The bending-torsion-stiffness ratio of flexure hinges with common and polynomial notch shapes

Das Biege-Torsions-Steifigkeitsverhältnisses von Festkörpergelenken mit Standard- und Polynomkonturen

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Abstract

In high-precision motion systems requirements with respect to the resolution and repeatability of motion are in the order of magnitude of nanometers and angular sub-seconds [1]. The lack of clearance and wear in flexure hinges [2] is advantageous in this context and contributes to a smooth and repeatable motion with the desired guiding accuracy [3]. Therefore, compliant mechanisms are widely used in precision engineering [4]. The common notch-shaped flexure hinges are bending-loaded and applied as materially coherent revolute joints in compliant mechanisms for mostly planar motions.

In practical applications the flexure hinges of those mechanisms are subjected to deviations e.g. by manufacturing tolerances which effect their shape, position and orientation. This causes non-planar load cases for the hinges within the compliant mechanism. Therefore, besides bending, torsional loads act on the hinge that change the path and reduce the mechanism accuracy. The obvious approach to mitigate the unappreciated non-planar motion of the compliant mechanism is the increase of the hinge thickness h to increase its torsional stiffness [5]. However, this is critical and limited since it is simultaneously increasing the bending stiffness and maximizing the strain in the hinges. Referring to literature, flexure hinges are mostly optimized with respect to either a low bending stiffness [6] or a high torsional stiffness [7] in addition to a high rotational precision [8]. A minimization approach aiming for a high torsional stiffness in combination with a low bending stiffness is not regarded so far. Therefore, this work introduces the bending-torsion-stiffness ratio as a novel combined performance criterion of notch flexure hinges. The influence of the notch shape on this criterion is investigated in dependence of three main geometric parameters, the hinge length l, minimum thickness h, and width w.

Based on a quasi-static geometrically non-linear 3D FEM simulation, the bending-torsion-stiffness ratio is investigated by simple kinematic considerations for an individual prismatic flexure hinge loaded with a combined bending and torsional moment at the free end face leading to typical discrete deflection angles of $\varphi_z = 1.0°$ and $\varphi_x = 0.1°$ (Figure 1).

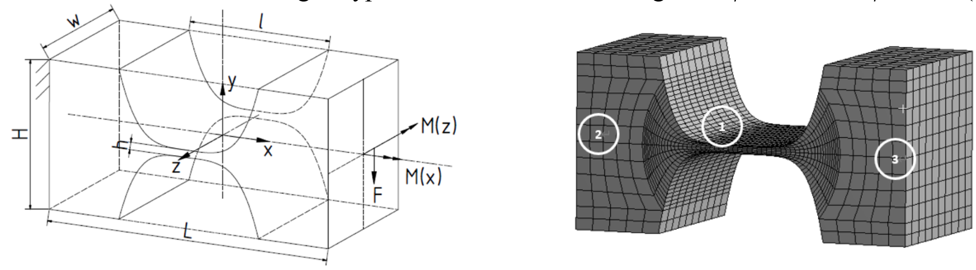

Figure 1 Flexure hinge model with geometric parameters and loads (l.); 3-zone mapped mesh strategy with notch refinement (r.)

Polynomial notch shapes are especially suitable due to their easily adjustable order [9]. Here, they are compared with two common shapes, the corner-filleted and semi-circular contour [2, 10], with regard to the proposed ratio (Figure 2). A full-factorial parametric study is performed with ANSYS Workbench in dependence of the notch shape and the geometric parameters. The desired minimization of the bending-torsion-stiffness ratio is based on a design of experiments due to the variation of the geometric hinge parameters and the contour as follows, which lead to 600 calculated designs:

- the hinge length ratio $\beta_l = \frac{l}{H}$, for three values of 0.5, 1.0 and 1.5 (while $\beta_L = \frac{L}{H}$ is fixed to 2);
- the hinge height respective thickness ratio $\beta_h = \frac{h}{H}$, for four values of 0.05, 1.0, 0.15 and 0.2;
- the hinge width ratio $\beta_w = \frac{w}{H}$, for five values of 0.25, 0.5, 1.0, 1.5 and 1.5;
- the hinge notch shape (corner-filleted, circular and polynomial contour of 2nd, 3rd, 4th, 5th, 6th, 8th, 12th and 16th order).

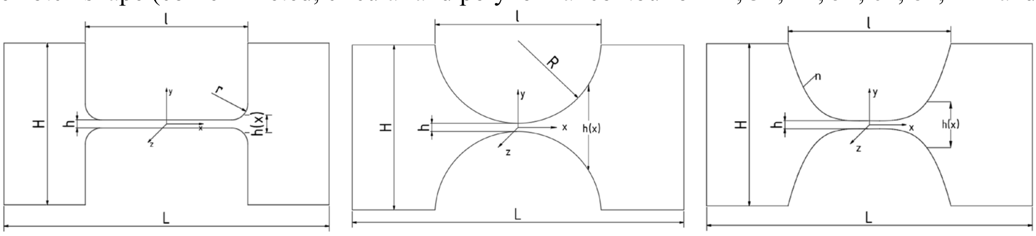

Figure 2 Corner-filleted contour with $r = 0.1l$ (l.); circular contour with $R = 0.5l$ (m.); polynomial contour with order $n = 4$ (r.)

Table 1 Contour-dependent FEM results of the bending stiffness k_B, torsional stiffness k_T and bending-torsion-stiffness ratio k_B/k_T, exemplarily shown for the smallest ratios that generally result for $\beta_l = 0.5$, $\beta_h = 0.05$ and $\beta_w = 2$ ($\varphi_z = 1.0°$ and $\varphi_x = 0.1°$)

Contour	k_B [Nmm/°]	k_T [Nmm/°]	k_B/k_T
CF	74.345	1781.242	0.042
C	223.040	11563.843	0.019
PC 2	386.421	29724.692	0.013
PC 3	208.642	13909.477	0.015
PC 4	147.559	8793.891	0.017
PC 5	123.341	6130.283	0.020
PC 6	112.601	4958.805	0.023
PC 8	101.693	4115.665	0.025
PC 12	89.662	2549.707	0.035
PC 16	86.822	2619.206	0.033

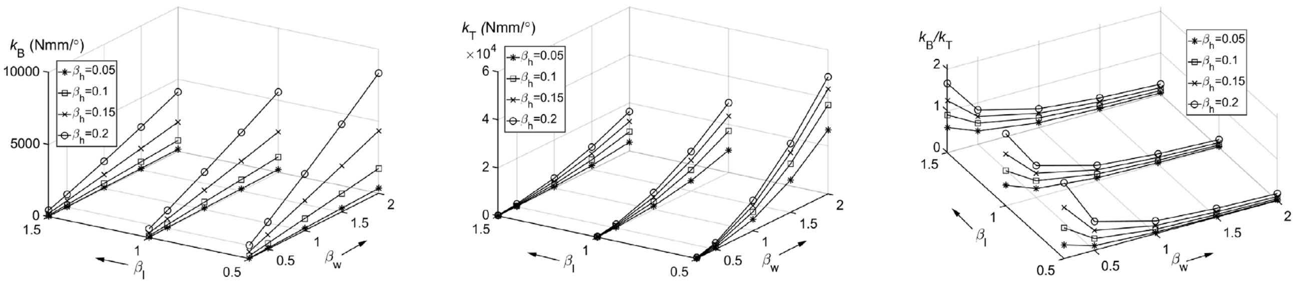

Figure 3 Parameter-dependent FEM results of the hinge stiffnesses for a polynomial contour of 2nd order: k_B (l.); k_T (m.); k_B/k_T (r.)

The results are exemplarily shown in Table 1 for all ten investigated hinge contours and the generally suitable geometric hinge parameters as well as in Figure 3 for the most suitable polynomial contour of 2nd order. The simulation results confirm the largest influence of the width *w* of a flexure hinge to reduce the bending-torsion-stiffness ratio by its increase. In addition, the ratio slightly decreases with the decrease of the hinge length *l* and minimal hinge height or thickness *h*. Furthermore, the notch shape or hinge contour can especially be used to minimize the bending-torsion-stiffness ratio. Of all contours, the contour based on quadratic polynomial functions shows the smallest bending-torsion-stiffness ratio for most investigated cases, especially for thin hinges (with a small minimal height or thickness *h*). On the other hand, an increased polynomial order results to the smallest stiffness ratio for thicker hinges (with high *h* values).

In conclusion, the potential of polynomial notch shapes to reduce the newly considered bending-torsion-stiffness ratio of flexure hinges has been shown with regard to precision engineering applications. In future work, the presented simulation results should be confirmed by analytical and experimental investigations [10]. Moreover, a more precise and more general optimization-based approach using rational exponents of power function contours may be implemented.

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