Essays on Optimal Insurance Demand

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Contents

List of Abbreviations

Bhvrl. Behavioral concepts

CARA Constant absolute risk aversion

Corr. Correlation

CPT Cumulative prospect theory

CRRA Constant relative risk aversion

DARA Decreasing absolute risk aversion

DT Dual theory

Emdl. Empirical model

EUT Expected utility theory

GEUT Generalized expected utility theory

HPLC High probability low consequence

LPHC Low probability high consequence

MPT Myopic prospect theory

MTU Markowitz type Neumann-Morgenstern utility function

NFIP National flood insurance programm

Pftb. Profitable

PT Prospect theory

SPM Stated preference method

Sub. Subsidized

Tmdl. Theoretical model

WTI Willingness to insure

WTP Willingness to pay

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Chapter 1

General introduction

Studying economic behavior under uncertainty it is common to use the theory of insurance demand (Schlesinger (2013)). While there exists a wide range of insurance economics this thesis concentrates on the optimal insurance demand as well as on the interaction of insurance and saving.[1]

The paper "Aspects of Rational Insurance Purchasing" by Mossin (1968) can be clearly seen as the start of the research regarding the optimal insurance demand.[2] The well-known results of the author determine the optimal insurance demand by including the topics optimal insurance coverage, risk aversion of the decision-maker and her/his financial status. Mossin (1968) argues that the decision-maker insures a risk completely if the insurance is priced in a fair way. An actuarially fair priced insurance implies that the premium required for the insurance is equal to the expected loss. Consequently, the expected profit for the insurer is equal to zero. As soon as the insurer requires a premium higher than the expected loss it is optimal to choose partial coverage only so that the decision-maker bears a part of the possible loss. The degree of risk aversion reveals how much coverage the decision-maker wants to buy and which amount s/he is willing to pay for it. If the decision-maker is highly risk avers s/he tries to insure a large part of the risk (of course, depending on the price for insurance) while less risk aversion leads to

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[1] For an overview about the topics regarding insurance economics see Loubergé (2013).

[2] According to Loubergé (2013) besides the work of Mossin (1968) also the studies of Borch (1962), Arrow (1963), Ehrlich and Becker (1972) as well as Joskow (1973) should be mentioned when talking about the theories of insurance. The author also mentions that the paper of Mossin (1968) is the seminal one regarding the optimal insurance demand.

less coverage. The impact of risk aversion on the demand for insurance thereby strongly depends on her/his financial status. Thus, decreasing risk aversion in combination with a higher wealth will lead to the case that the decision-maker is willing to pay less for insurance and also chooses less coverage.

Consequently, risk aversion is one of the crucial elements considering the optimal insurance demand. To better understand the behavior of the decision-maker concerning her/his risk aversion we therefore split up the overall risk preferences into time and risk preferences (Chapter 2). The risk preferences cover the risk aversion of the decision-maker described before and thus the aversion against a possible loss within a period. The time preferences are an additional element to specify the overall risk preferences and cover the risk aversion of the decision-maker over several periods. Hence, the aversion against changes in wealth over time.

The joint decision on optimal saving and insurance dates back to Dionne and Eeckhoudt (1984) who show that saving and insurance can be substitutes. We add to the literature on the joint decision using the described risk and time preferences. Thus, we are left with a more detailed analysis regarding the impact of risk aversion. Interestingly, we show that saving and insurance are substitutes depending on the risk preferences of the decision-maker. If this aversion against a possible loss increases the decision-maker prefers to insure instead to save. In contrast, the time preferences (the aversion against changes in wealth over time) only change the saving amount without influencing the insurance decision.

The applied theories are another important aspect within the theory of insurance demand. Over the years the famous results of Mossin (1968) were doubted, e.g. because of the expected utility approach which is sometimes not able to explain the behavior of the decision-maker properly (e.g. Loubergé (2013) and Chapter 3). Especially, the optimal insurance demand regarding low probability high consequence (LPHC) risks versus high probability low consequence (HPLC) risks reveals inconsistent behavior of the decision-maker by looking at theoretical and empirical work. On the one hand empirical studies detect a rather low insurance demand for LPHC-risks characterized by occasional but sever loss, e.g. caused by a flood or a thunderstorm. On the other hand one observes a high insurance demand for HPLC-risks which occur often but result in only small losses, e.g. a mobile phone or bicycle theft. These observations are not in line with expected utility theory (EUT) which predicts a high demand for catastrophic insurance compared to mobile phone or bicycle insurance and are referred to as ”insurance puzzle”.

In this sense, the thesis evaluates the literature regarding the insurance demand for LPHC- versus HPLC-risks to figure out which insurance is preferred (Chapter 3). Thereby, a necessary step is to analyze the most applied theories within literature. Therefore, we explain the theories based on EUT and prospect theory (PT). While EUT is one of the economic theories, PT belongs to the field of behavioral economics and integrates behavioral aspects to explain insurance decisions. As a further step, we then include other behavioral concepts, e.g. regret theory. We add to the existing literature by evaluating the explanatory power of the theories regarding the optimal insurance demand for LPHC- and HPLC-risks. Despite the weaknesses, EUT is still often used within the literature regarding insurance decisions to specify the optimal insurance demand. For this reason we use the basic insurance model (based on EUT) as a starting point for our analysis (Chapter 4). We complement the literature by showing within a model based approach that the insurance against LPHC-risks is indeed preferred. Afterwards, we explain the so-called "insurance puzzle", e.g. by using behavioral aspects. Thus, we are able show within a theoretical approach that both is possible: the optimal insurance demand can be higher for LPHC-risks but also for HPLC-risks.

In Chapter 2, we evaluate the optimal insurance demand by means of a dynamic model with two periods.[3] Besides the opportunity to insure the decision-maker can transfer wealth between the periods via saving. Hence, saving is used for consumption smoothing over time while insurance is needed for the within-period risk reduction. To economically interpret the insurance and saving decision we use a generalized mean-variance setup. This dynamic model allows to split up the overall variance into the between- and within-period variance. Consequently, we are able to interpret the time and risk preferences of the decision-maker. Thus, the differentiation allows us to better understand the optimal saving and insurance decision. The key findings comprise that the optimal insurance demand depends on the risk preferences (within-period variance) of the decision-maker only. The larger the aversion against within-period variance the higher the insurance demand and the lower the loss in the second period. In contrast, the optimal saving decision depends on her/his risk as well as time preferences (within- and between-period variance). Besides, the effect of risk and time preferences we show how the optimal insurance and saving decision depends on the size and probability of the loss and the characteristics of the financial

[3] The contents of Chapter 2 are joint work with Prof. Dr. Nicole Branger, Prof. Dr. Antje Mahayni and Dr. Nikolaus Schweizer.

market, e.g. the interest rate.

Chapter 3 reviews the literature regarding LPHC-risks versus HPLC-risks. The so-called ”insurance puzzle” describes the phenomenon that classical theories like EUT predict a higher optimal insurance demand for LPHC-risks while experiments and empirical observations show the opposite: the decision-maker does not behave in line with classical theories and prefers an insurance against HPLC-risks. Within the literature review we investigate this ”insurance puzzle” considering theoretical approaches, experiments and empirical work regarding the optimal insurance demand for LPHC- and HPLC-risks. The aim is to figure out if there exists a preference to insure one of the two risks. Therefore, we evaluate the mostly used theories to explain insurance decisions to point out their strengths and weaknesses. Additionally, we carve out impact factors which increase or decrease the optimal insurance demand. We also look at the probability range used to specify low and high probabilities, the insurance loading as well as experimental conditions. The analysis reveals that in line with EUT most of insurance literature states that there still exists a higher insurance demand for LPHC-risks.

In Chapter 4, we focus again on the so-called ”insurance puzzle” which implies that contrary to the predictions of EUT empirical studies document a rather low insurance demand for rare catastrophic risks (LPHC) and a rather high insurance demand for small but frequent risks (HPLC).[4] We explain this puzzle by means of mental accounting in a basic insurance model with two independent risks. To do so, we consider the optimization problems in which either LPHC- or HPLC-risk is insurable while the other risk is seen as (uninsurable) background risk. We find that the optimal insurance demand for LPHC-risks (with HPLC as background risk) can be larger or smaller than the optimal insurance for HPLC-risks (with LPHC as background risk). We provide an intuitive explanation by disentangling the opposing directional effects. Finally, we are able to explain the deviation between theory and experimental observations.

Finally, Chapter 5 concludes.

[4] The contents of Chapter 4 are joint work with Prof. Dr. Nicole Branger and Prof. Dr. Antje Mahayni.

Chapter 2

Optimal saving and insurance under generalized mean-variance preferences

2.1 Introduction

Risk and time preferences are crucial elements to explain the demand for saving and insurance. Intuitively, saving is important for income or consumption smoothing over time while insurance is needed for (within-period) risk reduction. In this paper, we analyze intertemporal risk management and saving decisions in a setup that disentangles risk and time preferences. In particular, we show how the optimal insurance and saving decision depends on the preferences of the decision-maker, the size and probability of the loss, and the characteristics of the financial market.

We consider a dynamic model with two points in time in which the decision-maker is endowed with some income today and tomorrow.[1] The income today is deterministic, while the income tomorrow is subject to the risk of some loss. The decision-maker can transfer wealth over time via a saving account with a deterministic interest rate. In addition, as in

[1] The literature on optimal consumption and saving in a model with more than one period dates back to Merton (1969). Briys (1986), Gollier (1994) and Moore and Young (2006) add optimal insurance to the continuous-time model of Merton (1969). Moore and Young (2006) show e.g. that optimal consumption depends on the planning horizon of the decision-maker whereas the optimal insurance does not.

the static basic insurance model, the decision-maker can buy insurance today to reduce the potential loss tomorrow.[2]

We assume that the decision-maker simultaneously chooses the optimal saving amount and insurance effort. This induces a dependence between the optimal saving and the optimal insurance demand which makes the analysis of the optimal decisions more involved. For example, one interesting question concerns the sensitivity of the optimal insurance demand w.r.t. risk aversion. In a model with insurance only, the standard approach is to evaluate the first-order condition of the more risk averse decision-maker, whose utility function is a concave transformation of the utility function of the less risk averse decision-maker, at the optimal decision of the less risk averse decision-maker.[3] The sign of the first-order condition then tells us something about the change in the optimal insurance demand. In a setup with insurance and saving, it only tells us something about the optimal insurance demand given the optimal saving of the less risk averse decision-maker. However, what we are interested in is the optimal insurance demand when savings are chosen optimally, too. The joint decision on insurance and saving thus hinders the application of the standard approach. The same problem occurs in the analysis of the sensitivity of the optimal saving decision w.r.t. risk aversion.

To overcome these problems, we rely on a generalized mean-variance approach which takes into account preferences for consumption smoothing but not for precautionary saving. The last assumption is restrictive. However, it arises (naturally) in different setups as for example used in Pratt (1964) and Maccheroni et al. (2013). More importantly, our main focus is on an in depth analysis of the joint effects of the aversion towards between- and within-period variance. We thus ignore the impact of higher order moments and multiple risks and leave in particular the analysis of prudence over time and states to further research.[4] In spite (and because) of our restriction, we are, by means of easy to inter-

[2] Analyzing the optimal insurance demand within a static basic insurance model already dates back to Mossin (1968). A general overview of the theory of insurance demand can be found in Schlesinger (2013).

[3] This standard approach is e.g. used in Dionne and Eeckhoudt (1985) who show that the level of self-insurance increases with risk aversion.

[4] Accounting for higher moments would introduce at least two additional preference parameters. This would introduce additional cases and interaction effects, but not add to the understanding of risk preferences over time and states. An analysis of the optimal insurance demand with multiple risks and higher-order risk attitudes can be found e.g. in Chi and Wei (2018). The interaction of prudence and saving is examined e.g. by Peter (2017).

pret closed form solutions, able to shed further light on the optimal saving and insurance demand and its sensitivities.

The basic idea which motivates our approach is that the expected utility of an income stream in a multi-period model can be rewritten as expected utility in a static model. To do so, we interpret the weighted average of expected utilities over time as an expectation of these utilities where the (outer) probabilities depend on the weights over time, i.e. on the subjective discount factors. The inner distributions are given by the distributions of wealth at each point in time. We can then apply the mean-variance approach to the reformulated problem. Furthermore, we decompose the variance of wealth into the expected within-period variance and the between-period variance of the expected periodic wealth levels over time.[5] The resulting preference functional only depends on the overall mean and the within-period and between-period variances of the adequately defined (pseudo) distribution and allows us to disentangle risk and time preferences.

We find that the optimal insurance demand depends on the aversion against within-period variance only. The larger this aversion, the more the decision-maker insures, and the lower the loss tomorrow. The optimal saving, in contrast, depends on both preference parameters. While a larger aversion against within-period variance unambiguously reduces the optimal saving (counterbalancing the already larger investment into insurance today), the impact of aversion against between-period variance depends on the risk-free rate. For a large (small) risk free rate, saving is attractive (unattractive), and a higher aversion reduces (increases) the optimal saving and brings them closer to zero.

With respect to the aversion against within-period variance, insurance and saving are thus substitutes: insurance increases and saving drops when this part of risk aversion increases. The aversion against between-period variance, in contrast, changes the saving only without having an impact on insurance.

We also analyze the impact of the other model and risk parameters on the optimal saving and the optimal insurance premium. The closed form solutions for the optimal insurance demand allow interesting insights about an ambiguous interaction of demand effects (a higher price of insurance reduces the insurance demand) and price effects (a higher price increases the cost today). First, a higher insurance loading and a lower interest

[5] An analogous procedure is found in the literature dealing with smooth ambiguity models (Maccheroni et al. (2013)). Here, the outer distribution is given by the distribution of an uncertain model parameter while the inner distributions are based on the given model parameter.

rate imply a higher price for insurance and a higher present value of one unit of wealth tomorrow. We show that this induces a higher investment into insurance and less saving if the loss size is very high, while the sensitivities are the other way round for a small loss size. Second, we show that a higher probability of a loss implies that the optimal insurance premium increases and the optimal saving decreases if the loss size is very high, while the reaction is the other way round for a small loss size. Finally, a larger loss size unambiguously increases the optimal insurance demand and crowds saving out.

Our paper is related to the literature on risk taking in general, optimal insurance demand, saving, and the literature on preference modeling, in particular dealing with mean-variance preferences. Without postulating completeness, we first refer to the seminal papers of Pratt (1964) and Kimball (1990).[6] For a general overview of the theory of insurance demand we refer to Schlesinger (2013) and the literature given therein.[7] We also want to mention Schmidt (1999) who considers the efficient risk-sharing and the dual theory of choice under risk. The literature incorporating insurance and saving decisions dates back to Dionne and Eeckhoudt (1984). Their work was one of the first which considers the joint decision in a two-period framework. One of the main results is that under decreasing risk aversion insurance and saving are substitutes. Menegatti and Rebessi (2011) study the interaction between prevention, saving and insurance. For the case of full (fair) insurance, insurance is used for risk reduction and saving for consumption smoothing. Hofmann and Peter (2015) consider the interaction between self-insurance, self-protection and saving. Without disentangling the effect of time and risk preferences the authors show that for the case of self-insurance and saving the same results as in a static model are achieved only if the decisions are made at the same time (saving is endogenous). Crainich and Peter (2019) examine the demand for insurance and saving. They show that insurance can be a complement or a substitute for saving depending on whether absolute risk aversion is increasing or decreasing in wealth. Finally, as mentioned above, a similar generalized mean-variance approach is also found in Maccheroni et al. (2013) in the context of a smooth model of ambiguity.

[6] Amongst other results, Pratt (1964) already shows that the usual mean-variance preferences arise naturally when approximating the certainty equivalent.

[7] For example, Schlesinger (1981) considers the optimal level of deductibility in insurance contracts. Initial random wealth is analyzed in Doherty and Schlesinger (1983). More recently, Mossin's Theorem given random initial wealth is tackled in Hong et al. (2011). The impact of background risk on insurance demand is analyzed by Fei and Schlesinger (2008).

The remainder of this paper is organized as follows. In Section 2.2, we introduce a (discrete time) dynamic version of the basic model of insurance demand where the decision-maker chooses her/his optimal level of insurance and saving. We state the mean-variance optimization problem in which we disentangle time and risk preferences. In Section 2.3, we derive closed form solutions for the optimal saving and insurance demand. Section 2.4 contains a detailed sensitivity analysis. In particular, we analyze the sensitivities with respect to the aversion to within- and between-period variance. Section 2.5 concludes.

2.2 Generalized mean-variance preferences setup

2.2.1 The problem

In the basic model of insurance demand a loss of $L > 0$ can occur with probability p while there is no loss with probability $1 - p$. Investing in insurance can reduce the loss size L, i.e. paying the insurance premium y reduces the loss (if it occurs) from L to $l(y)$ where $l(0) = L$.

In contrast to the static insurance model, we consider a dynamic model with two points in time where the decision-maker can buy insurance and save. There are two dates (or periods) $t = 0$ and $t = 1$, where the loss can only occur at $t = 1$. At $t = 0$, the decision-maker specifies a level of saving s, which yields interest r at $t = 1$.

In addition, as in the basic insurance model, the decision-maker can pay the insurance premium y at $t = 0$ to reduce the possible loss level at $t = 1$ from L to $l(y)$ where $l(0) = L$ and where $l(y)$ is decreasing in y.

Throughout the following, we assume a linear relation between $l(y)$ and y. The cost for reducing the loss from L to $l(y)$ are given by

$$y = (1 + \delta)\,\frac{p\,[L - l(y)]}{1 + r}$$

and the remaining loss size $l(y)$ with an insurance premium y is given by

$$l(y) = L - y\,\frac{1 + r}{p(1 + \delta)}. \tag{2.1}$$

In the actuarially fair case $\delta = 0$, the cost y are equal to the discounted expected payoff at time $t = 1$, discounted at the risk-free rate. A positive insurance loading $\delta > 0$ corresponds to the case where insurance is more expensive than its expected benefits, i.e. the decision-maker accepts a negative risk premium on insurance. To rule out arbitrage opportunities, we assume

$$\frac{p(1+\delta)}{1+r} < \frac{1}{1+r}, \tag{2.2}$$

i.e. the cost of a payoff equal to one in the loss-state are smaller than the cost of a payoff equal to one in both states.

The decision-maker specifies the levels of saving and insurance. With an investment of y into insurance and saving s, the wealth level $\hat{w}_0$ at time $t = 0$ and the state dependent wealth levels $w_1^{(L)}$ (loss) and $w_1^{(NL)}$ (no loss) at $t = 1$ are

$$\begin{aligned} \hat{w}_0 &:= w_0 - (s + y), \\ w_1^{(L)} &:= w_1 + (1+r)s - l(y) \ \text{ and } \ w_1^{(NL)} := w_1 + (1+r)s. \end{aligned}$$

The expected utility of the decision-maker is then given by

$$E[u(\hat{w}_0) + \beta u(w_1)] = u\left(\hat{w}_0\right) + \beta\left[p\, u\left(w_1^{(L)}\right) + (1-p)\, u\left(w_1^{(NL)}\right)\right]$$

where β is the subjective discount factor. Dividing by $1 + \beta$ does not change the ordering implied by the utility function and gives

$$\frac{1}{1+\beta} u\left(\hat{w}_0\right) + \frac{\beta\, p}{1+\beta}\, u\left(w_1^{(L)}\right) + \frac{\beta(1-p)}{1+\beta}\, u\left(w_1^{(NL)}\right).$$

This formula shows that we can reinterpret the above payment stream as a lottery $\tilde{W}$ with three possible outcomes. The distribution Q of $\tilde{W}$ is

$$\begin{aligned} Q(\tilde{W} = \hat{w}_0) &= \frac{1}{1+\beta} =: q_0, \\ Q(\tilde{W} = w_1^{(L)}) &= \frac{\beta p}{1+\beta} =: q_1^{(L)} \ \text{ and } \ Q(\tilde{W} = w_1^{(NL)}) = \frac{\beta(1-p)}{1+\beta} =: q_1^{(NL)}. \end{aligned} \tag{2.3}$$

2.2.2 Mean-variance preferences that separate risk and time

Consider the classical expected utility from an income (or consumption) stream W_i over the sequence of points in time $i = 0, 1, \ldots$, i.e.

$$U(W) = \sum_{i=0}^{\infty} \beta_i E[u(W_i)].$$

The random variables W_i have distributions P_i with means μ_i and standard deviations σ_i. Without loss of generality, we assume that the discounting weights β_i are normalized such that $\sum_i \beta_i = 1$.

The expected utility of the consumption stream is identical to the expected utility in a static model, i.e. it holds that

$$U(W) = E[u(W_I)]$$

where we first draw I from $0, 1, \ldots$ with probabilities $\beta_0, \beta_1, \ldots$ and then draw W_I from P_I.

We can then define a mean-variance preference functional $\Phi(\mu, \sigma)$ of the form $\Phi(\mu, \sigma) = \mu - \lambda\sigma^2$ where

$$\mu = E[W_I] = \sum_{i=0}^{\infty} \beta_i \mu_i,$$
$$\sigma^2 = \text{Var}(W_I) = \text{Var}(E[W_I|I]) + E[\text{Var}(W_I|I)] = \text{Var}(\mu_I) + E[\sigma_I^2]$$

and where we have used the conditional variance decomposition formula. The two terms on the right hand side are given by

$$\text{Var}(\mu_I) = \sum_{i=0}^{\infty} \beta_i \mu_i^2 - \mu^2 =: \sigma_b^2 \quad \text{and} \quad E[\sigma_I^2] = \sum_{i=0}^{\infty} \beta_i \sigma_i^2 =: \sigma_w^2.$$

$\text{Var}(\mu_I)$ captures the variation in average income over time. We later refer to it as between-period variance σ_b^2. $E[\sigma_I^2]$ is the average within-period variance σ_w^2.

In consequence, a straightforward way to separate time and risk preferences is to consider generalized mean-variance preferences given by the preference functional $\Phi(\mu, \sigma_b, \sigma_w)$ defined by

$$\Phi(\mu, \sigma_b, \sigma_w) := \mu - \lambda_b \sigma_b^2 - \lambda_w \sigma_w^2 \tag{2.4}$$

where the aversion of the decision-maker to between(within)-period variance is captured by the preference parameter λ_b (λ_w).

The above preferences can also be further motivated and found in literature in a different context, e.g. an ambiguity setup.

First, recall the seminal paper of Pratt (1964) who shows that the usual mean-variance preferences $\Phi(\mu, \sigma)$ arise naturally when approximating the certainty equivalent

$$CE_u = u^{-1}(E[u(W_I)]) \approx \mu - \frac{\lambda_u(\bar{w})}{2} \text{Var}(W_I)$$

where $\lambda_u(\bar{w}) = -u''(\bar{w})/u'(\bar{w})$ is the absolute risk aversion at some reference wealth level $\bar{w}$.

Maccheroni et al. (2013) show that generalized mean-variance preferences $\Phi(\mu, \sigma_b, \sigma_w)$ naturally arise in the context of smooth ambiguity.[8] In their interpretation, the distributions P_i are possible models for the one-period income W where each model is correct with probability β_i. A decision-maker with smooth ambiguity preferences first calculates the certainty equivalent of W for each fixed model P_i using a utility function u and then calculates the certainty equivalent of the resulting lottery over certainty equivalents using a second utility function v:

$$CE_{u,v} = v^{-1}\left(\sum_{i=0}^{\infty} \beta_i v\left(u^{-1}\left(E[u(W_i)]\right)\right)\right). \tag{2.5}$$

The resulting overall certainty equivalent $CE_{u,v}$ can be approximated by generalized mean-variance preferences of the form (2.4) analogously to Pratt's result on risk aversion in the small. If u and v are smooth and strictly concave, it holds that

$$CE_{u,v} \approx \mu - \frac{\lambda_v(\bar{w})}{2}\mathrm{Var}(\mu_I) - \frac{\lambda_u(\bar{w})}{2}E[\sigma_I^2] \tag{2.6}$$

where $\lambda_u = -u''/u'$ and $\lambda_v = -v''/v'$ denote the Arrow-Pratt measures of absolute risk aversion associated with u and v for some reference wealth level $\bar{w}$.

In our two-period insurance setting, the generalized certainty equivalent (2.5) takes the form

$$CE_{u,v} = v^{-1}\left(\frac{1}{1+\beta}v(\hat{w}_0) + \frac{\beta}{1+\beta}v\left(u^{-1}\left(p\,u(w_1^{(L)}) + (1-p)u(w_1^{(NL)})\right)\right)\right) \tag{2.7}$$

and a natural choice for the reference wealth is average wealth without insurance and savings given by $\frac{1}{1+\beta}(w_0+\beta(w_1-p\,L))$. Equation (2.6) then corresponds to the generalized mean-variance preferences we studied above.

2.3 Optimal saving and insurance

To use the mean-variance preferences introduced above, we have to calculate the overall mean, the within-period variance and the between-period variance of wealth $\tilde{W}$. The overall expected value μ and variance σ^2 depend on the insurance premium y and the saving decision s. We set

$$\mu(y,s) := E_Q[\tilde{W}] \quad\text{and}\quad \sigma^2(y,s) := \mathrm{Var}_Q[\tilde{W}]. \tag{2.8}$$

[8] An insurance setting with smooth ambiguity can be found in Gollier (2011).

The overall expected value can be decomposed into the (deterministic) value $\hat{w}_0$ at time 0 and the expected value μ_1 at time 1 where

$$\mu_1 = p\, w_1^{(L)} + (1-p) w_1^{(NL)}.$$

The overall expected value is then equal to

$$\begin{aligned}\mu(y,s) &= q_0(w_0 - s - y) + (1-q_0)\left(w_1 + (1+r)s - p\, l(y)\right)\\ &= \frac{1}{1+\beta}\left[w_0 + \beta w_1 - \beta p L + \left(\beta(1+r)-1\right)s + \left(\beta\,\frac{1+r}{1+\delta} - 1\right) y\right].\end{aligned}$$

It increases in saving s if saving is attractive, i.e. if $\beta(1+r) > 1$. Analogously, it decreases in the insurance premium y (and increases in the loss size l) if accepting the risk of a loss is attractive, which is the case if $\frac{(1+\delta)p}{1+r} > \beta p$, i.e. if the insurance premium today exceeds the the expected future loss.

The overall variance can be split up into the within-period variance and the between-period variance. The former only depends on the within-period variance at time $t = 1$ and is given by

$$\begin{aligned}\sigma_w^2(y) &= q_0 \cdot 0 + (1-q_0)\left[p\big(w_1^{(L)} - \mu_1\big)^2 + (1-p)\big(w_1^{(NL)} - \mu_1\big)^2\right]\\ &= \frac{\beta}{1+\beta}\, p(1-p)(l(y))^2. \end{aligned} \tag{2.9}$$

It depends on the size of the loss and thus on the insurance premium y only. The between-period (or time) variance of expected wealth is given by

$$\begin{aligned}\sigma_b^2(y,s) &= q_0 \hat{w}_0^2 + (1-q_0)\mu_1^2 - \mu^2\\ &= \frac{\beta}{(1+\beta)^2}\left(\mu_1 - \hat{w}_0\right)^2. \end{aligned} \tag{2.10}$$

It is driven by the expected increase in consumption $\mu_1 - \hat{w}_0$, which we denote by t and which in turn depends on both, the insurance level y and the saving decision s:

$$\begin{aligned}t &= w_1 + (1+r)s - p\, l(y) - (w_0 - y - s)\\ &= w_1 - w_0 + \frac{(1+\delta)p}{1+r} L + (2+r)s - \frac{2+r+\delta}{1+r}\, p\, l(y).\end{aligned}$$

It increases in saving (which shift consumption from today to tomorrow) and decreases in the accepted loss size.

We consider the generalized mean-variance preferences which are summarized in the following definition.

Definition 1 (Generalized mean-variance preferences) *The generalized mean-variance preferences are given by*

$$\Phi(y,s) = \mu(y,s) - \lambda_b \sigma_b^2(y,s) - \lambda_w \sigma_w^2(y). \tag{2.11}$$

The mean $\mu(y,s)$, the between-period variance $\sigma_b^2(y,s)$ and the within-period variance $\sigma_w^2(y)$ are defined in Equations (2.8), (2.9) and (2.10). The risk aversion parameters λ_b and λ_w specify the aversion against the between-period and within-period variances. The special case $\lambda_b = \lambda_w = \lambda$ results in the classical mean-variance preferences.

The optimal saving and the optimal investment into insurance can then be determined. They are given in the next proposition.

Proposition 1 (Optimal saving and insurance) *Under the generalized mean-variance preferences of Definition 1, the optimal insurance premium y^* and the optimal saving decision s^* solve the optimization problem*

$$(y^*, s^*) = \arg\max_{y,s} \mu(y,s) - \lambda_b \sigma_b^2(y,s) - \lambda_w \sigma_w^2(y).$$

They are given by

$$y^* = \frac{(1+\delta)p}{1+r}(L - l^*), \tag{2.12}$$

$$s^* = \frac{(w_0 - y^*) - [w_1 - p\, l^*]}{2+r} + \frac{t^*}{2+r} \tag{2.13}$$

where the optimal loss size l^* and the optimal expected increase of wealth t^* are

$$l^* = \frac{\delta}{2\lambda_w \frac{\beta}{1+\beta}(1-p)(2+r)}, \tag{2.14}$$

$$t^* = \frac{\beta(1+r) - 1}{2\lambda_b \frac{\beta}{1+\beta}(2+r)}. \tag{2.15}$$

Proof: The proof is given in Appendix A.1.

To get the intuition, we first consider the optimal loss size l^* and the optimal expected growth t^* of wealth. The optimal insurance premium y^* and the optimal saving s^* then follow from l^* and t^*.[9]

[9] Given s and y, the loss size and the expected growth of wealth are

$$l = L - \frac{1+r}{p(1+\delta)} y \qquad \text{and} \qquad t = w_1 + (1+r)s - p\,L + \frac{1+r}{1+\delta} y - (w_0 - s - y).$$

This linear system of equations can easily be solved for s and y as a function of l and t.

The loss size l chosen by the decision-maker has an impact on the within-period variance and the mean payoff. The within-period variance (2.9) increases in the loss size, which favors a small loss size close to zero. In contrast, the negative risk premium on insurance favors a large loss size l. There is thus a trade-off between achieving a small within-period variance by choosing a small l and achieving a high overall payoff by accepting a large l.

The aversion of the decision-maker to within-period variance is captured by the preference parameter λ_w. The larger λ_w, the more s/he worries about within-period variance and the smaller in turn the optimal loss size s/he accepts. The negative risk premium on insurance depends on the parameter δ. In the actuarially fair case, $\delta = 0$, the risk premium is zero. The decision-maker will then choose $l^* = 0$, thus full coverage. An increase in δ increases the optimal accepted loss size.

The chosen expected growth t of wealth has an impact on the between-period variance and on the mean payoff. The between-period variance (2.10) increases in the absolute value of t, which speaks in favor of choosing a t close to zero. Additionally, t has an impact on the mean payoff. If $\beta(1 + r) > 1$, saving is attractive, and the decision-maker profits from shifting wealth from today to tomorrow. In this case, the optimal t is positive and the negative impact of t on the between-period variance limits the size of t. If, in contrast, $\beta(1 + r) < 1$, the decision-maker profits from taking a loan, the size of t is again limited by the impact of t on the between-period variance.

The aversion of the decision-maker against between-period variance is captured by the preference parameter λ_b. The larger λ_b, the more s/he worries about between-period variance, and the smaller in turn the optimal expected growth s/he chooses in absolute terms.

The optimal insurance premium y^* and the optimal saving s^* then follow from l^* and t^*. The link between y^* and l^* simply reflects the pricing formula for buying insurance. The expression for s^* is more involved. It depends on both t^* and l^*. To get the intuition, first note that one additional unit of saving today increases the expected growth of wealth by $(1+r)+1 = 2+r$. This explains the last term $t^*/(2+r)$ in (2.13). With no insurance and no saving, the expected growth rate of wealth is $w_1 - p\,L - w_0$, and the decision-maker brings this expected growth to zero (before bringing it to t^*) by saving $-(w_1 - p\,L - w_0)/(2 + r)$. Finally, there is an indirect impact of insurance on the optimal saving. Each unit of money invested into insurance increases t by $(1 + r)/(1 + \delta) + 1$ and the decision-maker reduces the saving to offset this impact of insurance on t.

2.4 Sensitivity analysis

2.4.1 Risk preferences

The decision-maker is averse against between-period variance and within-period variance. The strength of this aversion is captured by λ_b and λ_w, which thus have an impact on the optimal growth of wealth and the optimal loss. We first look at the impact of these preference parameters on the optimal loss size and optimal insurance.

Proposition 2 (Impact of λ_w and λ_b on optimal loss size and insurance) *For the optimal loss size, it holds that*

$$\frac{\partial l^*}{\partial \lambda_w} \leq 0, \qquad \frac{\partial l^*}{\partial \lambda_b} = 0.$$

For the optimal insurance premium, it holds that

$$\frac{\partial y^*}{\partial \lambda_w} \geq 0, \qquad \frac{\partial y^*}{\partial \lambda_b} = 0.$$

In the special case $\lambda_w = \lambda_b = \lambda$, this gives

$$\frac{\partial l^*}{\partial \lambda} = \frac{\partial l^*}{\partial \lambda_w} + \frac{\partial l^*}{\partial \lambda_b} = \frac{\partial l^*}{\partial \lambda_w} \leq 0, \qquad \frac{\partial y^*}{\partial \lambda} = \frac{\partial y^*}{\partial \lambda_w} + \frac{\partial y^*}{\partial \lambda_b} = \frac{\partial y^*}{\partial \lambda_w} \geq 0.$$

Proof: The proof follows immediately from Proposition 1 and the optimal loss and insurance levels l^* and y^* given therein, cf. Equations (2.14) and (2.12).

Intuitively, the decision-maker uses insurance to smooth consumption over states and then uses saving to smooth consumption over time. Thus, the aversion λ_b against between-period variance has an impact on the optimal saving but not on the optimal demand for insurance. The latter depends on the aversion λ_w towards within-period variance only. The more risk averse the decision-maker is, the smaller the optimal loss size, and the higher the optimal insurance premium.

Recall that the classic mean-variance approach corresponds to the special case $\lambda = \lambda_b = \lambda_w$. Since there is no impact of λ_b on the optimal insurance demand, λ_w also drives the results in the classic mean-variance approach and the optimal insurance level (loss size) increases (decreases) in the overall risk aversion of the decision-maker.

Proposition 3 (Impact of λ_w and λ_b on optimal growth and saving) *For the optimal expected growth of wealth and the optimal saving, it holds that*

$$\frac{\partial t^*}{\partial \lambda_w} = 0, \qquad \frac{\partial t^*}{\partial \lambda_b} \leq 0 \Leftrightarrow \beta(1+r) > 1,$$
$$\frac{\partial s^*}{\partial \lambda_w} \leq 0, \qquad \frac{\partial s^*}{\partial \lambda_b} \leq 0 \Leftrightarrow \beta(1+r) > 1.$$

In the special case $\lambda_w = \lambda_b = \lambda$, this gives

$$\frac{\partial t^*}{\partial \lambda} = \frac{\partial t^*}{\partial \lambda_w} + \frac{\partial t^*}{\partial \lambda_b} = \frac{\partial t^*}{\partial \lambda_b}, \qquad \frac{\partial s^*}{\partial \lambda} = \frac{\partial s^*}{\partial \lambda_w} + \frac{\partial s^*}{\partial \lambda_b}.$$

Proof: The proof follows immediately from Proposition 1 and the optimal saving s^* and growth t^* given therein, cf. Equations (2.13) and (2.15).

The optimal saving depends on the aversion against between- and within-period variance. Thus, the aversion towards variation over time and also on the aversion towards variation over the states at time $t = 1$. We look at the more intuitive dependence on λ_b first. The larger λ_b, the more the decision-maker wants to smooth consumption over time. The difference t^* between expected wealth at time $t = 1$ and wealth at time $t = 0$ thus decreases in λ_b in absolute terms. The sign of this difference depends on the relation between the subjective discount factor β and the proceeds $1 + r$ from saving and thus on the attractiveness of saving. For $\beta(1 + r) > 1$, saving is attractive and the difference t^* is positive. An increase in λ_b then induces t^* to decrease and move closer to zero. For $\beta(1+r) < 1$, the argument goes the other way round, and the negative t^* increases in λ_b.

The dependence of the optimal saving on the aversion λ_w towards variation over states is surprising at first glance, given our intuition that the decision-maker uses saving to take care of variation over time and insurance to take care of variation over states. In line with this intuition, the optimal growth of wealth t^* is indeed independent of λ_w. However, saving does not only depend on the optimal growth t^*, but also on the optimal loss size l^*. A larger λ_w induces the decision-maker to increase his optimal demand for insurance. Wealth today then drops and expected wealth tomorrow increases. To counterbalance this (unintended) consequence of insurance demand, the decision-maker decreases her/his saving. Consequently, the optimal saving level decreases in λ_w.

Propositions 2 and 3 also give the joint reaction of y^* and s^* to changes in risk aversion. As just pointed out, a higher aversion towards within-period variance implies a larger insurance premium and less saving, so that saving and insurance are substitutes w.r.t. this parameter. A higher aversion towards between-period variance only changes the optimal saving, but has no impact on the optimal insurance premium. Finally, a higher overall risk aversion $\lambda = \lambda_w = \lambda_b$ implies a higher insurance premium and less saving if saving is attractive, so that saving and insurance are again substitutes. However, if saving is not attractive there are also parameter sets for which saving and insurance both increase in risk aversion and are thus complements.

2.4.2 Income effects

We next turn to the impact of wealth at $t = 0$ and $t = 1$ on the optimal decisions which easily follow from Proposition 1.

Corollary 1 (Impact of wealth levels on optimal decisions) *For the impact of wealth at $t = 0$ and $t = 1$, it holds that*

$$\frac{\partial l^*}{\partial w_0} = 0, \qquad \frac{\partial l^*}{\partial w_1} = 0, \qquad \frac{\partial t^*}{\partial w_0} = 0, \qquad \frac{\partial t^*}{\partial w_1} = 0,$$
$$\frac{\partial y^*}{\partial w_0} = 0, \qquad \frac{\partial y^*}{\partial w_1} = 0, \qquad \frac{\partial s^*}{\partial w_0} \geq 0, \qquad \frac{\partial s^*}{\partial w_1} \leq 0.$$

Neither the optimal loss size, nor the optimal expected growth of wealth depend on the given level of wealth. To see the reason, we go back to the motivation of the preference functional in Section 2.3. For the preference parameters λ_w and λ_b, it holds that $\lambda_w = -\frac{u''}{u'}$ and $\lambda_b = -\frac{v''}{v'}$, i.e. they capture absolute risk aversion which is thus assumed to be constant. For a constant absolute risk aversion, in turn, it holds that the optimal differences between wealth over states and over time do not depend on the level of wealth.

The optimal amount put into insurance, i.e. the amount needed to secure an insurance payoff equal to $L - l^*$ in case of a loss, also does not depend on wealth. However, the optimal saving s^* increases in w_0 and decrease in w_1. If w_0 increases, the expected growth of wealth decreases, and the decision-maker thus saves more to offset the impact of a larger wealth today. By a similar argument, s/he saves less (or takes a larger loan) to offset the impact of larger wealth tomorrow.

2.4.3 Price effects

The risk-free rate r and the insurance loading δ determine the price of insurance. They have an impact on optimal saving and optimal insurance via a demand effect (a higher price reduces the demand) and via a price effect (a higher price increases the cost today).

Corollary 2 (Impact of r and δ on optimal saving decision) *The dependence on the interest rate r and the insurance loading δ is given by*

$$
\begin{aligned}
&\frac{\partial l^*}{\partial r} \leq 0, && \frac{\partial l^*}{\partial \delta} \geq 0, \\
&\frac{\partial y^*}{\partial r} \leq 0 \;\Leftrightarrow\; l^* \leq \frac{2+r}{3+2r}\,L, && \frac{\partial y^*}{\partial \delta} \geq 0 \;\Leftrightarrow\; l^* \leq \frac{\delta}{1+2\delta}L, \\
&\frac{\partial t^*}{\partial r} \geq 0, && \frac{\partial t^*}{\partial \delta} = 0, \\
&\frac{\partial s^*}{\partial r} \geq 0 \;\Leftrightarrow\; A(l^*, t^*) \geq 0, && \frac{\partial s^*}{\partial \delta} \leq 0 \;\Leftrightarrow\; l^* \leq \frac{\delta}{2+2\delta+r}L
\end{aligned}
$$

where

$$
A(l,t) = \frac{\frac{(1+\delta)p}{1+r}\cdot\frac{3+2r}{1+r}\,L - \left(2 + \frac{3(1+\delta)}{1+r} + \frac{1+\delta}{(1+r)^2}\right)p\,l + \frac{2-\beta r}{\beta(1+r)-1}\,t + w_1 - w_0}{(2+r)^2}.
$$

To interpret the dependence of the optimal loss size l^* on δ and r, note that the price for insuring some given loss size increases in the markup δ and decreases in the interest rate r. A higher price for insurance implies that the decision-maker is willing to accept a higher loss. Thus, l^* increases in δ and decreases in r due to the demand effect.

The dependence of the optimal premium y^* today on the parameters δ and r is ambiguous. The pricing parameters do not only have an impact on the optimal payout $L - l^*$ of the insurance in case of a loss (demand effect) but also on the price paid for the insurance today (price effect). If δ increases, insurance becomes more expensive. As pointed out above, the higher price reduces the demand for insurance and the insurance payout $L - l^*$ decreases. This demand effect implies a smaller y^*. Additionally, the decision-maker has to pay more for a given insurance payout in case of a loss, which implies a larger y^* due to the price effect. The overall impact then depends on the trade-off between the demand effect and the price effect. For a high loss size L, the price effect eventually dominates, and the optimal y^* increases in δ. For the impact of the interest rate r, the argument goes the other way round and the optimal insurance premium y^* decreases in r for a large L.

The switching point can also be expressed in terms of l^* instead of L. The value of l^* for which the partial derivatives switch sign is given by $\frac{2+r}{3+2r}\,L$ (in case of r), and $\frac{\delta}{1+2\delta}\,L$ (in case of δ). These values are between 0 and L, so that both cases are indeed relevant. Since l^* depends on the aversion of the decision-maker against within-period variance, this aversion also drives the sign of the impact which r and δ have on the optimal insurance premium y^*. For a very risk averse decision-maker, l^* is small, and the price effect dominates. The optimal insurance premium then increases in the price the decision-maker has to pay for a given insurance contract, i.e. it increases in the loading δ and decreases in the risk-free rate r. For a not too risk averse decision-maker, the optimal loss size l^* is large, the demand effect dominates and the sensitivities are the other way round.

We next turn to the impact of r and δ on the optimal expected growth of wealth t^* and on the optimal saving s^*. The optimal expected growth of wealth depends on the interest rate only (which determines the trade-off between money today and tomorrow), but not on the price for insurance (which determines the trade-off between the insurance premium today and the possible loss tomorrow). A larger interest rate implies that saving becomes more attractive which induces the decision-maker to shift wealth from today to tomorrow, which in turn increases the optimal expected growth t^*.

Again, the impact on optimal saving is more involved. Look at the impact of δ first. As just pointed out, an increase in δ has no impact on t^*. However, it has an impact on the optimal amount put into insurance which increases in δ if L is large. The higher investment into insurance then crowds saving out and s^* decreases in δ.

For the impact of the risk-free rate on the optimal saving it is again the demand effect and the price effect which matters. First, an increase in r lowers (increases) the optimal insurance premium if L is large (small) which in turn crowds saving in (out). Second, a higher interest rate induces the decision-maker to save more and in line with this intuition, both t^* and $t^*/(2+r)$ increase in r. Finally, the decision-maker saves to counteract large differences in w_1 and w_0. Overall, optimal saving increases in r if L is large and decreases in r if L is small.

2.4.4 Risk effects

Finally, we look at the impact which the risk of a loss has on optimal saving and optimal insurance. The loss event is characterized by its probability p and its size L.

Corollary 3 (Impact of risk parameters on optimal decisions) *The impact of the probability and size of the loss are*

$$\frac{\partial l^*}{\partial p} \geq 0, \qquad \frac{\partial l^*}{\partial L} = 0,$$
$$\frac{\partial y^*}{\partial p} \geq 0 \;\Leftrightarrow\; l^* \leq (1-p)L, \qquad \frac{\partial y^*}{\partial L} \geq 0,$$
$$\frac{\partial t^*}{\partial p} = 0, \qquad \frac{\partial t^*}{\partial L} = 0,$$
$$\frac{\partial s^*}{\partial p} \leq 0 \;\Leftrightarrow\; l^* \leq \frac{(1-p)(1+\delta)}{2+r+\delta} L, \qquad \frac{\partial s^*}{\partial L} \leq 0.$$

The given loss size L has no impact on the optimal loss size and on the optimal growth of wealth, but mainly influences the given level of endowment (similar to w_0 and w_1). Since a larger given loss size increases the payout $L - l^*$ in case of a loss, the insurance premium y^* increases in L. The larger investment into insurance then crowds saving out, so that s^* is decreasing in L.

The loss probability p has an impact via the demand effect and via the price effect. The higher p, the more expensive a reduction of the loss size is and the higher thus the optimal loss size for the decision-maker. The demand for insurance thus drops which implies a smaller premium y^*. The second effect of the higher price is to increase the amount needed for insurance today, i.e. it implies a larger y^*. The trade-off between the demand effect and the price effect again depends on the size of the given loss. If L is large the price effect dominates and y^* is increasing in p. The switching point for the derivative depends on p. The larger p, the larger l^* and the smaller the bound $(1-p)L$. A large p thus makes it more likely that y^* decreases in p, i.e. that the demand effect dominates.

The optimal expected growth of wealth depends neither on p nor on L. The reason is that these risk parameters have no impact on the trade-off between deterministic payoffs today and tomorrow. They determine the given average levels of wealth today and tomorrow which does not change the optimal trade-off between wealth today and tomorrow since the absolute risk aversion is constant. In contrast, optimal saving depends on both parameters. Optimal saving decreases in L, since the induced larger demand for insurance crowds saving out. The impact of p is ambiguous. A larger p implies a larger optimal loss size and – if the optimal loss size is sufficiently smaller than the given loss – a higher insurance premium today which crowds saving out. Respectively, for a large L or a small l^* it thus holds that saving decreases in p.

2.5 Conclusion

We analyze a dynamic version of the basic model of insurance demand which also accounts for the possibility to transfer money between periods by saving. The decision-maker uses the insurance to optimal reduce risk and saving to optimize the relation between consumption today and tomorrow. To capture this intuition, we rely on preferences that disentangle time and risk preferences.

We use a mean-variance approximation of the decision-maker preferences. Decomposing the variance into the between-period and within-period variance allows us to naturally disentangle time and risk preferences and leads to generalized mean-variance preferences.

Our approach results in closed form solutions for the optimal saving and insurance demand that are tractable and easy to interpret. We show that a higher aversion w.r.t. within-period variance implies a larger insurance premium and smaller saving, so that insurance and saving are substitutes in this case. A higher aversion w.r.t. between-period variance has no impact on insurance but only on the optimal saving. Furthermore, we analyze the impact of the pricing parameters r and δ, the income levels w_0 and w_1, and the characteristics p and L of the potential loss. We find that there are also parameter regions in which saving and insurance both increase or both decreases in a parameter.

Chapter 3

On the insurance demand for low probability high consequence risks versus high probability low consequence risks - A literature review

3.1 Introduction

Studies regarding the optimal insurance demand date back to Mossin (1968) and his seminal article "Aspects of Rational Insurance Purchasing". As one of the first authors Mossin (1968) examines the optimal insurance demand of the decision-maker using expected utility theory (EUT). Over the years the literature on the optimal insurance demand developed and already reveals the weaknesses of EUT. Especially, experiments or empirical observations show that the decision-maker often does not behave in line with EUT. Anderson (1974) e.g. shows within a field study that there are still uninsured decision-maker although the insurance for a flood risk (low probability high consequence risk) is subsidized or fair. In contrast, according to Mossin (1968) and EUT the decision-maker will always buy the insurance when it is priced subsidized or fair. Quickly, new theories evolved to explain the behavior of the decision-maker, e.g. prospect theory (PT) by Kahneman and Tversky (1979). PT predicts that the decision-maker ignores low probability outcomes and thus does not insure the risk. The extension of PT, cumulative prospect theory (CPT) by Tversky and Kahneman (1991), uses a fourfold pattern to explain insurance decisions. According to this, the decision-maker is risk averse towards gains and risk seeking towards losses for high probability outcomes. For low probability outcomes s/he is risk averse towards losses and risk seeking towards gains. This implies, that the decision-maker should prefer an insurance against low probability high consequence (LPHC) risks over an insurance against high probability low consequence (HPLC) risks. In contrast, a lot of experiments and empirical observations show the opposite: the decision-maker prefers an insurance against HPLC-risks (e.g. Slovic et al. (1977), Friedl et al. (2014), Browne et al. (2015)). Consequently, there exists an ”insurance puzzle” regarding the optimal insurance demand for LPHC- versus HPLC-risks.

Within this literature review we want to look at this ”insurance puzzle” and answer the emerging question if there exists a preference to insure one of the two risks by reviewing the research regarding LPHC- and HPLC-risks. The preference thereby can be measured by the willingness to insure (WTI) or the willingness to pay (WTP). The WTI indicates if the decision-maker is willing to buy insurance while the willingness to pay indicates which amount s/he is willing to pay for the insurance.

This literature review is based on theories and behavioral as cognitive concepts. They may include models and experiments (field and labor). The theory based literature comprises EUT, PT and their extensions. Studies which use EUT to derive the optimal insurance demand of the decision-maker are e.g. by Mossin (1968) and Schlesinger (2013). Well-known extensions of EUT are generalized expected utility theory (GEUT) by Machina (1982) and dual theory by Yaari (1987). As already pointed out, EUT and their extensions are not always able to explain the behavior of the decision-maker, so that PT was developed which additionally includes behavioral aspects. PT by Kahneman and Tversky (1979), CTP by Tversky and Kahneman (1991) and myopic prospect theory by Benartzi and Thaler (1995) are in some areas better to explain the insurance demand of the decision-maker then EUT, but still show some weak spots. Other behavioral concepts such as regret theory (Bell (1982), Loomes and Sugden (1982) Fishburn (1984)) or cognitive biases try to fill in these gaps. Regarding cognitive biases we include e.g. ambiguity (Ellsberg (1961)) and mental accounting (Thaler (1999)).

The aim of this literature review is to point out if there exists a preference to insure LPHC- or HPLC-risks. Thereby, 14 of the 38 reviewed studies state a clear preference (more than 50% of the decision-maker buy the insurance) to insure the LPHC-risk. Whereas, only

8 of 38 the studies state a (weak) preference (only 50% or less than 50% of the decision-maker buy the insurance) to insure the HPLC-risk.

The remainder of this paper is organized as follows. In Section 3.2, we give a review of literature. Besides, we illustrate well-known results based on EUT, PT and discuss the violation of these results within insurance literature. Afterwards, we focus on other behavioral concepts as regret theory and cognitive biases and also discuss their standing within the literature. In Section 3.3, we answer the question if a preference to insure LPHC- over HPLC-risks exists. Thereby, we review the literature by distinguishing between three cases. The first comprises factors which increase the demand for LPHC insurance, the second includes the cases where a clear preference to insure LPHC-risks is found and the third comprises the cases where a weak preference to insure LPHC-risks is found or a preference to insure HPLC-risks. Section 3.4 concludes.

3.2 LPHC versus HPLC insurance demand

3.2.1 Literature review

For the literature review we integrate studies since 1974 until today. We included the literature regarding insurance decisions in the context of LPHC- (and HPLC-)risks. Of course, this review is not comprehensive and the choice is subjective. Nevertheless, the selection of studies should be adequate to give an overview of literature.

We categorize the literature according to the ”insurance puzzle” which implies that research based on classical decision theory like EUT predicts a higher insurance demand for LPHC-risks while experiments or empirical work show a higher demand for HPLC-risks. The column *type* indicates if the literature is experiment based (labor or field) and/or includes an empirical model (e.g. regression models) and/or a theoretical model (e.g. EUT). The column *concept* additionally shows which decision theory or behavioral concept is used to explain the demand.

The following Table 1 gives an overview of literature. It includes the author, context, task, type, incentive, insurance decision, concept, probability range and loading. The type of paper can be an experiment (field or labor), a theoretical model (Tmdl.) and/or an empirical model (Emdl.). In addition, the experiment can be performed within the scope of the national flood insurance program (NFIP) in the US.[1] The incentive refers to the payment for the participation in the experiment and it can be hypothetical or real money. The insurance decision distinguishes between factors that increase/decrease the demand for LPHC insurance, show a preference for LPHC insurance or a preference for HPLC insurance respectively no preference or an underinsurance for one of the two risks. Every preference (WTI and/ or WTP) above 50% will be referred to as a clear preference. A more precise subdivision states that the preference can be weak (under 50%), light (50-60%), medium (60-90%) or high (over 90%). Concepts include decision theories and other behavioral concepts. The decision theories involve expected utility theory (EUT), generalized expected utility theory (GEU), dual theory (DT), prospect theory (PT), cumulative prospect theory (CPT), myopic prospect theory (MPT), a Markowitz type Neumann-Morgenstern utility function (MTU), correlation (Corr.) and stated preference method (SPM). Other behavioral concepts (Bhvrl.) include e.g. regret theory, ambiguity and mental accounting. They will be discussed in detail in Section 3.2.3. The probability ranges between zero and one. The loading which the insurer requires for the insurance/contract can be subsidized (Sub.), fair or profitable (Pftb.).

[1] The national flood insurance program was developed to reduce flood damage by supplying a subsidized insurance, e.g. see Anderson (1974).

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Anderson (1974)	LPHC -Flood	WTI	Field, NFIP	/	Increasing demand for LPHC	/	/	Sub.
Slovic et al. (1977)	HPLC vs LPHC	WTI	Labor	Mostly hypo-tical	Preference for HPLC, 80% (p=0.25)	EUT,	0.001 -0.5	Fair
Baumann and Sims (1978)	LPHC -Flood	WTI	Field	/	Preference for LPHC, 60-86%	/	/	/
Schoemaker and Kun-reuther (1979)	HPLC vs LPHC	Binary choice, WTP	Field	Hypothetical	No preference for LPHC (<50%)	EUT, PT	0.01 -1	/ Sub.
Hershey and Schoemaker (1980)	HPLC vs LPHC	Binary choice, WTI	Labor, Tmdl.	Hypothetical	Preference for LPHC, 61-81%	EUT, MTU, PT	0.001 -0.999	Fair

(To be continued)

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Hogarth and Kunreuther (1989)	HPLC vs LPHC	WTP	Labor, Tmdl.	Real	Preference for LPHC (ambiguity avers decision-maker)	BEU, Bhvrl.	0.01, 0-0.9	/
Irwin et al. (1992)	LPHC	WTP, Vickery auction	Labor	Hypothetical vs real	Preference for LPHC, 75% (real incentives)	EUT	<0.1	/
McClelland et al. (1993)	HPLC vs LPHC	WTP, Vickery auction	Labor, Tmdl.	Real	More zero/ above expected loss bids for LPHC	EUT, GEU, PT	0.01 -0.9	/
Palm (1995)	LPHC	WTI	Field	/	Preference for LPHC*	/	/	/
Browne and Hoyt (2000)	LPHC -Flood	/	Field, NFIP, Emdl.	/	Increasing/decreasing demand for LPHC	/	/	/

(To be continued)

*Demand for LPHC insurance depends on perceived (biggest impact) and experienced (some impact) risk.

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Ganderton et al. (2000)	LPHC	WTI	Labor, Emdl.	Real	Increasing/decreasing demand for LPHC	EUT	0.001 -0.36, most below 0.1	Fair, pftb.
Theil (2000)	HPLC vs LPHC	WTI	Labor	/	Preference for LPHC, 50-70% (all premiums)	/	0.001 -0.25	Sub., fair, pftb.
Kunreuther et al. (2001)	Feeling of riski-ness	/	Labor	/	Improving evaluation of low probabilities	/	0.0000001 -0.002	Fair
Loubergé and Outreville (2001)	HPLC vs LPHC	WTI	Labor	/	Preference for LPHC, 65-80% (p=0.001 and p=0.01)	EUT	0.001 -0.5	Fair

(To be continued)

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Grace et al. (2004)	LPHC vs no LPHC	WTP	Field	/	More price elastic demand for LPHC as for non LPHC	/	/	/
Kriesel and Landry (2004)	LPHC -Flood	WTI	Field, NFIP, Emdl.	/	Increasing/decreasing demand for LPHC	/	/	/
Kunreuther and Pauly (2004)	LPHC (HPLC)	WTI	Tmdl.	/	Preference for HPLC (p=0.2 and p=0.25)	EUT	0.1, 0.2, 0.25	Pftb.
Papon (2008)	LPHC	WTI	Labor, Tmdl., Emdl.	Real	Increasing demand for LPHC	EUT, DT, MPT	0.04	Pftb.
Laury et al. (2009)	HPLC vs LPHC	WTI	Labor, Emdl.	Hypothetical vs real	Preference for LPHC, above 53%	CPT	0.01, 0.1	Sub., fair, pftb.

(To be continued)

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Landry and Jahan-Parvar (2011)	LPHC -Flood	WTI	Field, NFIP, Tmdl., Emdl.	/	Increasing/decreasing demand for LPHC	EUT	/	Sub., non sub.
Botzen and van den Bergh (2012)	LPHC -Flood	WTI, WTP	Field, Emdl.	/	No preference for LPHC, less then 50%	EUT, PT	0.0008, 0.002, 0.0025	/
Kousky and Cooke (2012)	LPHC	WTP	Tmdl.	/	Underinsurance of LPHC	EUT, Corr.	0.1, 0.01	/
Michel-Kerjan et al. (2012)	LPHC -Flood	WTI	Field, NFIP	/	Increasing/decreasing demand for LPHC	/	/	/
Schade et al. (2012)	LPHC	WTP	Labor, Emdl.	Real	Preference for LPHC, 64-87%	EUT, Bhvrl.	0.0001-0.2, 0.04	/

(To be continued)

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Botzen et al. (2013)	HPLC vs LPHC, Flood	WTP	Field, Emdl.	/	Preference for HPLC (unprotected floodplain)	EUT	/	/
Brunette et al.(2013)	LPHC	WTP	Labor, Tmdl., Emdl.	Real	Higher WTP for LPHC (ambiguity avers decision-maker)	EUT, Bhvrl.	0.2, 0.05 -0.35	/
Petrolia et al. (2013)	LPHC -Flood	Binary choice, NFIP	Field, Emdl.	Real	Preference for LPHC, 78%	EUT, Bhvrl.	0.1/0.9, 0.3/0.7, 0.5/0.5, 0.7/0.3, 0.9/0.1	/
Zaalberg and Midden (2013)	LPHC -Flood	Emotions, Pre-sence	Labor, Emdl.	/	Weak preference for LPHC	/	/	/
Brouwer et al. (2014)	LPHC -Flood	WTP	Field, Emdl.	/	Preference for LPHC, 62%	SPM	0.01, 0.0001 and higher	/

(To be continued)

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Friedl et al. (2014)	LPHC	WTP	Labor, Emdl.	Real	Preference for LPHC, 65%	EUT, PT, Bhvrl.	0.5	Sub., fair, pftb.
Gallagher (2014)	LPHC -Flood	WTI	Field, NFIP, Emdl.	/	Increasing/decreasing demand for LPHC	/	/	/
Ozdemir and Morone (2014)	LPHC	WTI, WTP	Labor, Emdl.	Real	Preference for LPHC, 70%	PT	0.01, 0.005	Fair
Petrova et al. (2014)	HPLC vs LPHC	WTP	Labor	Real	Increasing demand for LPHC	PT, Bhvrl.	0.1 -0.99	/
Atreya et al. (2015)	LPHC -Flood	WTI	Field, NFIP, Emdl.	/	Increasing demand for LPHC	Bhvrl.	/	/
Browne et al. (2015)	HPLC vs LPHC	WTI	Field*, Tmdl.	/	Weak preference for HPLC	EUT	/	/

(To be continued)

*Data from German insurer.

Table 3.1: Studies of literature review

Author	Context	Task	Type	Incentive	Insurance decision	Concept	Prob. Range	Loading
Kunreuther and Michel-Kerjan (2015)	LPHC	WTI, WTP	Labor, Tmdl., Emdl.	Real and large	Preference for LPHC	EUT, PT, Bhvrl.	0.04, 0.05	Fair, pftb.
Krawczyk et al. (2017)	LPHC	WTP	Labor, Emdl.	Real	Not clear	PT, Bhvrl.	0.05 -0.25	/
Kunreuther and Pauly (2018)	LPHC	WTI	Labor, Emld.	/	Preference for LPHC, 60% (decision-maker always insured)	EUT, Bhvrl.	0.04	/

3.2.2 Decision theory

Well-known results based on expected utility theory

Utility function Due to the St. Petersburg Paradox[2] Bernoulli (1954) developed that the utility of an outcome has to be extended by a subjective utility to explain empirical observations. The axiomatic explanation of the Bernoulli principle, which implies that the decision-maker wants to maximize expected utility and thus chooses the alternative with the highest expected value, dates back to Neumann and Morgenstern (1947). The authors define axioms to derive a preference functional.[3] The expected utility which represents the preferences of the decision-maker has to fulfill the axioms completeness, transitivity, continuity and independence.

Completeness implies that for any lottery x and y it holds that $x \succeq y$, $y \succeq x$ or $x \sim y$.

Transitivity signifies that if for any lottery x, y and z it holds that $x \succeq y$, $y \succeq z$ than it also has to hold that $x \succeq z$.

Continuity of the lotteries and a ranking of $x \succeq y \succeq z$ implies that there exists a probability p such that $y \sim p \cdot x + (1-p) \cdot z$.

Independence means that for the lotteries x and y with a ranking of $x \succeq y$ it has to hold for all lotteries z and probabilities p that $p \cdot x + (1-p) \cdot z \succeq p \cdot y + (1-p) \cdot z$.

For the case that the axioms are fulfilled the decision-maker is rational and chooses the option which yields the highest expected utility.

Risk aversion One of the most important works on the optimal insurance demand dates back to Mossin (1968). The paper ”Aspects of Rational Insurance Purchasing” addresses the problem of optimal insurance coverage. Using the expected utility approach, the author investigates the impact of risk aversion on the risk taking behavior of the decision-maker. The risk taking behavior determines if the decision-maker purchases an insurance and, if

[2] The St. Petersburg Paradox describes a gamble where the random variable has an infinite expected value. This also implies that the payout for the gamble is infinite. Regardless, the decision-maker is only willing to pay a small amount to participate in the gamble which shows that a rational decision-maker does not only decide based on the expected value (Bernoulli (1954)).

[3] If the preference functional of a random variable is represented by an utility function u it holds that $u(x) \geq u(y) \rightarrow x \succeq y$.

so, which amount. Important for this decision is the economic background of the decision-maker which is quantified by his/her initial wealth. The famous results of Mossin (1968) are that (1) decreasing risk aversion of the decision-maker leads to a lower maximum acceptable premium the higher her/his initial wealth, (2) actuarially unfair insurance coverage implies that full insurance coverage will never be optimal and (3) decreasing risk aversion of the decision-maker implies that the optimal insurance coverage will be lower the higher her/his initial wealth.

Initial wealth In his work "The Theory of Insurance Demand" Schlesinger (2013) deals among other things with the impact of risk aversion and wealth as background risk and the insurance loading on the optimal insurance demand in an expected utility framework. Regarding the initial wealth of the decision-maker and a positive insurance loading the author points out that for increasing initial wealth the optimal insurance demand will (1) decrease for decreasing absolute risk aversion, (2) be independent for constant absolute risk aversion and (3) increase for increasing absolute risk aversion.

Generalized expected utility theory The violation of the independence axiom within experimental and theoretical studies is the reason why Machina (1982) (and Machina (1983)) proposed GEUT as an alternative. Although, Machina (1982) shows that studies using EUT are still valid the author also points out that a very weak assumption on the functional form of the preference functional is sufficient to develop GEUT. This weak assumption is that the preferences of the decision-maker are "smooth", meaning that they are differentiable and thus locally linear. When ranking probability distributions linearity is identical to the maximization of expected utility so that GEUT can be used as an alternative to EUT. Nevertheless, the independence axiom will not necessarily be satisfied by this smooth preference function. Thus, no global Neumann-Morgenstern utility function exists but the local utility function will represent the preferences of the decision-maker.

Dual Theory The dual theory by Yaari (1987) is another modification of EUT. While EUT uses a utility function of wealth to describe the attitude towards risk of the decision-maker the author replaces this function with a probability distribution function. In other words, EUT is linear in the probabilities but not in wealth while it is the other way around for dual theory. This reversed decision rule allows the dual theory to test the robustness of results which are derived from EUT.

One of the biggest benefit of dual theory is the separation of the attitudes towards risk and the attitude towards wealth of the decision-maker. Under EUT risk aversion and diminishing sensitivity of wealth are synonyms. However, it is quite obvious that they are not. Risk aversion describes that an increase in uncertainty is bad for the decision-maker. In contrast, diminishing sensitivity towards wealth implies that a loss hurts more if the decision-maker is poor as when s/he is rich.

Well-known results based on prospect theory

Prospect Theory PT dates back to Kahneman and Tversky (1979) and is one of the most known theories which was developed as an alternative for EUT. One of the features of PT is the value function which accounts for the changes in wealth of the decision-maker. The value function, which uses a reference point as a starting point, is convex for losses and concave for gains. The marginal value for losses and gains decreases with their magnitude (diminishing sensitivity). The value function is also steeper for losses compared to gains. Implying, that the decision-maker is risk seeking in the case of losses and risk averse when it comes to gains. The steeper slope of the function for losses compared to gains indicates that the decision-maker is more sensitive to a reduction in wealth than to an increase. Using decision weights for each outcome leads to the weighting function. The weighting function accounts for the overweighting or ignorance of highly unlikely outcomes as for the neglection or exaggeration for high probability outcomes. Thus, PT is able to explain extreme probabilities and the resulting decisions made.[4]

Cumulative Prospect Theory CPT by Tversky and Kahneman (1992) as an extension to PT uses cumulative (instead of separable) decision weights and different weighting functions for gains and losses. The resulting fourfold pattern implies that the decision-maker is risk averse for gains and risk seeking for losses towards events with a high probability. For low probability events it is the other way around. The decision-maker is risk averse for losses and risk seeking for gains.

Myopic Prospect Theory The concept of myopic loss aversion dates back to Benartzi and Thaler (1995) and is based on the concepts loss aversion and mental accounting. Loss

[4] For an overview of prospect theory used in economics (insurance) see Barberis (2013).

aversion (Tversky and Kahneman (1991)) implies that, compared to a reference point, losses below this point appear larger then gains above this point. If the decision-maker has to decide between two options (small gain and small loss or larger gain and larger loss) where the choice between gain and loss has the same difference it plays an important role how the decision-maker evaluates the options small or large. If s/he views the options (small or large) as a difference among two disadvantages (in relation to the reference point) it will be weighted higher compared to two advantages. Mental accounting (Thaler (1985), Thaler (1999)) indicates how the decision-maker evaluates financial outcomes (see Section 3.2.3). In the presence of loss aversion the dynamic aggregation rules are very important because they are then not neutral. This means the decision-maker acts riskier if her/his performance is evaluated infrequently. Thus, myopic loss aversion is defined by loss aversion and a short evaluation period.

Langer and Weber (2005) study whether myopia always decreases the attractiveness of a gamble sequence or if it can change the evaluation of these gambles such that it results in a reverse pattern. The extension from myopic loss aversion to myopic prospect theory in combination with the risk profile of the decision-maker can give more insight of the decision-maker behavior. A decision-maker with a low degree of loss aversion decides independent of her/his myopia. Medium loss aversion as opposed to this plays a role in her/his decision and high levels can lead to a reverse effect. This means myopia can increase the attractiveness of a repeated gamble. Regarding LPHC investment opportunities myopia increases the attractiveness of repeated investing such that the time horizon is short. This implies for insurance decisions that the decision-maker is willing to take more risk or is risk seeking for a short time period.

Violation of decision theories within literature

Expected utility theory Regarding insurance decisions one of the most known theories is EUT as used by Mossin (1968) and Schlesinger (2013). Assuming that a risk averse decision-maker decides on different insurance choices by evaluating the expected utility of those s/he will choose the option which yields the highest expected value. For insurance decisions regarding LPHC- and HPLC-risks one can state that, if both risks have the same expected value and insurance loading, the decision-maker will always prefer an insurance for the LPHC-risk.[5]

Hence, EUT is still used in the literature involving insurance decision but it is not able to explain all of the decisions made. Thus, it leads to a violation of EUT looking at empirical work or experiments. This violation can be correlated with the insurance loading, risk aversion, wealth, loss probability and utility function.

The loading of the insurance contract can be subsidized, fair or profitable for the insurer. Regarding EUT the decision-maker will choose full insurance coverage if it is priced subsidized or fair. Otherwise, only partial or no coverage is optimal (Mossin (1968)). Interestingly, there are still uninsured decision-maker when the insurance is subsidized or fair (Anderson (1974), Loubergé and Outreville (2001)). For a profitable insurance, EUT predicts that only partial or no coverage is optimal. In contrast, there exist experiments which show that the decision-maker mostly chooses no or full coverage (Papon (2008)).

Investigating risk aversion, EUT is not able to explain a very high risk aversion of the decision-maker concerning insurance decisions in general (McClelland et al. (1993), Papon (2008)) or compared to a gamble situation (Hershey and Schoemaker (1980)).

EUT is also not able to explain why the decision-maker is more sensitive towards the loss probability than the loss size (e.g. Ganderton et al. (2000)). As a consequence, a variation of the loss probability has a stronger effect on the insurance decision.

The interaction of wealth and risk aversion plays an important role when it comes to insurance decisions (Mossin (1968), Schlesinger (2013)). Higher wealth can lead to the case that a possible loss is a smaller threat so that insurance becomes less attractive and the decision-maker decides to self-insure instead (Ganderton et al. (2000)). The impact of wealth on the optimal insurance demand also depends on the fraction of wealth which is at risk. An increase in wealth leads to a higher insurance demand if this additional income is not part of the wealth at risk (e.g. the home of the decision-maker). While an increase in wealth (holding all other factors constant) and an actuarially fair insurance will not affect the insurance purchase. In contrast, this is not the case for high insurance loadings. These high loadings as for LPHC insurance have an ambiguous effect depending on the risk aversion of the decision-maker (increasing or decreasing absolute risk aversion) and on the possible increasing loss through the higher wealth (Kousky and Cooke (2012)).

[5] This result was developed by Slovic et al. (1977) within a labor based experiment and supported by a field study by Kunreuther et al. (1978).

If the decision-maker compares her/his wealth to a peer group the correlation of risks is important. Thus, LPHC-risks which are correlated instead of HPLC-risks will imply a decreasing insurance purchase because the decision-maker suffers jointly in case of a loss (Friedl et al. (2014)).

Last but not least, turning to the utility function it can be observed that it is convex for losses. Therefore PT is more suitable to explain insurance decisions (Slovic et al. (1977)). For the comparison of LPHC- and HPLC-risks it is common to keep the expected loss of both risks constant. EUT as PT will then predict a higher optimal insurance demand for LPHC-risks. Contrary, it can be the other way around (insurance for the HPLC-risk is preferred) or there exists no clear preference to insure the LPHC-risk (e.g. Slovic et al. (1977), Schoemaker and Kunreuther (1979)).

Extensions of expected utility theory A Markowitz type of Neumann-Morgenstern utility function which is concave for low losses and convex for larger losses is able to explain why the decision-maker acts risk averse and risk seeking in the domain of losses (Hershey and Schoemaker (1980)). Still, it is not able to explain all of the decisions, e.g. underinsurance of risks.

GEUT predicts that the decision-maker is oversensitive towards low probabilities which results in an overestimation of these risks (McClelland et al. (1993)). Thus, it explains why the decision-maker acts in a certain way when insuring LPHC-risks.

The dual theory is able to state the results of a bimodal distribution of insurance decisions. The bimodal distribution in this case shows that the decision-maker mostly chooses no or full coverage for a profitable insurance (Papon (2008), McClelland et al. (1993)). Dual theory predicts such a behavior of a risk avers decision-maker for insurance with a positive loading. In contrast, EUT can only explain no or partial coverage (Mossin (1968)).

Tail dependence, fat tails and micro correlation lead to an increasing price for insurance (Kousky and Cooke (2009)).[6] Due to solvency constraints the insurer must charge

[6] These three instruments play an important role in risk management and for losses. A distribution characterized by fat tails implies that at the end of the distribution the loss probability decreases slowly compared to the severity of the loss. Tail dependence comprises that the probability of these severe losses occurring together is higher. In contrast, micro correlation (small correlations between risks) on its own are not dangerous but when these risks are accumulated they can be very severe (Kousky and

a higher value as soon as there are only small correlations between risks. Tail dependence or fat tails will eliminate the benefits of risk aggregation which also leads to an increasing price for insurance. Budget constraints of the decision-maker then result in an underinsurance of LPHC-risks (Kousky and Cooke (2012)).

Prospect theory As already pointed out PT and CPT are theories which are often used for a better explanation of insurance decisions. The well-known fourfold pattern of CPT predicts a risk averse decision-maker for gains and risk seeking for losses for HPLC-risks. For LPHC-risks it is the other way around. The decision-maker is risk averse for losses and risk seeking for gains (Tversky and Kahneman (1992)). This assumption is able to explain the behavior of the decision-maker regarding LPHC-risks (Ozdemir and Morone (2014)). However, both theories are still not able to illustrate every insurance decision.

The convex function for losses (implying risk seeking behavior) explains why the decision-maker fails to insure the LPHC-risk.[7] In contrast, one can argue that the utility function is not always convex for losses, such that the decision-maker is not mere risk seeking or avers in the context of losses.[8] At the same time PT predicts that the decision-maker overestimates low and underestimates large probabilities. This explains why the decision-maker is either risk averse or risk seeking regarding the probabilities (Hershey and Schoemaker (1980), Petrova et al. (2014)). For the case that both risks have the same expected loss PT predicts that the decision-maker prefers to insure the LPHC-risk. However, this result does not always hold such that PT is in some cases not able to explain insurance decisions (Schoemaker and Kunreuther (1979)). Comparing insurance contracts with different duration PT illustrates why the decision-maker does not buy a one year contract. For this short time horizon s/he is risk seeking (convex utility function for losses) and the benefit of saving money (instead of paying the premium for the insurance) rules out the costs for the expected loss (Kunreuther and Michel-Kerjan (2015)).

The extension of PT, CTP (Tversky and Kahneman (1992)), predicts that the decision-maker overestimates LPHC-risks (because s/he is risk averse towards low prob-

Cooke (2009)).

[7] The convex utility function should be used for probabilities ranging between 0.001 and 0.25 (Slovic et al. (1977)).

[8] Hershey and Schoemaker (1980) propose a utility function that is concave for low losses and convex for larger ones. Therefore, a Markowitz type Neumann-Morgenstern utility function is suitable.

ability losses). Thus, it implies risk aversion of the decision-maker regarding insurance decisions. As opposed to this one observes risk aversion for LPHC-risks, but also an underestimation of small probabilities (Krawczyk et al. (2017)). Thus, very small probabilities e.g. are rounded to zero (Laury et al. (2009)). As a result, CPT is also not able to explain all insurance decisions.

Including behavioral concepts as mental accounting, loss aversion, diminishing sensitivity (the decision-maker behaves risk seeking for losses) and probability transformation (probability weighting is not linear) myopic prospect theory can be better to explain insurance decisions compared to EUT, e.g. that a longer contract period increases the insurance demand. According to myopic prospect theory the decision-maker is willing to take a higher risk for a short time period. This explains why the decision-maker prefers insurance contracts with a longer period over a short one (Papon (2008)).

3.2.3 Other behavioral concepts

Regret theory

The regret theory dates back to Bell (1982), Loomes and Sugden (1982) and Fishburn (1984) and is developed as another alternative to EUT. Thus, it should help to better explain the behavior of the decision-maker where EUT fails. The pairwise choice which regret theory includes, violates the axiom transitivity. Regret means that the decision-maker worries on the one hand about the outcome s/he receives and on the other hand about the outcome s/he would have received if s/he had made a different choice. For the case that the chosen option is worse than the one which was not chosen the decision-maker feels the (negative) emotion of regret. Concerning insurance decisions, regret leads to less extreme choices (Braun and Muermann (2004)). This implies if EUT predicts that high levels of insurance are optimal for a regret avers decision-maker less insurance is optimal. For the case that EUT predicts that small levels of insurance are optimal the regret avers decision-maker will have a higher optimal insurance demand. The loading factor of the insurance thereby influences the direction of the impact of regret on the optimal insurance choice. Nevertheless, the critical insurance loading which indicates where the impact of regret changes form negative to positive is independent of the chosen amount of regret.

Cognitive biases

Ambiguity The concept of ambiguity dates back to Ellsberg (1961). The author uses the well-known urn gamble where the decision-maker has to decide between an urn with 50 red and 50 black balls and an urn with 100 red and black balls where the proportion of the balls is unknown. The decision-maker can bet on one of the colors and afterwards a ball is drawn randomly from the urns. Most decision-maker are indifferent between choosing red or black for both urns but they prefer the urn with the known probability over the unknown one. Thus, the decision-maker is ambiguity avers. This behavior of the decision-maker is in contrast with EUT because the subjective probability for the urn with the known probability is therefore greater, such that the probabilities of both urns do not add up to one in total (Fox and Tversky (1995)).[9]

Perceived risk The research about risk perception is rooted in the regarding cognitive psychology.[10] Slovic et al. (1982) point out that it is important to study risk perception in the context of decision-making regarding risks. The authors thereby focus on the definition of the term risk and examine what risky or not risky means for the decision-maker, how the decision-maker responds to new risks (theory of risk perception) and techniques that allow the decision-maker to react to risks. Slovic et al. (1982) find out e.g. that perceived risk is predictable and quantifiable so that risk perception between groups of decision-maker can be compared. Additionally, the authors point out that the term risk has a different meaning for various decision-maker, e.g. experts and non-experts.[11]

Regarding LPHC-risks Slovic et al. (1982) find out that the decision-maker has a low risk perception. Thus, the decision-maker does not worry about e.g. a flood and as a consequence s/he is not willing to insure the risk. The decision-maker also sees insurance as a form of investment and therefore anticipates a higher chance of getting the investment back by insuring HPLC-risks.

Social comparison Social comparison is closely related to inequity aversion of the decision-maker. Inequity aversion implies that the decision-maker dislikes unfair outcomes

[9] An overview regarding ambiguity can be found in Camerer and Weber (1992).

[10] For an overview regarding cognitive psychology see Slovic et al. (1982).

[11] For all findings regarding the aspects of risk perception see Slovic et al. (1982).

(and therefore renounces a part of her/his own welfare) which are evaluated in comparison to a neutral reference outcome (Fehr and Schmidt (1999)). The reference outcome thereby arises through social comparison with other decision-maker and leads to a social reference point. Consequently, the utility of the decision-maker depends on the own as well as on the utility of the other decision-maker (Richter et al. (2014)).[12]

For insurance against LPHC-risks which are correlated among decision-maker (Friedl et al. (2014), Richter et al. (2014)) social comparison influences the insurance demand. As a result, a social reference point can influence the insurance demand in such a way that insurance for LPHC-risk seems to be unattractive.

Mental Accounting Mental accounting dates back to Thaler (1999) and is equal to accounting within a company. With accounting or mental accounting the decision-maker is able to see their expenditures and control their spending.

Mental accounting involves three components. The first comprises the perception of outcomes and evaluation in decision-making. Mental accounting thereby provides the framework to do a cost-benefit analysis. The second includes the allocation of expenditures into categories and therefore mental accounts. The expenditures are thereby subject to budget constraints. The third involves the frequency of the account evaluation. This can be done e.g. daily, weekly or yearly. Especially, because money for different accounts is not fungible the assignment to the accounts plays an important role. Consequently, mental accounting influences the choices of the decision-maker and thus impacts consumption and insurance decisions.

Gamblers fallacy Gamblers fallacy (Tversky and Kahneman (1971)) describes that the decision-maker wrongly expects that a fair coin leads to the expectation if it drops for a certain time on one side it will be shortly drop on the other side. Besides the coin, another typical example would be a roulette. After a series of 10 times red the decision-maker will expected that now black is coming. This expectation is wrong because the likelihood of a fair draw is always 50%.

[12] Fehr and Schmidt (1999) refer to the literature regarding social psychology and sociology which studies the impact of social comparison for a long time. As a key finding the authors point out that the behavior and well-being of the decision-maker is influenced by relative material payoffs.

Affect heuristic The first work which points out that affect plays an important role in decision-making dates back to Zajonc (1980). Affect thereby is not only seen as a further step after a cognitive decision, but rather a first reaction in a decision process such that it is necessary for decision-making. Zajonc (1980) argues that with affect the decision-maker does not only see a house but s/he already evaluates it. The house can be e.g. ugly or pretentious. The author also points out that the decision-maker mostly decides not rational by evaluating all advantages and disadvantages because more often s/he decides by which car, house or job s/he likes more.

Affect is also used in the context of PT and the weighting function. Due to the fact, the preferences of the decision-maker depend more often on her/his affective reaction to a possible outcome of a risky choice then the psychological ones (Rottenstreich and Hsee (2001)). Affective outcomes thereby can be relatively affect-rich or affect-poor outcomes. Winning 100 Dollar to pay for a bill or for a dinner at a fancy restaurant will lead to totally different affects. Although, both winnings have the same value, the bill (e.g. internet-bill) will arouse less emotional reactions (affect-poor outcome) then the coupon for the restaurant (affect-rich outcome). The shape of the weighting function thereby depends on these outcomes and will be more s-shaped when the outcome is affect-rich then affect-poor.

For LPHC-risks, affect-rich outcomes, which evoke higher degrees of fear and hope, will lead to a higher overweighting of low probabilities and a higher underweighting of high probabilities (Petrova et al. (2014)).

Availability heuristic The availability heuristic dates back to Tversky and Kahneman (1973) and describes the evaluation of an event. It implies that the decision-maker analyzes the probability of an event by availability. It means that an event occurs more likely, e.g. has a higher subjective probability, the faster and easier the decision-maker can recall memories or imagine similar events. Availability is also related to the frequency of an event because more frequent events are the ones which are easier to bring back to mind. As a result, the use of the availability heuristic leads to biases because it influences the subjective probability of an event.

Projection bias The projection bias concerns the forecast of the future taste of the decision-maker. Thereby optimal decision-making depends on the future desires and wishes of the decision-maker which can differ from the current ones. These desires can be affected by e.g. mood, social influences and changes in the environment.[13] While the decision-maker can qualitatively imagine how her/his desires and wishes change s/he at the same time underestimates the dimension of this change. As a result, the projection bias e.g. leads to the case that the decision-maker orders too much food at the start of a dinner (Loewenstein et al. (2003)).

Probability neglect Probability neglect arises when the decision-maker thinks about worst case scenarios which provoke strong emotions. As a consequence, the decision-maker is not able to take into account the probability with which the worst case scenario occurs. Thus, emotions lead to probability neglect (Sunstein (2002)). Sunstein (2002) also points out that the decision-maker is not willing to learn about probabilities when making decisions. The author also refers to studies which conclude that a distinction between zero and low probabilities as significant differences in low probabilities are not beneficial for decision-making.

Behavioral concepts within literature

In the context of LPHC insurance and regret the decision-maker refuses partial coverage. The intuition behind it is, that the decision-maker considers that s/he regrets not choosing full coverage (Papon (2008)). Due to probability neglect (it will not happen to me), the decision-maker underinsures or does not insure the LPHC-risk. As a consequence, when the loss occurs s/he has no coverage and therefore will regret her/his decision (Michel-Kerjan et al. (2012)). Regret can also lead to a weighting of probabilities, e.g. the overestimation of small ones (which is in line with PT). Thus, LPHC-risks will be insured (Petrova et al. (2014)).

A decision-maker who is ambiguity avers prefers to know the exact probability of an event over an unknown one. Thus, ambiguity aversion implies that full insurance coverage is optimal if the insurance is priced fair. As soon as the insurer requires a premium for the insurance only partial coverage is optimal (Brunette et al. (2013)).[14] Turning to LPHC- and HPLC-risks one can state that the decision-maker is ambiguity avers regarding LPHC insurance decisions (Hogarth and Kunreuther (1989), Brunette et al. (2013),

[13] For a full list of influencing factors see Loewenstein et al. (2003).

[14] These results are in line with the ones of Mossin (1968).

Friedl et al. (2014)). In contrast, s/he prefers ambiguous situations concerning HPLC insurance decisions (Hogarth and Kunreuther (1989)). Logically, the insurance demand for the LPHC-risk is much higher when the risk is ambiguous.[15] Also the WTP of the decision-maker is higher for decisions under ambiguity compared to cases with no ambiguity involved. In addition, the WTP for an insurance against ambiguous risks is often higher than the expected loss (Hogarth and Kunreuther (1989), Schade et al. (2012), Brunette et al. (2013)) and is rarely zero.[16]

Perceived risk increases in the insurance demand against LPHC-risks (Atreya et al. (2015)). At least, it has an positive impact (if perception is high) on holding flood insurance (LPHC-risk) (Petrolia et al. (2013), Atreya et al. (2015)). The perception thereby influences the WTP for such a risk. Risk perception and an increase of the actual risk increase the WTP for the insurance (Botzen and van den Bergh (2012)).

Experience is closely linked to risk perception and also has a positive impact on the demand for LPHC insurance (Baumann and Sims (1978), Palm (1995), Browne and Hoyt (2000), Petrolia et al. (2013)). However, the higher the experienced loss the less insurance the decision-maker buys (Ganderton et al. (2000)). This effect disappears over time (Atreya et al. (2015)). In contrast, if no loss was experienced the less insurance the decision-maker buys (Papon (2008)).[17]

Beliefs of the decision-maker regarding LPHC-risks can change after the experience of such a risk. For the insurance demand this means it increases after e.g. a flood and then decreases again in the following years to its basis (Gallagher (2014)). In the context of the experience of other decision-maker, the decision-maker underestimates this information for her/his own decision. As a consequence, s/he does not update her/his beliefs. Thus, it plays an important role if the experience is self-experienced or by others (Krawczyk et al. (2017)).

Social comparison with other decision-maker influences insurance decisions. While LPHC-risks are correlated among each other this is not the case for HPLC-risks. Such a risk correlation has an impact on insurance decisions if the decision-maker cares about her/his wealth compared to others. For LPHC-risks it leads to a decreasing insurance

[15] 87% of the decision-maker insure the LPHC-risk when it is ambiguous compared to 64% when the risk is known (Schade et al. (2012)).

[16] Only 13% of the decision-maker are willing to pay zero (Schade et al. (2012)).

[17] This result is also in line with the availability bias (Papon (2008)).

demand because in case of a loss all decision-maker are affected (Friedl et al. (2014)). In contrast, the correlation has no impact on insurance decisions so that the decision-maker does not care about losses faced by others (Krawczyk et al. (2017)).

The concept of mental accounting is also used to explain insurance decisions. Expenses for insurance decisions are assigned to separate mental accounts. Assuming the decision-maker can decide about an insurance contract with an one or two year term and a fixed premium for each contract. The decision-maker prefers the two year contract because s/he can be sure that the insurance premium does not increase in the second year. As a result, the decision-maker adheres to her/his budget constraint (Kunreuther and Michel-Kerjan (2015)).

Gamblers fallacy predicts that the decision-maker buys less insurance if a disaster has occurred because s/he then underestimates the probability of the reoccurrence of such an event. Consequently, the decision-maker insures less as in the case of the absence of the event (Kunreuther and Michel-Kerjan (2015)).

Affect influences the description of an event. One can distinguish between affect-rich, neutral and affect-poor descriptions.[18] Very interesting thereby is the affect-rich description. It increases fear of a loss compared to a neutral description of an event. Thus, it increases the overestimation of small probabilities and so the insurance demand. In context of LPHC-risks, affect is in line with PT so that the decision-maker overestimates small and underestimates large probabilities (Petrova et al. (2014)).

One of the components of affect is worry which increases the demand for insurance. The WTP is much higher when worry is present. The effect of worry is stronger then the one of subjective probabilities for very low probability events (Schade et al. (2012)).

The availability bias implies that the decision-maker finds an event most likely to happen which can be easily brought back to mind (Papon (2008)). Consequently, the probability to insure such an event is higher.

The projection bias leads to an undervaluation of an insurance purchase. Insurance decisions are made previous to a possible loss. Making the insurance decision, the decision-maker does not focus on the moment when s/he really suffers from a loss. Thus, s/he

[18] Affect-rich outcomes elicit more hope and fear. The increase from 0% to 1% for a chance of winning elicits hope while the decrease from 100% to 99% elicits fear (for losses the outcome is reversed). Thereby, immediate affect results in fear or worry while anticipated affect evokes regret (Petrova et al. (2014)).

undervalues the importance of insurance while buying insurance (Kunreuther and Michel-Kerjan (2015)).

Probability neglect is another reason why the decision-maker buys no insurance or underinsures. It leads to a will-not-happen-to-me-attitude (Kunreuther and Michel-Kerjan (2015), Michel-Kerjan et al. (2012)).

Emotions can highly influence the insurance purchase against LPHC-risks. Thereby, the choice to be insured or to be not insured can not be explained by EUT. A lot of decision-maker who experienced a loss were very unhappy to be not insured so that they buy coverage afterwards. This behavior can be explained by regret. In contrast, insured decision-maker who did not experienced a loss were less unhappy about it so that they rarely change their insurance purchase decision. Consequently, emotions or in this particular case the feeling of unhappiness leads to a change in the insurance purchase (Kunreuther and Pauly (2018)).

3.3 Insurance demand for LPHC-risks

3.3.1 Does a preference to insure LPHC-risks exist?

Factors which increase the demand for LPHC insurance

Is there really a preference to insure LPHC-risks like EUT predicts or does the decision-maker prefer an insurance for HPLC-risks like experiments or empirical observations show?

To answer this question we distinguish between three cases. The first case involves factors that increase the demand (WTI and WTP) for LPHC insurance. The second case illustrates a clear preference to insure LPHC-risks which is the case if more than 50% of the decision-maker insure the risk or are willing to pay for it. The third case encompasses that no preference can be found to insure the LPHC-risk (less then 50% of the decision-maker choose the insurance), factors that decrease the demand for LPHC-risks or a preference to insure the HPLC-risk. The preference to insure HPLC-risks can also be a weak preference (less then 50% of the decision-maker choose the insurance). A few studies cannot be assigned to one of the three cases.

Starting with the first case which involves factors that can increase the demand for LPHC insurance respectively lead to a higher WTI or WTP, we found 13 of 38 studies which reveal factors that can cause a higher insurance demand.

The following Table 2 gives an overview of the impact factors. Factors can be divided in behavioral factors which are based on behavioral concepts and theories, personal factors, factors which are directly related to the LPHC-risk and insurance contract details. The last three categories can be related to classical theories like EUT as well as behavioral concepts and theories. An upward/downward arrow implies an increase/decrease of the factor which increases or decreases the insurance demand. A plus implies that the presence of the factor increases and/or decreases the insurance demand. Empty fields point out that these aspects are not discussed within the literature.

Pure behavioral factors, such as ambiguity aversion of the decision-maker, lead to a higher WTP for the insurance against LPHC-risks (Hogarth and Kunreuther (1989), Brunette et al. (2013)). Other behavioral factors such as perceived risk, expectation of the loss, recent losses, beliefs, experience of an uninsured loss, affect, regret and an availability bias increase the demand for LPHC insurance. If the perceived risk (Palm (1995), Browne and Hoyt (2000)) or the expectation of a loss (Landry and Jahan-Parvar (2011)) is high it increases the demand. Recent losses increase the demand for some time (Atreya et al. (2015)) and change the beliefs about such a risk (Gallagher (2014)). Thus, the insurance demand increases. The experience of an uninsured loss (Ganderton et al. (2000)) or of a loss which results in a small claim (Michel-Kerjan et al. (2012)) increases the demand as well. Affect (affect-rich description) increases fear of the decision-maker and thus the overweighting of small probabilities which leads to an increasing insurance demand for LPHC-risks (Petrova et al. (2014)). Regret has a positive impact on the chosen level of coverage for the insurance (Papon (2008)) as on the insurance of LPHC-risks itself (Michel-Kerjan et al. (2012)). The presence of an availability bias will increase the insurance demand for LPHC-risks because it positively influences the risk perception (Papon (2008)).

Furthermore, the insurance demand for LPHC-risks depends on personal factors. These can be risk aversion, wealth (income and financial status), age, education level, gender race (e.g. being female/ African-American), personal orientation and geographic factors or living conditions. While increasing risk aversion can increase the insurance demand for LPHC-risks it can be the other way around for decreasing risk aversion. Traditional EUT will predict that increasing wealth and increasing absolute risk aversion lead to a higher insurance demand for LPHC-risks (Schlesinger (2013)). In contrast, the higher loadings for an insurance against LPHC-risks can lead to an ambiguous effect also

Table 3.2: Overview of impact factors on insurance demand

Insurance demand for LPHC-risks...	**...increases**	**...decreases**
Pure behavioral factors		
Ambiguity	+	
Perceived risk, expectation of loss	↑	
Beliefs, recent loss	+	+
Experience of uninsured loss Affect (worry) Regret	+	
Social comparison, underestimation of risk		+
Gamblers fallacy Projection bias Probability neglect		+
Availability bias	+	
Budget constraints via mental accounting		+
Personal factors		
Risk aversion	↑	↓
Wealth, age	↑	↑
Education	↑	
Being female/ African-American Internal orientation, living space Direct debit, living in a dangerous area	+	
Directly related to LPHC-risks		
Risk correlation		+
Risk presentation Communication Prevention	+	
Insurance contract details		
Loss probability, expected loss	↑	↓
Positive loading	↓	↑
Contract length	↑	↑
Combined contract	+	+
Insurance as an investment	+	

depending on the risk aversion of the decision-maker (Kousky and Cooke (2012)). Risk aversion in general has a positive impact on the WTI LPHC-risks (Petrolia et al. (2013)) as well as on the WTP for such an insurance (Botzen and van den Bergh (2012)). A higher income, a higher financial status or in general a higher wealth of the decision-maker have a positive impact or increase the insurance demand for LPHC-risks (Baumann and Sims (1978), Browne and Hoyt (2000), Atreya et al. (2015)) as on LPHC- and HPLC-risks (Browne et al. (2015)). In contrast, the insurance demand can also decrease. A higher income increases the insurance demand, but this increase is not monotonic. For a very high income the decision-maker buys less insurance (Kunreuther and Michel-Kerjan (2015)), e.g. because s/he self-insures (Ganderton et al. (2000), Landry and Jahan-Parvar (2011)).

Likewise, age, a higher education level, internal orientation, being female, being African-American, living space and direct debit increase the demand for LPHC insurance. For age, on the one hand, an increase can lower the risk aversion and thereby the insurance demand (Botzen and van den Bergh (2012)). On the other hand it can increase the insurance demand for LPHC-risks (Atreya et al. (2015), Ozdemir and Morone (2014)). Other factors that positively influence the insurance demand (WTI or WTP) for LPHC-risks are a higher education level (Baumann and Sims (1978)), being female (Botzen and van den Bergh (2012)) and being African-American (Atreya et al. (2015)) and when the decision-maker is internally orientated (Baumann and Sims (1978)). Further factors that positively influence LPHC (and HPLC) insurance are living space and direct debit (Browne et al. (2015)). Geographical characteristics like living in an very exposed area to flood risk mostly do not have an positive impact on buying LPHC insurance (Palm (1995), Botzen and van den Bergh (2012), Michel-Kerjan et al. (2012), Browne et al. (2015)), but it has e.g. in costal counties (Atreya et al. (2015)).

Presentation or communication of the risk can highly influence insurance decisions. A numerical presentation of the probability increases the demand for LPHC-risks, whereas an underestimation of this risk happens when the loss probability is estimated (Krawczyk et al. (2017)). Moreover, a 3D presentation of a flood risk (LPHC-risk) increases the demand (Zaalberg and Midden (2013)). The communication of the risk and therefore the probability can increase the insurance demand for LPHC-risks. Risk ladders, which help the decision-maker to better understand the probability of an event, rank different probabilities with the help of a scale. They also combine the probabilities with risky events which are familiar to the decision-maker (Botzen and van den Bergh (2012)).

A prevention, like shoreline amoring, against a LPHC-risks, e.g. a flood, likewise increases the demand (Kriesel and Landry (2004)).

Factors that are related to the insurance contract such as the loss probability, expected loss, price/loading, contract period, combined contracts and viewing insurance as an investment can increase the insurance demand.

An increasing loss probability or expected loss (Ganderton et al. (2000)) increase the demand for LPHC insurance as well as subsidized insurance (Landry and Jahan-Parvar (2011)). Starting with the loss probability, the insurance demand increases with increasing loss probability (Slovic et al. (1977), Ganderton et al. (2000)). Contrarily, it decreases with increasing loss probability (Laury et al. (2009)).[19] As a result, it can be important if the incentives are hypothetical like for the increasing case or real like for the decreasing case.[20] One can detect a preference to insure LPHC-risks with a probability of 0.001, 0.01 or 0.1. Thereby, the WTI can be lower for lowest probability 0.001 compared to 0.01 or 0.1 (Hershey and Schoemaker (1980), Loubergé and Outreville (2001)). Contrarily, the WTP increases with decreasing probability. For a loss probability of 0.01 the WTP is normal distributed such that the decision-maker is willing to pay zero or around the expected loss. As a consequence, the decision-maker is often willing to pay more then the expected loss for the insurance (McClelland et al. (1993)). This also holds if the decision-maker knows the exact and very small probability. Hence, one third of the decision-maker are willing to pay zero, but the other ones are willing to pay more then the expected loss (Schade et al. (2012)).

Another important feature of an insurance contract is the premium (insurance loading) required. Thus, the insurance can be subsidized, fair or profitable for the insurer. In line with EUT, the well-known result implies that the insurance demand decreases with an increasing price for the insurance (Ganderton et al. (2000), Browne and Hoyt (2000), Laury et al. (2009)). The impact of the price for LPHC insurance is contradictory. On the one hand LPHC insurance can be preferred independently of the premium, but the WTI is the highest when the insurance is priced fair (Theil (2000)). On the other hand

[19] With real incentives the insurance demand increases with increasing loss probability. The loss probabilities range between 0.001 and 0.36 where the most are below 0.1 (Ganderton et al. (2000)).

[20] The replication of the experiment of Slovic et al. (1977) leads to the same result: the insurance demand is increasing with increasing loss probability (for hypothetical incentives and a loss probability between 0.001 and 0.5). The own experiment of the authors provides the result that the insurance demand is decreasing with increasing probability (for real incentives and a loss probability between 0.001 and 0.1)(Laury et al. (2009)).

the insurance demand for LPHC-risk e.g. flood risk is price inelastic meaning that it does not have a large impact on the demand itself (Landry and Jahan-Parvar (2011), Botzen et al. (2013), Atreya et al. (2015)).[21] A too expensive insurance can also lead to an under-insurance of LPHC-risks because of a personal budget constraint. The decision-maker is only willing to pay a couple of times of the expected loss and only when a large portion of wealth is at risk (Kousky and Cooke (2012)). Regarding subsidized insurance, the NFIP in the US moreover leads to a higher group of insured decision-maker, but there are still uninsured ones (Slovic et al. (1977)).

The length of the insurance contract is another important feature. A longer contract period leads to higher insurance demand (which is in line with myopic prospect theory) (Papon (2008), Michel-Kerjan et al. (2012), Kunreuther and Michel-Kerjan (2015)). The WTP thereby can be higher for a contract with a term of 5 to 10 years (Michel-Kerjan et al. (2012), Botzen et al. (2013)) but also lower for 15 years (Botzen et al. (2013)).[22]

One of the solutions to increase the insurance demand for LPHC-risks is to use combined insurance contracts. These contracts insure both risks at the same time and the decision-maker has only to decide about one contract. One can observe that the WTI is twice as high for combined insurance contracts (Slovic et al. (1977)). In contrast, the WTP can increase (Slovic et al. (1977)) or decrease (Schoemaker and Kunreuther (1979)) depending on the attractiveness of the contract.[23] Thereby, a WTP equal to zero for the insurance is higher for combined contracts in combination with an ambiguous risk (Schade et al. (2012)).

Viewing insurance as an investment (high chance of getting the premium payed back in the case of a loss), the WTI a LPHC-risk is twice as high (Slovic et al. (1977)).

As already stated by Schlesinger (2000), higher risk aversion leads to an insurance purchase (Kunreuther and Michel-Kerjan (2015)) as for LPHC-risks it has an positive impact on buying such an insurance (Petrolia et al. (2013)). While the decision-maker is

[21] In contrast, Grace et al. (2004) state that the insurance demand for LPHC-risks is more price elastic than for non LPHC-risks.

[22] Migration patterns are another explanation why some decision-maker only hold LPHC insurance for a short time (Michel-Kerjan et al. (2012)).

[23] For given contracts (fair) the WTP increases about 30% (Slovic et al. (1977)). The decision-maker is willing to pay less for combined insurance contracts. The loss probabilities thereby can adopt the values of 0.01, 0.02 and 0.06 (Schoemaker and Kunreuther (1979)).

willing to take some risk when it comes to decisions about losses in general (Schoemaker and Kunreuther (1979)) s/he is risk avers for low probability losses which is in line with PT (Krawczyk et al. (2017)). If a decision problem is formulated within an insurance context it will increase the risk aversion of the decision-maker (Hershey and Schoemaker (1980)). The risk aversion of the decision-maker has a positive impact on the WTP (Botzen and van den Bergh (2012)).

Clear preference to insure LPHC-risks

The second case illustrates a clear preference to insure LPHC-risks which is the case if more then 50% of the decision-maker insure the risk or are willing to pay for it. Referring to the second case, we found 15 of 38 studies which detect a clear preference (>50%) to insure LPHC-risks. Thereby, the preference is divided in a slight preference (between 50% and 60% choose insurance), a medium preference (between 60% and 90% choose insurance) and a strong preference (between 90% and 100% choose insurance). A medium preference to insure LPHC-risks is the most often observed case.

The WTI LPHC-risks with a medium preference thereby depends on the experience of such a risk, if the decision-maker takes the risk seriously, has a high income, has a high education level and is internally-oriented (Baumann and Sims (1978)). Moreover, it depends on the context in which the decision is presented within the experiment. For an insurance context and a loss probability between 0.001 and 0.1 a medium preference is observed. In that case, the WTI can be higher for 0.1 as for 0.001 (Hershey and Schoemaker (1980)). Comparing subsidized, fair and profitable contracts, the WTI is the highest for the fairly priced contract and a loss probability between 0.001 and 0.01. The subsidized contract leads as well to a medium preference to insure but is lower compared to the fairly priced contract. A slight preference is observed for the profitable contract (Theil (2000)). For the same range of loss probabilities likewise a medium preference can be found which slightly decreases for the lowest probability (Loubergé and Outreville (2001)). A medium preference for the WTI also reveals that the decision-maker concentrates more on the loss probability then on the loss size when making the decision (Ozdemir and Morone (2014)).

Real incentives, for the participation in the experiment, lead to a medium preference to insure LPHC-risks. The decision-maker then is willing to pay the expected loss and above. In contrast, the hypothetical payment leads to a WTP which lies below the expected loss. Still, the decision-maker shows a slight preference to insure LPHC-risks (Irwin et al. (1992)). As opposed to this, real incentives can only lead to a slight preference to insure

LPHC-risks (McClelland et al. (1993)). Within the context of flood insurance (LPHC-risk) a medium preference to pay for given contracts is detected (Brouwer et al. (2014)). Considering social comparison and that LPHC-risks are correlated (in contrast to HPLC-risks) a medium preference to insure the risk is found.[24] The decision-maker is willing to pay the expected loss or above for such an insurance (Friedl et al. (2014)). Living in a geographical area that is exposed to LPHC-risks, e.g. flood risk, leads to the case that the decision-maker shows a medium preference to insure the risk (Petrolia et al. (2013)). If the incentives for the experiment are real and large no underinsurance of LPHC-risks can be found there. For all examined premiums and probabilities buying insurance is slightly preferred (except for profitable insurance and a probability of 0.1) (Laury et al. (2009)).

Ambiguity influences insurance decision for LPHC-risks. The WTP is higher with ambiguity involved. This induces an ambiguity aversion of the decision-maker. With ambiguity involved the WTP is higher then the expected loss (Hogarth and Kunreuther (1989)). Either the risk is ambiguous or known the decision-maker shows a medium preference to insure the risk. It is noteworthy that for the ambiguous risk it is at the upper bound (nearly 90%)) and for the known risk at the lower bound (slightly over 50%) of the medium preference range. The WTP then is above the expected loss (Schade et al. (2012)) or is higher in general compared to the case where no ambiguity is involved (Brunette et al. (2013)).

Weak or no preference to insure LPHC-risks

The third case encompasses that no preference can be found to insure the LPHC-risk (less then 50% of the decision-maker choose the insurance), factors that decrease the demand for LPHC-risks or a preference to insure the HPLC-risk. The preference to insure HPLC-risks can also be a weak preference (less than 50% of the decision-maker choose the insurance). We found 8 of 38 studies which show that LPHC insurance is not always preferred.

Assuming a HPLC-risk with a probability of 0.2 or 0.25, the decision-maker shows a medium preference to insure the risk (Slovic et al. (1977)). A preference to insure HPLC-risks can also be detected by the highest expected utility. The decision-maker is confronted with the decision to purchase insurance or to purchase no insurance. Additionally, s/he can get information on the loss probability which comes at some costs. With low search

[24] This is in contrast to the case where only the correlation is considered (Section 3.1.3.).

costs to estimate the loss probability (because it yields the highest expected utility) one can state that the decision-maker then prefers HPLC insurance (Kunreuther and Pauly (2004)). HPLC-risks are uncorrelated compared to LPHC-risks. This leads only to a weak preference to insure HPLC-risks (in this case only a third of the decision-maker insure the risk but more then twice as much as for the LPHC-risk, Browne et al. (2015)). For HPLC-risks an ambiguity preference can be found so that with ambiguity involved less decision-maker insure the risk (Hogarth and Kunreuther (1989)). The WTP for HPLC insurance is lower compared to LPHC insurance (for which the decision-maker is willing to pay the expected loss or above). It is noteworthy that there exist less zero bids (the WTP of the decision-maker is equal to zero) for HPLC insurance. This leads to a higher number of insured decision-maker (McClelland et al. (1993)). Comparing two different flood risks, such as a LPHC-flood-risk (protected floodplain) and a HPLC-flood-risk (unprotected floodplain), one can state that the demand is higher for the HPLC-risk as well as the WTP. The communication of the risk thereby increases the WTP for the insurance compared to the case without risk communication (Botzen et al. (2013)).

A WTP, for given contracts for LPHC-risks, under 50% can be found for a loss probability ranging between 0.01 and 0.06 and under 0.001. Only the subsidized contract is attractive for more then 50% of the decision-maker. Furthermore, no clear preference to insure HPLC-risks can be found. Most of the decision-maker do not want to buy insurance for such a risk, although the insurance is subsidized (Schoemaker and Kunreuther (1979)). A WTI under 50% is due to a bad risk communication (Botzen and van den Bergh (2012)). A very short contract period (1 year) also leads to a very high underinsurance of LPHC-risks. Although, the contract is priced fairly only few decision-maker (4-7%) want to insure the risk (Kunreuther and Michel-Kerjan (2015)).

A low motivation to insure the LPHC-risk can be found when comparing the presentation mode of the risk. Thereby a 3D presentation (interactive participation of the decision-maker) slightly increase the insurance demand compared to a 2D presentation (Zaalberg and Midden (2013)).

Factors that decrease the demand for LPHC insurance or lead to an underinsurance are e.g. a higher distance to the shoreline (Kriesel and Landry (2004)), budget constraints of the decision-maker and a pricey insurance. An expensive insurance arises due to risk correlation of LPHC-risks. The correlation leads to a price increase because the insurer cannot profit from risk diversification (Kousky and Cooke (2012)). Underestimation of the risk, other expenses, social impact of suffering from a flood or a short time memory of the decision-maker are other factors (Michel-Kerjan et al. (2012)). The demand for LPHC insurance, e.g. a flood, decreases in the following years after a flood (Gallagher (2014)).

Impact of the experimental conditions

There exist differences in the experiments which can explain the diverse insurance decisions. These differences can emerge from the incentives which can be real or hypothetical, loss probability, formulation of the decision problem or the number of decision rounds.

If the incentives are hypothetical the insurance for HPLC-risks is preferred respectively no preference for the LPHC insurance can be found (Slovic et al. (1977), Schoemaker and Kunreuther (1979)). In contrast, differences in the questionnaire (Hershey and Schoemaker (1980)) can lead to the case that with a hypothetical payment the decision-maker prefers the insurance for the LPHC-risk (Hershey and Schoemaker (1980)). For the case that the decision-maker gets real incentives s/he prefers unambiguously the insurance for the LPHC-risk (Irwin et al. (1992), Laury et al. (2009), Friedl et al. (2014), Ozdemir and Morone (2014)). The decision-maker then is willing to pay zero, the expected loss or even above it (Irwin et al. (1992), McClelland et al. (1993)). This implies that the decision-maker over- or underestimates the risk. Real incentives and an increasing loss probability also increase the insurance demand for LPHC-rsiks (Ganderton et al. (2000)). A more period contract for LPHC-risks is preferred when the incentives are real (Kunreuther and Michel-Kerjan (2015)).

As shown, the payment received for the participation in the experiment plays an important role because it can influence the insurance demand, e.g. related to the loss probability. In this context the question arises which probabilities are low and which ones are high. While some studies include a probability of 0.25 into the range of low probabilities others view that probability already as a high one (see Table 1). Besides, the insurance choice can be different for a probability of 0.01 and 0.1. As a consequence, one should consider the probabilities below 0.01 as the low ones. Concerning decision-making, the loss probability is very important because the decision-maker concentrates more on loss probability then on loss size (Ozdemir and Morone (2014)).

Moreover, the formulation of the decision problem can be very important. An insurance situation of a decision problem versus a lottery can change the preferences of the decision-maker (Schoemaker and Kunreuther (1979)). It can also increase her/his risk aversion (Hershey and Schoemaker (1980)) and thereby increases the insurance demand.

The impact of the number of decision rounds in an experiment is ambiguous. On the one hand there is an enormous increase of the WTI with more decision rounds (Slovic et al. (1977)) and on the other hand there is no effect at all (Irwin et al. (1992)).[25]

3.3.2 Underinsurance of LPHC-risks

Reasons

Although, insurance is subsidized (Anderson (1974)) or supported by a program like the NFIP (Browne and Hoyt (2000)) there are still uninsured decision-maker. The question which comes to mind is why there are still uninsured decision-maker when it comes to LPHC insurance. The most frequently mentioned points are that there exists a threshold level or a reference point, the decision-maker is not able to process the probability which leads to an over- or underestimation or s/he totally dismisses the risk. The communication of the risk as well as the context, format or presentation are other important points to mention. Moreover, constraints like budget or solvency influence the insurance decision. Decision theories often fail to explain the behavior of the decision-maker, so that a lot of behavioral theories and concepts are used to explain why the decision-maker does not buy the insurance.

A probability threshold is often used to explain why the decision-maker buys no insurance or under-/overinsures. This threshold is different for every decision-maker. It could be the case that probabilities which are below some threshold are treated as if they are zero (Slovic et al. (1977)). Thus, the decision-maker fails to insure. The threshold could be also the minimum probability for which the decision-maker is willing to buy insurance (for a given loss, Ozdemir and Morone (2014)). A utility function with an inflection point is another kind of threshold. It describes a function which e.g. first increases and then at some point decreases. In case of loss probabilities, the later decrease can be found for the lowest probabilities. Thus, it leads to an underinsurance of the lowest probabilities (Loubergé and Outreville (2001)). A social reference point occurs if the decision-maker compares her/his situation with others. Social reference points decrease the insurance demand, especially for correlated risks such as LPHC-risks. If a loss occurs all decision-maker are affected, thus insurance becomes less attractive (Friedl et al. (2014)).

[25] For a loss probability of 0.05 and five instead of one decision round(s) the WTI increases form 58% to 94% (Slovic et al. (1977)).

Moreover, an underinsurance of LPHC-risks is created because the decision-maker is not able to handle information. This holds especially for probabilities (Schoemaker and Kunreuther (1979)). Besides, too little information on the probability also cause an underinsurance (Landry and Jahan-Parvar (2011)). A limited sensitivity towards LPHC-risks also influences the insurance demand (Schoemaker and Kunreuther (1979)). The sensitivity towards the loss probability is influenced by the expected loss and the insurance contract (Laury et al. (2009)). Very low loss probabilities are often neglected (Loubergé and Outreville (2001)) which results in an underinsurance of the risk. The overestimation of small probabilities and the underestimation of large probabilities which is in line with PT can cause an imbalance of insurance decisions (Hershey and Schoemaker (1980)). The effect thereby is greater for affect-rich outcomes (Petrova et al. (2014)).

The format, context and presentation of the loss probability are moreover relevant factors for the demand. The format can include the order of presentation of insurance decisions (e.g. urns, Slovic et al. (1977)), the probability (urn versus numerical decision problem), the loss and premium (monetary units versus undefined points) or the questions (group versus single, Theil (2000)). The context (insurance versus no insurance formulation of the decision problem) leads to contradictory preferences (Schoemaker and Kunreuther (1979), Hershey and Schoemaker (1980)). The decision-maker shows a greater risk aversion in case of the insurance context (Loubergé and Outreville (2001)). The way how the loss probability is presented to the decision-maker plays another important role. An underestimation of the LPHC-risk can be found if the loss probability is estimated by observation instead of a numerical presentation (Krawczyk et al. (2017)). Additionally, solvency and budget constraints lead to an underinsurance of LPHC-risks. In both cases, the insurance is too expensive for the decision-maker. Thus, s/he underinsures (Kousky and Cooke (2012)).

Regarding the experiment, differences in the questionnaire and the payment for the participation can influence insurance decisions. Therefore, an underinsurance can be caused by differences in the formulation of questions (Hershey and Schoemaker (1980)) or a hypothetical payment (Irwin et al. (1992)).

The underinsurance of LPHC-risks can also be explained by the fact that the risks are correlated. In this case the decision-maker cares about her/his wealth compared to others. Hence, insurance becomes less attractive and the decision-maker prefers the insurance for the uncorrelated HPLC-risk (Browne et al. (2015), Friedl et al. (2014)).

Especially, for a short contract period the decision-maker is willing to take some risk and therefore buys no insurance (Kunreuther and Michel-Kerjan (2015)).

Solutions

Within the literature there can be found various solutions for the afore mentioned problems in the context of insuring LPHC-risks. We now want to discuss some solutions which improve the numbers of insured decision-maker.

First, one has to enhance the understanding of low probabilities. A comparison point helps the decision-maker to better evaluate the probability and therefore choose the right amount of insurance (Kunreuther et al. (2001)). More information also enhance the understanding of probabilities (Kunreuther et al. (2001)) and increase the insurance demand (Botzen et al. (2013)). Thereby, the premium for the insurance does not improve the understanding of the risk compared to the probability (Kunreuther et al. (2001)). More information regarding the loss probability and the loading will improve the insurance demand. The reason is that the decision-maker has no additional search costs for the information and does not think that the insurance is too expensive (Kunreuther and Pauly (2004)).

Moreover, the adjustment of the contract time or combined contracts can improve the insurance demand. One can establish a lifetime insurance which is connected to the purchase of a house. This will prevent the underinsurance of LPHC-risk and that the insurance will be terminated early (Papon (2008)). A multi-year flood insurance connected with the property is another solution (Michel-Kerjan et al. (2012)) or a multi-year contract in general (Botzen et al. (2013)). Bundling coverage will lead to an increased loss probability because of the addition of probabilities. It could be the case that the probability is now above the personal threshold of the decision-maker such that s/he now perceives the risk. In this sense, the risk is now relevant and will be insured (Kunreuther and Pauly (2004)). Consequently, combined contracts (for HPLC- and LPHC-risks) can increase the insurance demand (Schoemaker and Kunreuther (1979)).

Regarding the experiment, it should be performed with real money as payment for the participation (Irwin et al. (1992)). It provokes a better feeling for riskiness and losing money.

Risk communication, e.g. using a risk ladder (Botzen and van den Bergh (2012)) and presentation, should be as detailed as possible to enhance the understanding. Thus, a 3D presentation within an experiment can increase the insurance demand (Zaalberg and

Midden (2013)).

Last but not least, the probability range should be chosen carefully. It is common to use 0.1 and below as a low loss probability. However, the results can be different for 0.1 and 0.01. Thus, it should be considered that a low probability is 0.01 and below. The same holds for the high probability. Some studies use values of 0.5 and above others already state that 0.25 is a high probability while others still use it for a low probability (see Table 1).

3.4 Conclusion

Within this literature review we figure out that a preference to insure LPHC-risks exists. Therefore, we reviewed 38 studies whereof 15 sate a clear preference ($\geq$ 50%) of the decision-maker to buy insurance for the risk. A (weak) preference ($<$ 50% or $\geq$ 50%) to insure the HPLC-risk is only found within 8 of the 38 studies.

The literature review reveals that different theories and concepts are able to explain partial aspects of the insurance demand. As a consequence, the review includes besides EUT, which is often used to explain insurance decisions, other theories and behavioral concepts. These theories like GEUT, PT and its extensions, a Markowitz type Neumann-Morgenstern utility function or dual theory also show some weaknesses. Thus, behavioral theories and concepts are integrated which give explanations for the underinsurance of LPHC-risks. The impact factors which increase or decrease the optimal insurance demand thereby can be pure behavioral factors, e.g. ambiguity or regret. Other factors which depend on decision theories as on behavioral theories and concepts are personal factors e.g. risk aversion or wealth of the decision-maker, factors which are related to the LPHC-risk or the contract like the risk correlation, loading or contract length.

To answer the question if there exists a preference to insure LPHC- or HPLC-risks we include these pure behavioral factors, personal factors, factors directly related to LPHC-risks or the insurance contract and distinguish between three cases: (1) factors which increase the insurance demand for LPHC-risks, (2) a clear preference to insure LPHC-risks which exists if 50% or more of the decision-maker are WTI or WTP for the insurance and (3) the case that no preference to insure LPHC-risks can be found or a preference to insure HPLC-risks exists.

Factors which unambiguously positively influence the insurance demand for LPHC-risks are ambiguity aversion of the decision-maker, experience of an uninsured loss, affect, worry, regret, availability bias, being female or African-American, internal orientation of the decision-maker, living space, direct debit, living in a dangerous area, a better risk presentation and communication, prevention activities and viewing insurance as an investment.

A clear preference (>50%) to insure LPHC-risks depends on the experience of such a risk, if the decision-maker takes the risk seriously, has a high income, has a high education level, is internally-orientated and lives in a geographical dangerous area. Social comparison with other decision-maker in combination with the fact that LPHC-risks are correlated also lead to the case that this kind of insurance is preferred. Moreover, an insurance context and a loss probability between 0.001 and 0.1 result in a preference to insure the LPHC-risk. The highest WTI LPHC-risks can be found for a fairly priced contract and a loss probability between 0.001 and 0.01.

A preference to insure HPLC-risks can be detected because HPLC-risks are not correlated. In addition, low search costs to estimate the loss probability lead to a high expected utility so that the decision-maker prefers the insurance.

The range of loss probabilities used within the studies starts from 0.0000001 till 0.99. The analysis reveals that for the low probability values of 0.01 and below should be used. For the high probability the values above 0.01 are a good choice.

For investigating the implementation of the experiments it is noteworthy that the incentive as well as the risk presentation are important for the insurance decision. Thereby, a real and large incentive lead to a higher insurance demand for the LPHC-risk. More information about the risk and other improvements which lead to better understanding of the risk also increase the insurance demand for the LPHC-risk. Consequently, all options that improve the conditions of the experiment in the way that it becomes more realistic for the decision-maker should be used. In this sense one gets a more realistic insurance decisions.

For further research real data of insurers should be used. The one experiment which was conducted with real data shows that the decision-maker prefers to insure HPLC-risks. This is in contrast to the main findings of our analysis. Thus, it would shed further light on which insurance is preferred.

Chapter 4

Optimal insurance demand - low probability, high consequence versus high probability, low consequence

4.1 Introduction

Empirical studies find a rather low insurance demand for ”catastrophic risks” characterized by occasional but severe losses.[1] In contrast, one observes a high insurance demand for risks which occur often but result in only small losses such as an insurance for mobile phones or against bicycle thefts.[2] These observations are not in line with expected utility theory (EUT) which predicts a high demand for disaster insurance compared to mobile phone or bicycle insurance.

In this paper, we explain this puzzle in a basic insurance model with mental accounting and background risk. We consider two types of risks. One risk occurs with low

[1] For example, Botzen and van den Bergh (2012) observe that a significant proportion of homeowners neglect the low-probability flood risk. Browne et al. (2015) point out that more than two times as many decision-maker have coverage for HPLC-risks than for LPHC-risks.

[2] Empirical examples are e.g. given in Browne et al. (2015). A literature review of experimental studies on optimal insurance demand can be found in Jaspersen (2016).

probability and has high consequences (LPHC), whereas the other risk occurs with high probability and has low consequences (HPLC). The decision-maker has decreasing absolute risk aversion (DARA).[3] We show that the decision-maker has a higher insurance demand for LPHC-risks than HPLC-risks when only one of these risks is present. In the more realistic case that both risks are present, we rely on mental accounting and assume that the decision-maker decides separately on both risks, treating the other risk as a background risk. We find that both results are possible: the decision-maker may have a higher insurance demand for the LPHC-risk, but s/he may also have a higher insurance demand for the HPLC-risk.

In a first step, we reconsider the basic insurance model. The decision-maker is endowed with an initial wealth w_0 which is threatened by a possible loss. A loss of size $L > 0$ can occur with probability p while there is no loss with probability $1 - p$. Investing the amount y into insurance reduces the loss size from L to $l(y, p)$ if a loss occurs. The insurance premium includes a loading factor δ such that it is equal to $(1 + \delta)$ times the actuarially fair premium (given by the expected payout of the insurance contract).

To compare the demand for insuring HPLC- and LPHC-risks, we assume that the expected impact of the loss is the same for both risks, i.e. $p \cdot L = c$ (constant). We show that the optimal investment into insurance is decreasing in the loss probability and increasing in the loss size. This implies a higher investment for insuring LPHC-risks than for insuring HPLC-risks.

In a second step, we turn to the case of two independent risks. The LPHC-risk occurs with a low probability $p^{(l)}$, while the HPLC-risk occurs with a high probability $p^{(h)}$ ($p^{(h)} > p^{(l)}$). We assume that the two risks have the same expected impact $c = p^{(l)} L^{(l)} = p^{(h)} L^{(h)}$, and that there are no differences in pricing in that both risks share the same loading factor δ for the insurance contract.[4]

The crucial point is that the decision-maker does not decide on both insurance contracts simultaneously due to mental accounting. S/he thus decides on the insurance

[3] With DARA, higher levels of financial wealth imply a lower absolute risk aversion (Pratt (1964), Arrow (1965)). Mossin (1968) has shown that for DARA, an increase in wealth leads to a decrease in the insurance purchase. As shown by Guiso and Paiella (2008), this case is also the empirically relevant one.

[4] The interested reader is referred to Kousky and Cooke (2012) who show that if insuring risks with loss distributions characterized by fat tails, micro-correlations or tail dependence, insurers need to charge a price that is many times the expected loss in order to meet their solvency constraint.

for the LPHC-risk while considering the HPLC-risk as uninsurable background risk and vice versa. In the benchmark case discussed above, s/he also decides on each insurance contract on its own but ignores the presence of background risk.[5]

We then solve for the optimal investment into insurance of the LPHC-risk (with HPLC-risk as background risk) and the optimal investment into insurance of the HPLC-risk (with LPHC-risk as background risk). For the resulting optimal insurance demand, we find that both relations are possible: the insurance demand for the LPHC-risk may be higher or lower than the insurance demand for the HPLC-risk. As we show in a numerical example, the higher demand for the HPLC-risk is observed when the initial wealth is small relative to the loss sizes and/or when the loss size of the LPHC-risk is very large. Overall, we contribute to the literature by showing that the combination of mental accounting and background risk can explain the puzzling higher demand for HPLC-risks than for LPHC-risks.

Our paper is related to several strands of the literature which aim to explain the empirically observed insurance demand, in particular with respect to LPHC- and HPLC-risks. Slovic et al. (1977) are one of the first using laboratory studies to show that decision-maker may prefer to buy insurance for HPLC-risks over buying insurance for LPHC-risks.[6] Subsequently, a lot of experiments were conducted in this field of research. McClelland et al. (1993) argue that decision-maker either do not pay attention to LPHC-risks at all or overestimate the risk. Ganderton et al. (2000) show that decision-maker are more sensitive towards the loss probability than the loss size. Laury et al. (2009) argue that decision-maker underinsure LPHC-risks because they can not distinguish between low- and zero-probabilities or because the insurance is too expensive.[7] Kousky and Cooke (2012) observe that a too expensive insurance is in conflict with the budget constraint of the decision-maker which leads to an underinsurance of LPHC-risks. Krawczyk et al. (2017)

[5] We could also use the case in which the decision-maker decides on both insurance contracts simultaneously as the benchmark case. Our chosen benchmark case is the most simple choice. It differs from the case discussed in the following by the treatment of the other risk only (ignored risk versus background risk).

[6] Slovic et al. (1977) point out that their results are in line with a field study by Kunreuther et al. (1977). The authors mention two explanations for their result: the decision-maker has a convex utility function over losses (instead of a (risk averse) concave function) or s/he does not care about very small probabilities.

[7] In an experiment, Laury et al. (2009) also show that decision-maker do not underinsure LPHC-risks when the incentives are real.

point out that low probabilities are underestimated and that the decision-maker is risk averse against such losses.

Besides the reasons for the underinsurance of LPHC-risks a lot of experiments try to explain the behavior of the decision-maker and give solutions to avoid an underinsurance. Schoemaker and Kunreuther (1979) find that the decision-maker takes more risk if the loss probability is small.[8] Kunreuther et al. (2001) as well as Landry and Jahan-Parvar (2011) point out that the decision-maker needs a lot of information to differentiate between low probabilities. Common probabilities or insurance premiums do not reflect the feeling of riskiness properly for the decision-maker. Schade et al. (2012) show that the decision-maker more likely insures LPHC-risks if the risk is ambiguous. Krawczyk et al. (2017) argue that social comparison between decision-maker does not have any positive effect on the insurance demand for LPHC-risks. Kunreuther and Pauly (2018) find that the underinsurance of LPHC-risks is influenced by emotions. The experience of an uninsured loss leads to the case that the decision-maker feels bad and thus buys insurance afterwards.

The literature discussing the optimal insurance demand dates back to Mossin (1968). The author shows that a risk averse decision-maker prefers full insurance coverage if the insurance is priced actuarially fair and partial insurance coverage in case of a positive loading factor. A lot of theories developed over the years extending this result and try to explain the demand for insurance. Interestingly, to the best of our knowledge, there exists no model which explains the anomaly that decision-maker prefer buying insurance for HPLC- over LPHC-risks if both risks are priced using the same loading factor.

The literature on background risk and insurance demand dates back to Eeckhoudt and Kimball (1992). The authors argue that the presence of an uninsurable background risk implies a higher optimal insurance amount for the insurable risk. Schlesinger (2013) confirms the findings of Mossin (1968) mentioned above also for an independent background risk.[9] For a non-independent background risk the author argues that the impact on the optimal insurance demand depends on different circumstances e.g. on whether the non-independent background risk exists in the loss and the no-loss state. Fei and Schlesinger

[8] Schoemaker and Kunreuther (1979) point out that this results from the limited sensitivity of the decision-maker towards events with low probabilities.

[9] Schlesinger (2013) adds that in case of an actuarially unfair insurance, a utility function that exhibits standard risk aversion (meaning that absolute risk aversion and prudence are decreasing in wealth) results in a higher optimal insurance amount.

(2008) show that a higher uninsurable background risk in the loss state leads to an increasing insurance demand for any prudent decision-maker while the opposite holds for a higher background risk in the no-loss state.[10]

The concept of mental accounting, as another approach to explain the demand for insurance, dates back to Thaler (1999). The author argues that *expenditures are grouped into budgets* and that these budgets are allocated to different mental accounts which are not fungible, i.e. substitutable. If the fungibility of budgets or money is violated this can have an impact on consumption and on insurance decisions. We add to this literature, by applying mental accounting to analyze the insurance demand for HPLC- and LPHC-risks.

The outline of the paper is as follows. In Section 4.2, we reconsider the basic insurance model and give a brief review of the results which are needed subsequently. In addition, we show that the decision-maker prefers to pay more for insuring LPHC-risks than HPLC-risks if the expected loss sizes as well as the loading factors coincide. In Section 4.3, we assume that the two risks, LPHC- and HPLC-risk, exist at the same time. We introduce mental accounting by stating two optimization problems with an insurance demand for either LPHC- or HPLC-risk while taking the other risk as (uninsurable) background risk. We show that the optimal investment into insurance for HPLC-risks can exceed the one for LPHC-risks, and numerically illustrate our results. Section 4.4 concludes the paper.

4.2 Basic insurance model and its implications for optimal insurance demand

We start with the basic model of insurance demand and show how to determine the optimal investment into insurance. We then briefly review some well-known results which will turn out to be useful afterwards. The section closes with a proof of the result that in the conventional setup, the decision-maker has a higher insurance demand for LPHC- than HPLC-risks.

[10] Fei and Schlesinger (2008) point out that for a higher uninsurable background risk in the loss state and an actuarially fair insurance more then full insurance coverage is optimal. This result is contrary to the one of Mossin (1968).

4.2.1 Basic model of insurance demand

We assume that the decision-maker is endowed with an initial wealth w_0 ($w_0 > 0$). In the basic model of insurance demand, a loss size of L ($0 < L < w_0$) can occur with probability p while there is no loss with probability $1-p$. Buying insurance can reduce the loss size, i.e. investing the amount y reduces the loss (if it occurs) from L to $l(y,p)$ where $l(0,p) = L$. The reduced loss size $l(y,p)$ and the costs for insurance y are given by

$$l(y,p) = L - \frac{y}{(1+\delta)p} \qquad \text{and} \qquad y = (1+\delta)p(L - l(y,p))$$

where δ denotes the loading factor. We assume that $\delta \geq 0$. For $\delta = 0$, the insurance is priced actuarially fair, while $\delta > 0$ implies that the insurer requires a premium above the actuarially fair one. In addition, we assume that $(1+\delta)p < 1$, which rules out arbitrage opportunities and ensures that a finite optimal loss size exists.[11]

The terminal wealth of the decision-maker depends on her/his investment into insurance and on whether a loss occurs. It is equal to $w_0 - y$ in case there is no loss, and equal to $w_0 - y - l(y,p)$ in case a loss occurs.

The decision-maker maximizes her/his expected utility of terminal wealth. Her/his utility function is u, and we assume that s/he is risk averse ($u' > 0, u'' < 0$). The optimal insurance demand then follows from

$$y^*(p, w_0) = \arg\max_y \left\{ p \cdot u(w_0 - y - l(y,p)) + (1-p) \cdot u(w - y) \right\}.$$

The following proposition summarizes the well-known optimality condition for the insurance demand.

Proposition 4 (Optimal insurance) *The first order condition for the optimal investment into insurance $y^*(p, w_0)$ is given by*

$$\frac{u'(w_0 - y - l(y,p))}{u'(w_0 - y)} = \frac{1-p}{\frac{1}{1+\delta} - p}.$$

Proof: For the sake of completeness, the proof is given in Appendix B.1.2.

For $\delta = 0$, full insurance coverage is therefore optimal, while for $\delta > 0$, partial coverage is optimal.

[11] For the proof see Appendix B.1.1.

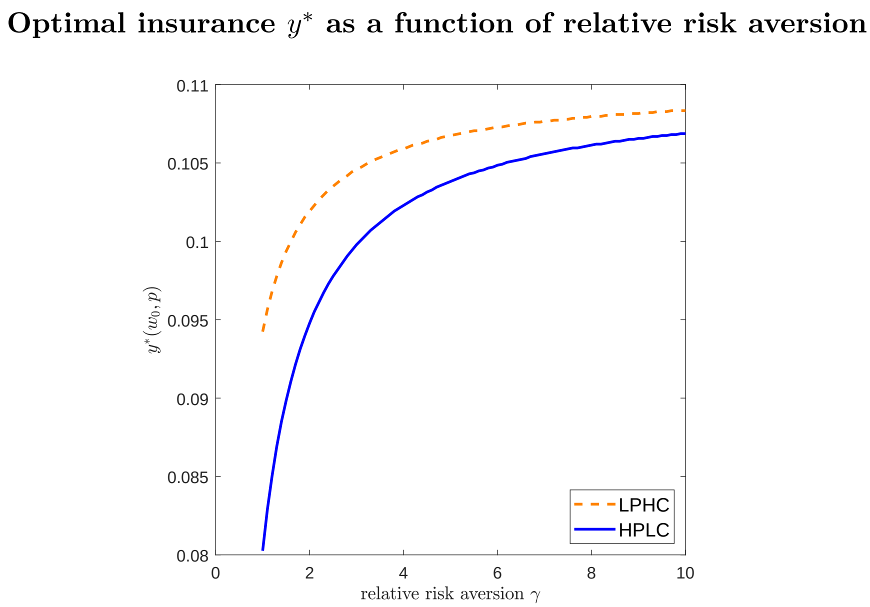

Figure 4.1: The figure gives the optimal investment of a CRRA decision-maker into insurance as a function of relative risk aversion for the HPLC-risk (solid blue line) and the LPHC-risk (dashed orange line). The remaining parameters are given in Table 4.1.

4.2.2 Optimal investment into insurance

We now review and illustrate some well-known results which turn out to be useful in the following analysis. We assume $\delta > 0$, such that partial insurance coverage is optimal.

Impact of risk aversion

An increase of risk aversion for all wealth levels implies an increase of the optimal investment into insurance (e.g. Schlesinger (2013)). We illustrate the impact of risk aversion in Figure 4.1 for the case of constant relative risk aversion (CRRA) utility. The parameters of the optimization problems with the HPLC- and LPHC-risk are given in Table 4.1. Figure 4.1 shows the optimal investment into insurance as a function of relative risk aversion γ. The values for γ are well within the range of values in Outreville (2014), e.g.. In line with the theory and with intuition, the optimal investment into insurance increases in the level of risk aversion.

Impact of initial wealth

For decreasing absolute risk aversion (which is the case for a CRRA decision-maker, e.g.), the optimal $y(p, w_0)$ is a decreasing function of initial wealth. The higher the wealth level of the decision-maker, the more s/he is willing to accept some risk, and the less s/he aims at reducing the risk of a loss by buying insurance. For constant absolute risk aversion (CARA)(with utility function $u(x) = -e^{-\gamma x}$), the optimal $y(p, w_0)$ is independent of the initial wealth. Finally, an increasing absolute risk aversion would imply that the optimal investment into insurance increases in the initial wealth.

Impact of an independent background risk

Uninsurable background risk in general has an impact on the optimal investment into insurance (Schlesinger (2013)). We assume that the risk that can be insured and the background risk are independent. It then again holds that full insurance is optimal for $\delta = 0$, while partial coverage is optimal for $\delta > 0$. For DARA and decreasing prudence, it furthermore holds that the optimal investment into insurance is increasing in the amount of mean-zero background risk.[12]

4.2.3 Optimal insurance demand for LPHC- and HPLC-risks

We now take a first look at the difference in the insurance demand for LPHC-risks (low probability for a high loss) and HPLC-risks (high probability for a low loss). To do so, we set $p \cdot L(p) = c$, i.e. we assume that the expected loss is the same for all risks $(p, L(p))$. Otherwise, our main results only rely on the assumption of decreasing absolute risk aversion, which is the empirically relevant case (e.g. Guiso and Paiella (2008)).

Proposition 5 (Optimal insurance as a function of loss probability) *Assume that the expected impact of the loss is constant, i.e.* $p \cdot L(p) = c$*. For DARA, it holds for the optimal investment* $y^*(p, w_0)$ *that*

$$\frac{\partial y^*(p, w_0)}{\partial p} < 0.$$

The optimal investment into insurance is thus larger for LPHC-risks than for HPLC-risks.

[12] The amount is e.g. measured by a scaling factor for background risk.

Benchmark parameters		
w_0	initial wealth	20
γ	relative risk aversion	8
δ	insurance loading	0.1
c	expected loss size	0.1
L^{HPLC}	HPLC-risk: loss size	6.80
p^{HPLC}	loss probability	c/L^{HPLC}
L^{LPHC}	LPHC-risk: loss size	12.75
p^{LPHC}	loss probability	c/L^{LPHC}

Table 4.1: Benchmark parameters

Proof: The proof is given in Appendix B.2.2.

Figure 4.2 gives the optimal investment into insurance as a function of the loss probability p for relative risk aversion $\gamma = 3$ and $\gamma = 8$. It shows that the optimal investment is decreasing in p. The decision-maker is more afraid of the risk of large but rare losses than of the risk of small but frequent losses, and the optimal insurance is larger the rarer the risk (and the larger its size).

In line with this intuition, in the example the optimal investment into insurance is larger for the LPHC-risk ($y = 0.10459$ for the less risk averse decision-maker, $y = 0.10795$ for the more risk averse decision-maker) than for the HPLC-risk ($y = 0.09978$ for the less risk averse decision-maker, $y = 0.10613$ for the more risk averse decision-maker).

4.3 Background risk

In the following, we consider the difference between the LPHC- and the HPLC-risk when both risks are present simultaneously. In particular, we want to know whether the decision-maker is always more anxious to insure LPHC-risks than to insure HPLC-risks.

4.3.1 Model setup

In the base case with one risk only, the decision-maker is more afraid of rare but large losses (LPHC-risk) than of small but frequent losses (HPLC-risk). If the expected loss

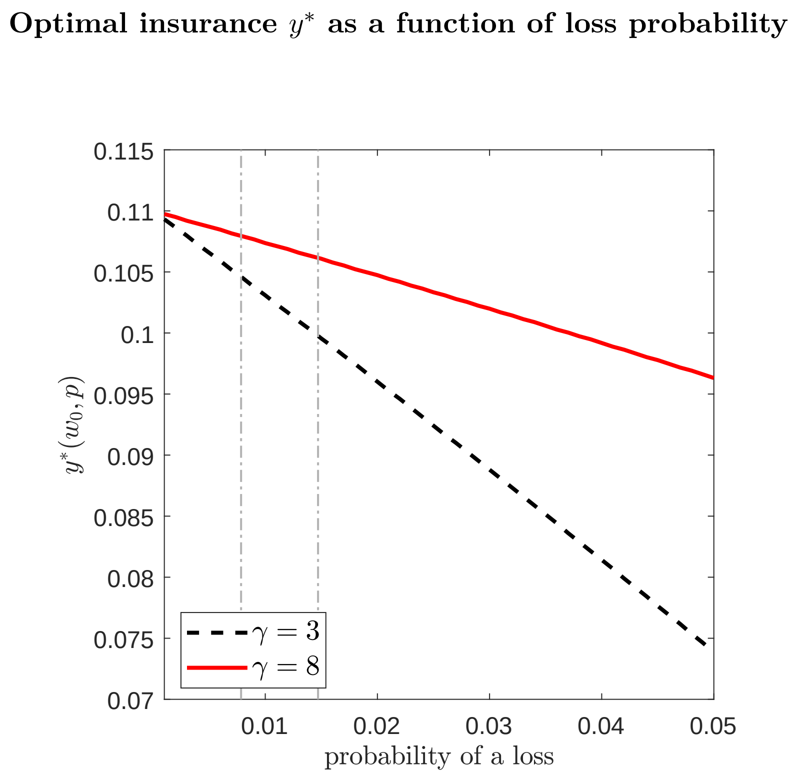

Figure 4.2: The figure gives the optimal investment into insurance as a function of the loss probability for a low risk aversion of $\gamma = 3$ (dashed black line) and a high risk aversion of $\gamma = 8$ (solid red line). The vertical lines mark LPHC-risk and HPLC-risk. The remaining parameters are from Table 4.1.

sizes coincide and the loading factor is the same for both risks, the optimal investment into insurance is thus larger for LPHC-risk than for HPLC-risk (cf. Proposition 5).

In order to explain the puzzle that the observed investment into insurance behaves the other way round, i.e. that there is a higher demand for e.g. mobile phone insurance than for e.g. flood insurance, we rely on mental accounting. With mental accounting, the decision-maker does not simultaneously decide on all insurance contracts. S/he rather chooses her/his optimal demand for a particular insurance coverage and treats the other risk as (supposedly uninsurable) background risk.

The difference between this case – in which the demand for HPLC insurance can exceed the demand for LPHC insurance – and the benchmark case considered before is the treatment of the other risk, which constitutes background risk now and was completely ignored in the benchmark case. Stated differently, the crucial point is the combination of mental accounting and background risk, but not the assumption that background risk can not be insured at all. We could also prespecify some y_b different from zero without changing our main result.

We are interested in the impact of LPHC- and HPLC background risks (p_b, L_b) on the insurance decision for the (p, L)-risk. We assume that $p \cdot L = c$ and $p_b \cdot L_p = c_b$, i.e. we fix the expected losses of the insurable risk and the background risk. The loading factor is the same for all insurable risks. Finally, we also assume that the insurable risk and the background risk are independent of each other.

The expected utility of the decision-maker is given by

$$E\left[u\left(W\right)\right] = E\left[u\left(w_0 - 1_b \cdot l(0, p_b) - y - 1_L \cdot l(y, p)\right)\right]$$

where 1_L and 1_b are the indicator functions for the insurable loss event and the uninsurable background loss event, respectively. The optimal investment into insurance $y(p, p_b, w_0)$ solves

$$y(p, p_b, w_0) = \arg\max_y E\left[u\left(w_0 - 1_b \cdot l(0, p_b) - y - 1_L \cdot l(y, p)\right)\right].$$

The uninsurable background risk (p_b, L_b) can be decomposed into a reduction of the initial wealth and a mean-zero background risk. With this decomposition, the initial wealth drops by the expected loss size $p_b \cdot L_b = c_b$ from w_0 to $w_0 - c_b$, and the mean-zero background risk is given by a loss of size $(1 - p_b)L_b$ with probability p_b and a gain of size $p_b L_b$ with probability $1 - p_b$.

4.3.2 Dependence of optimal investment into insurance on loss probabilities and initial wealth

In the basic model, the optimal investment into insurance is decreasing in wealth for decreasing absolute risk aversion, (cf. Section (2.1)). This results in the following bounds on the optimal investment into insurance.

Proposition 6 (Optimal insurance with background risk: Bounds) *The optimal investment $y(p, p_b, w_0)$ into insurance with an uninsurable background risk (p_b, L_b) solves the optimization problem*

$$\max_y E\left[u\left(w_0 - 1_b \cdot l(0, p_b) - y - 1_L \cdot l(y, p)\right)\right].$$

For DARA, it holds that

1. $y(p, p_b, w_0) > y(p, w_0)$

2. $y(p, p_b, w_0) > y(p, w_0 - p_b l(0, p_b))$

3. $y(p, p_b, w_0) < y(p, w_0 - l(0, p_b))$

Proof: The proof is given in Appendix B.3.1.

The first inequality states that, compared to the optimal insurance demand without background risk $y(p, w_0)$, the presence of background risk implies a larger optimal investment into insurance. This is due to (i) the lower expected initial wealth (which drops from w_0 to $w_0 - p_b \cdot l(0, p_b)$) and (ii) the additional variance of wealth. According to the second inequality, the optimal insurance demand also exceeds the optimal demand in the case where we account for the reduction of initial wealth by the expected loss size, but ignore the presence of mean-zero background risk. Finally, the optimal investment in case of a deterministic reduction of initial wealth by $l(0, p_b)$ is an upper bound for the investment into insurance (third inequality).

The next proposition gives the dependence of the optimal investment on the initial amount of wealth, the loss probability of the insurable event, and the loss probability of the background loss.

Proposition 7 *For the optimal investment $y(p, p_b, w_0)$ in case of background risk, it holds that*

1. $y(p, p_b, w_0)$ *is a decreasing function of wealth for DARA*

2. $y(p, p_b, w_0)$ *is a decreasing function of the loss probability p for DARA*

3. $y(p, p_b, w_0)$ *is a decreasing function of the background loss probability p_b when relative risk aversion is constant (and equal to γ) and $L_b > \frac{w_0 - y - l(y,p) + \gamma c_b}{1+\gamma}$*

Proof: The proof is given in Appendix B.3.3.

The first two results are inherited from the basic insurance model. With DARA, the decision-maker is willing to take a higher risk if her/his wealth increases. Thus, s/he is willing to hold on to a larger part of the risk $\left(p, \frac{c}{p}\right)$ and thereby reduces her/his investment into insurance.

The dependence of the insurance demand on the loss probability p reflects that the decision-maker is more afraid of large but rare losses than of small but frequent losses. It also holds in the case with background risk that the optimal investment into insurance is decreasing in the probability and increasing in the potential size of the insured loss.

The dependence on the background loss probability p_b is more involved. With background risk, the decision-maker suffers from the expected reduction in her/his initial wealth and from the additional riskiness of her/his terminal wealth. The first effect is the same for LPHC background risks and HPLC background risks, since we fix the expected loss due to background risk at c_b. However, LPHC-risks are again more severe for the decision-maker than HPLC-risks, so that the presence of a disastrous LPHC background risk increases the optimal insurance by more than the presence of a less severe HPLC background risk.[13]

4.3.3 Comparison of HPLC and LPHC if the other risk is background risk

We now compare the optimal insurance demand for LPHC- and HPLC-risks. We fix a low probability p_l and a high probability p_h, and we assume that $p_l L_l = p_h L_h = c$, i.e. we equate the expected losses. We then consider the following two situations where we switch the roles of the two risks from background risk to insurable risk and vice versa:

1. The LPHC-risk is insurable, while the HPLC-risk is the uninsurable background risk. The optimal amount of insurance is $y(p_l, p_h, w_0)$.
2. The HPLC-risk is insurable, while the LPHC-risk is the uninsurable background risk. The optimal amount of insurance is $y(p_h, p_l, w_0)$.

Consider the switch from the second case (LPHC is background risk) to the first case (LPHC is insurable). For the insurable risk, the loss probability decreases from p_h to p_l. When the decision-maker faces rare but large losses instead of the HPLC-risk, s/he increases her/his optimal investment into insurance. At the same time, the probability of the background risk increases from p_l to p_h. Background risk, which is now constituted of small but frequent losses, has become less severe, and the optimal investment into insurance drops. Overall, there are thus two opposing effects on the optimal investment into insurance. The optimal insurance demand for the LPHC-risk with HPLC background risk may thus be higher than the optimal insurance demand for the HPLC-risk in the

[13] In our numerical examples, the optimal investment into insurance is decreasing in p_b over the whole range.

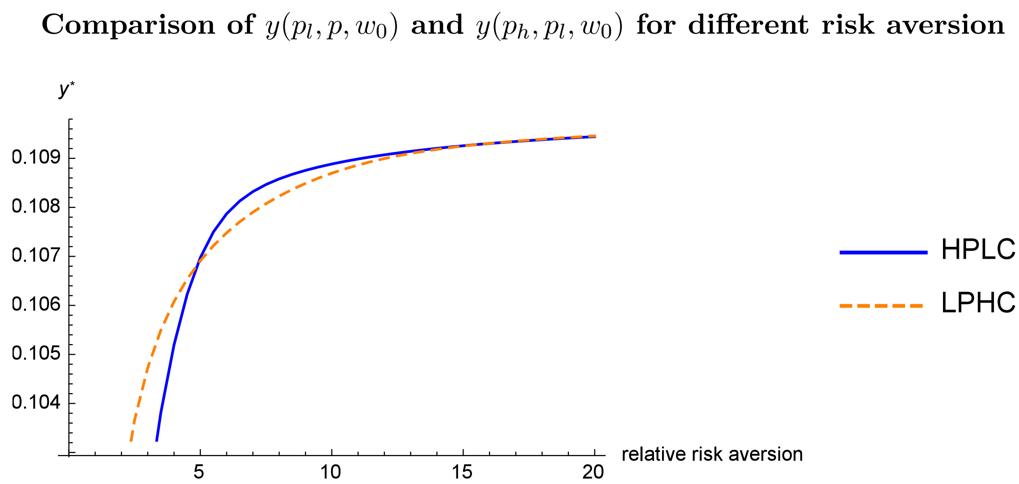

Figure 4.3: The figure gives the optimal investment into insurance as a function of the relative risk aversion. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 4.1.

switched case (if the more severe insurable risk dominates) or vice versa (if the less severe background risk dominates).

In most cases, the optimal investment into insurance is larger when rare but large losses (LPHC) can be insured than in the opposite case where small but frequent losses (HPLC) can be insured, i.e. it holds that $y(p_l, p_h, w_0) > y(p_h, p_l, w_0)$. There are, however, also cases where the optimal investment into insurance drops when the insurable risk switches from HPLC to LPHC.

To get the intuition, we consider a numerical example. The basic parameters for the following figures are given in Table 4.1. The utility function belongs to the class of utility functions with CRRA implying DARA. The constant level of relative risk aversion is denoted by γ.

Figure 4.3 depicts the optimal amount of insurance as a function of the relative risk aversion. It shows that the decision-maker chooses a higher insurance for large but rare losses (the standard result) if s/he has a small relative risk aversion. If her/his relative risk aversion increases, however, the result changes, and s/he now buys more insurance

Comparison of $y(p_l, p, w_0)$ and $y(p_h, p_l, w_0)$ for different values of p_l

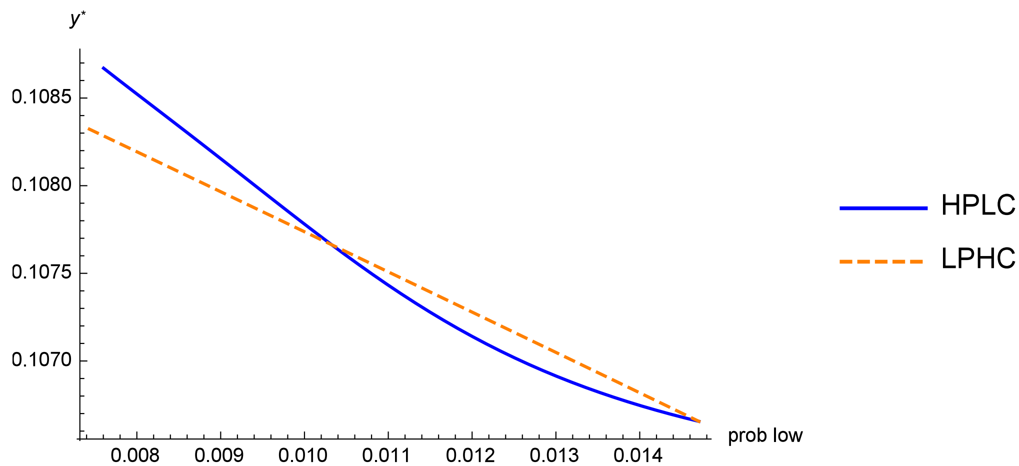

Figure 4.4: The figure gives the optimal investment into insurance as a function of the low probability p_l. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 4.1.

for small but frequent losses. For a high risk aversion, finally, the investment into insurance approaches its maximum for both types of risk, since the decision-maker decides to eliminate the insurable risk nearly completely.

Figure 4.4 gives the optimal amount of insurance as a function of the probability p_l of large but rare losses. Again, the standard result holds if the probability p_l is not too small. The more extreme the LPHC-risk gets, however, i.e. the smaller the loss probability and the larger the loss size, the larger the insurance of the HPLC-risk (with LPHC background risk) as compared to the insurance of the LPHC-risk (with HPLC background risk).

Figure 4.5 gives the optimal amount of insurance as a function of the initial wealth w_0. For high levels of initial wealth, we find the standard result that large but rare losses demand a higher investment into insurance than small but frequent losses. However, the result changes for low levels of initial wealth. Now, the optimal amount of insurance is larger for the LPHC- than for the HPLC-risk.

To get the intuition, we focus on the level of terminal wealth. The lowest value of

Comparison of $y(p_l, p, w_0)$ and $y(p_h, p_l, w_0)$ for different levels of initial wealth

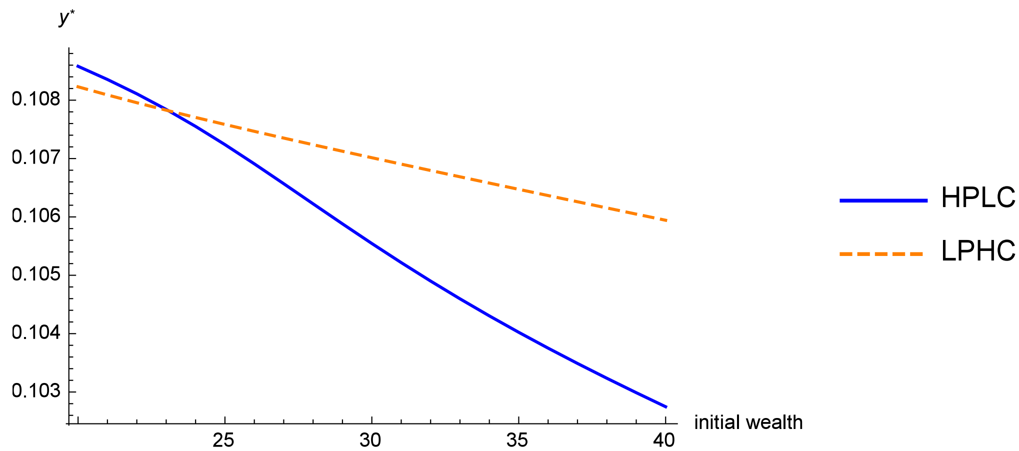

Figure 4.5: The figure gives the optimal investment into insurance as a function of the initial wealth w_0. LPHC (dashed orange line) denotes the case where large but rare losses (LPHC) can be insured, while HPLC is uninsurable background risk. HPLC (solid blue line) denotes the switched case where small but frequent losses (HPLC) can be insured, while LPHC is uninsurable background risk. The remaining parameters are given in Table 4.1.

terminal wealth is realized if both loss events occur simultaneously. Since the decision-maker suffers most from the large losses, s/he is particularly interested in achieving a somehow higher wealth in the worst case. S/he can do so by buying insurance against the risk. If the decision-maker can buy insurance for the LPHC-risk, the increase of wealth in the worst case is larger than when s/he can only buy insurance for the HPLC-risk. Since the loading is the same for both types of risk, this leads to the standard result that s/he invests more to protect herself against the insurable LPHC-risk than against the insurable HPLC-risk (where s/he is still left with the (remaining) uninsurable LPHC-risk).

However, this result can also change. If s/he has a small initial wealth w_0 or if the probability of the LPHC-risk p_l is very small (and the loss size $\frac{c}{p_b}$ thus very large), the worst case terminal wealth is very small and rather close to zero. In this case, the decision-maker desperately wants to improve her/his worst case wealth level. Buying insurance against the HPLC-risk can then become very attractive. Even if the decision-maker still has to cope with the remaining uninsurable risk, the insurance against the HPLC-risk increases the wealth in the worst case at least a bit. If this state is disastrous, the decision-maker would even be willing to accept a higher loading then the given one, and heavily buys HPLC insurance. When s/he buys LPHC insurance instead, a lower investment into insurance is needed to bring the worst case level of wealth to the same level as with HPLC insurance.

In summary, it is thus possible to explain the ”insurance puzzle” posed by a rather low demand for catastrophe insurance compared to the demand for HPLC-risks, e.g. given by mobile phones or bicycle thefts, in a basic insurance model with two independent risk by introducing mental accounting.

4.4 Conclusion

EUT predicts a high insurance demand for LPHC-risks and a smaller demand for HPLC-risks. Contrary to this result, empirical observations and experiments find a rather low insurance demand for LPHC-risks and a high demand for HPLC-risks. To the best of our knowledge, there is yet no theoretical model which explains the anomaly that decision-maker prefer buying insurance for HPLC-risks over buying insurance for LPHC-risks if these risks are priced with the same loading factor. In this paper, we explain this puzzle by the concept of mental accounting in a basic insurance model with two independent risks.

We assume that the decision-maker is confronted with two independent risks, LPHC and HPLC, at the same time. In case of mental accounting, the decision-maker decides about her/his two insurance demand separately while considering the other risk as an uninsurable background risk. S/he thus chooses her/his optimal insurance amount for the LPHC-risk with HPLC-risk as an uninsurable background risk and, in a second optimization, the optimal insurance demand for the HPLC-risk with LPHC-risk as an uninsurable background risk. We find that the optimal investment into insurance for HPLC-risks can exceed the optimal investment into insurance for LPHC-risks, explaining the documented ”insurance puzzle”. This holds true in particular for small initial wealth levels and rare risks with a high loss size.

Chapter 5

General conclusion

This thesis includes topics in the context of the optimal insurance demand. Especially, in combination with the optimal saving decision and the difference between LPHC- and HPLC-risks. All contributions should help to better understand the decisions regarding the insurance demand.

We look at the optimal saving and insurance decision by using generalized mean-variance preferences. This allows us to split up the overall risk preferences of the decision-maker into time and risk preferences which helps us to dig deeper in the topic of optimal insurance. We find out that the optimal insurance demand depends on risk preferences only while the optimal saving decision depends on both: risk and time preferences. Accordingly, larger aversion against within-period variance (risk preference) reduces the optimal saving. An increasing aversion against within-period variance also induces that saving and insurance are substitutes. Thus, for the case that the insurance demand increases saving drops. Furthermore, the impact of aversion against between-period variance (time preference) depends on the risk-free rate. For a large (small) risk free rate, saving is attractive (unattractive), and a higher aversion against between-period variance decreases (increases) the optimal saving and brings it closer to zero. As a result, the aversion against between-period variance changes the saving only without having an impact on insurance. We also analyze the impact of the other model and risk parameters on the optimal saving and insurance demand. The closed form solutions for the optimal insurance demand allow interesting insights about an ambiguous interaction of demand and price effects. In contrast of saving and insurance being substitutes, we find that there are also parameter regions in which saving and insurance both increase or both decreases in a parameter.

The so called ”insurance puzzle” implies that the optimal insurance demand for LPHC- versus HPLC-risks can deviate looking at theoretical and empirical work. Therefore, we review the literature and find out that still most of the studies reveal a higher insurance demand for LPHC-risks. Nevertheless, the one paper using real data from an German insurer detects the opposite: the optimal insurance demand is higher for HPLC-risks.

For this reason, we explain this phenomena within a theoretical model using the basic insurance model as a starting point. While the basic insurance model reveals a higher optimal insurance demand for LPHC-risks we are able to show that it can be the other way around by extending the model with the concepts of mental accounting and background risk. We find out that the optimal insurance demand for HPLC-risks exceeds the one for LPHC-risks for small initial wealth levels and rare risks with a high loss size. Consequently, we are able to explain the ”insurance puzzle”.

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Appendix A

Appendix to Chapter 2

A.1 Proof of Proposition 1: Optimal saving and insurance

The first order conditions for y and s are

$$\mu_y(y,s) - \lambda_b \sigma^2_{b,y}(y,s) - \lambda_w \sigma^2_{w,y}(y) = 0 \tag{A.1}$$

$$\mu_s(y,s) - \lambda_b \sigma^2_{b,s}(y,s) = 0 \tag{A.2}$$

where the (second) subscripts denote partial derivatives. The first order derivatives of the average overall wealth $\mu(y,s) = q_0 \hat{w}_0 + (1-q_0)\big[p w_1^{(L)} + (1-p) w_1^{(NL)}\big]$ are

$$\mu_y(y,s) = -q_0 - (1-q_0) p l'(y) \tag{A.3}$$

$$\mu_s(y,s) = -q_0 + (1-q_0)(1+r). \tag{A.4}$$

The first order derivative of within period variance $\sigma^2_w(y)$ in (2.9) are

$$\sigma^2_{w,y}(y) = 2(1-q_0)p(1-p)l(y)l'(y). \tag{A.5}$$

and the first order derivatives of between period variance $\sigma^2_b(y,s)$ in (2.10) are

$$\sigma^2_{b,y}(y,s) = 2q_0(1-q_0)(\mu_1 - \hat{w}_0)(-pl'(y)+1) \tag{A.6}$$

$$\sigma^2_{b,s}(y,s) = 2q_0(1-q_0)(\mu_1 - \hat{w}_0)(2+r). \tag{A.7}$$

Plugging (A.4) and (A.7) into the first order condition (A.2) gives

$$-q_0 + (1-q_0)(1+r) \quad = \quad 2\lambda_b q_0 (1-q_0)(\mu_1 - \hat{w}_0)(2+r)$$

which implies

$$\mu_1 - \hat{w}_0 \quad = \quad \frac{(1-q_0)(1+r) - q_0}{2\lambda_b q_0(1-q_0)(2+r)} =: t^*$$

With $q_0 = \frac{1}{1+\beta}$, we get

$$t^* = \frac{\beta(1+r) - 1}{2\lambda_b \frac{\beta}{1+\beta}(2+r)} \tag{A.8}$$

Plugging (A.3), (A.5) and (A.6) into the first order condition (A.1) gives

$$\begin{aligned} -q_0 - (1-q_0)pl'(y) \quad &= \quad 2\lambda_b q_0(1-q_0)(\mu_1 - \hat{w}_0)(-p\, l'(y) + 1) \\ &\quad + 2\lambda_w(1-q_0)p(1-p)l(y)l'(y) \end{aligned}$$

With the expression for the expected change in wealth t^*, we get

$$\begin{aligned} -q_0 - (1-q_0)pl'(y) \quad &= \quad \frac{(1-q_0)(1+r) - q_0}{2+r}(-p\, l'(y) + 1) \\ &\quad + 2\lambda_w(1-q_0)p(1-p)l(y)l'(y). \end{aligned}$$

Sorting terms gives

$$\begin{aligned} 2\lambda_w(1-q_0)p(1-p)l'(y)l(y) \quad &= \quad -q_0 - (1-q_0)pl'(y) + \frac{(1-q_0)(1+r) - q_0}{2+r}(p\, l'(y) - 1) \\ &= \quad -\frac{1+r}{2+r} - \frac{1}{2+r}\, p\, l'(y) \end{aligned}$$

Solving for $l(y)$ and using that $l'(y) = -\frac{1+r}{p(1+\delta)}$ then implies

$$l^* = \frac{\delta}{2\lambda_w \frac{\beta}{1+\beta}\,(1-p)(2+r)}$$

Equation (2.12) for y^* then follows by plugging l^* into the pricing formula for the insurance. The optimal savings s^* in Equation (2.13) follow from $\mu_1 - \hat{w}_0 = t^*$ and plugging in the expressions for expected future wealth μ_1 and current wealth $\hat{w}_0$.

A.2 Partial derivatives

For the following partial derivatives we need the optimal loss size l^* given by

$$l^* = \frac{\delta(1 + \frac{1}{\beta})}{2\lambda_w(1-p)(2+r)},$$

the optimal insurance demand y^* given by

$$y^* = \frac{p(1+\delta)(L-l^*)}{1+r},$$

the optimal difference between expected wealth at $t=1$ and wealth today given by

$$t^* = \frac{[\beta(1+r)-1]\,(1+\frac{1}{\beta})}{2\lambda_b(2+r)},$$

and the optimal savings s^* given by

$$s^* = \frac{(w_0-y^*)-[w_1-p\,l(y^*)]}{2+r} + \frac{t^*}{2+r}.$$

A.2.1 Partial derivatives w.r.t. risk preferences

The sensitivities of the optimal loss size with respect to the preference parameters are

$$\frac{\partial l^*}{\partial \lambda_w} = -\frac{l^*}{\lambda_w} = -\frac{\delta(1+\frac{1}{\beta})}{2\lambda_w^2(1-p)(2+r)} \leq 0, \quad \frac{\partial l^*}{\partial \lambda_b} = 0.$$

The sensitivities of the optimal insurance demand with respect to the preference parameters are

$$\begin{aligned}
\frac{\partial y^*}{\partial \lambda_w} &= -\frac{p(1+\delta)}{1+r}\cdot\frac{\partial l^*}{\partial \lambda_w} = \frac{p(1+\delta)}{1+r}\cdot\frac{l^*}{\lambda_w} = \frac{p\,(1+\delta)}{\lambda_w(1+r)}l^* \geq 0,\\
\frac{\partial y^*}{\partial \lambda_b} &= -\frac{p(1+\delta)}{1+r}\cdot\frac{\partial l^*}{\partial \lambda_b} = 0.
\end{aligned}$$

The sensitivities of the optimal growth with respect to the preference parameters are

$$\frac{\partial t^*}{\partial \lambda_w} = 0, \quad \frac{\partial t^*}{\partial \lambda_b} = -\frac{t^*}{\lambda_b} = -\frac{\beta(1+r)-1}{2\lambda_b^2(2+r)\frac{\beta}{1+\beta}}.$$

The sensitivities of the optimal savings with respect to the preference parameters are

$$\begin{aligned}
\frac{\partial s^*}{\partial \lambda_w} &= \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial \lambda_w} + p\,\frac{\partial l^*}{\partial \lambda_w} + \frac{\partial t^*}{\partial \lambda_w}\right] = -\frac{p(2+\delta+r)}{\lambda_w(1+r)(2+r)}\,l^* \leq 0,\\
\frac{\partial s^*}{\partial \lambda_b} &= \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial \lambda_b} + p\,\frac{\partial l^*}{\partial \lambda_b} + \frac{\partial t^*}{\partial \lambda_b}\right] = -\frac{\beta(1+r)-1}{2\lambda_b^2(2+r)^2\frac{\beta}{1+\beta}}.
\end{aligned}$$

A.2.2 Partial derivatives w.r.t. income

The sensitivities of the optimal loss size with respect to the initial wealth levels are

$$\frac{\partial l^*}{\partial w_0} = 0, \ \frac{\partial l^*}{\partial w_1} = 0.$$

The sensitivities of the optimal insurance demand with respect to the initial wealth levels are

$$\begin{aligned}\frac{\partial y^*}{\partial w_0} &= -\frac{p(1+\delta)}{1+r} \cdot \frac{\partial l^*}{\partial w_0} = 0, \\ \frac{\partial y^*}{\partial w_1} &= -\frac{p(1+\delta)}{1+r} \cdot \frac{\partial l^*}{\partial w_1} = 0.\end{aligned}$$

The sensitivities of the optimal growth with respect to the initial wealth levels are

$$\frac{\partial t^*}{\partial w_0} = 0, \quad \frac{\partial t^*}{\partial w_1} = 0.$$

The sensitivities of the optimal savings with respect to the initial wealth levels are

$$\frac{\partial s^*}{\partial w_0} = \frac{1}{2+r} \geq 0, \quad \frac{\partial s^*}{\partial w_1} = -\frac{1}{2+r} \leq 0.$$

A.2.3 Partial derivatives w.r.t. pricing parameters

The sensitivities of the optimal loss size with respect to the risk-free rate and the insurance loading are

$$\begin{aligned}\frac{\partial l^*}{\partial r} &= -\frac{l^*}{2+r} = -\frac{\delta(1+\frac{1}{\beta})}{2\lambda_w(1-p)(2+r)^2} \leq 0, \\ \frac{\partial l^*}{\partial \delta} &= \frac{l^*}{\delta} = \frac{(1+\frac{1}{\beta})}{2\lambda_w(1-p)(2+r)} \geq 0.\end{aligned}$$

The sensitivities of the optimal insurance demand with respect to the risk-free rate and the insurance loading are

$$\begin{aligned}\frac{\partial y^*}{\partial r} &= -\frac{p(1+\delta)}{(1+r)^2}(L-l^*) - \frac{p(1+\delta)}{1+r} \cdot \frac{\partial l^*}{\partial r} = -\frac{p(1+\delta)}{(1+r)^2}\left[L - \frac{3+2r}{2+r}\, l^*\right], \\ \frac{\partial y^*}{\partial \delta} &= \frac{p}{1+r}(L-l^*) - \frac{p(1+\delta)}{1+r} \cdot \frac{\partial l^*}{\partial \delta} = \frac{p}{1+r}\left[L - \frac{1+2\delta}{\delta}\, l^*\right].\end{aligned}$$

The sensitivities of the optimal growth with respect to the risk-free rate and the insurance loading are

$$\frac{\partial t^*}{\partial r} = \frac{2+\beta+\frac{1}{\beta}}{2\lambda_b(2+r)^2} \geq 0, \quad \frac{\partial t^*}{\partial \delta} = 0.$$

The sensitivities of the optimal savings with respect to the risk-free rate and the insurance loading are

$$\begin{aligned}
\frac{\partial s^*}{\partial r} &= -\frac{s^*}{2+r} + \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial r} + p\,\frac{\partial l^*}{\partial r} + \frac{\partial t^*}{\partial r}\right] \\
&= \frac{1}{(2+r)^2}\left[w_1 - w_0 + \frac{p(1+\delta)(3+2r)}{(1+r)^2}L - \frac{p\left[(1+r)(3+\delta+2r)+(1+\delta)(3+2r)\right]}{(1+r)^2}\,l^* + \frac{(2-\beta r)\left(1+\frac{1}{\beta}\right)}{2\lambda_b(2+r)}\right], \\
\frac{\partial s^*}{\partial \delta} &= \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial \delta} + p\,\frac{\partial l^*}{\partial \delta} + \frac{\partial t^*}{\partial \delta}\right] = \frac{p}{(1+r)(2+r)}\left[-L + \frac{2+2\delta+r}{\delta}\,l^*\right].
\end{aligned}$$

A.2.4 Partial derivatives w.r.t. risk of losses

The sensitivities of the optimal loss with respect to the loss size and the loss probability are

$$\frac{\partial l^*}{\partial p} = \frac{l^*}{1-p} = \frac{\delta\left(1+\frac{1}{\beta}\right)}{2\lambda_w(1-p)^2(2+r)} \geq 0, \quad \frac{\partial l^*}{\partial L} = 0.$$

The sensitivities of the optimal insurance demand with respect to the loss size and the loss probability are

$$\begin{aligned}
\frac{\partial y^*}{\partial p} &= \frac{1+\delta}{1+r}\,(L-l^*) - \frac{p(1+\delta)}{1+r}\cdot\frac{\partial l^*}{\partial p} = \frac{1+\delta}{1+r}\left[L - \frac{1}{1-p}\,l^*\right], \\
\frac{\partial y^*}{\partial L} &= \frac{p(1+\delta)}{1+r} \geq 0.
\end{aligned}$$

The sensitivities of the optimal growth with respect to the loss size and the loss probability are

$$\frac{\partial t^*}{\partial L} = 0, \quad \frac{\partial t^*}{\partial p} = 0.$$

The sensitivities of the optimal savings with respect to the loss size and the loss probability are

$$\begin{aligned}
\frac{\partial s^*}{\partial p} &= \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial p} + p\,\frac{\partial l^*}{\partial p} + l^* + \frac{\partial t^*}{\partial p}\right] = \frac{1+\delta}{(1+r)(2+r)}\left[-L + \frac{2+\delta+r}{(1-p)(1+\delta)}\,l^*\right], \\
\frac{\partial s^*}{\partial L} &= \frac{1}{2+r}\left[-\frac{\partial y^*}{\partial L} + p\,\frac{\partial l^*}{\partial L} + \frac{\partial t^*}{\partial L}\right] = -\frac{p(1+\delta)}{(1+r)(2+r)} \leq 0.
\end{aligned}$$

Appendix B

Appendix to Chapter 4

B.1 Basic insurance model with one insurable risk

B.1.1 Restriction $(1+\delta)p < 1$

The terminal wealth in the static insurance model is

$$w = \begin{cases} w_0 - y & \text{no loss} \\ w_0 - y - l(y,p) & \text{loss} \end{cases}$$

where the loss size and the amount paid for insurance are given by

$$l(y,p) = L - \frac{y}{(1+\delta)p} \qquad y = (1+\delta)p\,(L - l(y,p))\,.$$

The loss size $l(y,p)$ is positive for

$$y \leq (1+\delta)p\,L =: y_{max}.$$

We assume $\delta \geq 0$ and

$$(1+\delta)p < 1.$$

To see the reason for the latter restriction, consider the wealth as a function of $l(y,p)$:

$$w = \begin{cases} w_0 - (1+\delta)p\,L + (1+\delta)p\,l(y,p) & \text{no loss} \\ w_0 - (1+\delta)p\,L + [(1+\delta)p - 1]\,l(y,p) & \text{loss} \end{cases}$$

If $(1+\delta)p > 1$, terminal wealth increases in $l(y,p)$ in both states. It would then be optimal for the investor to choose an infinite loss size. This choice would also be optimal for $(1+\delta)p = 1$. For a finite optimal loss size, we thus need $(1+\delta)p < 1$.

B.1.2 Proof of Proposition 4: Optimal insurance

The expected utility is

$$E[u(W)] = p\,u(w_0 - y - l(y,p)) + (1-p)u(w_0 - y).$$

The investor wants to choose the optimal investment into insurance, i.e. the optimal $y^*(p, w_0)$. The first order condition is

$$\frac{\partial E[u(W)]}{\partial y} = 0.$$

This can be rewritten as

$$p\,u'(w_0 - y - l(y,p))\left(-1 + \frac{1}{(1+\delta)p}\right) + (1-p)u'(w_0 - y)(-1) = 0$$

$$\frac{u'(w_0 - y - l(y,p))}{u'(w_0 - y)} = \frac{1-p}{\frac{1}{1+\delta} - p}.$$

B.2 Basic insurance model with one insurable risk: Optimal insurance $y(p, w_0)$

The expected utility of the insuree is

$$E\left[u\left(w_0 - y - 1_L l(y,p)\right)\right] = p \cdot u\left(w_0 - y - l(y,p)\right) + (1-p) \cdot u\left(w_0 - y\right),$$

where 1_L is the indicator event for the loss to be insured, $L = \frac{c}{p}$ is the size of the insurable loss, and the loss size with insurance is given by

$$l(y,p) = L - \frac{y}{(1+\delta)p}.$$

The decision maker has to choose the investment into insurance y. The optimality condition is

$$g(y, p, w_0) = 0.$$

where

$$g(y, p, w_0) = \left(\frac{1}{1+\delta} - p\right) u'\left(w_0 - y - l(y,p)\right) - (1-p) \cdot u'\left(w_0 - y\right).$$

We want to determine the dependence of the optimal $y(p, w_0)$ on the loss probability and on the initial wealth. To do so, we start from the optimality condition. Taking the derivative of both sides w.r.t. the loss probability p gives

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial y} \cdot \frac{\partial y(p,w_0)}{\partial p} + \frac{\partial g(y(p,w_0),p,w_0)}{\partial p} = 0.$$

This equation can be solved for the partial derivative of y w.r.t. p:

$$\frac{\partial y(p,w_0)}{\partial p} = -\frac{\frac{\partial g(y(p,w_0),p,w_0)}{\partial p}}{\frac{\partial g(y(p,w_0),p,w_0)}{\partial y}}.$$

Similarly, one gets

$$\frac{\partial y(p,w_0)}{\partial w_0} = -\frac{\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0}}{\frac{\partial g(y(p,w_0),p,w_0)}{\partial y}}.$$

To determine these expressions, we need the partial derivatives of g w.r.t. its arguments y, p, and w_0, which we then evaluate for $y = y(p, w_0)$.

B.2.1 Partial derivatives of g

Partial derivative of g w.r.t. y

The partial derivative of g w.r.t. y is given by

$$\frac{\partial g(y,p,w_0)}{\partial y} = E\left[u''\left(w_0 - y - 1_L l(y,p)\right)\left(-1 + 1_L \frac{1}{(1+\delta)p}\right)^2\right].$$

It thus holds that

$$\frac{\partial g(y,p,w_0)}{\partial y} < 0.$$

Partial derivative of g w.r.t. p

The partial derivative of g w.r.t. p is given by

$$
\begin{aligned}
\frac{\partial g(y,p,w_0)}{\partial p} =& -u'\left(w_0-y-l(y,p)\right)+\left(\frac{1}{1+\delta}-p\right)u''\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}+u'\left(w_0-y\right)\\
=& -u'\left(w_0-y-l(y,p)\right)+u'\left(w_0-y\right)\\
&-\left(\frac{1}{1+\delta}-p\right)u'\left(w_0-y-l(y,p)\right)ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\\
=&\, u'\left(w_0-y-l(y,p)\right)\left[-1-\left(\frac{1}{1+\delta}-p\right)ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]\\
&+u'\left(w_0-y\right)\\
=&\left[-1-\left(\frac{1}{1+\delta}-p\right)ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]\\
&\times\frac{1}{\frac{1}{1+\delta}-p}\left[g(y,p,w_0)+(1-p)u'\left(w_0-y\right)\right]+u'\left(w_0-y\right)\\
=&\left[-\frac{1}{\frac{1}{1+\delta}-p}-ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]g(y,p,w_0)\\
&+\left[1-\frac{1-p}{\frac{1}{1+\delta}-p}-(1-p)ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]u'\left(w_0-y\right)\\
=&\left[-\frac{1}{\frac{1}{1+\delta}-p}-ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]g(y,p,w_0)\\
&+\left[-\frac{\delta}{1-(1+\delta)p}-(1-p)ARA\left(w_0-y-l(y,p)\right)\frac{l(y,p)}{p}\right]u'\left(w_0-y\right).
\end{aligned}
$$

For $y=y(p,w_0)$, the optimality condition $g(y(p,w_0),p,w_0)=0$ holds true. The partial derivative simplifies to

$$
\frac{\partial g(y(p,w_0),p,w_0)}{\partial p}=u'(w_0-y)\left[-\frac{\delta}{1-(1+\delta)p}-\frac{1-p}{p}ARA\left(w_0-y-l(y,p)\right)l(y,p)\right].
$$

It thus holds that

$$
\frac{\partial g(y(p,w_0),p,w_0)}{\partial p}<0.
$$

Partial derivative of g w.r.t. w_0

The partial derivative of g w.r.t. w_0 is given by

$$\begin{aligned}\frac{\partial g(y,p,w_0)}{\partial w_0} &= \left(\frac{1}{1+\delta} - p\right) u''\left(w_0 - y - l(y,p)\right) - (1-p)\cdot u''\left(w_0 - y\right)\\ &= -\left(\frac{1}{1+\delta} - p\right) u'\left(w_0 - y - l(y,p)\right) ARA\left(w_0 - y - l(y,p)\right)\\ &\quad + (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right)\\ &= -\left[g(y,p,w_0) + (1-p)\cdot u'\left(w_0-y\right)\right] ARA\left(w_0 - y - l(y,p)\right)\\ &\quad + (1-p)\cdot u'\left(w_0 - y\right) ARA\left(w_0 - y\right)\\ &= -\, g(y,p,w_0) ARA\left(w_0 - y - l(y,p)\right)\\ &\quad + (1-p)\cdot u'\left(w_0 - y\right)\left[ARA\left(w_0 - y\right) - ARA\left(w_0 - y - l(y,p)\right)\right].\end{aligned}$$

For $y = y(p, w_0)$, the optimality condition $g(y(p,w_0),p,w_0) = 0$ holds true. The partial derivative simplifies to

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0} = (1-p)\cdot u'\left(w_0 - y\right)\left[ARA\left(w_0 - y\right) - ARA\left(w_0 - y - l(y,p)\right)\right].$$

The sign of the partial derivative depends on the absolute risk aversion. It holds that

$$\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0}\begin{cases} < 0 & \text{decreasing ARA}\\ = 0 & \text{constant ARA}\\ > 0 & \text{increasing ARA}\end{cases}$$

B.2.2 Dependence of $y(p, w_0)$ on p and w_0

For the partial derivative of $y(p, w_0)$ w.r.t. the initial wealth w_0, it holds that

$$\frac{\partial y(p,w_0)}{\partial w_0} = -\frac{\frac{\partial g(y(p,w_0),p,w_0)}{\partial w_0}}{\frac{\partial g(y(p,w_0),p,w_0)}{\partial y}}.$$

This gives

$$\frac{\partial y(p,w_0)}{\partial w_0}\begin{cases} < 0 & \text{decreasing ARA}\\ = 0 & \text{constant ARA}\\ > 0 & \text{increasing ARA}\end{cases}$$

For the partial derivative of $y(p, w_0)$ w.r.t. the loss probability p, it holds that

$$\frac{\partial y(p,w_0)}{\partial p} = -\frac{\frac{\partial g(y(p,w_0),p,w_0)}{\partial p}}{\frac{\partial g(y(p,w_0),p,w_0)}{\partial y}}.$$

Both the nominator and the denominator of the fraction are negative, so we get that

$$\frac{\partial y(p, w_0)}{\partial p} < 0.$$

The optimal insurance is thus a decreasing function of the loss probability. In particular, it is larger for LPHC than for HPLC.

B.3 Two risks: HPLC versus LPHC

B.3.1 Bounds on optimal insurance demand

The expected utility of the insuree is given by

$$p_b E\Big[u\Big(w_0 - L_b - y - 1_L l(y,p)\Big)\Big] + (1-p_b)E\Big[u\Big(w_0 - y - 1_L l(y,p)\Big)\Big],$$

where 1_L is the indicator event for the loss to be insured. The first order condition for the investment into insurance y is

$$h(y, p, p_b, w_0) = p_b g(y, p, w_0 - L_b) + (1 - p_b) g(y, p, w_0) = 0.$$

We assume DARA. For $0 < y < y(p, w_0)$, both $g(y, p, w_0 - L_b)$ and $g(y, p, w_0)$ are positive. For $y(p, w_0) < y < y(p, w_0 - L_b)$, $g(y, p, w_0)$ is negative, while $g(y, p, w_0 - L_b)$ is still positive. For $y(p, w_0 - L_b) < y$, both $g(y, p, w_0 - L_b)$ and $g(y, p, w_0)$ are negative. This implies that

$$y(p, w_0) < y(p, p_b, w_0) < y(p, w_0 - L_b).$$

For the optimal $y(p, p_b, w_0)$, it holds that

$$g(y(p, p_b, w_0), p, w_0) < 0 < g(y(p, p_b, w_0), p, w_0 - L_b)$$

To determine the dependence of $y(p, p_b, w_0)$ on p and p_b, we proceed as in the preceding section. From the optimality condition, we get that

$$\frac{\partial h(y(p, p_b, w_0), p, p_b, w_0)}{\partial y} \cdot \frac{\partial y(p, p_b, w_0)}{\partial p} + \frac{\partial h(y(p, p_b, w), p, w_0)}{\partial p} = 0.$$

Solving for the partial derivative of y w.r.t. the loss probability p gives

$$\frac{\partial y(p, p_b, w_0)}{\partial p} = -\frac{\frac{\partial h(y(p,p_b,w_0),p,p_b,w_0)}{\partial p}}{\frac{\partial h(y(p,p_b,w_0),p,p_b,w_0)}{\partial y}}.$$

Similarly, the partial derivative of y w.r.t. the probability p_b of a background loss is

$$\frac{\partial y(p, p_b, w_0)}{\partial p_b} = -\frac{\frac{\partial h(y(p,p_b,w_0),p,p_b,w_0)}{\partial p_b}}{\frac{\partial h(y(p,p_b,w_0),p,p_b,w_0)}{\partial y}}.$$

B.3.2 Partial derivatives of h

The first order condition for the optimal insurance demand $y(p, p_b, w_0)$ is

$$h(y(p, p_b, w_0), p, p_b, w_0) = 0,$$

where the function h is given by

$$h(y, p, p_b, w_0) = E\left[u'(w_0 - 1_b L_b - y - 1_L l(y, p))\left(-1 + 1_L \frac{1}{(1+\delta)p}\right)\right]$$

and where 1_L and 1_b are the indicators for the loss events of the insurable risk and the background risk, respectively.

Partial derivative of h w.r.t. y

The partial derivative of h w.r.t. y is given by

$$\frac{\partial h(y, p, p_b, w_0)}{\partial y} = E\left[u''(w_0 - 1_b L_b - y - 1_L l(y, p))\left(-1 + 1_L \frac{1}{(1+\delta)p}\right)^2\right].$$

It thus holds that

$$\frac{\partial h(y, p, p_b, w_0)}{\partial y} < 0.$$

Partial derivative of h w.r.t. p

To determine the partial derivative of h w.r.t. p, we rewrite h as

$$\begin{aligned} h(y, p, p_b, w_0) &= p\, E\left[u'(w_0 - 1_b L_b - y - l(y, p))\left(-1 + \frac{1}{(1+\delta)p}\right)\right] \\ &\quad + (1-p)\, E\left[u'(w_0 - 1_b L_b - y)(-1)\right] \\ &= \left(-p + \frac{1}{1+\delta}\right) E\left[u'(w_0 - 1_b L_b - y - l(y, p))\right] - (1-p)\, E\left[u'(w_0 - 1_b L_b - y)\right]. \end{aligned}$$

The partial derivative of h w.r.t. p is then given by

$$\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p}\\
=&E\left[-u'(w_0-1_bL_b-y-l(y,p))\right]+E\left[u'(w_0-1_bL_b-y)\right]\\
&+\left(-p+\frac{1}{1+\delta}\right)E\left[u''(w_0-1_bL_b-y-l(y,p))\frac{l(y,p)}{p}\right]\\
=&E\left[-u'(w_0-1_bL_b-y-l(y,p))\right]\\
&+\frac{1}{1-p}\left[\left(-p+\frac{1}{1+\delta}\right)E\left[u'(w_0-1_bL_b-y-l(y,p))\right]-h(y,p,p_b,w_0)\right]\\
&-\left(-1+\frac{1}{(1+\delta)p}\right)E\left[u'(w_0-1_bL_b-y-l(y,p))ARA(w_0-1_bL_b-y-l(y,p))l(y,p)\right]\\
=&E\left[u'(w_0-1_bL_b-y-l(y,p))\right]\left(-1+\frac{1}{1-p}\left(-p+\frac{1}{1+\delta}\right)\right)-\frac{1}{1-p}h(y,p,p_b,w_0)\\
&-\left(-1+\frac{1}{(1+\delta)p}\right)E\left[u'(w_0-1_bL_b-y-l(y,p))ARA(w_0-1_bL_b-y-l(y,p))l(y,p)\right]\\
=&-\frac{\delta}{(1-p)(1+\delta)}E\left[u'(w_0-1_bL_b-y-l(y,p))\right]-\frac{1}{1-p}h(y,p,p_b,w_0)\\
&-\left(\frac{1}{(1+\delta)p}-1\right)E\left[u'(w_0-1_bL_b-y-l(y,p))ARA(w_0-1_bL_b-y-l(y,p))l(y,p)\right]
\end{aligned}$$

In the optimum, it holds that $h(y(p,p_b,w_0),p,p_b,w_0)=0$, so that

$$\frac{\partial h(y(p,p_b,w),p,p_b,w)}{\partial p}<0.$$

Partial derivative of h w.r.t. p_b

To determine the partial derivative of h w.r.t. p_b, we rewrite h as

$$\begin{aligned}
h(y,p,p_b,w_0)=&\,p_b\,E\left[u'(w_0-L_b-y-1_Ll(y,p))\left(-1+1_L\frac{1}{(1+\delta)p}\right)\right]\\
&+(1-p_b)\,E\left[u'(w_0-y-1_Ll(y,p))\left(-1+1_L\frac{1}{(1+\delta)p}\right)\right]
\end{aligned}$$

where $L_p = \frac{c}{p_b}$. The partial derivative of h w.r.t. p_b is then given by

$$\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p_b}\\
=& E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\right]\\
&- E\left[u'(w_0 - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\right]\\
&+ p_b\, E\left[u''(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\frac{L_b}{p_b}\right]\\
=& E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\right]\\
&- \frac{1}{1-p_b}\left[h(y,p,p_b,w_0) - p_b\, E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\right]\right]\\
&- E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)ARA(w_0 - L_b - y - 1_L l(y,p))L_b\right]\\
=& E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - 1_L l(y,p))L_b\right)\right]\\
&- \frac{1}{1-p_b}h(y,p,p_b,w_0)\\
=& p\, u'(w_0 - L_b - y - l(y,p))\left(-1 + \frac{1}{(1+\delta)p}\right)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b\right)\\
&- (1-p)u'(w_0 - L_b - y)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y)L_b\right)\\
&- \frac{1}{1-p_b}h(y,p,p_b,w_0).
\end{aligned}$$

To determine the sign of the partial derivative in the optimum, first note that $h(y(p,p_b,w_0),p,p_b,w_0) = 0$, so that the last term vanishes. A decreasing absolute risk aversion furthermore implies

$$\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b < \frac{1}{1-p_b} - ARA(w_0 - L_b - y)L_b.$$

This gives

$$\begin{aligned}
&\frac{\partial h(y,p,p_b,w)}{\partial p_b}\\
<& p\, u'(w_0 - L_b - y - l(y,p))\left(-1 + \frac{1}{(1+\delta)p}\right)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b\right)\\
&- (1-p)u'(w_0 - L_b - y)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b\right)\\
=& E\left[u'(w_0 - L_b - y - 1_L l(y,p))\left(-1 + 1_L\frac{1}{(1+\delta)p}\right)\right]\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b\right)\\
=& g(y(p,p_b,w_0),p,w_0 - L_b)\left(\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b\right).
\end{aligned}$$

It holds that $g(y(p, p_b, w_0), p, w_0 - L_b) > 0$. The inequality $\frac{1}{1-p_b} - ARA(w_0 - L_b - y - l(y,p))L_b < 0$ then implies that

$$\frac{\partial h(y, p, p_b, w)}{\partial p_b} < 0.$$

Rearranging the inequality gives the condition $L_b > \frac{w_0 - y - l(y,p) + RRA(w_0 - L_b - y - l(y,p))c_b}{1 + RRA(w_0 - L_b - y - l(y,p))}$, which simplifies to $L_b > \frac{w_0 - y - l(y,p) + \gamma c_b}{1+\gamma}$ in case of a constant relative risk aversion γ.

Partial derivative of h w.r.t. w_0

The partial derivative of h w.r.t. w_0 is given by

$$\begin{aligned}
&\frac{\partial h(y, p, p_b, w_0)}{\partial w_0} \\
=&E\left[u''(w_0 - 1_b L_b - y - 1_L l(y,p))\left(-1 + 1_L \frac{1}{(1+\delta)p}\right)\right] \\
=&E\left[-u'(w_0 - 1_b L_b - y - 1_L l(y,p))\left(-1 + 1_L \frac{1}{(1+\delta)p}\right) ARA(w_0 - 1_b L_b - y - 1_L l(y,p))\right] \\
=&-\left(\frac{1}{1+\delta} - p\right) E\left[u'(w_0 - 1_b L_b - y - l(y,p)) ARA(w_0 - 1_b L_b - y - l(y,p))\right] \\
&+ (1-p)\, E\left[u'(w_0 - 1_b L_b - y) ARA(w_0 - 1_b L_b - y)\right].
\end{aligned}$$

For a decreasing absolute risk aversion, it holds that

$$ARA(w_0 - 1_b L_b - y - l(y,p)) > ARA(w_0 - 1_b L_b - y)$$

which implies

$$
\begin{aligned}
&\frac{\partial h(y,p,p_b,w_0)}{\partial w_0}\\
&< -\left(\frac{1}{1+\delta}-p\right)\,E\left[u'(w_0-1_bL_b-y-l(y,p))ARA(w_0-1_bL_b-y-l(y,p))\right]\\
&\quad + (1-p)\,E\left[u'(w_0-1_bL_b-y)ARA(w_0-1_bL_b-y-l(y,p))\right]\\
&= -\,E\left[u'(w_0-1_bL_b-y-1_Ll(y,p))\left(-1+1_L\frac{1}{(1+\delta)p}\right)ARA(w_0-1_bL_b-y-l(y,p))\right]\\
&= -\,p_bE\left[u'(w_0-L_b-y-1_Ll(y,p))\left(-1+1_L\frac{1}{(1+\delta)p}\right)ARA(w_0-L_b-y-l(y,p))\right]\\
&\quad -(1-p_b)E\left[u'(w_0-y-1_Ll(y,p))\left(-1+1_L\frac{1}{(1+\delta)p}\right)ARA(w_0-y-l(y,p))\right]\\
&= -\,p_bg(y,p,w_0-L_b)ARA(w_0-L_b-y-l(y,p))-(1-p_b)g(y,p,w_0)ARA(w_0-y-l(y,p))\\
&= -\,p_bg(y,p,w_0-L_b)ARA(w_0-L_b-y-l(y,p))-(1-p_b)g(y,p,w_0)ARA(w_0-L_b-y-l(y,p))\\
&\quad +(1-p_b)g(y,p,w_0)ARA(w_0-L_b-y-l(y,p))-(1-p_b)g(y,p,w_0)ARA(w_0-y-l(y,p))\\
&= -\,h(y,p,p_b,w_0)ARA(w_0-L_b-y-l(y,p))\\
&\quad +(1-p_b)g(y,p,w_0)\left[ARA(w_0-L_b-y-l(y,p))-ARA(w_0-y-l(y,p))\right].
\end{aligned}
$$

For the optimal $y = y(p, p_b, w_0)$, it holds that $h(y(p, p_b, w_0), p, p_b, w_0) = 0$, and $g(y(p, p_b, w_0), p, w_0) < 0$. Since the difference of the absolute risk aversion is positive for DARA, we can conclude that

$$\frac{\partial h(y(p,p_b,w),p,p_b,w_0)}{\partial w_0} < 0.$$

B.3.3 Dependence of $y(p, p_b, w_0)$ on p, p_b and w_0

With the signs of the partial derivatives of h in the optimum, we get

$$
\begin{aligned}
&\frac{\partial y(p,p_b,w)}{\partial p} < 0,\\
&\frac{\partial y(p,p_b,w)}{\partial p_b} < 0 \text{ for } L_b > \frac{w_0-y-l(y,p)+\gamma c}{1+\gamma},\\
&\frac{\partial y(p,p_b,w)}{\partial w_0} < 0.
\end{aligned}
$$

In line with intuition, the optimal investment into insurance is thus decreasing in the loss probability of the event to be insured. Furthermore, if the background risk is ”disastrous enough”, the optimal investment into insurance increases when it becomes even more disastrous (the worse the background risk, i.e. the more the investor has to deal with rare but large background losses, the higher the insurance needed for the insurable event).

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