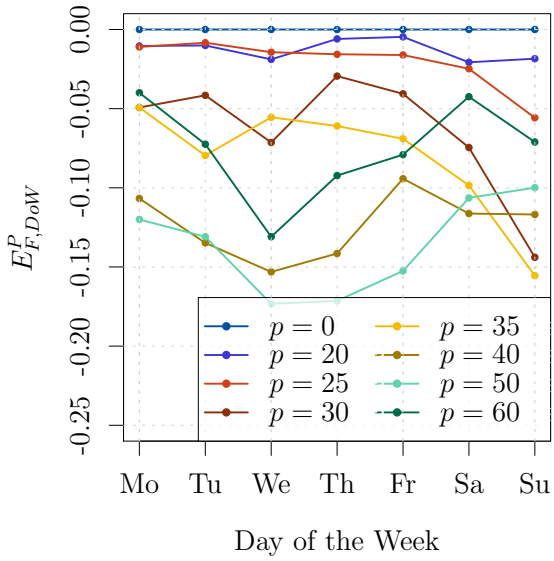
(a) Slope coefficients $ls(p, h)$



(b) Point elasticities of demand E_F^p



(c) Average monthly elasticities



(d) Day-of-the-week elasticities

Figure 10: The analysis of demand elasticities

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5 Conclusion

The present thesis is built on three academic papers. These papers provide a comprehensive study of econometric modeling and forecasting with application in electricity markets. More specifically, the thesis places an emphasis on the analysis of wholesale auction supply and demand curves (also known as sales and purchase curves) in electricity markets. The thesis makes four global contributions to academic literature. First, it is shown that econometric modeling and forecasting of the curves can be successfully used to both deeper understand the underlying structure of an electricity market as well as to make relatively accurate electricity price and volume forecasts. Therefore, the thesis fills an existing gap in the academic literature, as the wholesale auction curves in electricity markets were relatively unstudied. Second, the thesis shows that econometric modeling of wholesale supply and demand curves allows best aspects of both statistical and structural methods to be combined. Therefore, the thesis can be placed on the intersection between these two methods and bridges the gap between them. Third, several novel econometric models were elaborated within this thesis, thus underlining its novelty and presenting multiple opportunities for further research. They will be discussed at length in what follows. Fourth, besides its methodological and academic contribution, the thesis elaborates on real-world economic and policy implications of models' results. Hence, the thesis can be of interest to both researchers and practitioners alike.

The remainder of the conclusion will be split into two parts. The first part will be dedicated to three discussions, each related to a corresponding academic paper presented in the thesis. These discussions will consist of a brief overview of papers' results as well as of descriptions of further research perspectives provided by these papers. The second part will elaborate on a potential opportunity for further research beyond those presented by the three papers. This opportunity has already been given investigation and is related to the application of wholesale auction curves for the purposes of data transformation.

5.1 Discussion of the Obtained Results

5.1.1 X-model: Further Development and Possible Modifications

Results' overview

The present paper demonstrated that combining the findings of [Ziel and Steinert, 2016] and [Coulon et al., 2014] allows the X-model to be significantly improved. The first improvement is a substantial increase in execution speed (approximately 3 times faster). This is achieved because the demand curve in the modified X-model is represented by only a single point. Therefore, as opposed to the original X-model which needs to construct $M_D > 1$ time series models to make a forecast for the demand curve, the modified X-model relies upon only one model. To put these numbers into perspective, the original X-model forecasts $M_D = 16$ points on the demand curve.

Second, the modified X-model is proven to be more robust towards outliers present in the original data. This improvement occurs due to the fact that the only supply curve is constructed via cumulating volume forecasts $X^c_{S,d,h}$ for price classes $(c > 1, ..., M_S)$ over volume $X^1_{S,d,h}$ in the first price class $c = 1$. Therefore, if there is an outlier in one price class $c < M_S$ then the part of the supply curve $(c, ..., M_S)$ will be affected because this part is accumulated on top of volumes $X^c_{S,d,h}$ in price class c. Contrary to the original model, this effect does not occur on the demand side in the modified X-model because the demand curve is inelastic. Moreover, the fact that the impact of outliers on the model's performance is reduced is especially important in the post-COVID market, where the amount of price spikes, in particular negative ones, will heavily impact the performance

of forecasting models (see e.g. [Zhong et al., 2020] or [Ghiani et al., 2020]).

Third, the modified X-model yields more accurate forecasting results. An increase in accuracy is achieved due to the two previously described improvements: having only one forecast for the demand curve means that the overall amount of forecasting errors is minimized, while a greater robustness towards outliers means that the model is more precise generally. As is shown in the paper's results, the modified X-model has significantly lower MAE and RMSE values than the original X-model (the modified X-model outperforms even an equally weighted mixture of the modified and original X-models). Furthermore, the better performance of the modified X-model has been confirmed by the conducted DM-test.

Further Research Options

There are several potential directions for further research provided by this paper. The first one is related to minimizing the issue of outliers. As was mentioned earlier, the current version of the X-model accumulates volumes $X^c_{S,d,h}$ for price classes $(c > 1, ..., M_S)$ over volume $X^1_{S,d,h}$. It is, however, also possible to accumulate volumes $X^c_{S,d,h}$ for price classes $(c, ..., c < M_S)$ over volume $X^{M_S}_{S,d,h}$, i.e. the volume in the last price class $c = M_S$. Furthermore, it is possible to compute an average of these two options (accumulated over $X^1_{S,d,h}$ and over $X^{M_S}_{S,d,h}$) to determine the final forecast for the supply curve and thus minimize the influence of outliers. A further potential step would be to neglect a simple arithmetic average of the two options in favor of determining optimal weights of each of these options in the final forecast for the supply curve.

The second option would also allow the impact of outliers to be minimized. As follows from the paper, the X-model creates volume forecasts for volume differences in each price class relative to volume $X^1_{S,d,h}$. Instead of using these differences, it is also possible to create M_S straightforward volume forecasts for each price class. In this case, it is important to ensure (e.g. via a separate condition) that volume $X^c_{S,d,h}$ forecasted in price class $c > 1$ is grater or equal to volume $X^{c-1}_{S,d,h}$. Otherwise, the supply curve can not be constructed. The main advantage of this option is that the cumulative impact of outliers described in the previous paragraph is being minimized or even vanishes.

The third option is to aim directly on improving the selection process of the price classes. Currently, there is a simple method with an application of an equidistant volume grid to an average price curve. Instead, it is possible to create an algorithm that will select only points which are most important for the composition of the curve. Potentially, these points may be concentrated (a) in the middle sector of the supply curve because equilibrium prices usually occur in this sector, (b) in sectors of the supply curve where the curve bends or (c) towards the extremes of the curves where relatively large bids are being placed. As a result, the optimal amount of price classes $\mathcal{C}$ will (a) ensure that the forecast is accurate enough and (b) that price classes are not too close to each other, thus increasing the probability that two forecasts for points c and $c+1$ yielding almost identical results and becoming redundant.

The fourth option is to implement a small fine-tuning adjustment after the forecast for both curves has been conducted. This adjustment can be done similarly to the paper "The Impact of Renewable Energy Forecasts on Intraday Electricity Prices", i.e. through horizontally shifting both supply and demand curves. In other words, if to denote shift magnitudes for supply and demand curves as $\boldsymbol{\kappa}_{d,h} = (\kappa_{S,d,h}, \kappa_{D,d,h}))$, the shift magnitudes can be determined by a non-linear least squares optimization problem in the following exemplary form: $\widehat{\boldsymbol{\kappa}}_{d,h} = \arg\min_{\kappa \in \boldsymbol{R}^2} (P^{DA}_{d,h} - P^{xmod^{combined}}_{d,h})^2$, where $P^{DA}_{d,h}$ denotes a day-ahead price and $P^{xmod^{combined}}_{d,h}$ is the model's price for time point d, h.

5.1.2 The Impact of Renewable Energy Forecasts on Intraday Electricity Prices

Results overview

The contribution of this paper is threefold. First, the paper presents a novel approach to econometric modeling and forecasting of the wholesale auction curves. The model assumes that the difference between transformed day-ahead and intraday wholesale supply curves is driven only by errors in wind and solar power forecasts, i.e. by differences between actual and forecasted values of wind and solar supply.[3] Then, the model subtracts adjusted sizes of forecasting errors from the day-ahead supply curve, thereby shifting this curve horizontally. The optimal size of the adjustment is derived by means of a relatively straightforward non-linear least-squares optimization problem. The shifted day-ahead curves thus constitute the forecast for for intraday supply curves. In turn, intersections of the shifted curves with the demand coincide with the forecast for intraday prices. Moreover, there is a key advantage of the model relative to conventional non-linear benchmark models. Given that it is possible to immediately see the contribution of each of the modeled parameters into a supply curve's shift size, the model enables a clear and straightforward interpretation of results.

Second, the obtained results allow us to conclude that the impact of wind and solar power forecasts on intraday electricity prices is non-linear. This conclusion was made because quadratic benchmarks were better than the linear ones, while the constructed model significantly outperformed conventional benchmarks of both linear and non-linear types. Furthermore, besides the main analysis, there is an auxiliary study outlined in the paper. This study is based on a numerical example and shows that an increase of wind and solar power capacities will inevitably lead to a growth in the corresponding errors in wind and solar power forecasts. The impact of these errors on price volatility, as follows from the study, is non-linear too.

Third, on the grounds of the results there are six economic and policy implications presented in the paper. These implications are aimed on reducing the non-linear impact of forecast errors on intraday prices, as this impact may lead to some substantial market turmoils. The reduction of the impact can be achieved, among others, through reducing either the amount of forecast errors or the volatility of renewable energy infeed. Therefore, there are the following arguments provided in the paper: (a) selecting locations of new wind and solar power plants such that the correlation between the power supply of these plants with that of older plants is minimized, (b) diversifying the supply sources of renewable energy, especially given the fact that the infeed of wind and solar power plants is negatively correlated, (c) expanding cross-border infrastrcture and cross-border intraday trading opportunities to avoid country-specific bottlenecks, (d) generally improving the quality of renewable energy forecasts, (e) taking greater advantage of flexible generation technologies and better demand side management to keep the merit-order curve winder and more elastic for longer periods of time and (f) introducing a better renewable energy curtailment management, especially e.g. centralized public disclosure of information regarding upcoming curtailments.

Further research options

The first way to further enhance the performance of the model would be to include a shift of the demand curve, as only the supply curve is shifted in the current version of the model. As a result, the model will not only approximate intraday supply, but also intraday demand. There are several ways to implement this shift. On the one hand, it is possible to split the amount of wind and solar forecast errors between the supply and demand sides, i.e. to use $\chi(W_t^{\Delta}+S_t^{\Delta})$ and $(1-\chi)(W_t^{\Delta}+S_t^{\Delta})$

[3] As has been mentioned in the paper, actual values were taken as the best approximation of last-minute intraday market values.

as non-adjusted shift sizes of the supply and demand curves, respectively. The value of χ may be either set to 0.5 by default or be determined by means of another optimization. On the other hand, given that the demand curve is inelastic, it is possible to implement a time-series volume forecasting model in the spirit of the X-model.

The second way would be to solve the optimization problem via a more sophisticated optimization tool. Currently, the `R`-package `optim` with method `BFGS` (that is a quasi-Newton algorithm) is used to determine the vector $\boldsymbol{\beta}$ of coefficients. This algorithm suffers from the problem of local optimum. A more advanced technique, e.g. metaheuristic-based genetic algorithm in a spirit of those described in the work by [Yang, 2010], will potentially significantly improve the performance of the model.

Third, the obtained results showed that the model, despite being unconventional, can be successfully used for intraday price forecasting. However, due to a lack of available data, the model currently uses actually realized values of wind and solar power infeed (denoted by W_t^A and S_t^A, respectively) to determine the sizes of forecast errors. Therefore, in order to be applied for intraday price forecasting in classical sense, the model's inputs need to be adjusted. Instead of using actually realized values, wind and solar power forecasts made in the intraday market must be put into the model. Moreover, besides considering only the errors in wind and solar power forecasts, further additional parameters, i.e. autoregressive components or load forecasts should be added to further improve forecasting performance of the model. Furthermore, as has been mentioned earlier, combining the X-model with the idea of shifting the auction curves can potentially yield further positive results.

Finally, an auxiliary study provided in the paper can be further expanded into a separate full-fledged analysis. Currently, the study is based on dummy data and a number of very simplistic assumptions. Using actual empirical data in this study will allow the real-world non-linearities in the impact of forecast errors on electricity prices arising from newly build wind and solar capacities to be quantified. Furthermore, on the grounds of empirical data the actual impact of newly added capacities on intraday price volatility can be evaluated. Another potential way to convert this auxiliary model into a separate study is to improve this model's underlying assumptions. Currently, a very simplistic correlation structure is used. This structure can be replaced with a more sophisticated one, e.g. with a set of spatially correct assumptions regarding actual locations and correlations of existing wind and solar power plants. Moreover, other energy sources (both renewable and non-renewable) can be incorporated into the model alongside wind and solar power, thereby providing grounds for creating a sound fundamental model of an electricity market.

5.1.3 Determining Fundamental Supply and Demand Curves in a Wholesale Electricity Market

Results overview

Analogously to the paper summarized in the previous subsection, the present paper also elaborates a novel econometric model. This model manipulates the wholesale auction curves with the purpose of decomposing them into individual orders of market participants. The need in this decomposition arises from the fact that contributions of each individual market participant to the auction curves (i.e. their bid and ask orders) are not publicly revealed in the German electricity market, as opposed to e.g. the Italian one. Therefore, the model allows the bidding structure of the German electricity market to be investigated.

The decomposition is carried out as follows. First, it is assumed that market participants in an electricity market are represented by three umbrella types: a Supplier (who can place orders only

into the wholesale supply curve), a Retailer (who can place orders only into the wholesale demand curve) and a Utility (who can place orders into both wholesale supply and demand curves). Second, several assumptions regarding trading behavior of these three market participants are introduced. Among these assumptions is e.g. that only the Utility has an opportunity to purchase electricity in the wholesale market (i.e. to put arbitrage orders into the wholesale demand instead of actually producing these orders). Third, findings of [Knaut and Paulus, 2016] are recalled. Following this paper, there are two ways to aggregate individual orders of market participants into wholesale supply and demand curves. These ways differ in that one of them prohibits the Utility to purchase electricity in the market. As a result, two equilibria can be constructed: one includes Utility's arbitrage orders and one does not.[4] Fourth, an iterative optimization algorithm is set up. A brief stepwise description of this algorithm is as follows: (a) take the initial empirical wholesale supply and demand curves, (b) split these curves in some proportions between the three market participants, thereby obtaining supply and demand schedules of each individual market participant, (c) use these individual supply and demand schedules to construct an equilibrium that excludes Utility's arbitrage orders, (d) check the differences between the obtained equilibrium volumes and the actual load values, (e) reiterate with different proportions mentioned in (b) to minimize the difference mentioned in (d).

The model obtained in the end is a novel fundamental model of an actual electricity market. This model has several advantages. First, the model excludes arbitrage orders from the initial empirical wholesale auction curves. In other words, the model leaves only actual not-speculation-driven transactions with physical assets on buy and sell sides. Therefore, the model serves as a more accurate representation of the "true" ("fundamental") market equilibrium, as it only contains orders that actually are expected to be realized. Second, the model explicitly indicates the contribution of each type of market participant into the final supply and demand curves. This allows the structure of an electricity market to be scrutinized. Third, the model's demand curve lies in between the initial elastic wholesale demand curve and a perfectly inelastic curve as described in [Coulon et al., 2014]. Thus, the derived demand curve retains only a part of original elasticities, which is to be expected because some of the orders in the initial wholesale demand curve are of arbitrage nature. Fourth, the model can be utilized to relatively accurately predict actual electricity load. Finally, besides the methodological contribution, the paper also provides a study of demand elasticities in the German electricity market, presumably only the second one after that in [Knaut and Paulus, 2016].

Further research options

There are several research opportunities that this paper provides. First, an application of the model to electricity markets other than the German one may yield noteworthy results. One of the most interesting markets would be the Italian Power Exchange (IPEX). The regulation, structure and dynamics of this market is relatively similar to those of the German one (see e.g. [Chapman and Itaoka, 2018], [Gianfreda and Grossi, 2012] and [Shah and Lisi, 2020]). However, contrary to the EPEX SPOT SE, the IPEX reveals orders of each individual market participants in the wholesale auction curves. In other words, the composition of the wholesale caution curves is publicly available in Italy. Therefore, it is possible to conduct real-world tests of the model and evaluate its accuracy on the grounds of the data from the Italian electricity market.

Second, the model currently assumes only three types of market participants. However, in the paper by [Knaut and Paulus, 2016] each of these types was represented by two players. Therefore,

[4]The former equilibrium is in fact the one represented by empirical wholesale supply and demand curves.

it is possible to extend our model for the case of multiple players in each type, e.g. two Suppliers, two Retailers and two Utilities. This will potentially further increase the precision of the model, albeit will come at a high computational cost. As a somewhat compromise solution, one can construct a model with several Utilities but only one Supplier and one Retailer. Such approach has potential because one can assume that Utilities have different trading behaviors. For example, one of the Utilities may decide to purchase electricity in the wholesale market only if the market price is at least some value v below the Utility's internal price. Moreover, as another potential research direction, one can compare various versions of the model with different assumptions regarding market participants' number and trading behaviors.

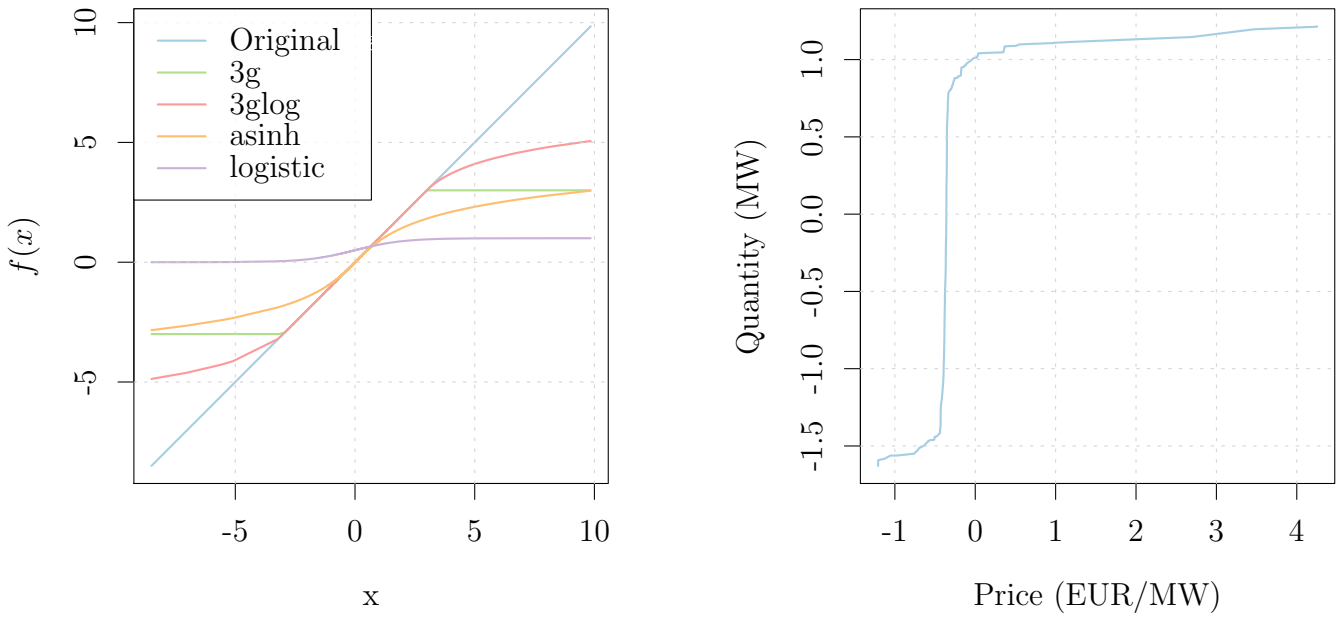
Third, technical improvements to the model can be introduced. First, analogously to the second research option described in the previous subsection, the present model is also estimated using on a package-based solution, although a relatively advanced one (library `GA` in `R`). Nevertheless, an improvement of the results can be achieved by application of a more sophisticated technique tailored to the specific needs of the model. Second, the current fundamental demand curve is essentially split into two parts: elastic and inelastic. One can smoothen the transition one part to another. Given that this transition will occur in the area of the market's equilibrium, the final results can be positively affected. Third, more optimization parameters can be incorporated into the model. Among such parameters are those for determining must-run parts of the supply and demand curves at prices $P_{\min}$ and $P_{\max}$, respectively. These parts are typically largest in the curves, which is why adjusting the sizes of these parts will potentially improve the accuracy of the model. From this perspective, both curves will be shifted through altering the sizes of their must-run parts, which is an idea similar to the shifts of the wholesale auction curves described in the previous subsection.

Fourth, the model and its underlying intuition can be applied in a variety of settings. For example, as the above results demonstrated, the model can be used for the purposes of load forecasting. Therefore, the model can be tested against conventional load forecasting benchmarks or become a part of a combined load forecasting model. Another option would be to utilize the model for the benefit of a more economic- or policy-oriented study. For instance, it is possible to introduce transaction costs into the model or determine such size of transaction costs at which the willingness of the Utility to trade in the wholesale market is optimally limited. Besides the transaction costs, the impact of the curtailment of renewable energy supply can be studied on the grounds of the model, e.g. by artificially altering the must-run part of the supply curve.

5.2 Potential Further Research

There are, of course, a lot of other ways to utilize the wholesale supply and demand curves for further research purposes. One such way has been investigated and its results are promising. More specifically, one can use the wholesale supply curve for the purposes of data transformation and variance stabilization. In line with the work in [Uniejewski et al., 2017], the idea is that the shape of the wholesale supply curve is similar to the shapes of typical variance stabilization functions. Therefore, it is rational to assume that using the curve or its modified versions to transform initial electricity price data will allow the forecasting quality to be improved. The results of the conducted analysis indicate that this is indeed the case. Furthermore, an example of several transformation classes is depicted in Figure 4(a), while an example of an inversed day-ahead wholesale supply curve is presented in Figure 4(b).

To prove the above statements, a data set of German wholesale electricity prices and volumes from the EPEX spot SE was considered. Furthermore, supplementary data regarding predicted wind and solar infeed as well as the forecasted electrical load values were taken into account.



(a) Visualisation of several popular transformations, all scaled such that $f(0) = 0$ and the slope at $x = 0$ is 45 degrees

(b) Scaled version of the inversed wholesale supply curve on 17-02-2018 23:00:00

Figure 4: Popular variance transformations (left) and a scaled version of the inverse wholesale supply curve (right)

From this perspective, the data set was as the one used in the paper "The Impact of Renewable Energy Forecasts on Intraday Electricity Prices", with the considered time frame being 01.01.2017 to 31.12.2018. Then, to proceed with the analysis, the data set was transformed using several variance stabilization techniques. Afterwards, a benchmark forecasting model was applied to each of the transformed time series. Each forecast was evaluated afterwards using the conventional MAE and RMSE measures.

The benchmark forecasting model is a simple expert model that was estimated using the LASSO-technique (package `glmnet` in `R`) and has the following specification

$$\widehat{Y}_{d+1,h} = \widehat{\beta}_{d,h,0} + \sum_{n=1}^{7} \widehat{\beta}_{d,h,n} Y_{d-n,h} + \sum_{k=1}^{7} \widehat{\beta}_{d,h,7+k} W^F_{d-k,h} + \sum_{j=1}^{7} \widehat{\beta}_{d,h,14+j} S^F_{d-j,h} \tag{2}$$

$$+ \sum_{i=1}^{7} \widehat{\beta}_{d,h,21+i} load^F_{d-i,h} + \varepsilon_{d,h}, \tag{3}$$

where $Y_{d,h}$ stands for the transformed price $P_{d,h}$, d and h are time indices, $W^F_{d,h}$ and $S^F_{d,h}$ are day-ahead forecasts of wind and solar infeed, $load^F_{d,h}$ is a day-ahead load forecast and $\varepsilon_{d,h}$ is an error term. Given that the main goal is to compare different variance stabilization techniques, the model was kept relatively simple, albeit the model includes a lot of autoregressive lags and is LASSO-estimated.

The following variance stabilization techniques were considered. First, the conventional *asinh* transformation was taken as the main benchmark. *asinh* was given special attention because it was proven by [Uniejewski et al., 2017] to be one of the best applications to electricity price data.

The *asinh* transformation is technically defined as follows

$$Y_{d,h}^{asinh} = asinh(P_{d,h}) = \log\left(P_{d,h} + \sqrt{P_{d,h}^2 + 1}\right) \tag{4}$$

with the inverse transformation being defined as $P_{d,h} = \sinh(Y_{d,h})$.

Second, the inverse wholesale supply curve at time point d, h. The inverse of the curve is taken to plug in price values instead of volume values. The transformation is defined as $sup1$ and its definition reads

$$Y_{d,h}^{sup1} = Sup_{d,h}^{-1}(P_{d,h}) \tag{5}$$

with the inverse function being defined as $P_{d,h} = Sup(Y_{d,h})$. Therefore, speaking technically, whenever the wholesale supply curve is applied as a tool for data transformation, a volume forecast is to be carried out on a transformed data, which would then be converted back into the forecast for day-ahead price.

Third, an average of the inverse wholesale supply curves denoted with *avgsupr* and computed over some period of time r. The main aim in this case is to incorporate past information into the data transformation. More specifically, this transformation is determined as

$$Y_{d,h}^{avgsupr} = \left(\frac{1}{r}\sum_{l=1}^{r} Sup_{d-l,h}^{-1}\right)(P_{d,h}) \tag{6}$$

where r is the amount of past days over which the average wholesale supply curve is calculated. In the current analysis, it is assumed that $\boldsymbol{r} = (28, 56, 112)$. The selection of days r was not performed using any specific method. The inverse function is thus $\left(\frac{1}{r}\sum_{l=1}^{r} Sup_{d-l,h}\right)(Y_{d,h})$.

To evaluate the performance of the benchmark forecasting model when applied to different transformed data sets, a rolling-window study with 24 hour step was used. The in-sample time frame is the year 2017, the out-of-sample is 2018. The obtained MAE- and RMSE values are summarized in Table 1 below. Furthermore, to ease the comparison, a simple naive benchmark with the formula $P_{d+1,h} = P_{d,h}$ was added into the Table.

	naive	no transformation	*asinh*	*sup*1	*avgsupr* 28	*avgsupr* 56	*avgsupr* 112
MAE	9.9705	5.8321	5.4424	5.4696	**5.3440**	5.3699	5.3435
RMSE	15.7523	9.9617	9.5208	9.975	9.6352	9.6082	**9.5087**

Table 1: The obtained MAE- and RMSE-test results.

As can be seen from Table 1, the proposed variance stabilization methods with an application of the adjusted wholesale supply curve outperforms the conventional *asinh* technique in terms of both MAE and RMSE measures. Furthermore, it is important to note that the lowest MAE value is achieved in the method *avgsupr* with $r = 28$, while the lowest RMSE value is in the case of *avgsupr* with $r = 112$.

The obtained results indicate clearly that using the wholesale supply curve for the purpose of data transformation and variance stabilization is an idea well worth studying further. In fact, there are several ways to extend the above analysis and develop a full-fledged study out of the idea. In order to do so, several steps can be undertaken. First, a comparison of the models by means of the DM-test or the pinball score or other forecasting methodologies is to be done.

Second, general improvements to the considered techniques can be made. On the one hand, it is possible to incorporate other options for better tailoring the wholesale supply curves for the purpose of data transformation. For example, instead of a simple average over r days, one can consider an autoregressive process to derive a better tool for transforming the data. The resulting transformation can be defined as

$$Y_{d,h}^{avgsupregr} = \widehat{\beta}_0 + \left(\sum_{m=1}^{r} \widehat{\beta}_m Sup_{d-m,h}^{-1} \right) (P_{d,h}) \tag{7}$$

with an inverse process being $P_{d,h} = \widehat{\beta}_0 + \left(\sum_{m=1}^{r} \widehat{\beta}_m Sup_{d-m,h} \right) (Y_{d,h})$. On the other hand, it is also possible to incorporate a more theoretically profound method for the selections of days r used to determine an average of the wholesale supply curve. Third, the model can be evaluated on the grounds of the more recent data, especially including the data from 2020 with the impact of the coronavirus pandemic and the resulting price spikes. Especially interesting would be to see how the transformation tackles the issue of negative price spikes.

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