Proceedings of the 14th International Conference on Technology in Mathematics Teaching – ICTMT 14

Essen, Germany, 22nd to 25th of July 2019, University of Duisburg-Essen

ICTMT 14
ESSEN
2019

- *Developing Visions*
- *Enhancing Assessment*
- *Inspiring Learning and Teaching*
- *Networking of Theories*

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ICTMT 14: INSPIRING LEARNING AND TEACHING, NETWORKING OF THEORIES, ENHANCING ASSESSMENT & DEVELOPING VISIONS

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THEMATIC STRUCTURE

The 14th International Conference on Technology in Mathematics Teaching – ICTMT 14 – was organized by the University of Duisburg-Essen. It was held in Essen, Germany, from the 22nd to 25th of July 2019.

The local organizing committee decided to give the conference a thematic structure by identifying four different fields of research and development. For the first three fields (Inspiring Learning and Teaching, Networking of Theories, and Enhancing Assessment), two plenary speakers were invited to present their viewpoints on these central topics. The theme "Developing Visions" provided a large and fruitful open space for developing visions after a stimulating impulse. Hence, the conference especially called for papers regarding the following topics:

Inspiring learning and teaching: The integration of scientific knowledge and research findings into practice faces the specific challenge that people not following this scientific discourse are likely to miss the outcomes in both research and practice. Therefore, there is a need for developing strategies to inspire teaching and learning in accessible ways. We especially ask for research findings that suggest and study examples of the teaching and learning of mathematics with digital technology as well as innovative solutions of scaling and sustaining impacts.

Networking of theories: Research on the use of technology in mathematics education often requires collaborative actions involving different theoretical perspectives. Hence, there is a need to communicate and network different theoretical approaches. Therefore, it is necessary to study theoretical frameworks focusing on integrating technology into teaching and learning to recognise their similarities and differences. As a result, it is possible to connect, compare and contrast a variety of theoretical frameworks in mathematics education.

Enhancing Assessment: Digital technology has the potential to enhance both summative and formative forms of assessment. This conference theme addresses questions concerning the type of skills assessed, the underlying goals, the tasks in use and questions of validity and reliability of assessment.

Developing Visions: With the development of new technologies (for example augmented or airtual aeality), mathematics educators face the challenge of examining whether these technologies support mathematics learning and – if yes – how they can be integrated into the mathematics classroom. Furthermore, there is a need to develop visions of how an integrated technology can be used to support learning in the future. This conference theme especially calls for contributions either supporting the development of such visions or relating to a meaningful integration of new technological developments into the classroom.

This thematic structure continues to organize the conference's proceedings. We chose four different sections in this book which cover the whole range of submissions contributed to the conference.

HEART OF THE CONFERENCE: DISCUSSION & COOPERATION

An important decision was made by providing much space for discussion and cooperation during the conference. Therefore, the plenary sessions were structured quite differently compared to previous conferences. For each conference theme, two keynote speakers were invited to give plenary speeches. After the two plenaries, the participants had time for small group discussions. They were asked to summarize their discussions in keywords via an Audience Response System. Figure 1 shows an example of such a summary which was the product of the group work in regards to the keynotes on the theme 'networking of theories'. The summaries were used to engage participants in a whole-group discussion at the end of each plenary session.

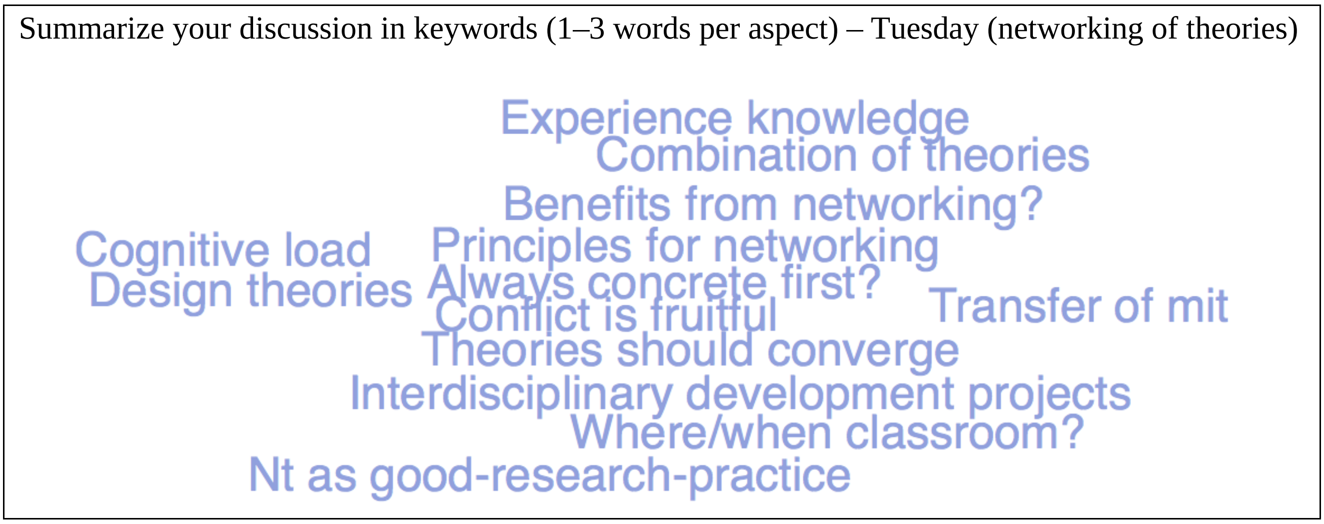

Figure 1: Collected discussion points on the conference theme Networking of Theories

Overall, this approach to incorporate discussion and cooperation during the plenary sessions allowed participants to explore our main conference themes from different viewpoints and in regards to various educational, social and national backgrounds. Emerging discussion points and ideas were then continued in other sessions (e. g., paper or poster presentations) during ICTMT 14.

This type of an open and cooperative approach to the conference's plenary sessions became most clear in regards to the theme of 'developing visions'. In this session, conference participants were provided with a longer time period to discuss future visions for the use of technology in mathematics education. A summary of the session's outcome can be found in these proceedings (Ruchniewicz, Göbel, & Pohl in these proceedings).

AN EYE INTO THE FUTURE – A FORUM FOR INNOVATIVE TECHNOLOGY IN THE MATHEMATICS CLASSROOM

As a new element, the ICTMT 14 offered an innovative forum for a conversation between researchers, teachers and emerging technology developersfocusing on aspects of mathematics education. By bringing these different communities together, the local organizing team aimed to:

- provide a space for teachers and researchers to see and try out emerging technology,
- enable technology developers to engage in constructive discussions about their technology and its implementation,

- develop opportunities for cooperative and collaborative future research pilots and development of new projects.

The forum included a formal presentation of each company and a subsequent open forum in order for the companies to get the chance to showcase their product at an exhibition stand.

EVALUATION

The evaluation of the conference highlighted the spirit of the ICTMT conference series. The following contributions of participants gives impression to this spirit of exchange and collaborating:

- *The conference was small - but HIGHLY focused!*
- *It was a great experience. This is for me the first time at an international conference that there is time to go into deep details in the presentation and still enough time for discussions. Also, the placement of the posters in the center and the possibility to view them throughout the whole conference (managed to read the last ones on the last day) was highly appreciated. I also very much liked the way you organized group discussions in connection with the key note speaker presentations.*
- *There were many opportunities to contribute ideas and experiences.*
- *The focused plenaries with different voices and the dedicated time to discuss these was a highlight! The contributions from the companies was also really enlightening.*
- *There is a need for workshops that contribute to professional development.*
- *The discussion groups after key notes were great ways to explicate our thinking and learn new ideas.*
- *The last part, discussing in small groups about the three main topics, was great. Every member of our group was able to put a part of his own work into the discussion. It would have been great to spend more time on this task.*

OUTLOOK AND THANKS

The local organizing team first has to thank all the participants for their contributions that enabled us to host this very insightful conference. Special thanks go to Paul Drijvers and Alison Clark-Wilson for supporting the process of organizing and for their stimulating thoughts throughout the preparation process and throughout the conference.

Conference Paper

Part 1: Plenary sessions

DEVELOPING VISIONS – A SUMMARY OF IDEAS FROM ICTMT 14 PARTICIPANTS ON FUTURE DEVELOPMENTS REGARDING THE USE OF TECHNOLOGY IN MATHEMATICS EDUCATION

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One of four main conference themes – developing visions – addresses the challenge for mathematics educators to integrate new technologies meaningfully into the practice and research of teaching and learning mathematics. It aims to encourage visualizations of new possibilities for using affordances of technology to, e.g., foster sense-making of mathematical contents, create successful learning environments and teaching scenarios, innovate new ways to obtain empirical findings, or develop research-based theories. In this article, we summarize ideas of ICTMT 14 participants on possible future developments regarding the use of technology in mathematics education. These ideas have emerged as a product of a cooperative discussion session during the conference, in which groups of participants addressed three main questions together: What might the future of 1) teaching and learning mathematics, 2) technological innovation, and 3) mathematics education research look like?

Keywords: developing visions, technology, mathematics education

INTRODUCTION

With the development of new technologies (e.g., augmented or virtual reality), mathematics educators face the challenge of examining whether these technologies support mathematics learning and – if so – how they can be integrated into the mathematics classroom. Furthermore, there is a need to develop visions of how technology can be used to support teaching and learning mathematics as well as educational research in the future. This is why one of four main conference themes of ICTMT 14 was called “developing visions”.

In order to elaborate, discuss and share ideas on possible future developments regarding the use of technology in mathematics education, we held a plenary session in form of a collaborative “open space” during ICTMT 14. While for each of the remaining conference themes – enhancing assessment, inspiring learning and teaching as well as networking of theories – plenary talks were given by two keynote speakers, the theme of “developing visions” demanded a more inclusive approach. Instead of having keynote speakers present insights into this conference theme from their perspectives, the aim was to utilize the expertise and creativity of all participants and spark cooperation among them. During the plenary session, a short input was given by Bärbel Barzel and Florian Schacht (University of Duisburg-Essen, GER) together with Paul Drijvers (Utrecht University, NL). They raised three questions which were then discussed in groups of 4–10 conference participants. The groups documented their discussion outcomes using an online Google document which was shared among participants. Their notes build the foundation for this article, in which we interpret and summarize the ideas that emerged regarding the following questions:

1) What will future technology-rich mathematics teaching and learning look like?
2) What will technology look like and how will it impact future learning goals and curricula?
3) How will mathematics education research be affected by technological innovation in terms of agenda, theories, tools and practices?

VISIONS FOR TEACHING AND LEARNING MATHEMATICS

Summary of the session input by Barzel, Drijvers, and Schacht

High-quality education can be viewed as one of humans' most valuable possessions. This can be made apparent, for example, by considering the Millennium Development Goals [MDGs] formulated by the United Nations [UN] at the beginning of this millennium to reduce extreme poverty. One of the eight goals is to "Achieve universal primary education" (UN, 2015). However, the question of what constitutes high-quality education is not easy to answer. Praetorius, Klieme, Herbert, and Pinger (2018) suggest that quality teaching is formed by three basic dimensions: *classroom management, student support,* and *cognitive activation* (Praetorius et al., 2018). Integrating technology into the mathematics classroom can influence all three of these dimensions. This is why currently the challenge for mathematics educators lies in rethinking curriculum goals and teaching styles in regard to the use of digital tools. Teachers as well as students need to accumulate knowledge about a) technologies themselves, e.g., how to use them, and b) learning mathematics with technology. This presents a challenge as a large diversity of digital tools are already available for mathematics education.

On the one hand, current technologies can be categorized into more general digital tools. These can either focus more on work or everyday life, for example, technologies that foster communication or the presentation of content, or aim specifically at the use in education. Examples of such tools are student response systems, digital textbooks, or learning platforms. On the other hand, mathematics-specific technologies are available, which again are either more suitable to be used in economical or everyday contexts (e.g., computer algebra systems or statistics tools) or aim at the implementation in education such as specific-purposed tools (e.g., applets) or general-purposed tools (e.g., dynamic geometry systems).

Besides the question of how to best implement different types of technology into the mathematics classroom, visions for teaching and learning mathematics in a digital era might include ideas about new theories used to describe learning processes. While social constructivism is currently the most commonly used learning theory among mathematics educators, it might not be able to explain technology-based learning processes fully. Other theories, such as humanism or connectivism, are starting to emerge and might be more suitable to describe the process of gaining knowledge in the digital era. Connectivism, for example, was developed in regard to e-learning environments and, thus, "views learning as a network phenomenon influenced by technology and socialization" (Goldie, 2016, p. 1064). It is based on the idea that people process information by forming connections between different fields, ideas and concepts. In addition, it emphasizes learning as taking place in connected learning communities, in which thoughts can be shared via words, images or other multimedia formats. Thus, abilities to access and filter relevant information, being aware of the diversity of different peoples' opinions as well as making decisions about one's own learning process are essential in such e-learning environments (Goldie, 2016).

Barzel, Drijvers, and Schacht (2019) summarize their vision by stating that in the future students should be:

- aware of the role of mathematics in the world,
- able to use mathematics flexibly and confidently,
- creative and open-minded problem-solvers,
- engaged and willing to solve the problems of the future.

Thus, they see the need for the teaching and learning of mathematics to:

- be engaging, exploratory and focused on sense-making,
- be supportive, focused on individual needs,
- integrate technology to support a) students' understanding of mathematical concepts, b) the integration of real-life situations in classrooms (e.g., using simulations or augmented reality), and c) both individual and cooperative work.

First group's discussion outcomes

The first group of ICTMT 14 participants to discuss the question of how a technologically-rich teaching and learning of mathematics might look in the future consisted of: Gulay Bozkurt (Eskisehir Osmangazi University, TR), Maxim Brnic (Westfälische Wilhelms Universität Münster, GER), Patrick Ebers (Universität Duisburg-Essen, GER), Maria Fahlgren (Karlstad University, SE), Christos Itsios (University of Duisburg-Essen, GER), Bea Kristinsdóttir (University of Iceland, IS), Matthias Müller (TU Braunschweig, GER), Hana Ruchniewicz (University of Duisburg-Essen, GER), and Stefanie Schallert (Johannes Kepler University Linz, AT).

This group's discussion mainly focused on collecting ideas about how to scale-up research results and examples of "good practice" to make use of technological affordances in many mathematics classrooms. Furthermore, some challenges of trying to introduce technologies into more classrooms were addressed. Changes were seen to be necessary in the following areas:

Teachers' professional development (PD):

- The increasing availability of online PD courses offers the possibility for teachers to repeat content and improve their skills and knowledge about using technology to teach and learn mathematics independent of time and location.
- As new innovations are mostly brought into practice via PD courses, there is a need to show their efficacy, especially in regards to students' learning outcomes.
- There is a need to identify quality criteria of "good" tasks and materials for teachers to select useful digital learning environments or tasks for their classrooms.
- Web 2.0 technology offers new opportunities for teachers to work collaboratively, e.g., in online professional learning groups (PLGs).
- Video-recordings of real classrooms using digital technologies can be used to demonstrate examples of "good practice" to other teachers.
- Technology offers teachers the opportunity to share own experiences of using digital tools in their mathematics classrooms (e.g., via self-reports or video-recordings) and receive peer- or expert-feedback on their teaching. Thus, the element of micro-teaching could be implemented into PD courses.
- A challenge of integrating new technologies into mathematics classrooms is the speed of technological development: while researchers and teacher educators are addressing how to integrate digital tools, new technologies emerge.
- Online PD can help to reduce the number of required (mathematics) teachers. In order to reach the goal to provide quality education and promote lifelong learning to all children, the UNESCO Institute of Statistics [UIS] predicts that, globally, we need 9.8 million additional primary and 22.3 million additional secondary school teachers by 2020 (UIS, 2016).

Textbooks:

- Textbooks still represent one of the main influences for school teachers in planning mathematics lessons. Therefore, the use of technology needs to become apparent in textbooks or there even needs to be a shift towards using digital textbooks.

- A “choose your own textbook” approach might be easy to handle for teachers. The idea is that teachers can choose a number of applets, tasks, etc. for a specific topic to create their own digital textbook for a mathematics lesson.
- PD courses or workshops are needed on “How to use a digital textbook?”.
- Not enough research results are available on “How to use digital textbooks?”. Especially, as their structure and elements differ from pen-and-paper textbooks. For example, if digital textbooks are only composed of applets, will the diversity of classroom activities be reduced and, thus, student engagement be reduced?
- Quality criteria and typologies on digital textbooks are needed to describe their potentials and restraints.

Interdisciplinary and out-of-school Learning:

- The use of technology offers new possibilities to engage students in doing and learning mathematics outside the typical classroom setting. For example, they might play a game on their mobile phones at home, which teaches them mathematical contents.
- More opportunities for seamless learning: currently there is a disconnection between the students’ use of digital technologies in their everyday life (digital natives) and at school (use of own mobile devices is often forbidden).
- Technologies might change the way we arrange or design classrooms. The vision of the discussion group is that there will be mixture of “open spaces” and “quiet areas” in schools rather than the traditional “desks facing the front of the classroom” set up.

Subject matter – choosing what to teach:

- As future teachers are taught at universities, one has to focus more on how to implement the use of technology in higher education. Many institutions still focus on using paper-and-pencil methods in undergraduate courses.
- Guidelines on “What to teach students?” in regards to learning mathematics through and with technologies are needed and the mathematics education research community should take part in that discussion.

Second group’s discussion outcomes

The second group discussing visions for the use of technology in teaching and learning mathematics consisted of: Lynda Ball (University of Melbourne, AU), Bärbel Barzel (University of Duisburg-Essen, GER), Giulia Bernardi (Politecnico di Milano, IT), Paula Beukers (University of Groningen, NL), Domenico Brunetto (Politecnico di Milan, IT), Mats Brunström (Karlstad University, SE), Zeger-Jan Kock (Eindhoven University, NL), Maria A. Lepellere (university of Udine, IT), and Marios Pittalis (University of Cyprus, CY).

Aim of mathematics education:

This group started their discussion by thinking about the goal of quality mathematics education. Their idea was that the aim of education – as well as the way of reaching it – is to find balance between each of the following six pairs: conceptual and procedural knowledge; paper-and-pencil and computer technologies; modelling real-life situations and acting within mathematical contexts; guided and open teaching approaches; individuality and cooperation; lower-order and higher-order skills. Teachers should be able to view mathematics education from a “meta-perspective”. This means, for example, that they are able to create opportunities for students to relate new to previously learned knowledge, make connections, stress sense-making over the accumulation of skills and are able to re-balance their teaching according to specific situations and lesson goals.

More cooperation between teachers and researchers is needed:
A main idea, that emerged from this group's discussion, was that teachers and researchers should work together more cooperatively to implement technology in the teaching and learning of mathematics and in return learn more about learning processes that take place when new technologies are used in the mathematics classroom. The group stressed that we need critical analyses of *how* technology is used to do teach and learn mathematics with and through digital tools.

Out-of-school learning:
Similar to the first group, the second also saw a potential of new technologies in offering new learning opportunities, especially in out-of-school or informal settings.

New contents are needed for 21st century skills:
One point of this group's discussion was that new technologies will demand different skills. Gravemeijer (2012) addresses the need for mathematics education to change in order to teach students 21st century skills. He explains that changes in education tend to happen slowly while both our society and work environment change rapidly. Thus, there is "an 'achievement gap' between what schools […] are teaching and what students will need to succeed in today's global knowledge economy" (Gravemeijer, 2012, p. 30). Mathematical contents, such as modelling real-life situations using variables and using statistics to overlook an increasing amount of data, will be more important in the future (Gravemeijer, 2012).

Affordances of using technology in teaching and learning mathematics:
- Digital technologies allow for students to explore mathematical contents (e.g., by interactive or linked representations of mathematical objects).
- Digital technologies allow for students to gain higher-order skills by activating them cognitively.

Chance to change attitudes:
A problem for mathematics educators to engage students is that a bad attitude towards the subject is oftentimes accepted socially. It seems positively connoted to be "bad at maths". Using technology to learn mathematics might change students' attitudes towards the subject. It offers opportunities to highlight the "beauty of mathematics".

Teacher Education:
If the aim is to use technology to support the teaching and learning of mathematics, the pre- and in-service education of teachers becomes crucial. They need to learn, for example, how to select digital tools, how to create tasks and activities with these tools, how to make use of their affordances, which constraints might become apparent, how to structure lessons, etc. The goal for teachers must be to be critical and know when and how to use digital tools.

VISIONS FOR TECHNOLOGICAL INNOVATIONS AND THEIR IMPACT

Summary of the session input by Barzel, Drijvers, and Schacht

When the task is to predict how technologies of the future might look like, history has shown that it is often far from accurate. Examples can be found plentiful in motion pictures and literature. One example is the cover art by Robert Tinney for the 1981 April issue of Byte Magazine (Morgan, 1981), which depicts a future smartwatch including a tiny keyboard and floppy disk drive. Even though it is stated to be a humorous in regard to the design and functionalities of modern smartwatches, it shows the way people think of future technologies. McCracken (2014) summarizes in a TIME magazine article: “We tend to think that new products will be a lot like the ones we know. […] Oftentimes, we don’t dream big enough” (McCracken, 2014). Thus, the 1981 envisioned wrist computer included floppy disks as it was the known way for data storage. However, some historical predictions envision future technologies partly correctly. One example given by Barzel et al. (2019) is from the movie franchise *Back to the future.* In the second movie (Canton & Gale, 1989), technological innovations such as smart glasses and face recognition were predicted, which are nowadays available (Barzel et al., 2019; Hill, 2015).

These examples of more or less accurate predictions show that it is not so easy to plan for the future. Nevertheless, envisioning future technologies can produce exciting ideas. Emerging technologies which are also used in education are, for example, augmented and virtual reality (Swidan et al., 2020). But with the emerging technologies used for education, one has to address the difficult relationships between private and public companies as well as academia. Funding through external companies is beneficial, but the research should be independent and objective regardless for whom the research is conducted (Barzel et al., 2019).

Barzel et al. (2019) concluded their input regarding the visions for technological innovations and their impact with the question: “Which technology supports the teaching of mathematics in the future?” and asked for criteria, examples and connections. To support the discussion, the following starting points were given:

- What will future technology look like?
- What should future technology look like?
- How will we use future technology in the (mathematics) classroom?
- How does/will technology impact on learning goals and curricula?
- Cooperation between private and public companies and academia?

First group’s discussion outcomes

The first group discussing future technologies and their impact consisted of Peter Boon (Numworx, NL), Alison Clark-Wilson (UCL Institute of Education, UK), Florian Schacht (University of Duisburg-Essen, GER), and Anna Shvarts (Utrecht University, NL):

They focused on the question “*What are the (future) mathematical practices which technology can support?*” in their discussion and noted the following six practices.

- The first practice concerns **shared collaborations** – which could lead to collaborative productions and even to fundamentally new goals.
- But new media also necessarily leads to the question of how conceptualizations of mathematical content beyond mathematics changes through them.
- The possibility that conceptualization changes through the media connects to research that **cognitive functions** (attention, perception, writing abilities) are changing together with technology. For the use of smartphones, it has even been shown that the sensory representation in the brain is influenced by the usage (Gindrat et al., 2015).
- As the usage of smartphones and other media increases and many high school students have access to them (for Germany 99 % of 15-year old students have access to smartphones; Feierabend et al., 2018), something called **open schools** could emerge. These are schools where everyday reality and the classroom are not separated opposed to school where the use of smartphones is forbidden. If the future technology is used in a beneficial way it might help in making mathematics knowledge meaningful.

- **Goals** are changing with technology-supported transformations in society. Thus, it has to be asked and reflected if the way how mathematics is **used** is still current or what aspects of mathematics need to be outsourced to technology completely.
- Digital technologies have already changed and might continue to change **social norms** by connecting a diversity of people worldwide in real-time. Thus, the appropriate way of interacting, communicating, cooperating, or learning together will be influenced.

Second group's discussion outcomes

The second group to discuss the questions raised by Barzel et al. (2019) regarding future technologies consisted of Rotem Abdu (Haifa University, IL), Ana Donevska-Todorova (Humboldt-University of Berlin, GER), Eirini Geraniou (UCL Institute of Education, UK), Miriam Romberg (University of Duisburg-Essen, GER), Andreas Trappmair (Johannes Kepler University Linz, AT), Sylvia van Borkulo (Utrecht University, NL), and Johanna Zöchbauer (Johannes Kepler University Linz, AT).

The second group's discussion revolved around four parts that have to be considered and discussed in the frame of future technology used in mathematics education. These four parts concern the transformation of already existing technologies, new technologies, ethical issues and delaying factors for implementation in the mathematics classroom:

1. The aim is to **transform already existing technologies** and find ways to integrate them in the mathematics classroom in different ways regarding different technologies.
    - Intelligent tutoring systems should be transformed and adaptive as well as personalized features included, so that they may become more efficient in mathematics education practices.
    - But one also has to convince the teachers of the effectiveness of these "new" technologies, so that they are actually introduced into their classrooms. This has been difficult in the past and so the effective technologies are not implemented in real classrooms.
    - One also has to catch up with updates of existing software and teachers need to develop digital competences to manage the changes in their classrooms.
    - Augmented and virtual reality should be included in the mathematics classroom, as they become more widely available (Swidan et al., 2020).

2. Apart from transforming the already existing technologies, there are also **new technologies appearing in industry**, these might lead to new ways of teaching and learning mathematics.

- For this goal interdisciplinary approaches in development of appropriate software for school mathematics should be undertaken to bridge the gap between industry and education more quickly and effectively.
- One example of this technology is artificial intelligence, which might offer more adaptive technology for education such as intelligent tutoring systems that give explicit feedback to the learners (e.g., ASSISTments; Roschelle et al., 2016).
- But one also has to question, if all new and emerging technology can and should be used for education and in particular in mathematics. Do they all match or meet our educational needs?

3. An important point to consider, when using technology in school are **ethical issues**:

- There are controversial issues, e.g., microchips utilized to enhance connectivity between a learner and the world, but they also raise data privacy issues, when using the software in class.
- Ethical issues can also emerge, when cooperating with big technology players. The question remains, if we can afford to ignore them, because of their global reach and the resulting possibility to scale access to education globally. How do we ensure an ethical, equitable, high quality mathematics education "offer" that is accessible to all?

4. While discussing how to implement existing technology and emerging technology in the mathematics classrooms around the world, there are a number of **delaying factors** that have to be considered.

- One of the biggest problems is non-existent school infrastructure for the implementation of technology. For the use of software, there is often a need for internet access to work properly, which might not be available. Also hardware in sufficient numbers, so students can use it effectively, is often expensive and public schools might not have the funds to acquire the necessary hardware.
- Often there is a perceived need of "big" scientific data to evidence impact and successful implementation, before stakeholders advocate the use of new technologies in education.
- Depending on the country, more or less strict educational policies are in place. If these include or exclude technology use specifically, it can lead to significantly delayed introduction of beneficial technology into classrooms.
- Even if the infrastructure and education policies enable the use of technology in a timely manner, teachers need to be educated through PD programs on how to use the new technology meaningful. As these PD programs first have to be developed themselves, this leads to a delay in using the technology meaningful.

VISIONS FOR MATHEMATICS EDUCATION RESEARCH

Summary of the session input by Barzel, Drijvers, and Schacht

The third theme for discussion focused on the impact of technological innovations on the mathematics education research in terms of agendas, theoretical frameworks, and tools. In terms of research agendas, several questions show the considerable variety of research directions focusing on technology:

- Can and should mathematics education research focus on students' higher-order thinking skills and/or on mathematical literacy, both regarding the significance of technology for the learning processes?
- Can and should embodiment with and through digital technology as well as embodied instrumentation be a fruitful research investigation?
- How can student-student- and/or student-teacher-collaboration be affected while using digital technology? How does the language change due to the use of technology?
- What are the relationships between mathematical and computational thinking? How does computational transposition during the design and implementation of computer learning environments transform the knowledge to be taught through these technological means?

In order to investigate in these research agendas, there is a growing need for adequate theoretical frameworks. Accordingly, one must look at available theoretical approaches that will help us to understand and exploit the affordances of digital technology for mathematics education. To do so, a look at theories from mathematics education but also from cognitive science, cognitive ergonomics, game research, human-machine interaction, etc. is auspicious. For the purpose of dealing with the diversity of theories, exploring the possibilities of interactions between theories, such as contrasting, coordinating, and locally integrating them, serves as a beneficial method (Bikner-Ahsbahs & Prediger, 2014).

In addition to looking at the variety of theories, the technological devices at disposal – for researches and for learners – have increased largely in the last couple of years. Focusing on the perspective of researches, there are more than a handful of different tools that (are likely to) affect researchers' work and research outcomes, such as:

- Eye tracking and finger tracking devices,
- Automated transcription,
- Automated coding in software for qualitative data analysis,
- Multiple camera recording and motion tracking,
- Data logging and learning analytics,
- Hand writing recognition.

Overall, technological progress will influence mathematics education research. The question is how will and should mathematics education research be affected by technological innovation, in terms of agendas, theories and tools. This issue was discussed by two groups.

First group's discussion outcomes

The first group discussing future technologies and their impact on mathematics education research consisted of Angelika Bikner-Ahsbahs (University of Bremen, GER), Florian Berens (University of Goettingen, GER), Eleonora Faggiano (University of Bari Aldo Moro, IT), Federica Mennuni (University of Bari Aldo Moro, IT), Laura Ostsieker (University of Applied Sciences Cologne, GER), Benjamin Rott (University of Cologne, GER), and Moritz L. Sümmermann (University of Cologne, GER).

They raised a variety of sub-questions regarding the main impact of technology on research agendas, frameworks, and tools ranging from the issue of technological enhancements (e.g., smartwatches, smart glasses, artificial legs, etc.) – leading to an alternative view on our body – to treating technology as an opportunity for changes instead of a black box in order to realize the innovation technology brings to learning mathematics.

Along these lines, one of the main points of discussion was put on the change in mathematics learning through technology and, therefore, on artificial intelligence as well as new tools helping researchers to manage and process data collection, coding, etc. Although the benefits of such tools were not argued but seen as a real chance, the group agreed on concentrating on better planning what and how data is collected in terms of being careful not to trust automated analyses too much. With that being said, the group saw the starting point of research projects in problems and research questions and in particular not in the collected data. This is due to the fact that a lot of data can be collected during on-going research, which is easy to collect and to analyze, but possibly leading to negligible data as well. Therefore, the focus in mathematics education research should be theory-driven.

Furthermore, the group discussed issues of ethics as well as conference standards which might change due to technological innovations and, therefore, affect research principles. One aspect was the influence of platforms like *arXiv* to discuss manuscripts before publication: How can we, as a research community, ensure that experienced reviewers look at articles? But also: Will new technologies be developed that will change international conferences? Will we still travel around the world? At the moment, technologies like Skype cannot replace personal meetings.

Finally, issues of computational thinking, statistical thinking, dealing with big data, and data literacy were considered as they will also address teaching issues:

- New topics will have to be researched (machine learning, networks, graph theory, statistics, big data, etc.).
- New technologies will have an impact on curriculum design and research.

Second group’s discussion outcomes

The second group to discuss the questions raised by Barzel et al. (2019) regarding the influence of technology on mathematics education research consisted of Cecilie Carlsen Bach (Aarhus University, DK), Christine Bescherer (University of Education Ludwigsburg, GER), Paul Drijvers (Utrecht University,NL), Rikke M. Gregersen (Aarhus University, DK), Karina Höveler (Westfälische Wilhelms Universität Münster, GER), Mathilde K. Pedersen (Aarhus University, DK), and Marianne Thomsen (Aarhus University, DK).

This group discussed four different facets regarding the impact of technological innovations on the mathematics education research in terms of agendas:

- Bridge the gap between using physical manipulatives and digital technology and investigate on how to exploit the potentiality of their synergic use.
- Address mathematics education and societal issues (e.g., equity or diversity).
- Teacher professional development, for example, How to use technology to better monitor and understand all students’ learning in the classroom?
- How to prevent researchers from re-inventing the wheel?

Regarding theoretical frameworks, the following aspects were addressed:

- Input from neuroscience research?
- How to benefit from the different theoretical lenses to look at the same set of data?

Focusing on research tools, it was concluded that researchers will need their own professional development; for research practices, ethical issues and the community’s responsibility towards the society was addressed.

CONCLUDING REMARKS

The collaborative “open space” on developing visions at ICTMT 14 showed many current and possible future developments regarding the use of technology in mathematics education practice and research. The points addressed in the discussion groups reveal that many issues raised are not country specific but common themes around the world. The three main questions of the discussion regarding future teaching and learning, technological developments and research practices encouraged the conference participants to share a wide variety of goals, challenges and visions.

Regarding the teaching and learning of mathematics, it became apparent that the use of technology affords the exploration of new content areas (e.g., modelling, statistics, big data), materials (e.g., digital textbooks, interactive learning environments) and ways to connect mathematical contents, disciplines, learners, teachers and researchers. However, the difficulty for teachers to select beneficial technology and to orchestrate their use in the classroom remains. This is why there is not only a need for guidelines and quality criteria on what and how to teach with and through digital tools, but also for PD courses and pre-service teacher education. Such teacher trainings could be supported by the use of technology in that it connects teachers (e.g., online PD courses, online PLGs) and gives them the opportunity to share examples of good practice or receive peer- or expert-feedback on their teaching. Finally, both discussion groups stressed a need for more cooperation between teachers and mathematics education researchers in the future.

Regarding future developments in technology, both groups discussed how this might change the conceptualization of mathematical contents. Educators must consider emerging technologies (currently e.g., virtual or augmented reality, adaptive features, artificial intelligence) in light of their affordances and constraints for learning mathematics. Technological developments are seen to support, for example, collaborations, cognitive functions, or the exploration of new mathematical contents. However, challenges of using technology in mathematics education include scaling-up scientific findings, providing schools with the necessary infrastructure (e.g., WIFI access), or ethical issues in regards to data privacy as well as cooperation between educators and commercial companies. Nevertheless, such collaborations could have great impact in terms of new technological developments for mathematics education.

Regarding the use of technology in mathematics education research, groups addressed that not only research agendas, but also research frameworks and tools might change. With technological advances new mathematical contents (e.g., machine learning, statistics) will become more important and, thus, need to be investigated as they become part of curriculums. In addition, the question of how to best educate teachers on the use of technology needs to be researched further. Concerning research methodologies technology can support the collection, managing, processing, coding and analyzation of data. Moreover, approaches from other disciplines such as neuroscience might be introduced to mathematics education. Finally, technology might impact the work process of researchers as it offers, for example, platforms to share and review scientific work, or change the way researchers exchange ideas in the future. However, both groups stressed that technological developments come with ethical issues that need to be addressed.

Over all three topics it became evident that ICTMT 14 participants stress the importance of using technology to explore and connect mathematical ideas and actively engage in mathematical activities. In particular, possibilities to connect and collaborate through technology were addressed. Furthermore, the education of mathematics teachers (pre- and in-service) on how to use digital tools efficiently in their classrooms was a common discussion point. Besides these opportunities to support mathematics education, groups identified ethical issues that need to be considered by mathematics educators. Finally, the discussions showed a demand for more collaboration between teachers, teachers and researchers as well as researchers and commercial companies. The local organizing team of ICTMT 14 hopes that the "open space" and discussion on developing visions serves as a starting point for this purpose.

ACKNOWLEDGMENTS

The ideas presented here were gathered during the discussion groups as stated in each section. The authors of this summary article hope that all thoughts and ideas were restated in their intended way. We thank all participants in the ICTMT 14 developing visions discussion groups for their valuable input.

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AUTOMATED FEEDBACK FOR MATHEMATICAL LEARNING ENVIRONMENTS

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Digital learning environments that offer well-designed feedback have the potential to enhance mathematics education. Building such a system is typically a huge and complex undertaking. Generating informative feedback at the level of steps a student takes requires the encoding of expert knowledge about the problem domain in software. The software component that processes this knowledge is traditionally called a domain reasoner. Such a reasoner can produce various types of feedback, for example about the correctness of a step, common errors, hints about how to proceed, or complete worked-out solutions.

In this paper, we highlight the main domain reasoner components that are responsible for generating feedback: rules, problem-solving procedures, normal forms, buggy rules, and constraints. Examples are drawn from the Digital Mathematics Environment (DME), which uses feedback generated by specialized domain reasoners for solving equations and structuring hypothesis tests. Similar techniques have also been used in tutoring systems for domains outside mathematics.

Keywords: intelligent tutoring systems, feedback generation, expert domain knowledge

INTRODUCTION

An intelligent tutoring system (ITS) helps students with learning a particular topic. It typically does this by offering learning material to study, tasks to solve, and by providing various kinds of help. The help provided by an ITS may take several forms: it might be through sequencing the tasks in a way that suits the student, providing scaffolding at the level of the student, giving elaborated feedback on the steps a student takes towards a solution for a task, helping a student take a next step towards a solution, etc.

Most ITSs do not need a teacher to provide help to a student: they automatically calculate the feedback on the work of a student, or a hint for a student. What do these systems need for calculating feedback, and how do they do this? A typical ITS has an expert knowledge module that contains the information necessary to calculate feedback and hints. In the last decade we have developed an approach to construct expert knowledge modules for a variety of ITSs, based on problem-solving strategies and rewriting and refinement steps. This paper describes our approach to developing expert knowledge modules, which we call domain reasoners. We show which components we need in our domain reasoners, and how we use domain reasoners in various ITSs. We will not describe how to design a full-blown environment for learning and practicing mathematics (such as the Digital Mathematical Environment (Drijvers, Boon, Doorman, Bokhove, & Tacoma, 2013)), nor how or when the calculated feedback is presented to a student (VanLehn, 2006), nor which kind of feedback is most effective (Bokhove & Drijvers, 2012).

We believe that explicitly describing the expert knowledge module concepts of an ITS can help designers and developers of ITSs, which often are complex software systems. Our approach can be applied in many domains, and the generality of the approach gives some guarantees about the consistency and completeness of the feedback provided.

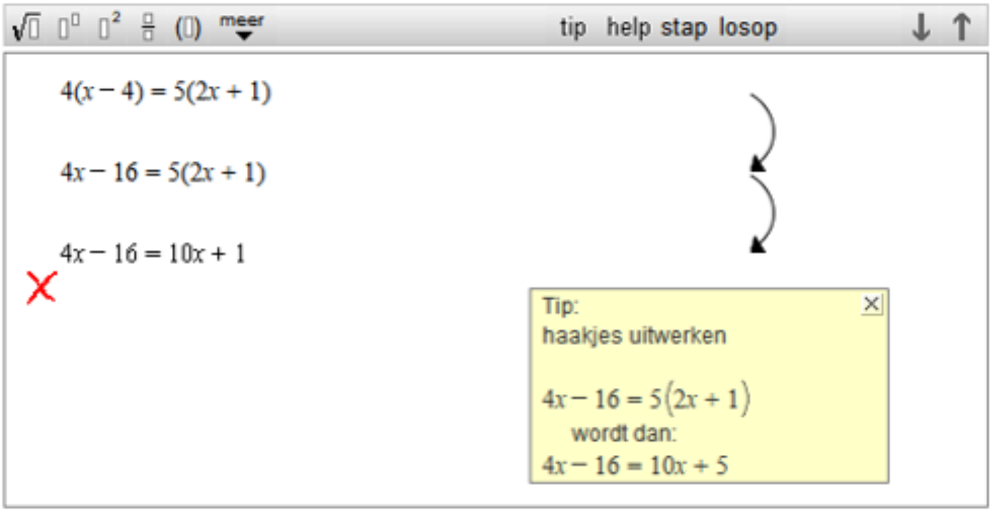


Figure 1: Inner-loop feedback, presented in the Digital Mathematics Environment

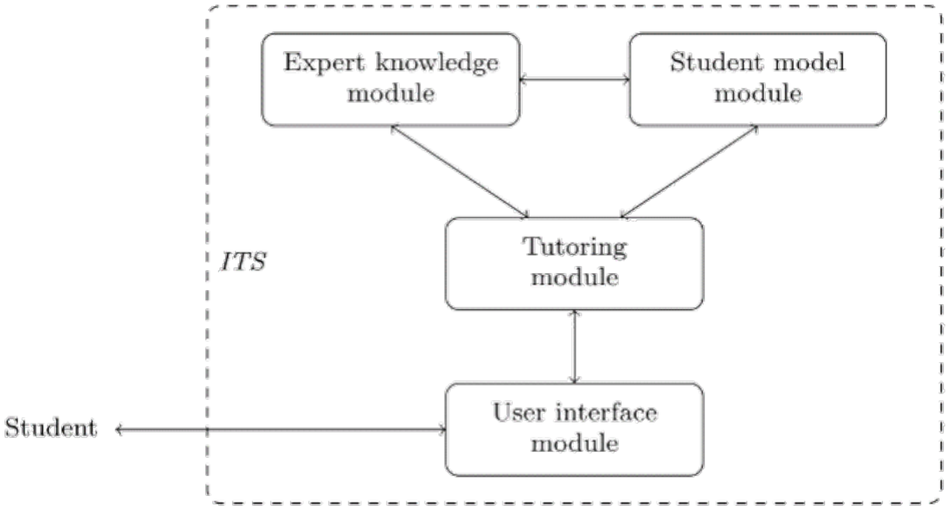


Figure 2: Classic structure of an ITS, decomposed into four components

In the remainder of the paper, we first introduce ITSs. We then show how we represent expert domain knowledge, and give many examples of ITSs in which we have used our approach. We conclude by expressing the paper's main points, and by identifying trends and challenges for the future.

INTELLIGENT TUTORING SYSTEMS

What is an Intelligent Tutoring System (ITS)? In his seminal paper, VanLehn (2006) explains that the behavior of such a system is structured around two loops. The outer loop concerns itself with solving one task after another. Feedback for the outer loop could, for example, suggest the next suitable task to solve. The inner loop considers the steps for solving one complex, multi-step task. Figure 1 gives an example of feedback at the inner loop: it combines feedback about correctness, high-level help ('remove parentheses'), and a bottom-out hint. In this paper, we only focus on feedback for the inner loop.

The four-component architecture shown in Figure 2 is the classic decomposition of an ITS in four parts (Nkambou, Bourdeau, & Psyché, 2010). The decomposition helps with assigning responsibilities: there are modules for user interaction, for pedagogical strategies (the tutoring module: e.g., which hint facilities to offer), for modeling the current knowledge of a student, and for expressing expert domain knowledge. Although this conceptual architecture can be helpful for understanding the inner workings of ITSs, clear interfaces and communication protocols between the modules are missing, and in practice, one often finds monolithic systems instead of separate components.

Feedback and hints are generated by the expert knowledge module. Following Goguadze (2011), we use the term domain reasoner for the part of the module that can 'reason about problems'. This reasoning includes knowledge about the objects in a domain (e.g. expressions, equations), how these objects can be manipulated, and how to guide manipulation to reach a certain goal (Bundy, 1983).

For mathematical learning environments, computer algebra systems (CAS) can do part of the domain reasoner's job. Such systems are powerful tools that are great at evaluating expressions. However, these tools have not been designed for providing feedback, and using built-in equality for comparing expressions can be very subtle. Hence, specialized domain reasoners have an advantage in generating feedback.

Narciss (2008) distinguishes the following widely used feedback types:

- knowledge of performance, e.g. percentage of tasks solved correctly;
- knowledge of result/response, e.g. correct or incorrect;
- knowledge of the correct response, which provides the correct answer;
- answer-until-correct feedback and multiple-try feedback, which provide extra opportunities after an incorrect answer;
- elaborated feedback, which provides additional information besides correctness and the correct answer.

Similar feedback types have been described by Shute (2008). Domain reasoners provide feedback services that are derived from the feedback types (Heeren & Jeuring, 2014). Intuitively, a feedback service is just a request-response communication pattern that exposes the capabilities of the domain reasoner. Services can correspond to the inner loop or the outer loop. Examples of service requests are: *Am I finished? Give me a next-step hint or worked-out solution. Is my step correct* (step diagnosis)? If yes: *does the step bring me closer to a solution?* If no: *is it a common mistake?*

EXPERT DOMAIN KNOWLEDGE

We use the IDEAS framework[1] for constructing domain reasoners: IDEAS (Interactive Domain-specific Exercise Assistants) is a generic, open-source software framework that can be used for expressing expert domain knowledge and for calculating automated feedback based on this knowledge. The framework is independent of the problem domain. Table 1 summarizes the main expert domain knowledge components, and how these components are used for calculating feedback. Certain feedback types require combinations of knowledge components, such as the checking procedure for steps (the 'diagnose' feedback service) that is used by the domain reasoner for feedback on the structure of hypothesis tests (Tacoma, Heeren, Jeuring, & Drijvers, 2019). The following sections discuss the knowledge components.

[1] http://hackage.haskell.org/package/ideas

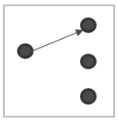
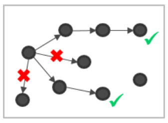
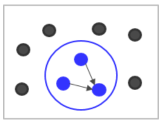
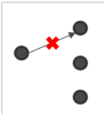
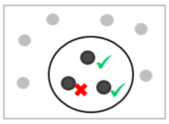
component		*used for*
rules		recognizing steps; suggesting possible next steps
problem-solving procedures		recognizing the solution strategy; detecting detours; providing next-step hints; providing worked-out examples
normal forms		recognizing steps; deciding whether in finished form or not; rewriting atypical expressions (e.g. x+(-5) to x-5)
buggy rules		detecting common mistakes
constraints		checking properties or attributes; reporting violations

Table 1: Five knowledge components for expert domain knowledge, and how these are used

Rules

Rules specify the steps (manipulations) that are allowed: these steps can be rewriting steps (e.g. replacing 5 + 2 by 7) or refinement steps (e.g. adding a line to a proof). For example, consider the rule for distributivity:

$$\forall abc\ .\ a(b + c) \rightarrow ab + ac$$

With this rule, the step $5(x + 2) \rightarrow 5x + 10$ can be taken towards reaching a solution. Rules can be used for recognizing steps, or for suggesting possible next steps.

When coding rules into software, it should be clear where the knowledge ends up. Preferably, the rule is programmed as a rewrite rule (Baader & Nipkow, 1997), which can be coded using a datatype-generic approach (Van Noort et al., 2010) as follows:

```
rule "distr" $ \a b c -> a * (b + c) :~> a * b + a * c
```

Observe the similarities between the rule's specification and its implementation. The explicit representation of the rewrite rule allows for further analysis and transformation, such as Knuth-Bendix completion for finding missing or conflicting rules (Knuth & Bendix, 1983), support for associative-commutative rewriting, rule inversion, automated testing, documentation, etc.

Problem-solving procedures

Whatever aspect of intelligence you attempt to model in a computer program, the same needs arise over and over again (Bundy, 1983): the need to have knowledge about the domain; the need to reason with that knowledge; the need for knowledge about how to direct or guide that reasoning. Problem-solving procedures describe sequences of rule applications that solve a particular task (Heeren & Jeuring, 2017), and thus address the third type of need. For instance, a common procedure for adding two fractions is: *(1)* find the lowest common denominator (LCD), *(2)* convert fractions to a form with the LCD as denominator, *(3)* add the resulting fractions, and *(4)* simplify the result. We have developed a domain-specific language for specifying such procedures explicitly. This language provides a rich vocabulary for accurately specifying procedures, and introduces composition operators for combining simple procedures into complex composites. Examples of such operators are sequence, choice, repeat, try, prefer, and somewhere. With this language, the procedure for adding two fractions can be defined as:

FindLCD ; many(somewhere Convert) ; Add ; try Simplify

where FindLCD, Convert, Add, and Simplify are rules. An example of a step-wise derivation for this procedure is:

$$\frac{1}{2}+\frac{4}{5} \xrightarrow{\text{FindLCD}} \frac{1}{2}+\frac{4}{5} \xrightarrow{\text{Convert}} \frac{5}{10}+\frac{4}{5} \xrightarrow{\text{Convert}} \frac{5}{10}+\frac{8}{10} \xrightarrow{\text{Add}} \frac{13}{10} \xrightarrow{\text{Simplify}} 1\frac{3}{10}$$

The first step finds 10 as LCD, which is used in the conversion steps that follow. Problem-solving procedures can be used for feed forward (providing next-step hints and worked-out examples), or to recognize the approach followed by a student and to detect possible detours. Problem-solving procedures help with following the steps of a model solution conform the model-tracing paradigm (Anderson, Boyle, Corbett, & Lewis, 1990). The composite structure of procedures also allows for decomposing procedures into parts, and to tailor feedback based on this decomposition.

Normal forms (equivalence classes)

Normal forms define classes of expressions that are treated the same, and select one canonical element for such a class (Heeren & Jeuring, 2009): for example, $10 + 5x \approx 5x + 10 \approx 5x + 5\cdot 2$, for which $5x + 10$ is usually considered the standard notation. In mathematics, equivalences concerning associativity, commutativity, basic calculations, simplifications, etc. are often implicitly assumed for relations dealing with which expressions are equal, equivalent, similar, indistinguishable, etc. Equivalence classes make the granularity (step size) of a task explicit (McCalla, Greer, Barrie, & Pospisil, 1992). For example, given the rule for distributivity, $5(x + 2)$ and $5x + 10$ should be distinguishable.

Normal forms can be used for rewriting atypical expressions (e.g. replacing $x + (-5)$ by $x - 5$), and for deciding whether an expression is accepted as the final answer for a task or not (e.g. should $\sqrt{12}$ and $2\sqrt{3}$ be distinguishable). Normal forms can also be used for recognizing steps and rules.

Buggy rules

Buggy rules describe common mistakes and enable specialized feedback messages when detected. Consider, for example, a buggy rule for distribution:

$$\forall abc\ .\ a(b + c) \rightarrow ab + c$$

This buggy rule can be used for detecting the common mistake in $5(x + 3) \rightarrow 5x + 3$. More examples of buggy rules are the sign mistake in $5x = 2x + 3 \rightarrow 7x = 3$, and Hennecke's collection of 350 buggy rules for the fraction domain (Hennecke, 1999). Buggy rules are often associated with a sound rule.

Constraints

Constraints are based on the theory of learning from performance errors (Ohlsson, 1996) and can be used for checking properties and attributes, and reporting violations. Constraints have a relevance condition and a satisfaction condition: on violation (when relevant, but not satisfied), a special message can be reported. An example of a constraint is: if the equation is linear (relevance), then the equation's right-hand side should not contain variable x (satisfaction). The corresponding constraint message would report that 'the equation is not yet solved because x appears on the right'.

EXAMPLES OF DOMAIN REASONERS

In this section we discuss examples of domain reasoners that have been developed with our approach, and their problem domains (see Figure 3). We will explain which knowledge components are used for generating feedback, and highlight some specifics of the problem domain.

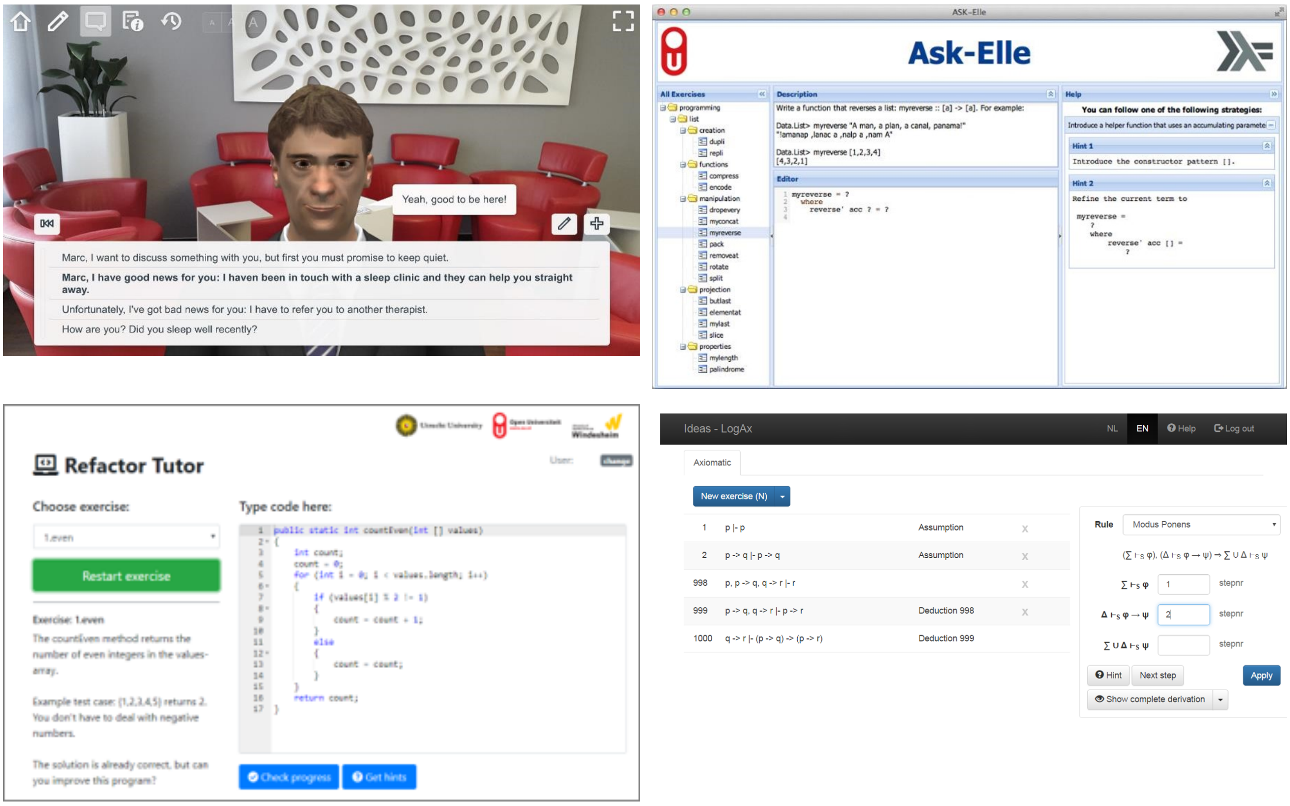


Figure 3: Examples of domain reasoners. Top row: Communicate! for training communication skills, and the functional programming tutor Ask-Elle. Bottom row: the Refactor Tutor for learning how to refactor imperative programs, and LogAx for constructing axiomatic proofs

Advise-Me. The Advise-Me assessment software analyses free-text input for math story problems (Heeren et al., 2018). The problems target mathematical competencies for setting up algebraic expressions and equations, and simplifying them. The software first extracts the mathematical expressions from answers, and then uses an analyzer that tries to recognize the solution steps (rules) and the high-level approach (problem-solving procedure). Normalizations are used for recognizing steps and buggy rules.

LogAx. LogAx is a tutor for constructing axiomatic proofs (Lodder, Heeren, & Jeuring, 2017), for example for proving that $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$. Proofs are constructed in two directions: by formulating assumptions and combining these (forwards), or by working backwards from the goal. A directed acyclic multi-graph (DAM) is built that represents multiple proofs, and from this DAM a problem-solving procedure is generated. Feedback messages report subgoals and rules that can be applied. A student selects which rule she wants to apply and fills in a template for that rule. After an unanticipated step by the student, the DAM and the problem-solving procedure are re-generated to facilitate giving feedback on subsequent student steps.

Ask-Elle. The functional programming tutor Ask-Elle lets students practice with defining small functions in Haskell (Gerdes, Heeren, Jeuring, & van Binsbergen, 2017). Holes can be used for unfinished parts in the program: a student can ask for hints on how to complete these holes. A problem-solving procedure is generated from multiple annotated model solutions: the procedure recognizes different solution approaches and can give feedback on this. The tutor relies on an extensive normalization procedure for recognizing many variations of the model solutions. For programs that cannot be recognized, constraints are used to test input-output correctness.

Refactor Tutor. This tutor lets students practice with refactoring small programs that are already functionally correct. The tutor uses a problem-solving strategy that imitates step-wise refactoring strategies by experts. Feedback is provided in a hint tree that can be expanded to see more detailed hints. Buggy rules capture common mistakes, such as logical errors in rewriting conditions. Similar to Ask-Elle, the tutor uses program normalizations for recognizing functionally equivalent programs, and constraints that perform input-output testing for detecting incorrect changes.

Communicate! The serious game Communicate! supports practicing interpersonal communication skills (Jeuring et al., 2015), for example between a health care professional and a patient. A virtual character is presented, and the player is offered a menu with sentences to choose from. Feedback is provided during the conversation (e.g. the flow of the conversation and emotions shown by the virtual character) and afterwards. Conversations can be scripted with a scenario editor. Scenarios are translated into problem-solving procedures.

CONCLUSION AND FUTURE WORK

In this paper we have presented our approach to automatically generating feedback in mathematical learning environments and Intelligent Tutoring Systems. To give better feedback, the approach is based on explicitly representing expert domain knowledge in such systems. We discussed which knowledge components we use for generating feedback, and explained that the step-size of a task is an often ignored, but very relevant aspect. Step-size can be controlled by defining normal forms, and using the hierarchical structure of problem-solving procedures.

Designing domain reasoners with feedback services simplifies the construction of ITSs. Feedback services result in loosely-coupled, reusable software components. The design follows the 'separation of concerns' design principle and localizes expert domain knowledge. Feedback services can be derived from the popular feedback types that have been described in literature. The presented approach can be applied to a wide range of problem domains.

For the future, we observe the following trends and challenges:

– Literature reports that every one hour of instruction that uses an ITS takes 200–300 hours for authoring content (Murray, 2003). There are design trade-offs for building an ITS. For example, supporting only one task simplifies feedback generation compared to supporting a full class of problems (e.g. solving quadratic equations), but reduces reusability and maintainability. We believe that software technology can help with developing high-quality, reusable solutions.
– There is a trend towards data-driven intelligent tutoring systems (Koedinger, Brunskill, Baker, McLaughlin, & Stamper, 2013). These systems use AI techniques to generate feedback from collected data, and typically scale well. This trend raises questions about the need for explicit

expert domain knowledge. Hybrid solutions that only use collected data when expert domain knowledge cannot provide feedback may combine the best of both worlds.

- There is a need for further adaptation and personalization, both for the inner loop and the outer loop. This requires models for mastery learning (e.g. techniques for Bayesian knowledge tracing), and more advanced techniques to use the information from such models in domain reasoners.
- Designing tools with automated feedback for less-structured problem domains, such as software design and learning foreign languages, is challenging, especially compared to the structured domain of mathematics.

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SEEING THE ENTIRE PICTURE (STEP):

AN EXAMPLE-ELICITING APPROACH TO ONLINE FORMATIVE ASSESSMENT

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ABSTRACT

Attempts to reform mathematics teaching in the past decades have endorsed the view that mathematical reasoning and strategic competence are central learning goals. The affordances of technology for creating learning environments that nurture mathematical reasoning have been developed and investigated for the past decades. Recent efforts bring diagnostic tools from cognitive research to technological platforms that automatically assess students' work to provide immediate feedback. Such feedback is used for assessment, to support conceptual adaptive learning, and to help teachers with formative assessment in the classroom. Yet, automatic feedback has usually been limited to tasks that require procedural interactions that do not necessarily offer teachers opportunities to assess various dimensions of conceptual understanding. In the past few years, across design-based research cycles, we developed and tested an innovative online assessment approach and supporting platform (STEP) that provides meaningful immediate feedback. We used STEP to study the affordances of example-eliciting tasks (EETs) and articulate design principles for an online example-based formative assessment environment. The article focuses on the rationale and principles of the development and of the design-based research. I analyze cognitive aspects of the conceptual online feedback and of our attempts to articulate the potential of technology for making EETs less demanding for teachers to manage.

Keywords: Automated formative assessments, Example-eliciting task, Elaborated feedback, Task design patterns

TECHNOLOGY-BASED ASSESSMENT: NEED FOR NEW PERSPECTIVES

There is agreement that to have immediate and long-lasting effect on teaching and learning, online educational assessment should: (a) be aligned with institutional goals and views; (b) rely on strong evidence that can be analyzed online; (c) use data to analyze student learning within the norms and the environment in which they are learning, and across a variety of learning situations and contexts; and (d) capture the social aspects of learning by providing tools for analyzing submissions of large groups or of an entire class, or by reporting on the roles of individual students in dialogic interactions, rather than just on one individual student (Shepard, Penuel and Pellegrino, 2018).

Formative assessment of student skills and understanding, as well as more recent efforts to bring diagnostic tools from cognitive research to technological platforms, support teachers in various ways that include: (a) tools that help teachers collect and present differences in student work to the whole class (e.g., Clark-Wilson (2010) with TI-Nspire); (b) environments that provide feedback and suggest expected misconceptions (e.g., Heffernan & Heffernan (2014) report on ASSISTments); (c) dynamic technology-based approaches that analyze student work online to follow solution processes and provide feedback (e.g., Sangwin & Köcher (2016) use STACK) or construct student model (e.g., Advice Me project Heeren et al. (2018)); and (d) learning analytics tools that analyze big data online in anticipation of a near future in which assessment and instruction are one process (e.g. Wise, Vytasek, Hausknecht and Zhao (2016; Cope & Kalantzis, 2015).

The presentation focus on the Seeing The Entire Picture project (STEP) that investigates innovations in formative assessment in mathematics teaching through the use of automatic assessment technology, to provide teachers with diagnostic competence in everyday classroom assessment, and to provide students with online feedback. Specifically, the research and development investigate the potential of technology for making example-eliciting tasks (EETs) less demanding for teachers to manage, thereby supporting students' inquiry-centered learning.

EXAMPLE GENERATION IN MATHEMATICS TEACHING AND LEARNING

Mathematical reasoning is the backbone of many reform-based curricula and may include processes of conjecturing, pattern recognition, generalizing, comparing, contrasting, separating, and validating. Example generation is a crucial aspect of many of these processes, and as such it is both a process and an indicator of mathematical reasoning (Zaslavsky & Zodik, 2014). Watson and Mason (2005) have studied student "response spaces"—collections of learner-generated examples that fulfill a specific requirement. Given suitable exemplification tasks, such response spaces can provide insight into the correctness and richness of students' concept images (Sinclair, Watson, Zazkis, & Mason, 2011). EETs need to be focused on mathematically important conceptual structures, whose characteristics are revealed by the examination of multiple examples and of how they relate to each other. Such tasks can be especially powerful when they build on existing research, which has identified persistent alternative conceptions whose presence can be identified in examples submitted by students. Because student responses to EETs can provide teachers with information about a variety of concept definitions and concept images that students hold, teachers can use them to make instructional decisions. Yerushalmy, Nagari-Haddif, and Olsher (2017) have articulated certain design principles for such tasks, based on interactive diagrams that provide multiple representations of mathematical objects (e.g., functions, tangent lines), together with dedicated tools for generating examples that can be automatically assessed. Realized as design patterns (DPs), these design principles are conceived as ways to describe similarities across tasks in what they ask of students, and what can be algorithmically analyzed.

EETs are not yet commonly used as tools for formative assessment of students' conceptual understanding in classrooms. A possible reason for the infrequent use of such tasks in school is that tasks that can elicit many different correct and incorrect solutions are demanding for teachers to assess. With a wide range of potential solutions, students need elaborated feedback that goes beyond indications of correctness, and teachers must use, and often expand, their own mathematical knowledge to examine and provide feedback to each solution; the time needed to routinely review classroom responses is liable to overwhelm teachers.

EETs AS A TOOL FOR ONLINE FORMATIVE ASSESSMENT

STEP was designed and developed to support EETs that are formulated to be implemented as online assessment tasks based on specific DPs. Following Mislevy, Haertel, Riconscente, Rutstein, and Ziker (2017), the DPs developed offer a several different approaches that can be used to obtain evidence about reasoning processes. The STEP platform (Olsher et al., 2016) is built around interactive diagram (ID) that students use to produce and submit their examples. The IDs (the dynamic construction and tools) offer means for the exploration of tasks with multiple solutions. The two main components of STEP EETs are claims that define the goal and examples submitted as answers. The submitted example is an instance of exploration with the given diagram and tools. According to Buchbinder and Zaslavsky (2011), the presence of both the claim and of multiple examples can provide important insights into the learners' understanding of a concept, as well as of their understanding of the logic of the coordination of examples, and of claims that are central to mathematical argumentation. STEP makes possible the online monitoring of work on rich EETs, followed by analysis focused on the student's submitted work. At the same time, the student is expected to save, revisit, and review more self-generated examples and explanations in the allocated personal space. The submitted examples can be analyzed automatically to classify various characteristics of the mathematical object included in it, as well as heuristics that were probably used to construct them. Users (teachers, researchers, or developers) can specify the mathematical characteristics required of examples that are deemed correct, and the mathematical characteristics expected in typical incorrect examples. But because we cannot assume that the algorithmically determined indication of correctness of the answer is a comprehensive one, based on these predefined specifications, STEP can indicate only whether the examples meet task requirements. STEP characteristics, however, can provide information to help answer the following questions: What other characteristics do the solutions have, beyond the correctness? Do the answers represent familiar misconceptions? Do they indeed support the chosen claim, and if yes, how? How similar or different from one another were the examples generated by the same student or by different students? The elaborated feedback is constructed out of these characteristics and reported to the teacher in various formats (see Olsher et al., 2016, and Olsher, 2019, for details).

A *STEP* EXAMPLE

The following task analysis demonstrates the central aspects of designing assessment with EETs. The task lists four conditions: two conditions of relations between two quadratic functions f(x) and g(x), and two conditions of properties of g(x). f(x) is given by a function expression and graph: f(x)=-2x^2+4x+5. g(x) can be set using three independent sliders controlling the coefficients of the function expression. The goal is to find and submit three examples where g(x) and f(x) satisfy the maximum possible number of conditions. The answer consists of the marked chosen statements and the examples that demonstrate the choice. The conditions are: (1) The graph of f(x) intersects the graph of g(x) in exactly one point; (2) The two functions have the same symmetry axes; (3) g(x) passes through the origin (0,0) of the system; and (4) The function g(x) has a minimum. There are three possible triplets that fulfill the requirements. Here are examples for each triplet.

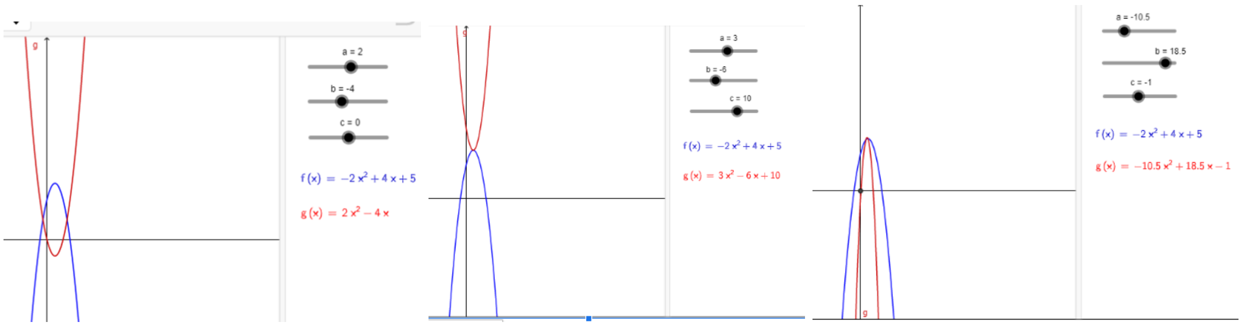

Figure 1. Answers submitted to meet the requirements of conditions 2, 3, 4 (left, g(x)=2x^2-4x), of 1, 2, 4 (center, g(x)=3x^2-6x+10), and of 1, 2, 3 (right, g(x)=-10.5x^2+18.5x-1).

Note that the submission presented in Figure 1 (right) 'looks right' and might be filtered as possible answer, whereas g(x)=-10.5x^2+18.5x-1 is incorrect. The platform collected and automatically analyzed the students' answers (Figure 2), based on predefined characteristics and predefined thresholds for accuracy (Table 1). The submission presented at the first row of Figure 2 is of a student who simplified the task and constructed a claim relating to two conditions. Two of the examples are correct instances of the claim: There is a function g(x) that passes through the origin and its graph intersects the function f(x) in exactly one point. In the three examples presented on the second row, the values chosen for the coefficients are accurate options that may suggest that the student attended to the symbolic aspects of g(x): g1(x)=1x^2-2x+8, g2(x)=10x^2-20x+17, and g3(x)=3x^2-6x+10. The submission presented in Figure 2 (middle of the bottom row) also suggests that the student focused on the visual aspects by adjusting the slider to graph a 'peculiar' graph through the maximum of f(x), whereas the function expression is inaccurate.

The criteria for online analysis may be organized into three groups, as shown in Table 1.

Table 1: A partial list of STEP analysis criteria

Meet the requirements: Marked vs. submitted example	**Example characteristics**: Of mathematical objects	**Example characteristics**: Of methods of construction - heuristics
Marked 3 valid triplets Submission includes Demonstration of the marked statements	The functions share symmetry line The graph of g intersects the origin f(x)=g(x) at exactly one point g(x) is a function with minimum The graphs do not intersect The graphs intersect in two points The graphs intersect in the extremum	Solved simpler task (less than 3 conditions) "Peculiar" example Sketch (looks right but not correct symbolically) Easy to compute coefficients

	The intersection is not in the extremum Marked invalid triplet Marked 4, 2 or 1 conditions Examples demonstrating the marked **and** additional conditions Example demonstrating **part of** the marked conditions.	

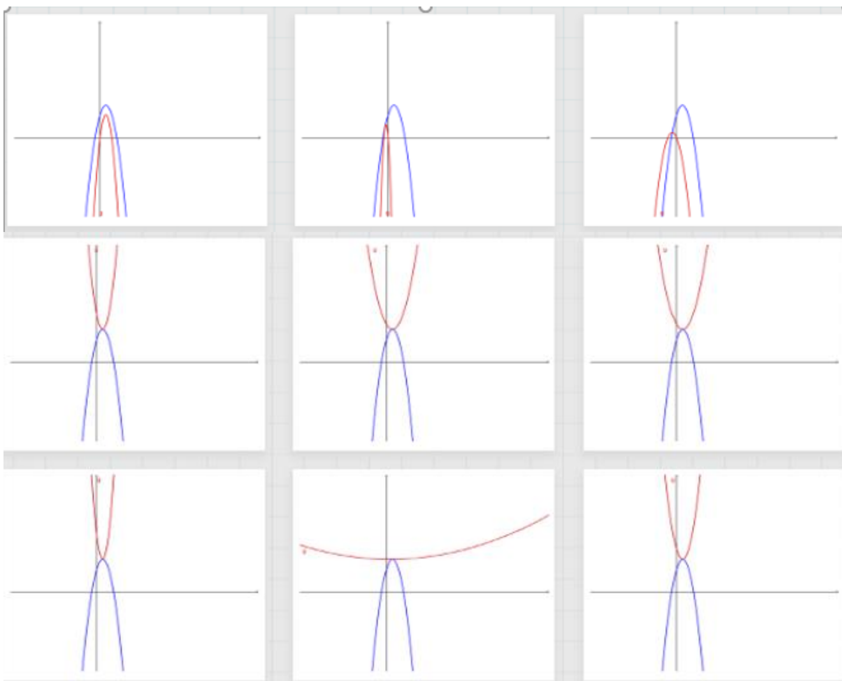

Figure 2. A collection of three submissions (a student's submission consists of one row)

WHAT IS IT AN EXAMPLE OF?

To summarize, the guiding principles and terms are: (a) each of the tasks requires the construction of example(s) for one or more statements; (b) the example is a static instance created by using the interactive diagram with specifically designed tools; and (c) the context of the example is set by a given or constructed claim, and the example could be provided by different mathematical input channels, which are part of the interactive diagram. In the task shown above, the requirement was to submit examples, but the claim is not explicitly stated. The task requires students to find out what is the maximum number of conditions that can form a claim, then support the answer by submitting existential examples. The interactive diagram is created in a multiple-linked representation (MLR) environment, and the means to create the example (construct the resulted graph) are sliders that can be dragged to independently change each parameter of g(x).

Returning to the question that guides our efforts: What are the structures and design patterns of tasks that could assess students' mathematical conceptions? Table 2 is an attempt to create a conceptual organization of the different tasks designed and studied so far. The demonstrated task is an example of the marked cell in Table 2. Next, I will describe the central aspects of this organization and explain why they are chosen as sources for assessment.

Table 2: Interactions that form the context of an answer to an EET

Mathematical content	**Provide evidence for:**			
	A given claim		**A given set of conditions**	
Input modalities	Provide an example(s)	Support or refute	**Specify the claim**	**Specify the claim** with reflecting feedback
Embodied: Supporting sensual attempts				
MLR: Supporting experimentation with reflecting feedback			X	

Assessing the interactions with input modalities

Using an interactive diagram in an MLR environment, and accepting answers in a variety of forms, encourages students to first "sense" the problem, control their actions and reflect on them during assessment, observe the effect of what they have created, and decide which of the examples should be submitted (Yerushalmy et al., 2017).

(i) The embodied input channel

such as sketching with a pen tool (Yerushalmy et al., 2017), fitting a graph by dragging a set of points (Nagari-Hadif, 2019), sketching lines or points related to graphs (e.g., tangent) using dedicated tools (Nagari-Hadif, 2017), using ready-made iconic constructs that serve as building blocks of graphs, or constructing and dragging soft constructions of geometric shapes or graphsFree-hand sketching (e.g., sketching with a pen tool (Yerushalmy et al., 2017), fitting a graph by dragging a set of points (Nagari-Hadif, 2019), sketching lines or points related to graphs using dedicated tools (Nagari-Hadif, 2017)) involves a sensory attempt that is as close as possible to a hand-drawn paper-and-pencil sketch, and can provide insights into students' conceptions and the mathematical resources they use. Each of these options may lead students to particular functions, constructions, or characteristics (e.g., continuity), and affect the learner-generated example space, even if other examples may be accessible in a different setting. The digital environment does not provide reflecting online feedback, but sketches are analyzed automatically, and STEP provides elaborated feedback consisting of the identified characteristics (Yerushalmy et al., 2017).

(ii) Input that generates reflecting feedback

The design of tasks based on MLR offers students an environment that encourages inquiry through systematic experimentation with embedded mirror feedback. For the designer of the task, MLR offers opportunities to control the interaction, for example, by determining the leading input representations, and by doing it in a way that reflects the purpose of the task (Naftaliev & Yerushalmy, 2017). For the designers of STEP assessment tasks, MLR is a source for characterizing the mathematics of the skills to be assessed. Below are a few design principles used and studied across cycles of design-based research:

a. Based on cognitive research related to MLR, we designed the assessment as an activity that progresses from using sensory/embodied knowledge, manipulating soft constructions or freehand drawings, toward experimenting with the interactive diagram to produce robust constructions or generalization with mathematical symbols. MLR has been the appropriate setting for designing a task that presents a cognitive conflict between representations that teachers may find valuable.

b. We designed the assessment to create learning opportunities in the course of experimentation. For example, the challenge of constructing an accurate symbolic representation, when stated and assessed within the MLR environment, should be considered a constructive assessment activity that can engage students in productive trial and error.

c. We designed the assessment in a way that attends to the student's preferences. Giving a choice of input types makes it possible to assess student's preferences, suggesting where each student's strengths lie. For example, when given the choice of sketching or using a symbolic function expression, many students prefer sketching, even if they do not necessarily provide examples that fit the requirements of the task. This design also allows assessing the tendencies of an entire class, or to follow a student along several activities and identify whether this tendency amounts to a habit.

Assessing the interactions with the given claims

The tasks that Yerushalmy and Nagari-Haddif studied (Nagari-Haddif & Yerushalmy, 2015; Nagari-Haddif, 2017) fall into three main categories: proving existential claims by means of appropriate examples, refuting universal claims by means of counter-examples, and demonstrating a definition by means of examples and non-examples. Although the first two categories (which are logically equivalent) require only a single example, in some cases students may be instructed to submit many different examples. What students consider "different" is in itself an indicator of mathematical reasoning. Certain types of examples suggest a richer concept image. These include degenerate functions or non-prototypical functions, and the attributes of the mutual relations between mathematical objects (e.g., an altitude coinciding with one of the vertices of the triangle). Students' choice of representations and transitions between them also serve as an indication of proficiency, strategy preference, or both. According to a key design principle of EETs, the number of examples required is part of the definition of a correct answer. The majority of STEP tasks ask for submission of a few examples as an answer. Requesting three different supportive examples enriches the response space, making possible an automatic analysis that distinguishes between the different characteristics appearing in each response and in the group response space. This design encourages students to move out of the comfort zone of the first immediate example, with the freedom of doing it in various ways. The request for several different examples also aims at motivating students to search for special or "peculiar" examples (in the sense of Mason & Watson, 2001), such as boundary examples or degenerate cases. The search for such examples must involve observation of the wide range of possible examples.

Assessing students' constructions of claims

The task described above did not provide a claim that needed to be exemplified. It is an example of another category of STEP DPs, characterized by tasks that require students to decide what can be constructed into an existential claim based on the given conditions, statements, and information. Such a decision must involve more than mere exemplification reasoning. In particular, it requires classification skills needed to establish a conjecture that could be a valid existential claim. Students experience the properties stated in a given condition by interacting with the ID, after which they may complete the converse or complementary process of classifying all the functions having those properties. Next, they may decide which of the given conditions can be grouped into a compound class of properties, and state a compound existential claim. Doing so in different situations may be considered an essential mathematical reasoning process (Mason, 2001), and it extends the range of example-based reasoning skills that we attempt to assess. To support the processes involved in formulating the claim and to decrease the cognitive load of analyzing the different classes that each condition defines, we developed another DP that allows stating the claim by choosing from a given set of relations. In this DP, each checked relation is reflected in the interactive diagram in a way that limits the domain of examples to those for which the checked relation is true, enabling interaction with more complex mathematical objects..

FURTHER LOOK AT USES OF EXAMPLE-BASED FORMATIVE ASSESSMENT

This concluding section looks at studies suggesting innovative perspectives on the use of example-based assessment for offloading part of the burden of teaching inquiry-based mathematics. Briefly, we will look at using technology to facilitate learning formats that make it easier for teachers to support personalization of learning. Whereas personalization refers to supporting the agency of students over ideas by providing online domain-specific evidence to the teacher, and providing elaborated conceptual feedback to the student.

Adaptive example-based assessment

Adaptive assessment methods are formative in that they adaptively adjust assessment level to the students' level or to their personalized learning preferences. Luz and colleagues (for details, see Luz, 2019, and Soldano, Luz, Arzarello, & Yerushalmy, 2019) suggest a different approach to adaptive assessment of an inquiry-based learning process. We introduced a dynamic geometry environment (DGE)-based activity that prompts students to generate examples for an unknown statement whose properties can be gleaned from the construction in the DGE. We asked students to find strategies that would lead them to the desired result (e.g., a conjecture or a proof of a conjecture that describes the statement embedded in the construction). Rather than starting with a claim, here the process starts with an activity that generates examples; for example, competing in an inquiry game in which the student and the computer assume different roles, is a way that motivates the generation of examples. Students go back and forth between cycles of identifying and formulating patterns and generalizations, as they struggle with conditions formulated in mathematical or natural language.

Evidence-based discussions

When using rich tasks in the classroom, teachers usually base the discussions either on their expectations of the student's work, or preferably, on actual student examples. Several studies explored the different ways in which teachers used topic-centered learning analytics. Olsher and Abu-Raiya (2019) claimed that when teachers have access to analytics regarding the mathematical characteristics of student answers, which are not limited to student mistakes, they use these insights in classroom discussions, expanding the range of discussions based on student work beyond correctness. Another example (Olsher, 2019) describes how a teacher uses online assessment to classify different examples into categories in a guided inquiry lesson. The classified examples are then used in a class discussion to refine student conjectures. Another opportunity for inquiry discussed by Hess-Green & Olsher (2018). The interactions with many, at times extreme, examples often generate opportunities to attend to the definitions. As all the examples submitted are of a mathematical object, and as mathematical objects are well defined, the automatic assessment of the examples requires precise specification of the algorithm used for the identification of these mathematical concepts.

Automatic grouping

As suggested above, fostering learning by inquiry with a DGE in the mathematics classroom is often inhibited by the teachers’ limited ability to interpret and support learning by several students at once (Clark-Wilson & Noss, 2015). Group learning is one way to foster DGE-based inquiry learning (e.g., Stahl, 2009). At the same time, the formation of the groups is yet another activity that requires investment on the part of teachers. Grouping students is usually done over extended timeframes and often with minimal considerations of the students’ work on a given task (Lou et al., 1996). Abdu, Olsher, and Yerushalmy (2019) studied the use of the STEP platform for multi-faceted automatic analysis of student answers to a single activity, to create a method that groups them in ways that are mutually beneficial to all students in the group, and provide teachers with recommendations on how to group students. This is part of a larger project, in which it may be possible to offer the teacher suggestions based on automated grouping using STEP data on students’ work.

A FINAL NOTE

The pedagogical opportunities of research and implementation of Learning Analytics (LA) in the domain is yet to be explored. Shifting the attention from the big data to design of models that will allow LA be part of pedagogical *interventions* (rather than *implementation*s as termed by Wise et al., 2016) has the potential to introduce an important change in formative assessment.

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AN ONLINE RESOURCE TO INSPIRE LEARNING AND REFLECTION ON TEACHING: THE POTENTIAL OF THE SER

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This paper will outline a new STEM Education Resource (SER) which was initially targeted for use with year 9 and 10 students. The SER aims to 'inspire' learning by developing students' independence and their capacity to plan and conduct real life investigations. The challenge is to ensure that there is adequate focus on 'M' in a STEM investigation and this is going to be evidenced through students' analysis, interpretation and presentation of evidence to support conclusions. The design of the resource can prompt teachers to consider ways to develop the investigative skills of their students, thus there is the potential to 'inspire' teachers to reflect on their teaching through use of the resource.

Keywords: Mathematics education, STEM, secondary school

STEM IS A FOCUS IN AUSTRALIA

Australia has a national STEM education strategy, which is in place from 2016-2026 (Education Council, 2015). In Australia, education is state based so the existence of a national strategy which has the support of all states highlights the importance placed on improving STEM outcomes across the country. There are three areas of national action in the STEM education strategy:

- "Increasing student STEM ability, engagement, participation and aspiration" (p.8)
- "Increasing teacher capacity and STEM teaching quality" (p.8)
- "Building a strong evidence base" (p.9)

Thus, there is a national imperative to improve STEM outcomes, increase the number of students studying STEM in later years of secondary schooling and there is recognition of the importance of building teacher capacity in order to achieve these goals.

OVERVIEW OF THE SER (STEM EDUCATION RESOURCE)

The goal of the SER was to provide an online resource with the capacity to improve the investigative competence of year 9 & 10 students through supporting students in conceptualising and carrying out STEM-based investigations. The SER had the potential to inspire learning and teaching by:

- developing students' independence and their capacity to plan and conduct real life investigations.
- prompting teachers to reflect on their teaching through consideration of use of the SER to develop the investigative skills of their students.

It was necessary to consider design features which could promote and develop students' independence in planning and conducting real life investigations. Independence was fostered by requiring students to make decisions, implement their approach to solve a problem and then articulating their thinking. For example, the inbuilt functionality of the SER prompted practices such as consideration of how to unpack a given scenario (which was pre-populated into the system by the teacher), followed by student identification of a problem to solve based on the scenario. Appropriate data to answer the problem then needed to be identified, as well as strategies to find or collect the required data. The SER supported group collection of data, however, students within a group could choose which subset of data they would use in their analysis. Decisions needed to be made about how to display selected data to assist in finding an answer for the problem. Independence was also fostered through the ability for students to keep a record of their thinking, findings and interpretation through the online system. Requiring students to decide on evidence to support their solutions and the production of a report to substantiate a solution to the given problem encouraged students to consider what is necessary to appropriately communicate a solution.

OVERVIEW OF DESIGN AND TRIALLING OF SER

Figure 1 provides an overview of the design and classroom trials of the SER. The SER was developed in an iterative fashion, where features were modified or changed as the system was developed. It was trialled in two teacher workshops and during the classroom trials. In the first teacher workshop there were ten teachers from the three project schools. Three teachers volunteered to use the SER with one of their classes and these teachers were present in the second teacher workshop. In between trials with teachers and students there were multiple iterations in response to project team trials.

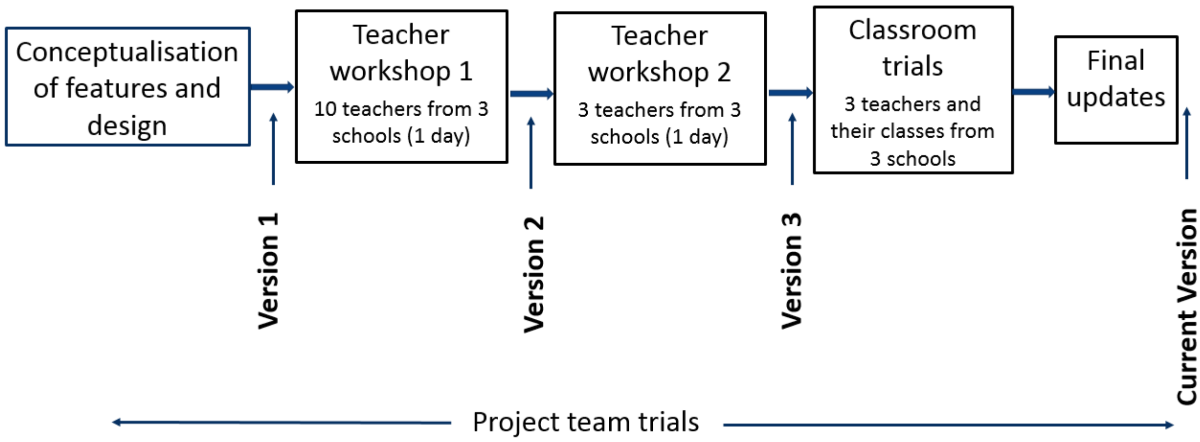

Figure 1. Overview of design and classroom trials

USER INTERFACE

There were design decisions in conceptualising both the content of the resource and the structure of the user interface. The user interface would impact engagement with the SER, so the system needed to be easy to navigate and use so that the educational goals could be realised. Decisions were made about which features would be fixed and unable to be adapted by the teacher and where there would be choice of features or the facility for the teacher to add tasks to focus students on particular aspects of the investigation.

Figure 2 shows part of the teacher interface, where teachers had the opportunity to create master versions of projects which could be shared with other teachers in the same school ("School projects"), access active projects being used with current classes ("My active projects"), set up sensors and Raspberry Pis for class use ("Admin") and organise student groups to work on projects. In order to support classroom implementation of the resource the team wanted to provide a resource for teachers where aspects could be customised to enable teachers to implement a range of investigations for students.

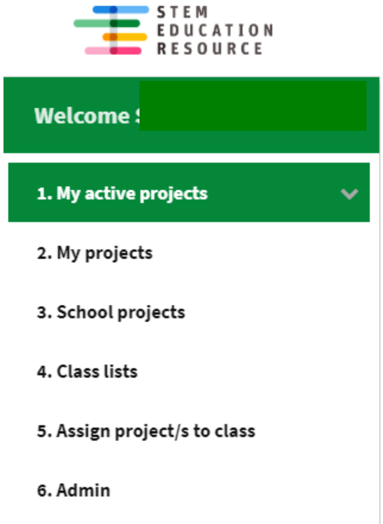

Figure 2. Overview of the teacher interface menu

To cater for a range of implementations of STEM the SER was designed with the functionality to be used by students in one subject (e.g. in their maths class) or across several subjects (e.g. where the data and work for a project could be accessed by students online in multiple classes). Enabling projects to be shared across classes supported interdisciplinary projects.

The student interface consists of five modules which support students in identifying a problem, collecting appropriate data, analysing and interpreting data, providing a solution supported by evidence and reflecting on the investigative process. Students can view their work for all modules and print a PDF of their written responses and any saved graphs, etc.

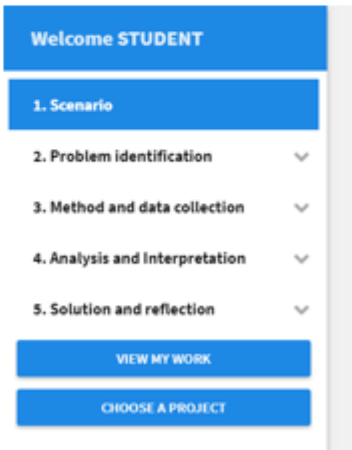

Figure 2. Overview of the student interface menu

Each module provides students with information to support them in their investigation. For example, in the “Problem identification” module students have to identify a problem to solve based on the scenario provided by the teacher. As part of this, students identify the explicit and implicit information to help them to understand the scenario. To support this, students are provided with an explanation about explicit and implicit information and they are then asked to identify and record this information for the scenario they are investigating. Students are expected to state a problem to solve, based on the scenario, and make a conjecture to predict what the answer might be.

In module three, students need to identify and collect data to assist in solving their problem; this data can be from a variety of sources including existing data sets, measurements from sensors or data collected via surveys, etc. When completing the modules students can collect data, produce graphs/tables/summary statistics and record their thinking and reflections on the investigation through the given prompts.

In the final module students produce a solution to their problem, supported by evidence from within the other modules. In this module students are asked to reflect on their solution, as well as the investigation as a whole. Students are asked to comment on 'things I know for next time' and advice they would give a friend who had to do the same investigation. This reflection is intended to promote metacognition about the investigative process.

SER SUPPORTING CLASS DISCUSSION

Technology displays have been identified as playing an important role in enabling discourse in the mathematics classroom (Ball & Barzel, 2018). The SER supports communication by enabling groups of students to access common datasets, so that they can compare and contrast representations and analysis of data with other members in their group. The ability to view technology displays can facilitate student discussion about selection of appropriate subsets of data to answer a given problem, choice of graphs, tables and statistics to provide evidence for a solution and reasoning to communicate thinking.

In addition to facilitating discussion in small groups, the SER can also enable teachers to promote discussion at a class level. Classroom display of student work using technology has been shown to support engagement with mathematics through consideration of cases which prompt mathematical discourse (Clark-Wilson, 2010). Using the teacher interface, it is possible for the teacher to access the data collected by all groups of students, as well as each individual student's responses to prompts requiring written reasoning. The ability for the teacher to show up to four displays at one time using a split screen (Refer to Figure 2) can provide prompts for class discussion through:

- multiple representations of one data set, to compare the appropriateness or usefulness of different graphical representations
- graphical representation of multiple data sets for comparison
- displays of tables of data or summary statistics
- display of student written responses for discussion (accessed through 'View prompts')
    - this can facilitate discussion of effective reasoning and use of mathematical terminology and ideas to support the investigation

Metacognition can be promoted through reflection on strategies (Baker, 2013); in this case the choice, representation and analysis of data can be used to promote student reflection about their strategies for finding a solution to their problem. Mathematical discourse can be promoted through consideration of choices when displaying data, as well as what the 'best' graph may be for answering a given problem and through comparison of students' reasoning for responses to prompts.

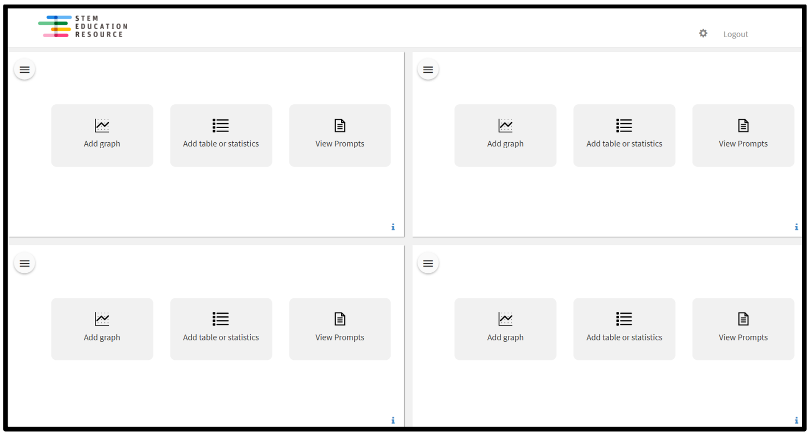

Figure 3. Four data displays - prompt class discussion

MATHEMATICAL FUNCTIONALITY

English (2015) highlighted the need to ensure that mathematics had appropriate importance in STEM. One challenge in developing the SER was to ensure that there was the potential to support a focus on 'M' in STEM investigations. The actual investigations were to be determined by the teacher, so the design features of the SER needed to support the use of mathematics in investigations. Major functionality identified as crucial to investigation was the ability to collect and tabulate data, produce graphs and find some summary statistics. This approach was supported by English (2016), in a commentary on STEM education, when she suggested that mathematics has a key role in enabling students to work with and critically interpret data in context. The SER was designed to support such data interpretation through the functionality within the system.

The SER does not contain a computer algebra system (CAS), or a calculator, so if students want to use these technologies to support their investigation they will need to be used independently of the resource. The ability to upload pdfs enables students to include screendumps, graphs or images from other technologies, as well as hand drawn graphs or pen-and-paper solutions to problems. This can assist in students' communication of the reasoning for their solutions to problems.

CLASSROOM IMPLEMENTATION OF THE SER

To prepare for classroom implementation of the SER it was necessary to introduce teachers to the features of the latest version of the SER during the second teacher workshop. The intention was to ensure that teachers were familiar with any updated functionality and the use of data collection tools, namely the sensors and Raspberry Pis. Teacher workshop two also had time for teachers to develop draft scenarios for use in their schools. This gave teachers an opportunity to discuss potential scenarios across the three schools and to work on descriptions to be used with their students.

The SER was trialled in three classes. The participants were three teachers from three different schools and the students in the classes where they used the SER. The three classes were: a year 8 class, a year 9 class and a group of students drawn from year 7 & 8 classes (in one school).

SNAPSHOT OF STUDENTS' USE OF SER FROM THE TRIAL

This section provides a snapshot of some initial findings from the trial of the SER. Sample student responses are presented to highlight some potential of the SER to foster investigation and promote communication about investigations. It is important to note that there were limitations in the data collected, as not all students had the opportunity to complete all aspects of the given tasks, due to timetabling constraints, but for the students who did manage to complete all tasks the sample responses show that there is potential for the SER to provide support for investigations involving data.

Graphs prompted classroom discussion of mathematics in context

The inbuilt functionality of the SER supported analysis of data and production of graphs. When students included graphs to support their investigation, the SER prompted them to explain what the graph shows, as well as explain how the graph contributes towards a solution to their problem (Figure 4). These two aspects ensured that students were considering any graphs produced in the context of the entire problem, not in isolation from the problem.

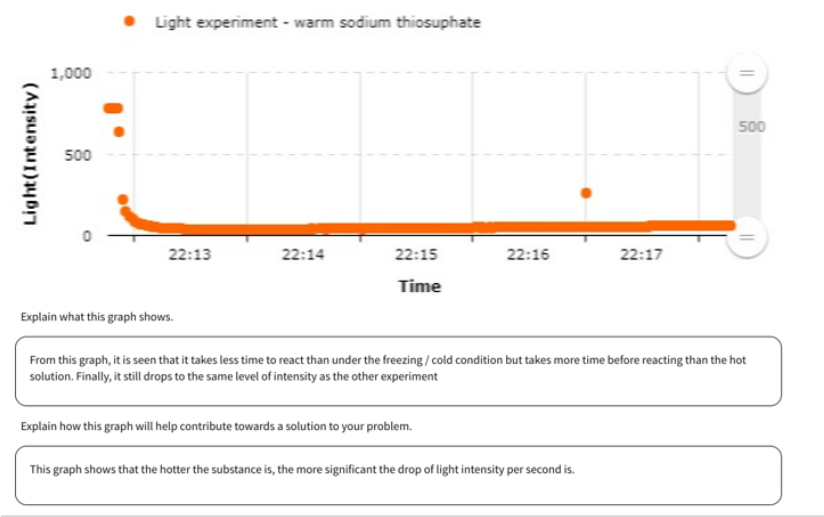

Figure 4. Sample student graph with responses to two prompts

Students identified further opportunities for analysis, including alternative displays and calculations. This implies that students engaged with the investigative process, considering how well the chosen displays helped with providing a solution to their problem.

Having students give written responses to prompts appeared to be positive and links to mathematical language were evident in some responses. One student used understanding of the shape of the graph of exponential functions to describe results, even though exponential functions had not been covered in mathematics yet.

> "…The results collected…seem to defy the normal and expected by going back up again after reach[ing] it's lowest point. This means it is most likely that there was an uncontrolled variable in there (sic) practical as most results took on the shape of an exponential. The main trend seen in the rest of the data is this steep slope from the moment the liquid is introduced. This slope looks steeper the hotter the liquid is."

This suggested that where problems afforded students the opportunity to use mathematics other than the current topic, the SER could have the potential to support this through the structure of the prompts presented to students.

Student reflection on the investigative process

One goal of the SER was to develop students' independence and their capacity to plan and conduct real life investigations. Achievement of this goal relies on students being able to reflect on both the current investigation and the investigative process, thus drawing on metacognitive knowledge. As students complete the modules there are points where they are asked to reflect on their work and on the investigation. In the small trial carried out to check the efficacy of the SER there were a range of reflections produced; some of which showed that students were being critical of their choice of data display, in the context of providing good displays to enable the problem to be answered effectively. For example, one student noted the need to consider choice of data sets, while another noted the need to collect multiple data sets to support the solution of a problem:

> "In order to improve the display of data, it would be better if my set(s) of data are combined together and compared at the same time, figuring out the difference of the reaction time of different experiments."

> "Try collecting multiple data sets of data so that you have enough data to work with and solve your problem".

The importance of accuracy in dealing with data and providing appropriate evidence to support solutions was emphasised:

> "Furthermore, during the data collection process, it is important to keep the consistency, accuracy and precision at all times. When answering the question, use graphs and tables to interpret the data and support your "argument" or "hypothesis"."

In reflecting on what students would need to know for the next time they used the SER, there were comments that focussed on aspects of a mathematical investigation, namely the need to understand a problem, collect appropriate data and use mathematics to support answers.

> "Firstly, it is highly essential to understand your aim and what you are going to investigate. This means you have to firstly identify and analyse your problem, trying to "problem-solve" it."

> "Make sure the evidence you provide an actually support your claims"

These comments suggest that the SER prompted consideration of more than technical issues, which might be expected when students are learning to use a new technology. A focus on evidence supporting the claims made could be capitalised on when students are working on other mathematical tasks where reasoning needs to be supplied for a solution to a problem.

CONCLUDING REMARKS

The SER was found to be useful, with positive feedback from teachers and students. The engagement by teachers and students was generally high, with teachers very keen to use the SER in subsequent years.

Based on the data we have we can't say that the SER improved investigative skills, as we have not collected longitudinal data at this stage. But, students' reflection on the investigative process points in a positive direction. One challenge for use of the SER is in developing scenarios at an appropriate level where students can have genuine choice of problem to solve.

The initial trials of the SER suggest that there is potential for capacity building with regards to students' ability to carry out investigations. Future investigations will consider types of scenarios to be used with the SER to focus on M in STEM; an important consideration if we are to include STEM investigations in secondary school.

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INSPIRING LEARNING AND TEACHING OF FUNCTIONAL THINKING BY EXPERIMENTS WITH REAL AND DIGITAL MATERIALS

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Inspiring learning means to aim for promoting skills, knowledge, and interest. In a pre-post-test intervention study (N = 282, two experimental groups: real materials vs. digital materials, control group) we investigated whether experiments with real materials or digital materials based on GeoGebra are an appropriate measure to promote the functional thinking of sixth graders. Even though both types of material led to a significant increase in functional thinking, the increase of the digital material group was significantly higher. Digging deeper into data we found, that the use of either material has a different impact on the learning of functional thinking. To inspire teaching we use a concept of long-term in-service teacher training, were the analysis of video vignettes from student work processes within our online learning environment ViviAn is one measure among several others.

Keywords: teaching-learning laboratories, functional thinking, experiments, real and digital materials, video vignettes

1 TEACHING-LEARNING LABORATORIES INSPIRE LEARNING AND TEACHING

In German-speaking countries, teaching-learning laboratories (In German they are called "Lehr-Lern-Labore" (Priemer & Roth, 2019).) are getting more and more common when learning and teaching are to be inspired for students and pre- as well as in-service teachers at universities, especially in STEM-subjects. Research-based learning is a cornerstone of the work in teaching-learning laboratories. The Mathematics Laboratory "Maths is more" (cf. www.mathe-labor.uni-landau.de) at the Landau campus of the University of Koblenz-Landau comprises three closely linked pillars that are necessary components of every teaching-learning laboratory (cf. Roth 2019):

(1) It is initially a students' laboratory in which entire school classes work in groups (four students each) within three 90 minutes slots on a curriculum topic in the sense of research-based learning (cf. Roth & Weigand 2014). On the basis of workbooks which contain written work instructions and in which the students record their procedures and results (cf. Roth, Schumacher & Sitter 2016, p. 195), the learners work independently with real and digital materials in learning environments according to Vollrath & Roth (2012, p. 151). The preparation and follow-up of the laboratory work take place in mathematics lessons at school and are supervised by the mathematics teacher.

(2) In addition, the Mathematics Laboratory "Maths is more" is a research laboratory to which (almost) all research activities of the working group Mathematics Education (secondary level) in Landau refer. This applies in one dimension from a pure perspective (Basic Science) with the purpose to understand the nature of mathematical thinking, teaching and learning to an applied perspective (Engineering) with the purpose to use such understandings to improve mathematics instruction (cf. Schoenfeld, 2000) and in another dimension from research on teaching mathematics in schools to university didactic research. A research project from this spectrum is presented in section 2.

(3) Last but not least, the Mathematics Laboratory "Maths is more" is also a teaching laboratory in which pre-service teachers apply and reflect their theoretical knowledge and skills in a practical way in the sense of research-based learning. They design learning environments for the students' laboratory based on their knowledge of mathematics and mathematics education they gained at university

and support as well as diagnose students’ work on those learning environments. One means to interconnect practical work in the Mathematics Laboratory “Maths is more” to courses [1] for pre- and in-service teachers is the use of our self-developed video-tool ViviAn described in section 3.

2 INSPIRING LEARNING: EXPERIMENTS WITH REAL OR DIGITAL MATERIALS

Inspiring learning means to aim for promoting skills, knowledge, and interest of students. Especially in the case of topics that are relevant during the whole mathematics curriculum like functional thinking, it is important to inspire learning. Functional thinking can be described on the one hand by means of the three aspects mapping, covariation, and function as an object (Thomson 1994, Vollrath, 1989). On the other hand, functional thinking of students can be concluded if they are able to deal appropriately with forms of representation for functions like tables, graphs, formula, and situational descriptions and to change and translate between those representations (Nitsch, 2015). Experiments give the possibility to facilitate students’ understanding of functional relationships and foster their functional thinking by a scientific discovery process. This process includes generating hypotheses, testing hypotheses by performing experiments, and reflecting the results (De Jong 2005; Reid et al. 2003). Besides the use of real materials, studies also investigate and recommend the use of digital materials (Goldstone and Son 2005; Jaakkola et al. 2011). For an overview of arguments in favor of the use of real materials or digital materials respectively to foster functional thinking see Lichti and Roth (2018). Despite those existing arguments it has not been clear yet, whether (1) learning environments based on experiments with real materials or digital materials (GeoGebra) developed to foster the functional thinking of sixth-graders lead to significant effects on their functional thinking and (2) if those effects differ in both settings. Furthermore, we wanted to find out, if (3) learning environments based on experiments with real materials have a different effect on functional thinking than experiments with digital materials (GeoGebra). We decided to do an intervention study with students at the end of grade six as we wanted to be sure that it is our intervention, that eventually triggers the development in functional thinking. As in German mathematics curricula explicit teaching of functional relationships begins with grade seven, students would not have heard about it during mathematics lessons.

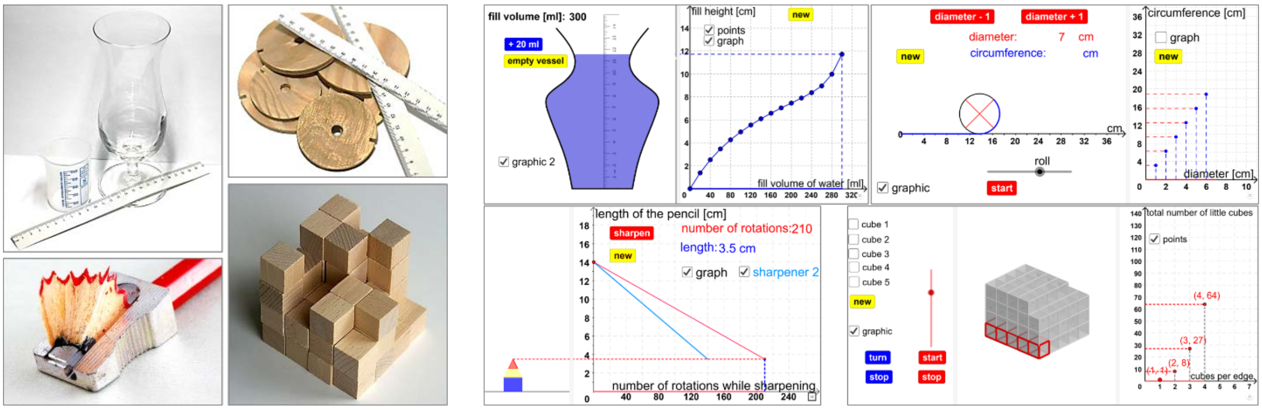

Figure 2.1: Four situations namely filling vessels, rolling circles, sharpening pencils, and building cubes, have been chosen for experiments with real materials (left) and digital materials (right)

To deal with the mentioned research questions four contexts have been chosen that could be used to perform experiments with real and digital materials respectively (cf. Fig. 2.1): (1) *Filling vessels* → relationship between the fill volume and the fill height of a vessel; (2) *rolling circles* → relationship between the diameter and circumference of a circle; (3) s*harpening pencils* → relationship between the number of rotations while sharpening a pencil and its remaining length; (4) *building cubes* → relationship between the number of little cubes fitting on the edge of a big cube and the number of little cubes needed to build the big cube. They cover different functional relationships, do not focus on linear functions only (de Beer et al. 2015) and offer the opportunity for the students to use the same actions and procedures when dealing with both types of material. This was essential, as two experimental groups namely *real material* (EG1) and *digital material* (EG2) had to be compared. Furthermore, they led to experiments that were practicable for students of grade 6 in both settings. It would for example not have been feasible for 30 students aged 11–12 years to experiment with burning candles at the same time. The complexity of the digital material also had to be appropriate for the students, as they should be able to use it intuitively.

After choosing contexts, we designed identical or at least equivalent tasks for both settings to guide the students through the experiments. We designed those tasks aiming to cover the three aspects of functional thinking according to Vollrath (1989) and using tables, graphs, and situational descriptions as forms of representation for functions. The syntactical form of representation (formula) was omitted, as students of grade 6 are not yet familiar with it.

The students were guided through the intervention using different types of tasks. We distinguished tasks for (i) estimating, (ii) experimenting, (iii) understanding the respective context, (iv) understand-ding the graphic form of representation, (v) applying (related to the results of the experiment) and (vi) transferring (tasks to the same context that go beyond the experiment). The tasks of the contexts at the beginning of the intervention focused on the task types (i)-(iv). The tasks on contexts of the second half of the intervention focused on types (v) and (vi). It was important to make the tasks in both settings equivalent or, if possible, identical. In addition, it had to be ensured that the students did not come to completely wrong conclusions. Therefore, there were help cards and solutions integrated into the sequence of tasks. The entire setting - the real materials, digital materials, and tasks - was tested for its usability and equivalence in a preliminary study. For this purpose, 30 students worked in groups on the tasks using the respective medium while they were being filmed. The video recordings were evaluated and based on the results the media and tasks were optimized.

For more information concerning the tasks as well as the way in which the digital materials were carefully constructed using GeoGebra (www.geogebra.org) to meet the requirements of the inter-vention study see Lichti and Roth (2018)

In a preliminary study, it was examined whether the real material, digital material, and tasks are suitable for students aged 11-12 and whether the processing time differs depending on the choice of medium. It turned out that the students need the same time to solve the tasks, regardless of the choice of medium. Based on the results of the preliminary study, an intervention study in a pre-post-control-group design was carried out shortly before the end of the school year 2015/16 ($N = 282$). 11 classes were involved in the intervention study. The students of each class were randomly assigned to the two experimental groups (EG1: $N = 111$; EG2: $N = 123$). Two further classes formed the control group (CG: $N = 48$). Figure 2.2 provides an overview of the intervention study.

A test to measure functional thinking was constructed and validated in which functional thinking was operationalized according to the three above named aspects, using tables, graphs, verbal descriptions as forms of representation in which functional relationships occur. The appropriate use of and change between these forms was considered an indication of functional thinking. After developing items to test functional thinking, their fit to the operationalization was controlled (expert rating: $N = 2$, $\kappa = 0.86$). Thereafter, a test was created and implemented among students at the age of 12 to 13 which is grade 7 in Germany ($N = 221$). Using item response theory, we controlled for Rasch scalability. Our test showed an expected a posteriori (EAP/PV) reliability based on plausible values of EAP/PV = 0.77. For further information on the test and the test-items see Lichti and Roth (2019).

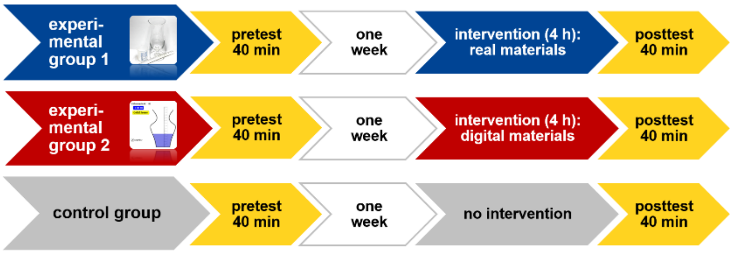

Figure 2.2: Setting of the intervention study: Students of 11 classes were randomly distributed on class level to two experimental groups, two further classes formed the control group ($N = 282$).

The students who participated in the intervention worked 4 school hours (45 minutes each) in individual work on equivalent tasks to functional contexts, only the material (real vs. digital) they used differed. Directly after the intervention, the post-test was processed and the data collected in the pre- and post-tests were evaluated using item-response theory. By means of virtual persons, the item difficulties were estimated, whereas the personal ability values were estimated by means of a 2-dim. Rasch model and 10 plausible values (Rost, 2004). A mixed ANOVA (between-Factor: Intervention, within-Factor: Time) (Field et al., 2013) was then calculated to compare functional thinking in pre-test and post-test.

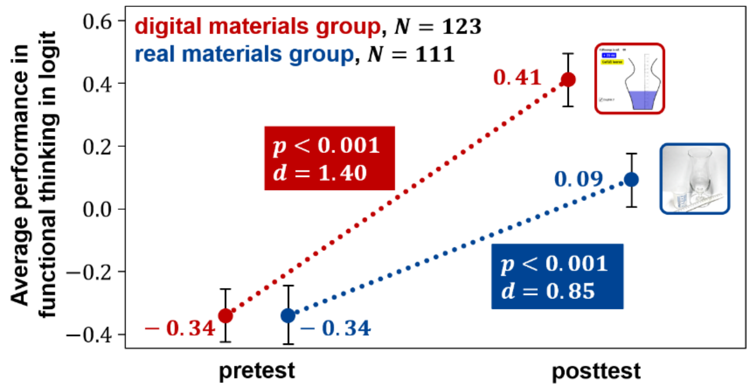

Figure 2.3: Mean test results in the functional thinking test presented in Logit (Effect size: Cohen's d)

The *control group* ($N = 48$) was analyzed with a paired Wilcoxon-Signed-Rank test. The analysis showed that the test without intervention did not have a significant effect on students' functional thinking ($V = 423$, $p = .091$, $d = .26$).

Experimental groups (EG): The mixed ANOVA resulted in two significant effects: First, there was a main effect of time $F(1, 11.79) = 36.90$, $p < .001$, $\eta_p^2 = .554$. Therefore, the functional thinking of the experimental groups considered as one group increased significantly with a large effect from $M = -.34$ logits ($SD = .035$) up to $M = .25$ logits ($SD = .06$). The pairwise t-test with Bonferroni correction led to the following results: The EGs did not differ before the intervention, but the functional thinking of both groups increased significantly from pre- to post-test (*real materials group*: $t(110) = -9.42$, $p < .001$, Cohen's $d = .85$; *digital materials group*: $t(122) = -16.46$, $p < .001$, Cohen's $d = 1.41$). Second, based on the mixed ANOVA there was an interaction effect between time and EG ($F(1, 25.820) = 8.856$**, $p = .006$, $\eta_p^2 = .090$). In comparison, the increase in both groups from pre- to post-test is significantly different (cf. Fig. 2.3). The functional thinking improved significantly more in the digital material group (EG2) than in the real material group (EG2) with a medium effect ($\eta_p^2 = .09$)

To sum up, the functional thinking of students in grade 6 can be promoted using real and digital materials. Although both types of material generate a significant increase in functional thinking, the increase generated by digital materials is significantly higher. There seem to be differences in the way both types of material influence functional thinking. In order to understand the outcomes, a closer look at these differences is necessary. Consequently, the consideration of processes in functional thinking seemed appropriate. The following analysis of written processing results of students on tasks in functional contexts under consideration of the influence of real and digital materials is done to provide answers to this question.

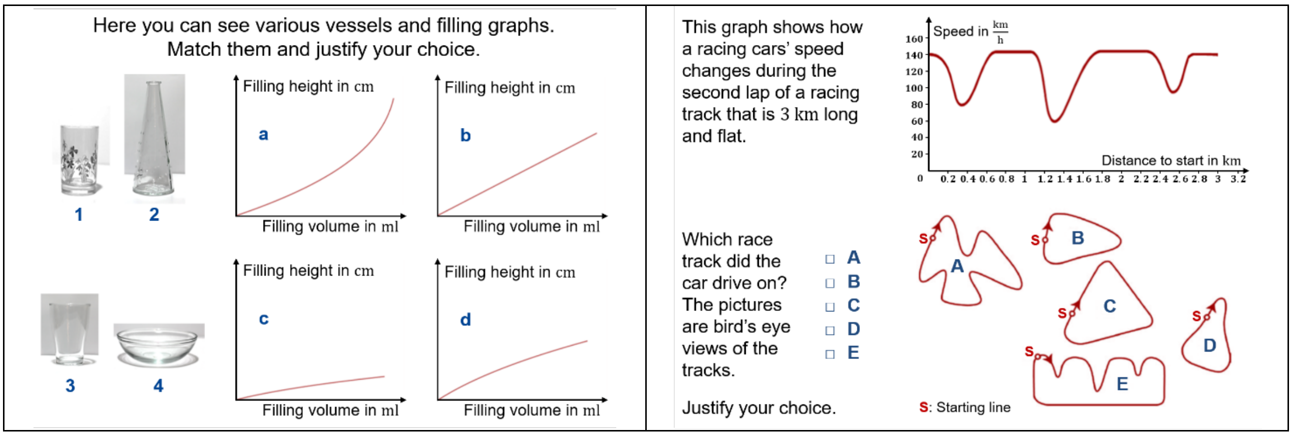

Figure 2.4: Task *filling vessels* (left) and task *racing car* (right)

The task *filling vessels* (cf. Fig. 2.4 left) was used during the intervention and the task *racing car* (cf. Fig. 2.3 right) was part of the follow-up test. *Filling vessels* requires the allocation of images of vessels (photos) to the corresponding filling graphs, *racing cars* the allocation of one of five race tracks to the corresponding speed graph. In both cases a depicted situation (photos, abstract representation of the race track from a bird's eye view) must be brought together with the respective graphic representation of the underlying functional context. In addition, the students had to justify their assignment. These justifications were examined by means of qualitative content analysis (Mayring, 2008), in which inductive categories were formed and confirmed by experts. Two category systems, one for each task, were created. Two raters used those category systems to code the students' justifications. Individual arguments could be provided with several categories. The interrater reliability was checked with Cohens Kappa (κ). The categories (1) shape of the vessel (κ = .97), (2) course of the graph (κ = .87), (3) state (κ = .97) and (4) change (κ = .93) were relevant for the task "filling vessels". While (1) and (2) reveal themselves in terms of content, in (3) the students had an actual state in view (the graph is steep.), in (4) they focused on a change (the vessel becomes narrower towards the top.) For the task of racing cars, the categories (1) speed and curves (κ = .92), speed (κ = .93), type of curves (κ = .87) and (4) graph-as-picture error of interpretation were used. (1) includes that the students argued with their everyday knowledge about the relationship between curves and speed (a car has to drive slower in curves), the categories (2) and (3) describe that the students worked with speed based on the graph or recognized that the track they were looking for had to have three different curves. (4) contains a typical misconception with the graph-as-picture error. This was followed by a χ^2 test comparison of category frequencies between students who had worked with real or digital materials to promote their functional thinking.

The comparison of the frequencies of the categories of the task *filling vessels* showed that students of the *real materials group* argued significantly more frequently with the shape of the vessel ($\chi^2 = 4.16$, $p = 0.04$) and states ($\chi^2 = 4.36$, $p = 0.04$). The students in the *digital materials group*, on the other hand, used the graph significantly more frequently in their argumentations ($\chi^2 = 6.62$, $p < 0.01$) and focused significantly more often on variation ($\chi^2 = 6.95$, $p = 0.008$).

When considering the task *racing car* we found that students in the *real materials group* chose the wrong solution significantly more frequently despite correct reference to their everyday knowledge about curves and speed ($p = 0.022$*). This could be due to problems interpreting the graph appropriately or difficulties with the representation of the race tracks. Tacking into account, that the graph-as-picture error did only occur in the *real materials group* suggests the assumption that problems with the interpretation of the graph are the cause for the difficulty in bringing everyday knowledge together with the task. The same was true for the category Speed ($p = 0.042$*). With regard to the type of curves category, it became clear that recognising the need for different types of curves seemed to be particularly important for the proper handling of the task. Out of 21 students who argued on the nature of the curves, 20 chose a correct solution. Also noteworthy was the appearance of the graph-as-picture error. The graph-as-picture error occurred in 36% of the students in the real materials group but by none of the students in the digital materials group. These findings led to a number of hypotheses regarding the processing of tasks on functional relationships. Three of these hypotheses focus on the aspects according to Vollrath. Thus it was assumed that (1) the real materials group had an advantage with regard to the promotion of the understanding of *mapping* since these students increasingly argued with states. In contrast, the digital materials group seemed to have an advantage in promoting the understanding of (2) *covariation* and (3) *function as object*. (2) resulted from the focus of the digital materials group on change and the course of the graph, (3) from the lower difficulties the students revealed in linking situation and graphical representation in the racing car task.

In order to verify the validity of these hypotheses, data from pre- and post-tests were used to measure the effectiveness of the intervention. For this purpose, the increases in the mean solution rates with regard to each test item were considered separately for the real materials group and the digital materials group. The items were grouped according to whether the real materials or digital materials group had achieved a larger increase in the mean solution rate. There were 9 items for the real materials group and 17 items for the digital materials group. These were distributed clearly in different ways between the groups with regard to the aspect function as object: 2 function-as-object items were found in the real materials group, 7 in the digital materials group. This was interpreted as a further indication for hypothesis (3).

An analysis of the items grouped in this way followed in order to identify commonalities between them that could explain the achievement of a larger increase in the respective group. It was noticeable that the 9 items that were allocated to the material group often dealt with the identification of pairs of values and thus indicated an advantage for the group with regard to the mapping aspect. The 17 items of the digital materials group, on the other hand, increasingly included anticipation and comparison of gradients and rates of change. This provided evidence for the hypotheses (1) and (2). It was also noticeable that the digital materials group achieved a larger increase in all items using tables. Perhaps, since students in this group only had to “click”, they had more free capacity to deal with the table as a form of representation. In contrast, the real material group achieved a larger increase in items of the cube context, which required a spatial understanding. Apparently, this kind of understanding could be promoted better by the use of real materials.

In summary, it must be stated that real materials and digital materials have different influences on the processing of tasks in functional contexts. There are indications that digital materials can improve the understanding of the aspects of covariation and function as object, while materials can improve the understanding of the mapping aspect. Furthermore, the work with certain contexts and the handling of forms of representation also seem to depend on the choice of the material used for the promotion. A comprehensive promotion of functional thinking should therefore include working with real materials as well as with digital materials to inspire learning, since both types of material seem to complement each other perfectly.

3 INSPIRING TEACHING: VIDEO VIGNETTES FOR THE ANALYSIS OF TEACHING AND LEARNING PROCESSES

ViviAn is an acronym that stands for “**vi**deo **vi**gnettes for the **an**alysis of teaching processes”. The ViviAn learning environment (cf. www.vivian.uni-landau.de and Roth 2019) was developed because there was no diagnostic tool that met all the following requirements:

(1) Pre- and in-service teachers who process diagnostic tasks for video vignettes of group work processes of students should essentially have the same information on the situation presented as the supervising teacher of the group of students would have.
(2) Edits for diagnostic tasks are entered into a text field directly next to the corresponding tasks and automatically saved.
(3) After completing a diagnostic assignment individually, pre- and in-service teachers receive feedback from experts which they can compare to their own answers.
(4) Pre- and in-service teachers can carry out the diagnostic assignments at a time of their choice within a specified time window at a location of their choice.
(5) There is a user administration that allows lecturers to individually approve video vignettes (also via adjustable time intervals) and allows them to be used only after receipt of a signed data protection declaration.
(6) Lecturers can call up in an overview which video vignettes have already been processed by the individual pre- or in-service teacher in the course [1].
(7) All video data of the persons depicted are protected, stored on a server of the University of Koblenz-Landau under the complete control and exclusive administrative access of two administrators of the working group Didactics of Mathematics (secondary levels).

In particular, the first four points of the enumeration led to the development of the ViviAn learning environment and its interface. The functional scope of these is explained below.

Figure 3.1 presents the interface of the ViviAn learning environment. A video vignette is embedded in the center, showing an excerpt of approximately three minutes from an authentic group work phase of students in the Mathematics Laboratory “Maths is more”. The video player allows you to start, stop, fast-forward and rewind within the video vignette at any time. The group work process was recorded in bird’s eye view. The selected camera perspective supports the observation of the entire learning group as well as the focus on individual learners. Furthermore, all actions on the material are clearly visible. In phases in which the students essentially work with digital material, the screen recording (video of the students’ screen actions) is displayed at the center of the ViviAn learning environment. To ensure that the interaction of the learners at the table is not lost, the recording of the previously described camera perspective is displayed in reduced size in the lower-left corner of the video. In order to be able to clearly assign the verbalizations to the learners in the video, the person who is currently speaking was marked (cf. "S3" in Figure 3.1). This mark contains the identifier of the student. All students in the video vignettes were provided with one of the identifiers S1, S2, S3 or S4 starting from the bottom left clockwise in order to be able to identify them unambiguously for diagnostic tasks and answers.

Figure 3.1: Interface of the learning environment ViviAn (cf. www.vivian.uni-landau.de)

A window can be opened above the video vignette via the "*Learning environment: Topic and Goals*" button. In this window, the subject and learning objectives of the learning environment in which the students are working in the video are briefly displayed. This information should be called up at first for an initial overview. To the right of the video vignette in the "*Meta level*" box, pre- or in-service teachers can access information that a teacher in the classroom usually has. The "*Student Profiles*" button retrieves information about the learners in the video (including age, grade and type of school attended). Below this is the seating plan on which the learners (for clear communication) are numbered S1, S2, S3, S4 from left to right. This is meant to facilitate the overview of the events and access to individual students.

Below this is the button "*Temporal classification*". It opens the content plan of the three 90 minutes slots in which the students work in the students' laboratory. This enables the pre- or in-service teachers to see which learning objective is to be achieved by the task and which contents the learners are working on before and after the situation shown in the video vignette.

To the left of the video in the box "*Student level*", materials from the learning environment that learners work with or produce during the learning process can be retrieved. The "*Task*" button opens the task that the learning group processes in the video in a pop-up window (cf. Fig. 3.2). With the button "Materials" photos of the real material with which the learners work in the displayed situation can be called up in a pop-up window (cf. Fig. 3.2). If the students in the video use digital material it is shown directly in the pop-up window. The digital material, usually a GeoGebra-based simulation, has the full range of functions and can be used by the pre- or in-service teachers in the same way as the learners in the video vignette. In this way, their actions can be understood in the best possible way.

The “*Student documents*” button displays the written work results of all learners from the video. These can be selected via tabs and arranged so that any edits can be compared in pairs (cf. Fig. 3.3). This allows further access to the respective depth of reflection of the individual students.

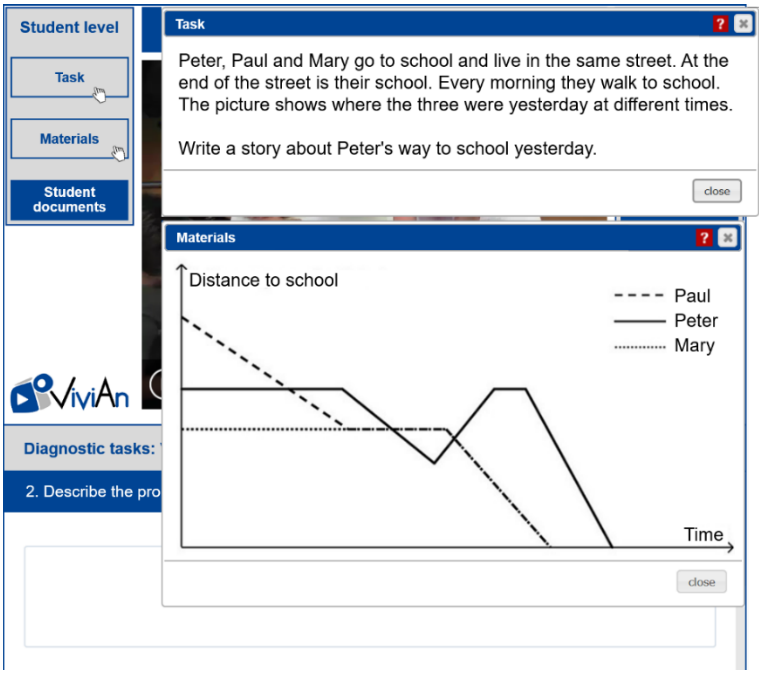

Figure 3.2: ViviAn-interface with open windows *Task* and *Materials*

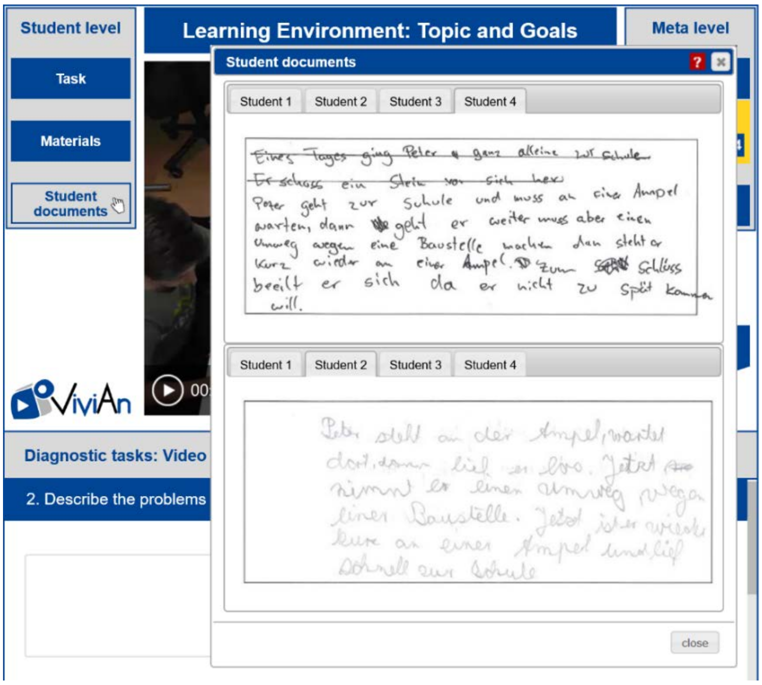

Figure 3.3. ViviAn-interface with open window *Student documents*

The “*Diagnostic tasks*” button at the bottom right of the ViviAn interface calls up the diagnostic tasks, which are then displayed individually below the video (cf. Fig. 3.4). The diagnostic assignments in the ViviAn learning environment each focus on a specific content aspect of mathematics learning in the situation presented in the video vignette, for which the necessary theoretical facets and pedagogical content knowledge were previously discussed in-depth in the course [1]. This shifting of the main focus of diagnosis to theoretical aspects previously addressed in the course [1] is intended to help pre- and in-service teachers become accustomed to carrying out their diagnoses on a theoretical basis. The diagnostic tasks are either open items that have to be answered in free text format or consist of a combination of a closed and an open item. The closed items are single-choice and multiple-choice questions. In order to encourage the pre- or in-service teachers to deal intensively and in detail with the situation, each closed item is followed by a question in free text format, which requires a justification of the previously chosen answer. Typical work assignments within ViviAn, which are intended to help train pre- and in-service teachers’ ability to diagnose group work situations of students (cf. Bartel & Roth, 2019), invite them to

- work on the tasks of the students,
- describe observations,
- interpret observations and give reasons for these interpretations (basic ideas, students’ ideas, etc.),
- propose and justify adaptive teaching.

If a pre- or in-service teacher has entered a text in the box for the answers to diagnostic tasks (see Fig. 3.4), answered the single-choice and multiple-choice questions, if applicable, and sent them by clicking on the box “*Next*” down on the page, a feedback page opens below the video (see Fig. 3.5).

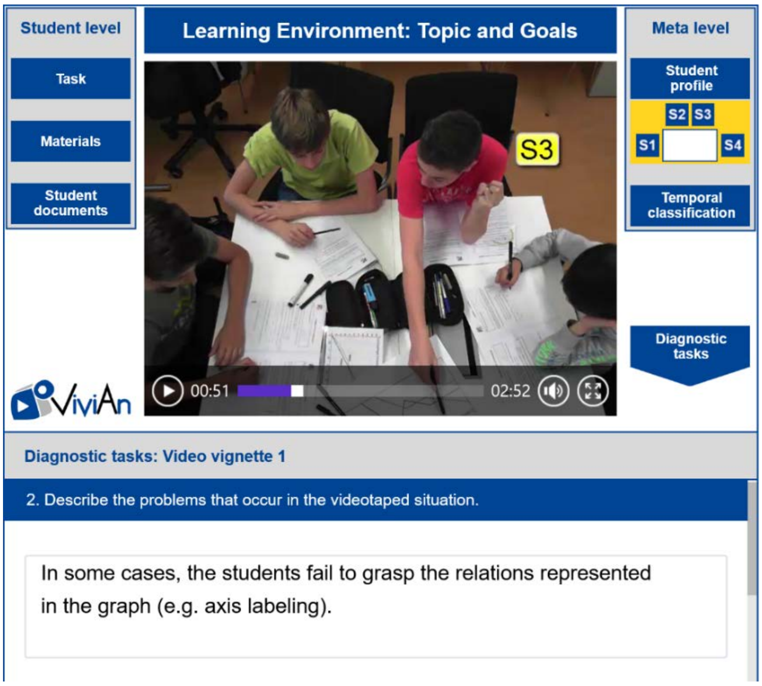

Figure 3.4: ViviAn-interface with open diagnostic task and a provided answer

The following is displayed on this page:

(1) The diagnostic task that the pre- or in-service teacher just completed,
(2) the text that the pre- or in-service teacher has just entered, as well as the single-choice or multiple-choice questions, if applicable, with markers for the selected answers,
(3) expert diagnoses as feedback on both closed and open questions. These are short texts containing analyses as well as corresponding justifications for the respective diagnostic assignments.

Figure 3.5. ViviAn-interface with feedback to an answer of a pre-service teacher

In this way, pre- and in-service teachers can compare their answers with expert answers and thus reflect on it. In two independent studies, Bartel and Roth (2017) and Enenkiel and Roth (in print) were able to show that the diagnostic abilities of teachers can be significantly improved in this way.

In order to enable feedback in the form of expert diagnoses, all video vignettes were first transcribed. The transcripts were then analyzed using mathematics education literature on the aspect of interest, e.g. learning the concept of function. Based on this and already completed processing of the videos, both by pre-service an in-service teachers in the context of a preliminary study and by researchers in mathematics education, an analysis with corresponding justifications was then carried out for each diagnostic task. Before being used in ViviAn, the correctness of this analysis was checked by at least three researchers in mathematics education and, if necessary, sharpened.

In this paper, different aspects of inspiring teaching and learning were addressed, starting with teaching-learning laboratories, continuing with the use of real and digital materials to learn functional thinking, and ending with the use of the learning environment ViviAn to train diagnostic competences of teachers to inspire their teaching. The inspiration can be manifold, but, as should be shown in the article, it has to meet various criteria in order to be successful.

NOTES

1. A *course* is in the case of pre-service teachers a lecture on mathematics education, in the case of in-service teachers a teacher training course.

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NETWORKING OF THEORIES RECONSIDERED

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New directions of design for mathematics education may require insights from multiple resources including different theoretical perspectives. Rather than as a problem, the Networking of Theories conceptualizes this diversity as having potential for teaching and learning. Over the past decade more insight has been gained into how theories may work together and what this may offer. This paper provides an overview on research in which the Networking of Theories is used as a research practice including the notion of theory. By examples it highlights networking strategies and methodologies as ways to link theoretical perspectives and their potentials systematically. Reflecting on a recent design study that includes technology as part of a multimodal perspective, a case of networking theories across disciplinary boundaries to integrate technological as well as educational design is also considered as part of interdisciplinary design research.

Keywords: Networking of Theories, multimodality, notion of theories, design research, epistemological gap

A CASE OF NETWORKING OF THEORIES IN MATHEMATICS

The networking of theories seems to be a practice implicitly used in mathematics. To begin with, I would like to illustrate a case of local integration where an algebraic perspective uses geometrical objects to build an algebraic structure. Let us consider a square and its symmetry mappings, four rotations and four reflections across the symmetry axes of the square. Composing these mappings will shape symmetry mappings of the square, too. Hence, the set of these symmetry mappings establishes a dihedral group, an algebraic structure concretized by a set of geometric objects with an operation. This illustrates a Networking of Theory case of locally integrating two theoretical perspectives. Geometrical as well as algebraic features of the square beyond this specific situation are not in play.

NETWORKING OF THEORIES IN MATHEMATICS EDUCATION

The term Networking of Theories has emerged during the Thematic Working Group on theoretical perspectives at CERME 4 in 2005 as a reaction to the growing diversity of theories and paradigms in mathematics education and the problems it encounters (Artigue, Bartolini Bussi, Dreyfus, Gray, & Prediger, 2005). This diversity seems to be an intrinsic feature of the field because of the diverse philosophical traditions, the distinct cultural, institutional and social situations of the educational systems in the field. The problem was not the growing diversity itself, but the field's resulting fragmentation, which causes problems of communication between theory cultures, of how to integrate research results coming from different theory traditions, and of how to deal with the complex nature of the research objects such as teaching and learning in the classrooms, in research as well as design.

At CERME 4, the Networking Theories Group was founded to explore the potentials and pitfalls of the Networking of Theories (see Bikner-Ahsbahs & Prediger, 2014; Kidron, Bosch, Monaghan, & Palmér, 2018). This group used a common data source offered by the Turin research group to investigate how to connect the five theories present in the group by case studies, discussed results in the subsequent CERME theory groups and at PME conferences and published a book edited by Bikner-Ahsbahs and Prediger (2014). New concepts have emerged in this work, for instance a *common epistemological sensitivity* among theories made it possible to compare and contrast different kinds of conceptualizing context (Kidron, Artigue, Bosch, Dreyfus, & Haspekian, 2014). Networking of Theories has forerunners who did not explicitly use this term but were already engaged in research in a similar manner, for example Artigue (see Kidron & Bikner-Ahsbahs, 2016) and Bauersfeld (1992). However, Artigue and Bosch (2014) have also shown that the Networking of Theories as conducted in the Networking Theories Group was still at a stage of craft knowledge, which lacked a theoretically informed discourse on a meta-theoretical level. Additional case studies from outside this group added results, for example about different ways of conceptualizing mathematical objects (Font Moll, Trigueros, Badillo, & Rubio, 2016) or comparing the notion of practices with CAS from the instrumental and the Onto-semiotic approach (Drijvers, Godino, Font, & Trouche, 2013).

The Networking of Theories is also used in empirical research. For example Bikner-Ahsbahs and Kidron (2015) describe how the concept of General Epistemic Need (GEN) was built by a cross-methodology conducted from the two theoretical perspectives involved. Tabach, Rasmussen, Dreyfus and Hershkowitz (2017) have linked an individual and a collective perspective to investigate knowledge construction in an inquiry based classroom. The research coordinated key concepts of the two theories involved and showed their complementary nature for constructing knowledge.

Meanwhile, in design research the Networking of Theories is considered as a way of including different theoretical approaches into design cycles. For example Kouropatov and Dreyfus (2017) have used two theories, Proceptual Thinking and Abstraction in Context, for designing a task-based curriculum for teaching the integral as an accumulating function in three design stages, predesign, the initial design and re-design, and final design. Another interesting example on design research is shown in the project ReMath, where a common framework was established to link digital tools with different theoretical perspectives of design research studies. In ReMath a new networking methodology was implemented, *cross-experimentation* (Artigue, & Mariotti, 2014).

Previous definitions of the notion of Networking of Theories focussed on the connecting of theoretical perspectives (Bikner-Ahsbahs, & Prediger, 2010). But Kidron and Bikner-Ahsbahs (2015), Kidron et al. (2018) and also the book edited by Bikner-Ahsbahs and Prediger (2014) have worked out a more comprehensive understanding of this notion. After about fifteen years of research crucial elements of the Networking of Theories as a research practice have become visible. Based on this body of research I would like to propose five defining features: the Networking of Theories as a research practice

- investigates how specific theories work and may work together
- for a specific purpose
- by creating a dialogue, linking and building relations between (at least) parts of theoretical approaches in a methodological sound manner and
- reflect on this networking practice
- while respecting the identities of the theoretical approaches involved.

This definition builds on the assumption that the diversity of theoretical approaches is an intrinsic and enriching feature in the field of mathematics education. As any research practice, also this one may evolve by research, be further explored, and expanded to address new aims and provide new kinds of questions and methodologies.

Since theories are the objects of the Networking of Theories, the notion of theory should also be a topic of contemporary research.

NOTIONS OF ‘THEORY’

Unfortunately, the field does not agree in what the term *theory* means (Assude, Boero, Herbst, Lerman, & Radford, 2008). But there are some commonalities. Mason and Waywood distinguish between background theory and foreground theory as relative categories (1996):

- A *background theory* “is a (mostly) consistent philosophical stance of or about mathematics education which plays an important role in discerning and defining what kind of objects are to be studied, …. “ (p. 1058).
- *Foreground theories* are mostly local theories in mathematics education: “…. because [of] the foreground aim of most mathematics education ….. .” (p. 1056).

Constructivism as an individual learning perspective could be taken as a philosophical stance, which considers learning as individual construction allowing to discern what can be taken as research objects, for example mental models. Investigating how a mental model of a function concept is built would be a question addressing a foreground aim that may lead to a foreground theory on learning functions. However, this perspective does not allow for conceptualizing teaching and learning mathematics as a cultural activity like in activity theory. As activity theory considers teaching-learning as an irreducible entity (see Shvarts & Abrahamson, 2019), a foreground theory on functions within activity theory might be gained by exploring how the common labor in teaching and learning allows function concepts be built by actions and their goals being supported or mediated by tools.

In sum, any theory can be regarded as a lens with a language that consists of concepts and claims, but how these parts are structured depends on the specific notion of theory.

Radford (2008) for example proposes that „a theory can be seen as a way of producing understandings and ways of action based on:

- A system, *P, of basic principles,* which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
- A *methodology [methodologies], M*, which includes techniques of data collection and data-interpretation as supported by *P*.
- A set, *Q, of paradigmatic research* questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified)“ (Radford 2008, p. 320, emphasis in the original).

He uses the triplet (P, M, Q) as a representation of this notion of theory and enlarges it towards a dynamic understanding of theory: a theory develops by research results *R* that may inform the “way of producing understandings and ways of action”. As a final shortcut of such a dynamic understanding of *theory* he introduces the quadruplet [(P, M, Q), R] (see Radford, 2012).

In Radford’s description of theories Networking of Theories means building relations among their parts. What does it mean? Given two theories are used to investigate a common question by a common methodology that encompasses the methodologies of each of the two theories, this may lead to a new concept if a common principle can be included into the two theories. In the latter case we may have developed a local integration linking all the three parts (P, M, Q) of the theories. This kind of networking will now be illustrated by the first example described in Sabena, Arzarello, Bikner-Ahsbahs, & Schäfer (2014).

EXAMPLE 1: THE EPISTEMOLOGICAL GAP

During the first meetings of the Networking Theories Group a short episode was analyzed where the students explored the exponential function $y = a^x$ with Capri. The students observed the graph of this function (Fig. 1) with a secant when the teacher says: How does the exponential function grow for very big x?

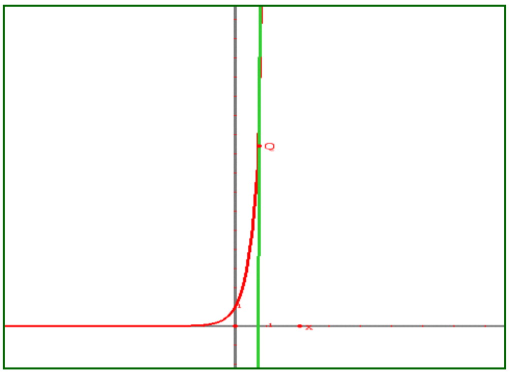

Fig. 1: Graph of $y = a^x$ (Sabena, Arzarello, Bikner-Ahsbahs & Schäfer, 2014, p. 181)

The first two utterances are:

(#1) G: … but always for *a* very big this straight line (pointing at the screen), when they meet each other, there it is again…that is it approximates the function very well

(#2) T: what straight line, sorry?

……

The following interaction between the teacher and the student G was investigated by Sabena et al. (2014), it happens in three phases (see Sabena et al. 2014, pp. 182-184).

Phase 1

G's view: the graph of the function can be approximated by a vertical line (#1-3)

Fig. 2 G shows that x is going up as part of the points on the graph

Phase 2

Teacher (#4): Will they meet each other?

Fig. 3 The teacher introduces with gestures how the graph and a vertical line should meet.

Phase 3

G (#5-7): That is, yes […] It makes so

Fig. 4 Dissonance between G's gestures with the gestures of the teacher

This episode was analyzed from two perspective: (1) the semiotic game the teacher played with the students in the teaching of the exponential function with Capri was analyzed by the semiotic bundle concept and (2) the epistemic process in the interaction was analyzed to explore if an interest-dense situation could emerge and why or why not.

The semiotic bundle consists of three semiotic sets and their developing relations as the bundle evolves, the sets of "words, gestures and representations [of the exponential function $y = a^x$] in the Cabri file (Sabena, 2014, p. 186). In the semiotic game, the teacher tries to introduce new features of the content by tuning with the student's words and at the same time using gestures to address the new aspects he wants to be part pf the learning (see Fig. 2-4).

(#8)

Fig. 5 The teacher shows in line 8 how the graph should cross vertical lines

(#16)

Fig. 6 The teacher shows that for big x that the vertical line at x should be crossed by the graph

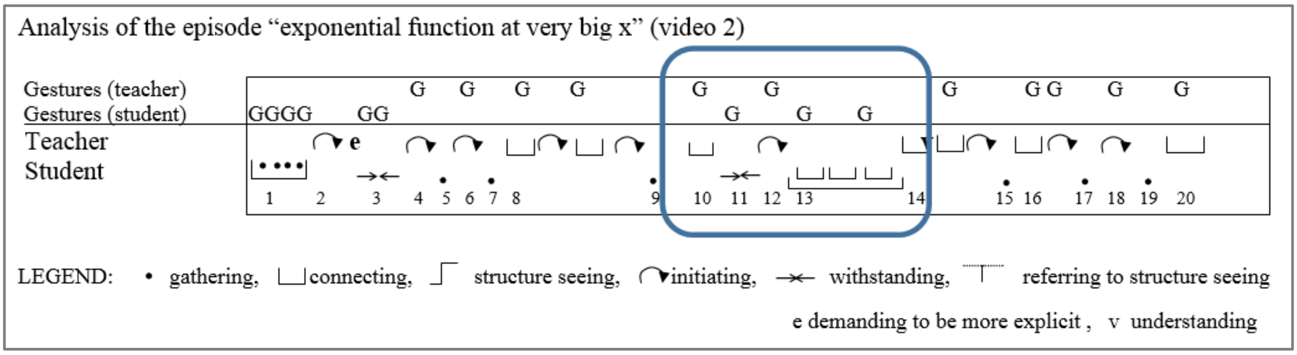

Fig. 7 The analysis diagram of the episode by the use of the epistemic action model (following line numbers). G indicates relevant gestures.

The second perspective investigates the interactions to characterize the epistemic process; and if and how interest-dense situations emerge. Interest-dense situation are specifically fruitful in that in the epistemic process the students achieve structure-seeing (Bikner-Ahsbahs & Halverscheid, 2014). The analysis uses an epistemic action model to reconstruct this epistemic process shaped by three epistemic action. It assumes that the epistemic actions *gathering* and *connecting* mathematical meanings must take place before *structure-seeing* is possible. If the students begin to deepen their engagement into the mathematical task by these actions an interest-dense situation emerges while catching and keeping students' interest for a while, at least until they see and explore mathematical structures.

Fig. 7 shows the analysis diagram of the epistemic process. Until line 9 the teacher tries to initiate his own view (see the arrows) while the student briefly gathers ideas (points in the diagram). At line 10 the situation changes because the student begins to withstand the teacher's view (double arrow) pointing to the screen in line 11. The big rectangle in the middle of the diagram highlights the part where an interest-dense situation emerges: an intense gesture exchange and a complex engagement by the student by *connecting* several mathematical ideas happens. This situation finishes with the following two utterances:

Student G: that is, at a certain point … that is if the function (image G-00:57) increases more and more, more and more (image G-00:59) then it also becomes almost a vertical straight line (image G-1:03)

Teacher: eh, this is what seems to you by looking at (pointing gesture)

The student G tries to describe his perceptive view in that he thinks that for very big x the graph of the exponential function grows with x and approximates finally a vertical line (this is not true but the screen gives this impression to the student). The teacher emphasizes that the screen gives a wrong impression: "this is what seems to you by looking at [the screen]" without taking into account how the student could achieve this evaluation himself. What the teacher just expresses is that what the student perceives is wrong trying then to describe what is true. At this moment, the student stops to engage himself and just gives short one-word answers. The interest-dense situation dries up.

Starting the process of Networking of Theories

The separate analyses of the two perspectives showed: Although the semiotic game was successful, the student did not profit from it, the emerging interest-dense situation dried up. The question why this happened could not be answered at the beginning. Both, Ferdinando Arzarello and Angelika Bikner-Ahsbahs, met to deeply become re-engaged with analysing the data, adding data, trying to make the theoretical views mutually explicit, and compare and contrast them. In the attempt to translate each interpretation into the other language both became aware that there was something missing in both approaches, an epistemological perspective which would explain what we finally were aware of, the *epistemological gap* between the teacher's and the student's epistemological views. While the student showed in his gestures that he associated big x with the top location of the points on the graph the teacher did not look at the graph, instead he argued on the basis of formal mathematics and its limit concept of calculus (Fig. 2-4, Fig. 5, 6). The two views, *perceptive fact seen on the screen* and *formal mathematics*, shape the epistemological gap that the student could not bridge by himself. How such a gap can be turned into a learning opportunity, would be a question for design research.

This case shows a local integration by adding a new concept at the boundary of the two theory cultures, the epistemological gap, which involved the three criteria of theory as proposed by Radford (2008) (see Sabena et al., 2014):

- A common research question: Why does the teacher-student interaction not work?
- A common theoretical principle: Adding an epistemological dimension by including the concept of the epistemological view into both approaches.
- A common methodology: Including both kinds of analysis into a common methodology.

NETWORKING STRATEGIES

Fig. 8 represents a revised version of the networking strategies landscape developed based on the contributions of the CERME theory groups (Prediger, Bikner-Ahsbahs, & Arzarello, 2008). The landscape does not consider the two poles, *ignoring others* and *unifying globally*, as belonging to networking practices. On the contrary, the diversity of theories is regarded as richness in the field and connecting theories as a way of how theory cultures may interact. Between the two poles, pairs of networking strategies are located according to their degree of integration. Integrating locally as described in the example above is one of the two strategies of integration. The other one is synthesizing which the Networking Theories Group has not yet observed.

The local integration of the two perspectives, *semiotic bundle* and *interest-dense situation*, did not happen by chance, it had been prepared by a process of mutual understanding, comparing and contrasting ideas and concepts of the two approaches. In this process, mutual understanding was more than just a first step. Radford conceptualized this strategy as an attempt to translate own theoretical aspects into the other language which probably is not possible and therefore results in unpacking implicit assumptions, blind spots or the lack of an important principle. By combining and coordination both approaches in the process of analysing the data, the experts arrived at producing a common question, a common additional principle and merged their methodologies into a common one, thus, yielding a case of local integration. This local integration was possible because the two perspectives already shared common assumptions, e.g., constructing knowledge is a micro process expressed semiotically. During the whole process, mutual understanding was improved by permanent reflections which Akkerman and Bakker (2011) call perspective taking and making, a learning mechanism of boundary crossing between the two theory cultures (Fig. 8).

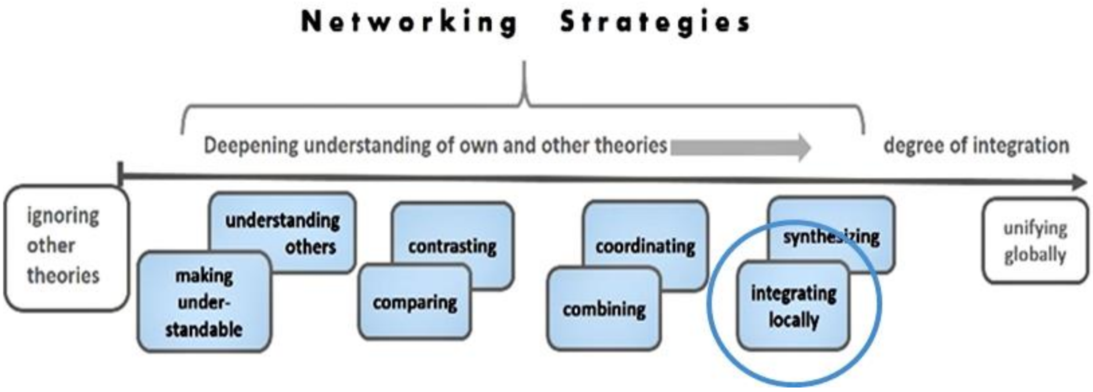

Fig. 8 Networking strategies (Bikner-Ahsbahs, 2016, p. 34, revised version from Prediger et al. 2008, p. 170)

EXAMPLE 2: DESIGN OF A TECHNOLOGICAL LEARNING SYSTEM

Algebra is a gatekeeper for mathematics and therefore should be accessible for all students. For that reason, tangible manipulatives and digital tools have been developed to support learning algebra. But many digital tools have shown conceptual weaknesses (see e.g. Janßen, Reid, & Bikner-Ahsbahs, 2019, in press). The MAL-project (MAL: Multimodal Algebra Learning) attempts to overcome these weaknesses by addressing learning algebra in a multimodal and conceptual way. With this aim in mind, computer scientists (from human computer interactions) and mathematics educators are working together from the beginning to design an algebra tiles system (MAL-system, MAL: Multimodal Algebra Learning) for the teaching and learning of linear equations in a multimodal way.

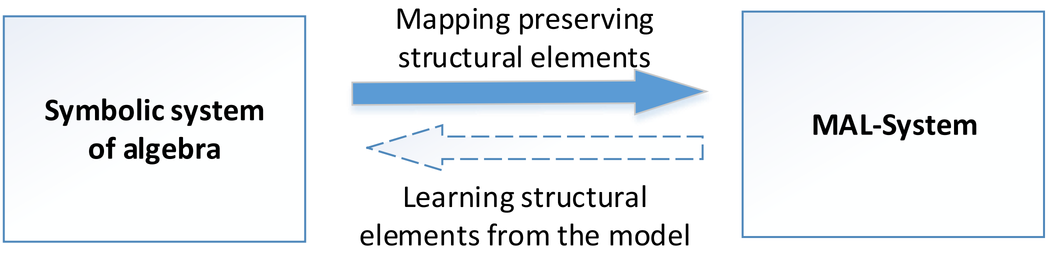

Fig. 9 Conceptualising the design process by mapping (see Janßen et al., 2019)

Based on the interface theory developed by Goguen (1999) the design of a didactic model may be regarded as a mapping from the symbolic system of algebra to an algebra tiles model (MAL-System) that preserves key structures (Fig. 9) (see Janßen et al. 2019).

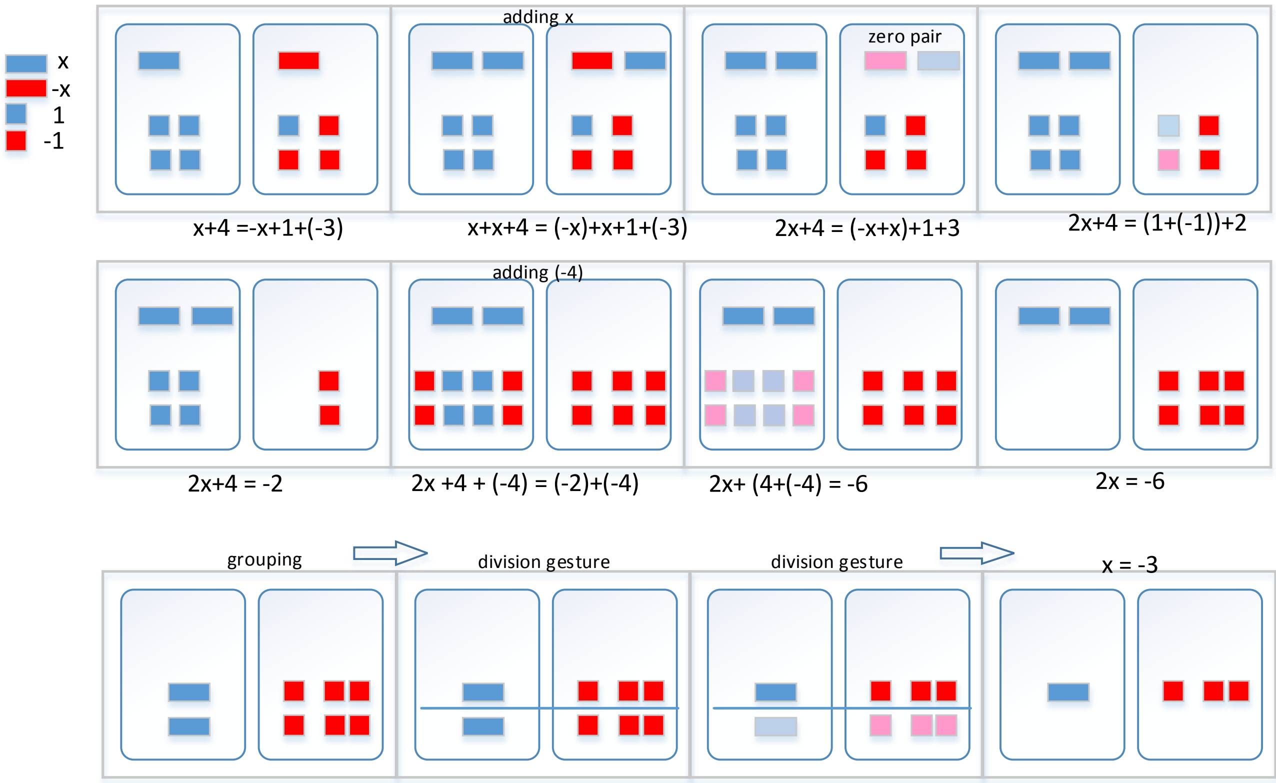

Fig. 10 How people may solve an equation with the MAL-system

These structures are extracted from epistemological analyses, and modelled in the technological design. Variables, numbers and algebraic expressions are represented by tiles, the two signs (+, -) are represented by colors and operations by actions with the tiles (Fig. 10). Variables and numbers are distinguished by the shapes of tiles, rectangles are used for variables and squares for unit tiles. Main features of the current version of the MAL-system are shown in Fig 10: addition, subtraction, zero pairs, division to solve linear equations.

While designing the didactic model is regarded as a mapping from the symbolic system of algebra to the target technological design learning goes into the opposite direction (Fig. 9): By working with the MAL-system students shall first learn the key structures, and then expand them beyond the restrictions the didactic model has. For this purpose, Activity theory (Leontjew, 1979; Shvarts & Abrahamson, 2019) is added to frame teaching-learning for designing and researching tasks for the MAL-system which are explored in teaching-learning situations to understand the functioning of the MAL-system and to inform revisions of technological as well as educational design in the next step.

Final remarks on design and research on the MAL-system

The current MAL-system as presented in Fig. 10 has been explored with four pairs of students of grade eight who already have learned how to solve linear equations. However, the use of the division gesture was new to them, hence, helped to understand the division of equations conceptually (see Janßen, Vallejo-Vargas, Bikner-Ahsbahs, & Reid, submitted). Fig. 11 shows one pair of students working on division tasks. The two students have grouped tiles on the two sides of the mat of the MAL-system in a way the MAL-system is not able to handle. Using the division gesture (see Fig. 10) for dividing by 2, that is drawing a line on the multi-touch display with a finger which separates the two groups on each side of the mat, would have required to split two tiles into halves. However, the MAL-system does not allow for splitting tiles although this should be theoretically possible. This situation indicates the students' need to go beyond the restrictions of the MAL-system, hence, to emancipate from it. This aspect points to one of four layers on which investigating the teaching-learning with the MAL system is conducted:

Layer 1: Learning the MAL-system
Layer 2: Learning with the MAL-System
Layer 3: Learning to link the MAL-symbolic expressions with the algebraic expressions
Layer 4: Learning to emancipate from the MAL-system

Fig. 11 Splitting tiles by dividing by 2 with the MAL-System (see Janßen et al., submitted)

Teaching and learning of algebra with the MAL-system is not yet investigated profoundly enough. But our pilot studies have pointed to four layers to be considered in research. Layer 1 and layer 2 address the instrumental genesis when using the MAL-system (Artigue, 2002; Drijvers et al. 2013). Layer 3 looks in more detail at forms of translations between the two semiotic systems (Duval, 2008) including feedback. Finally, layer 4 describes the transformation of the students' relations with the instrument when they emancipate from it.

The networking of theories may offer a way to link these four layers of learning through a common framework of different theoretical elements. Following this path would require an in-depth understanding of the theoretical approaches involved. This may be achieved by using the strategies of mutual understanding, comparing and contrasting before considering combining and coordinating them. In this process, principles and assumptions of the theoretical approaches should become visible, the way the perspectives behind shape what can be taken as research objects, what the researchable questions are and the way they address methodological affordances when the teaching and learning of algebra with the MAL-system is researched.

DISCUSSION

Looking at the topic of the Networking of Theories, Bakker's review (2016) has opened up the interesting perspective of considering it as boundary crossing between different theory cultures for which boundary objects are identified. If we look back on the development of the concept of epistemological gap, the video of the episode was our data source and the boundary object both groups could use. During research, we have revised the given transcript to make it a piece of data for both theoretical approaches, hence, data was made to serve for the local integration.

According to the MAL-project, the design of the MAL-system is our (dynamically changing) boundary object. For example, structural elements from an epistemological analysis of solving linear equations were offered to the computer scientists, but these elements had to be structured in a certain way, thus perspective taking and making was an important part of boundary crossing. Before the computer scientists could develop the product we conducted a paper prototyping study to investigate beforehand how our ideas would work together to avoid developing a product which would not be educationally useful, thus, perspective taking and making was systematically included.

The main challenge in the MAL-project was boundary crossing across the disciplines and this involved different notions of theory. The Networking of Theories was part of it. In the design process, the theory by Goguen (1999) and an epistemological theorizing of the mathematics informed the designing process, and Activity Theory assisted in researching the use of the design to understand and inform the next cycle of design research. Once, the MAL-system is ready for use in teaching and learning in school, research including several theoretical perspectives into a comprehensive research frame would be indispensable, hence, follow a Networking of Theories path.

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Part 2: Developing Visions

LEARNING MATHEMATICS WITH A DIGITAL TEXTBOOK AND ITS INTEGRATED DIGITAL TOOLS: THE KOMNETMATH PROJECT

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Although research on the use of (printed) mathematics textbooks and digital tools exists, there is still a high need for research on digital mathematics textbooks with integrated digital tools that are used by teachers and students in mathematics lessons. Since 2019, the project KomNetMath researches the impact of a regularly used digital mathematics textbook at German secondary schools. The actual use, the impact on students' beliefs towards the use of digital mathematics textbooks as well as the influence on mathematical competencies are of special interest.

Keywords: digital mathematics textbook, digital tools, use of digital textbooks

THEORETICAL BACKGROUND

Mathematics textbooks play an important role for didactic aspects of teaching organisation (Rezat 2010). In recent years, the number of studies dealing with digital textbooks has increased (e.g. Pohl & Schacht 2017, Froitzheim et al. 2016). Pohl and Schacht (2017) analysed the structural characteristics and elements of digital mathematics textbooks and the structural elements which students use. First results show that a digital textbook differs from a traditional (printed) textbook in respect of its dynamic structural elements. These dynamic structural elements characterize the potential and the technical possibilities of digital mathematics textbooks that a traditional textbook cannot provide. For the impact on the students and teacher's use of the digital textbook and on the students' learning process, in consequence of the expansion capabilities of digital mathematics textbooks with integrated digital tools, there is still a lack of research. In addition to that, the influence on students' beliefs towards the use of digital mathematics textbooks is a research gap. The digital mathematics textbook used in the project KomNetMath is also characterized by dynamic elements due to its integrated digital tools. For example, this digital mathematics textbook contains GeoGebra applets, drag and drop tasks and direct feedback functions.

The textbook is regarded as an artefact in the classroom that is an important resource for teachers as well as learners. It is employed as an instrument for different activities such as preparing lessons or learning from the textbook (Rezat 2010). Thereby a textbook can influence teaching directly or indirectly (Matić & Gracin 2016). "Whenever a textbook is explicitly or implicitly used in the classroom one can think about it as an influential factor" (Johansson 2006, p. 9). Most studies on the use of mathematics textbooks differ in terms of the teachers' use (e.g. Johansson 2006) and the students' use (e.g. Rezat 2009), whereby research results mainly refer to traditional textbooks. Studies on the utilization of a regularly used digital textbook can be found in other disciplines. Froitzheim et al. (2016) examined the use and acceptance of a digital computer science textbook. Teachers reported a frequent explicit and implicit use of the digital textbook. It was used as workbook in the classroom (direct use) and in lesson planning (indirect use). Furthermore, the study revealed a high student acceptance of the digital textbook.

In the project KomNetMath we survey what the digital mathematics textbook is used for in the classroom and assess to what extent the beliefs towards the use of the digital mathematics textbook change when using a digital mathematics textbook over an extended time period.

RESEARCH QUESTIONS

If teachers and students use a digital mathematics textbook with digital tools in the classroom over a longer period of time, the following research questions emerge 1) How do teachers and students use the digital mathematics textbook in teaching and learning? 2) How do the students' beliefs towards the use of the digital mathematics textbook change when using one?

METHOD AND OUTLOOK

From the beginning of the project in February 2019 onwards, 16 mathematics courses use a digital mathematics textbook instead of a traditional one in the classroom. One goal is to track the actual use of the digital textbook in class. For this purpose, teachers fill out lesson reports for each lesson. The teachers note down on a timeline at what part of the specific lesson the digital textbook is used. In this way, the extent of digital textbook's use can be specified. Furthermore, the teachers record what the digital textbook is used for in the classroom, whereby they have to make a distinction between explicit and implicit use. An implicit use would be the case if teachers use the textbook for lesson planning. If teachers or students use the digital textbook as a workbook directly in class, it would be an explicit use. They also have to write down the main tabs of the digital textbook that have been applied as well as the class arrangements and organisational forms of each lesson. The beliefs towards the use of the digital mathematics textbook will be measured in a pre- and posttest design. Students will get an identical questionnaire at the beginning (pretest) as well as at the end of the study (posttest). The test comprises items about students' attitude, scepticism, curiosity and acceptance towards the used digital textbook. The pilot study in 2019 includes testing the teacher lesson reports and the belief test in a first pass, thereby collecting quantitative data. In the main study, students and teachers will use the digital textbook for one school year in 2019 and 2020. When students and teachers regularly use a digital textbook, the question arises if there is an impact on the development of mathematical competencies. Another (upcoming) goal of the project KomNetMath is to compare the use of a digital and traditional textbook in relation to the progress of mathematical competencies.

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AN AUGMENTED REALITY APPLICATION TO ENGAGE STUDENTS IN STEM EDUCATION

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This paper proposes a Mobile Augmented Reality application to engage students in STEM education. With a design research methodology, the SolarSystemGO game, initially designed to promote learning about our Solar System, is being developed by undergraduate higher education students supervised by higher education teachers from a Portuguese polytechnic. After several cycles of implementation of the game (design research cycles), which took place over two academic years, we argue that the SolarSystemGO game can be a resource to develop mathematical contents in formal and informal learning environments. Based on our research and experience in the field, we sustain the importance of developing Mobile Augmented Reality games that engage students and motivate them to learn interdisciplinary subject matters adequate to primary school syllabus.

Keywords: STEM education, augmented reality, hands-on, mobile technologies, gamification.

INTRODUCTION AND THEORETICAL FRAMEWORK

The European report Rocard et al. (2007) identifies an alarming decline in students' interest for STEM (Sience, Technology, Engineering and Mathematics), which will compromise future careers in these subjects and consequently Europe's future potential for innovation. More recently, another European report (European Schoolnet, 2018) and other studies all around the world (Breiner, Harkness, Johnson e Koehler, 2012; Office of the Chief Scientist, 2016) continue to identify the same problem.

To face this situation, several authors argue the need to promote STEM integration sustaining that the world needs skills related to these subjects to correspond to the increasing challenges of our century (Breiner et al., 2012; Kim & Bolger, 2017; Office of the Chief Scientist, 2016). In particular, Stohlmann (2018) states that mathematics should be more emphasized in STEM education. In this regard, STEM should be used to innovate the teaching of mathematics (Fitzallen, 2015) as well as to improve its performance (Stohlmann, 2018). In fact, integrative approaches among STEM subjects have positive effects in student's performance, with better results at the primary school level (Becker & Park, 2011). The same authors state that integrating Mathematics with Science, Technology and Engineering leads students to develop meaningful connections between these topics.

Concerning technology, it has potential to integrate mathematics and to promote student's motivation and meaningful learning (Costley, 2014). In a preliminary work, the authors of this paper conclude that technology resources are effective to capture children's attention and can engage students to learn mathematics and science, according to the school syllabus (Costa & Domingos, 2017).

The mathematics education community also recognizes the importance of developing research about integrating technology in mathematics teaching and learning. In fact, latest advances in technology have the potential to enhance the implementation of integrative approaches involving mathematics (Stohlmann, 2018). However, the same author realizes that, despite the advantages of this approach, there is the need to develop more research on this matter. In this regard, at the 10th Congress of European Research in Mathematics Education (CERME 10) it was recognized the need for focusing on emerging technologies such as Augmented Reality in future European Researchers in Mathematics Education (ERME) congresses (Trgalová, Clark-Wilson, Weigand, 2018).

Augmented Reality (AR) is an emerging topic that has been gaining prominence due to its potential to combine the real world with virtual objects (Azuma, 1997; Hwang, Wu, Chen, & Tu, 2016) and to engage students in practice-based activities (Fotaris, Pellas, Kazanidis, & Smith,2017). Huang, Hui, Peylo and Chatzopoulos (2013) introduce Mobile Augmented Reality (MAR), referring that the big improvement of mobile devices enables to boost AR by providing easier and more attractive access to this tool.

However, there is limited research in primary education and a gap in the literature about the use of Augmented Reality (AR) within mobile games and applications (Koutromanos, Sofos, & Avraamidou, 2015). In our research we also did not find any studies that include interdisciplinarity among mathematics and other disciplines in the context of MAR games technologies in primary education. This is the main reason we believe our study will contribute to research by highlighting mathematics in a MAR game initially designed to learn about our Solar System. In this regard, our research question is: how to engage students to learn mathematics in the framework of a MAR astronomy game designed to promote learning about our Solar System?

This study is an extension of a previous work about a MAR astronomy game entitled "SolarSystemGO" (Costa et al., 2018), where the authors conclude that this strategy is effective to capture children's attention and promotes learning of subject matters such as astronomy. In this paper, we begin by introducing the background and framework of this study. The following sections concern the methodology and data analysis. Finally, findings of our research and future work are presented.

BACKGROUND AND FRAMEWORK OF THE STUDY

This study is inserted into a broader STEAMH (Science, Technology, Engineering, Mathematics and Heritage) project (Costa & Domingos, 2018) entitled Academy of Science, Arts and Heritage (AcademySAH) and coordinated by the first author of this paper. Created in 2013 at the Instituto Politécnico de Tomar (IPT) in central Portugal, the AcademySAH is a pedagogical intervention project (http://www.academiacap.ipt.pt/) at the primary school level that focuses on establishing a constructivist approach on students' knowledge with the supervision of faculty members in a laboratory environment (Costa & Loureiro, 2016).

One of the intervention areas of this broader project is the design of hands-on experiments and prototypes intended to engage students to learn about STEAMH and to support primary teachers in fulfilling their mission of teaching. In this regard, it welcomes projects of higher education students under the supervision of the team's project staff (faculty members). It is in this context, that several mobile games are designed to promote STEM learning as is the case of the MAR SolarSystemGO (SS_GO) game (Costa et al., 2018).

In this section, we describe the MAR SS_GO game and propose to consider the concepts of relative scales, distances and sizes, in order to explore mathematics in the framework of our Solar System.

The MAR SolarSystemGO game

The SS_GO game consists in a kind of planet's hunt, where the players, starting from a given coordinate (the Sun), and guided by an AR application (app), try to find the orbits of the Solar System planets and satellites in the least possible time, answering questions concerning each of the celestial bodies they encounter (Costa et al., 2018). At each stage (finding the orbit, "hunting" the planet and answering the question) the players score points (Figures 1 and 2). The player with the best score and the best time wins the game.

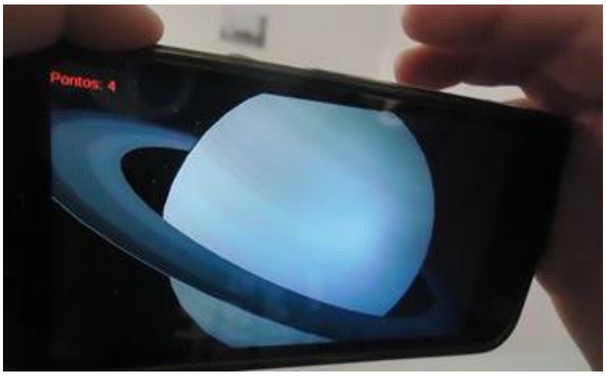

Figure 1. Saturn. This player has four points.

Figure 2. Neptune's question: Blue is my colour. How long lasts my day?

The first version of this game was designed in the school year 2016/2017 by higher education students as part of their final-year project in Digital Content Production (Master's degree) and in computer engineering (Bachelors' degree). A second version was developed in the school year 2017/2018 by three computer engineering students (Manso, Costa, Patrício, & Carvalho, 2019). This new version includes a Web platform that enables the teachers to interact with the system and include contents adequate to school syllabus, such as introducing the information about the planets and the questions according to the grade level of their students. Currently a third version is being developed by two other computer engineering students to improve its performance in order for anyone in the world to be able to play it. Also, it is necessary to provide the teachers with an easier access to the Web platform. When this is achieved our final target is to include the SS_GO game in a teachers' Continuing Professional Development Programme.

Working relative scales, distances and sizes in the framework of our Solar System

Planets' shape and the caracterization of their orbits in our Solar System are included in the curricular contents of primary education in many countries such as Portugal. Most of the time, this subject matter is approached based on images from school textbooks, where either planets' dimensions either their orbital radius around the Sun are not at scale (Costa & Silva, 2016). To achieve a better knowledge of our Solar System, the head of the STEAMH project (first author of this paper) launched the challenge of developing the modelling of the Solar System in the city of Tomar (at Portugal) at scale. Inspired in MiMa project (www.mathematicsinthemaking.eu), it was decided to build the Sun and planets at scale and place them in Tomar city, as if the Sun had one meter of diameter. Civil enginneers from IPT designed and built the Sun with one meter of diameter and placed it in the IPT campus at the following coordinates: 39°36'02.4"N 8°23'27.9"W (Figure 3).

The planets were modelled in clay by children, at the Conservation and Restoration laboratory, during their participation in Easter 2016 holidays at the IPT campus (Figure 3). Before starting the activity, the head of the AcademySAH presented a brief introduction about our Solar System inquiring the participants about its dimension. One question was: If the Sun had one meter of diameter what would be the size of the planets? After listening to children's opinions, it was explained to them that they were going to build the planets at scale according to the Sun's diameter of one meter.

Figure 3. The Sun and the planets built by children in Easter 2016 holidays.

After building the planets, it was necessary to choose the locations in the city to place the planets according to their distance from the Sun, at scale. In this task, we used internet to mark the orbital radius of each planet searching for strategic places known by the citizens, as presented in figure 4. The first four planets are in the IPT campus. Neptune is about 3 km from the IPT campus.

Figure 5. Modelling the Solar System on Tomar municipality considering the Sun with one meter of diameter.

We also propose to discuss other Sun's diameters such as half of a meter, a basket-ball or even a tennis ball. For each case, teachers can reproduce the same activity developing mathematical tasks and discussing the planets sizes and where they could be placed in the map of their city. But the STEAMH's project team is always designing new activities and one constrain of the above modelling is that the planets are static, which does not represent the reality of our Solar System because the planets circulate around the Sun. This is the reason we decided to use MAR technologies in order to promote a better understanding of our Solar System.

METHODOLOGY

The MAR game is being improved every school year according to the implementation tests performed with the targeted public. For this reason, we use a Design Research (DR) methodology. This is in line with Reeves (2006) who argues the "need for a better approach to educational technology research" (p. 95) and sustains that educational technologists should undertake DR methodologies". Design-based researchers "make fundamental commitment via close collaboration (…) to developing interactive learning environments in the contexts in which they will be implemented" (Reeves, 2006, p. 98). This context is designed and systematically changed by the researchers. DR is relevant for educational practice and "the research always incorporates systematic educational design processes" (Plomp, 2013, p. 17). In this premise, it presupposes the implementation of several cycles of DR. In our study, we intend to design and develop a MAR game in order to promote students' engagement to learn about STEM. In this regard this is a design research development study (Plomp, 2013).

Event	Duration	Date	Location	Children	Teachers
Christmas holidays	1 h	2016	Polytechnic campus	15	-
Summer holidays	3 h	2017	Polytechnic campus	17	-
Children's Day	30 min	June 1, 2018	City Garden	76	11
Summer holidays	3 h	June 2018	Polytechnic campus	19	-
Primary school class	2 h	October 2018	Santarem district	19	2

Table 1. Game implementation tests over two school years

In this research, we use a qualitative methodology and an interpretative approach (Cohen, Lawrence, & Keith, 2007). Participants are children aged 7 to 14 years old and the teachers who accompanied them during the experiences (Table 1). Data collection includes participant observation, interviews and questionnaires. First author of this paper is a participant observer and the second author is responsible for the triangulation and validation of the collected data.

DATA ANALYSIS AND DISCUSSION

In this section, we discuss the five cycles of design research of the SS_GO game that occurred for two school years. With a DR methodology, after each experience with primary students, the game is upgraded to improve it, in order to be implemented in the next DR cycles. One of the challenges of this game is to design the Solar System at scale (for example as if the Sun had one meter of diameter or if it was the size of a basketball, amongst other scales), in order to provide the users with an idea of the Solar System dimension, by relating the Sun's diameter with the planets' diameters and their orbits (orbital rays).

The first cycles of Design Research in informal learning contexts (school year 2016/2017)

Because of space limitations it is not possible to present in detail each cycle of DR. For this reason, we present the DR phases of the first cycles (Tables 2 and 3) and the summary of the following cycles.

DR phases	DR activity	Outcome
Preliminary research phase	A need to design and develop a MAR game. Several meetings with the researchers and designers' team to discuss the needs, literature review, and what and how to design the game.	*SolarSystemGO* prototype.
Prototyping phase	The researchers and designers' team develop several simulations in order to test the prototype. Literature review to find out how to improve the prototype and more meetings to promote its development.	Recommendations to improve the prototype. Prototype indoor version.
Summative evaluation phase	First experience with the target public: primary school students. Participant observations, questionnaires and interviews to the students are performed. Reflection and discussion about this experience.	2016 Christmas school holidays in the Polytechnic Campus. Upgrade the prototype in order to design an outdoor version.

Table 2. DR phases of the *SolarSystemGO* game – 1st cycle (1st semester of 2016/2017).

First implementation test to evaluate the impact of the SS_GO game with primary school students occurred in the IPT campus during children's 2016 Christmas school holidays. This first experience was an indoor version with the mobile device. This version had several limitations. For example, the users had to wait a while for the orbits and planets were not easy to track. Only one older student finished the game. Almost all the other students, specially the youngest, gave up playing after ten to fifteen minutes looking for planets that were not easy to find or to track.

This first experience was very important for the game designers (the higher education students) to understand the limitations of the software related to the game's performance and what needed to be corrected and improved for the next implementation test. One important challenge was to develop an outdoor version. After the game designers improved the game, a second test was performed at 2017 Summer holidays at the IPT. Table 3 represent the DR phases of the 2nd cycle of the SS_GO game.

DR phases	DR activity	Outcome
Preliminary research phase	Developing the SS_GO prototype in order to design an outdoor version that engages students. Meeting with the researchers and designers' team to discuss problems related the game's performance.	SS_GO prototype upgraded: outdoor version.
Prototyping phase	The researchers and designers' team make several simulations in order to test the prototype. Literature review to find out how to improve the prototype and more meetings to promote its development.	Recommendations to improve the prototype. Prototype upgraded.
Summative evaluation phase	Second experience with primary school students. Participant observations, questionnaires and interviews to the students are performed. Reflection and discussion about this experience.	SS_GO engages students and promotes learning about the Solar System. Developing the SS_GO prototype in order to be performed outside the IPT Campus.

Table 3. DR phases of the SS_GO game – 2nd cycle (2nd semester of 2016/2017).

In the experience performed in the 2nd cycle, students completed all the tasks assigned to them and answered to all the questions included in the game, showing a great motivation and engagement to play it (Costa et al., 2018). These results lead the researchers to prepare the 3rd cycle with the aim to upgrade the game in order to be performed outside the IPT campus.

Next cycles of design research (school year 2017/2018)

In the school year 2017/2018, another group of computer engineering students continued to improve the SS_GO game. Two more cycles of DR (3rd and 4th) occurred in informal learning contexts and were very useful for the improvement of the software until finally it was prepared to be performed in a primary school (5th cycle of DR). The formal experience took place with a group of twenty 4th grade students at a primary school (Manso et al., 2019). The activity started in the classroom with a power point presentation about the Solar System, including planet's dimensions and its characteristics. After this presentation, the class was organized in groups. Each group had a mobile phone with the SS_GO app and went outside to play the game. After finishing the game, they returned to the classroom to find out who's the winner. In order to understand the new architecture of this version and the results of the questionnaires applied to assess the impact of the game in the participants, the reader may see Manso et al. (2019).

In the end, students and their teacher answered a questionnaire. Students answered that they enjoyed playing the game and would gladly repeat the experience. In fact, all of them finished the game and answered all the questions about the planets. The teacher referred that the SS_GO game was very adequate to her students and that it was a way of providing the students with a real experience that improves their learning about the Solar System. Also, she was very excited with the possibility of interacting with the Web page in order to design her own information and questions.

FINAL CONSIDERATIONS AND FUTURE WORK

This paper proposes a MAR app to promote interdisciplinarity in the framework of STEM education. The SS_GO game is being developed with a DR methodology by undergraduate higher education students from a Portuguese polytechnic. After several cycles of implementation of the game in informal and formal learning environments, we verified that this game can engage children to play it and to promote STEM learning. Also, primary teachers consider that the game promotes students' interest to learn school contents. In particular, we argue that this game can be a resource to explore mathematics, namely working the notions of relative scales, distances and sizes.

Our next focus is to develop more mathematical tasks in the framework of the SS_GO game, in order to engage the students and to promote learning about this subject matter. For example, more than working scales with the big numbers of our Solar System it is possible to develop other topics by providing information and mathematical problems using the Web platform. Next DR cycles will be focused on this matter. Final target is to introduce this game in a Continuing Professional Development Programme in the framework of the broader STEAMH project. Based in our research and experience in the field, we sustain the importance of developing MAR games that engage students and motivate them to learn about STEM subject matters adequate to primary school syllabus.

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COMPARING DIGITAL AND CLASSICAL APPROACHES - THE CASE OF TESSELLATION IN PRIMARY SCHOOL

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The relevance of the 3D-printing technology in mathematics education has largely increased in the last years. This triggers the discussion about the benefits of digital approaches and the relation of the digital and the classical approaches. This relation will be pursued in this article using the example of tessellation in primary school mathematics. The basis of the study is a three-day workshop at a primary school in North Rhine-Westphalia (Germany) in which students worked on the topic in a classical way and with 3D-printing technology. Reflection sheets, videographed group interviews and other data material are evaluated using the method of qualitative content analysis according to Mayring (2000) with a special focus on the theory of subjective domains of experience according to Bauersfeld (1983).

Keywords: 3D-Printing Technology, CAD-Software, Qualitative Content Analysis, Nibbling Technique, Tessellation

INTRODUCTION

In recent years, the 3D-printing technology has attracted increasing attention in mathematics education practice and research (c.f. Dilling, 2019, Ng, 2017, Panorkou & Pratt, 2016, Witzke & Hoffart, 2018). It can be used to develop many new and alternative approaches to mathematical topics. The following article shows how the 3D-printing technology can be used appropriately in primary school to teach the topic of tessellation. The comparison with a classical approach to this topic illustrates the opportunities and challenges of the use of digital media in mathematics class.

3D-PRINTING IN MATHEMATICS EDUCATION

The use of 3D printing technology offers many opportunities to enrich teaching. A particular focus can be placed on the qualitative concept building. Sustainable basic ideas can be developed by working with 3D-printing-technology and 3D-printed models. This results in interesting connections to the approach of subject matter didactics. Three options for using the 3D-printing technology in the classroom can be distinguished (cf. Witzke & Hoffart, 2018):

- The technology is used to reproduce existing material.
- The teacher develops individual material for the use in mathematics lessons and the students retrace the developmental process.
- The students develop 3D-printed objects on their own in the mathematics classroom.

This article focuses on the students working individually with CAD software in class. Thus, the role of CAD software as a digital mathematics tool with its parallels to Dynamic Geometry Software is particularly relevant (cf. Dilling, 2019).

Concerning CAD applications, direct and parametric modeling methods can be differentiated. Direct modeling applications enable the user to compile three-dimensional objects of geometric basic bodies (cuboids, cylinders, etc.). Those can be moved on the work plane, directly modified by dragging parts of it (points, surfaces, edges) and finally connected by Boolean operators (union, subtract, etc.). Parametric modeling is based on two-dimensional sketches in a selected drawing plane. The elements of the sketches are fully defined by inserting dimensionings and relationships (perpendicular, parallel, concentric, etc.). The created two-dimensional sketch is extruded subsequently (developed into a volume body). The use of direct modeling applications is generally easier and more intuitive, but it is much simpler to create complex objects with parametric modeling software.

In the workshop on the topic of tessellation described in this article, the students used the parametric modeling software SketchUp. The results of Panorkou & Pratt (2016) show that this software is appropriate for working with primary school students. It enables them to draw the tessellation-unit in two dimensions (figure 1/a). Afterwards, this drawing can be extruded for 3D-printing (figure 1/b).

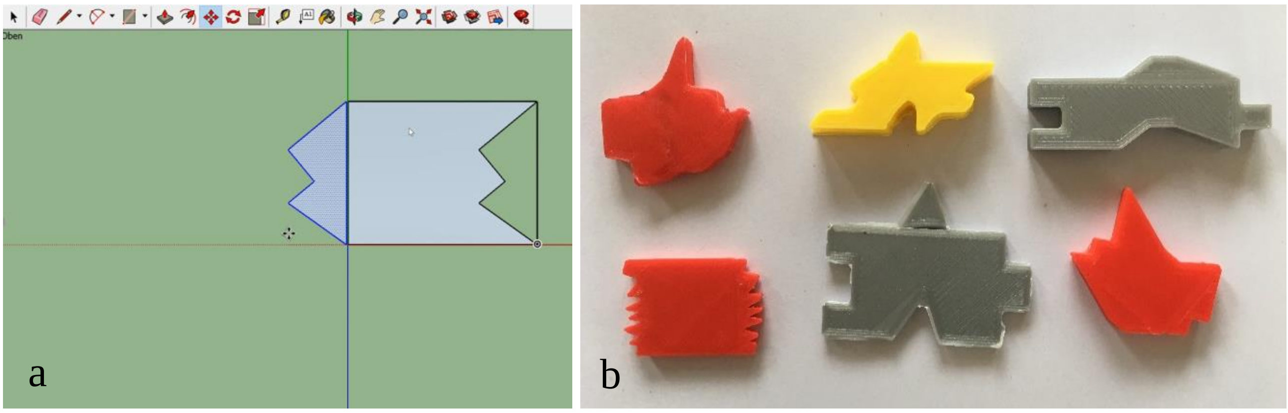

Figure 1: Creation of a tessellation-unit with SketchUp (a) and the 3D-printed objects of the students (b)

The various phases of the 3D printing process (design, construction with CAD, printing, use of objects) and the other parts of the lesson are often very different contexts for the students' development of knowledge. The theory of subjective domains of experience (SDE) is appropriate for the description of such processes of knowledge development and is used for the analysis in this study. According to Bauersfeld (1983) human experience is gained always linked to a context and can be described as separated concerning their situational link. The individual experiences are memorised in separate SDE. An SDE includes the cognitive dimension of an experience as well as motor skills, emotions and valuations. The "society of mind" forms the totality of the SDE of an individual. They are organized non-hierarchical, cumulative and compete for activation within this system. The repetition of a similar situation leads to a consolidation and thus to an effective activation of an SDE. A generalization of concepts occurs through the active attempt to recognize equalities in two different SDE, e.g. by establishment of an analogy. This forms a new SDE that enables a networking of the original SDE.

TESSELLATION IN PRIMARY SCHOOL

A tessellation is the tiling of a plane with no overlaps and no gaps using geometric shapes. Different activities are connected with this topic in the mathematics class in primary school. These include the description and analysis of tessellations, the classification of tessellations, the drawing of tessellations with stencils or on squared paper, the merging of prepared shapes and the production of individual tessellations with the nibbling technique (c.f. Franke & Reinhold, 2016). The study described in this article focuses on the creation of tessellations using the nibbling technique.

The nibbling technique can be used to develop complex elements for tessellation from basic shapes. To do this, a line is drawn between two adjacent corners within the shape. Depending on the basic shape (e.g. rectangle or triangle), the resulting piece can be attached to the opposite or adjacent side. The nibbling technique can be done by handicrafts with pen, scissors and paper as well as with digital media.

According to Eichler (2009), different competences can be promoted with the topic tessellation. On the one hand, this includes mathematical content (examining and classifying shapes, comparing and measuring lengths, angles and surfaces, naming and displaying reflections, rotations and displacements, etc.). On the other hand, different goals of personality development can be pursued (e.g. fantasy and creativity, accuracy and personal responsibility, etc.). Nevertheless, tessellations are not explicitly part of the North Rhine-Westphalian curricula (c.f. MoE NRW, 2008).

EMPIRICAL STUDY

Methodology and Conditions

The benefit of the use of 3D printing technology in the field of tessellation was to be investigated in an empirical study on the basis of a three-day workshop at a primary school in North Rhine-Westphalia. The 24 students of a fourth grade were already familiar with the topic since the previous academic year.

On the first day of the workshop, the term tessellation was repeated using various examples. A definition in the form of “A tessellation is the tiling of a plane with no overlaps and no gaps using geometric shapes.” was developed together. Afterwards, the basic shapes suitable for tessellation were explored in groups. The students formed the initial shapes for the application of the nibbling technique. After the introduction, the students used pen, scissors and paper to create stencils with the nibbling technique (figure 2/a). The stencils were used to draw tessellations on a large sheet of paper (figure 2c).

The second day, the students worked in groups with the 3D-printing technology to create tessellation-units (figure 2/b). They drew those units with the CAD-software “SketchUp Make”. The software was not introduced in advance, since most functions are intuitively understandable. Afterwards, the created virtual models were 3D-printed to enable the enactive work with them.

On the last day of the workshop, the students received their 3D-printed tessellation-units. These were merged to tessellate a section of a plane (figure 2/d). A written reflection brought together the different situations of the workshop and made them accessible for an empirical investigation. In addition, the students developed posters to present their work to their classmates.

The empirical study focused on the following two research questions:

- What are the characteristics of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp?
- What are the characteristics of drawing a tessellation with a handcrafted stencil and merging a tessellation with 3D-printed units?

A variety of different types of data was collected in order to investigate the research questions. These include the video recording of the workshop with three cameras, the recording of the group work on the computer with the screen and webcam recording function, the results of the students (reflection sheets, posters, tessellations, drawings) as well as the video recording of concluding group interviews. The analysis described in this paper focuses on the statements made by the students during the interviews.

The videotaped material was transcribed according to the rules of Dresing & Pehl (2015) and translated into English by the authors for a deeper analysis. The generated data was then categorized using the method of qualitative content analysis according to Mayring (2000). The summarizing content analysis is performed in four steps. The first step includes the detailed description of the data material. In the second step, the relevant parts of the text are summarized (paraphrasing). The paraphrases are generalized at a defined level of abstraction. The number of generalized statements is then reduced several times by increasing the level of abstraction and removing statements of the same meaning. The statements are compiled in a system of categories in the third step. This system is checked based on the material in the fourth step.

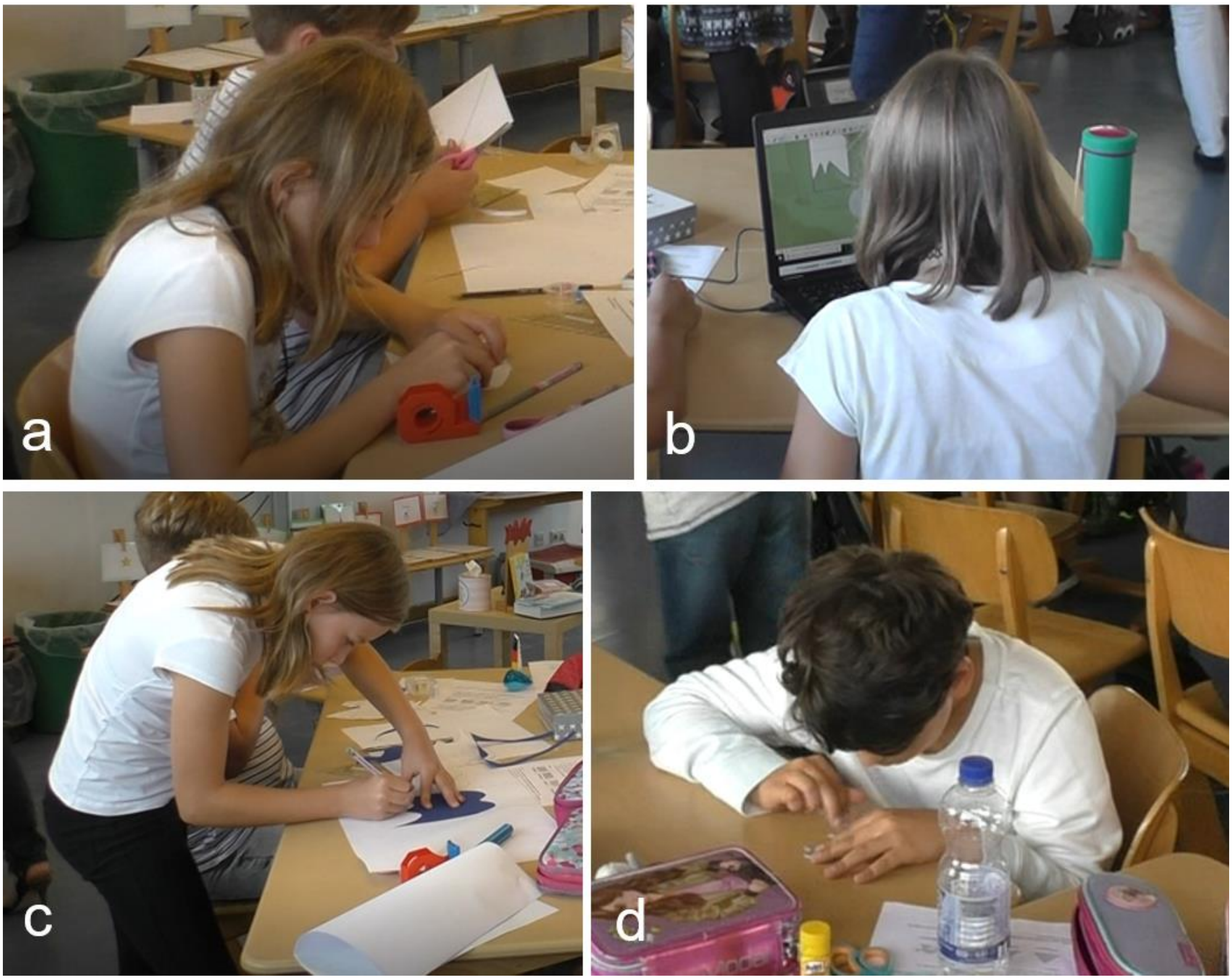

Figure 2: Students draw a tessellation stencil by hand (a), draw a tessellation-unit with SketchUp (b), draw a tessellation with a handcrafted stencil (c) and merge a tessellation with 3D-printed units (d)

Results and Interpretation

Eight categories could be formed inductively based on the data material in order to answer the first research question on the comparison of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp. The system of categories is presented in table 1.

Category (with a Computer and by hand)	*Paraphrase*
C1: Precise vs. not precise	Working with the computer is more precise and less error-prone.
C2: Fast vs. slow	Working with the computer is faster.
C3: Virtual-enactive vs enactive	Working with the computer is virtual-enactive while drawing by hand is actually enactive.
C4: Errors can be corrected vs. cannot be corrected	Errors can be corrected with the computer while errors cannot be corrected that easily by hand.
C5: Group work vs. individual work	Working with the computer takes place in groups while drawing by hand is individual.
C6: Aesthetic result	The results of both approaches are aesthetic.
C7: Motivation and concentration	Both approaches enable focussed and motivated learning.
C8: Problems and errors	There were problems and errors with both approaches.

Table 1: System of Categories for the comparison of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp

The statements of the students on the differences and similarities of the two approaches were very meaningful. Many of the statements concern the precision of both approaches (C1). The computer is described as more precisely in terms of drawing shapes and lines as well as in terms of joining shapes together. However, inaccuracies often occur when using pencil, scissors and paper.

Student 4: You could draw a bit more precisely.
Interviewer: Where was it possible to draw more precisely?
Student 4: You could make the rectangle more exact.
Interviewer: On the computer?
Student 2: Mhm. ((nods))

(Interview 1)

Student 4: Yes, because I think if you stick it together properly with your hand, it's not as neat as on a computer.

(Interview 1)

Other statements of the students concern the duration and extent of both approaches (C2). Working with a computer is much faster and less time-consuming because drawing is easier and the shapes do not have to be cut out. The 3D printer handles the production of the objects.

Student 10: Because you can draw better with it than with your hand.
Student 9: And then you can do a lot more things faster.

(Interview 3)

Student 14: And with the computer I thought it was cooler, because you didn't have to put the scissors in the corners and cut it out like that, but you could just press a button and then you could get it right out.

(Interview 5)

The relationship between the two forms of actions was also in the focus of the students' statements (C3). Working with pen, scissors and paper is perceived as a real action in which sensations play a role. In contrast, working with the computer is different because only the buttons are pressed and the computer performs the actions itself. These statements of the students can be described with the dimensions virtual-enactive and actually enactive described by Hartmann, Näf & Reichert (2007).

Student 13: There you could also have the sensation and see how to cut. There is no sensation on the computer. You just type things or just draw with your finger. You have to press this top row all the time to do something new or something like that. That was, I would say it sucks because with scissors, pencil and paper, there you could simply get the things out of the pencil case and draw along and cut and done.

(Interview 5)

The students consider the possibility of correcting errors with the undo function to be a significant advantage when working with the computer (C4). This is not that simple when doing things with a pen and scissors.

Student 7: Well, that will stay forever - you can't just erase it like on a computer.
Student 6: But you can erase it. ((looks at student 7))
Student 7: But not when you've cut it in.
Student 6: Yes.

(Interview 2)

One last difference between the two approaches is observed in the social form (C5). On the one hand, the work with pen, scissors and paper is done individually. On the other hand, computer work takes place in groups.

Student 2: And with the computer everybody has actually done something of it (...), so what she also said. We have/ So it isn't just created by me or by (...) her ((points to Student 4)) and that's where we did it together so I made the nibble (…) I don't know who made the rectangle anymore. ((looks at other students))
Student 1: That's what I did. ((answers))

(Interview 1)

In addition to the differences that the students notice in the two approaches, there are also some similarities. These include, among other things, the aesthetics of the results (C6). Individuality is particularly appreciated in the handcrafted products, while stability and accuracy are emphasized in the computer-assisted production.

Student 1: Somehow more beautiful because you know that you did it yourself. With the computer you can say "Yes, I did it myself" but only on the computer. Somebody else can do that as well, it's not just from you.

(Interview 1)

Student 14:	But in the end it looks really nice.
Student 13:	Yes, and it's more stable.
Student 14:	As paper. ((looks at student 13))

(Interview 5)

According to their own statements, the students were motivated in both approaches and worked with concentration (C7). However, errors and problems occurred with both approaches (C8).

With reference to the theory of subjective domains of experience, two contexts can be identified. These are determined by the approaches with the computer and with pen, scissors and paper. The students' statements first show some differences in the approaches that emphasize different properties of the mathematical content tessellation. However, it can also be stated that the students were able to recognize a structural equality in the developed SDE and formed a comparative SDE.

Three categories were built in order to answer the second research question, which focuses on the comparison of the drawing of a tessellation with the handcrafted stencil and the merging of 3D-printed tessellation-units. The system of categories is presented in table 2.

Category	*Paraphrase*
C1: No difference between both approaches	The students do not see any difference between the two approaches.
C2: Different sizes	The stencil and the unit have different sizes.
C3: Merging is faster	Merging the units is faster than drawing with the stencil.

Table 2: System of Categories for the comparison of the drawing of a tessellation with the handcrafted stencil and the merging of 3D-printed tessellation-units

Almost all students do not see any difference between the two approaches (C1). The differences mentioned do not affect the structure of both approaches. They concern the different sizes of the stencil and the unit (C2) as well as the duration of both approaches (C3). The stencils are bigger and thinner, whereas the 3D-printed units are smaller and thicker. Merging units is generally faster than drawing with a stencil. Thus, the two approaches of drawing a tessellation by hand and merging 3D-printed units seem to result in the formation of a comparative subjective domain of experience.

Interviewer:	Do you think it's something else or don't you think it matters?
Student 6:	Um, actually I think it's the same and you? ((looks at student 5 and student 7))
Student 7:	Yes.
Interviewer:	Does it matter whether you draw or merge?
Student 5:	No.

(Interview 2)

Student 13:	So it takes a little longer to draw, but if you have them and you're sticking them together, it takes half a minute? ((shrugs his shoulders)) That's very fast.

(Interview 5)

Student 11:	Different is of course that it/that the forms are thicker ((S2 nods approvingly)) but smaller, with the paper it is bigger but thinner. So there is no advantage.

(Interview 4)

SUMMARY AND OUTLOOK

The results of the empirical study show the opportunities of the 3D-printing-technology to support mathematics learning. In addition to the different chances of the approach to tessellation with digital media (i.e. precision, duration), various challenges arise (i.e. virtual enactivity, technical problems). Especially the parallel use of the digital and the classical approaches has led to an effect of synergy by the development of comparative subjective domains of experience. These enabled the students to get to know different facets of tessellation and combine the different advantages. The reflection on the characteristics of the two approaches led to the adoption of a meta-level and the linking of contents and methods.

It turns out that the use of 3D printing technology can be successful in primary school mathematics teaching. Many other contents of primary school mathematics also seem to be suitable for the use of 3D-printing technology. Further studies have to be conducted to investigate the use of this technology in greater depth.

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STUDENTS' CHOICE AND PERCEIVED IMPORTANCE OF RESOURCES IN FIRST-YEAR UNIVERSITY CALCULUS AND LINEAR ALGEBRA

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With a plethora of (digital) resources available to first year engineering students studying mathematics, it becomes increasingly important to understand which resources are preferably selected by students. In this study we investigated (1) which student clusters can be distinguished in the perceived importance of resources to study first year mathematics courses; and (2) how these clusters can be related to selected student characteristics and the perceived importance of resources at high school. We conducted a survey among Calculus and Linear Algebra students (N=403) at a Dutch university of technology. In terms of data analysis, we used "community analysis" based on a social network approach. Results showed that three clusters could be identified; these are portrayed and discussed. In one cluster interactive digital resources for formative assessment were considered of some importance, while students in the other clusters preferred traditional curriculum resources.

Keywords: student resources; university mathematics; community analysis; student cluster

INTRODUCTION

Undergraduate university students have a plethora of digital and traditional resources at their disposal to study mathematics. Several studies have addressed undergraduate university students' learning with and use of particular resources to study mathematics (Anastasakis, Robinson, & Lerman, 2017; Howard, Meehan, & Parnell, 2018). These studies focused on the use of resources provided by the university, such as textbooks, live lectures, online lecture videos and mathematics support centres. Anastasakis et al. (2017) noticed a lack of empirical studies on the full range of resources students use to learn mathematics (those provided by the university and those selected by the students themselves). They studied this range of resources, in relation to the students' goals, among second year undergraduate engineering students. In this paper, we report on a survey study on the selection and perceived importance of digital and traditional resources, conducted among first-year engineering students at a Dutch university of technology. We identified groups of students across different mathematics courses who held similar views on the resources they considered the most important. Moreover, we compared selected student characteristics in these groups (e.g. importance of high school resources).

We expected that different mathematics courses, in terms of student body and organizational context, may lead to a different selection and use of resources. Therefore, we studied the resources, and their importance, in first term Calculus (CS) courses, offered to all first-year students at three different levels (CS A, B and C), and a Linear Algebra (LA) course, offered only to mathematics and physics students, in a first-year Bachelor College programme. We also investigated, retrospectively, the selection of resources and their importance in high school, because high school experiences might have had an impact on first-year engineering students' approaches to learning (Cook & Leckey, 1999).

In this paper, we first outline the theoretical frame of "resources" (and their use), we review research literature related to "high school to university transition", and student selection of resources, as lenses to investigate the role of resources in students' mathematics learning. Second, we describe our methodology, data collection strategies, and methods of data analysis. In the third section we present our findings and discuss our results, followed by, fourth, the identification of potential implications for the practice of university mathematics learning and teaching.

THEORETICAL FRAMEWORK

The lens of resources

We assume that the ways university students study mathematics are influenced and shaped by their use of the various resources at their disposal. Gueudet and Pepin (2018) have defined student resources as anything likely to re-source ("to source again or differently") students' mathematical practice, leaning on Adler's (2000) definition of mathematics "re-sources" (used by teachers). This definition emphasizes that resources are not limited to material artefacts. Associated with the "resources approach to mathematics education" (Trouche, Gueudet, & Pepin 2019), Pepin and Gueudet (2018) have proposed a classification of (curriculum) resources in order to understand and discuss their selection and use. For this paper, we distinguish between (1) *curriculum resources*, which are proposed to students and are aligned with the course curriculum, (2) *general resources*, which students might find/access on the web, and (3) *social resources*. The traditional textbook belongs to the curriculum resources, and so do digital documents (e.g. lecture notes) and interactive digital resources (e.g. digital homework and practice systems). Interactivity allows digital curriculum resources to play a role in personalized formative assessment practices, helping students to understand their progress and take the next step in learning (Pepin & Gueudet, 2018). In terms of social resources, we refer to formal (e.g. tutor-student conversations) and casual human interactions (e.g. conversations with friends, or social media contacts). At times it is difficult to distinguish between curriculum and social resources: for example, live lectures and tutor group meetings are proposed to students and aligned with the curriculum, but they clearly also have a social component.

RELATED RESEARCH

Transition from secondary to tertiary education

In terms of mathematics learning, the transition from high school to university is challenging for many students (Pepin, 2014). High school mathematics prepares students to work on a precise and narrow set of tasks, for which they follow the teacher or use worked examples in textbooks for guidance. Hence, completing many exercises appears to lead to success in tests and examinations (Gueudet, 2008). However, in the transition to university, students experience an number of discontinuities (Artigue, 2016), and a change in the (implicit) rules and expectations related to studying and learning (Pepin, 2014). In particular (a) the mathematical content is introduced faster; (b) more autonomy is expected; (c) the levels of generalization and abstraction are higher; (d) the mathematical approach is more formal; and (e) the institutional cultures at high school and university are different (Artigue, 2016; Gueudet, 2008). Moreover, at university students have to autonomously manage the various resources to learn mathematics, but high school does not prepare them well for this task (Williams, Black, Davis, Pepin, & Wake, 2011).

Student selection of resources

Several studies have focused on student selection among a limited number of resources (e.g. the choice between live lectures and video recordings). Howard et al. (2018) studied first-year Business

Studies students' choices to attend live mathematics lectures, watch short online videos, or combine the two. They identified four usage patterns: students focusing on the videos, students focusing on the live lectures, dual users of videos and live lectures, and switchers. Students with a weaker mathematical background tended to belong to the group of dual-users. However, students may need guidance to combine the use of various resources into an effective study strategy (Inglis, Palipana, Trenholm, & Ward, 2011).

Regarding the selection among a wider range of resources, Anastasakis et al. (2017) made an inventory of the resources selected and used by students when studying mathematics modules, and related these to student learning goals. Participants in this study predominantly focused on success in examinations, and selected resources accordingly. The most widely used resources were those that the university provided, and students' own notes. The use of mathematics textbooks was specifically linked to the study of worked examples, which were said to help students to prepare for examinations.

In our earlier qualitative and quantitative studies (Kock & Pepin, 2018; Pepin & Kock, 2019) we identified the resources considered important by students (a) at the end of high school, (b) at university in three CS courses, and (c) at university in an LA course. At high school a limited range of mostly traditional resources was considered important by the students. These included first of all the textbook, followed by worked solutions, past exam papers, the graphical calculator and the teacher. From focus group interviews we identified a number of *actual student study paths* for resource use at university, that is, ways in which students arranged and orchestrated the resources to study mathematics. Students' perceived importance of resources at university had similarities with that at high school, although general web-based resources gained some importance (e.g. Kahn-academy videos). Moreover, we identified differences between the importance of resources in the four courses (3 CS, LA), which to some extent could be explained by the nature and organization of these courses. However, from these studies it did not become clear if there were clusters of students, characterized by similar views on the importance of resources.

Research questions

We propose the following research questions for this study:

1. What student clusters can be distinguished in the perceived importance of resources to study first-year mathematics (CS and LA) courses?

2. How can these clusters be related to selected student characteristics and the perceived importance of resources at high school?

METHOD

Context

The study took place at a university of technology in the Netherlands, with a student body of approximately 13,000 engineering students. The university offers 15 bachelor courses related to technology and engineering. We collected data on the importance of resources in two first-year first term courses: Calculus (CS); and Linear Algebra (LA). CS was offered to all first-year students (approximately 2000), and was differentiated at three levels of increasing perceived difficulty and formality of mathematics: CS A, CS B and CS C, selected according to students' majors and preferences. LA was only taken by mathematics and physics students (approximately 150). The university offered a Digital Learning Environment (DLE), digitally giving access to text resources (e.g. lecture notes), as well as access to videotaped lectures and, in the case of Calculus, access to an interactive homework/practice system for formative assessment.

Data collection strategies

A survey was designed and administered to all first-year engineering students of the university at the end of the first quartile of the 2017-2018 academic year. The survey was administered on paper to selected students in one of the final first term lectures and electronically to the remaining first-year students. We received 446 responses in total, of which 403 responses remained (after removing largely incomplete responses).

The survey contained items on the frequency, importance, and use of resources for studying mathematics (a) at high school, and (b) at university. In particular we asked students what they considered the five most important resources to study mathematics, from a list of relevant resources based on the course catalogue, interviews with lecturers and students, and the literature (e.g. Anastasakis et al., 2017). The list included traditional and digital curriculum resources, digital general resources and social resources. Additional information (age, gender, mathematics course, high school mathematics results, mathematics self-efficacy at university) was collected to relate the data on resources to student characteristics. The items were partly amended from surveys used in the Transmath study in the UK and Norway (e.g. Pampaka, Pepin, & Sikko, 2016).

Analysis

To investigate if any clusters were present in the perceived importance of resources across the university cases, we followed the community analysis procedure described by Brunetto (2017), based on a social network approach. We chose this method, because it is said to produce meaningful results in spite of the distribution of the survey responses, which did not allow for the classical forms of cluster analysis. To our knowledge, this application is relatively new in an educational context (Brunetto, 2017), although it is recognized as a powerful tool in other contexts, such as criminal networks (Calderoni, Brunetto, & Piccardi, 2017). The method consisted of three steps.

First, we created an undirected, weighted network between students based on their responses to the item concerning the five most importance resources. We emphasise that we did not consider this student network as a real network of social relations, but as a tool to analyse commonalities among students based on their survey responses. In the network, students were represented as nodes. We created a link between two nodes (i.e. two students), if they chose the same resource; the more important the resources were to the students, the stronger the link. To be precise, the weight of a link was calculated as follows: (a) the five most importance resources selected by a student were given a score between 5 (most important) and 1 (least important); (b) for the same resources selected by two students, the corresponding scores were multiplied; (c) the weight of the link between the two students was calculated by summing the multiplied scores. Responses to other items in the survey, such as the students' mathematics self-efficacy, represented further attributes of the nodes.

Second, we looked for possible clusters in the network, that is, groups of nodes characterised by comparatively large internal connectivity (Calderoni et al., 2017). In general terms, a cluster consists of nodes that tend to connect more with the other nodes of the same cluster than with the rest of the network. In our context, a cluster would include students with a similar pattern concerning the importance of resources. We adopted the Louvain method (see Calderoni et al., 2017) to identify clusters (or communities) of students according to their responses to the survey. The Louvain method partitions the set of nodes based on the optimization of a particular quantity, the network modularity Q. Given a partitioning of a network into clusters ($C_1,C_2,\ldots,C_k$), the modularity Q is by approximation the (normalized) difference between the total weight of links internal to the clusters C_k, and the expected value of such a total weight in a randomized null network model suitably defined. We evaluated the quality of the clusters that were identified by the Louvain algorithm by calculating the persistence probability α_k, which measures the cohesiveness of the cluster C_k. The significance of α_k is identified by the standard z-score (Calderoni et al., 2017). In this way, we were able to identify clusters in the network, as well as their "goodness".

Third we compared the average perceived importance of resources and other student characteristics in the resulting clusters using Analysis of Variance (ANOVA). The statistically significant differences between variables made a qualitative characterization of the clusters possible.

RESULTS

Community analysis and identification

The community analysis, employed on this network, identified three clusters (or communities). These were non-trivial because of their persistence probability α_k around 0.4 and high z-score (see Table 1).

Table 1. Details of the community analysis

Cluster	Number of students	α_k	z-score
C1	93	0.38	29.26
C2	138	0.48	25.58
C3	172	0.49	12.52

Using ANOVA and Games-Howell post hoc tests, we compared the mean perceived importance of resources among the three clusters to characterize them, confirming their non-triviality. Table 2 shows only the significant differences resulting from this comparison. In particular, the last column informs about the statistically significant characterization of the three clusters.

Table 2. Comparison of mean perceived importance of resources among the three clusters

	Mean (SD); scale 0 – 5			
Resource	C1	C2	C3	Post Hoc test
Textbook	1,56 (1,7)	4,36[a] (0,87)	1,12 (1,3)	C2 > C1 & C3
Worked solutions	0,86[a] (1,3)	2,13 (1,6)	1,92 (1,8)	C1 < C2 & C3
Materials created by the teacher (e.g. lecture notes)	1,00 (1,6)	1,00 (1,5)	2,77[a] (2,2)	C3 > C1 & C2
DLE, homework and practice environment	0,55 (1,3)	0,67 (1,4)	1,35[a] (1,9)	C3 > C1 & C2
Past examinations	0,71[a] (1,3)	1,42 (1,7)	1,88 (1,9)	C1 < C2 & C3
General online videos[b]	0,35[b] (1,0)	0,76 (1,5)	0,70 (1,4)	C1 < C2 & C3
Teacher explanation of content	3,91[a] (1,4)	0,19 (0,5)	0,27 (0,7)	C1 > C2 & C3
Teacher explanation of problem solving	2,26[a] (1,9)	0,20 (0,7)	0,30 (0,8)	C1 > C2 & C3
Working together, outside class	0,19[a] (0,8)	0,40 (0,9)	0,60[a] (1,3)	C1 < C3

Notes: [a]: p<0,01 in post-hoc test, [b]: p<0,10

Cluster 1 (C1) students considered *teacher explanation of content* and *teacher explanations of problem solving* as the most important resources whilst *worked solutions*, *past examinations*, and to a lesser extent *general online videos* were less important, than in the other clusters. Students in cluster 2 (C2) considered the *textbook* as the most important resource. In cluster 3 (C3) *materials created by the teacher (lecture notes), worked solutions*, *past examinations* and the university's *DLE (homework and practice environment)* were perceived as the most important resources.

Further characterisation of the clusters

The network was created based on students' perceived importance of resources. In the subsequent analysis we exploited other characteristics of the cluster members to enhance the picture. We compared the different clusters in terms of the *different courses*, the *background variables* measured in the survey, and the *relative importance of resources at high school*, measured retrospectively.

The clusters consisted of students from all four mathematics courses (see Table 3), with the exception of C2 among LA students. This is understandable, as C2 consisted of students who considered the textbook as the most important resource, and in LA no textbook was used.

In the courses CS A and CS C, more students belonged to C2 (textbook oriented) than to C1 (lecturer oriented) and C3 (oriented to teacher created materials and DLE). In CS B, more students belonged to C3 than to the other clusters. LA students belonged to C1 and C3 in equal numbers. In general, C3 was the largest cluster, followed by C2 and C1.

Table 3. The three clusters and the mathematics courses

Course	C1	C2	C3	Total
CS A	19	50	18	87
CS B	44	77	126	247
CS C	7	10	5	22
LA	23	1	23	47
Total	93	138	172	403

The membership of male and female students to the different clusters was investigated per course because the distribution of male and female students across courses was not equal. We used SPSS Crosstabs and Chi-Square tests. No significant differences were found, except for the case of CS B, where 23% of male (more than expected), and 11% of female students (less than expected) belonged to C1, while 44% of male (less than expected) and 60% of female students (more than expected) belonged to C3 ($\chi^2(2)=8{,}27$, $p=0{,}016$). No significant differences were found between the clusters regarding mathematics self-efficacy or high school pedagogy.

The most important resource at high school for the students in all clusters was the textbook. Based on ANOVA, the students in C1 reported a significantly higher importance of teacher explanations at high school than the other clusters ($p<0{,}01$). They reported a significantly lower importance of past exam papers than the other clusters ($p<0{,}05$). No significant differences were found regarding the importance of other resources at high school. The students in C2 perceived less meaningful connections in the university learning environment than the other clusters ($p<0{,}01$). However, this was only due to the CS A members of this cluster.

CONCLUSION AND DISCUSSION

To answer the first research question, we found that three student clusters could be distinguished: a cluster of students who considered explanations by the lecturer the most important resource (C1, the smallest cluster), a cluster who considered the textbook the most importance resource (C2), and a cluster who considered other curriculum materials (teacher materials, worked solutions, past examinations, the DLE) the most important resources (C3, the largest cluster). Members of all clusters were present in the four courses, with the exception of C2 in LA (as in LA no textbook was used). However, different proportions of students belonged to the three clusters in the different subjects. Concerning the second research question, we found that concerning teacher explanations, members of C1 on average perceived these as more important in high school than members of the other clusters. Moreover, we found that relatively more female CS B students belonged to C3, and less to C1, while relatively more male CS B students belonged to C1 and less to C3. Digitally provided materials (e.g. lecture notes, past examinations) had some importance, in particular in C2 and C3. However, the DLE did not rank among the most important resources (though relatively more important in C3). In an earlier study, we found that for some students the formative assessment provided by the DLE was not sufficiently aligned with the final exam (Pepin & Kock, 2019).

The results provide quantitative support for the claim that students choose different study paths to study mathematics at the start of university (Pepin & Kock, 2019), that is, they put different resources at the centre of their approach. Selected students who relied on the textbook at high school, did so also at university. The university lecturer took an important role for those students who considered the teacher explanations as relatively important in high school. Apparently for these students their high school study practices remained the default position. Another group relied on new resources in the university environment: the teacher materials, and to some extent the DLE. However, membership of C2 or C3 could not be related to the selected student characteristics we included in our survey. Further research needs to clarify if other student characteristics are relevant, or if the selection of resources at the start of university is a process with little predictability, due to the new situation students encounter at university and the plethora of resources available.

The results provide quantitative support for the claim that curriculum resources and course organizations effect student study practices (Pepin & Kock, 2019), as the distribution of cluster membership varies per course. Further research among a larger group of students is needed to investigate differences within courses between different lecturers or student majors. Such differences might also explain the different cluster memberships of male and female students in CS B, for which we now have no further explanation.

The most important resources for members of C1 and C2 were traditional curriculum resources (Pepin & Gueudet, 2018), while members of C3 valued resources belonging to a more blended approach. In many cases the resources mix to study mathematics does not appear optimal (Inglis et al., 2011). It is relevant to monitor the changes over time in the selection and importance of resources, as digital resources receive more emphasis and course developers begin to guide students on resource selection.

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EMPOWERING TEACHERS TO ENGAGE STUDENTS IN MATHEMATICAL LEARNING THROUGH DIGITAL COMPETENCE SCENARIOS

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Twenty-first century students are exposed to technology from a young age and build up digital competencies throughout their school years. However, research has shown that students do not have opportunities to put into action their digital competencies when learning mathematics. This poster presents an example activity that integrates mathematical learning in a broader scenario within the CRISS platform that enables the guided acquisition and certification of digital competencies.

Keywords: digital competencies, mathematical competencies, teaching scenarios

INTRODUCTION

In the digital era, students' acquisition of digital competencies is viewed as a necessity. In 2013, the European Commission stated that all students need to acquire digital competence before completing their compulsory education and therefore be better prepared to join the labour force. In the same year, the European Commission's DigComp framework on digital competencies for citizens was published (Ferrari, 2013). This is structured around a number of main areas, namely Information, Communication, Content criterion, Safety and Problem Solving. A *digital competency* is described as "the set of knowledge, skills, attitudes […] required when using ICT and digital media to perform tasks; solve problems; communicate; manage information; collaborate; create and share content; and build knowledge effectively, efficiently, appropriately, critically, creatively, autonomously, flexibly, ethically, reflectively for work, leisure, participation, learning, socialising, consuming, and empowerment" (Ferrari, 2012, p. 43). However, its application and adaptation to schools is not that straightforward. Thinking about mathematics classrooms in particular, these general digital competence areas cannot be directly linked to the mathematical context and in fact not all of them are of equal importance with regards to certain mathematical competencies (Geraniou & Jankvist, 2018).

Teachers are expected to support students' acquiring mathematical knowledge and mathematical competencies, but at the same time they are challenged to employ technology to promote digital competencies. To make things even more complicated, teachers are often not convinced that digital technologies can have a better impact on students' mathematics learning compared to non-digital resources and are hesitant in using them in their practice due to their perceptions, attitudes, professional development experiences and technical or pedagogical support networks (e.g. Clark-Wilson, Robutti & Sinclair, 2014).

The EU-funded H2020 project CRISS [1] came together to address some of the issues raised above, particularly the need to document and certify digital competences in primary and secondary schools. The CRISS platform provides opportunities for students to acquire digital comptencies through broad scenarios and associated activities that encourage creativity and collaboration within a particular subject or even across the curriculum. Teachers are able to configure these scenarios and through learning analytics monitor their students' progress and support them in their journey to certification using the CRISS digital competence framework that is based on DigComp (cf. Guardia, Maina & Julia, 2017).

A SCENARIO IN THE CRISS PLATFORM

As an example activity of a scenario that integrates mathematical and digital competencies, we focus on a particular instantiation of the well-known Monty Hall probability problem, which is about examining the best strategy for winning a prize based on probabilities. Students are encouraged to search for a simulation presenting the game show, compare and contrast the chances to win for each option and design a gameshow for peers. They are asked to reflect on the problem first on their own (personal inquiry), then search for a simulation on the internet in pairs (e.g. Scratch or Geogebra) and use it to solve the probability problem collaboratively. Their final task is to recreate this problem using a programming environment such as Scratch or GeoGebra and design their own simulation. The teacher chooses the most interesting simulation and asks the pair of students who created it to 'host' a game show for their peers. Through this process, the scenario addresses the following digital competencies from the CRISS framework: (a) Collaborating through digital technologies, (b) Planning, searching and critically selecting data, information and digital content, (c) Developing digital content, and (d) Programming and configuring digital tools, applications and devices and some mathematical competencies from the Danish KOM project (Niss and Højgaard, 2011): (a) reasoning competency by following a line of inquiry, (b) problem tackling competency, (c) modelling competency, (d) symbol and formalism competency and (e) modelling competency.

The CRISS scenarios are currently being piloted at schools. In this presentation, we will share the project's early findings and showcase how the two frameworks, i.e. CRISS and KOM, can be used to analyse the interplay of digital and mathematical competencies. We therefore envisage addressing how teachers integrate digital competencies in their practice and in students' mathematical learning.

NOTES

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ENHANCING CHILDRENS' ARGUMENTATION SKILLS IN PRIMARY SCHOOLS USING DIGITAL LEARNING TOOLS – INTERPRETATIVE ANALYSIS OF A FIRST DRAFT LEARNING ENVIRONMENT

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Within the project "Prim-E-Proof", digital media learning environments (Open Source Applets on Tablet PCs) are designed. The aim is to support argumentation and proving skills through the iconic and interactive visualization of reasoning processes in primary school. The focus of the project is on supporting classical teaching and learning processes with digital learning environments. In this paper selected scenes of the argumentation process of two pupils using a first draft learning environment (Platz, 2019) are analysed using interpretive methods.

Keywords: Educational Media, Primary Schools, Argumentation.

INTRODUCTION

> *"[…] The marginal place of proving in elementary school is seriously problematic. There are at least two reasons for that. First, elementary students are deprived of opportunities to learn deeply in mathematics. Second, when students ultimately encounter proving in secondary school or university it feels alien to them rather than a natural extension of their earlier mathematical experiences at elementary school." (Stylianides, 2016, p. 1)*

To foster a proof understanding the project "Prim-E-Proof" investigates the basics of proof understanding to derive conclusions for the proof understanding in higher education. That is why it aims to go beyond the mediation of argumentation competences in Primary School. But is this possible? Asking the same question as Stylianides (2016): "As these young children engage in mathematical reasoning, how does that activity connect to and prepare them for understanding proof in more advanced mathematics, a challenging topic even for older students and adults?" (Stylianides, 2016, Foreword). Pedemonte (2007) derives, that a structural change is required for the construction of a deductive proof, that is from abductive or inductive steps to deductive steps. A first draft learning environment using digital media was developed and tested in an empirical pilot study (Platz, 2019). The result was, that we did not succeed to awaken a need for proof (Kothe, 1979) in the pupils using this learning environment. Therefore, it was not possible to reconstruct a structural change for the construction of a deductive proof (Pedemonte, 2007). Nevertheless, a thorough analysis of the argumentation process of the pupils who participated in the study can provide indications for the optimization of the learning environment. In the present paper selected argumentation sequences of two fourth grade pupils are analysed using interpretive methods.

OBJECTIVES

The aim of the overall project is to create learning environments to support argumentation skills through the iconic and enactive visualization of reasoning processes. A central media aspect is to make the handling with teaching and learning material computer-detectable and thus, to infer optimized learning environments. The focus is on supporting classical teaching and learning processes in primary education with both digital and non-digital learning environments. As a first step, arithmetic proofs are focussed e.g. the sum/ difference of an even/ odd number and an even/ odd number is even/ odd. The objective of this paper is to give an interpretation of the argumentation process of two pupils while working with the first draft learning environment (cf Platz, 2019). For this purpose, digital learning tool specific patterns of action and opportunities (as well as risks) for promoting learners' reasoning skills are identified.

THEORETICAL BACKGROUND

For the analysis of students' proof and problem-solving activities exist several findings and theories. Selected theories are presented below and interpreted in the light of the specific form of support for such processes through digital learning environments.

Proof and Reasoning

Reasoning is not a new phenomenon and has been scientifically studied, especially in the recent century. A peculiarity of reasoning in elementary school mathematics lessons is that arguments are not just recited by a single person to justify their opinion. Rather, all learners are "usually involved in interaction processes that produce an argument in the entirety of their actions" (Krummheuer & Brandt, 2001, p. 18, Translation by the authors). Different structural schemes are used to analyze arguments (Budke & Meyer, 2015). In this article, argumentation processes will be represented in a model which is a combination of the approaches of Toulmin (1958), Toulmin, Rieke and Janik (1979), Miller (1986), Aberdein (2005), Pedemonte (2007) and Reid, Knipping and Crosby (2011), see Figure 1 (cf Platz, 2019). Toulmin's layout (1958) forms the basis of the developed model. Toulmin himself applied his layout to mathematics (cf Toulmin et al., 1979) and Aberdein (2005) applies Toulmin's layout to multi-step proofs. Over and above that Toulmin's model can be utilized as methodological tool to compare proof and argument (Pedemonte, 2007). To enable to grasp the entire process of argumentation and proof development, the refutation of arguments needs to be involved (Reid et al., 2011) as well as the structure of the argumentation process (Miller, 1986).

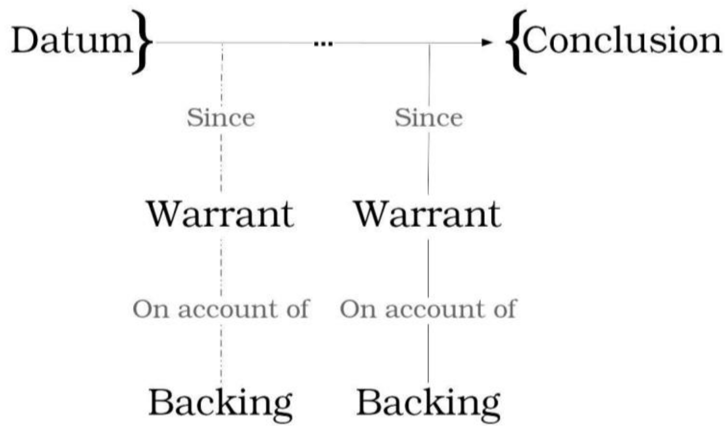


Figure 1. General argumentation scheme (cf Platz, 2019); the diagram can be read like a timeline from left to right; the dashed lines mark refutation trials of the assertion

Digital Learning Tool

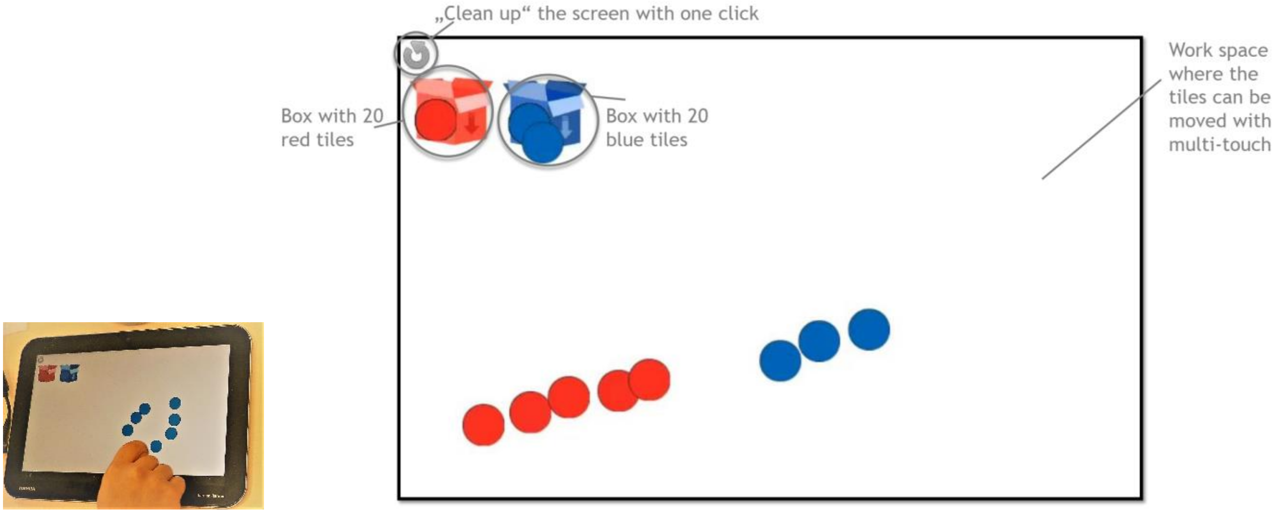


Figure 2. Child working on the freely available "(Reversible) Tile Applet" (http://www.melanie-platz.com/WPA/) and Screenshot of the Applet with explanations of the functionality.

The developed digital tool "(Reversible) Tile Applet" (see Figure 2) is developed for Tablet PCs and requires a browser which allows for multi-touch (e.g. Firefox). In contrast to existing apps and applets with tiles, no structuring aids are given in order to ensure a certain openness of the use of the applet. The applet can thus be used in the function as a medium of argumentation and proof. Due to the two-dimensional screen, the tiles cannot be stapled or lifted up. The current version of the applet acts almost as substitution of analogue tiles and can therefore be classified in the first step of enhancement referring to the SAMR-Model (Puentedura, 2010). Compared with analogue tiles, the organizational handling of the applet is more suitable for everyday use as it can be quickly provided and disposed off and single tiles cannot disappear (cf Platz, 2019). In contrast to real material, a modified applet would allow to generate any number of tiles. This would offer the potential to be used as a fact in a chain of arguments: No matter how many tiles are additional created to the existing one, the total number is always even. As the number of tiles increase, pairs of tiles can always be formed. In the learning tool used in the study design, only 40 tiles (20 red, 20 blue) were made available to the pupils to avoid the distraction of the pupils due trying to generate the most tiles possible and not working on the task given to the pupils. This concern can be supported by the reaction of one pupil in the test group: "You have to do that with more tiles. Because 40 is, we can already do it, we already count up to one million. These are too few then." Nevertheless, due to the potential of this feature of an arbitrary number of tiles, it needs to be tested in further empirical studies if these concerns are justified. Furthermore, a central media aspect shall be to make the handling with teaching and learning material computer-detectable and thus, to derive optimized learning environments. Multi-touch-"gestures" can be made detectable with the help of digital media, and automated evaluation and help selection are supposed to be provided. The idea is to support on the one hand the teacher by receiving feedback on the individual learning process of the pupils and through decision support on the when and how to start a discussion in a classroom context supporting argumentation processes. On the other hand, the pupils are supported through individualized task and support design and tailored help suggestion. With implementation of these functionalities, the first step of transformation according to the SAMR-Model (Puentedura, 2010) can be reached, which is modification, i.e. the technology allows for significant task redesign and enables other possibilities to handle heterogeneity in a school class. In order to optimize the developed learning tool and the learning environment, Design Science research is performed which is grounded in the philosophy of pragmatism (cf March & Smith, 1995). The Artifacts are the developed learning environments applying digital learning tools to support argumentation skills in primary school.

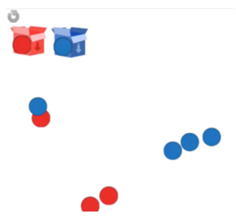
STUDY DESIGN

An empirical pilot study on the first draft learning environment (Platz, 2019) was performed with a fourth-grade school class of a German regular school containing 23 pupils (aged 10-11 years). The school class was separated into groups of six or respectively five pupils. Three tablets were made available where pupils worked in groups of two using one tablet and discussing their ideas and results. In each of the four cases, one group of two pupils of the three groups of pupils was videographed and the tablet screen was screen-casted, both with audio recording. Due to the teacher, the pupils had experience with working with analogue tiles and they knew how to control a tablet PC, but they had never worked with the applet or on proving tasks. A worksheet was given to the pupils: http://www.melanie-platz.com/ES_1/Task_even-and-odd.pdf. The main task the pupils were concerned with in the learning station is the following: *If you add two odd numbers together, you always get an even number. Is that correct? Give reasons!* An example on how a preformal proof for the task above could look like is made available at the following link: http://www.melanie-platz.com/ES_1/Preformal-Proof_even-and-odd.png. Each group worked on this task for 20 minutes. The recorded material was transcribed and sections were extracted, where information on the argumentation process could be discerned. In this paper, we take a closer look at selected scenes of the argumentation process of the two pupils Liam and Veit (names changed for data protection). The analysis of the data serves the purpose of reconstructing the arguments of the learners (user perspective). On the basis of the use by the learners, optimization possibilities for the learning environment can be determined in a next step (developer perspective). In order to reconstruct the user perspective, the transcript sections are analyzed with the help of interpretive methods (eg Bauersfeld, 1980, Voigt, 1995) with regard to argumentation structures.

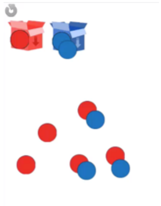
EMPIRICAL RESULTS

Scene 1

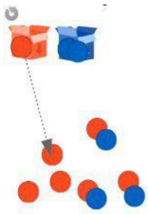
In the following scene Liam and Veit work on the given task: *If you add two odd numbers together, you always get an even number. Is that correct? Give reasons!* Right before the beginning of the scene, the two students decide to edit the task 5+3. The following dialog starts:



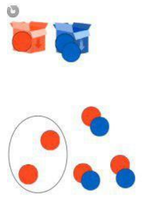
1	Liam:	This can be put away. (*puts the blue tile on the upper left side back in the blue box. Afterwards, Liam shoves the remaining red tile next to the other red tiles. Hereafter, he creates two more red tiles and also shoves them next to the other red tiles*) first like this and then- (*Liam pairs each of the blue tiles to a red tile. Therefore, he rearranges the order. Two red tiles have no corresponding partner.*) Two remain, again even! (*laughter*)



2	Interviewer:	Exactly, that is, what can you do with those two?
3	Veit:	// Put away?
4	Liam:	Another pair! //
5	Interviewer:	Another pair, exactly!
6	Veit:	Red and Red.
7	Liam:	Red. (*creates another red tile and pairs it with one red tile*)



8	Interviewer:	Ah, no, you do not have to put another one, this is already a pair.
9	Liam:	Oh, I see.
10	Interviewer:	Or?
11	Veit:	Yes, because we could do it like this and this (*shoves the two remaining red tiles together like the other pairs*)



12	Interviewer:	Okay, great!
13	Veit:	We could also put this away. (*shoves a red tile in direction of the red box*) And just get a blue one, works too (*shoves the red tile back to the other red tile*).
14	Interviewer:	Okay, exactly

Liam shows a structured approach in turn 1 ("first like this and then"): First, the task is illustrated color-coded with the tiles. Then, corresponding pairs of blue and red tiles are formed. For the remaining red tiles, Liam uses the quantity to support the statement that an even number is the result. At the request of the interviewer (Turn 2), Liam expresses the possibility of pairing the remaining tiles (Turn 4). Veit responds and gives permission to pair up two red tiles (Turn 6). Liam uses this option to create a new red tile (Turn 7). Why did Liam create a new red tile in Turn 7? One possible interpretation is that Liam has already accepted the fact of the even result and now wants to increase the red summand to show the phenomenon of the even result for arbitrary odd summands. However, in contrast to his procedure in Turn 1 he does not explain this and the interpretation is not confirmed in the following turns. Probably opposed to the creation of new tiles, Veit explicates his idea of pairing in Turn 11, as hinted at in Turn 6: The two red tiles can be easily paired and no more tiles need to be created. This procedure is accepted by the interviewer. Therefore, and according to interaction patterns in primary school (cf. Voigt, 1995), the argumentation process ends. In this scene, the phenomenon formulated in the theoretical background can be observed: arguments are manifested in the interaction by several learners (Krummheuer & Brandt, 2001). In the present scene, facets of an argument are taken up and modified by interactants and altogether an argumentation chain manifests itself in the conversation. Since, interactants refer to (suspected) rules of the other one: Veit seems to assume Liam's rule of pairing: "pairing is only possible from blue to red tiles" in Turn 13 and therefore probably explains his approach from Turn 11 with the explication that one can also replace a red tile with a blue tile.

The reconstructed argumentation process can be represented in the presented argumentation scheme, see Figure 3.

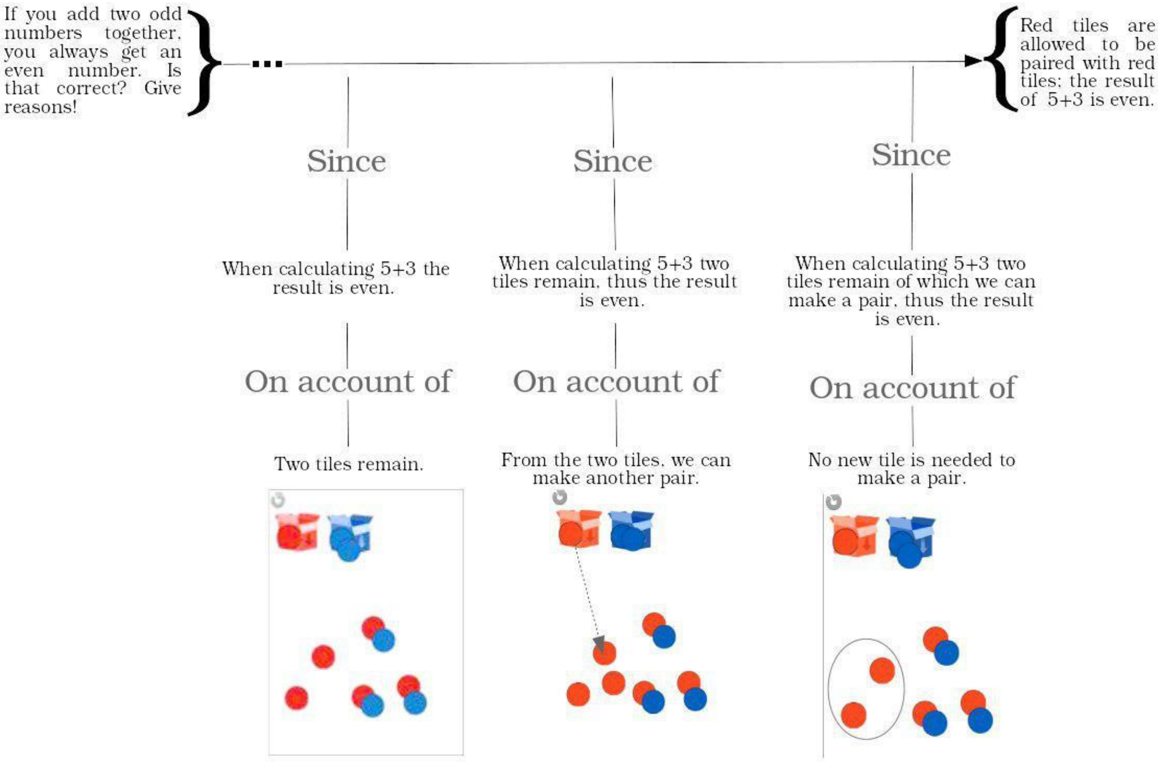


Figure 3. Representation of selected scenes of the argumentation process of Liam and Veit; the diagram can be read like a timeline from left to right; screenshots with markings visualize the action of the pupils while speaking (cf Platz, 2019)

Scene 2

Immediately following the first scene, the dialogue continues as follows:

15	Interviewer:	then maybe just try with a (.) larger number so with (.) seven plus.
16	Veit:	uh
17	Liam:	//Five.
18	Interviewer:	Then maybe seven plus three // or something like that.
19	Veit:	seven plus eighty

20 Interviewer: Try seven plus three, if this (.) what happens then?

21 Liam: Seven three.

22 Veit: Well ten.

23 Liam: Three blue ones.

24 Veit: // Ah

25 Liam: Three four // five six seven plus three. (...) Look seven plus three. (..) Oh, I have to allocate it, like that (..) that and again like that and that (inc.) that that, okay, we have it!

26 Veit: Yes.

27 Liam: Easy!

Here is a much faster procedure than in the first scene to watch. Liam seems to have accepted the rule that red tiles are allowed to be paired with red tiles, and is quick to do that in Turn 25. Striking is the repeated use of the structured procedure from the first scene: First, the tiles have to be laid in sorted order by color. Then, the pairing has to be done. Perhaps Veit would like to do a complicated task and is under-challenged (Turn 19) but does not adhere to the requirement of adding two odd numbers, this interpretation is supported by quickly naming the even result ten in Turn 22. Ten is an even number, why should we now deal with this, the solution is clear. Overall, the argument from Scene 1, developed in the interaction and accepted by the interviewer, is correctly applied to a new task.

CONCLUSION

In our analysis facets of the (scientific) correct argumentation in the interaction have been worked out and the developed argumentation scheme has been successfully applied to the presentation of the interpretively worked out results. Likewise, possibilities for a generalization of the utterances of Liam (Scene 1) and Veit (Scene 2) have been identified. Some questions require further processing in the research project. Thus, Liam and Veit do not fully exploit all the possibilities to support their argumentation: For instance, the fact that a larger number of tiles (up to 40) can be generated in the developed app is not applied to Liam's and Veit's argument. This seems to be a problem of domain specificity (cf. Carrraher, Carraher & Schliemann, 1985 for pupil's handling with analogue manipulatives): which backings of arguments are supported by the tools offered? Which not? The orientation of learners to several options of backings seems to be a very important matter, since digital tools allow other possibilities than analogous ones. In a next step the developer perspective will be focused in order to derive optimization measures for the applet and the learning environment on the basis of the reconstruction of the user perspective presented in this paper.

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SUPPORTING ALGORITHMIC APPROACH TO BASIC SCHOOL MATHEMATICS BY PROGRAMMING TASKS

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The paper describes some findings from a programming task book project. The book contains tasks on arithmetic (operations, calculating the value of numeric expressions, common fractions etc), number theory (factors, prime numbers, GCD, LCM, Euclid algorithm), constructions in planar geometry, algebra (linear equations and inequalities, systems of linear equations, polynomials), some types of nonroutine tasks. The paper also brings up the necessary information processing that is not explicit when mathematical tasks are solved by paper-and-pencil method: parsing of algebraic expressions, finding the coordinates of intersection points by planar constructions, programming of algorithms for the tasks that appear in textbooks only in the form of one single numerical example. Some warnings are given about the impact of "brute force" computer solutions of reasoning-oriented tasks.

Keywords: teaching of programming, factorization, algebraic manipulation, geometry constructions, nonstandard tasks

INTRODUCTION

All developed countries complain about lack of specialists of Information Technology, including programmers. The universities try to take in more IT students and prepare more IT specialists. In order to create the necessary prerequisites, countries experiment with introduction of elements of programming in the school syllabus. Thereby it is quite natural that programming will be taught by the teachers of mathematics (in many countries their training contains certain amount of programming) and even as a part of the mathematics subject. Using mathematics-oriented tasks can help the mathematics teacher to enter the world of programming education. Mathematics textbooks contain many tasks that can be easily reformulated as programming tasks. On the other hand, to prepare more people for IT studies, the algorithmic side of the subject could be brought into greater attention in the teaching of mathematics.

The first major initiative on using programming in teaching of mathematics was LOGO programming (Papert, 1980; Feurzeig & Papert, 2011). LOGO creates a microworld for mathematical experiments. Later the most popular school programming language has been Scratch (Scratch; Benton, Hoyles, Kalas, & Noss, 2016). There exist tens of publications about projects on teaching concrete topics of mathematics using LOGO, Scratch or similar languages. They describe nice programming tasks that are often not just mathematics but can be solved using some mathematical knowledge. Some analyse of learning mathematics through programming is given in (Misfeldt & Ejsing-Duun, 2015). The tasks of our task book are taken directly from the mathematics textbooks. In every chapter of the task book the student should implement or use for solving the tasks the textbook algorithms and solution methods.

Exploring the exercises and theory in textbooks, we see that some problem types are presented together with solution algorithms (like long multiplication or division, operations with fractions, solution of linear equations, some constructions in geometry). However, other (also completely algorithmic) types are presented just with particular numeric examples (Lints, 1981, page 182):

How to measure 7 litres of water using 3- and 5-litre vessels?

One of the possible reasons for this is that the algorithm can be too labour-intensive to execute by hand. In such a case, execution of algorithms on the computer facilitates developing more algorithms for Basic School mathematics. This article describes a project for creating a task book of programming tasks based on the mathematics tasks from the textbooks for grades 1-8 in Estonian schools (age of 7-16 years). The task book formulates programming problems that correspond to different task types in mathematics.

The author has tried to include in the task book not only the most interesting or most important task types but all the task types that have considerable algorithmic character. The resulting task set is the author’s answer to the question, What is the algorithmic content of the Basic School mathematics? Unfortunately, this answer is too voluminous for such a short paper. The paper gives a brief overview of the themes of the tasks and concentrates then on the issues that can be not very obvious for the teachers but can be important when using the tasks. We consider some issues of expression manipulation and geometry construction tasks, discuss the possibility of solving reasoning-oriented tasks by “brute force” programming and point to the algorithms for the task types that appear in the textbooks in the form of one single numerical example.

THEMES OF THE TASKS

A large part of the algorithmic content of school mathematics is presented explicitly in the form of algorithms for various types of standard tasks or example solutions. In arithmetic and algebra, the textbooks formulate, for example, algorithms for long multiplication or division, factorization of integers, operations with fractions, solution of linear equations and equation systems, operations with monomials and polynomials. Basic school geometry contains algorithms for bisection of a segment or an angle, for construction of parallel or orthogonal line, construction of a triangle from given three elements. Obviously, we can reword these problems as programming tasks and require writing programs that do the same work. At the time of writing this paper, our collection of mathematics-inspired programming tasks contains 194 items extracted from about 20 Estonian textbooks of different authors. The order of tasks mirrors the places where the underlying mathematical formulation of the task first appears in a textbook for the respective grade. In many cases, the tasks contain more than one variant and some of them can belong to the textbook(s) of higher grade(s). Often the tasks on a mathematical topic begin with a series of preparatory technical tasks. For example, the chapter on geometry constructions begins with drawing segments, rays and lines based on two given points and with drawing circles with a given centre and a given radius or passing through a given point. Some of the subsequent tasks require finding the coordinates of intersection points of two lines, of a line and a circle, and of two circles. Using subroutines for these tasks, it is possible to program school algorithms for bisection of a segment or an angle, for drawing a parallel or an orthogonal line, etc.

The tasks contain computerized variants of tasks from the following topics of the mathematics syllabus:

1. Arithmetic: operations, calculating the value of numeric expressions, decimal digits, equalities and inequalities;
2. Number theory: factors, prime numbers, GCD, LCM, Euclid algorithm;
3. Constructions in planar geometry. Different classes of triangles and quadrangles;
4. Common fractions;
5. Linear equations and inequalities, systems of linear equations;
6. Operations with monomials and polynomials;

7. Solution of some types of nonstandard tasks, using
    a) nested loops,
    b) breadth-first search.

There are also some smaller topics containing (comparatively routine) tasks on

8. Calendar, weekdays, clock;
9. Units of measurement.

Our task book does not contain programming technique exercises that belong to general introductory programming courses. We assume that, before solving our tasks, students have completed a course/chapter on the technical side of programming in some language. The tasks are oriented not to "toy" languages (Logo, Scratch etc) but for "proper" programming (Python, Java, C etc). Working on our tasks also requires more mathematics than studied in grades 1, 2, … that correspond to the underlying textbook tasks. By integration of subjects of programming and mathematics the task book can be used as a source of programming tasks for students. But the teacher can also use a store of implemented solutions for demonstrations or organization of student experiments. Partially or defectively implemented solutions can be used for focusing the attention on special cases in definitions or algorithms.

In the following sections of the paper we discuss the task settings and investigate what additional mathematics and algorithms are necessary for computerized solution of problems in particular topics. Programs for the arithmetic and algebra tasks should combine Basic School mathematics with parsing of expressions. Geometry constructions use the drawing commands. But they are based on coordinates of the points. We establish what amount of analytic geometry would have to be implemented for different construction steps. We describe yet the algorithms for solution of two types of nonroutine problems from the textbooks and analyse the impact of programming on reasoning-oriented tasks.

ARITHMETIC AND ALGEBRA. PARSING AND EXPRESSION MANIPULATION

We discuss here the tasks that belong to computer algebra. The first major topic of our exercises is calculation of the value of arithmetic expressions composed of integers. We start with first-grade exercises containing only one operation, like $2+5$, $4-1$, etc. The tasks of higher grades contain expressions of growing complexity. In our usual paper and pencil calculations and expression manipulations, we extract decimal numbers, operation signs, parentheses and variables in the expressions almost without formulating explicit rules for this. The textbooks contain explicit rules for the order of operations. Some books can give a list of operation signs and tell that letters of the Latin alphabet represent variables. To solve the same tasks on a computer we should formulate the syntactic rules explicitly, apply some general or task-specific algorithm for parsing the expression, and finally implement a calculation algorithm that solves the actual problem. For learning to use syntactic restrictions we have included tasks with various contents of expressions (different sets of possible operation signs, possibility of unary operations, parentheses). For organization of calculation process we recommend implementing both bottom-up and top-down approaches. Note here also that some programming languages have standard functions (for example, `eval` in Python) that count the value of an expression immediately. For learning/creating the necessary algorithms, our tasks prohibit application of such functions.

The task book contains several chapters with expression manipulation tasks (operations with fractions, solution of equations, inequalities and equation systems, operations with monomials and polynomials). The textbooks present detailed algorithms for solution of many task types and the programs should just implement them. But the students discover also that some textbook algorithms are not complete. They leave some decisions to the user (e.g., what unknown to isolate first, …). Some algorithms are not described explicitly but should be extracted from a series of examples.

For expression manipulation tasks we use the same expression treatment instruments as in arithmetic but do not need any new tools outside of school mathematics algorithms. The input data of number theory tasks (factorization, prime numbers, GCD, LCM, Euclid algorithm, etc.) consist of one or more integers. The programs for these tasks do not need syntactic supplements. For them it is sufficient to implement just the known mathematical algorithms.

GEOMETRY CONSTRUCTIONS

In the context of mathematics education, programming of geometry construction algorithms means programming of some construction blocks of a dynamic geometry system. The student learns what mathematics is working inside programs like Geogebra. Considering ruler and compass versus computerised solution of elementary mathematics construction problems, we discover that they are based on different information and even on different mathematics. Constructions with paper and pencil belong to Basic School mathematics while writing respective computer programs in general-purpose programming languages requires application of analytic geometry.

Classical ruler and compass constructions do not require any calculations. The intermediate and final results of the constructions are received and used in their graphical form. For drawing a line or a circle we can place the edge of the ruler, or the needlepoint of the compass, at a freely chosen point on a plane, at a point belonging to a curve or at an intersection point of two curves. The task of construction of a point can be completed simply by declaring that the answer is the intersection point of certain two curves.

In programming languages the commands for elementary construction steps (like drawing a point, segment or circle) are based on coordinates, for example, `circle(centre,radius)`. Points, lines and circles can be drawn only when the program "knows" their numerical parameters. This means that information processing of the program that utilises a construction algorithm differs significantly from information processing of the student who makes the same construction on paper. For example, the program for bisection of the angle ABC can consist of the following steps (Figure 1):

1) Draw the circle $c1$ with centre B and radius $R1 = min(|AB|, |CB|)$.
2) Let D be the intersection point of AB and $c1$. Find coordinates of D.
3) Choose the radius $R2 > R1$,
4) Draw the circle $c2$ with centre at D and radius $R2$.
5) Let E be the intersection point of CB and $c1$. Find coordinates of E.
6) Draw the circle $c3$ with centre at E and radius $R2$.
7) Let F be the intersection point of $c2$ and $c3$. Find coordinates of F.
8) Draw the line through B and F – bisector of ABC.

In ruler and compass construction, the calculation steps 2, 5 and 7 are not necessary and the minimum of two radiuses in step 1 can be found on paper without calculating the values of radiuses.

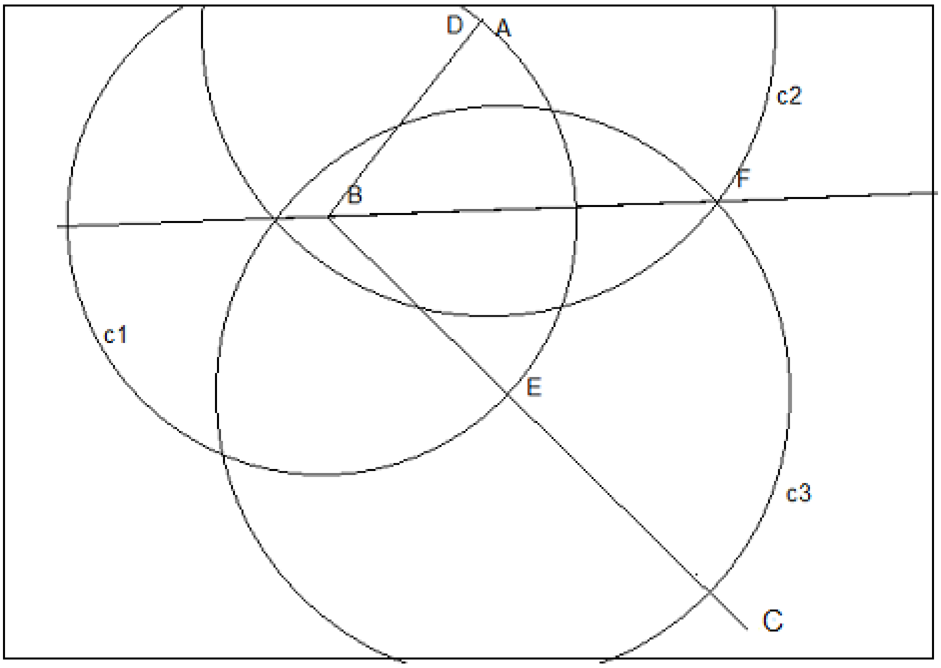

Figure 1. Construction of angle bisector

Note the possibility of an intermediate approach, implementing an interactive program that does not calculate the coordinates of D, E and F but asks the user to point them by mouse (like in dynamic geometry). However, our task book contains only tasks for completely automated solution.

To demonstrate the coordinate-driven character of the drawing commands, we start the chapter on constructions with elementary tasks on drawing of points, segments and circles using different input data (for example, drawing the circle with a given centre and one point on the circle or radius). A particular task can require one or more commands, depending on the programming language. Crucial tasks for modelling ruler and compass constructions are finding the coordinates of intersection point(s) of two lines, a line and a circle and two circles, but also finding the distance between two points. The first task requires solution of a linear system of equations and the fourth uses the Pythagorean theorem. They should be feasible for school students. The second and third tasks are normal exercises in university analytic geometry courses, but their worked solutions should be comprehensive. We have included web links to the theory and detailed mathematical solutions of both tasks so that they can be converted to (branching) programs that find the coordinates. We recommend finalizing the programs for the technical tasks described here in the form of subroutines (functions) that can be used in subsequent constructions. This makes the jump from ruler and compass constructions to their computer implementations easier.

The main goal of our construction chapter is creation of computer programs that model on screen the Basic School ruler and compass algorithms. We saw that, compared to original tasks, this can require additional calculations. The programs should also be capable of handling special cases because the textbook algorithms do not speak about coincidence of points, vertical and horizontal lines, etc. On the other hand, for some tasks the coordinate-based solution can be simpler than doing the same without coordinates. For example, the midpoint of a segment can be received without any construction. We can get the coordinates of the midpoint (as an average) and then use just one command to draw the point. The same can be done for drawing a parallel or an orthogonal line. Together with any task requiring modelling the ruler and compass construction we can ask for a possible easier way to get the result.

REPLACING ASTERISKS OR LETTERS WITH NUMBERS

Many textbooks contain entertainment-style tasks requiring students to replace asterisks or letters with decimal digits and to get a known mathematical structure, for example, a long multiplication scheme as in the left part of Figure 2 (Telgmaa &Nurk, 2002, page 82).

```
   **                 ab
x  52              x  52
  ---                ---
   **                 cd
+ **               + ef
  ---                ---
  *7*                g7h
```

Figure 2. Task of restoring long multiplication

The idea behind such tasks is to make the student analyse what can we conclude about the digits when we take into account the operations between the numbers. For example, our first conclusion can be that the first digit of the first factor should be 1. Otherwise, the first subsum would be a 3-digit number. Further, the first digit of the first subsum can only be 2 or 3 and the second digit of the second subsum can only be 5 or 0. Only the combination of 2 and 5 gives the sum 7, etc.

Consider now what do we get if we ask the student to write a computer program that solves this task. The coarsest approach would be choosing eight variables for the digits replacing the eight asterisks in the figure (Fig 2 right) and writing eight nested loops where the variables $a, \ldots, h$ take the values from 1 to 9 for the first digits of the numbers and from 0 to 9 for the remaining variables. For each combination of the values of $a, \ldots, h$ the program should check whether multiplication of $10a + b$ with 52 gives the subsums and the final result that correspond to the values of other variables. In reality we only need 7 loops, as $d = h$. The task can be solved already by this brute force approach. Execution of the body of the loop 10^7 times is possible even using a regular laptop. However, the program can be significantly accelerated. The subsums and the product are defined by the values of the factors. We can seek for a and b such that multiplication of $10a + b$ with 52 gives a 3-digit result where the subsums are 2-digit numbers and the second digit is 7. These conditions can be expressed as $52(10a + b) < 1000$, , $2(10a + b) < 100$, $5(10a + b) < 100$ and $mod(div(52(10a + b), 10), 10) = 7$.

For general evaluation of the situation we can tell:

1) The conditions for the digits in long multiplication and other similar schemes are easily expressible in programming languages;
2) The structure of the program is trivial and the program works quickly;
3) Using the loops can be successfully combined/accelerated with using the logically derived conditions;
4) If the student implements only a part of the conditions in the program, then the extraneous solutions can be quite easily recognised and removed.

From this we can conclude that programming is a powerful instrument for solution of tasks of this and similar types. Solving such tasks can persuade students to learn programming. On the flip side, solution by computer allows replacing mathematical thinking with quite routine composing of the loops. Even more, if the student finds a program for some other task of this type, then it would be quite easy to adapt. In some sense, using computers can ruin a nice type of bonus problems in the textbooks. The teachers must be prepared at least for permanent modification of task conditions.

TASKS WITH HIDDEN ALGORITHMIC ESSENCE

We investigate here another type of nonstandard tasks that can be solved by programming. As early as in the grade 1 textbook (Lints, 1981, page 182), we find the following task:

How to measure 7 litres of water using 3- and 5-litre vessels?

The key to finding the solution is that by filling the 3-litre vessel from a full 5-litre vessel, 2 litres of water remain. Adding it to 5 litres we get 7 litres. There are some other tasks of similar kind in this textbook and in textbooks for higher grades. The textbooks formulate the tasks with concrete numbers of different coins/vessels/… and their sizes. They rely on ingenuity of the brightest students and do not speak about solving such tasks in a general case.

Educated programmers and students who have trained for programming olympiads know that such tasks can be solved using breadth-first search. Every stage in water measuring process can be described by a triplet of numbers (a, b, c) where $a \in \{0, \dots, 3\}$ is the amount of liquid in the 3-litre vessel, $b \in \{0, \dots, 5\}$ is the amount of liquid in the 5-litre vessel and c is the amount of liquid in the vessel for the result. The algorithm finds consecutively the stages that can be reached with 0, 1, 2, etc., steps (pouring operations). The initial stage (0, 0, 0) corresponds to 0 steps. With step 1 we can get (3, 0, 0) and (0, 5, 0). From them we can get with step 2 the stages (0,3,0), (0,0,3), (3,5,0), (3,2,0) and (0,0,5). Further we can construct the stages that need 3, 4, 5, … steps. The task is solved when we get to a stage where the third component is 7. There is no need to construct stages where the third component is higher than 7+5. The search can be finished when at some $n \epsilon N$ no stage gets this number of steps. Our textbook task has two 5-step solutions: (0,0,0), (0,5,0), (0,0,5), (0,5,5), (3,2,5), (3,0,7) or the same with last two stages (0, 0, 10), (3, 0, 7).

The preceding paragraph shows that this task type has a solution algorithm. However, school mathematics does not present such tasks in an algorithmic manner for two reasons. The algorithm uses a data structure (multi-dimensional array) that is too complex for Basic School. Further, the necessary solution time and the required amount of memory grow quickly with increasing input data. The calculation can be executed by hand only if the amount of input data is fairly limited and the number of necessary steps is small. Solving the tasks on a computer has also restrictions of the same kind but the examples from textbooks are computer-solvable. We can demonstrate their algorithmic character. Unfortunately, we must state here again that using computers can change the status of reasoning-oriented tasks.

CONCLUSIONS

Our project of conversion of mathematics tasks to programming exercises gave us about 200 quite natural-looking programming problems. From the perspective of mathematics education:

- Considering mathematical problems as programming tasks is a much better demonstration of the idea of an algorithm. Instead of nondeterministic sets of conversion rules (for operations with fractions or polynomials, for solution of equations and equation systems, etc.), we see these rules being executed in the order that is prescribed by the program. It is possible to change the order and compare different variants.
- Programming of solution algorithms requires a fairly detailed understanding of mathematical theory – definitions and possible special cases, textbook algorithms. It is possible to check this understanding by supplementing the tasks with test data for complex cases.

Many schools and teachers use computer algebra and dynamic geometry software in mathematics teaching. Programming of tasks that correspond to the commands of such programs tells the students what is inside of mathematical software.

- It is possible to emphasize details of algorithms (for example, checking only the numbers up to square root of n by finding the factors of n) and the computational complexity. Note that for paper and pencil calculations this can be more important than for using computers.
- Execution of algorithms on a computer enables them to be applied to much bigger numbers and bigger data. It also facilitates development or at least discussion of solution algorithms for some new types of problems.

Some issues need attention of the teachers.

- In case of certain topics, the program should do some work that is not explicit in paper-and-pencil solutions (understanding of expressions, calculation of coordinates on the plane). This can require some new algorithms and new mathematical knowledge. When supervising such programming, the teacher should have a clear understanding of potential needs that may arise.
- The teacher must know which algorithms have incomplete or example-based formulations available for students in textbooks.
- Some reasoning-oriented tasks, which are quite common in textbooks, can be solved by rather trivial or standard programs. The teachers should be informed about such opportunities.
- On the other hand, programming can help the teachers when they have need to create fresh versions of tasks mentioned in last two sections. Programming enables to evaluate the existence and number of solutions of new versions.

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IMPLEMENTING AUGMENTED REALITY IN FLIPPED MATHEMATIC CLASSROOMS TO ENABLE INQUIRY-BASED LEARNING

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When learning mathematics following flipped classroom approaches, students most often have to watch videos as homework before class. Teachers should consider not consistently using a video where a new concept is introduced at the beginning of the learning process. If it suits the learning material, learners could first explore the problem independently and afterwards they could watch a video to consolidate their findings. By implementing augmented reality (AR) activities in flipped classroom scenarios, inquiry-based learning can be fostered. But, designing such sequences can be challenging for educators. Thus, this paper describes the development of teaching model templates for AR-based flipped classroom scenarios. Using a design-based research approach, further research activity will focus on the evaluation of the proposed teaching model templates to investigate how mathematics teachers adapt the templates to their teaching needs.

Keywords: augmented reality, flipped classroom, inquiry-based learning, student-centred learning

INTRODUCTION AND THEORETICAL BACKGROUND

The development of AR applications (for instance GeoGebra AR) provides new possibilities to support learning processes in mathematics education. Though Samuelsson (2006) accentuates that it is not only using technologies that should be central but above all the way how new technologies are used in mathematics education that determines the orientation and thus the success of mathematics lessons - namely utilising technologies for discovering and exploring mathematics. Johnson et al. (2011) point out that AR can be used especially to support inquiry-based learning through exploration. Learners need enough time to explore new phenomena at their own pace. The implementation of AR activities in flipped classroom environments can lead to more in-class time, which can then be used for exploration.

Azuma (1997) defines AR as an interactive real-time technology, which combines and registers real and virtual objects in real 3D space. Spatial problems, which are difficult to grasp in 2D, can be presented and explored in 3D using AR (Woods et al., 2004). Furthermore, Seichter (2007) found out that AR supports the development of spatial abilities, and according to Pemberton and Winter (2009) the use of a collaborative AR environment can support students in acquiring a deeper understanding of concepts by generating knowledge in groups and reflecting on their experiences. Moreover, several studies (Dünser & Hornecker, 2007; Lamanauskas et al., 2008) indicate that AR activities can increase students' motivation.

Lage, Platt and Treglia (2000) define the key-term "inverting" of flipped classroom as follows: "Inverting the classroom means that events that have traditionally taken place inside the classroom now take place outside the classroom and vice versa" (p. 32). Therefore, in traditional flipped classroom environments, students usually have to watch videos before class, and the in-class phase is used for different student-centred, group-based or problem-solving activities. More and more research projects (e.g., Esperanza, Fabian & Toto, 2016) demonstrate that flipped classroom approaches can improve students' achievement in learning mathematics. But rather than merely presenting facts or worked-out examples, the learning process in a flipped mathematics classroom can also be initiated by posing questions or problems, thus fostering inquiry-based learning. According to Bloom's taxonomy (1956), inquiry-based learning is fundamental for the development of higher-order thinking. However, only a few studies (e.g. Raouna & Lee, 2018) are available on how to design learning activities in a flipped classroom that encourage higher-order conceptual thinking. Moreover, few attempts (e.g., Bujak et al., 2013) have been made to investigate the role of AR in mathematics teaching, and many unanswered questions remain about the integration of AR activities/environments in mathematics education. For instance, how to integrate AR learning activities effectively in blended learning scenarios such as flipped classroom environments is still unknown (Ibáñez & Delgado-Kloos, 2018). So, it is important to investigate how AR-based flipped classroom sequences should be designed to enable inquiry-based learning in mathematics education.

In order to achieve the research aim, teaching model templates are developed with regard to existing literature. The proposed teaching model templates should act as a guide to assist secondary mathematics teachers designing AR-based flipped classroom scenarios focusing on inquiry-based learning.

From the design perspective, the development of the templates adopts design-based research (Reimann, 2011) as a study method to improve practice and advance the theory of learning. According to Bakker (2018), research and development in this research methodology are closely linked. Through various iterative design-cycles, the teaching model templates will be applied in practice, and then refined. However, the goal is not to validate and compare the two templates during a field trial.

DESIGN OF THE TEACHING MODEL TEMPLATES

In the following section, two recently developed teaching model templates for AR-based flipped classroom scenarios are presented. These templates are built on inquiry-based pedagogies. We use inquiry-based learning as an umbrella term and regard problem-based learning as a specific way of learning through inquiry.

Inquiry-based teaching model template

The development of inquiry goes back to John Dewey, who developed it in the twentieth century (Dewey, 1938). His model is based on five cyclical phases: "asking questions, investigating solutions, creating new knowledge as information is gathered, discussing discoveries and experiences, and reflecting on new found knowledge" (Crippen & Archambault, 2012).

According to Albion (2015), the lowest common denominator of all inquiry-based pedagogies is a "big question" that serves as a starting point. This question is sometimes also described in a broader sense as a problem or project. In inquiry-based learning environments, teachers support the learning process and provide knowledge. However, the teachers' role as a knowledge provider should not be the most crucial aspect in such learning scenarios.

In science education, the 5-E instructional model is a widely used inquiry-based learning pedagogy; it employs five phases: Engage, Explore, Explain, Elaborate and Evaluate (Bybee, 2009). The five stages are described as follows:

Phase 1 (Engagement): includes the activation of prior knowledge and should help students to become engaged in new concepts through promoting curiosity.

Phase 2 (Exploration): provides learners with hands-on exploration activities upon which they formulate concepts, processes and skills.

Phase 3 (Explanation): involves student's explanations of an aspect of their exploration experiences, and also provides opportunities for teachers to introduce a concept, theory or principle that may guide learners toward a more in-depth understanding.

Phase 4 (Elaboration): facilitates the transfer of concepts to new closely related situations to help students develop deeper understanding.

Phase 5 (Evaluation): engages students to self-assess their understanding and offers opportunities for educators to evaluate students' progress toward achieving educational goals.

For developing the inquiry-based teaching model template for AR-based flipped classrooms, the 5-E instructional model was used. Table 1 presents the designed teaching model template, which includes a description of different activities in the classroom ("in-class") and outside the physical learning space of the classroom ("out-of-class").

5-E model phase	"Out-of-class" activities	"In-class" activities
1. Engagement	Teacher introduces the educational scenario to provoke curiosity and tries to activate prior knowledge using digital material (e.g. interactive video with integrated questions). Students go through the provided material at their own pace and note any questions that arise.	Teacher leads classroom discussion, and the question for investigation is developed. Students engage in the classroom discussion.
2. Exploration	Teacher presents the AR environment to be explored. Students prepare for class by inspecting the environment presented.	Teacher supports the exploration process and encourages learners to formulate concepts, processes and skills based on their experiences. Students explore the AR environment and share their findings with the class.
3. Explanation	Teacher introduces relevant concepts or theories that might have escaped students' notice to foster deeper understanding. Students study the provided material.	Teacher and students utilise the concept and the experiences to describe and explain the phenomenon and answer the initial question.
4. Elaboration	Teacher describes new situations when appropriate using digital media (e.g. video). Students try to identify new situations.	Teacher promotes elaboration. Students apply the knowledge gained to new situations (when appropriate in an AR environment).
5. Evaluation	Teacher provides self-assessment for learners.	Teacher uses formal assessment to evaluate students' progress towards

	Students engage in the self-assessment task to reflect on their learning process.	achieving educational goals.

Table 1. 5-E instructional model template

Table 1 shows the inquiry-based teaching model template suggests teaching/learning activities for each 5-E phase. This template can act as a blueprint for teachers, which can be further adapted to their own teaching needs and preferences. As flipped classroom approaches propose, learner-centred activities have been included in the teacher-supported face-to-face sessions to encourage higher-order thinking, whereas the "out-of-class" phases are aimed at students' engagement with the resources provided.

The AR environment implemented in the exploratory phase is intended to enable inquiry-based learning and should be only used if it is beneficial for the learning process. Moreover, in the stage of elaboration, students should have the opportunity to exchange their thoughts in a collaborative AR scenario. The use of such an environment should help learners to develop a deeper understanding of concepts, as Pemberton and Winter (2009) suggest.

Problem-based teaching model template

In the 1960s, problem-based learning evolved from innovative medical education in North America (Boud & Feletti, 1997). As in the case of inquiry-based learning, the roots of problem-based learning can be found in Dewey's philosophy, whereby inquiry-based learning can be considered as a parent (Albion, 2015). Hmelo-Silver et al. (2007) describe problem-based learning as opportunities for learners to work collaboratively in groups on problem solutions, learn in a self-directed manner, apply the knowledge gained to new situations, and reflect about what they have learned and the effectiveness of the strategies used.

The selection of an interdisciplinary problem is crucial for the success of problem-based learning. Equally crucial for success is a teacher who accompanies the learning process. Problem-based teaching approaches have much in common with the above-mentioned inquiry-based pedagogies. In both learning environments, students construct their understanding through experiencing and reflecting. Therefore, both approaches can be seen as constructivist ways of learning; however, what characterises problem-based learning is the role that teachers play. In inquiry-based learning scenarios, teachers are facilitators of learning and knowledge providers, whereas in problem-based learning environments teachers do not offer information regarding the problem (Savery, 2006).

In the context of mathematics education, different studies (e.g., Padmavathy & Mareesh, 2013) showed that problem-based learning is effective in improving students' understanding and the ability to apply newly learnt concepts in real life. Thus, the described teaching model template described below is based on the problem-based learning framework presented by Eggen and Kauchak (2012). Implementing problem-based learning is conducted in four phases that are outlined here:

Phase 1 (Problem Presentation): includes the representation and modelling of the interdisciplinary problem to be investigated. It is essential to activate prior knowledge in this phase.

Phase 2 (Problem Solving Strategy Development): engages learners to formulate and select the strategy for solving the interdisciplinary problem.

Phase 3 (Problem Solving Strategy Implementation): involves the implementation of the selected strategy for solving the interdisciplinary problem.

Phase 4 (Discussion & Evaluation): embraces reflections on students' solutions based on the teacher's and peers' feedback.

Table 2 presents the problem-based teaching model template that has been developed; it includes a description of activities in the classroom ("in-class") and beyond the physical learning space of the classroom ("out-of-class").

Problem-based learning phase	"Out-of-class" activities	"In-class" activities
1. Problem Presentation	Teacher presents the interdisciplinary problem to provoke curiosity and shares digital support material for modelling the problem. Students try to identify the problem to be solved and study the material offered.	Teacher leads classroom discussion to eliminate ambiguities and supports learners in modelling the problem. Students model the interdisciplinary problem with AR technology if it facilitates the problem-solving process.
2. Problem Solving Strategy Development	Teacher provides collaborative AR environment (when appropriate) for learners. Students think about suitable problem-solving strategies and share their first thoughts in the environment provided.	Teacher supports learners during the development of the problem-solving strategies but should not offer additional information regarding the problem. Students discuss and select the optimal strategy for solving the problem in groups.
3. Problem Solving Strategy Implementation	Teacher provides feedback on the selected optimal strategy. Students exchange their selected strategy for solving the problem with other groups and give feedback.	Teacher facilitates the implementation of the selected strategy. Students implement the selected problem-solving strategy based on the teachers' and peers' feedback and record their findings.
4. Discussion & Evaluation	Teacher presents possible solutions or an optimal strategy using digital media (e.g. interactive video with integrated questions). Students study resources provided and note any questions or remarks that arise.	Teacher leads classroom discussion based on learners' questions and remarks and gives feedback. Students engage in the classroom discussion and reflect on their learning process.

Table 2. Problem-based learning template

Similar to the previously presented inquiry-based teaching model template, the template described in Table 2 can be seen as a design guide for educators describing activities linked to each phase of the problem-solving cycle. To help teachers orchestrate the learning scenario, the template also suggests a distribution of the aforementioned activities inside and outside the classroom according to flipped classroom approaches.

Modelling the problem in an AR environment can be helpful. AR allows spatial problems that are difficult to detect in 2D to be displayed and investigated in 3D (Woods et al., 2004). The representation of the problem with AR technology has been integrated in the teaching model template in the phase of the problem presentation; however, AR should only be used if it adds value to the problem-solving process. Furthermore, learners can apply the knowledge acquired in new situations in an AR environment if the use of AR is appropriate and beneficial.

CONCLUSION AND FUTURE RESEARCH ACTIVITIES

Inquiry-based and problem-based learning are considered as student-centred approaches where learners actively ask questions, collaboratively solve problems, and reflect on their own experiences. In AR environments, learners can learn by inquiring. Combining AR with flipped classroom scenarios, teachers can offer enough in-class time for exploration. But, designing such scenarios can be challenging for educators.

The teaching model templates presented aim to provide a design guide for teachers when implementing AR activities in flipped mathematic classrooms and can foster inquiry-based and problem-based learning. Furthermore, the developed templates can act as guidelines for the flexible implementation of flipped classroom approaches. More specifically, for the construction of different mathematical meanings, students should first be allowed to explore the problem independently instead of watching a concept being explained in a video. Afterwards, a video can be provided as homework to consolidate the findings. Therefore, depending on the learning content, flipped classroom approaches should not be continuously used as a form of direct instruction in mathematics teaching.

The teaching model templates are considered a priori in an on-going project. Therefore, future research will have to determine whether the teaching model templates provided facilitate mathematics teachers to design AR-based flipped classroom scenarios. According to a design-based research approach, the next step is to implement them in practice and refine them. Moreover, it would be interesting to investigate how teachers adjust and further adapt the developed templates towards meeting their own teaching needs.

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USING INTERACTIVE H5P-VIDEOS TO REDUCE STUDENTS' ERRORS CAUSED BY MISCONCEPTIONS

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This poster describes a research project that aims to explore the impact and effectiveness of interactive videos that are designed to induce cognitive conflicts and promote a conceptional change in the context of differential calculus. Therefore, an instructional ECRR-sequence (elicit-confront-resolve-reflect) is implemented within an interactive H5P-video. We use this kind of automated, time- and location-independent instrument to reduce students' errors that are caused by misconceptions.

Keywords: interactive H5P-videos, misconceptions, conceptual change, ECRR

INTRODUCTION

Typical errors of pupils in mathematics are well investigated (Malle 1993; Kieran 1992) and systematic errors of secondary school students are often caused by a lack of understanding the concepts of school algebra (Tietze 1988). Moreover, these errors can be established (Kersten 2015) and then hinder students to succeed in university math courses (Altieri 2016). Errors can be indications for underlying misconceptions, so that it becomes important to consider students' prerequisites and aid students to change concepts that are not adequate.

Possible approaches to foster a change of existing concepts are instructional strategies that cause students to experience a cognitive conflict of their existing knowledge in contrast to new concepts. Several studies have investigated the impact of such strategies within traditional classroom intervention designs (for an overview, see: Chow and Treagust 2013), but there are only few studies that use digital tools. Therefore, this poster presents a digital intervention design based on videos that include different types of interactions provided by the software H5P.

THEORETICAL BACKGROUND

According to a constructivist view, a learner develops knowledge through interactions with its environment by building or reorganizing cognitive structures (Piaget 1976). Within this process, it is natural that an existing cognitive structure may not be adequate or in line with a specific perception. In such cases of inconsistency, a cognitive conflict can arise and at its best – if necessary – induce a conceptual change.

In the context of mathematics, the absence of questioning the applicability of concepts can cause a lack of cognitive conflicts, which can therefore result in overgeneralizations. An example of an overgeneralization is the improper use of linear reasoning, often referred to as "illusion of linearity" (De Bock et al. 2002). Coming from everyday life experience, this concept is intuitive, students tend to rely on it without reflecting its limitations and expand the domain of applicability inconsiderately (Verschaffel and Vosniadou 2004).

An established cognitive conflict strategy in physics education is the ECR (*elicit-confront-resolve*) sequence that has been shown to be effective in face-to-face interventions (McDermott 2001). Engelman (2016) added a reflection phase (R) and has proven its effectiveness when it was integrated in a computer-based learning environment.

CONFLICT-INDUCING INTERACTIVE H5P-VIDEOS

We transfer the cognitive conflict instructional ECRR-sequence to mathematics and implement it partly into interactive videos in order to reduce typical students' errors in differential calculus that are based on the illusion of linearity. In a first step (*elicit*), the error is detected via a diagnostic E-Assessment. In a second step (*confront*), the inconsistency of two alternative ways to calculate the derivative is uncovered in a video sequence. We use interactive exercises to cause students to become aware of the conflict. The *resolve*-phase consists of another video sequence in which the correct concept is justified by further explanations. The *reflect*-phase contains a set of initial exercises.

PILOT STUDY

We conducted a qualitative pilot experiment in order to get insights, how students experience the conflict-inducing part of the video. In videotaped sessions first-year students worked on the interactive video and were interviewed afterwards. First results have shown that the design of the confront-phase needs to be improved in order to present the inconsistency more explicit. This seems important to foster a rejection of the misconception and to evoke the desired conceptual change.

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EMBODIED INSTRUMENTATION: REIFICATION OF SENSORIMOTOR ACTIVITY INTO A MATHEMATICAL ARTIFACT

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The poster proposes a theoretically grounded vision on the position of mathematical artifacts in a learning trajectory. Based on instrumental, radical embodied, and culture-historical approaches, we propose a new design principle for computer-based design sequences: reification of enacted coordinations*. A design study for trigonometry learning is described.*

Keywords: instrumental genesis, embodied design, reification, trigonometry, design research

THEORETICAL FRAMEWORK

New dynamic technology leads to a great increase in the amount of tools designed to enhance students' mathematical reasoning. However, it is still unclear in which cases new tools are capable to actually enhance conceptual understanding. The research aim of this poster is to elaborate on a theoretically grounded vision on the position of a new artifact in a learning trajectory. This vision is inspired by radical embodied, instrumental and culture-historical approaches to the use and development of the artifacts. From an instrumental perspective, the difficulty that students might encounter by outsourcing part of their mathematical activity to an artifact comes from the *dialectics between pragmatic and epistemic value* of the instrumented action *schemes* (Artigue, 2002). In the instrumental genesis process, students need to understand the mathematical meaning behind the appropriated techniques. From a radical embodied perspective we match *a scheme* with the notions of a sensory-motor coordination. We consider conceptual understanding as a coordination of mathematical actions with a variety of artifacts, in consonance with Radford's definition of knowledge "as an ensemble of culturally and historically constituted embodied processes of reflection and action" (Radford, 2013, p. 10). As knowledge itself understood as to be reified (or, metaphorically speaking, crystallized) labor, we recognize cultural artifacts as the reified traces of historically preceding activities. In our educational designs we emulate this historical process by provoking students to disclose a sine function as a depiction of coordination between a sine and an arc length of the unit circle. From the elaborated vision, instrumental actions with an artifact (e.g. sine graph) contribute to mathematical understanding if this mathematical artifact emerges for a student as a reification of her own sensory-motor coordinations.

DESIGN RESEARCH: FROM AN EMBODIED EXPERIENCE TO THE ARTIFACTS

Our work elaborates the design research presented by Rosa Alberto at CERME 2019 (Drijvers, in press) and suggests the series of interactive activities within the embodied action-based design genre (Abrahamson, 2014) that stimulate students to connect a unit circle and a sine graph. The iterative design cycles were recorded on video, audio and by an eye-tracker. The design attempts led us to a new design principle, namely *reification of the previously enacted coordinations* in a following mathematical artifact. Here we present the final sequence that was designed with the guidance of this principle: a sine graph is reified from a student's sensory-motor coordination and further embedded into technological instrument.

Phase 1: coordination of the length of an arc on the unit circle and the length of the segment on the *x*-axis. The task was posed as a motor problem in which students moved one point along a unit circle and another one along a straight line. Color feedback would signal the moments when two passed distances were equal. At first the student would reflect on their coordination as “moving hands with the same speed”. Then additional visual cues would push them from a “same-speed” strategy, towards keeping the length of an arc on the circle equal to a line segment. The phase finishes by manipulating an artifact that automatically shows a point on the x-axis that corresponds to the point on the unit circle. In this way, the enacted coordination was reified in the artifact.

Phase 2: coordination of the heights of a point on the unit circle and a point on the Cartesian plane. In this phase the point on the Cartesian plane was automatically adjusted horizontally according to the point on the unit circle, as reification of the coordination from the previous phase. At first the students manipulated one point on the unit circle, and the vertical position of another point on the Cartesian plane, thus directly coordinating the heights of two points. The direct enactment made it very easy to notice the relation. Further, they manipulated only one point on the Cartesian plain with horizontal and vertical degrees of freedom, while keeping it at the same height with the automatically adjusted point on the unit circle. This solution allowed them to focus on the enacted trajectory, that they finally drew, thus reifying their previous enactments.

Phase 3: coordination of two coordinations. In this phase students were invited to coordinate the movement on the unit circle and the horizontal and vertical movements of a point on the Cartesian plain. Finally, the students drew a sine graph as they combined two coordinations from the previous phases. This target artifact appeared as reification of coordinated enactment of two previously established coordinations. The students were able to explain how the sine graph was constructed and how might be used, so we claim the artifact emerged within the system of instrumental actions.

CONCLUSIONS

An initial design principle of embodied design required a continuous color feedback on the spatially articulated prospectively mathematical sensory-motor actions (Abrahamson, 2014). In a few iterative design cycles we supplemented it by another, instrument-oriented principle: *a previously enacted coordination is reified in the mathematical artifact* for the next phase. Later the coordination that was fixed in the artifact might be released and enacted again for the further coordination with another feature. This approach can be seen as an embodied alternative to the classical instrumental orchestration of a pre-given artifact. Further research is needed to understand if an artifact, as it is reified by a student, is properly and solidly coordinated with mathematical actions, thus conceptually understood and can be involved into student’s instrumental activity.

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USABILITY IMPROVEMENT OF A MOBILE GRAPHING CALCULATOR APPLICATION

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Previous research has established that the integration of mobile technologies and related mobile applications in mathematics education can enhance learning in various ways. Additionally, there is a growing body of literature that recognises the importance of the usability of educational mobile applications. Thus, it is important to study how these mobile applications can be designed and developed in order to best support the impact on educational achievements. This paper presents an overview of possible usability issues of mobile graphing calculator applications and provides recommendations on how to overcome these concerns. The contribution of this paper is to propose helpful suggestions for designers and developers of mobile mathematics applications in order to develop easy to use applications that best support mobile technology enhanced learning in mathematics education.

Keywords: GeoGebra, Graphing Calculator, Mobile, Usability

INTRODUCTION

The integration of Information and Communication Technology (ICT) in educational settings has gained a lot of interest. Previous research indicates that the integration of technology in educational environments is essential for high-quality mathematics (Burrill, 2011; NCTM, 2008; Pimm & Johnston-Wilder, 2005). Thus, there exists an increased demand for teachers to develop learning environments that enable opportunities for the integration of technological tools and digital learning resources in their classroom settings (Bachmair, 2013; Kerres, Heinen & Stratmann, 2012; Lim, Zhao, Tondeur, Chai & Tsai, 2013). Because of the advancing popularity of mobile devices, especially smartphones, among students (MPFS, 2018) great opportunities are offered to complement traditional education with these devices and enrich students' learning in various ways. However, it is particularly important to develop mobile applications with high usability in order to exploit the full potential of mobile learning in mathematics classrooms and to best support learning outcomes. In this regard, Deegan and Rothwell (2010) highlight the importance of cognitive load theory. The authors point out that a poor usability of the deployed mobile application can cause extraneous cognitive load and thus waste the limited resources of the learners' working memory. Respectively, high usability of educational mobile applications can reduce the cognitive load, because learners can focus on the educational content instead of directing cognitive load toward using the application (Deegan & Rothwell, 2010; Reis et al., 2012). For this reason, the main purpose of this paper is to present possible usability issues of mobile graphing calculator applications and to provide suggestions for an improved user interface and interaction design. In order to achieve this objective, the GeoGebra Graphing Calculator (2019) application for smartphones was investigated (see Tomaschko & Hohenwarter, 2018). Therefore, qualitative and quantitative evaluation methods were used. Hence, a set of usability issues that learners encountered while interacting with the mobile application were identified. The revealed study findings were considered for a comprehensive revision of the mobile GeoGebra Graphing Calculator application. As a result, the usability of the mobile application could be improved. This paper presents the revealed usability concerns as well as recommendations and hints that could be considered from designers and developers of mobile mathematics applications in order to increase the impact on educational achievements

GEOGEBRA GRAPHING CALCULATOR

In this section, the GeoGebra Graphing Calculator (2019) application is introduced. This application was investigated on a smartphone in order to reveal potential usability issues of mobile graphing calculator applications (see Tomaschko & Hohenwarter, 2018) that are presented in this paper.

GeoGebra[1] provides a set of dynamic mathematics software tools that are used by millions of students and teachers worldwide. These open source applications can support the teaching and learning of geometry, algebra, spreadsheets, graphing, statistics, and calculus. GeoGebra was first developed as desktop application for the operating systems Windows, MacOS X, and Linux and later as HTML5 web application running in all modern browsers. In 2015, the development of fully native applications for the platforms Android and iOS has been started. These applications are based on the existing source code of GeoGebra's mathematical algorithms, however the user interface was modified in order to address the special requirements of smartphones, in particular their smaller screen size.

The GeoGebra Graphing Calculator (2019) application offers typical functionalities of a graphing calculator such as an equation editor, a virtual keyboard, and a set of geometrical construction tools. After starting the application, the graphics and algebra views are displayed on the screen. The algebra view provides an equation editor and a custom virtual keyboard to facilitate the input of complex mathematical expressions (see Figure 1 left). In addition to the algebraic possibility to construct new mathematical objects, geometrical construction tools are offered that can be used to construct new objects within the graphics view (see Figure 1 middle). All of the created objects are represented in both views (see Figure 1 right) and are dynamically connected, which means that modifications in one view are immediately displayed in the other one as well (Kimeswenger & Hohenwarter, 2015).

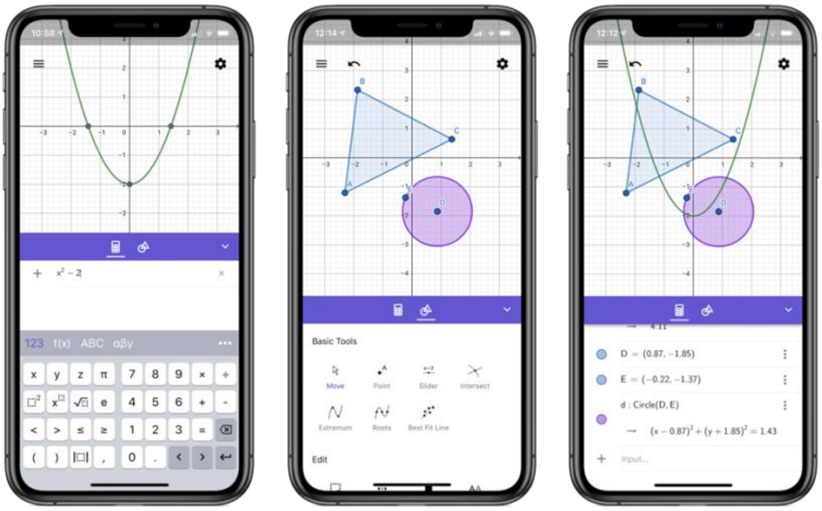

Figure 1. Screenshots of the GeoGebra Graphing Calculator (Version from February 2019)

SUGGESTIONS FOR USABILITY IMPROVEMENTS

Based on previous usability studies (see Tomaschko & Hohenwarter, 2018), the GeoGebra Graphing Calculator was extensively redesigned and revised. As a result, a measurable improvement of the

[1] GeoGebra. www.geogebra.org, last visited Feb. 14, 2019.

application's usability could be achieved. In this section, we describe the revealed usability concerns and present solutions on how to overcome these issues. Even though the undergone study and revision were undertaken with the GeoGebra Graphing Calculator application, they are relevant for any educational application that offer the input of mathematical expressions or geometrical construction tools.

Math Keyboard and Equation Editor

The GeoGebra Graphing Calculator application provides an equation editor and a custom virtual keyboard for mathematical input. As this is one of the most important features of a graphing calculator, it is essential to guarantee high usability and to aid users in typing mathematical expressions.

In order to improve the input of mathematical expressions and reduce potential sources of errors it is essential to offer appropriate labels for all keys that are displayed on the keyboard. What we have learned from a previous study, it is important that all symbols that are not inserted into the equation editor after pressing a certain key should be clearly distinguishable from characters and symbols that are actually inserted. If the keyboard would offer a key for the power of two labeled as x^2, users would expect that the character *x* as well as the exponent is inserted into the equation editor, which is not the case. Similar problems can encounter if other characters are used in order to indicate a placeholder. As shown in Figure 2 (left) the keyboard offers a key for the input of an exponent labeled as a^x. However, users do not recognize the purpose of this button. These problems can be rectified by using special placeholders such as dashed squares. Figure 2 shows a comparison of a keyboard that uses characters (left) and dashed squares (right) as placeholders.

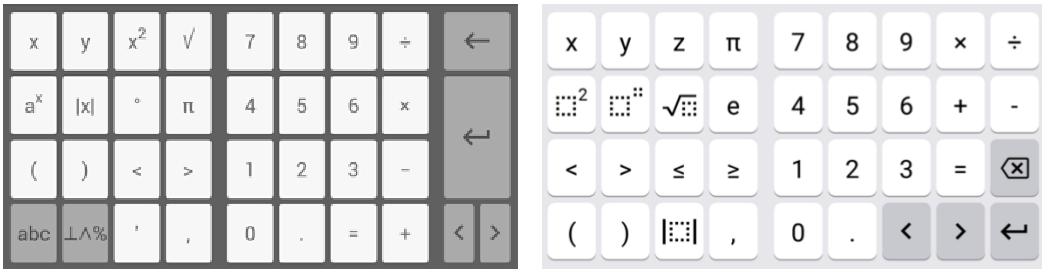

Figure 2. Keyboards that use characters (left) and dashed squares (right) as placeholders

To optimize the input of exponents, we suggest for specific use cases to automatically align the cursor again to the baseline after a specific user input. Otherwise, it is more likely that users keep typing without recognizing that the cursor is still set to superscript. This requires to revise the input and to remove the already typed characters. For this reason, we suggest that the cursor is again aligned with the baseline as soon as the user presses the plus or minus button (except in parenthesized expressions, including square root or absolute value, and as algebraic sign). Because a functional equation of the form $x^{number+number}$ would be unusual in secondary school, this expression could automatically be recognized and set to the more common form of $x^{number} + number$. The same applies to subtractions of numbers in such exponents. However, as divisions and multiplications are commonly used in such exponents in schools, the cursor should stay superscript in these cases.

In order to further improve the interaction with the equation editor, we suggest to support an interaction with the equation editor as it is known from common text fields on mobile devices with a touchscreen. This includes for example the precise positioning of the cursor by tapping with the finger at a specific position. Additionally, touch gestures such as holding and dragging to open a zooming lens (see Figure 3) could be offered to facilitate the precise movement of the cursor. Thereby, the zooming lens is slightly shifted upwards, so that it is not covered by the finger.

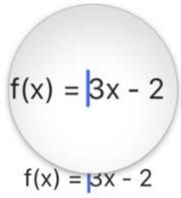

Figure 3. Zooming lens

If the mobile application offers a possibility for calculating an integral, we recommend to offer multiple options to conduct this operation. Users tend to search the keyboard for an integral button as they may already know from traditional graphing calculators. Thus, we suggest to add an integral button on the keyboard. For applications that offer geometrical construction tools, it could also be considered to offer a tool for the integral.

Geometrical Construction Tools

Based on our prior research studies (Tomaschko & Hohenwarter, 2018), we can recommend the use of a grid layout for construction tools, especially if the mobile application offers a large number of different tools. For the representation of the tools it is important to provide an image as well as caption text. Particularly for novice users it is important to offer the name of the tool, as offering images only may not be indicative enough. In contrast, already experienced users tend to search for the already known icon of the tool. Figure 4 shows an example of a grid layout with construction tools that display an icon as well as the name of the tool.

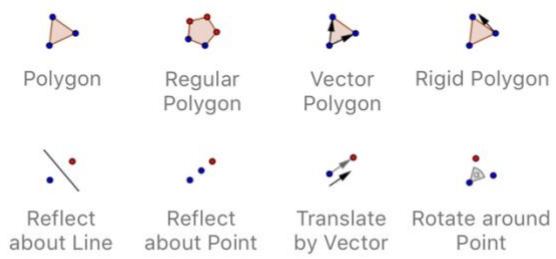

Figure 4. Representation for Geometrical Construction Tools

For mobile applications that offer a set of construction tools, it should also be considered whether to use sticky tools or not. The GeoGebra Graphing Calculator application uses sticky tools, which means that after a tool has been selected it "sticks" and can be used multiple times. However previous research investigations (Tomaschko & Hohenwarter, 2018) have shown that if a tool is used to create multiple objects directly one after the other, the tool is almost always selected again. Therefore, it could be considered to automatically deselect the tool after the object is created. This could further improve the usability, because with sticky tools users often "forget" that a tool is still selected and accidentally create new objects while they are trying to move an existing object or zoom the graphics view. This requires additional interactions of the user, as they have to reverse these unintentional constructions.

For the construction of an angle, the GeoGebra Graphing Calculator offers different possibilities: by selecting three points, two segments, two lines, etc. Thereby, the order of selecting these objects is relevant. However, it often happens that users select the objects in the wrong direction and thus construct the exterior instead of the interior angles. For simple objects such as triangles this could be solved by defining a default interval between 0° and 180° for the angles. This guarantees that always the interior angles are constructed, no matter in which direction the objects were selected.

For more complex tools that require multiple steps to be completed, users require further support in the construction process. GeoGebra offers short information texts describing how a certain tool can be used. However, this is not invariably helpful. In order to create a parallel line for example, it is required to first create a line. Only after the line is created the *Parallel Line* tool can be used to select the previously created line and to construct a new point through which the parallel line should go. To improve the construction process for such complex tools, a possible solution could be a step-by-step guide for users (see Figure 5).

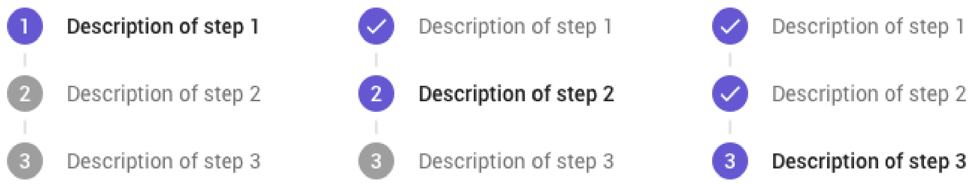

Figure 5. Step-by-Step instructions

Further Improvements

Even though the GeoGebra Graphing Calculator application offers two possibilities to delete existing objects, it is not always easy for users to find these functionalities. GeoGebra allows to delete objects using the delete tool from the tools view or using the small cross displayed at the end of each algebra view row. From previous research (Tomaschko & Hohenwarter, 2018) it could be observed, that users first try to select the object within the graphics view or to open the corresponding properties of the object. For this reason, we suggest to provide a small context menu for each object as soon as it is selected from the graphics view (see Figure 6). This context menu should contain an opportunity to delete the object and also frequently used properties. This way of providing access to the properties of an object can further improve the usability of the application. Another opportunity would be to long tap an object in order to open a properties view of the object. However, this may not be sufficient for users, as they are often interrupted in their construction process while moving objects or the graphics view because the touch gesture is recognized as a long tap and the properties view is opened.

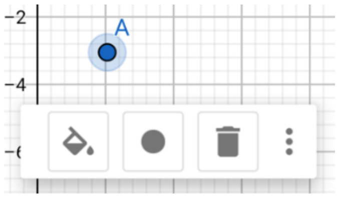

Figure 6. Context menu for objects

While users are creating new objects within the graphics view, they often try to zoom the graphics view by pinch-zooming while a construction tool is active. In this case, it is important to automatically recognize such gestures and to not apply the currently selected tool. Because otherwise, instead of zooming the view, new objects would be created. In this case users would have to interrupt their construction process and delete the accidentally created objects before being able to continue.

CONCLUSION

This paper presented several usability issues students may be facing while interacting with a mobile graphing calculator application which were found in previous eye-tracking research studies. Furthermore, recommendations on how to best design such applications for educational purposes in order to improve the usability of the mathematics application were given. The main contributions of this paper is to provide suggestions for an improved interaction and user interface design. Hence, we hope that our presented study findings will be informative for other researchers and designers willing to develop easy to use mathematics applications and to further extend the research on the usability of mathematics applications.

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DRIVING AUGMENTED REALITY: GEOGEBRA'S NEW AR FEATURES IN TEACHING MATHEMATICS

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Over the last few years augmented reality has matured from a niche technology with sophisticated hardware in special laboratories to a mobile mass product requiring only standard smartphones. Research on this topic raised some crucial difficulties that come along with the adoption of augmented reality especially for education. This contribution aims to describe in part how the newly developed features of the GeoGebra Augmented Reality mobile application can be used in mathematics education. Previous problems of augmented reality applications are discussed, examples for in-class tasks are presented and possible future research projects in the field are outlined. Thus, this contribution can inspire other researchers and educators to design their own augmented reality applications or related content for learning and teaching mathematics.

Keywords: augmented reality, GeoGebra, geometry, education

INTRODUCTION AND THEORETICAL BACKGROUND

Augmented reality (AR) can be defined as a technology that connects the real world with virtual information on top of it so that both coexist in the same frame. Without replacing the real-world information, the user has continuous and implicit control of the virtual information (Mehmet, 2012; Azuma, 2001). Over the past few years, technological progress in the field of AR thrived. Platforms like ARKit[1] and ARCore[2] allow AR's adoption and development for standard smartphones. Thus, the adoption for education also seems inevitable.

In parallel with the increasing research interest, various AR technologies and multiple problems connected with them came into being. Several systematic literature reviews of the past two years have come to the conclusion that the existing AR technologies suffer from information overload (Cardenas-Roblade, 2018; Suh, 2018; Chen, 2017; Akcayir, 2017), cognitive overload (Cardenas-Roblade, 2018; Suh, 2018), distraction of attention (Cardenas-Roblade, 2018; Suh, 2018), general technical difficulties (Akcayir, 2017) or even motion sickness when using so-called head mounted displays (Suh, 2018).

Many of the early AR technologies also require extensive technological know-how of the educator or depend on sophisticated hardware. Even attempts of making mobile AR devices have been made, as can be seen for example by the setup for the Austrian "Studierstube" project (Szalavari, 1998) in Figure 1. It shows, that for generating a simple AR model many different hardware components had to be used, which today are almost all combined in a standard smartphone.

[1] See: https://developer.apple.com/arkit/ . *Accessed 27 February 2019*
[2] See: https://developers.google.com/ar/ . *Accessed 27 February 2019*

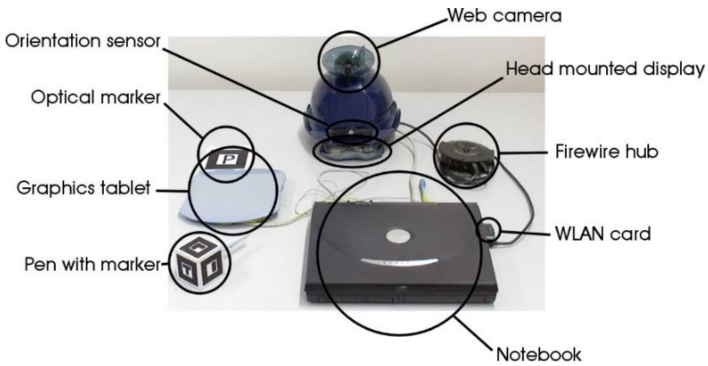

Figure 1. Setup for the early mobile AR system "Studierstube"[3].

However, these early setups are not suitable for mass-adoption in schools. Thornton et al. (2012) point out that AR presents many entry points for educational applications, especially when used on mobile phones. The GeoGebra Augmented Reality mobile application – both available for iOS[4] and Android[5] – intends to offer an easy to use but multi-functional alternative to existing AR technologies and aims to tackle the problems stated above. Hence, this paper investigates the following two questions which will be discussed theoretically on the basis of selected examples:

1. *For which topics of mathematics education can the new GeoGebra AR features be used?*
2. *How do these features tackle the previous drawbacks of AR known from the literature?*

NEW FEATURES OF GEOGEBRA AR

Version 2.0 of the application "GeoGebra Augmented Reality[6]" for iOS already had numerous features convenient for in-class usage, such as the depiction of basic pre-defined solids, 3D function-graphs or even exotic objects like the Sierpinski Pyramid or Klein's Bottle, depicted in Figure 2.

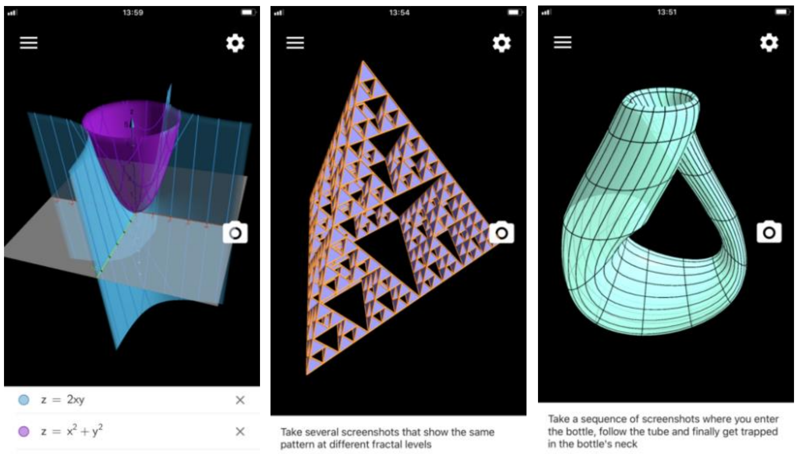

Figure 2. Examples from the GeoGebra AR app 2.0 on iOS: 3D functions, Sierpinski Pyramid, Klein's Bottle

Kerawalla et al. (2006), however, point out that educators would like to have more control over the digital content shown in AR. Therefore, we decided to adjust and enhance the GeoGebra mobile

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[3] See: http://studierstube.icg.tugraz.at/media/images/mobile/setupexplained.jpg . *Accessed 26 February 2019*
[4] See: https://itunes.apple.com/us/app/geogebra-augmented-reality/id1276964610?mt=8 . *Accessed 27 February 2019*
[5] See: https://play.google.com/store/apps/details?id=org.geogebra.android.g3d . *Accessed 27 February 2019*
[6] Available since 26 March 2018

application in the following ways as described in Table 1. These additional features should help address the above-mentioned problems of AR. We try to tackle information and cognitive overload by enabling easy filtering of information, altering AR models and functions in a similar way as in previously known GeoGebra desktop versions and preparing for lessons and sharing the activities in class via the geogebra.org online platform.

Also, we hope to support more widespread in-classroom-adoption for mathematics education and reducing general technical difficulties such that teachers as well as students can work in the well-known GeoGebra environment using nothing more than a standard smartphone.

Feature	**Short description**
Direct input	3D models can be transferred from other GeoGebra applications and 3D models can be created in the AR view
Access to known features	Well-known features of GeoGebra can be accessed directly in the AR application and be used to modify the AR objects
Point-and-click / drag-and-drop	In the centre of the AR view a point can be generated, which can be used to select virtual objects such as points, lengths, …
Real-time rendering	Coefficients of functions can be altered through sliders and the 3D functions render in real-time
Labelling	Properties of virtual objects such as lengths, areas, volumes, names, … can be depicted in the AR view
Measurement	Properties of virtual objects such as lengths, areas or volumes can be measured using the AR view

Table 1. Short descriptions of the new features of the Android GeoGebra AR mobile application

Except for the “point-and-click / drag-and-drop” feature all features described in Table 1 and later on in this paper are already available in the latest Android version of GeoGebra’s 3D Graphing application, version 5.0.526.0, which can be downloaded since February 2019 from the Google Play Store. The “point-and-click / drag-and-drop” feature is currently in development and will be available soon both for iOS and Android.

USE IN EDUCATIONAL CONTEXT

In this section we present several examples on how these described new features can be used in mathematics education. The basic ideas are described, and afterwards detailed examples are given. All the following examples shown in Figures 3 to 7 are screenshots from the Android version 5.0.526.0 of the 3D Graphing AR application which has been viewed on an inexpensive Android smartphone (Pocophone F1, 6GB RAM, 64GB ROM, Snapdragon 845 with up to 2.8GHz).

Modelling

With the features of GeoGebra students can now model virtual 3D objects on top of real-world objects directly in the AR application. With the help of the measuring / labelling tool it is also possible to determine the real values of given objects, e.g. investigating the volume of a prism, considering appropriate grid scaling. For example, one task could be to calculate the volume and the surface area of a glass vase. Figure 3 illustrates how one can create the base area of the glass vase using the “Polygon” tool. In a second step utilising the “Extrude to Prism” tool, the volume of the prism can be created by simply dragging upwards on the screen with one finger. Again, these tools work in the same way as the tools in GeoGebra’s desktop version.

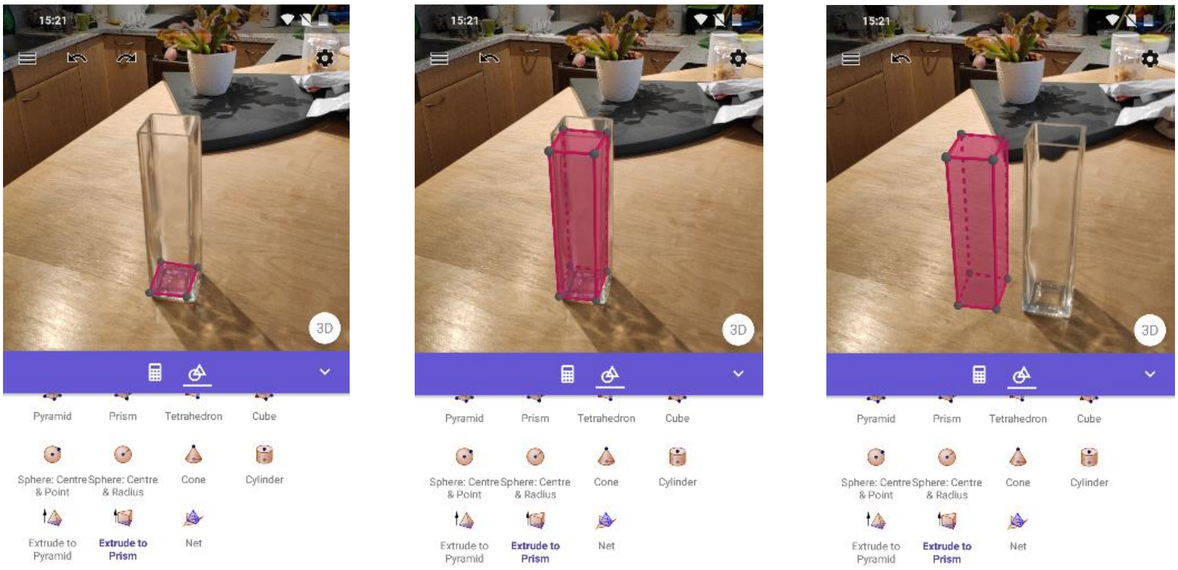

Figure 3. Modelling the volume of a glass vase

Later on, this digital representation of the vase volume can be analysed further. For instance, the net of the prism can be generated, using the “Net” tool or the value of the volume can be displayed and labelled to the prism using the “Volume” tool (see Figure 4). We believe, that these tools enable educators to quickly demonstrate certain properties of different bodies (e.g. surface area, geometrical properties of the volume, etc.). Also students can use these tools to discover geometrical properties with their smartphone in an intuitive way.

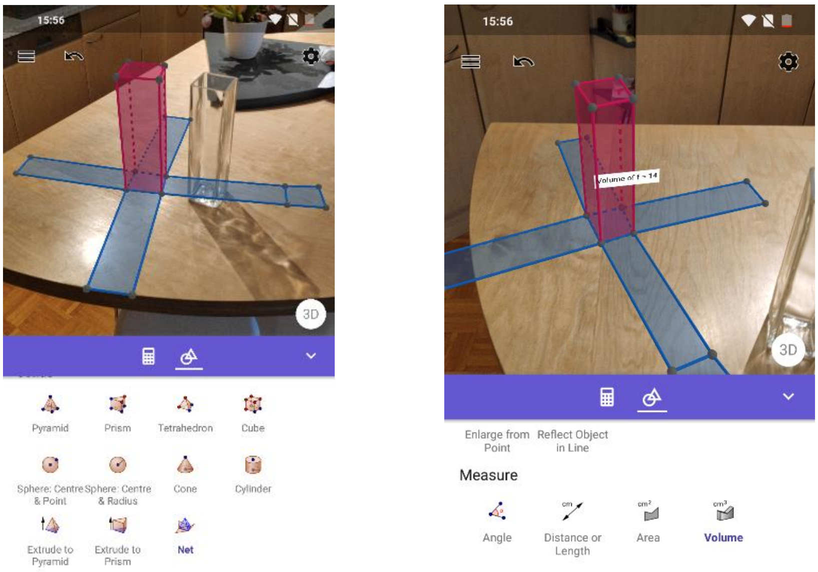

Figure 4. Generating the net of the prism, label with the volume of the prism

Exploring geometry

An extensive part of Austrian lower-secondary geometry education[7] focuses on developing spatial abilities as well as knowing and recognizing special properties of different bodies. Students can use the GeoGebra AR application to develop their skills in these sectors by getting a “deeper look” at certain bodies and their properties. This has yet to be tested but previous work (Kurtulus, 2011) already shows the benefits of using a simple 3D modelling program over pencil and paper in the development of spatial skills. Also, when drawing a diagonal section of these bodies, their real angles and lengths are distorted which makes it even more difficult for students to estimate the real-world properties of the object.

Figure 5 shows an oblique prism with a regular hexagonal base area. Using different measurement tools like “Area”, “Distance or Length” or “Angle” and with a single tap on the screen onto the desired object, our application delivers the asked value. Additionally, when selecting the “Point” tool, a white dot appears in the middle of the screen that snaps to the depicted objects automatically when in reach. Thus, for example coordinates of certain points can be displayed as seen in Figure 6.

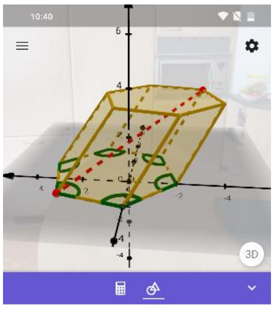

Figure 5. Oblique prism

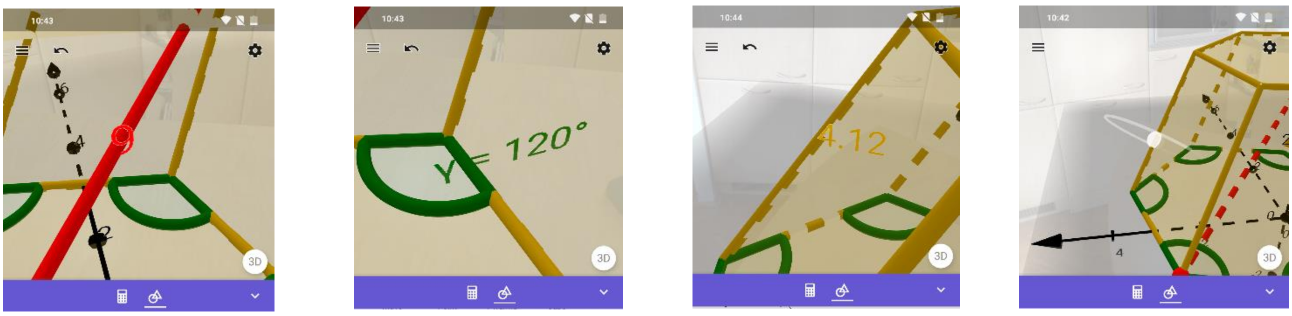

Figure 6. Measuring lengths, areas, angles or coordinates. The background opacity has been set higher in these examples to get a clearer view on the objects.

Exploring conic sections

The new features of the GeoGebra AR application can also be used in higher-secondary education when dealing for example with conic sections. The basic setup is depicted in Figure 7: a simple

[7] See: https://www.ris.bka.gv.at/GeltendeFassung.wxe?Abfrage=Bundesnormen&Gesetzesnummer=10008568 . *Accessed 26 February 2019 (only available in German)*

cone and a plane which can be moved parallel using a simple slider intersect in this activity. When moving the plane, the intersection thus changes from hyperbolas to parabolas and ellipses.

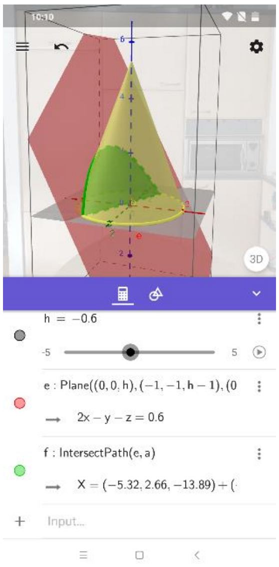

Figure 7. Cone sections

Additionally, within the application the equations for the plane and the intersection can be seen. By using different intersection planes or more sliders to alter the plane, also circles could be created.

CONCLUSION

In this contribution we present new features for the GeoGebra augmented reality mobile application and visions on how to implement these features in mathematics education. We explicitly state, that these approaches are experimental and conceptual, therefore have yet to be researched in follow up studies. Regarding our two initially posed questions, we can come to the following conclusions:

For which topics of mathematics education can the new AR features be used?

The newly developed features can be used in various parts of mathematics education. Different aspects of geometry like creating and analysing digital models of real-world objects or discovering certain properties of special bodies like volumes, areas, lengths or angles can be tackled. Also, when giving attention to spatial geometry in higher-secondary education our features for 3D functions may be useful. However, this will need to be investigated in future work with students.

How do these features tackle the drawbacks of AR known from the literature?

We think that the novel features of our application bring several advancements to mobile AR technology. Because they are built up on the established GeoGebra interface and use the same commands and tools as the desktop versions, we believe that adoption should be quick and thus help to tackle the problem of cognitive overload. Information overload is approached by the different labelling and layout options, e.g. displaying only single highlighted properties or altering the opacity of the in-app background. As our application can be downloaded and operated on any standard smartphone, general technical difficulties should typically not occur.

FUTURE WORK AND LIMITATIONS OF THIS CONTRIBUTION

The adoption of AR in mathematics education is of further interest. Following the latest developments of the GeoGebra AR application and our claims in this paper, we want to test the following hypothesis: *The use of the GeoGebra AR mobile application is intuitive even for less experienced users. Its use furthermore enhances the geometrical understanding of lower-secondary students.*

To test these claims, we are going to evaluate the changes made in our application through observations and interviews in a closed environment to determine student's learning outcomes on given tasks, their motivation and the usability of our application. In Austria students typically encounter technological aid – calculators and / or computer algebra systems – for mathematics education at the age of 12-13. Younger Austrian students typically have little knowledge on the usage of technology in mathematics education. Therefore, we are planning on observing students of the age group of 11 to 12. The observations will have two different foci: first the user experience, second enhancing geometrical skills. Students are audio-video-recorded while they work on specific tasks intended to test the operability of the application. These observations and an additional, slightly modified, system-usability-scale questionnaire (Brooke, 1986) will be evaluated quantitively. Additionally, our observations will focus on the development of geometrical skills of the observed students. Therefore, we provide sample tasks that the students should solve using the GeoGebra AR application. Referring to an open-ended approach (Kwon, 2006) the students are asked to solve the stated problems without any further explanation. In a subsequent interview, explanations to their findings and problem-solving strategies are recorded. These interviews will later be coded and the performance of the students (accuracy of the solution, time needed to solve the task and performance on similar tasks) will be evaluated.

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DEVELOPING A LIVE SESSION FEATURE FOR GEOGEBRA FOR TEACHING AND LEARNING MATHEMATICS

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The aim of this poster is to present suggestions how a new live session feature for GeoGebra could look like, which can be used either to share GeoGebra resources with the students or to collect the responses in a convenient way and clear format on a dashboard and to use them to start or guide a classroom discussion. The suggestions are based on the review of already existing online tools, a literature review and semi-structured interviews with experts.

Keywords: Connected classroom technologies, online tools, student response system, live session

INTRODUCTION AND BACKGROUND

On the GeoGebra platform there are more than one million resources freely available for math and science (GeoGebra, 2019a). Teachers can search for an existing classroom resource or create one on their own. The teachers share the materials with their students and the students start to work on it. Now there are three different activity elements available, where the students are advised to create or enter something: applet, open question and multiple-choice question. Within the applet the teachers can choose between different perspectives such as geometry, algebra, spreadsheets, graphing, statistics and calculus or search for an already existing, public GeoGebra applet that is available on the GeoGebra platform. Now the teachers have the possibility to share the material in the GeoGebra group, share the link or insert in on other platforms (GeoGebra, 2019b).

By using the GeoGebra activities in the classroom just by sharing them with the link and not by using any special online classroom collaboration environment, it is hardly possible to access a useful insight to the students' understanding of the task. The teachers have no chance to see and monitor all students' constructions or responses at a glance nor to use those results to start or guide a classroom discussion. Giving formative feedback to every single student during the lesson is a challenge for teachers and the students may miss valuable support and on-going feedback.

DEVELOPING NEW IDEAS

To help teachers to improve their teaching by using GeoGebra activities as well as to help students to improve their learning we want to develop a new system that supports mathematics teaching better than existing systems. To gather ideas how this new system could look like, we collected information from different sources. Firstly, we made a review of several online tools that can be applied in mathematics education and focused on the classroom collaboration features. For this review we have chosen the online tools Classkick (www.classkick.com), Desmos (www.desmos.com), Nearpod (www.nearpod.com) and UniDoodle (www.unidoodle.com) which can be used in a classroom setting by teachers to provide activities and to collect students' responses in real time.

Additionally to this review, we made also a literature review and gathered information about student response systems (SRS), classroom response systems (CRS) and connected classroom technology (CCT), which appear in the literature under different names, but have in common that they facilitate the communication between teachers and students as well as display the student responses in real time (Fies & Marshall, 2006; Irving, 2006; McLoone, Kelly, Brennan, & NiShe, 2017). Besides we were not only interested in how the tools work and in their advantages for teachers and students. But also in how the teachers can use the power of those tools for teacher noticing, especially technology-mediated teacher noticing (Walkoe, Wilkerson, & Elby, 2017) in mathematics education as well as for orchestration in e-learning environments (Weinberger & Papadopoulos, 2016).

Moreover, we also conducted several semi-structured interviews with experts, who are using different online tools in their mathematics teaching complementary to GeoGebra activities. The results of the reviews and the interviews are used to develop the suggestions for a live session feature in GeoGebra.

POSTER / FINDINGS

On the poster we will present suggestions how this new system could look like. Primarily, it should be easy for the teachers to share GeoGebra resources with students. Moreover, teachers have a dashboard which visualizes students' progress of their activities and the responses in a clear format. The teachers should be able to switch between an overview of the whole class and the progress of individual students seamlessly. Each of the three available activity elements appears in a different visualization on the dashboard, where the teachers have a summary of the whole group as well as single answers at one glance. The power of the new live session feature should be that teachers have the possibility to see in real time and at a glance all work done in GeoGebra by their students.

However, the focus is not only on the collection and the display of students' responses. Besides, the teacher can organise and use the collected results for further classroom activities such as starting or guiding a classroom discussion of the varying responses or analysing errors that students may have made. Additionally, the results can be saved by students to record their learning progress.

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Part 3: Enhancing Assessment

TOWARDS AUTOMATED GROUPING: UNRAVELING MATHEMATICS TEACHERS' CONSIDERATIONS

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ABSTRACT

What are mathematics teachers' considerations in grouping students, and how could automated formative assessment systems help them in doing it? In this study, nine teachers were asked to use data on students' performance in a mathematics task, derived from an automated formative assessment system, to create pairs in which students could contribute to their peers. We called this grouping strategy "complementary." The teachers were also asked to explain their considerations for each grouping. We found two main grouping strategies in addition to the complementary one: based on similar answers ("similarity"), and based on dissimilar answers, in which one student performed better than another and could teach the other ("hierarchy"). Findings show that despite the experimenter's request to group students based on complementarity, teachers mostly grouped based on other considerations, at times even grouping students whose answers were complementary using hierarchical considerations. In some cases, different teachers formed the same groups of students based on different grouping strategies. The findings confirm the hypothesis that informed grouping may be challenging for teachers, and may benefit greatly from an automated pairing system.

Keywords: informed grouping; differentness; linear functions; personal example space; automated formative assessment.

INTRODUCTION

How do mathematics teachers group students? What kind of information should they use and how may this information help them in forming groups? What are their considerations as they do so? This information is crucial to our quest, in which we aim to use data about students' mathematical work, generated by an automated formative assessment system, to perform informed grouping of students. Thus, we asked mathematics teachers to groups students using data retrieved from an automated mathematics formative assessment system, and explain their reasons for these groupings. The paper begins with a literature review on formative assessment, collaborative learning, and informed grouping. It continues with a description of the study, followed by our main findings. We end with a discussion of the findings, with particular focus on how these results may influence the design and research of future automated grouping systems.

THEORETICAL BACKGOUND

Group learning

Group learning is a fundamental pedagogical practice (Dillenbourg, 1999), although it is not always effective (Barron, 2003; Hoyles, Healy & Sutherland, 1991) and requires teachers to take into account various considerations (Webb, 2009). An important practice is informed grouping, allocating students to groups based on predefined dimensions and knowledge of learners' abilities along these dimensions. Researchers who design and study informed grouping are concerned with the different dimensions along which grouping should be carried out, and how these dimensions may best inform grouping for it to be beneficial to students.

Groups of students can be either homogeneous or heterogeneous (e.g., Maqtary, Mohsen, & Bechkoum, 2017). A homogeneous group is usually comprised of learners who are considered similar along some measures such as age, gender, or grades. When students learn in a homogenous group, it is more likely that they make progress at the same pace and are able to solve similar tasks of about the same difficulty (e.g., Connor et al., 2013). There are two types of heterogeneous grouping: hierarchic and complementary. In a hierarchic group one student is stronger than the other(s), can lead the interaction, and probably guide the other. In a complementary group the differences between the learners are such that members can either combine their capabilities to solve tasks, or teach each other what they know to expand their knowledge on a certain topic (e.g., Gutierrez-Santos et al., 2017).

Automated assessment

Recently, several tools have been designed to automate the process of formative assessment, particularly in mathematics education (e.g., Stacey & Wiliam, 2012). Formative assessment systems contribute to learning and instruction by giving detailed yet idiosyncratic feedback to learners, and providing teachers with an overview of the learning in the classroom. In the present study, we were interested in grouping based on students' performance in a particular mathematical task.

We used a formative assessment system to automate the process of assessing students' performance with the Seeing the Entire Picture (STEP) platform: a domain-specific formative assessment system developed in our laboratory (Olsher, Yerushalmy, & Chazan, 2016; Yerushalmy, Nagari-Haddif, & Olsher, 2017). With STEP, mathematical questions are posed to students using dedicated GeoGebra-based applets. Students solve the problem and submit their answers, for example, as a finite solution to a question, as a set of examples supporting or refuting a mathematical claim, or as examples of a mathematical idea. STEP collects the submitted answers and characterizes them based on their mathematical properties and on their correctness. The system then uses this elaborated data to report back to students (for further learning) and to the teacher (for further instruction). For example, students may be asked to choose three pairs of points and build three linear functions on these points using GeoGebra. When submitting answers, the system can provide information on the properties of the students' answers, such as whether the function increases or decreases, or whether one of the points chosen is on the Y axis. The system can also provide information on correctness, for example, indicating whether the functions generated passes through the selected points. Using this information, teachers can decide which students need further instruction, and of what type.

The answers submitted by the student through STEP in the above example of linear functions is a representation of the student's personal example space: a repertoire of available examples with regard to a certain mathematical idea (Sinclair, Watson, Zaskis, & Mason, 2011). A learning goal for individual students may be to extend their personal example space. One promising path for widening one's personal example space would be conducting a dialogue with a teacher, a peer, or perhaps an object.

The objective of our research project is to design, implement and test a support system for informed grouping. Therefore, the following study aims to unravel teachers' considerations, in grouping students using data obtained from STEP. We asked: (a) How do teachers use STEP data to group students? and (b) What were their considerations for grouping students?

METHODOLOGY

Population

Twenty graduate students in mathematics education, all of them mathematics school teachers at various levels, who attended a course on automated formative assessment in mathematics education, participated in stage 1 of the experiment. Twelve of the students participated also in stage 2. Participation was voluntarily.

The task

Students were presented with a GeoGebra-based applet (Figure 1) and given the following instructions: "Choose (red) points with the '*new points*' button to build a linear function whose graph passes through the points you have chosen. If you think it is not possible, explain why. Submit three examples of three different pairs of points." The problem was designed to probe the personal example space of learners: they can choose their pairs of points of their liking, and build linear functions on them. The presentations of pairs of points in the applet was only partially randomized: the probability of the applet presenting some cases was higher than that of presenting some other cases. For example, the case "one point is on the Y axis" appears in 25% of cases; the case "both points have the same Y value" appeared in 15% of cases.

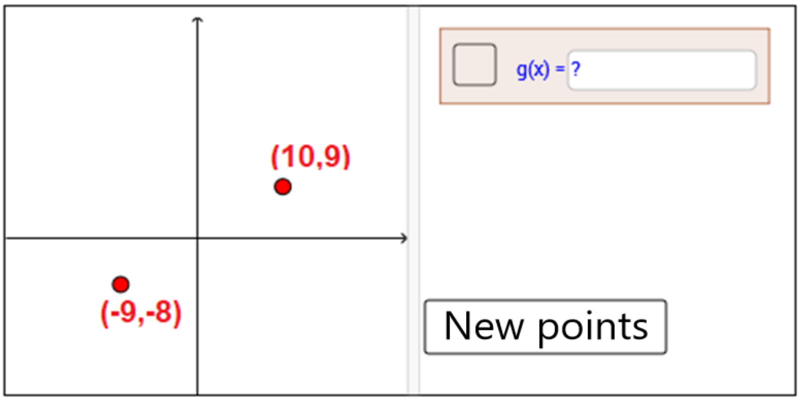

Figure 1: GeoGebra-based applet for the activity.

Procedure

Stage 1. The teachers solved the activity described above (Figure 1). Every answer contained three examples that represented their "personal learning space" (although their actual personal example space may be larger). Next, the teachers participated in a whole-class discussion of the various solutions to the problem submitted by the different participants.

Since teachers and students are the same individuals, but we need to distinguish between them, we will refer to "students" when discussing personal example spaces using pseudonyms (e.g., Arale, Yochi); and to "teachers" (e.g., Teacher a, Teacher g) when discussing the grouping being performed.

Stage 2 was carried out a week later. Twelve teachers of the 20 decided to participate in this stage. Before the beginning of the lesson 12 teachers received printouts of 20 answers (i.e., the personal example spaces of the 20 students in stage 1). In addition, every teacher received an experiment form, in it a table with three columns and eight rows. At the headline of the rows printed "student A", "Student B" and "why were they paired?"

First, the teachers were asked to take five minutes to think of possible ways to group students based on these answers, without producing anything in writing. Second, the teachers participated in a 50-minute lecture and discussion about the underlying principles of grouping, the concept of a personal example space, and how it may support complementarity in grouping. Third, the teachers were asked to look at the printouts of the answers, presented under fictitious “student” names, and group the students by pairs, entering each pair in a table and providing the reason for the grouping: “Group the students based on their answers. The answers should be complementary. The objective is to expand the students' personal example space, assuming the two students will later learn together. Explain your choice.” Fourth, the teachers participated in a discussion in which they presented some of groupings and explained their rationale for them. The lecturer of the course did not participate in the experiment.

Data collection

Nine of the twelve teachers who agreed to participate in the experiment agreed to submit the experiment forms (the completed tables). The researchers were blind to the process of submission. The teachers submitted a total of 53 groupings (average 5.78). See Table XY for the number of groupings per teacher.

Data analysis

The first author created definitions for, and coded the data based on, three strategies for grouping: similarity, hierarchy, and complementarity. Based on these definitions, the second author coded 20% of the data, and discussed it with the first author, and the two identified in the data two sub-strategies of hierarchic grouping. The three authors then looked again at the definitions and the coding; together they reached a finer articulation of each strategy. Finally, the first author recoded all the data according to the resolution by the three authors.

FINDINGS

We start this section providing quantitative data about the distribution of groupings done by the teachers (see table 1), followed by an explanation of the way they manifest in the context of the given STEP activity and an example for how they were manifested. This findings section ends with an illustration of a case study that sheds light on the complexity of this grouping-task. Table XY lists the groupings submitted by the nine teachers, and the categories to which the groupings were assigned.

Table 1. Type and distribution of groupings by teachers

Teacher	**Number of groupings**	**Similarity**	**Hierarchy**		**Complementarity**
			PES Size	Correct answers	
Teacher a	**7**	2	3		2
Teacher b	**5**		1		4
Teacher c	**7**	1	1	1	4
Teacher d	**8**		5	2	1
Teacher e	**6**		5		1
Teacher f	**4**		1	1	2
Teacher g	**7**		2	3	3

Teacher h	**4**		4		
Teacher i	**5**	1	1	3	
Total	**53**	**4**	**23**	**10**	**17**

Table XY shows that the most popular grouping type (33 out of 53) was hierarchy-based. Hierarchy was the one strategy used by all teachers. Complementarity and similarity were used by fewer teachers, seven and three correspondingly. Teachers a and c have shown the most versatility by grouping based on all three grouping strategies.

Strategy A: Similarity

In four cases (7.5%), three of the teachers chose to group students based on similarity. According to this strategy, students who displayed few differences based on one or more of the analyzed dimensions were grouped together. The teachers who used this strategy would often propose an assignment for them as a group, to expand the personal example space of both students in the same direction. This can be accomplished by correcting a common incorrect answer or by asking the learners to expand their similar personal example spaces toward a certain dimension. For example, Teacher a explained why he proposed to group Haviva and Arale (all names are fictitious) together: “Both of them created increasing functions, so I would like them to sit together to expand their example spaces and create more examples that are different from each other.”

Strategy B: Hierarchy

In 33 cases (62.3%), the teachers chose to group students based on hierarchy. According to these strategies, the teacher considered the submission of one student to be superior to that of another, and aimed for the better student to teach the weaker one. We identified two types of such hierarchical relationships determined by the teachers: (a) Correctness (10 cases, 18.9%), when the solutions of one student were correct and those of the other were not, in which case the student with the correct answers was expected to teach the one with the incorrect answers. For example, Teacher g explained why she grouped students Yochi and Haviva together (figure 2): “Yochi solved correctly and with variance [between answers]. Haviva didn’t solve the exercises correctly, so it’s possible that the first will teach the second.” (b) Size of the personal example space (23 cases, 43.4%), when large differences were found between the personal example spaces of the students. A particular case may be one in which the personal example space of one student is fully included in that of another. For example, Teacher e and Teacher h grouped Haviva and Arale based on hierarchy. Teacher e explained: “Arale provided correct examples although all were very similar (increasing functions), but Haviva does not seem to be able to create all of the examples, so he can receive help from Haviva.”

Strategy C: Complementarity

In 17 cases (32.1%), seven of the teachers chose to group students based on complementarity. According to this strategy, grouping was based on some differences between the partners, so that each learner may learn from the other and that both may expand their personal example space. Grouping according to this strategy was based on various degrees of differentness. We identified two types of such relationships determined by the teachers. (a) Functions orientation: according to this grouping strategy, at least one possible orientation of a linear function (up, down, constant) was not present in the submissions of one student, but it was present in the submission of the other. (b) Choice of points: the criterion upon which grouping was made was related to the points chosen by the students—for example, solving for functions that have points on the axes or for functions that are built out of points in different quadrants.

Note that strategies (a) and (b) are rather similar, but the *explanations* for the grouping given by the teachers were either based on the outcome (function orientation) or on choice. We did not, however, distinguish between these strategies because they were rather close to one another. For example, Teacher f grouped Arale and Ayelet together, explaining that "Arale could teach Ayelet about the intersection [of the function] with the axes, and Ayelet could teach Arale about [choosing] a point on the X axis [and] an example of a constant function."

In some of the cases, the personal example spaces of the two students were complementary, but nevertheless the teacher saw the relationship between the two students as hierarchic. We categorized these cases as hierarchic, but noted the fact that automated grouping systems would have categorized these cases more accurately.

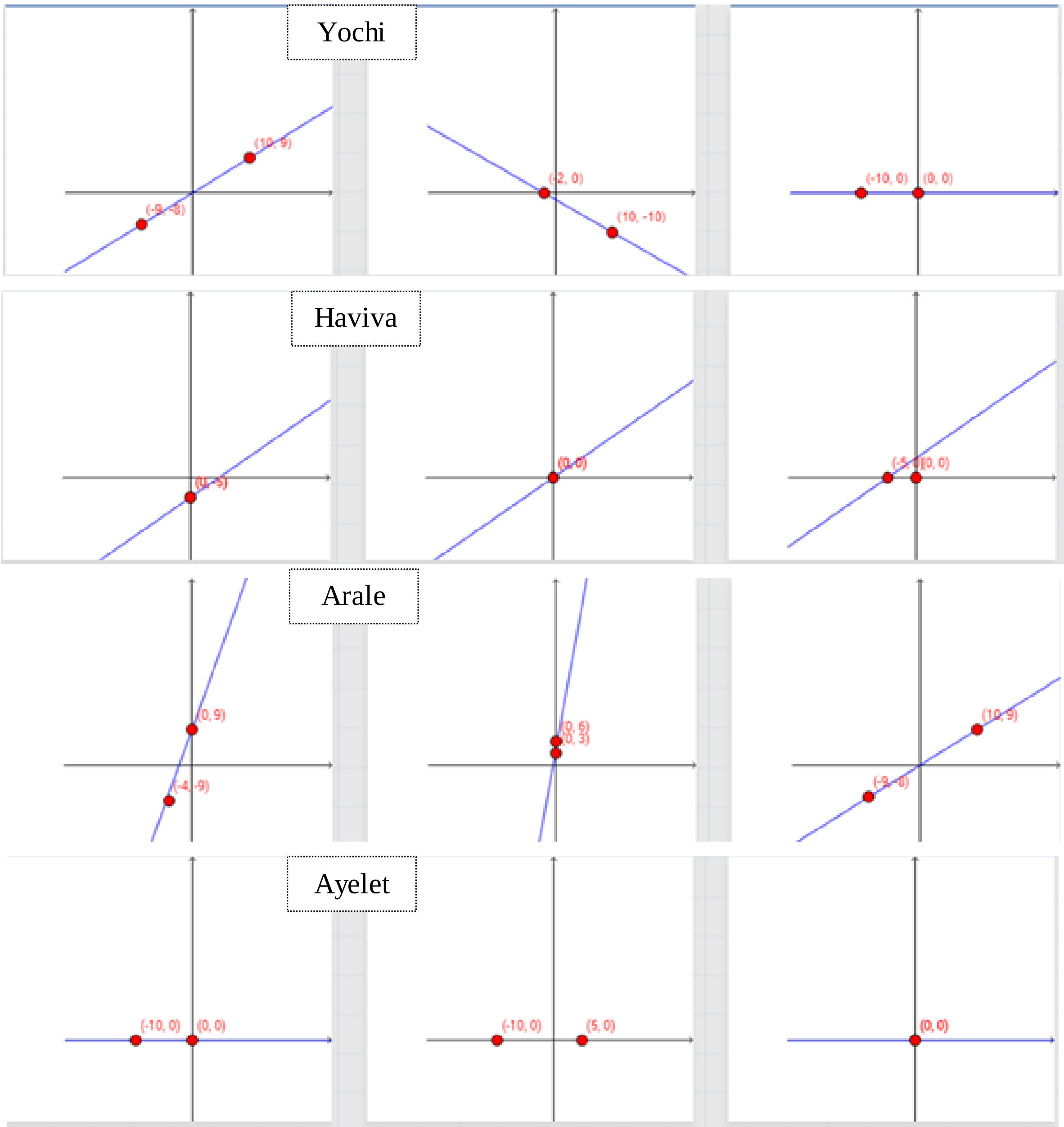

Figure 2. Four personal example spaces of four students.

Same grouping, different strategy

In nine cases, certain groupings were chosen by more than one teacher. In two instances, teachers' explanations were coded as the same strategy, and in seven instances as different strategies. In the examples above Haviva and Arale were grouped together by Teacher e and Teacher h based on hierarchic considerations, and by Teacher a based on similarity considerations. Five of the groupings appeared twice, three by three teachers and one by four teachers. One possible explanation for this unlikely finding is that five of the groupings were based on answers that were close physically in the printout of the answers.

DISCUSSION AND CONCLUSIONS

In a quest to use data from a mathematics-specific automated formative assessment system in order to perform informed groupings, we found the need to unravel teachers' ways of grouping students using such data. We asked, (a) How do teachers group learners into small groups based on STEP data? and (2) What are the teachers' considerations in making these groupings?

We produced three main findings. First, we made a new distinction between grouping strategies obtained from the literature and showed how they manifest in the context of a STEP-based activity. Second, we learned that despite the experimenters' request to group students in a way that each student will be able to teach the partner, with a planned bias toward complementarity, teachers grouped the students based mostly on other considerations, despite the fact that complementarity was present in the students' answers. This result appears to indicate an epistemological gap that teachers need to close. Most of the teachers were able to perform hierarchic grouping based on dimensions other than correctness, which suggests that grouping based on complementarity may be more complex. Third, we found that different teachers grouped students based on different considerations, suggesting that grouping by humans is not an objective process, and allows for diverse approaches.

These findings contribute to the hypothesis that informed grouping may be challenging for teachers, because it requires knowledge of the learning task, choosing dimensions for grouping, assessing learners based on these dimensions, and deciding how their performance in chosen areas should inform grouping. We argue that automated grouping systems and (mathematics) domain-specific automated formative assessment platforms such as STEP can together provide a satisfactory solution for informed grouping in mathematics. Processing large chunks of data in short time may overwhelm teachers, undermining their efforts to achieve data-informed groupings. Informed grouping could therefore benefit from automation. It could, just as well, free the teacher to address other aspects of learning, such as supporting the communication between the learners. As several computer scientists have shown, automated grouping algorithms can improve our ability to assess students' performance and provide informed suggestions for grouping (e.g., Abnar, Orooji, & Taghiyareh, 2012). In the case of STEP, informed grouping requires deep understanding of a students' personal example spaces, which STEP is already capable of achieving.

Complementary grouping helps avoiding the homogeneous vs. heterogeneous dichotomy. Complementarity is based on heterogeneity, that is, on difference, but it is also homogeneous because both learners could contribute to the learning of the other. With hierarchic grouping, one student may be categorized as inferior to another, so that one student can teach the other and help expand the other student's example space. In the case of complementarity, however, a student who may have displayed a narrower personal example space than the other student, may be still able to contribute to the work of the stronger student and help expand the stronger student's example space.

The following questions remain to be answered: Is complementary grouping superior to grouping based on similarity, hierarchy, or both? And if yes, what are the dimensions along which complementarity should be achieved with respect to students' personal example spaces? In our effort to automate the process of grouping, we sought to understand the ways in which teachers used STEP-based data to propose ways of grouping. We believe that the present study has contributed to creating an effective automated grouping systems using automated formative assessment systems.

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ENHANCING FORMATIVE ASSESSMENT PRACTICES IN UNDERGRADUATE COURSES BY MEANS OF ONLINE WORKSHOPS

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In this paper, we report a research study concerning the use of online e-learning platform for enhancing formative assessment strategies through which the students are asked to provide feedback on their peers' productions and are therefore activated as instructional resources for one another. In playing the assessor role, students are prompted in developing continuous reflections on their learning processes. The activity has been experimented at undergraduate level in two different Universities in Italy, with courses with different mathematical contents. We describe the design of the activity and analyse the first experimental results.

Keywords: formative assessment; peer review; blended learning; online workshops.

INTRODUCTION

Shared experiences and literature review (Larreamendy-Joerns & Leinhardt, 2006) show evidence of the integration of online instruction practices at University level. The benefits of such practices seem to be mainly found in the freedom of the students to move at their own pace. Nonetheless, there is not enough literature reporting the actual added-value of online support with respect to the traditional face-to-face (f2f) instruction. Then, the key question becomes how online platforms can be used for formative assessment (FA) in blended learning, and how students accept and use them.

Research has also widely shown the effectiveness of a correct implementation of FA practices for improving mathematical learning in school context (Hattie, 2009). In the last decade, mathematics education research has been focusing on FA as a teaching practice (e.g. the Mathematics Assessment Program: http://map.mathshell.org), also with the support of new technology (see the FaSMEd Project: https://microsites.ncl.ac.uk/fasmedtoolkit/). Research has focused mostly on primary and secondary school levels in f2f context. Undergraduate contexts with a large numbers of students are a great challenge for adapting such models of FA processes, where the teacher-students interactions, also at one-to-one level, may play a key role.

Our investigation takes place at the University level in a blended learning context, that is a learning environment where traditional f2f lectures are supported by further teaching/learning activities conducted through a web-based platform[1]. In the light of previous experience on e-learning platform to engage students themselves as responsible of their own learning process and of their peers (Albano, 2011; Albano & Pierri, 2014), in this paper we report on the use of a specific advanced tool of the web-based platform Moodle (called "workshop") in order to support peer work in the view of online FA, intended as "the application of formative assessment within learning online and blended settings where the teacher and learners are separated by time and/or space and where a substantial proportion

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of learning/teaching activities are conducted through web-based ICT" (ICT stands for Information and Communication Technologies) (Gikandi et al., 2011, p. 2337). Our focus is first on the didactic function and, accordingly, the choice and the didactical design of the tools to ensure that the desired result is achieved (Chevallard & Ladage, 2008). The core of the design is that students are asked to provide feedback on their peers' productions and are therefore activated as instructional resources for one another, though cycles of online FA workshops. In the following, we present the theoretical model for formative assessment, which guided our design. We then describe the experimental settings (involving two different Universities in Italy, at the University of Salerno and at the University of Torino), and present our didactical design. Data analysis is still on-going: first results and implications are presented in the paper; additional results will be provided in the conference presentation.

FORMATIVE ASSESSMENT WITH TECHNOLOGY

FA or assessment *for* learning (contrasted to assessment *of* learning) includes activities and practices enacted by teachers with the aim of improving students' learning. As largely shared within education literature, FA is conceived as a teaching method, where "evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers, to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have taken in the absence of the evidence that was elicited" (Black & Wiliam, 2009, p. 8).

Typical FA activities are therefore those through which students have the opportunity to verify their own learning levels, plan and implement, in interaction with the teacher and classmates, the strategies necessary to achieve the learning goals. In order to base FA on solid foundations, Wiliam and Thompson (2007) considered three key processes identified by Ramaprasad (1983): (a) establish where the learners are in their learning; (b) establish where they are going in this process; (c) establish how the goal can be reached. Traditionally, the teacher is held responsible for each of the three processes, but it is also necessary to consider the role that students can play themselves and their peers, who are usually their classmates. By crossing the three key processes with the different 'agents' (teacher, classmates, pupil), Wiliam and Thompson (2007) have elaborated a theoretical framework for FA, highlighting that it can be developed through five key strategies:

(A) *Clarifying and sharing learning intentions and criteria for success;*
(B) *Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;*
(C) *Providing feedback that moves learners forward;*
(D) *Activating students as instructional resources for one another;*
(E) *Activating students as the owners of their own learning.*

In this model, three different agents intervene in FA practices: the teacher, the learner, and the learner's peers. The teacher is responsible for clarifying learning objectives and criteria for success, which become assessment criteria (key strategy A), for organising class activities and discussions in which she can have evidence of pupils' understanding (key strategy B) and for providing feedback to enable students to progress in learning (key strategy C). Feedback concerns the information that the student receives about her performance and is undoubtedly one of the most important tools for building a bridge between actual and expected learning. Following the definition of Ramaprasad (1983), feedback only becomes formative if the information given to the student is used in some way to improve her performance. It is therefore important that the feedback goes beyond a simple green or red 'traffic light' for the student, which would merely orient the student's behaviour, and that it rather shows her what any errors, deficiencies, inaccuracies and possibly what may cause them. If the teacher succeeds in placing herself in the student's proximal development zone (Vygotsky, 1978), structured feedback can guide her in dealing with those tasks she is not (yet) able to deal with on her own (Shepard, 2000). Beside the teacher, learners have important roles also, both in understanding the learning objectives and criteria for success (key strategy A), and in taking responsibility for the learning of their fellow students and themselves (key strategies D and E).

Research has shown that activating formative assessment practices is highly demanding for teachers, and recent projects have investigated how technology may be exploited to support them in school classroom (e.g. the European FaSMEd project - Improving Progress for Lower Achievers through Formative Assessment in Science and Mathematics Education; Aldon & Sabena, 2015). Considering the different *agents*, the five *key-strategies* and how *technology* may support FA processes within educational contexts, a three-dimensional framework for the design and implementation of technologically-enhanced formative assessment activities has been proposed (Aldon, Cusi, Morselli, Panero & Sabena, 2017; Cusi, Morselli & Sabena, 2017). The framework is represented in the chart[2] in Figure 1.

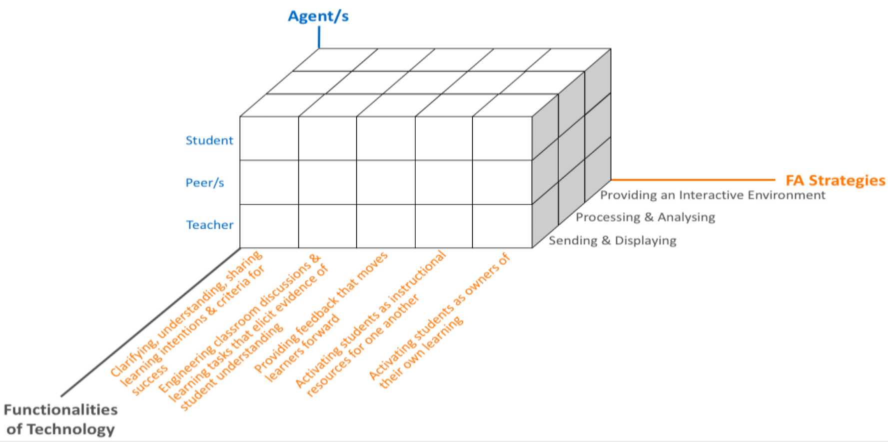

Figure 1. Chart of the FaSMEd framework, where FA strategies, agents, and functionalities of technology constitute three fundamental dimensions

FaSMEd model identifies three main *functionalities* through which technology could support FA:

(1) *Sending and displaying*, e.g. sending and receiving messages and files, displaying and sharing screens or documents to students;

(2) *Processing and analysing* data collected during the lessons, e.g. showing the statistics of students' answers to polls or questionnaires, or the feedbacks given directly by the technology to the students when they are performing a test;

(3) *Providing an interactive environment*, in which students can interact to work individually or in groups on a task or to explore mathematical/ scientific contents (e.g. the use of specific software where it is possible to dynamically explore specific mathematical representations).

In our research study, we refer to the sending-and-displaying and the processing-and-analysing functionalities within undergraduate level and investigate how an online platform may be exploited to promote mathematics FA processes involving the three agents—teacher, students and peers— and in particular peers in a blended modality. Students attend traditional classes of the f2f course and participate in online activities concerning formative assessment.

THE RESEARCH STUDY

[2] We thank Wright (Newcastle University) for the digital version of the chart and Ruchniewicz (University of Duisburg-Essen) for its adaptation.

Research context and methodology

We experimented online FA workshops experimented in two different Universities in Italy, at the University of Salerno and at the University of Torino, in undergraduate mandatory courses with different mathematical contents. Initial trials started in years 2017-2018 and allowed us to gauge the workshop. In this paper we report on the theoretical design, the feelings of the students of the initial experiments and give some flavours of the outcomes of the current ones (year 2018-19).

At the University of Salerno, FA online workshops constitute a support to a traditional f2f course of Geometry, Algebra and Logic for freshmen students in Computer Engineering (second term). The content of the course mainly concerns linear algebra, whose learning difficulties are well known. The course foresees two written examination tests, one mid-term and one final, that are prerequisite to access the oral exams. Four online workshops have been submitted, two before each of the written tests, conceived as summary exercises in view of the test to be done. For each session, about 40 exercises have been prepared, and each of them has been submitted to groups of 3 or 4 students.

At the University of Torino, five FA online workshops have been carried out during a course of Mathematics and Mathematics Education for freshman prospective primary teachers. The course focuses on arithmetic and early algebra and one of its major goals developing teachers' argumentative competence in these topics. In each workshop, students have received a collection of 3 to 5 exercises or problems to solve, related to the topics discussed during f2f classes. Workshops have been open also to students not attending the classes.

In both cases, in FA online workshops students have been asked to solve problems/exercises and to upload their solutions within a given deadline. Then all the assignments and their solutions have been redistributed to the students, so that each student would receive a certain number of them. Students have been expected to assess them according to the given criteria of correctness, completeness, and clearness. As a final step, they have received a structured feedback from the teacher.

FA workshops have been proposed to the students as a learning support and participation was not mandatory. Data have been collected by means of the platform facilities, such as reports and repository of materials associated to students' profile. We have used quantitative analysis for investigating the effect that the online formative assessment strategy has had on the students in terms of participation to the activity in the context of the two Universities.

In order to include the students' point of view, we have submitted a questionnaire concerning the students' feelings about their participation to the activity and on the perceived effects on their learning process. The questionnaire has focused on the three steps of the FA workshops: problem solving, assessment, feedbacks. Concerning the first two, we have investigated how much difficult the tasks have been perceived by the students and which difficulties they faced. Then we have asked about the usefulness of the teacher and the peers' feedbacks. On the affective side, we have explored the students' feelings when they assumed the role of the assessor as well as of the assessed. Moreover, we have collected their views on the influence that their participation in this activity has had on the learning of the subject, the method of study and the success of the examination. Finally, suggestions have been asked for improving the activity.

Didactical design of FA online workshops

The online FA workshops are focused on peer review processes implemented by using the "workshop module" of the web-based platform Moodle, followed by a structured feedback by the teacher. FA workshops allow students, on the one hand, to upload the solution of specific problems/exercises and, on the other hand, the automatic and anonymous redistribution of the productions, to be assessed by other students, related to a specific topic of the course (sending-and-displaying functionality of technology). Assessment is guided by specific criteria established by the teacher (FA strategy A). The submission consists of plain text and optional attached files including the students' answer with respect to a specific topic assigned by the teacher.

Overall, the FA activity realized by means of the support of the online platform is carried out according to three sequential phases:

- *Planning*: the teacher configures setting and creates assessment forms and instructions (FA strategy A). The structures of the *Workshop* module make possible to: set the times of the various tasks; define the assessment criteria (correctness, completeness, clearness); distribute a certain number of products for each student, excluding self-assessment (we chose 3); deliver peers' feedback to students; make evident to the students the scores obtained on the basis of the scores received from their peers (processing-and-analysing functionality of technology).
- *Solving*: each student receives a problem, with the following request:

  Solve the problems: "*Solve the following problems and for each question, give correct, clear and complete answers, explaining the reasoning behind and referring the theoretical properties used. Then upload your product, being careful to leave it anonymous."*

  Once solved the problems, the student uploads her product, set anonymous (in order to avoid any bias), which will be automatically and randomly redistributed to three peers.
- *Assessing*: each student assesses the productions of three peers, grades them and provides feedback (FA strategy C, applied to students rather than teacher) according to the following request:

  Assess and grade the received products: "*Correct and assess the three products you received randomly from the system. The assessment is expected to be carried out according to the three criteria: correctness, clarity and completeness. For each criterion, you will be asked to give a numerical rating from 0 to 10 and to give accurate feedback. You will also be asked to give overall feedback on the work examined."*

  The criteria have been detailed as shown in the following:

  - *Criterion 1 (correctness)*: "*For each exercise, assess whether there are any errors in the solutions or solving process and whether all the answers have been given. Are the theoretical references correct, if any? Are mathematical symbols correctly used?"*
  - *Criterion 2 (clearness):* "*For each exercise, assess whether the solution is clearly and unambiguously expressed and whether the solving process is shown and comprehensible. In other words, assess if the solving process does express clearly, precisely and unambiguously its content."*.
  - *Criterion 3 (completeness)*: *"Assess whether all the solutions have been given. When required, assess whether the processes are complete or whether there are lacking parts or gaps in reasoning, or unjustified conclusions*."

Students are asked to give to their mates two types of feedback: an analytical one, based on the three chosen criteria, and a synthetic one, expressed with a numerical grade similar to the marks given in the final exams. We note that the request of grading the peers' works is a structural component of the Moodle workshops and cannot be avoided.

Once the students have completed all the previous steps, the teacher provides a structured feedback, making available (in a shared folder on the platform) some tasks carried out very well and that meet all the established criteria as well as some tasks failing to reach the foreseen criteria, either because they contain typical mistakes, or incomplete answers, or unclear arguments, equipped with appropriate comments (FA strategies A and C).

Outcomes and discussion

In this section we give an overview of the effect that the online FA strategy has had on the students in terms of participation to the activity in the context of the two Universities as well as students' feeling. In particular, at the University of Salerno (UniSA), four workshops (WS1, WS2, WS3, WS4) have been delivered to the students. As shown in the bar chart on the left (Fig. 2) and taken into account that the average number of students attending the course is 130 (orange line), we can register a high and constant participation to the activity (blue line). A positive participation has also observed in the course at the University of Turin (UniTO). Here, five workshops have been delivered to students. The bar chart on the right (Fig. 2) gives information about the level of participation (green line). That is very high if we consider that the average number of students attending the classes is about 110, compared to the total number of course participants (200 – blue line).

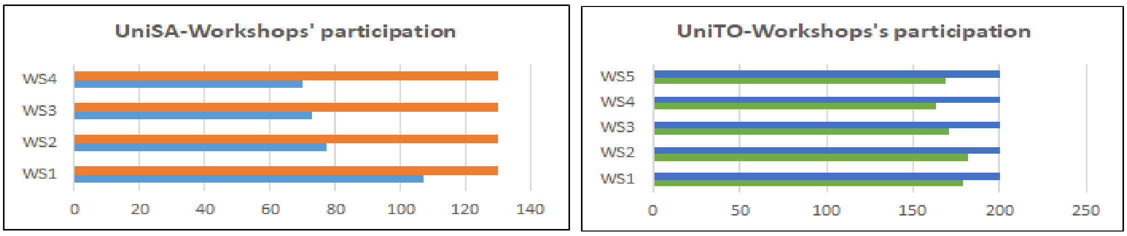

Figure 2. Workshops' participations

More in depth, we show, starting with two samples of students from the University of Turin, how the FA strategy could enhance the students' argumentation competence according to the peers' assessment. Specifically, we consider the feedbacks received from the students' assessors referred to workshops WS1 and WS2, whose topics of these workshops are quite similar. Figure 3.a shows the marks assigned to the student S1 for each criterion from the three peer assessors, named val1 (navy line), val2 (orange line) and val3 (grey line), referred respectively to WS1 and WS2. Analogous data are shown in Figure 3.b for the student S2 (the three peer assessors are again named val1 – green line, val2 – blue line, val3 – yellow line, but they are not the same students as in the case of S1).

Figure 3. Analysis of the assessment received by the students S1 and S2

It is evident how in moving from WS1 to WS2 the performance of the student S1 in the *Solving* phase is considerably improved from a point of view of correctness, clearness and completeness. Indeed, in WS1, the student S1 receives the following feedback [...*Suggestions: 1) Read the assignment; 2) Not to do your homework just for doing something, it is useless for you and also for me who I am* *correcting your productions; 3) Whoever corrects isn't in your head, so write down all.; 4) If you have really put a lot of effort into it, you have a lot to catch up on. …*]. In WS2, as the three evaluation criteria are judged positively by all the assessors, we can assume that all suggestions have been taken into account by the students.

Concerning the student S2, we highlight the precise feedback given with respect to the correctness criteria. If we analyse the marks given by the assessor val2 in WS2 (Fig. 3.b), we can observe that S2 received the same marks for clearness and completeness criteria but a lower mark for correctness justified by the following sentence "*The exercises have been completed with clarity and completeness of content but unfortunately some parts of the exercises have been set in the wrong way and this has compromised the validity of some reasoning expressed to answer adequately and correctly to the requests of the various deliveries*". So, it is evidence the attention that assessor students pay in reviewing the tasks she received with respect to the assessment criteria set by the teacher.

Regarding the questionnaires, we focus here only on following three aspects:

Feeling about the role of assessing. Many students have had the experience of helping their classmates: "*In some tasks, I have tried to steer the student examined towards the correct execution of the exercise, motivating with the right theoretical references where necessary. In those situations, I felt comfortable with the idea of being able to help*". This goes alongside feeling responsible, not only for her own learning, but above all for peers' one. The downside to the sense of responsibility was to feel not up to the task that some students reported.

Impact on content learning. Among the benefits of participating in the workshops, the main ones consist in acquiring regular rhythm of studying, the self-assessment of what understood during the lectures as well as the identification and recovering of gaps that might be.

Impact on learning approach. The students report that the workshops made them realize how to study the subject, leading them to a more in-depth theoretical study. Benefits are also recognized from peers' comparison, allowing to develop correct and formal solving methods, focusing in particular on arguing the answers and justifying the choices made.

CONCLUSIONS

Students are facing exams in different sessions and the questionnaires' scrutiny is not finished yet. Thus we do not have a complete picture about the actual impact of FA online workshops on their exam performance and data analysis is still on-going. However, we may trace the first considerations and implications from the study. The outcomes of the questionnaires show that most of the students recognize the possibility of comparing and interacting with peers as the main advantage of the FA online workshop. It concerns in particular the role of assessor. The investigation of the difficulties faced in the *Assessing phase* shows that the role of the assessor has made the students aware of three main critical aspects concerning mathematics learning: (a) the need of clearness and of completeness when describing the solution to a problem; (b) the attention to the communicative dimension; (c) the need of a deep understanding of the topic to be able to correctly perceive the severity of an error and to provide help. On the affective level, among the benefits of the FA online workshop crucial elements emerge, such as confidence with the subject as well as autonomy.

Almost all the students said that, if they were course teachers, they would combine classroom lessons with online FA workshops. The students have also indicated as a positive point the use of the correct solutions posted by the teacher, as a means to improve their work as assessor in next sessions of the workshop. Some students suggest that having such optimal models at their disposal before their own assessment on the problems at stake could support them in the assessor role. We are considering this feature as an element of re-design for the future FA workshop cycle.

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STUDENT MODELS TO GENERATE AUTOMATED FEEDBACK ON INTERMEDIATE STEPS IN SOLVING MATHEMATICAL PROBLEMS

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Digital technology can help with providing detailed assessment of mathematical competences. We present the Advise-Me project, in which evidence of the level of mastery is collected from free-form input, without restricting user interaction. We discuss the novel components of our approach, such as the use of a domain reasoner and Bayesian networks for open algebra problems, and an upcoming evaluation study in three countries.

Keywords: Step-based assessment · Free-form input · Solution strategies · User modelling

ADVISE-ME RESEARCH SUMMARY

Low achievement in mathematics is a common concern for all European countries. An extensive review of research evidence on what works for children with mathematical difficulties has concluded that interventions should ideally be targeted towards an individual child's particular difficulties (Dowker, 2004). General research on feedback (Hattie, 2009; Kluger & DeNisi, 1996) states that feedback is most effective if it is about the task, reports about correctness, and builds on changes from previous trials. Learning environments used for stepwise solving homework are just as effective as individual tutors (VanLehn, 2011), and have the added advantage of giving teachers information about progress of individual students.

In this poster, we present the Advise-Me project. The objective of this project is to develop flexible support for detailed diagnostics of mathematical competences of students, and to use this in existing testing and practicing environments in mathematics education. The actions of a student when working in a digital environment can be collected in, and interpreted by, a so-called student model. It is essential that this model not only analyses final answers, but also intermediate steps (VanLehn, 2011). We thus want to analyse intermediate steps to get precise diagnostic information. The interface should not restrict students; we therefore allow free-form input.

For the Advise-Me project ten open tasks on setting up and solving algebraic equations are developed, in which students can choose their own solution strategy. Figure 1 (left) shows one such task. Different solution strategies are specified in a domain reasoner that is based on the Ideas framework. The domain reasoner recognizes the solution approach, but also (intermediate) steps and mistakes. This information is interpreted by task and student models, which estimate the competences of the student based on Bayesian inference. The inference results can be reported to students and teachers, and used by a task sequencer to select a new task. Figure 1 (right) shows the information flow (Heeren et al., 2018).

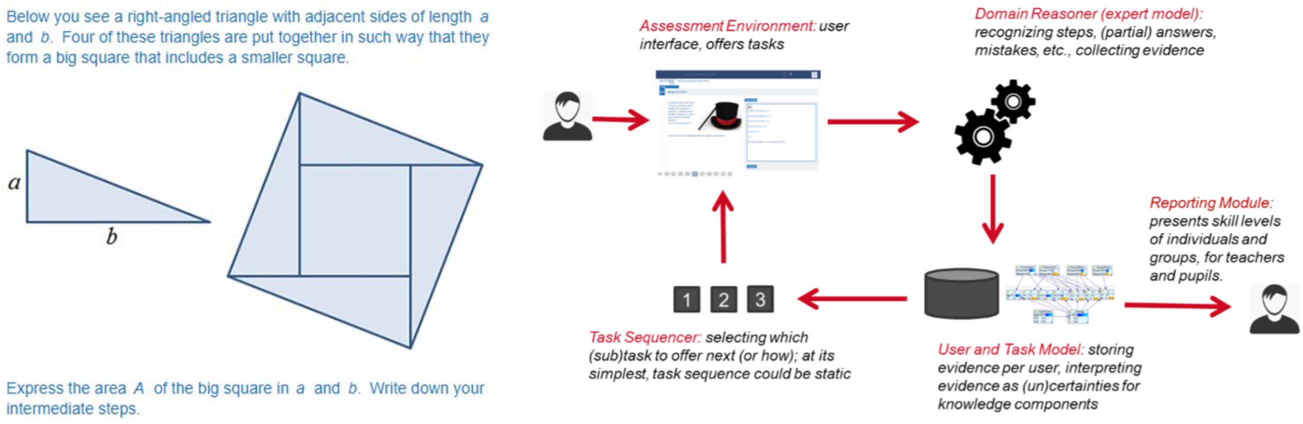

Figure 1. Task used in pilots (left) and information flow (right)

To test the feasibility of our approach, we organized pilots in which 62 students from France, Germany and the Netherlands participated. The collected data helped us to improve the tasks, the domain reasoner and the task and student models. In spring 2019 we ran more extensive evaluation studies with 358 14-15-year old students. They worked on the tasks within the Dutch Digital Mathematics Environment or the French Pépite software. The analysis of the collected data will include a comparison of machine and human scoring, leading to an improved student model.

CONCLUSION

We have developed a novel approach to the assessment of mathematical competences in a digital environment, with free-form student input. Key components in this approach are the domain reasoner for recognizing solution strategies and information about steps, and student and task models for interpreting this information. Upcoming studies will evaluate our approach.

NOTE

The Advise-Me project has received funding from the European Union's ERASMUS+ Programme, Strategic Partnerships for school education for the development of innovation, under grant agreement number 2016-1-NL01-KA201-023022. For more information, visit http://advise-me.ou.nl.

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CLUSTERING STUDENT ERRORS

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This paper gives an in-depth description of the research design in the pursuit of clustering of students' performance when solving different types of linear equations. The student performances are clustered using data from 457,185 answers to equation tasks, made by 37,585 students, distributed across 3,438 unique linear equations in a digital learning environment. The tasks consist of different categories of linear equations. The clustering analysis contributes to the development of an online tool to provide the teachers with easy accessible formative assessment. At this point, the attempt to cluster the students' performance have not yet been successful, meaning that no clusters are found. Instead, a description of how the pursuit of these clusters will continue is presented alongside the research design.

Keywords: Students' difficulties, linear equations, unsupervised learning, clustering, formative assessment.

INTRODUCTION

This paper presents the research design for utilizing a large amount of data to assess and accommodate students' mathematical difficulties when they are working with linear equations in Danish lower secondary school, more specifically the 6th and 7th grade (12-14 years old). Overall, this paper describes a research design that is based around the clustering of students working with linear equations in an online learning environment. An initial attempt was made, that unfortunately did not bear fruit. The project stems from a collaboration between two doctoral candidates associated with two independent industrial PhD projects. The authors collaborate across academic institutions with the common goal of generating knowledge about whether or not standard digital mathematical tasks on an online platform can serve as a non-disruptive diagnostic tool to generate easily accessible formative assessment for the teachers in Danish lower secondary school. The ideas and the design described are research work in progress. Therefore, only initial results and anticipated outcomes will be presented alongside the design.

The Danish private company Edulab develops and maintains an online mathematical learning platform for the Danish K-10 schools. In Denmark, 75% of the K-10 schools subscribe to Edulab's learning platform, which effectively means that 600,000 students have access to these digital learning materials. During the last 12 years, Edulab has developed the online mathematical learning platform, called *matematikfessor.dk*. The platform performs primarily as a teacher-driven supplement to a learning material, but the students also have access in order to explore the mathematical content on their own. Every day Danish school students collectively give answers to 1.5 million tasks on the online learning platform. This creates a unique opportunity to implement didactical research results regarding students' difficulties with equations directly into practice. The two industrial PhD projects aim to provide Edulab with a research-based tool to reveal and capture students' mathematical difficulties when working with linear equations in order to provide teachers with valuable information hereof. The assumption is that standard digital mathematical tasks (already implemented on the platform) can serve as a substitute for a diagnostic test, based on the idea that a large enough amount of data together with research-based diagnostic verification can generate significant information on students with difficulties when working with linear equations. This leads to the following overall research aim:

How can an online diagnostic tool for lower secondary school be designed, utilizing existing research findings on mathematical difficulties when working with equations in order to provide the teachers with significant dynamic formative assessment?

In order to approach the answer to the above question, we have chosen to explore possibilities based on the vast amount of data that Edulab has collected so far from students answering textbook like standard tasks, involving equations, in their online environment. Therefore, we have posed the following research question that will be the main research question for this paper:

To what extent can the existing categorization of standard textbook linear equations serve as a mean for generating clusters of students with a large amount of data?

By existing categorization, we mean the levels of distinctions we are able to make based on the tasks already implemented on the planform.

THEORETICAL BACKGROUND

Terms like misconceptions, alternative conceptions, or errors have been used in the mathematics education literature in the past describing children's beliefs, conceptions and problem solving strategies (J. L. Booth, McGinn, Barbieri, & Young, 2017; L. Booth, 1984; Linsell, 2009). When we mention children's difficulties working with equations in this particular context, the main focus is to describe common misinterpretations or common conceptual limitations that children have about linear equations, for example, the role of the equal sign, the role of the literal symbols or the ability to apply different problem solving strategies.

From reviewing the literature, it is concluded that the mathematics educations community knows a great deal about children's conceptions and errors working with the elements of algebra (Rhine, Harrington, & Starr, 2018). Linear equations have not received the same attention, but some studies have dealt with the strategic aspect and the concrete errors children make when working with equation solving (Herscovics & Linchevski, 1994; Kieran, 1985, 1992; Linsell, 2009). Linsell (2009) presents tests and interviews with the purpose of uncovering the difficulties children have with solving equations as well as the origin of these difficulties. Together with the strategic difficulties that arise when students have to solve equations, there are of course also difficulties with the individual sub-concepts of linear equations. The interpretation of the equal sign becomes a natural conceptual centrepiece when addressing the overall concept of equations (Kieran, 1981). Alongside the equal sign comes the algebraic elements of literal symbols, numbers and operators (L. Booth, 1984; Küchemann, 1981). When talking about the concept of equations, one also has to take more intangible aspects such as truth-value and solution into consideration.

Following categorization of the types of linear equations is adapted from Vlassis (2002) based on abstraction level and the partition presented in Filloy and Rojano (1989).

- Concrete arithmetical equations: Arithmetical equations that consist only of natural numbers and only include a single occurrence of the unknown. (e.g. $ax + b = c, \quad a, b, c \in N_0$)

- Abstract arithmetical equations: Equations with the unknown in one member, which require certain algebraic manipulations, because of the presence of negative integers or several occurrences of the unknown. ($a_1x \pm \cdots \pm a_n \pm b_1x \pm \cdots \pm b_nx \ \pm c, \quad a_1 \ldots a_n, b_1 \ldots b_n, c \ \in Z, \ \ n \in N$)

- Algebraic equations: Equations similar to the abstract arithmetical equations, but with the exception of occurrences of the unknown on both sides. ($ax \pm b = cx \pm d, \quad a, b, c, d \ \in Z$)

Linsell (2007) lets us divide this categorization, into equation types based on the number of steps it will take to solve the equations by normal transformations. Within the concrete arithmetical equations, we find the one-step equations, divided into containing small and large numbers. The abstract arithmetical equations contain (in this context) two- and three-step equations. It is also possible to have two-step equations within the algebraic equations, but for the most part in this context, they will require further transformational steps. The categorization of the equations used for analysis, taken from the online platform, is presented in table 1.

Label	Form	Steps	Number of tasks
1	$ax + b = c, \qquad x, a, c \in \{1,2, \ldots ,9\}, b \in \{0,1, \ldots ,9\}$	One/Two-step	106
2	$ax + b = c, \qquad x, a, c \in N \ b \in N_0$	One/Two-step	812
3	$ax + b = c, \qquad x, c \in Z \ b \in N_0, a \in N$	One/Two-step	501
4	$ax + b = cx + d, \qquad a, b, c, d, x \ \in N$	Two/Three-step	1478
5	$ax + b = cx + d, \qquad a, c \ \in N \ x, b, d \ \in Z$	Two/Three-step	519

Table 1 - Categorization of linear equations on the online platform

The reason for having this categorization of the linear equation is that the online platform currently only give us the opportunity to make the above distinction at this point. Notice that equations labelled 1, 2 and 3 are arithmetical, but are only considered abstract if e.g. c is a negative number. The equations labelled 4 and 5 are considered algebraic equations.

METHODOLOGICAL CONSIDERATIONS

Design research (Barab & Squire, 2004) will pave the way for the knowledge generated through iterations of the work with the initial clustering of students performance solving different types of equations and the selection of the standard textbook tasks, about equations, implemented on the online learning platform.

In order to answer the main research question for this paper, the design goal is the analysis of student interaction with the chosen items on the platform, fitting the categorization based on the literature. The categorization of the equations mentioned in the theoretical background is to be utilized in order to analyse the clusters when found. In the following section, we describe methodological considerations as well as the process of and theory behind the approach for the clustering of the different types of students based on the answers given to the equations presented in the theoretical background and the specific errors the student make while working with these equations.

CATEGORIZING AND CLUSTERING OF ERRONEOUS ANSWERS AT EDULAB

This section contains our current approach to the categorizing and clustering of the errors the students make when using the portal. Before the approach can be described in detail, it is necessary to shortly establish how the current system at Edulab works.

The primary activity students engage in at Edulab's online learning platform is answering tasks. The tasks can be either multiple-choice or writing an answer in an input field. The multiple-choice tasks are structured such that each task always has five possible answers, while the free form input fields can only contain a number. Each task is associated with a lesson e.g. "Equations with a single unknown, plus minus", which is the lesson introducing them to equations with only one unknown, containing only simple plus or minus operations. A lesson is a categorized element consisting of a video-clip introducing the content and related tasks. Each lesson is further associated to a topic, e.g. "Equations with unknowns" which is more a general element than the lesson.

Edulab's platform contains a collection of over 1 million tasks. Each task is therefore not "handmade", and there is no meta-information related to the task on what different erroneous answers can indicate of possible underlying mathematical difficulty related to equations. Despite the lack of meta-information, we hypothesize that with a large number of erroneous answered tasks from the users on the portal, we can infer what the erroneous answers might indicate of student difficulties.

Distribution of erroneous answers

Before we elaborate on how we intend to use the erroneous answers, we first investigate how the erroneous answers are distributed for each task to ensure that not all erroneous answers are equally likely to be chosen by the students, as this will require us to infer the meaning of a very large number of erroneous answers. If some erroneous answers are very unlikely, we will not try to utilize these, as the information for these are very rare.

We will focus on the distribution of erroneous answers for a lesson related to equations, which only contain multiple-choice tasks. We use log data collected for the school year 2018-2019 for students attending 6^{th} and 7^{th} grade. The log contains 457,185 answers to tasks, provided by 37,585 students, in the lesson containing the equation tasks mentioned in table 1, distributed across 3,416 tasks. We remove all tasks that received less than 150 answers leaving us 197 tasks and 379,315 answers. To investigate the distribution of erroneous answers we apply the following for each task:

1. Compute the number of times each answer is answered by the students for each task
2. Normalize it to a probability distribution
3. Sort the probabilities from most probable to least probable (On all answers the most probable answer is the correct answer)

We are left with a probability distribution for each task, which we want to cluster, to see how the answers of the students vary across tasks. To do the clustering we use affinity propagation (Frey & Dueck, 2007), which finds candidate "exemplars" of the data points, which can be used as a general example for a cluster. We use standard parameters with damping being 0.5, and the preference is the median of the affinities. For the affinity measure, we use the well-known Jensen-Shannon divergence, as the data points we wish to cluster are probability distributions. Doing this we find 3 clusters:

1. Cluster 1 are tasks that primarily are always answered correctly, with very few errors.
2. Cluster 2 are tasks that have 1 particular wrong answer, which receives most of the erroneous answers, when a student chooses an incorrect answer.
3. Cluster 3 are tasks that have 2-3 wrong answers, which receive most of all the erroneous answers, when a student chooses an incorrect answer.

Cluster 1 have 88 of the tasks, cluster 2 have 39 and cluster 3 have 70. Our assumption that not all erroneous answers are equally likely is thereby supported.

We have done a similar analysis on a collection of other lessons, which also have input field answers, which shows that the answers from input fields also are focused on few very likely erroneous answers, and a large tail distribution of almost random answers.

Co-occurrence of erroneous answers

In the previous section we established that not all erroneous answers are equally likely, therefore the erroneous answers are focused on a smaller subset of possible erroneous answers. In the following, we establish how the erroneous answers are going to be utilized.

Each erroneous answer to a task gets a unique ID, if the erroneous answer has received more than some threshold, $\boldsymbol{P}$, of the answers for the given task. All erroneous answers, which occur often, will therefore have a unique ID, while we ignore the erroneous answers, which receive little attention. The reason for this is twofold, if all erroneous answers get an ID, the space of erroneous answers will be very large, and it is thus difficult to infer the meaning of each erroneous answer as they occur rarely.

Doing this, each student will have a sequence of erroneous answers which have been given an ID, $e_1, e_2, \ldots, e_k$, ordered by when the student gave the answer. Based on this sequence we can now construct a co-occurrence matrix, which is a MxM where M is the number of erroneous answer IDs. Each row and column correspond to an erroneous answer, and the entry at the i'th row, j'th column is a counter of how often the i'th erroneous answer occur together with the j'th erroneous answer in a student sequence. What it means for two IDs to occur together is a matter of choice, e.g. if there is a sequence of student errors over a long time horizon, then the definition of two erroneous answers to occur together can depend on the time between the two erroneous answers. By doing this, only erroneous answers, which occur closely together in time, are considered a pair. On the other hand, all IDs for the same student can also be considered as occurring together if only a small number of lessons are considered, or the time horizon is small.

We use data for a single lesson (the same as in the previous section) which focuses on equations to construct the co-occurrence matrix, where we set the threshold $\boldsymbol{P}$=5%. We only include students who have made more than three mistakes on the lesson. This is early work, and for the final work, we wish to include a long series of lessons related to equations, but these are currently being deployed, and the data needs to be collected.

For this project we have no labels and are therefore limited to unsupervised learning (Hastie, Tibshirani, & Friedman, 2009), which is the paradigm of machine learning of learning some underlying structure of the data. As an initial experiment, we wanted to investigate if there was any clusters in the co-occurrence matrix, as this would indicate the existence of errors that mostly occurred together.

To explore the existence of clusters, we employed t-SNE (Maaten & Hinton, 2008), which is a powerful non-linear clustering technique, which tries to find a mapping for each data points to a new space, such that elements which are close in the original space, are also closed in the mapped space.

This exploration did not reveal any clusters, and we therefore did not manage to find any errors that occur primarily together.

When we get the full dataset, where students interact with equations over a much larger variety of lessons, we will repeat the following exploration. This is further elaborated on in the final section of the paper. In addition, the co-occurrence matrix allows for embedding based strategies of the errors. An embedding of an error would be some function $F(e_i)$ which maps the error to some real space of dimension N. The embedding would be such that errors who occur often together will be more similar than errors than does not occur together. This can be done using the algorithm Glove (Pennington, Socher, & Manning, 2014) which is used widely for finding embeddings for words, and work directly on a co-occurrence structure like ours.

CONCLUSION AND FUTURE DESIGN

The main focus of this paper was to answer the question; to what extent a categorization of standard textbook linear equations could serve as a mean for generating clusters of students with a large amount of data. Unfortunately, the clusters are not yet found. In the following section, we elaborate on the next step of the research design in order to accomplish finding the clusters.

To link the content of this paper to the overall research question, the following section will describe the future possibilities for finding clusters. With the right strategy, it will be possible to push these new tasks to the users of the platform, both for the teachers to assign to their students and for the students to explore on their own. In table 2 is presented a new possible categorization of the content on the platform that involves solving linear equations. This content already exists but have not yet been put to full use, meaning that the amount of answers given to these tasks are yet too sparse.

Label	Form	Type	Steps	Number of tasks
A	$x + a = b, \quad a \in N, b \in Z\backslash\{0\}$	Arithmetical	One-step	910
B	$ax = b, \quad a, b \in N$	Arithmetical	One-step	467
C	$x - a = b, \quad a \in N, b \in Z\backslash\{0\}$	Arithmetical	One-step	432
D	$a - x = b, \quad a \in N, b \in Z\backslash\{0\}$	Arithmetical	One-step	493
E	$ax + b = c, \quad a, b, c \in N$	Arithmetical	Two-step	431
F	$\frac{x}{a} = b, \quad a \in N, b \in Z\backslash\{0\}$	Abstract Arith.	One-step	426
G	$x \cdot \frac{a}{b} = c, \quad a, c \in Z\backslash\{0\}, b \in N$	Abstract Arith.	One-/two-step	587
H	$\frac{a}{x+b} = c, \quad a, c \in Z\backslash\{0\}, b \in N$	Abstract Arith.	Two-/three-step	755
I	$ax + b = cx + d, \quad a, c \in N\ b, d \in Z\backslash\{0\}$	Algebraic	Two-/three-step	DBT
J	$ax + b = cx + d, \quad a, c \in Z\ b, d \in Z\backslash\{0\}$	Algebraic	Two-/three-step	DBT

Table 2 – Future categorization of linear equations on the online platform

The future goal is to use the same clustering approach but with the new, more refined variety of linear equations. Furthermore, every type of equation comes in five different difficulty levels. This gives us another opportunity for selection and distinction. This means that instead of just having the 10 categories presented in table 2 we possibly have 50 levels of distinction instead of the 5 we had for the first iteration of the clustering attempt. We believe that having 50 levels of distinction between the different types of linear equations will be a step in the right direction in order to achieve the student clustering. As mentioned in the methodological considerations having this categorization of the types of linear equations will serve as a mean to analyse and interpret the clusters when found.

FINAL REMARKS

The overall research aim of the project is to provide Edulab with a tool to support teachers in their teaching. The utilization of the vast amount of data Edulab are able to collect shall pave the way for the development of this assessment tool. The project's vision is that Danish mathematics teachers, based on an easy accessible formative assessment source, can be given a unique opportunity to organize, plan and complete their teaching (Palm, Andersson, Boström, & Vingsle, 2017). The future goal is to set up a continuous categorization of all students using Edulab's platform in the 7th grade in order to "catch" students with conceptual or strategical difficulties when working with linear equations on the platform.

Inspired by the work presented in Linsell (2009), the goal is to develop a series of online questions in order to uncover the strategies the students use or the transformational difficulties the students experience, when working with the different types of linear equations. Together with the categorization of students, the second goal is to provide teachers with information of students' strategies as well as typical errors their students make when solving different types of equations. This information should be in the form of auto-generated 'formative assessment reports' for each student that the tool has identified and verified as having conceptual difficulties working with linear equations, describing also the student's behaviour and the difficulties present based on research findings on the identified difficulties. The formative assessment report should contain procedures and guidelines for how the teacher can approach remediation of the student's identified difficulties.

An alternate way to proceed could be to provide the student suggestions for teaching materials on the online learning platform for further learning on their own. The suggestions could be video lectures addressing their concrete subject in which they have difficulties.

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ORCHESTRATING WHOLE-CLASS DISCUSSIONS IN MATHEMATICS USING CONNECTED CLASSROOM TECHNOLOGY

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This paper reports on the planning of a pilot study where the aim is to develop and investigate teaching practices, using connected classroom technology (CCT), to support formative classroom practices in mathematics. The focus is on the design of a teaching unit including a whole-class discussion drawing on students' computer-based work. The paper outlines both generic and topic-specific theories underpinning the design. Moreover, findings from a previous study, in terms of student responses to a task developed for a dynamic mathematics software environment, are used in the planning. One important issue addressed in this paper is the sequencing of student responses to display and use as a basis for whole-class discussions.

Keywords: connected classroom technology, mathematics education, whole-class discussion

INTRODUCTION

In a concluding paragraph of a survey of the research on technology use in upper secondary mathematics education, Hegedus et al. (2017) present some practically oriented questions to consider in the future. For example,

> … is there, or could there be, a taxonomy for orchestrating student digital work? How can the teacher make best use of student created contributions? What new opportunities of interaction are there between the teacher and the students and what is the role of the teacher within these new forms of interactions? (p. 32)

These types of question have been addressed by studies investigating the educational use of the type of technology referred to as Connected Classroom Technology (CCT). Irving (2006) defines CCT as "… a networked system of personal computers or handheld devices specifically designed to be used in a classroom for interactive teaching and learning." (p. 16). Technological development in this area has led to more sophisticated teaching tools to support richer classroom discourse. For example, Clark-Wilson (2010) found how teachers who used a CCT system over a period of a few months developed their teaching towards new types of formative assessment practices. When describing the most 'desirable' feature of CCT, the teachers emphasised the enhanced opportunity of monitoring students' computer-based work during the lesson. For example this helped the teachers to identify 'interesting screens' to share with the whole class to promote a rich classroom discourse. Although the literature identifies promising opportunities provided by CCT, there is still a challenge for teachers in orchestrating (in the sense used by Hegedus) these types of teaching practice.

This paper reports on preparation for a project, located in a Swedish upper secondary school, which aims to develop methods and procedures for using CCT to support formative classroom practices in mathematics, and so to develop design principles to guide teaching activities using CCT. In this paper, we focus on the design, for a pilot study, of a teaching unit using CCT to support a formative classroom practice in mathematics. The design is guided by both generic and topic-specific theories.

GENERIC THEORIES GUIDING THE DESIGN

In the pilot study, a teaching unit will be trialled. The design of this teaching unit is guided by three generic theories: first, the theoretical framework for formative assessment (FA) by Black and William (2009); second, the three technological functionalities identified by the European project *Formative Assessment in Science and Mathematics Education* (FaSMEd) (e.g. Cusi, Morselli, & Sabena, 2017); and finally the model of five practices as a tool for "orchestrating productive mathematical discussions" by Stein, Engle, Smith, and Hughes (2008). The framework for formative assessment (Black & Wiliam, 2009) consists of five key strategies and three agents (teacher, learner, peer). The FA strategies are

(1) clarifying and sharing learning intentions and criteria for success;
(2) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
(3) providing feedback that moves learners forward;
(4) activating students as instructional resources for one another; and
(5) activating students as the owners of their own learning (p. 8).

While the teacher takes the initiative in the first strategy, it is also important that students are actively engaged, e.g. by discussing the learning intensions. The second strategy aims to identify current student understandings and misunderstandings by making their thinking visible. To do this, the teacher can use various tactics, e.g. using carefully designed tasks that explicitly require students to explain their thinking. However, it is not enough to make students' thinking visible; as the third strategy indicates, the teacher also needs to use this information to provide feedback that guides learners towards the learning goals. The fifth strategy involves helping students to become aware of their own learning process so as to be able to perform self-assessment and self-regulation. Finally, in the fourth strategy, to act as learning resources for one another, students need to learn to collaborate, e.g. to provide as well as receive peer feedback (Black & Wiliam, 2009).

The literature on formative assessment has emphasized the potential of technology to enhance FA strategies in mathematics education (Clark-Wilson, 2010; Irving et al., 2016). The FaSMEd project investigated different aspects of the use of digital technology to promote formative assessment and developed a three dimensional theoretical framework. Besides the five key strategies and the three agents introduced above, the framework identifies a third dimension consisting of three technological functionalities: (a) sending and displaying, (b) processing and analysing, and (c) providing an interactive environment (Cusi et al., 2017).

The first functionality facilitates teacher – student and student – student communication. The second functionality concerns different kinds of management and analysis of data. Finally, the third functionality concerns interactive environments in two ways; technology that enables students to explore mathematical relations individually (or in small groups) and technology that provides a shared platform for whole-class collaboration.

Cusi et al. (2017) emphasize the challenge for teachers to plan whole-class discussions based on students' computer-based work. Accordingly, they complemented the FaSMEd framework with the five practices proposed by Stein et al. (2008):

(1) anticipating likely student responses to cognitively demanding mathematical tasks,
(2) monitoring students' responses to the tasks during the explore phase,
(3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase,
(4) purposefully sequencing the student responses that will be displayed, and

(5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas. (p. 321)

Although this model focuses on how to follow up students' previous work on problem solving tasks, results from a previous study (Fahlgren & Brunström, 2016) indicate that the model might be useful when following up students' computer-based work as well.

To summarize, the generic theoretical frames that will underpin the design of a teaching unit are the five key strategies for formative assessment (FA-1 to FA-5), the three technological functionalities (TF-1 to TF-3), and the five practices for orchestrating whole-class discussions (TP-1 to TP-5).

GENERIC DESIGN PRINCIPLES FOR A TEACHING UNIT

The planned teaching unit consists of three stages: *introduction*, *pair work*, and *whole-class discussion*. A description of the stages and how elements from each theory have been taken into account is presented below.

Introduction. The teacher introduces the activity by clarifying the purpose (FA-1) and providing necessary instructions for performing the activity. The material, primarily in terms of e-worksheets, is delivered (TF-1) to pairs (or small groups) of students.

Pair work. While the students are working on the computer-based activity (TF-3), they are prompted to send (TF-1) their responses to each subtask as they proceed. Students will be encouraged to agree on a common response to each subtask. The reason for this is twofold. Primarily, this obliges the students to communicate their mathematical reasoning to each other. Moreover, in this way the number of responses for the teacher to monitor is reduced.

The teacher's role during this stage is to monitor (exploiting TF-1 to carry out TP-2 as a basis for subsequent FA-2) the students' work in two ways; on one hand the teacher follows all the students' progression across the whole activity, and on the other hand, the teacher monitors all students' responses to a particular item.

The most critical issue for the teacher during this stage is to identify and select appropriate student responses (exploiting TF1 to carry out TP-3) to display and use as a basis for the whole-class discussion. Moreover, the teacher has to decide the sequencing of these responses (TP-4).

Whole-class discussion. During this stage, the teacher purposefully displays the selected student responses so as to provide an instructive basis for the whole-class discussion (exploiting TF-1 to support FA-2, FA-3, FA-4, FA-5 in carrying out TP-5). Preferably, the teacher has had the opportunity to think about possible student responses in advance (TP-1), and hence, has prepared some useful questions to discuss in relation to different types of student responses. To conclude the activity, we suggest asking some evaluative (e.g. multiple-choice) questions (exploiting TF-1, TF-2 to support FA-2). In this way, the students are provided a further opportunity to clarify or consolidate their mathematical thinking. For the teacher, on the other hand, this supplies information regarding the students' current understanding, which will be useful in planning the subsequent teaching.

TOPIC-SPECIFIC CONSIDERATIONS AND THEORIES GUIDING THE DESIGN

In planning the teaching unit, careful attention needs to be given to the handling of student responses to the mathematical task in play. In this respect, we draw on a previous study (Fahlgren & Brunström, 2018) which investigated how small but potentially significant changes in wording might influence students' explanatory responses in a dynamic mathematics software (DMS) environment.

These student responses were collected in a study involving 229 tenth grade students, from eight different classes, at an upper secondary school in Sweden. The students worked on a sequence of tasks designed for a DMS environment. In this particular task sequence, students were introduced to graphical representations of quadratic functions written in the standard form $f(x) = ax^2 + bx + c$, and encouraged to examine the visual effect on the graph when changing the value of a parameter by using a slider tool. The students had previously worked with linear functions, linear equations and on solving quadratic equations algebraically.

The task sequence included ‘explanation tasks’ where students were asked to explain their empirical observations made in the DMS environment. As a basis for planning the teaching unit to be used in the pilot study, we used findings relating to Task 1c (see Figure 1).

Task 1

(a) Investigate, by dragging the slider *c*, in what way the value of *c* alters the graph. Describe in your own words.

(b) The value of the constant *c* can be found in the coordinate system. How?

(c) Give a mathematical explanation why the value of *c* can be found in this way.

Figure 1. The first task including a request for an explanation (subtask c)

Briefly, the analysis of students’ responses to Task 1c identified the following elements of explanation (see Table 1) which could feature in a particular response.

Code	Explanation element
A	$x = 0$ gives $y = c$
B	*c* can be found where the graph intersects the *y*-axis (i.e. repeats the answer to the previous subtask)
C	Comparing with the standard linear equation, $y = kx + m$
D	*c* behaves like/corresponds to *m*
E	*c* is the constant term
F	*c* is independent of *x*
G	*c* is independent of *a* and/or *b*
H	solves for *c*
I	Providing example
J	Referring to the DMS feedback

Table 1. The categories of explanation element in Task 1c

These findings provide useful information about what kind of student responses to expect during this particular activity and within a similar context. To simplify the findings appropriately for the pilot study, we have dropped the categories G and H because there were very few answers in these categories. We have also merged the categories C and D, E and F, and I and J into the categories C/D, E/F, and I/J respectively.

Based on these findings, and the assumption that all categories are represented among the student responses in a class, we suggest the sequencing in Table 2 when displaying different student responses as a basis for the whole-class discussion.

Order	Category	Comments
1	B	Since this is a repetition of the answer to the previous subtask, it might form a base for a whole-class discussion concerning the distinction between a description and an explanation in mathematics.
2	I/J	These responses encourage discussions about the suitability of using examples as mathematical explanations.
3	C/D	This links to a topic that the students' have previously studied, and so are already familiar with, and provides a base for discussions about similarities and differences between the linear function and the quadratic function.
4	E/F	That *c* is the constant term and independent of *x*, naturally follows from the foregoing discussions.
5	A	Finally, the valid mathematical explanation is introduced and discussed

Table 2. The suggested sequencing for displaying student responses.

To investigate the presence of the different categories in the eight different classes, a further analysis was made. In this analysis, each single student response was categorized according to an order of precedence, based on the sequencing above. So, for example, a student response that only include the explanation element B, is categorized as B, while a student response that include both explanation element B and C or D, is categorized as C/D. Table 3 provides an overview of the presence of student responses across the eight classes, organized in the sequence suggested above.

Class	B (only)	I/J	C/D	E/F	A	Total
1	3	8	12	1	3	**27**
2	5	3	13	2	1	**24**
3	2	8	5	6	4	**25**
4	1	1	18	3	3	**26**
5	0	5	7	6	6	**24**
6	4	3	12	3	2	**24**
7	2	5	17	5	0	**29**
8	2	2	13	12	2	**31**
Total	**19 (9%)**	**35 (17%)**	**97 (46%)**	**38 (18%)**	**21 (10%)**	**210**

Table 3. The presence of the categories of student responses in the eight classes.

Table 3 indicates that all categories are represented in six out of eight classes. In particular, while the first response (B only) is relatively rarely offered (by around 10% of students overall) it is present in 7 of the 8 classes; the next response (I/J) is offered (by nearly 20% of students overall) in every class; the following response (C/D) is offered (by between roughly 20% and 70% of students) in every class, indicating the prevalence of the linear analogy; the penultimate response (E/F) is offered (by nearly 20% of students overall) in every class; and while the final 'desirable' response (A) is relatively rarely offered (by 10% of students overall) it is present in 7 of the 8 classes.

Functions and graphs

The literature points out various aspects to consider in the teaching of functions and graphs. One important aspect is the distinction between a 'local' and a 'global' view of functions (Leinhardt, Zaslavsky, & Stein, 1990). The local view is characterized by a point by point attention where the focus is on specific values of a function. A global view, on the other hand, draws attention to global features of a function such as the shape of its graph or the structure of its closed form equation. In Task 1a (see Figure 1), the focus is on how c alters the graph, which might promote a global view. In contrast, Task 1b encourages a local view by directing students' focus on a specific point in the coordinate system.

PLANNING FOR A WHOLE-CLASS DISCUSSION

The main aims of the whole-class discussion are to (a) discuss what constitutes an appropriate mathematical explanation, (b) to clarify that $x = 0$ when a graph intersects the y-axis (local view), and (c) to clarify how changing the constant term translates the whole graph vertically (global view).

Following the sequencing outlined in Table 2, and provided that all the categories of student responses (in Table 3) are present, we suggest that the categories are displayed one by one (by one or several examples of student responses). Of course, the teacher must be flexible in the orchestration of the discussion, however, we propose the following step-by-step guidance including some questions to discuss.

(1) Category B
What is the distinction between Task 1b and Task 1c? (i.e. what is the distinction between a description and an explanation in mathematics?)

(2) Categories I and/or J
Could 'providing an example' (Category I) be regarded as an explanation?
Could referring to the DMS feedback (Category J) be regarded as an explanation?

(3) Categories C and/or D
What do m in $f(x) = kx + m$ and c in $f(x)=ax^2 + bx + c$ have in common?

This discussion might provide a natural link to the next category.

(4) Categories E and/or F
Could the explanation be strengthened? Why does this mean that the graph intersects the y-axis when $y=c$?

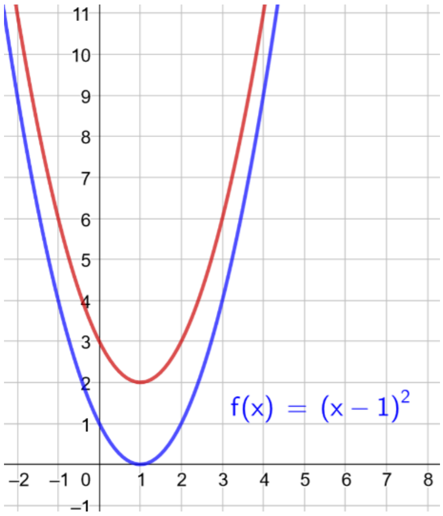
Preferably, this discussion leads into the final (and most desirable) response.

(5) Category A
These discussions should lead to a class agreement on what constitutes an approprate explanation in this particular case (Task 1c).

To conclude the activity, we suggest using some follow-up questions. First, to consolidate students' understanding that $x = 0$ gives the y-intercept (local view), we recommend the following question:

What are the coordinates of the point where the function $f(x) = (x – 2)^2$ intersects the y-axis?

As a second follow-up question, we suggest the multiple-choice question in Figure 2.



Use the lower graph, $f(x) = (x-1)^2$, to figure out which function the upper graph shows.

- $f(x) = (x-3)^2$
- $f(x) = (x+1)^2$
- $f(x) = (x-1)^2 + 2$
- $f(x) = 2(x-1)^2$

Figure 2. The second follow-up question.

The intention behind the latter question is to introduce the more general feature that changing the constant term translates the whole graph vertically (global view).

CONCLUDING REMARKS

The planned teaching unit will be trialled in four classes during the spring 2019. Preferably, the students would have access to both the DMS environment and the instructions and questions on the same screen display. However, we have not yet found any technical solutions that fulfill these pedagogical needs. Therefore, the students will use two computers; one displaying an e-worksheet and one displaying the DMS environment, in this case *GeoGebra*. After testing different available software systems, we decided to use the *Desmos Classroom Activity* as an e-worksheet where students can submit their responses.

Data in terms of field notes from classroom observations, audio recordings from the classroom and from teacher interviews, and screen recordings of the teachers' computer will be collected. In the analytical process, both generic and topic-specific theories will be used.

Reconsidering the potentials suggested in the literature on technology-enhanced strategies for developing formative classroom practices in mathematics (Clark-Wilson, 2010; Cusi et al., 2017; Irving et al., 2016), several questions appear. For example, what particular CCT features are useful? And how could these features be utilized to provide the teacher with appropriate information about their students' mathematical thinking during the lesson, and to support the whole-class discussion? These and related questions will guide the analysis. At the time of the conference, we will be able to report and discuss some results from the pilot study.

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DESIGNING ONLINE FORMATIVE ASSESSMENT THAT PROMOTES STUDENTS' REASONING PROCESSES

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Automated online formative assessment of students' work in a rich digital environment has the potential to support and develop students' reasoning process. Previous studies have presented the challenge of designing e-tasks. Here we focus on the challenge of designing a personal online formative assessment that supports the students' reasoning process. A common type of online formative assessment is elaborated feedback. We provide a design principle for elaborated online feedback of students' work on an online example-eliciting task (EET) using the Seeing the Entire Picture (STEP) platform. We demonstrate two cases of attribute isolation elaborated feedback (AIEF) design, and the case of a pair of students who used the AIEF to support their reasoning process.

Keywords: automatic personal feedback, formative assessment, formative assessment design

INTRODUCTION

Online Mathematics Formative Assessment

Technological developments offer important tools for the assessment of students working in rich digital environments. We use the term "formative assessment" in the sense described by Black and Wiliam, (1998). Automated online formative assessment has the potential to promote reasoning processes by improving the learners' mathematics thinking and skills (Shute, 2008). We are interested in exploring the main design principles of an online formative assessment tool that supports students' reasoning processes. Researchers distinguish between two main types of online formative assessment feedback: verification and elaborated feedback. Verification feedback provides simple information about whether or not the student's answer is correct. Elaborated feedback provides an explanation of why a response is or is not correct. According to a meta-analysis by Van der Kleij, Feskens, and Eggen (2015), elaborated feedback is more effective than verification feedback for higher-order learning outcomes in mathematics. Shute (2008) identified the following six different types of online elaborated feedback: Attribute isolation, Topic contingent, Response contingent, Hint/cues/prompts, Bugs/misconceptions, Informative tutoring. In this study we used: "attribute isolation elaborated feedback" (AIEF), which consists of observations on the requirements of the task and on its mathematical characteristics, including the nature of mathematical objects and actions involved, and the mathematical reasoning processes entailed.

The Challenge in Designing Elaborated Feedback

The challenge in designing AIEF lies in presenting the information to the students in a rich digital environment in a way that supports their reasoning process. To design such elaborated feedback, we needed an environment that supports mathematical inquiry and reasoning, as well as automatic online elaborated feedback. For this purpose, we used the Seeing the Entire Picture (STEP) platform (Olsher, Yerushalmy, and Chazan, 2016). STEP is an environment that supports example-eliciting tasks, where students are asked to construct examples in a multiple linked representations (MLR) environment that supports their answer. Example eliciting is a vital element in the reasoning processes, and may also be indicative of the students' mathematical reasoning (Zaslavsky and Zodik, 2014). The STEP platform supports exemplification generated by work with interactive diagrams.

Yerushalmy, Nagari-Haddif, and Olsher, (2017) stressed the importance of example eliciting as an e-task design principle. According to Yerushalmy et al. (ibid.), asking students to submit as different as possible examples encourages them to develop a rich and varied example space. The example space can be automatically analyzed to provide feedback to students. The STEP platform, therefore, constitutes a novel pedagogical tool that supports reasoning processes as well as rich feedback. Yerushalmy et al. (2017) formulated the principles of e-task design, but to support the students' reasoning processes it is necessary to also design the feedback students receive.

In the present study we focused on conjectures, which are a key component of mathematical reasoning. Conjecturing is the process by which students raise conjectures, refute or dismiss some of the conjectures, and choose the conjectures they want to justify. During conjecturing, students enhance their ability to prove (Boero, Garuti, and Lemut, 2007). Here we present a design principle for AIEF that promotes the students' conjecturing process. The study consists of three parts. In the first part, we exhibit the design principle for AIEF. In the second part, we demonstrate the AIEF design principle in two different contexts. In the third part, we describe the case of a pair of students who used the elaborated feedback to support their reasoning process.

AIEF DESIGN PRINCIPLE

Before designing the AIEF, we must design the EET. The aim of the EET design was to support the students in raising conjectures, which is the beginning stage of the conjecturing processes. To this end, we designed the EET following Yerushalmy et al. (2017). Specifically, for each task, students were asked to submit three representative examples, as different as possible. Additionally, we asked the students to formulate a conjecture as general as possible.

The STEP platform provides various formats of personal feedback. For this study, we used AIEF, which consists of a list of mathematical characteristics of the task (e.g., the figure is a rectangle, the figure has equal sides). The list of characteristics was prepared in advance, as part of the AIEF design. STEP can analyze the submitted work, and mark the identified characteristics of the submitted example in blue (here presented by circles) and the non-identified ones in gray, automatically producing the AIEF (see Figures 3 and 4 for student submissions and AIEF). We hypothesized that giving the students elaborated feedback that consists of three lists of characteristics (one for each submitted example) that differ in the indication whether or not the characteristics of each example were identified, enables students to analyze the differences and similarities between the submitted examples, and in this way support the conjecturing process.

Our goal for the AIEF design was to support students in shifting from special cases to a general conjecture. In other words, we sought to support the process in which students observe special cases of conjectures (from among the conjectures they raised when working on the EET), choose relevant information to formulate the general conjecture, and finally choose or formulate a general conjecture. To this end, the guiding principle was to design a rich and varied list of characteristics that would enable STEP to identify characteristics of the students' submissions; these include special cases of the conjecture as well as relevant and extraneous information regarding more general cases. Below we demonstrate such a design.

DEMONSTRATIONS OF AIEF DESIGN

We present two cases of AIEF design in different contexts, based on different types of information regarding the general conjecture. Each case begins with an explanation of the EET design and continues with an explanation of the AIEF design.

First EET Design Example

The first EET example was formulated as follows: "A, B, C, and D form the quadrilateral ABCD. They are all dynamic and can be dragged. If possible, create 3 examples that are as different as possible from each other, in which the perpendicular bisectors to the sides of ABCD meet in a single point. In the dialogue box formulate a conjecture as general as possible of the conditions in which this happens." Figure 1 shows a screenshot of the task applet.

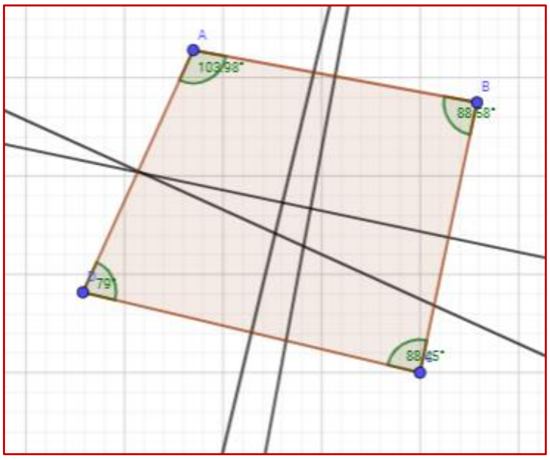

Figure 1. First task applet

The EET has been found effective for raising conjectures (Olsher, forthcoming). We used the Geogebra software to design the task applet. To enable students to focus on building quadrilaterals, the features of Geogebra were not available for them, and only points A-D were draggable, while maintaining the perpendicular bisectors. To support eliciting examples of various types of quadrilaterals, the applet provided measurements of squares and angles. Asking the students to create three examples as different as possible encouraged the construction of a personal example space that could be assessed and analyzed with STEP to produce elaborated feedback for each example submitted.

First AIEF Design Example

To design a rich and varied list of characteristics that would enable STEP to identify the characteristics of the students' submissions, first we analyzed the general conjecture: The perpendicular bisectors to the sides meet in a single point if and only if the opposite angles are supplementary. We observed that the formulation of the general conjecture contained the attribute: "opposite angles are supplementary."

Assuming that the students would also submit special cases of the conjecture, we listed all the types and attributes of quadrilaterals in which the perpendicular bisectors to the sides meet in a single point. To include extraneous information, we also predefined types and attributes of quadrilaterals that have a hierarchical relationship with the special cases. Thus, we choose to include the following types and attributes of quadrilaterals: "All angles are equal," "Square," "Rhombus," "Trapezoid," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," "Sum of opposite angles is 180º," and "All sides have equal length." For example, suppose the students submitted a square and STEP would identified the following characteristics: "All angles are equal," "Square," "Rhombus," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," "Sum of opposite angles is 180º," and "All sides have equal length." By doing so, STEP identified a list of special cases of the conjecture, as well as relevant and extraneous characteristics.

We hypothesize that this kind of AIEF can support the students' inquiry and conjecturing process. As explained in the EET design, the students were asked to submit three examples. The STEP platform can analyze each example and produce AIEF that consists of three lists of characteristics that differ from each other by the indication whether the characteristics of each example were or were not identified. The underlying principle in giving students such elaborated feedback is that they will further engage with the different mathematical characteristics of their examples, enabling them to analyze and communicate the differences and similarities between their submitted examples, which supports the conjecturing process.

Second EET Design Example

The second EET was formulated as follows: "f(x) (blue) and g(x) (green) are linear functions. h(x) is the product function of the two linear functions. Drag the blue and green points to create new functions. What kind of product functions appears when multiplying two linear functions? Create three examples that are as different as possible from each other. Formulate in the dialogue box a conjecture as general as possible of the product functions that appears." Figure 2 shows a screen of the task applet:

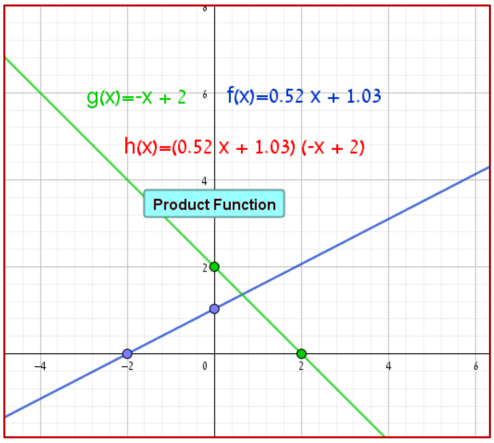

Figure 2: Second task applet

The task was chosen because the product function of two linear functions has a potentially varied and rich example space. Similarly to the previous task, the Geogebra features were limited to dragging the two linear functions and to the "Product Function" button, which supports the immediate construction of the product function. Similarly to the previous task, students were asked to submit three examples as different as possible.

Second AIEF Design Example

To design a rich and varied list of characteristics that would enable STEP to analyze the characteristics of the students' submissions, we considered the following possible general conjecture: The product function of two linear functions is either a linear function or a quadratic function that has at least one zero point. When deconstructing this statement, we noted that the formulation of the general conjecture contained, among others, several attributes that are relevant for this general conjecture. We included all the relevant attributes in the list of characteristics. For example, the information that the two linear functions and the product function share the zero points could be relevant for conjecturing that if the product function is quadratic, it has at least one zero point. To contain other mathematical characteristics that students may choose to address or take into account in their conjectures, we included other attributes of the functions, such as: "The two linear functions have the same slope tendency" and "The product function has a maximum value," which could serve in other conjectures, with different levels of generality for this given applet and setting.

As a consequence, we predefined the following types and attributes: "The product function has one zero point," "The two linear functions are rising," "The two linear functions have different slope tendencies," "The two linear functions are descending," "The two linear functions have the same slope tendency," "The product function has two zero points," "The product function is a quadratic function," "The product function has a maximum value," "The product function has a minimum value," "The product function has no zero points," "The two linear functions and the product function have the same zero points," "The product function is a linear function." For example, suppose the students submitted a product function of two linear functions that have the same slope tendency and different zero points. STEP would identify the following characteristics: "The two linear functions rising," "The two linear functions have the same slope tendency," "The product function has two zero points," "The product function is a quadratic function," "The product function has a minimum value," and "The two linear functions and the product function have the same zero points." In this case, the students are expected to observe the information and further engage with the different mathematical and relevant characteristics of their examples, in the process of formulating a general conjecture.

A SUPORTING EXAMPLE FOR THE AIEF DESIGN PRINCPLE

In the current section we demonstrate a supportive example for the AIEF design principle. To that end, we present a case of a pair of 9th grade students, Ella and Anna, chosen randomly from high schools in Israel. They performed the first activity, on perpendicular bisectors. First, they performed the online EET. To be able to check whether the students used the feedback to produce new conjectures, we gave the students the option to go back to the task, resubmit new examples, and formulate a new conjecture. The data were collected using the STEP platform. The students' work was recorded using the Camtasia screen recorder and video editing software, to triangulate with elements collected automatically. The three examples were analyzed by STEP to produce AIEF. Figure 3 shows a screenshot of the student submissions and the online AIEF they received:

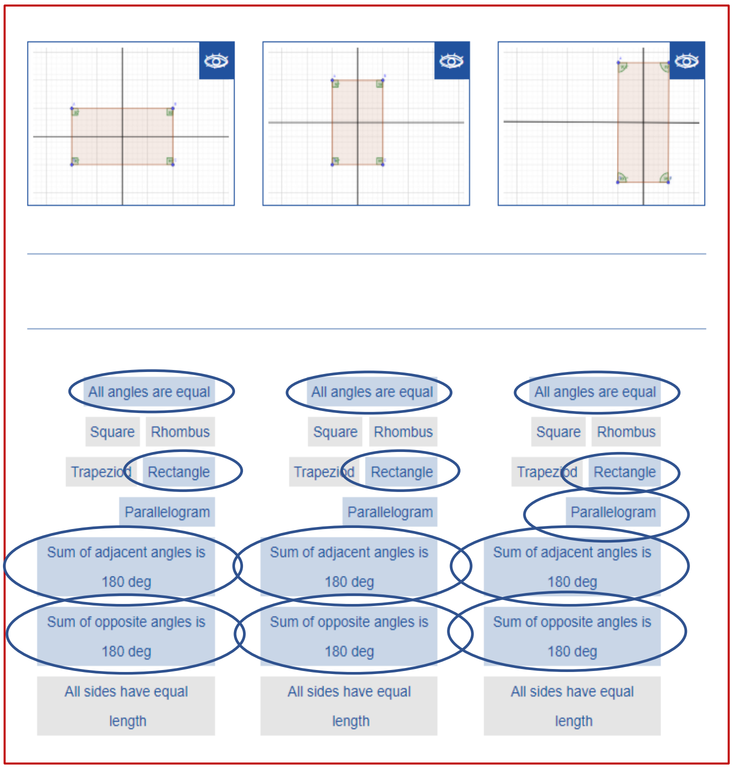

Figure 3. First set of submissions and feedback to students

As expected based on previous experiences and the literature, the students submitted a general conjecture that contains more conditions than necessary in the context provided. It is a special case consisting of alterations of a single shape: a rectangle. We sought to produce a rich list of characteristics that would enable STEP to identify various characteristics of the students' submissions. As can be seen in Figure 3, STEP identified the same five characteristics for each example: "All angles are equal," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," and "Sum of opposite angles is 180º."

Below are some of the students' reactions to the feedback:

Anna:	All the feedbacks are the same.
Ella:	Yes. When you submit a rectangle.
Anna:	No. But it doesn't have to be like that. It can also be a parallelogram [pointing at the attribute: "parallelogram" that appears in the feedbacks].
Ella:	So it happens every time there is a pair of equal sides.
Anna:	No. Whenever there is a pair of equal and congruent sides in the quadrilateral.

At this point, the students could use the characteristic "rectangle" to verify the conjecture they formulated (rectangles are shapes in which the perpendicular bisectors to the sides meet in a single point). Instead, they mentioned the type "parallelogram," and the attributes "equal sides," and "congruent sides," and used them to formulate a new conjecture. As explained in the AEIF design section, we predefined parallelogram as fitting only special cases of parallelograms, not providing a general claim that fits the required conditions. The students used this information to formulate a new conjecture.

The students had the option of returning to the task and resubmitting new examples, and they returned to the applet. First, they refuted the conjecture that parallelogram bisectors to the sides of ABCD meet in a single point. Based on their first action, we came to the conclusion that they returned to the applet to investigate the parallelogram shape, which led us to the conclusion that predefining the parallelogram supported the continuity of the conjecturing process.

Next, the students found several examples in which the perpendicular bisectors to the sides of ABCD meet in a single point. Confused by the variety, they decided to submit three examples and to get ideas from the feedback. In other words, they deliberately sought help from the feedback in formulating conjectures. Figure 4 shows a screenshot of the students' submissions and the elaborated feedback they received.

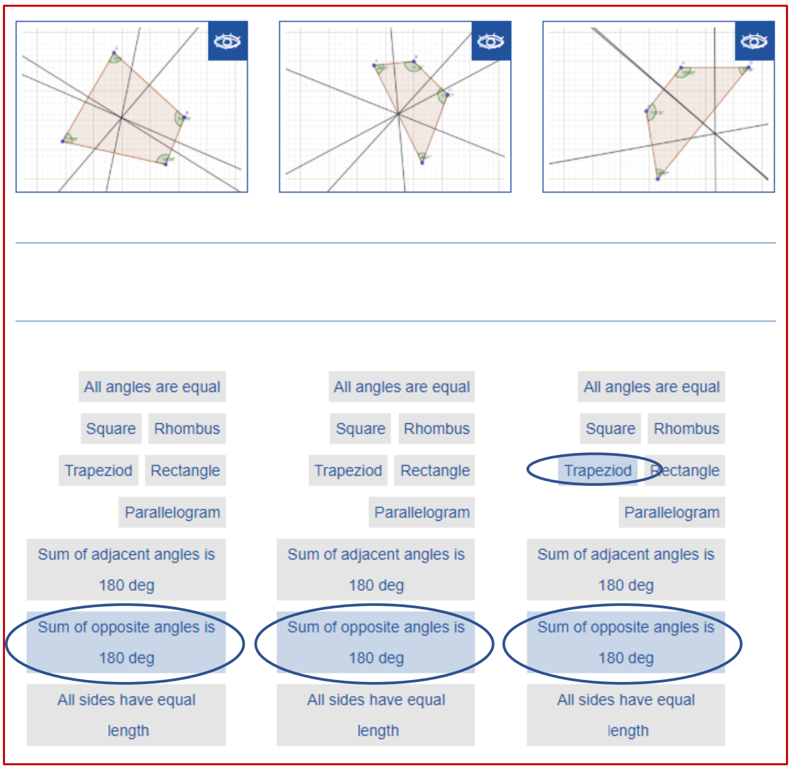

Figure 4. Second set of submissions and feedback to students

The students' reaction to the feedback was as follows:

Ella: Well, that explains it.

Anna: Opposite angles are supplementary.

Ella: This is our assumption. Opposite angles are supplementary.

The students used the feedback to formulate the general conjecture. Indeed, the AIEF dictated a single general conjecture. It is reasonable to assume that without this particular design of the AIEF, the students might not have proceeded beyond formulating different general conjectures that do not arrive to the most general level of conjecturing for this applet and setting.

DISCUSSION

The aim of our study was to offer a design principle for online formative assessment that supports the students' conjecturing process. The two design cases that were presented, provided different types of information regarding general conjectures. For example, in the first case, the students could generalize from one attribute appearing as part of the list of attributes of the AIEF. In the second case, the students could generalize from several characteristics appearing as part of the list of attributes of the AIEF. In this way, we seek to demonstrate the design principle of AIEF manifest in different contexts.

Our goal for AIEF design was to support students in moving from special cases to a general conjecture. The supporting example of the pair of students suggest that the AIEF supports the conjecturing process, which started with the students' specific conjecture and moved toward formulation of the general case. The students also succeeded in shifting from formulations that consist of shapes (rectangle, parallelogram) to those consisting of attributes (opposite angles are supplementary). We suggest that the AIEF design has the potential to support the reasoning process of the students. Shute (2008) argued that automated online formative assessment has the potential to improve the learners' mathematical thinking and skills. We believe that AIEF, appearing as a well-designed component of an online formative assessment system, has the potential to improve the students' conjecturing skills.

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USING SILENT VIDEO TASKS FOR FORMATIVE ASSESSMENT

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Silent video tasks are currently being developed in collaboration with four mathematics teachers in different upper-secondary schools in Iceland. In silent video tasks, teachers invite students to watch a short silent mathematics film and ask them to work in pairs to prepare and record their voice-over to the video. Next, teachers listen to students' solutions and plan a follow-up lesson in which they initiate a whole-group discussion. The data collection focused on the teachers' expectations and experiences with the silent video task. Thus, the teachers were interviewed before and after assigning the task, and after the follow-up lesson to better understand their experiences. This paper focuses on one of the four participating teachers and discusses the potentials of silent video tasks as means of formative assessment.

Keywords: Silent video tasks, use of new technologies, formative assessment, thinking classroom.

INTRODUCTION

The idea of students recording a commentary to silent mathematics videos was initiated and first tested in a Nordic and Baltic collaboration project for mathematics teachers and teacher educators (Hreinsdóttir & Kristinsdóttir, 2016). The silent videos were short animated films without text or sound, created with the digital geometry software GeoGebra and screen recording technology. The current research project is aimed at i) further developing the silent video tasks' instructional sequence, ii) defining what characteristics a silent video must have for it to be feasible to use in a silent video task, and iii) gaining insight into teachers' expectations and experiences with using the silent video task in their classrooms.

Four mathematics teachers in different upper-secondary schools in Iceland took part in the research project, all of which used the silent video task as a summary of a previously taught topic. A silent video task requires teachers to either make or select a short silent mathematics film to present to their students. In turn, the teacher hands out the link to the video for further viewing and splits the students into pairs to discuss, prepare and record their voice-over to the video. After preparing a follow-up lesson by listening to the students' solutions, the teacher leads a whole group discussion on basis of some example student solutions and addresses topics such as precision in word use.

During the data collection the task was developed further in collaboration with teachers with the goal to support teaching and learning, i.e. following the direction of design-based research where the focus is set on concrete tools and the tool should be sharable and usable for different teachers (Lesh & Sriraman, 2010). The data analysis informed the further development of the silent video task instructional sequence and contributed to a clearer definition of the characteristics that silent videos require in order to be suitable as silent video tasks.

ENCOURAGING DISCUSSION

Silent video tasks are designed to encourage students to communicate verbally about mathematics, to inform teachers about their students' conceptual understanding, and to make a ground for group discussion, e.g. about precision in language use. Not only do silent video tasks encourage classroom discussions about mathematics, they also seem to open teachers' eyes to students' ability to use new technologies (Kristinsdóttir et al., 2018). Different discourses get exposed in the silent video task solutions. Some of the student-produced discourse may not be mathematically correct or fulfil what is expected according to the mathematics curriculum. By listening to different solutions and discussing them with the whole group, the silent video task has a potential to become a mediator between the different types of discourses, i.e. it might shorten the knowledge or meaning gap described by Leung (2017). Keeping the follow-up group discussion in the task instructional sequence enables teachers in collaboration with students the opportunity to narrow or bridge the gap between students' intuition based mathematical world and the formal mathematical world. This gap exists because students rather use their own presumed mathematical ideas and discourses in learning activities that promote discussion and not the more formal mathematical concepts (Leung, 2017).

In order not to restrict this open task in any way, no list of concepts for students to address gets handed out. If students find it hard to start recording, a suggestion for them to draft a script can be made. The aim is to assess students' different levels of understanding of the previously studied concepts and to uncover possible imprecise language use, misconceptions, and/or misunderstandings; laying the groundwork for a whole group conversation on selected topics. This requires the teacher to listen to the student recordings and prepare a group discussion for the follow-up lesson. In the follow-up lesson some (randomly or strategically) selected solutions are presented and discussed, and topics such as problems arising from imprecise language use and possible misconceptions or misunderstandings can be addressed and clarified. After the whole group discussion, students can be asked to write about their experiences.

MOTIVATION AND THE ICELANDIC CONTEXT

The mathematics curriculum for upper secondary schools in Iceland describes goals regarding the knowledge, skills, and competences in mathematics that students are expected to reach partly or fully at each of four competence levels. At all levels, schools are expected to provide students with a foundation for understanding in mathematics, i.e. the aim is to not only get proficient in calculation methods (procedural fluency) but also to reach some level of conceptual understanding. The competence goals also include some adaptive reasoning as students are expected to understand and interpret the explanations and arguments of others without prejudice, showing respect and tolerance (Ministry of Education, Science, and Culture, 2011).

Nevertheless, interviews with teachers and observations in classrooms in nine Icelandic upper secondary schools revealed that the mathematics teaching practice was mostly limited to expository methods and recitation, drill, and practice (Jónsdóttir et al., 2014) and this was confirmed in a more recent study by Sigurgeirsson et al. (2018) where mathematics lessons stood out among other subjects taught in upper secondary schools for lack of diversity in the teaching methods used. Given these conditions, junior teachers might like to implement changes in the teaching practice. In Iceland, however, a study by Eiríksdóttir and Jóhannesson (2016) revealed that if school policy did not support changes in the teaching practice, the senior faculty in the group of mathematics teachers determined whether changes would be implemented or not, sometimes limiting possibilities for change and causing frustration for junior teachers. This aligns with findings from other international studies (Thurlings, Evers, & Vermeulen, 2015).

As one of the participating teachers in the teaching experiment conducted with silent videos in the previously mentioned Nordic-Baltic teacher-researcher collaboration project, I (the first author) became interested in developing further the tasks and their implementation in mathematics classrooms at upper secondary school level. After the international conference ICME-13 in Hamburg, I realised that what I had experienced in my classroom whilst working on and discussing the silent video task with the students was a *thinking classroom*, i.e. a classroom organized in a way that students are expected to think and given opportunities to think via activities and continuous discussions (Liljedahl, 2016). I was interested whether the silent video tasks would awaken teachers' interest in practices that support a thinking classroom. Of course, I was aware that many teachers probably had not heard of the characteristics of a thinking classroom. Yet, by listening to what teachers had to say about their experiences, connections might be recognized. Also, my intention was set to consider how different parts of the process of assigning the task could be made clearer, and it especially seemed quite important to find a good way to organize the follow-up discussion.

SILENT VIDEO TASKS AS FORMATIVE ASSESSMENT

We agree with Suurtamm et al.'s (2016) claim that the primary purpose of assessment is to improve student learning of mathematics. Silent video tasks give students an opportunity to explain to others and/or to receive explanations from their classmates do so. They also have the potential that students become aware of the fact that once they are able to explain to others what they have learnt, they also improve their understanding. One of the students who completed the silent video task commented the following in an online survey: "You don't know the material well enough if you cannot explain it to others in a good [understandable] way."

Silent video tasks that serve as a summary of previously worked-on topics can be used as formative assessment, supporting students' learning of mathematics. This is partly because after the follow-up lesson, teacher decisions about the next steps in instruction are likely to be better founded and the process of assigning a silent video task then would fulfil Wiliam's definition (2011, p. 43) of formative assessment:

> An assessment functions formatively to the extent that evidence about student achievement is elicited, interpreted, and used by teachers, learners, or their peers to make decisions about the next steps in instruction that are likely to be better, or better founded, than the decisions they would have made in the absence of that evidence.

Wright, Clark, & Tiplady (2018) list six potentials that technology-based formative assessment strategies have to support learning: providing immediate feedback, encouraging discussion, providing a meaningful way to represent problems and misunderstandings, giving opportunities to use preferred strategies in new ways, help raising issues that were previously not transparent for teachers, and providing different outcomes feedback (Wright et al., 2018, p. 219) and the silent video supports all but the first one since the feedback is not immediate but finds place in the follow-up lesson.

The knowledge involved in assessing students' solutions to a silent video task and preparing the follow-up lesson is manifold. To name a few things, teachers need to be aware of listening with an open mind, to notice and address any lack of precision in word use, to identify misunderstandings, to point out where the students might be able to dig deeper, and where to change the timing of information given or even offer more information.

Instead of unpacking mathematics for students, like would be done if the teacher would prepare a mathematics video with sound, in a silent video task the students are asked to add their commentary to the video. This connects to what Mason (2002) describes as disturbance that can trigger development in the case that it is seen as an opportunity for learning. It becomes the students' role to take apart mathematical concepts and present them in such a way that it might enable other students to gain access to the mathematics shown in the video. An inner monologue is started, and an attempt is made to make this inner monologue visible by bringing it out and into the conversation. The teachers get the role to recognize solutions or parts of solutions that should be addressed in the follow-up lesson. The unpacking of mathematics thus becomes a shared responsibility.

Mason (2002) notes on support that if one starts to do things for other people, it is easy to fall into the trap of doing always more and at the same time it is more likely that people will expect you to do things for them. This is especially true of new things that people expect will take great effort and time and foresee themselves doing it rather slowly and inefficiently; then it is of course much more convenient to get someone else to think and do the exercise! To the student all this relates to the concept of learner autonomy, i.e. the ability to take charge of one's own learning, and to the teacher this relates to the delicate task of keeping students in flow, challenging them such that they do not get bored and yet not so much that they give up (Csikszentmihalyi, 2014).

RESEARCH DESIGN

In my research project the focus is set on teacher expectations to and experiences with using silent video tasks in class. Another goal set was to develop further the process of assigning silent video tasks and to define what characteristics a silent video should have. I worked with four mathematics teachers in different upper secondary schools in Iceland who assigned a silent video task to their 17-year-old students in fall 2017.

I prepared a two-minutes-long silent video with focus on the unit circle (see https://ggbm.at/BfRqGSKq) for the teachers to show to their students. Next, I prepared semi-structured teacher interviews (Brinkmann & Kvale, 2009) to conduct before and after the assignment of the silent video task, and after the follow-up lesson. The video was pilot tested with some students and the interview questions were also pilot tested with an upper secondary school mathematics teacher. The teacher interviews were to be my main data source, however I also prepared two short questionnaires for students regarding their experiences of a) recording a voice-over to the silent video, and b) taking part in the whole-class discussion. Each questionnaire included an open comment field and five short questions to be answered on a Likert-scale.

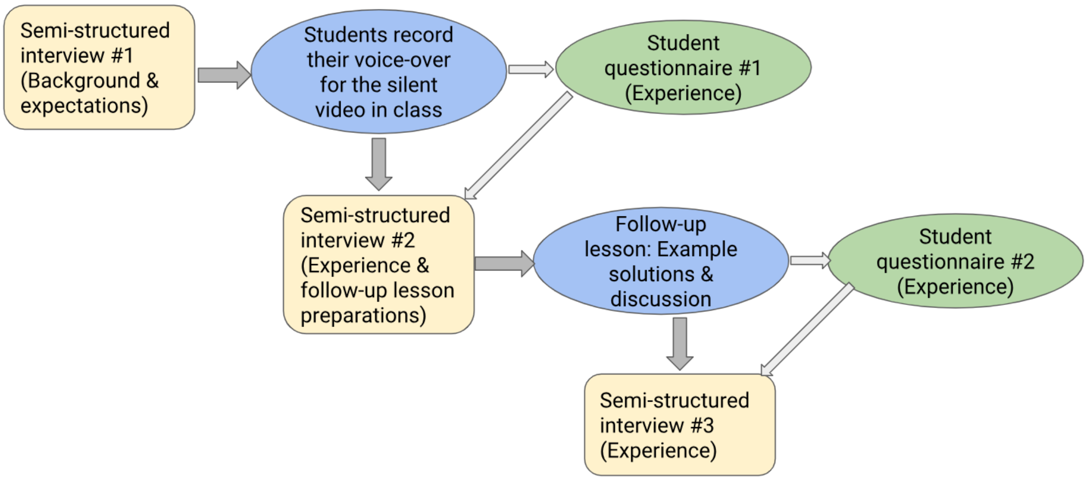

Figure 1. This flow chart shows the research design workflow. Each teacher was interviewed before and after implementing the silent video task in class, and after the follow-up lesson. Despite not being the focus of the study, the short online questionnaires for students are also shown.

The reason for including a students' questionnaire was that it seemed important to me to be able to refer to students' voices in the teacher interviews, e.g. in the event of a teacher receiving solutions that were very different from expected. In this case, referring to the students' answers on whether they had understood the task might be helpful. Another reason was to partly triangulate the data so that both student and teacher perspectives could be considered.

There are thirty upper secondary schools in Iceland that prepare students for studies at university. In fall 2017, I contacted and found teachers in six such schools who were willing to try out the silent video task with their 17-years-old students. The order in which I contacted the schools was randomly generated but since teachers could reject participation, the sample was self-selective. One of the six teachers dropped out because less than five students signed up for her trigonometry course. Another teacher participated only in the first interview and quit participation after that; giving lack of both time and students' interest as the reason.

Four teachers took part in all three interviews and were given the following pseudonyms (*gender*, *teaching experience in years*) Gauti (*m*, *9*), Lilja (*f*, *13*), Magni (*m*, *4*), and Snorri (*m*, *37*). Their students answered the questionnaires with answer rate 86% (before the follow-up) and 70% (after the follow-up), and their solutions to the task were collected. The questionnaire answers were partly used as input in the second and third teacher interviews. Both the student solutions and the teacher interviews were transcribed verbatim and words with special emphasis were underlined. The teachers did not assign the silent video task simultaneously. Practical information that was gathered from the teacher experiences (e.g. the previously mentioned "it is easier to let the students record their voice-over with their mobile phones") was communicated by the researcher to the other teachers on the go in order for each teacher to have the best information available at each point in time on how to assign the task and how to prepare the follow-up lesson. The order in which the teachers assigned the silent video task is reflected in the alphabetical order of their pseudonyms.

DATA ANALYSIS

As the focus was set on the task development, the teacher and the teachers' expectations to and experiences with using the task, I decided to analyse the data by using open coding. Some results of this work regarding the use of technology in the mathematics classroom were discussed in a conference paper (Kristinsdóttir, 2018) and in this paper we will focus on Lilja, who was the only teacher who got the discussion properly started in her classroom. This is not surprising, considering that orchestrating a discussion in a follow-up lesson using some student solutions to view and discuss is a difficult task (Stein & Smith, 2011), and the fact that the procedural tasks normally used in the Icelandic mathematics classrooms did not evoke any need for discussion.

FINDINGS AND DISCUSSION

To prepare the follow-up lessons, together with the researcher, teachers (each at different point in time) listened to the students' solutions. They were invited to think aloud, and the goal was not to assign grades to the student solutions, but rather to find possible topics to address in a whole-group discussion and to prepare feedback for the students.

Lilja described in the interview how students at her school all had little, but still some experience of taking part in mathematical group discussions. The other three teachers described little or no experience with group discussions and as Cobb (2000) points out, students are influenced by the classroom microculture that they participate in. It influences what it means to them to know and do mathematics and it is of course hard to suddenly change students working culture and thus violate the didactical contract (Brousseau, 1997) between teacher and students that is present in the classroom. Since Lilja's students were not used to working independently on tasks of a kind that they had never seen before, it created tension. Her students were especially upset about the random assignment into groups, the very short oral instructions for the task and the transfer of the responsibility to solve possible technological issues to the students themselves, i.e. the enhancement of student autonomy

Lilja: Then they just “But wait a minute, aren’t you going to help us?” “No, you will figure this out”

After a short discussion, her students understood that the random assignment into pairs was fair. Lilja would, in retrospect, not have liked to rob students of the joy of getting past some technical obstacles themselves. It caused tension but it was a productive struggle. In Lilja’s opinion it also would be an important step in the teachers’ preparation to try to record a solution to the task before assigning it.

Lilja: Mmm.. I think that the teacher would profit from recording herself beforehand and check [...] then she puts herself in the kids’ place [...] I tried it myself and I found it too short [...] it is good to understand this stress factor before assigning it.

This was also reflected in interviews with other teachers. All four teachers noticed and identified things that they had expected but were missing in some of the students’ solutions.

Lilja: ...only thing that surprised me yes there were some who did not <u>mention</u> cosine and sine at all.

The fact that some students only focused on the coordinate system, the moving point, and/or the line segments, but never used the word sine or cosine in their voice-over was something that surprised all four teachers. Furthermore, when students mentioned the trigonometric functions, some further explanations or type of phrasing were noted missing. Lilja, Gauti and Snorri all noted that their students would have no problems solving simple procedural trigonometry tasks. However, being able to solve the equation $\sin(x) = 0.6$ not necessarily implies understanding of the topic. In this way the task helped raise an issue that was previously not transparent to teachers: Students often had not given much thought to the definition of the sine and cosine functions.

A similar pattern was recognized regarding not all student pairs making a clear distinction between a circle and a unit circle in their solutions. When students referred to these two concepts precisely, teachers found and mentioned some other deficit. However, Lilja managed in her follow-up lesson to get the discussion going around this exact topic. She started by showing examples from two volunteering groups. Both solutions had some errors in them, and she discussed with her students that every solution had something that could be done better, tiny errors or imprecise word use. Her students got into a heated debate regarding at which point in time it is acceptable to say that the circle is a unit circle

Lilja: ...and they found it unbelievable that I interfered… that I did not approve of when they said “This is a unit circle” at the start before it was visible that it was a unit circle because they have already watched it all and then of course they see that it becomes a unit circle… and they found it strange that I should mention this and got quite heated about it […] “Why do you say that we cannot say it at the start?”

This was a starting point for a discussion about mathematical definitions. Also, her students discussed hard whether to focus on the length of the line segment or to rather refer to the projected point on the corresponding axis. That discussion gave an opportunity to point out the value of different approaches, and address some difficulties, bearing in mind that a length is always positive.

Lilja was tempted to give her own version of a solution in the follow-up lesson. Especially in order to illustrate that every second could be used to explain something. Teachers noted that in many cases, students’ solutions contained surprisingly many seconds of silence and that there was some lack of dialogue practice. This lack of exercise with talking about mathematics was most often blamed when deficits came up and at some point, during the interview, all teachers expressed that they were convinced that their students knew better than they showed in their solutions.

Lilja: There was something that got messed up… When they were talking, I think they were thinking it correctly just they are not used to talking

Apart from that, the fact that it was an open task with no direct questions was mentioned as a possible factor causing students to forget to mention what the teachers considered important aspects and the teachers all had their suggestions for changing the task, making it less open.

CONCLUSION

Out of the eleven practices supporting a thinking classroom described by Liljedahl (2016) Lilja used the following six: begin with an open problem, assign students visibly randomly into groups, give short oral instructions, answer only keep-thinking questions, build student autonomy, and formative assessment. The other teachers used two to three of those. Her experience with using the silent video task was positive and she, like the other three teachers, noted that it helped to break up the normal teaching routine. Lilja said it was possible but not sure that she would use such a task again. The reason being that it was "not the most important input into [her] teaching". This could relate to what the other three teachers noted on the task not preparing their students for the exam. In this regard, it is important to note, that in Iceland there is no standardized examination for upper-secondary school and each school decided how and what to test.

Silent video tasks seem to have a potential to be used as part of formative assessment practices, shedding light on differences in conceptual understanding and making classroom discussion around that issue possible. This will be developed and tested further in the next cycle of the research project.

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THE USE OF QUIZZES ON MOODLE FOR TEACHING DIFFERENTIAL EQUATIONS TO ENGINEERING STUDENTS

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This paper explains the design and psychometric quality of a battery of multiple-choice quiz as formative e-assessment for teaching differential equations. It was a preliminary experiment conducted inside a second course of calculus (Mathematical Analysis 2) for about 200 students in Managerial and Electronic Engineering at University of Udine. A survey was also administered to evaluate the appreciation of the students about the quizzes. From the survey, we found out that the quizzes were well chosen, and they discriminated adequately between higher and lower student subject knowledge. Moreover, they helped students to find gaps in their preparation and to keep up with the lessons.

Keywords: e-learning, quizzes, engineering students, differential equations, psychometric analysis, survey

1. INTRODUCTION

Formative assessment is defined “as the iterative processes of establishing what, how much and how well students are learning in relation to the learning goals and expected outcomes in order to inform tailored formative feedback and support further learning, a pedagogical strategy that is more productive when role is shared among the teacher, peers and the individual learner” (Gikandi, Morrow & Davis, 2011).

According to the OECD “students need considerable, regular and meaningful feedback; teachers need it to understand who is learning and how to orchestrate the learning process” (Dumont, Istance & Benavides, 2010) and e-assessment can provide systematic, timely, rich and personalized feedback (Albano & Ferrari). In these way, online formative assessment can play a crucial role in enhancing learning by creating improved learning environments that motivate students to actively engage and regulate their studies (Chung, Shel & Kaiser, 2006).

One advantage of regular quizzes is that they encourage students to keep up with the course content as it is presented, (Sales-Morera, Arauzo-Azofra, & Garcia-Hernandez, 2012; Angus & Watson, 2009). Additionally, online quizzes provide immediate feedback to students about their level of learning (Hattie & Yates, 2014), and help the tutor identify parts of the course content that students are struggling with and that require further explanation in class. Angus and Watson (2009) found out that regular, low-stakes, online quizzes improved student learning, as evidenced in the final exam. However, it is essential to bear in mind that the whole process should be permanently revised and updated (for the analysis of a bridge course see Lepellere, Cristea & Gubiani 2019).

The concept of a differential equation (DE) in Italy is introduced in high school (especially in science-based schools) and will be an important subject of study throughout the University, especially in the degrees of science and engineering. It has strong correlations with many mathematical concepts including functions, derivatives, integrals, etc. Therefore, students should understand these concepts in order to understand DEs and vice versa; if they understand DEs conceptually, they will understand these concepts better. Research has revealed that students hardly even understand what a differential equation is and what it is for (Raychaudhuri, 2008). The experience has indicated that an engineering perspective may improve both students' motivation for learning mathematics and students' ability to transfer their mathematics learning to engineering contexts (Pennell, Avitabile & White, 2009; Varsavsky 1995). Moreover, in mathematics education, researchers have suggested that instructional design capitalize on students' intuitions by using realistic settings for DE problems (Rasmussen & King, 2000; Rasmussen, 2001).

Differential equations, for Management and Electronic Engineering students at the University of Udine, is a topic taught inside Mathematical Analysis 2: a second course in calculus (9 credits corresponding to 72 hours of lessons), held in the second semester of the first year. It is preceded by Mathematical Analysis 1 (12 credits) and Linear Algebra (6 credits) and it is simultaneously with Physics 1. Mathematical Analysis 2 includes topics (in the order presented during the lessons) as: first and second order differentials equations; integrals on curves; differential and integral calculus in several variables; surfaces and integral calculus of vector fields; systems of linear differential equations with simple studies of stableness; Fourier series. Unfortunately, many students end up preparing the exam in the second year after having successfully passed Mathematical Analysis 1 and Linear Algebra exams slowing down their studies. In the current academic year, it was decided to propose quizzes on Moodle to encourage them to keep up with the lessons and participate in the two partial tests proposed one at half semester and the other one at the end. Moodle (Modular Object-Oriented Dynamic Learning Environment) is an e-learning platform designed to stimulate interactivity between teachers and students used at university of Udine. We used the quizzes not to assess students, but to make them study more, to encourage them not to postpone their studies. Several applets have also been provided with GeoGebra to facilitate the graphical representations of the concepts.

In this contribution we limited ourselves, for the sake of brevity, to the design and the psychometric analysis of ten quizzes about first and second order differential equations. We have also proposed to the students a survey to test how the quizzes were dealt with and whether they served their purpose.

The research questions addressed by this study are therefore:

1. Were the quizzes of a suitable level of difficulty and discriminate well between higher and lower mathematical abilities?
2. Did the use of quizzes on Moodle help the students to find gaps in their preparation and to keep up with the lessons? How do they tackle quizzes on Moodle and deal with the doubts that emerged during the study?

2. METHODS

In engineering careers, the study of Differential Equations is one of the core subjects for undergraduate students. It is not a new subject for undergraduate students in Italy, since they already have a first approach in secondary school.

Students received 10 hours of instruction in first and second order differential equations, 8 hours of exercises class and 2 hours of applications of the subject for engineer and physics.

A researcher in mathematical analysis, a researcher in geometry and a didactic collaborator expert in physics and engineering applications were involved.

From the wide range of tools offered by Moodle, we focused on the multiple-choice quiz format. The aim of the quizzes was not to assess students, but to make them study more, to encourage them not to postpone their study, and to provide more balance in their study program; and to make students more aware of their level of understanding (often students only realise that they cannot solve the exercises when they get the first test, in the middle of the semester). The quizzes were available in

Moodle and the staff repeatedly reminded students that the aim of the quizzes was to help their study of the subject and be aware of their level of understanding.
To answer the first research question we use the psychometric analysis method, for the second we use a survey. We start to present the design of the quizzes proposed.

2.1. THE QUIZZES

Six batteries of quizzes were administered online, every two or three weeks, one for each chapter of the course. They were not randomly generated and did not have a time limit to answer the questions, but it was mandatory for the students to reach a certain level to enter the midterm test. We have designed questions to supervise students' progress articulated in specified learning outcomes and skills: knowledge, comprehension and applications.
The ten quizzes designed for first and second order DEs were the following:

1. **Recognizing equations with separable variables**: Students often find the exercises on books or on the net already grouped according to the typology, consequently they can use the solution formulas without paying the right attention. Here we wanted to force them to think on the structure of the equations with separable variables. Furthermore, the advantage of quizzes is the fact that the equations must not be solvable as in standard exercises.
2. **The use of initial conditions**: Here we focused on a critical passage (remove the absolute value) in a differential equation with separable variables and a parametric Cauchy condition. By isolating the critical passage, we allowed the students to concentrate and analyze in depth the question linked to the initial conditions. In a complete exercise, students sometimes "content themselves" with studying only a particular case or removing the absolute value because "the exercise is too hard however the teacher will see that I understood".
3. **Recognizing a solution from the graph**: In order to understand the definitions and the theorems involving differential equations, students need to handle graphical representations.
4. **The use of the Overlapping Principle**: It was mainly used to review the additive property. Once again, the structure of the quiz allows us to show "long" solutions without the student loosing hours doing tedious calculations.
5. **Similarity or variation of constants method?** Analogous to quiz 1, but now for second order differential equations. They had to identify whether one is forced to use the method of variation of constants.
6. **Resonance yes or no**: Instead of giving them one of those horrible summary tables that are often used by the students, we designed a quiz to make most cases appear where the resonance phenomenon occurs. Here as well, the advantage of the quiz is that it does not require boring calculations but only to focus on the problem.
7. **Choose the particular solution of first order DEs**: Since students tend to use the pre-packaged formula, we required a specific intermediate result to force them to use the method of variation of the constant.
8. **Forcing piecewise function**: This question was made to underline the fact that, in the case of forcing piecewise function, different constants are needed for each case and subsequently the solutions must be connected. Here also, the problem was that, for relatively long exercises, students sometimes have a drop in attention in the final step or "settle" for a partial result.
9. **Qualitative study (with the help of a GeoGebra applet)**: The question wanted to guide the students to the use of the GeoGebra software in the analysis of a qualitative study where it is not possible to calculate an explicit solution (for example in the study of models applied to real cases).

Sometimes the engineers' studies concentrate only on the explicit solution, they do not analyze the result obtained.

10. **Resonance in RLC circuit**: We wanted to force the students to study the RLC circuit. Usually when they see the "applications" they think: "this question can't be asked in the exam, so I don't need to study it" or "I can study this topic at the end of the course since I'll only need it for the oral part of the exam".

Quiz questions were not randomly generated, so all students got the same questions and naturally, students shared the solutions with each other. To avoid unfairness, it was strongly emphasised that quizzes were important to students, to allow them to test themselves and get feedback on their level of understanding.

2.2. THE PSYCOMETRIC ANALYSIS

Moodle offers a wide range of resources to carry out a psychometric analysis of quizzes, we utilized the Facility Index (FI), the Discrimination Index (DI) and the Discrimination Coefficient (DC) (Martins, 2018; Blanco, Estela, Ginovart & Saà, 2009). FI describes the overall difficulty of the questions, it represents the ratio of users that answer the question correctly. In principle, a very high or low FI suggests that the question is not useful as an instrument of measurement. There are two descriptors to measure effectiveness, DI and DC, both ranging from -1 to +1. The DI index provides a rough indicator of the performance of each item to separate high scores vs. scorers. The DC is a correlation coefficient between scores at the item and at the whole quiz. In both cases, positive values indicate items that discriminate proficient learners, whereas negative indices mark items that are answered best by those with lowest grades, hence not helping to discern between the good and the bad performers. In short, these coefficients can be used as powerful methods of evaluating the effectiveness of the quiz when assessing differentiation of learners. The advantage of using DC over DI is that the former uses information from the whole population of learners, and not just the extreme upper and lower thirds as DI. Thus, this parameter may be more sensitive to detect item performance.

2.3. THE SURVEY

The other research question of this study was: are the quizzes a fair and effective tool to increase students' learning? To answer, we proposed the following queries:

- Q1: Do you think the use of quizzes on Moodle has helped you to find gaps in your preparation?
- Q2: Do you think the quizzes on Moodle have helped you to keep up with the lessons?
- Q3: How do you tackle quizzes on Moodle? (respond with the utmost sincerity)
- Q4: How did you deal with the doubts that emerged during the study?

3. RESULTS

3.1. THE PSYCOMETRIC ANALYSIS

In this section we analyze, through the psychometric quantity FI, DI and DC, the quality of the assessments. They help to answer whether the questions were appropriate, well chosen to demonstrate concepts, of a suitable level of difficulty and whether they discriminate well between higher and lower mathematical abilities.
The results were summarized in the following table:

	Questions	FI	DI	DC
1	Recognizing equations with separable variables	88.37%	51.34%	58.53%
2	Use of initial conditions	84.16%	40.97%	48.12%
3	Recognizing a solution from the graph	85.15%	41.60%	50.91%
4	The use of Overlapping Principle	79.70%	46.59%	52.85%
5	Similarity or variation of the constants	76.73%	61.47%	64.37%
6	Resonance yes or no	**68.07%**	57.67%	59.71%
7	Choose the particular solution of first order DEs	80.69%	49.00%	56.84%
8	Second member defined in two pieces	80.69%	52.69%	61.40%
9	Qualitative study (GeoGebra applet)	75.87%	65.25%	**66.95%**
10	Resonance in RLC circuit	78.71%	47.86%	54.79%

Table 1. Psychometric results

For the psychometric analysis of the ten quizzes involved, as seen in Table 1, FI ranges from 68% to 88% says that the quizzes were quite easy for the students. Moreover, most of the questions show high values for DC, yet lower values for DI, this means that the quizzes were excellent. An ideal item will be the one which has average FI between 50% and 70%, high discrimination (Dp > 30%), most of the items met the criteria of acceptable difficulty level and good discrimination. The most difficult quiz has been the recognition of the resonance phenomenon. While the one with worst discriminatory power has been the use of initial conditions.

3.2. THE SURVEY

In this section we present the results of the survey. From 202 students who participated in the quizzes, 148 responded to the survey. Of these, 120 participated to the midterm test.

The answers to the question "Do you think the use of quizzes on Moodle has helped you to find gaps in your preparation?" are summarized in Figure 1, left side. 42 students answer "Very", 77 "Little enough", 26 "For nothing" and 3 "I don't know". To the question "Do you think the quizzes on Moodle have helped you to keep up with the lessons?" (Figure 1 right side) 22 students answer "Very", 82 "Little enough", 41 "For nothing" and 3 "I don't know".

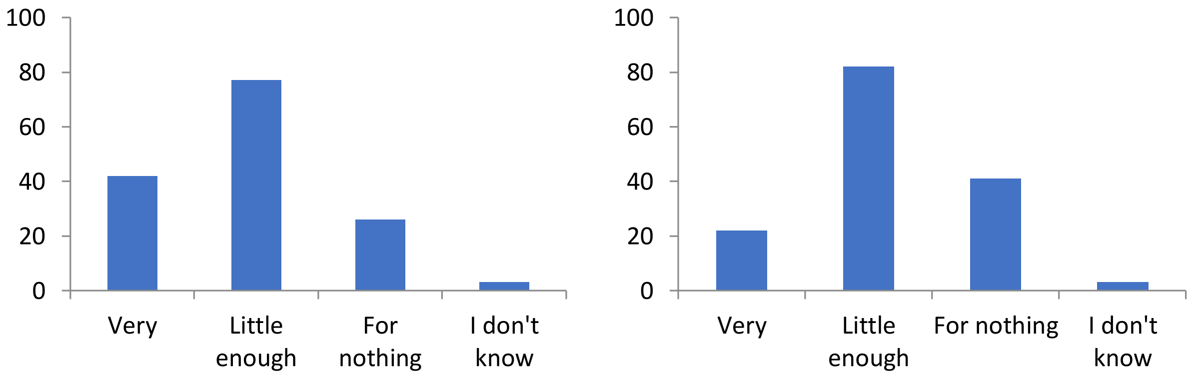

Figure 1: On the left Q1: Do you think the use of quizzes on Moodle has helped you to find gaps in your preparation? On the right Q2: Do you think the quizzes on Moodle have helped you to keep up with the lessons?

The answers to the question “How do you tackle quizzes on Moodle?” are summarized in Figure 2, left side. 49 students answer “Alone”; 72 “Alone, then I double check my answers with my colleagues' before submitting them”; 19 “With the colleagues”; 4 “Copy”; 3 “Other”. Finally, to the question “How did you deal with the doubts that emerged during the study?” (Figure 2 right side) 14 students answer “Asking professors”; 54 “Asking the colleagues”; 36 “Using books”, 42 “Consulting online material”.

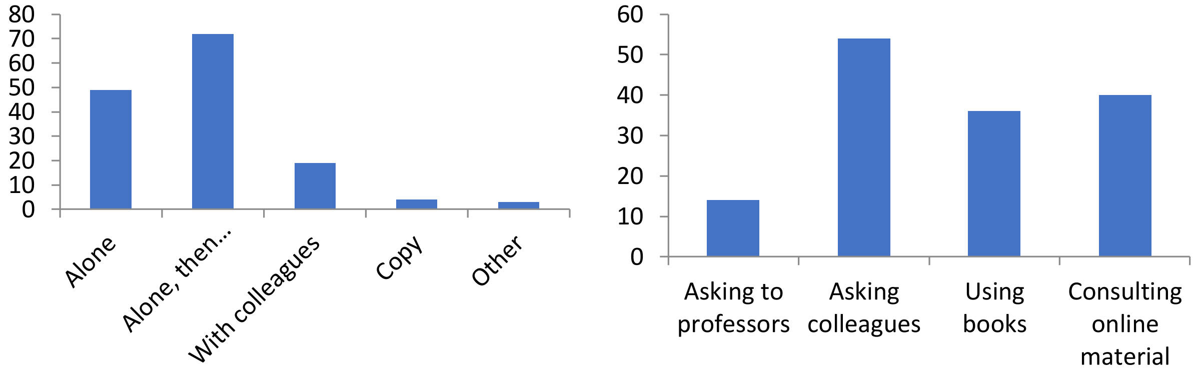

Figure 2: On the left Q3: How do you tackle quizzes on Moodle? On the right Q4: How did you deal with the doubts that emerged during the study?

The results of the survey show that the students appreciate the use of the quizzes: they consider them useful to highlight their gaps and to keep up with the study of the course. In addition, many students prefer to deal with colleagues for fear of making mistakes and lost so the possibility to participate to the midterm test. The comparison with the colleagues is also widely used by students to resolve their doubts, followed by consulting online material and then the use of books. Nonetheless asking to professors is the least chosen, there have been frequent requests for clarification especially at the end of the lessons principally about the subject of the lesson just taught.

CONCLUSIONS

As a preliminary experience regarding the use of Moodle quizzes we can say that they surely helped most of the students to keep up with the lectures. The fact that the students had to reach a certain grade with the quizzes to be able to take part in the mid-term tests, led them to collaborate even more among themselves. In light of this, psychometric analysis rightly presents coefficients that exceeds the expectations. We used the quizzes not to assess students, but to make them study more, to encourage them not to postpone their studies. The quizzes were designed to highlight the most important aspects of the various topics discussed in the lessons and to remark the crucial points where students usually get into trouble. Next year the quizzes will be repeated after a review based on the results of the psychometric analysis and the performance of the students to the exams.
The collaboration between students is a positive fact and it will be encouraged even more next year. For example, we are thinking to propose specific activities such as working group projects to exhibit to the whole class.
The results of the partial tests were encouraging, more than 50% (versus 30% of the previous year)

of those enrolled in the first one passed the exam but we cannot correlate this result only to the use of the quizzes. The participation to the mid-term test was higher in this academic year than in the previous one too.

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CONSTRAINED SKETCHING ON A GRID: A LENS FOR ONLINE ASSESSMENT OF DERIVATIVE SKETCHING

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In this study we aim to characterize the challenge related to sketching functions by constrained sketching on a grid. Our research question is as follows: What are the characteristics of sketching graphs by dragging points vertically in assessment e-tasks? Specifically, which functionality does this design support? Towards this goal we explore how students construct a sketch of f'(x) based on a given graphic representation of f(x), and vice versa*, how they construct a sketch of a function based on the graphic representation of its derivative. The analysis of 114 submissions of high-school students, support the formation of assessment tasks' design principles for drawing a sketch. The sketch that students construct makes it possible to characterize each submission with respect to the actions the students took at critical or non-critical points; to whether they attend to certain points separately or to domain and to conjecture about various concept images.*

Keywords: automatic, assessment, derivative, design, principles.

INTRODUCTION AND THEORETICAL FRAMEWORK

Multiple representations are often important components of rich tasks. The ability to identify and represent the same element numerically, graphically, and algebraically has been one of the goals of reform in calculus education (Berry & Nyman, 2003). Tasks designed within multiple representation environments have the potential to support the formative assessment of problem-solving processes, to catalyze ideas, and to provide indications of students' perceptions and concept images (as described by Tall & Vinner, 1981). Tracking and studying choices of representation made by students during problem solving are likely to inform us about the students' interests, preferences, and difficulties (Ainsworth 1999). Solving interactive tasks that involve multiple representations supports understanding by the teacher of the student's concept image because it invites students to consider mathematical concepts in relation to their properties (e.g., Even 1990). Interactive diagrams are a primary means of designing such problems (Naftaliev & Yerushalmy 2013).The concept of derivative is one of the main ones in school mathematics. It is an epistemologically and psychologically difficult to understand, and can benefit from work with an especially designed interactive multiple linked representations (MLR) learning environment. Construction e-tasks (Nagari-Haddif & Yerushalmy 2018; Yerushalmy, Nagari-Haddif, & Olsher, 2017) require students to construct examples that satisfy given conditions by the optional use of technological affordances, such as symbolic expressions and sketches. Using the Seeing the Entire Picture (STEP[1]) online assessment platform, in our design of technology-based rich assessment tasks, we provide means for generating examples in multiple representations, to support calculus problem-solving, and to mirror reasoning processes. STEP enables students to submit examples of mathematical objects constructed as interactive diagrams (based on Geogebra). One of our efforts focuses on the design of tasks that require freehand construction of the graph of a function in ways that facilitate automatic analysis of submissions. We found that a design that allows students to choose in which representation to submit the answer, either by freehand sketching or through a symbolic expression that is automatically graphed, offers a mathematically appropriate means of expression. Most often, such design encourages students to start by "sensing" the problem using freehand sketching, and provides informative indication of the mathematical knowledge of the student (Yerushalmy et al., 2017). Accordingly, calculus tasks in

STEP are designed to include several tools that provide different ways of expressing mathematical ideas. This design principle assumes that for many students, symbolic expressions are not the only choice of communication, that it is important to retain the natural communication of mathematical ideas through freehand drawing, and that such tasks increase the ability to make informed assessment decisions about students' work. Sketches, which are less accurate mathematical representations than formal drawings constructed using symbolic expressions, in other words, than a neat graph, are often part of problem solving and can support the analysis of problem-solving reasoning. Free-hand sketching provides students with great flexibility in answering questions (Yerushalmy et al. 2017), and poses challenges when teachers attempt to interpret the sketches as mathematical objects that are part of the expected solution to the task (tangency points, functions, asymptotes, etc.). In the current report we extend our view on sketching functions and focus on constrained sketching on a grid. When drawing a freehand sketch, students might ignore certain points for various reasons, and sketch the entire graph as a curve made up of infinite many points. In the current design, however, students drag a finite number of equally spaced, connected horizontally. In this respect, the design is more restrictive, but it requires students to pay attention to the accuracy of the points, and makes it possible to assess students' perceptions. Figure 1 shows an example of sketching a function by dragging points vertically. The connected red points are placed horizontally at the upper edge of the screen (Figure 1 (a)). To construct a sketch, it is necessary to drag the red points vertically (Figure 1 (b)). (One can imagine a series of beads constrained to move along parallel vertical paths on a Cartesian grid). It is possible to evaluate the locations of the dragged points using the coordinate system and the zoom in and out tools (the coordinates are marked numerically).

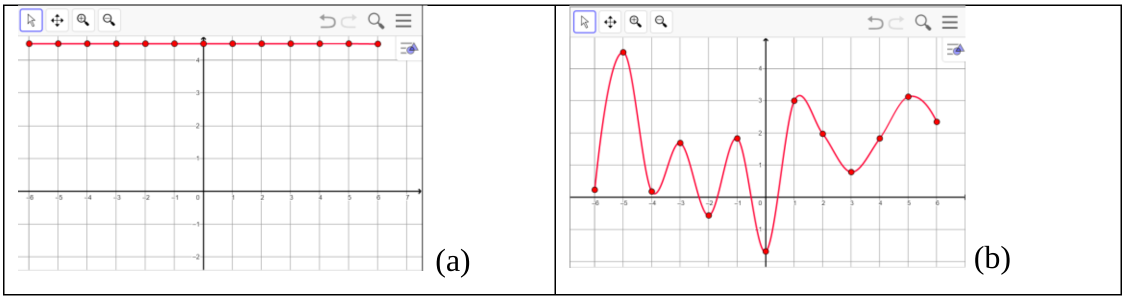


Figure 1. Example of sketching a function by dragging points vertically.

Asking students to construct a sketch by dragging vertically points should allow to learn about their intentions regarding the place of each point (up to a certain deviation).

CONTENT OF THE E-TASK

The relationship between a function and its derivative is a central theme of calculus. High-school students tend to assume that there are partial resemblances between a function and its derivative. These assumptions lead them to ascribe similar global features to both (such as increasing/ decreasing sign), generally focusing on either the function or the derivative rather than on the relation between them (Nemirovsky & Rubin 1992). One of the most useful and instructive applications of derivative is to aid in determining the maximum and minimum values of a function. Problems of optimization and can be formulated as problems of finding the maximum, minimum, and inflection points of a function. Focusing on such significant or critical points of the function can lead to ignoring other non-critical points, which may be a sign of a problem in understanding the derivative as a function that represents a collection of slopes of lines at various points, not necessarily critical ones. This example illustrates why we are interested in formulating design principles for drawing a sketch that **assigns the same weight to critical and non-critical points.** Traditionally, when analyzing function characteristics in calculus, we calculate several critical components: extremum points, intersection points with the axis, increasing and decreasing domains, inflection points, concave upward and downward domains. While sketching, these critical characteristics are preserved. The design of the task that requires creating sketch should allow analyzing and assessing these and other characteristics that students perceive to be important (including mistakes).

RESEARCH GOAL AND QUESTIONS

We aim to characterize the challenge related to constrained sketching on a grid[2]. Our research question is as follows: What are the characteristics of sketching graphs by dragging points vertically in assessment e-tasks? Specifically, which functionality does this design support? Towards this goal we explore how students construct a sketch of f'(x) based on a given graphic representation of f(x), and *vice versa*, how they construct a sketch of a function based on the graphic representation of its derivative. The study seeks to determine whether students follow the (approximately) correct value of critical and non-critical points and analyze other submission characteristics.

METHODOLOGY, RESEARCH TOOLS, AND DATA ANALYSIS

Design-based research (DBR), the methodology for studying the innovative principles of assessment, is characterized by an iterative cycle of design, implementation, analysis, and redesign. The present study is part of a larger research project on the principles of innovative assessment designs in an MLR environment. The data consisted of **several cycles** of students' submissions, each cycle examined a design pattern that was refined and re-examined in the next cycle. Using a DBR methodology, we conducted a study focusing on an activity called "The relationship between the graphs of a function and its derivative." We use this activity as a research tool, consists of three tasks. Data for this report consist of submissions by 114 high school Israeli students aged 16-17, who volunteered to anonymously solve the tasks. All students were enrolled in the most advanced high school calculus course. They all used the same curricular resources, but were taught by different teachers in different schools. They were all conversant with the basic graphing technology. Before the experiment, students participated in a preparatory session to familiarize themselves with the STEP environment. Each student submitted an answer for the first two tasks, and 109 out of the 114 students submitted an answer for the third task. Below we describe each task, its design considerations, and possible solutions. For each task we present findings and data analysis.

TASK 1: EXPERIMENTATION WITH THE MEASURING MECHANISM

Description and design rationale

Task 1 includes a dynamic diagram of function f(x) (Figure 2). The requirement of the task is to drag the red tangency point and place it so that the slope value of the tangent line is approximately 1.5, and to attach an appropriate screenshot. The two possible correct answers are shown in Figure 2.

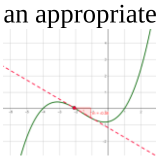
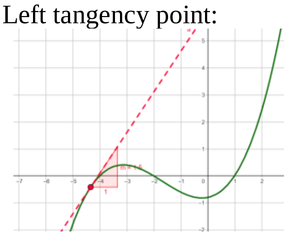
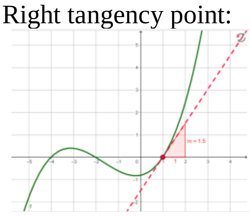
The task:	**Two possible correct answers:**	
The function f(x) is described in the dynamic diagram. Drag the red tangency point and place it so that the slope of the tangent line is approximately 1.5. Attach an appropriate screenshot. 	(a) Left tangency point: 	(b) Right tangency point: 

Figure 2: Task 1 and its possible correct submissions.

Adding grids and the numeric value of the tangent slope (measuring mechanism) to the task convey the message that the required answer needs accuracy. This is a preparatory task aimed mainly to familiarize students with the dynamic diagram that appears also in task 2 (see Nagari-Haddif 2017).

Findings and data analysis

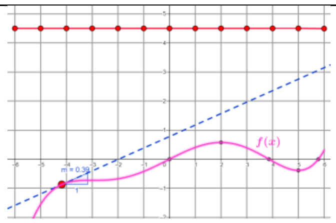
All the students submitted correct answers for task 1. Most students (74%) chose to submit a right tangency point (Figure 2 (b)), which may imply that students have a preference to drag points from left to right. The result raises questions about further research regarding students' general and personal tendency for a specific side of a graph. To answer this question, it is necessary to analyze additional similar tasks, following the process by which students reached their solutions, in addition to their final submissions. Tasks 2 and 3 required submitting a sketch drawn by dragging 13 equidistant points vertically in a closed domain (Figure 1). In task 2, students were asked to produce a sketch that represents the derivative of the given function in the presence of measuring mechanism (Figure 3), and in task 3, they were required sketch a function whose derivative was given in a dynamic diagram (no symbolic expression was provided in any of the three tasks) using approximate measurements (Figure 6).

TASK 2: CONSTRAINED SKETCHING IN THE PRESENCE OF MEASURING MECHANISM

Description and design rationale

In this task, the function f(x) is described in the dynamic diagram (Figure 3) in the domain $-6 \leq x \leq 6$. Students were asked to construct, as accurately as possible, the derivative function by dragging each of the 13 red points vertically to sketch the derivative of f(x). The task supports the requirement for accuracy by providing the numeric value of the tangent slope in each point (Figure 2), by adding the grids, and by marking the scale values in the coordinate system. Sketching the derivative requires dragging each point to the y value measured by the slope tool. The construction is discrete, and therefore it may encourage students to be accurate in the numeric value of each dragged point. The measuring mechanism may help students be more precise. Therefore, incorrect values of the locations of the points in the sketch suggest that students may have difficulty in relating the slope of the function to the derivative function. We placed 13 points in the given domain $-6 \leq x \leq 6$ at intervals of one unit apart: $x = -6, -5 \dots, 5, 6$. The purpose of this design is to help students calculate the slope between two adjacent points $a, a+1$ as the difference of values of y $(f(a+1) - f(a))$ rather than as the quotient $\frac{\Delta f}{\Delta x}$.

The function f(x) is described in the dynamic diagram (pink). Drag the red points so that the resulting red curve is the derivative of the pink function f(x). Try to be as accurate as possible. Attach an appropriate screenshot.



The correct answer:

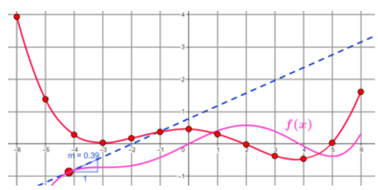
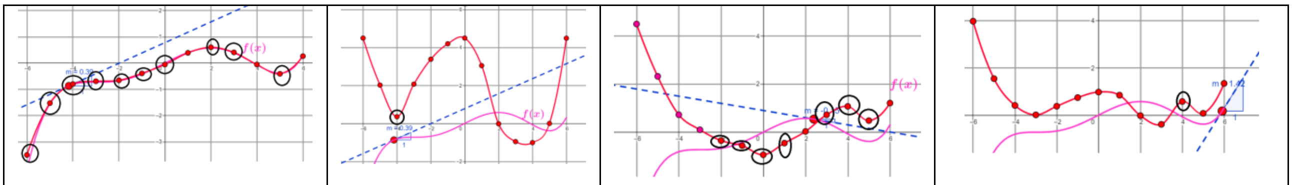


Figure 3. Task 2 and possible correct submissions.

By analyzing the sketches submitted by the students it is expected to characterize students' submissions. We were able to derive the following types of information from the submissions: (a) setting critical points to zero points only suggests that students do not make the connection between points with non-zero slope and the derivative; (b) sketches that represent incorrect values but with the correct sign (negative or positive) suggest that students connect between the increasing (decreasing) domains of the function and the positive (negative) domain of the derivative, but might ignore concavity upward and downward of the function domain; (c) sketches that follow correct increasing and decreasing domains but contain mistakes in values suggest that students make the connection between the concavity upward and downward of the function domain, and the increasing and decreasing domains of the derivative. These characteristics can be analyzed automatically.

Findings and data analysis

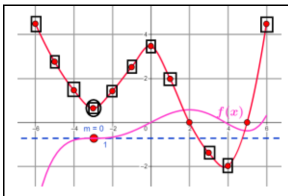
Findings and data analysis: The vast majority of the submissions have the correct signs (see Figure 4 (b), (d)). This is not surprising, because the relationship between the function and its derivative is studied in Israeli schools, with especial emphasis on the relationship between increasing (decreasing) domains of f(x) and positive (or negative) domains of f'(x). Most students knew that a critical point of f(x) is a zero point of f'(x): 93 (81.6%) students submitted a zero point at x=-3, which is an inflection point of f(x) with a zero slope (see Figure 5 (BON53)); 107 (93.9%) students submitted a zero point at x=2, which is a maximum point of f(x) (see Figure 5 (BON53, DALII20)); 95 (83.3%) students submitted a zero point at x=5, which is a minimum point of f(x) (see Figure 5 (BON53, DALII20)). Most of the students identified the correct sign, and although they could deviate by 0.3 from the exact value of each non-critical point, fewer students by far were able to find its correct value. For example, students BON53, DALII20 (Figure 5) submitted sketches with correct sign of 10 non-critical points (out of 10) and incorrect values of f'(x).



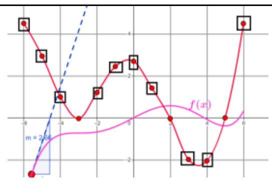
(a) IRIN5 (b) BON29 (c) GH8 (d) GH30

Figure 4. Task 2: Examples of submissions with incorrect signs of the y value (circled in black).

From the perspective of a single submission, most students (84 students, 74%) submitted 9-13 correct y values per submission. The students DALII20, BON53 (Figure 5) submitted correct signs for most dragged points, but the values of these points were not precise at all. Their explanations reinforce the impression that the mistakes are not incidental. Students appear to have recalled a "recipe" regarding to the connection between the sign of the derivative and the increasing or decreasing domain. When focusing on each point separately, however, without referring to other points, it is possible to miss other important characteristics of the solution, such as the increasing and decreasing domains of the derivative function (which are affected by the concaving upward and downward domains of the original function, and vice versa). Students’ mistakes in the value or sign of any point may affect the correctness of the increasing and decreasing domains of the derivative function. Student GH8 (Figure 4) appears to have ignored the increasing and decreasing domains of the functions. Student IRIN5 (Figure 4 (a)) constructed a sketch that follows the original graph of f(x) instead of constructing f'(x). This result is consistent with the findings of Nemirovsky and Rubin (1992), according to which students tend to assume resemblances between the behaviour or appearance of a function and its derivative. In submissions with only one or two incorrect signs, in some cases the shape of the constructed derivative function is similar to the correct shape (for example, the sketch of Figure 4 (b)), but in other cases it is completely different (Figure 4 (d),). Generally, students constructed a sketch that decreases in the -6<x<-3 domain (94.7%), increases in the -3<x<0 (88.6%) domain, decreases in the 0<x<4 domain (83.3%), and increases in the 4<x<6 domain (86.8%). Other students submitted decreasing domains instead of increasing ones, increasing domains instead of decreasing ones, combined domains instead of decreasing/increasing ones (GH30 at 0<x<4, 4<x<6, Figure 4).



DALII20: 11 dragged points with incorrect values, 1 dragged point with incorrect sign.



BON53: 10 dragged points with incorrect values, 0 dragged points with incorrect sign

Figure 5. Task 2: Examples of submissions. Dragged points with an incorrect sign for the y value are circled, and those with incorrect y values are marked by a square

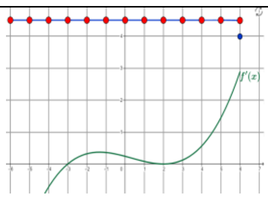
In sum, most students submitted a fairly accurate graph, with almost all points dragged to the correct place of the coordinate system. This confirms that in some cases we can expect accuracy, and students can sketch relatively accurately. Some students, however, do not appear to have been concerned with the precise value of the derivative, but only with its sign. They did not seem to have perceived the tangent as a construct that can help them measure the location of the points and is useful for defining the derivative function in any given point on the graph (where it was provided). These students referred only to the critical points, and in non-critical points, only to the sign of the derivative, although their slope tool allowed them to read the slope at any point, and although they had available a coordinate system with marked ticks that provided accuracy. This strategy resulted in different shapes of the derivative sketch, including incorrectly increasing or decreasing the domains of the derivative function (for example Figure 4 (b), (d)). Although it is possible to construct the sketch as a collection of precise points, these points should not be analyzed only separately. To assess additional characteristics of students' submissions, such as increasing or decreasing domains, the points should be analyzed in relation to all the other points (because if one point is out of place, it changes the shape of the entire graph).

TASK 3: CONSTRAINED SKETCHING USING APPROXIMATE MEASUREMENTS

In this task, students were asked to drag the red points vertically to sketch the function f(x) that passes through the blue point (6,4), and whose derivative f'(x) is described as the green graph in the dynamic diagram (Figure 6). They are asked to be accurate as much as possible. We support his requirement for accuracy by adding the grids and by marking the scale values in the coordinate system. Because the expression of f' (x) was hidden from the students' eyes, we cannot expect the same accuracy as in task 2. Indeed, we expected students to construct the function f(x) based on the values of the derivative function at each of the 13 points. Therefore, the approximate derivative should be based on the "neighbors" of each point. To avoid analyzing errors resulting from technical inaccuracies, we defined a more lenient requirements for the endpoints: (a) for x=6, the slope is bigger than 2: f(6)-f(5)>2; (b) for x=-6, the slope is smaller than -2.5: f(-5)-f(-6)<-2.5. By analyzing the submitted sketches, we expected to distinguish between students who paid attention to (a) domains (the slopes of each endpoints); (b) critical points (x=0,2,-3); (c) domains for which the function is increasing and decreasing; and (d) concaving upward and downward.

The task:

With reference to function f'(x) (in green), drag the red points so that the red curve produces function f(x), which passes through the blue point (6,4). Attach an appropriate screenshot.



Possible correct answer

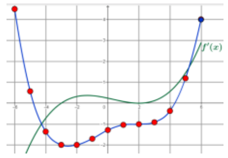


Figure 6. Task 3 and its possible correct answer.

Findings and data analysis: A hundred and nine out of the 114 participants submitted an answer for this task. Some students mistakenly focused on the zero points of the derivative and interpreted them as extremum points, apparently arbitrarily (Figure 7). An unusual and surprising finding, which raises some concern, was that about 10 students (9%) interpreted the intersection point of the derivative with the y axis as an extremum point (Figure 7 (c), (d)). This finding is particularly worrisome because there is no connection between the point of intersection of the derivative with the y-axis and the function, and it may indicate a real difficulty students have understanding the basic meaning of a slope of a function and its relation to the first derivative. Some students submitted sketches with a W shape, which had at least two extrema with similar y values (Figure 7 (c)). This may explain some of the submissions in which an extremum point occurs at x=0. All the W-shaped sketches had an extremum point at x=0.

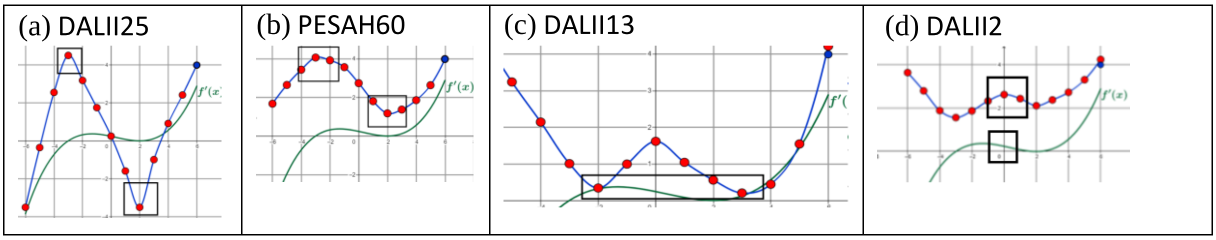


Figure 7. Sample of submissions.

In sum, in this task, dragging the points on a coordinate system with marked ticks that provide accuracy, enabled students to construct a relatively accurate function for the given derivative (albeit less so than in task 2). We analyzed a large number of attributes. Teachers, however, may choose to ignore some of the attributes, such as domains concaving upward and downward, and focus on others, such as decreasing and increasing domains. The task illustrates the problematic concept of "correctness" and the importance of analyzing the attributes of the submissions, which provide a rich picture of the student's knowledge and have the potential to promote learning and teaching.

DISCUSSION

This design of tasks requiring students to sketch functions using a vertically draggable points supports the following functionality: (1) Helps students focus on the required place of each dragged point, to make possible the accurate assessment of students' perceptions. Asking to drag a finite number of points on a coordinate system with marked ticks, together with other optional measurements, helps students drag the points accurately, up to a certain standard deviation. The required accuracy depends on pedagogical and technical factors, such as the content and the given measuring mechanisms. (2) The sketch that students construct this way makes it possible to characterize each submission with respect to the actions the students took at critical or non-critical points, as they considered certain points separately or a collection of points (a domain). The design of this type of sketching tool makes it possible to characterize the submissions along various concept images and to conjecture whether they were concerned with the value of each dragged point, or with the value of only the critical points. Their choice of value for each dragged point may suggest which points they perceive to be critical (if, for example, students submit only a few points with precise values, it may indicate that they perceive these points to be more important or critical than the others). It is recommended that task designers make draggable the important points, which students may perceive as critical, such as the intersection point with the y axis, as demonstrated in tasks 2 and 3. In addition to assessing the correctness of the solution and the accuracy of each point, it is possible to evaluate other attributes of the solution, such as the domains in which the function is increasing and decreasing, and the ones in which it is concaving upward and downward. It is also possible to identify misconceptions described in the literature. When comparing the shape of the sketches with that of the given function by assessing the difference between the y value of each dragged point and the corresponding y value of the given function, phenomena that are familiar from previous research (of Nemirovsky and Rubin, 1992, (Figure 4 (a)) and new surprising phenomena (Figure 7 (c), (d)) were revealed. When dragging two non-adjacent points, students may accidently create unintended extremum points between them, which might change unintentionally other characteristics of the functions such as increasing and decreasing domains. Finally, trying to generalize, this sketching tool, which was demonstrated here being lens to assess students understanding of the relationship between a function and its derivative, is suitable for other tasks that have a single or a few possible continuous solutions; for tasks where the accuracy of the solution is important for assessing the students' knowledge; for constructing a function in relation to another function, for example, constructing f(ax), f(x+a), f(x)+a for a given f(x); and for tasks that require to create a sketch of a continuous function that meets given requirements. Analyzing task characteristics, as demonstrated above, affects teaching practices by providing feedback to students and teachers, but this is beyond the scope of this article.

NOTES

1. Seeing the Entire Picture (STEP) is a formative assessment platform developed at the Mathematics Education Research and Innovation Center, at the University of Haifa. Details about this platform are available at www.visustep.com.

2. This study is part of a doctoral research carried at the MERI center, the University of Haifa and supervised by Prof. Michal Yerushalmy.

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TEACHER'S ATTENTION TO CHARACTERISTICS OF PARABOLA SKETCHES: DIFFERENCES BETWEEN USE OF MANUAL AND AUTOMATED ANALYSIS

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This study compares the different characteristics of student answers addressed during classroom discussion by mathematics school teachers in various settings. The study examines 3 9th grade mathematics teachers from the same school in Israel, all of whom taught the topic of quadratic functions. We examined and analyzed the teachers' choices and sequencing of the different characteristics of parabola sketches in a classroom discussion about student answers. The teachers analyzed student answers either manually or using the Seeing the Entire Picture (STEP) automatic formative assessment platform. Findings suggest a difference between the characteristics that teachers addressed in the classroom discussions in the two conditions. In the manual setting, teachers focused on incorrect features of the example, whereas in the automatic setting they focused on characteristics that emphasize different mathematical dimensions.

Keywords: *formative assessment, quadratic function, automatic assessment, classroom discussions, function sketches*

INTRODUCTION

Conducting mathematics lessons based on the ideas of students and the analysis of their answers enables a meaningful dialog. To achieve this, teachers cannot settle for merely watching the students or reacting to correct answers (Black & William, 2014). Instead, they are required to provide feedback in the form of a class discussion that promotes formative assessment interactions. Under these conditions, teachers choose examples for the discussion without prior planning, or focus on the student's mistakes (Stacey, 2009). Many online platforms address this challenge by providing teachers with an immediate status of students' submissions and with further analysis of predefined characteristics of student answers. In this study, we examine the potential of a platform of this type, the Seeing the Entire Picture(STEP) platform (Olsher, Yerushalmy, & Chazan, 2016), when teaching the topic of quadratic functions. We examine the characteristics of student answers addressed by the teachers in the class discussion, and compare the characteristics addressed when the teachers analyzed the answers manually with the characteristics addressed when the answers were automatically analyzed and the teachers were able to access the results.

THEORETICAL BACKGROUND

In the early stages of digital education, a great deal of attention was paid to technological attributes. By contrast, these days, most of the approaches to using technology adopt an integrated perspective, in which aspects such as social interactions, educational strategies, and the teachers' role are being assessed (Bottino, 2004). Many technological platforms for formative and summative assessment have been developed, and they are influenced by these approaches. Examples include TI-Nspire, ASSISTments, and DESMOS. These digital platforms inspired a shift in the teacher's role. In computer-mediated teaching, where student answers are automatically assessed (Koedinger, McLaughlin, & Heffernan,2010), teachers are released from the time-consuming task of evaluating student submissions, and can concentrate on their main function, which is guiding the class discussions in accordance with the students' needs (Yerushalmy &Elikan, 2010).

A key attribute of digital learning platforms, apart from enhancing classroom interaction, is the nature of the tasks. Mathematicians stress the importance of examples and of tasks that encourage example creation (Watson & Mason, 2002). The solutions to example-eliciting tasks (EET) can reflect the student's understanding, and suggest difficulties or misunderstandings of mathematical concepts (Zaskis & Leikin, 2007).

Quadratic functions are a central topic in school mathematics. They provide ample opportunities for EETs, and reveal many student difficulties (Dreyfus & Eisenberg, 1984). By providing students with rich EETs in a digital environment that supports automatic analysis of student answers and their classification according to different mathematical attributes (Olsher et al., 2016), we can reduce the teachers’ workload, helping them to more thoroughly assess the characteristics of their students' work.

METHODOLOGY

This study examines the pedagogical potential inherent in teaching the topic of quadratic functions using STEP, as well as its influences on class discussions and changes in their contents. The two research questions are: What are the characteristics of 9th grade student submissions in response to EETs on the topic of the intersection between a parabola and a point or a line, on which teachers chose to base class discussions? Are there any differences between the characteristics chosen by the teachers when they assess student answers manually and automatically?

Sample

Three certified mathematics teachers from the same Israeli school, teaching 9th grade, participated in this study. Two of the teachers hold a bachelor's degree in mathematics and are currently studying for their master's degree: one studies mathematics, the other mathematics education. Both were in their second year of teaching. The third teacher holds a Bachelor's degree in statistics, and this was her fourth year of teaching.

Research tools

The research tools included an online activity delivered on the STEP platform, which included two tasks, and the students’ answers for these tasks.

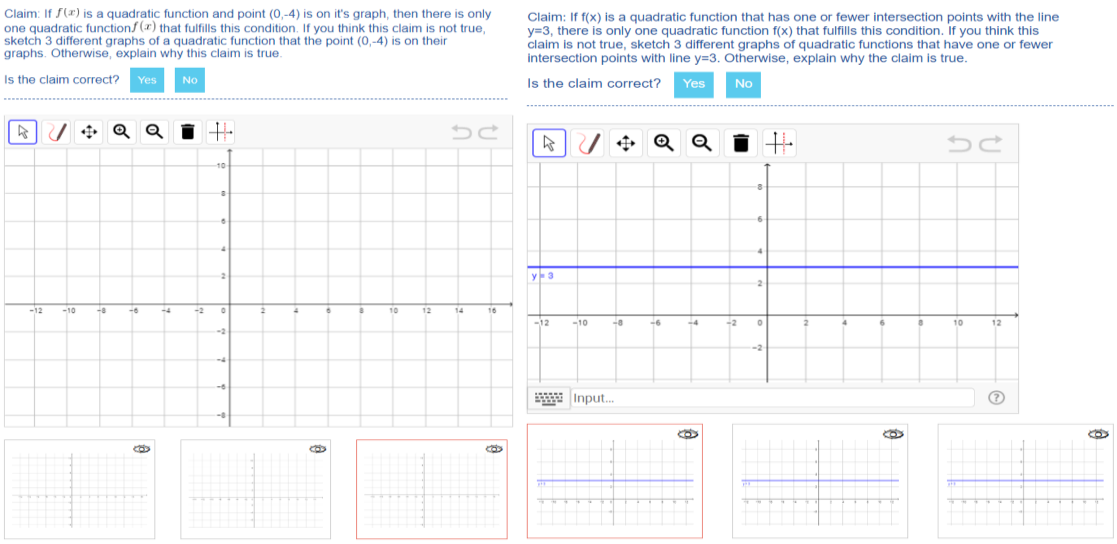

Figure 1. Tasks 1 and 2 as they appear on the student interface of the STEP platform

The students completed the tasks, which require sketching different parabolas with certain characteristics. For example, in the second task (Figure 1), the students were required to assess whether there is more than one quadratic function that intersects the line y=3, and to sketch 3 different examples for such functions, if any exist. The platform collected and automatically analyzed the students' answers (Figure 2), based on predefined characteristics. The characteristics may be categorized into three groups, as shown in Tables 1 and 2 for the two tasks.

Incorrect example characteristics (1I)	Correct example characteristics (1C)	Cross-example characteristics (1E)
i sketch does not describe a function	i sketch has min at (0,-4)	i all 3 sketches have minimum points
ii sketch has a vertex at (-4,0) || (4,0) || (0,4)	ii sketch has max at (0,-4)	ii all 3 sketches have maximum points
iii sketch does not describe a quadratic function	iii sketch vertex is not (0,-4)	iii each of the 3 sketches has a different (1C) char.
iv sketch has a V-shape		iv sketches have different parameter (where y=ax^2)
v asymmetric sketch		
vi partial sketch		

Table 1. Student answer's characteristics for task 1

Incorrect example characteristics (2I)	Correct example characteristics (2C)	Cross-example characteristics (2E)
i sketch intersects y=3 at 2 different points	i sketch has min at y=3	i none of the sketches have a vertex at y=3
ii sketch does not describe a function	ii sketch has max at y=3	ii each sketch has a different number of intersection points with y=3
iii sketch does not describe quadratic function	iii sketch has max below y=3	iii all sketches have a vertex at y=3
iv partial sketch	iv sketch has min above y=3	
	v sketch has vertex at y=3 and on the y axis	

Table 2. Student answer's characteristics for task 2

The first group includes various characteristics of incorrect examples: for example, sketches that do not represent functions (Figure 2, column 2Iii), or do not represent quadratic functions (Figure 2, column 2Iiii). The second group includes various characteristics of correct examples, such as sketches with a minimum at y=3 (Figure 2, column 2Ci), or sketches with a vertex at y=3 and on the y axis (Figure 2, column 2Cv). The third group includes characteristics that apply to all three submitted examples, for example all of the submitted sketches have a minimum point.

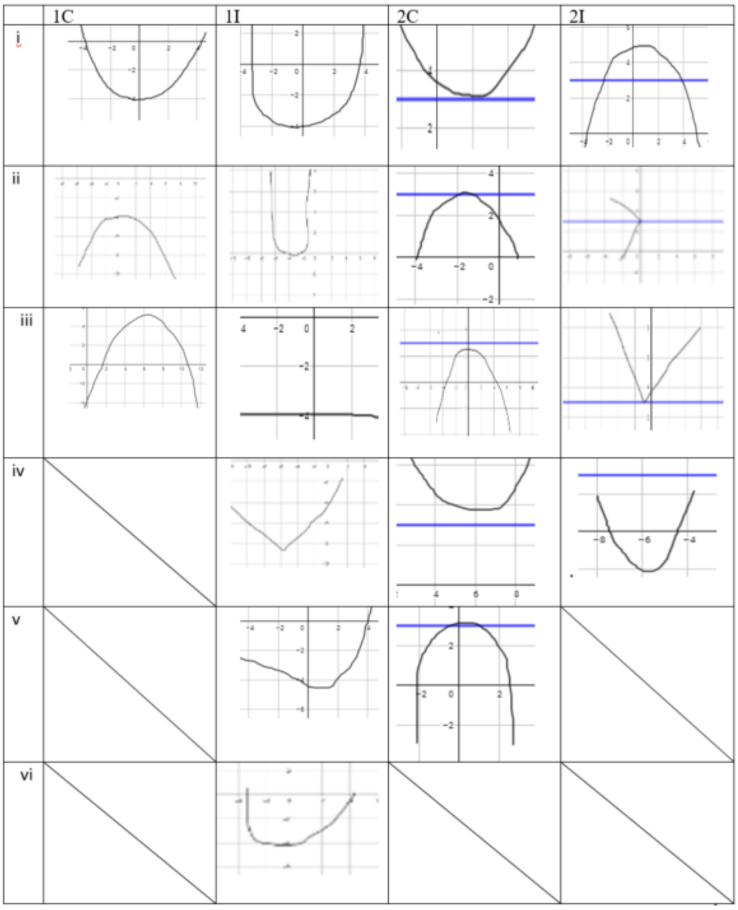

Figure 2. Examples of student answers for tasks 1 and 2, grouped by characteristics

The automatic analysis enables the teacher to filter the answers based on the characteristics and present only the answers that contain a given set of one or more characteristics to address during classroom discussion.

The student answers were presented to the teachers, who were asked to categorize them during an interview. After the teachers categorized the examples, they were shown the predefined categories within STEP. Next, the teachers chose the order in which they would address the characteristics of the answers in their classroom discussions.

Following the interview, teachers performed the activity in their classrooms, as part of their routine teaching sequence, and conducted a discussion after the students completed the activity. Then the teachers were interviewed once again to clarify various decisions and actions that were noted during the lesson.

Data sources and analysis

Data sources include student answers to the tasks, two interviews with each of the teachers (before and after the class discussion following the students' submissions), and classroom observations conducted by the second author.

To describe our findings, we chose an analytical framework that regards professional teaching as incorporating unique knowledge. Therefore, we use the teaching episodes reported here as manifestations of various types of pedagogic content knowledge in mathematics, specifically common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and student (KCS), and knowledge of content and teaching (KCT) (Ball, Thames, & Phelps, 2008).

In our specific setting, the different types of knowledge are assessed using different methods. We asked teachers to solve the tasks assigned to the students, to assess their mathematical knowledge and skills in the topic at hand, which is described by Ball et al. (2008) as CCK. Next, we asked the teachers to estimate what types of answers their students will submit, which reflects their KCS (*ibid.*). To assess SCK (*ibid.*), we presented the teachers with student answers for each task, asking them to categorize and interpret the answers. To assess teachers KCT (*ibid.*), we asked them to choose various characteristics, to address in their classroom discussion to achieve efficient teaching, and later observed the actual discussions.

FINDINGS

The findings show that there was a difference between the categories of the characteristics teachers chose to address during the classroom discussion on quadratic function sketches, and in the order in which these characteristics were discussed in the two different settings: when teachers analyzed the answers themselves and when their students' answers were automatically analyzed.

Below we describe the four types of teachers' pedagogical content knowledge (CCK, KCS, SCK, and KCT), as they were manifested at the different stages of the study. We begin by briefly discussing the teachers' CCK on the topic at hand when solving the tasks. Next, we describe and illustrate different aspects of KCS, as manifested in the teachers' expectations from student answers. We proceed to the various characteristics the teachers identified in the students' answers and compare them with the teachers' expectations, to shed light on the teachers' SCK. Finally, we report on the choices and sequencing of the various characteristics in the classroom discussion, and compare the manual and automatic analyses, to demonstrate the teachers' KCT.

CCK

None of the three teachers had difficulty in solving the tasks of the activity. Teachers demonstrated CCK that is appropriate for the topic of quadratic functions, specifically sketching quadratic functions that meet the given conditions.

KCS

Expected answers for the first task included all three correct example characteristics for the task (1Ci-iii, Table 1). The teachers expected three out of the total of 13 research-based predefined characteristics for task 1 (Table 1), all from the same category (correct example characteristics).

Expected answers for the second task included all five correct example characteristics for the task (2Ci-v, Table 2). In contrast to the first task, the teachers also expected answers with incorrect example characteristics (2Ii, 2Iiv, Table 2). All the teachers expressed concern about students not fully understanding the term "at most," which is implied by the description of "one or fewer." In general, the teacher's expectations were similar with respect to correct example characteristics, whereas in their expectations of incorrect example characteristics, one of the teachers expected only characteristic 2Ii, and the other two teachers also expected characteristic 2Iiv. In conclusion, the teachers expected 6 out of a total of 12 research-based predefined characteristics for task 2 (Table 2), from two different categories (correct and incorrect example characteristics).

SCK

Analysis of actual student answers (as opposed to predicting the students' answers) led teachers to notice the expected characteristics in both tasks. Teachers discovered and categorized other characteristics that they did not initially expect.

In their analysis of student answers to the first task, teachers also noticed five incorrect example characteristics out of six predefined characteristics, such as sketches that do not represent a quadratic function (1Iiii, Table 1). They did not notice the incorrect example characteristic of an asymmetric sketch. Teachers also noticed two out of the four cross-example characteristics, such as the three sketches submitted with different distances from the symmetry line (1Eiv, Table 1). Each of the three teachers discovered additional characteristics (four, five, and six) to what they had expected before being exposed to the student answers.

In their analysis of student answers to the second task, teachers also noticed all 12 predefined characteristics, from all three categories. The teachers categorized a new characteristic in the form of the shape of the graph, and identified sketches of quadratic functions in the shape of the letter V. Similar to the analysis of answers to the first task, the teachers also addressed cross-example characteristics. Two of the three teachers discovered additional characteristics to what they had expected before seeing the student answers (seven and five additional).

KCT

When asked to sequence the characteristics that they would address in a classroom discussion after manually analyzing the students' submission for task 1, two teachers initially decided to attend incorrect example characteristics, seeking to address sketches that did not represent functions at all, or that did not represent quadratic functions. The teachers considered these mistakes to be severe because they indicated lack of understanding of the concept of functions in general, and of quadratic functions in particular, therefore the teachers intended to place these at the top of the list of topics to be discussed in class. Table 3 shows the first three characteristics that were chosen by the teachers in each of the settings for task 1. In contrast to her peers, the third teacher decided to initially address the correct example characteristic of having a vertex at the requested point, because she expected these to be the most common sketches, and intended to move on later to incorrect example characteristics from which her peers started (Table 3).

	Manual analysis			Automatic analysis		
	Teacher 1	Teacher 2	Teacher 3	Teacher 1	Teacher 2	Teacher 3
1st characteristic	1Ii	1Ii	1Ci	1Iiii	1Iiii	1Iiii
2nd characteristic	1Iiii	1Iiii	1Ii	1Ci	1Ciii	1Ciii
3rd characteristic	1Iii	1Ci	1Iiii	1Cii	1Ci	1Ci

Table 3. Sequencing of characteristics for discussion by teachers for task 1

When asked to sequence the various characteristics that they would address in a classroom discussion after manually analyzing the students' work on task 2, similarly to task 1, two of the teachers (1 and 3) chose to first discuss incorrect example characteristics. They wanted to begin by discussing sketches that did not represent quadratic functions, then quadratic functions that had two intersection points with line y=3, similarly to task 1, because in their opinion these mistakes indicate lack of understanding of the concept. The two teachers also stated that if they discussed the two tasks in the same lesson, they would skip the characteristic of sketches not representing functions in general. By contrast, teacher 2 chose to discuss correct example characteristics that are related to the mathematical content of the task. The teacher said:

> In my opinion, there will not be many examples of sketches above or under the line owing to the fact that students do not fully understand the concept. So it is important to start with this characteristic, and then proceed to answers that have a vertex on the line, which will be the common student answers.

Table 4 shows the first three characteristics that were chosen by the teachers in each of the settings for task 2.

	Manual analysis			Automatic analysis		
	Teacher 1	Teacher 2	Teacher 3	Teacher 1	Teacher 2	Teacher 3
1st characteristic	2Iiii	2Civ	2Iii	2Civ	2Cii	2Ii
2ndcharacteristic	2Ii	2Ciii	2Iiii	2Ciii	2Ciii	2Ci
3rdcharacteristic	2Ci	2Ci	2Ii	2Cv	2Civ	2Cii

Table 4. Sequencing of characteristics for discussion by teachers for task 2

When teachers used the automatic analysis by the STEP platform in their classrooms, the sequencing changed. They chose to discuss less incorrect example characteristics than they did in the case of manual analysis, and focused more on correct examples characteristics, as shown in Table 3 for task 1 and in Table 4 for task 2. All the teachers noted this difference, as shown in the description of task 1 by teacher 1:

> There was nothing critical in the student answers with sketches that are not functions, but there was more of a problem with functions that are not quadratic, which is why this characteristic [sketch does not describe a quadratic function] turned out to be less important than others.

The teachers started with what they found to be a critical mistake of the students, then moved on to sketches of quadratic functions that intersect the requested point, as shown in Table 3. Teachers 2 and 3 chose to start with functions that intersect the requested point but not in their vertex, then went on to functions where the requested point was in the vertex. Teacher 2 explained:

> I decided to address a characteristic that was less present in the students' work. I noticed that while the students were working, most of them thought that the point should be a vertex… This is why it was interesting to expose this characteristic to students who did not think of it.

Teacher 1 chose to begin with sketches that had a vertex at the requested point, although this characteristic was not one of her top three when analyzing the students' work manually.

In task 2, only teacher 3 discussed one incorrect example characteristic, in contrast to 3 examples she chose when analyzing the students' work manually. She said:

> I noticed that there were no submissions under or above the line… A lot of sketches intersecting the line in two points… This is why I decided to begin with this critical and common mistake revealed by the automatic analysis.

By contrast, teacher 1 did not discuss any incorrect example characteristics, as opposed to two such characteristics when analyzing the students' work manually. She explained her choice:

> When the students worked on the task, I noticed that most of them did not sketch a function that is above or under the line [without touching it], which is why I started with this characteristic. I checked all the characteristics and saw that there was only one answer with a sketch above the line… Then I asked the students: "What do you think? Is this sketch correct or incorrect? And a discussion arose around the meaning of "at most."

Teacher 2 changed only the sequence between the same characteristics she chose when manually analyzing the students' work, because of the commonality of the characteristics in the student answers.

DISCUSSION

The findings of this study show a difference between the characteristics teachers address when conducting a discussion about student answers after having manually analyzed their work, and the characteristics they address when student answers are automatically analyzed. The literature suggests that teachers select examples for discussion in an unsystematic way or cling on to student mistakes (Stacey et al., 2009). This claim finds support in our findings when teachers manually analyzed student answers. But when teachers had access to automatic analysis of student submissions, they changed their sequencing of characteristics. Teachers moved from characteristics that focus on incorrect features of the examples, to those that emphasize different mathematical dimensions of the topic being taught, in this case, quadratic functions.

This study introduces the novel notion of topic-centered learning analytics: mathematical characteristics of student answers that are not limited to student mistakes. The findings suggest that teachers use these insights in classroom discussion when they are available, expanding the range of student work-based discussions in the mathematics classroom.

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AUTOMATED FEEDBACK AT TASK LEVEL: ERROR ANALYSIS OR WORKED OUT EXAMPLES – WHICH TYPE IS MORE EFFECTIVE?

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This paper reports on a small-scale quantitative study conducted in a middle school in Germany that compared the effects of two types of feedback on reactivating procedural skills with fractions. Tasks and feedback were implemented in a STACK-based digital learning environment that allowed randomization of numerical and graphical elements of a task as well as automated analysis of student responses to each of the numerical or graphical variations of each task. Due to a small data basis, observations are statistically not verified, but nevertheless point to unexpected results: Especially low achievers seem to benefit more from the error analysis type feedback than from feedback that provided fully worked out solutions. If true, this result suggests that for reactivating and practising skills, error-based feedback is more effective than worked out examples.

Keywords: Automated Feedback, Computer Aided Assessment, Error Analysis, Worked Out Examples

TYPES OF FEEDBACK AT THE TASK LEVEL

"Feedback is information provided by an agent (e.g., teacher, peer, book, parent, self, experience)." (Hattie & Timperley, 2007). In the view of a learner, feedback is good feedback when he or she perceives it as "advice for action" (Ras, Whitelock and Kalz, 2016). One may expect a learner to make even better use of the feedback when it is adapted to individual needs and expectations. Following Ras et al. (2016), the quality of adaptive feedback depends on how the feedback content adjusts to the level of cognition and the personality traits of the learner.

Adaptation to cognitive features and personality traits

Whether feedback is supportive depends on the cognitive disposition of the learner. According to Johnson and Priest (2005), merely communication that an answer is correct or not could be sufficient to prompt the learner to revise and correct his answer, if he or she is an expert. Also, drawing the attention to obvious mistakes in calculation and reasoning can encourage a focussed revision of the solution when the learner has already some relevant knowledge in the area (see Ras et al., 2016). Novices, on the other hand, need feedback of an explanatory and supportive kind, such as scaffolding, which offers specific tips for correcting the presented solutions (Rittle-Johnson and Koedinger 2005). In the case of obvious ignorance of the novice, worked out examples are also conceivable as a basis for further attempts to solve the problem. In terms of performance disposition as a persistent personality trait, Shute (2008) gives similar suggestions for conceptualizing feedback, with timing playing a central role here. Information for low achievers should be made promptly, those for high achievers should be delayed to allow time for revision.

Error analysis vs. worked out examples

Recent research has shown that the use of worked out examples leads to comparably high learning gains (cf. Renkl, 2002). This is explained from the view of cognitive load theory that students can focus on what is important by reducing the need to occupy the learner's mind with performing single computational steps of the procedure involved (Sweller 1994, Scherrmann, 2016). Worked out examples are suitable especially at the beginning of skill acquisition (Kirschner, Sweller & Clark, 2006). However, while novices profit from worked out examples, this might not hold for experts since the solving procedure shown in the worked out example could interfere with the students' own strategies (Renk & Atkinson 2003).

Making errors is generally regarded as an essential part in the learning process. “An 'error' is a fact or process deviating from a norm, which makes it possible in the first place to discern the correct norm-related fact as an opposite to the erroneous fact or process." (Oser and Hascher, 1996). Thus it seems surprising that in the past there has not been much research on the use of error analysis in feedback (Mory, 2004). With growing availability of computer-based assessment systems (e.g. STACK, Moebius, or Viris) there seems to be a growing interest in research on effects of digital feedback based on a detailed automated analysis of a student’s response. In fact, Livne, Livne and Wight (2007) have shown that while both humans and computers were very good at scoring performance, computers were better in identifying error patterns. “Adaptive feedback information can easily be facilitated within a computer-based instruction environment, where the computer can record and analyze the types of errors being made and give appropriate feedback based upon error types" (Mory, 2004).

RESEARCH INTEREST AND QUESTIONS

The goal of this study is to compare two types of feedback “error analysis” and “worked out examples” with respect to effects on students reactivating and practising their skills in performing simple mathematical operations. For this purpose, the study aims at providing answers to the following four questions:

Q1: Is there a difference between error analysis type and worked out example type feedback with respect to acceptance?

Q2: Is there a difference between error analysis type and worked out example type feedback with respect to performance?

Q3: How does – with respect to each error analysis type and worked out example type feedback – the achievement level correlate with performance?

Our implementation of the error analysis type feedback contains a fully worked out example, too, that becomes accessible only after a delay of 60 sec., and only after the student explicitly decides to view it. The following question seeks to find out whether some students provided with error analysis feedback make extensive use of the worked out examples.

Q4. How does – with respect to the error analysis type feedback – the use of the delayed worked-out example correlate with performance?

THE STUDY

Tasks and feedback were part of the digital learning environment platform of the Heidelberger MatheBrücke (mathebruecke.pinkernell.online), which is Debian based Moodle server with the plugin STACK, a computer algebra aided assessment system (Sangwin, 2013). It allows randomizing mathematical questions as well as analysing student response as to predefined error patterns, which then is being used for error related feedback. Hence, STACK seems a suitable digital learning environment for reactivating and practising mathematical skills that students had an introduction to before.

Tasks

The tasks were for practising simple mathematical operations with fractions. They contain reducing and expanding fractions, adding, subtracting, multiplying and dividing fractions, and switching between graphical and numerical representations of fractions. Each task is randomized, i.e. when the student reloads the task it comes with different numerals or graphics.

Feedback types

Each task also comes with an automated feedback routine that is based on an automatic mathematical analysis of the student's answer. Two type feedbacks were implemented. For describing the feedback content we refer to Narciss´ (2008) classification of feedback components (table 1):

Table 1: Content related classification of feedback components (Narciss, 2008)

Simple components of feedback	Knowlede of performance (KP)
	Knowledge of results (KR)
	Knowledge of the correct result (KCR)
Elaborate components of feedback	Knowledge about task constraints (KTC)
	Knowledge about concepts (KC)
	Knowledge about mistakes (KM)
	Knowledge about how to proceed (KH)
	Knowledge about metacognition (KMC)

The worked-out example type feedback first informs the student about whether the answer is correct or wrong (KR). If wrong, it also tells what the correct solution is (KCR) and gives a model step-by-step solution of the problem (KH), cf. Fig.1.

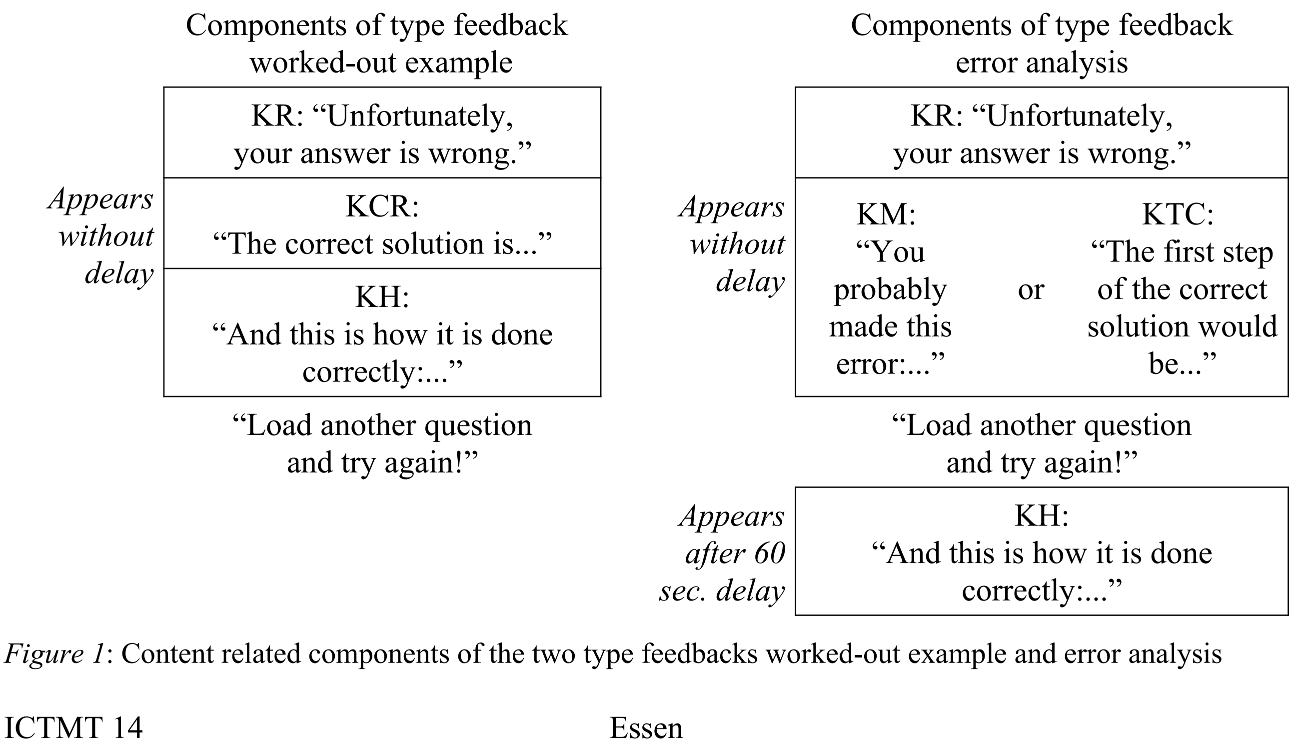

Figure 1: Content related components of the two type feedbacks worked-out example and error analysis

The other feedback type, error analysis, also begins with informing the student that the given answer is correct or wrong (KR). Then, if the student made an error that was previously implemented based on the CAS-based automated error analysis, the student is told what error is made (KM). If the error has not been implemented in the error analysis, the student receives a first step of a suitable procedure (KTC). In both cases, the student then is asked to think about it and try again. Since students of all achieving levels will be part of the error analysis feedback group, we expect some students that rely on a worked-out example. After one minute delay a button will appear which – after click – provides access to a worked-out example.

DESIGN

The treatment was administered to two classes (7th and 8th grade) of a German middle school in the state of Hessia. Each class consisted of 15 pupils between 12 and 14 resp. 13 and 15 years age. All students had been introduced to fractions in grade 6. The students were distributed to two treatment groups such that low and high achievers (identified by their math grade) and were about equally distributed within each groups. In one group the students would be working with tasks with the error analysis type feedback, in the other with tasks with the worked-out example type feedback. One week before treatment the students were given a performance test on fractions which consisted of 8 items that were very similar to the tasks from the treatment. The treatment took place in a computer pool room of their school and lasted about 60 min., after which they filled in an acceptance questionnaire (Cheung & Vogel, 2013, adapted to the digital feedback material of this study as object of acceptance) and, for the error analysis type feedback group only, two use-of-feedback items which asked whether they generally waited for the worked-out example to appear. The posttest consisted of numerical variations of the 8 pretest items and was administered one week after treatment.

RESULTS AND DISCUSSSION

Of the 30 pupils of both classes, 29 took part in pretest, treatment and acceptance as well as use-of-feedback measures. Of these, 4 did not do the posttest, so eventually 25 took part in all testing and treatment measures. Cronbach's alphas for the 10 acceptance items and the two use-of-feedback items were .858 and .574, resp. The data was analysed using non-parametric tests. Except in one case, all tested differences or correlations were not significant. Since all the data seems to point towards a rather unexpected result, we give the details in full.

Q1: Is there a difference between error analysis type and worked-out example type feedback with respect to acceptance?

Fig. 2 shows that there is a very high grade of acceptance of the automated feedback administered in this study. It also indicates that there seems to be a difference between how students rate the two types. Students that worked with type feedback error analysis show a slightly higher grade of acceptance than those that were given the worked-out example type. However, the difference is not significant (Mann-Whitney: $U = 73.5$, $z = 1.35295$, $p = .17702$).

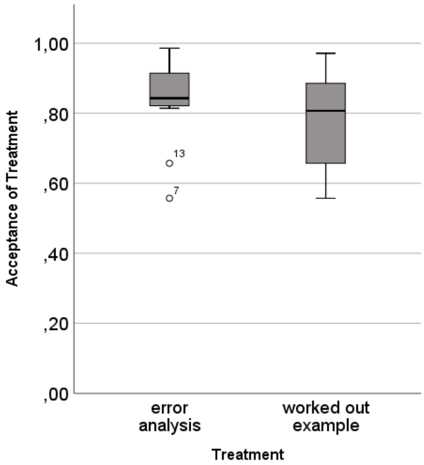

Figure 2: Acceptance rates of digital feedback by students of the two different type feedback treatment groups

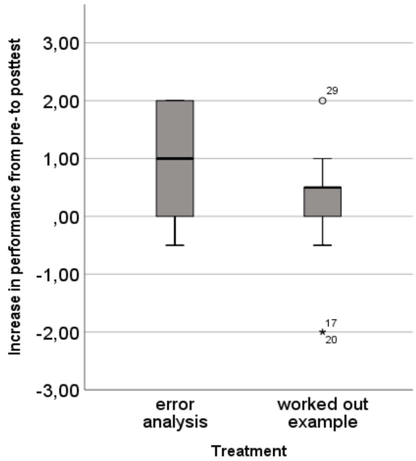

Figure 3: Increase in performance from Pre- to Posttest in the two different type feedback treatment groups

Q2: Is there a difference between error analysis type and worked-out example type feedback with respect to performance?

Fig. 3 shows that students from the treatment group "error analysis" show a slightly higher increase in performance from pre- to posttest. Again, the difference is not significant (Mann-Whitney: $U = 52.5$, $z = 1.35982$, $p = .17384$).

Q3: How does – with respect to each error analysis type and worked-out example type feedback – the achievement level correlate with performance?

While for the worked-out example group the two scales do not seem to correlate (Spearman-Rho: $r = .008$, $p = .98$, cf. Fig. 5), there is a high correlation between decreasing math grade and increase in test performance for the error analysis-feedback group (Spearman-Rho: $r = .73$, $p = .007$, cf. Fig. 4). It seems that high achievers as a whole do not seem to profit as much from the automatic feedback of both types as the low achievers, which could be explained by a possible ceiling effect due to a pretest that, for some pupils, was too easy. Yet for low achievers the feedback type seems to make a difference, which rather unexpectedly speaks in favour of the error analysis type over the worked-out example type feedback.

Q4. How does – with respect to the error analysis type feedback – the use of the delayed worked-out example correlate with performance?

The error analysis type feedback also provided, after a delay of 60 sec., a worked-out example. After treatment, students of this group were asked whether they generally waited for the worked-out example or started with new tasks right after they were given an error analysis.

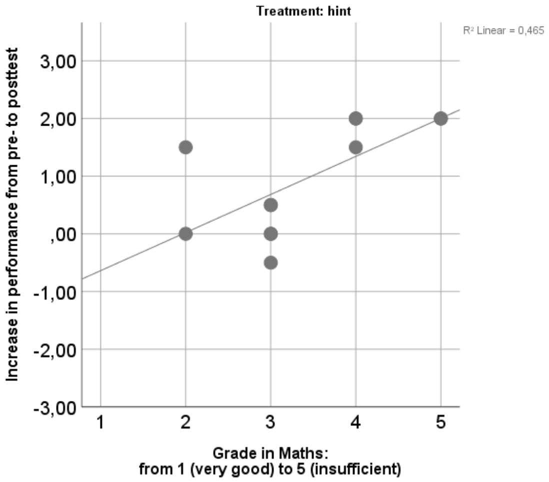

Figure 4: Correlation between increase in performance from Pre- to Posttest and math grades of students from the error analysis type feedback group

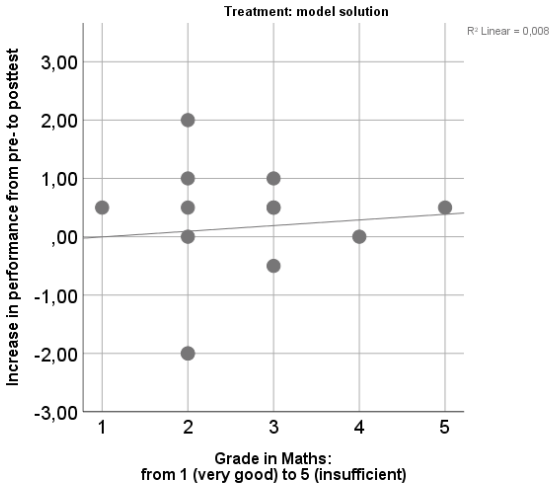

Figure 5: Correlation between increase in performance from Pre- to Posttest and math grades of students from the worked-out example type feedback group

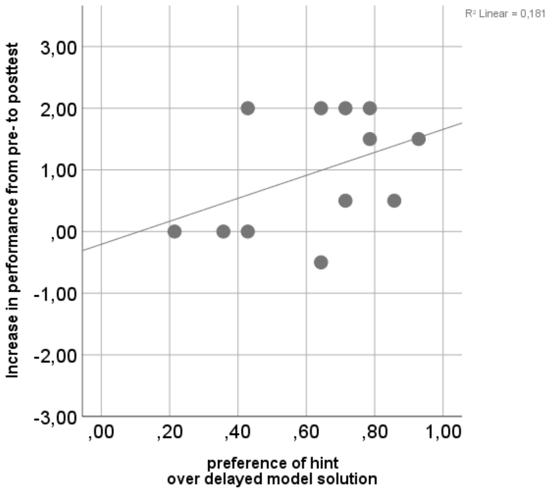

Figure 6: Correlation between increase in performance from Pre- to Posttest and feedback use by students from error analysis type feedback group

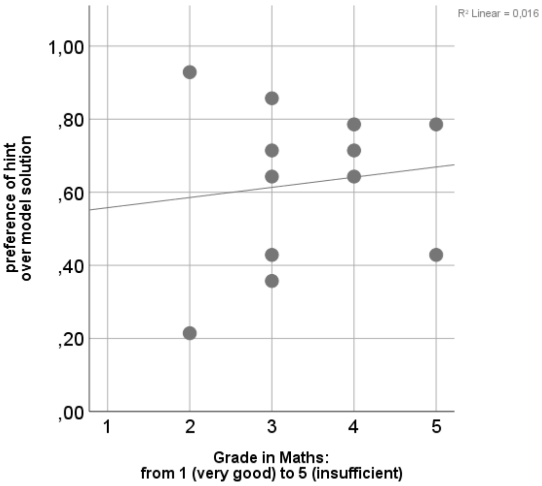

Figure 7: Correlation between feedback use and math grades of students from error analysis type feedback group

Fig. 6 shows that those students who generally did not wait for worked-out examples tend to have a higher performance increase than those who occasionally did (Spearman-Rho: r = .391, p = .208). In fact, there is no correlation between math grade and effective use of the error analysis type feedback (Spearman-Rho: r = .089, p = .784, cf. fig. 7): Within the error analysis feedback group there were high graders as well as low graders who preferred working with the immediately given error analysis or were waiting for the worked out example.

CONCLUSIONS

Due to a very small data base and nearly all observations not being statistically verified, conclusions need to be treated with caution. However, all data seems to point to a rather unexpected result: It seems that low achievers benefit more from error analysis type feedback than from worked-out examples. If this is true, it probably is due to the fact that the treatment was about reactivating skills that had been acquired before. Hence students could not be considered novices any more but experts, for which a worked-out example interfered with the solving strategies they already had in mind, or for which a worked-out example simply did not provide any new information. When students already have some basic understanding of operating with fractions, information reduced to the point of errors made has a better effect on enhancing possibly erroneous or incomplete knowledge. To conclude, it seems that feedback based on an error analysis of the students' responses should be preferred over worked-out examples in the traditional sense in phases of reactivating and practising skills that are already known to students.

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WHEN DIDACTICS MEETS DATA SCIENCE

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The Department for Evaluation, Prospective and Performance (DEPP) at the French ministry of education provides deciders with data analysis on the state of the French educational system. French students' performance is measured through Large Scale Assessments (LSA). From 2018, DEPP's LSA instruments are computer based when it comes to assess secondary school students. In mathematics, new test instruments have been developed embedding digital tools in support of the mathematical task. This paper presents the results of a study aiming at profiling student's mathematical activity, based on a "big-data" analysis of the digital traces left in technology enriched interactive items by a sample of 9-graders in France. Results give hard evidence of how data science in education can benefit from the mixed contribution of three paramount aspects: technology, analytics and didactics.

Keywords: digital, assessment, big data, didactics

CONTEXT AND RESEARCH QUESTIONS

CEDRE (Subject related sample based assessment cycle) is a sample based large scale assessment aiming at measuring students' abilities in Mathematics at the end of Grade 9 every 5 or 6 years in France. Constructed and designed by the Department for Evaluation, Prospective and Performance (DEPP) at the French ministry of education, its framework is based on the national French curriculum in mathematics. First administered in 2008 and 2014, CEDRE will be administered again in May 2019. This new cycle will be computer based for the first time. As trends must be secured so comparison with previous cycles is guaranteed, a large part of test instruments are similar to formerly paper based items. However, the DEPP developed interactive items, very different from more classical item formats, in order to fully profit from the potentialities of assessing with digital tools (Stacey, 2013). Offering students the possibility to use digital tools during the assessment may outsource (Drijvers, 2018) basic procedural work to such tools. Therefore opportunities are given to students to better engage higher order skills such as devising a strategy or mathematical thinking. To what extend can logdata analysis better inform on students' performance in LSA and explain achievement? Can it participate to categorize students' productions and procedures, allowing didactical interpretation and profiling? Log data files issued from this pilot test have been analysed using a combination of machine learning methods enriched by the results of an *a priori* didactical analysis of the mathematical task in each item.

A PRIORI DIDACTICAL ANALYSIS

A conceptual framework was designed in order to *a priori* analyse the mathematical task in interactive items. This analysis appeared necessary from a methodological point of view in order to draw hypothesis and allow defining variables of interest with respect to the potential mathematical activity in the items. The didactical frame was structured around three main questions: How does mathematical knowledge need to be adapted in order to answer the question? What tools' utilization is necessary to solve the problem? How do student/machine interactions influence the mathematical task?

Determining what mathematical knowledge are involved in items is a preliminary necessity to task analysis. Beyond listing them, we have to identify and describe the way they must or could be operated and what operation's adaptions are necessary to achieve the tasks (Robert (2005) et Roditi-Salles (2015)). This level of analysis can be a first step towards determining choices students have to make, the number of step required, types of errors etc.

These very specific types of tasks are set in a technological environment and embark digital tools. Taking into account digital technology leads us to address two complementary aspects: digital tools utilizations and human/machine interactions.

Tool use can be described in reference to the instrumental approach (Rabardel (1995)). This approach distinguishes a tool from an instrument. The tool gets the status of an instrument when it is used by the student as a mean to solve the problem. In this case, its use is decomposed into "*cognitive schemes containing conceptual understanding and techniques for using a tool for a specific type of task*" (Drijvers (2012)). The analysis will then focus on identifying and describing utilization schemes potentially involved in the task. Rabardel distinguishes two types of utilization schemes: "*usage schemes, related to "secondary tasks" (...) and instrument-mediated action schemes,(...) related to "primary tasks" (...) [which] incorporate usage schemes as constituents.*" (Ibid, p 83)

Interactions and feedback are important features of problem solving situations, especially in an assessment context. Being immerged in a digital environment gives birth to specific kind of human/machine interactions. Most of them are intentional and planned by developers when designing the assessment environment, others are not, but all somehow carry information students can grasp to proceed into the solving process. Laborde (2018) distinguishes two types of feedback with regards to technologically enriched situations: one issued from task specific digital tools, another being the "teacher's voice". This last type of feedback is meant to help students catch, adapt and retain information given in the environment. In a summative assessment context it should be very limited as it could interfere with the objective of measuring students' ability. Nonetheless it can be considered paramount in a formative orchestration. If a summative assessment platform as the one used at the DEPP can be considered as a *non-didactical* environment, where the teacher's voice is supposed to be absent, we yet consider this environment as potentially allowing machine/student interactions. In his Theory of didactical situations, Brousseau (1998) separates three levels of feedback depending on the nature of the environment's mathematical reaction: the feedback can either reflect on actions, formulations or validations. This last model has been used to help describing item/student interactions in DEPP's interactive items.

APPLYING THE FRAME TO A SPECIFIC TASK ANALYSIS: "TREE GROWTH".

In the exercise used in the study, two nonlinear functions model two different tree growths. Both are given in linked numerical and graphical representations (Stacey (2013)). Students act in the numerical representation (a table of values) entering the age of the trees in months. A calculation tool returns the corresponding tree heights and a graphing tool spots the points in the graph. Both actions are realized when student press the "calculate and graph" button. By default, values for 300, 500 and 600 months are given. Additional tools can be used: a pencil, an eraser (not allowing erasing given information), a length tool, a compass, a non-scientific calculator. The question can be translated as: "At what age (other than 0 month) do both trees have the same height?"

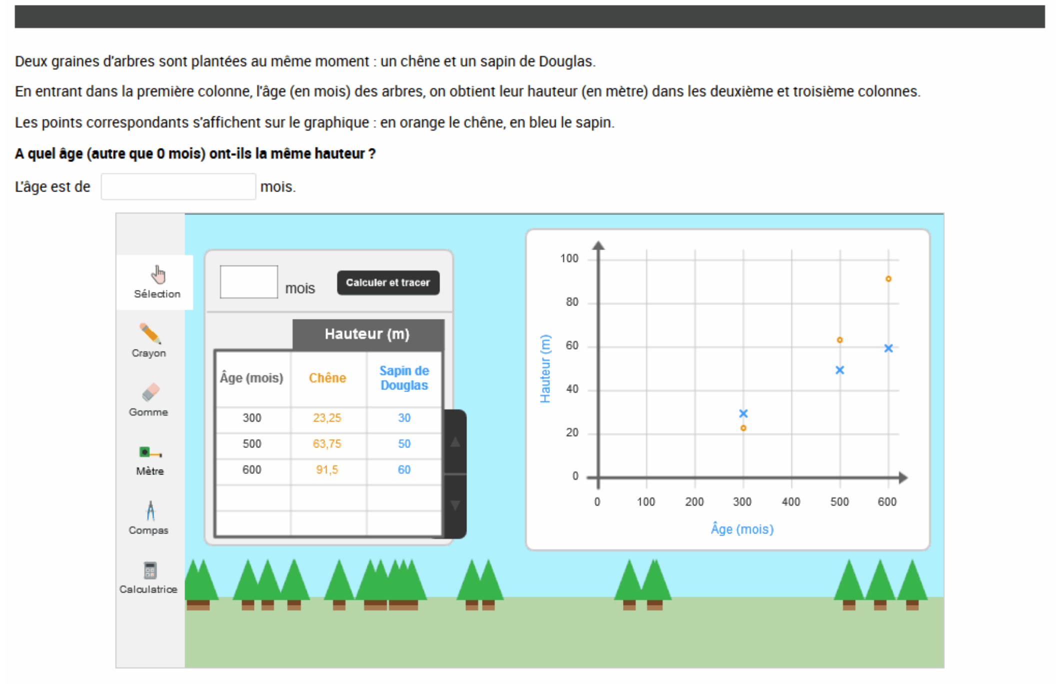

Figure 1: Interactive item "tree growth"

From Grade 9 students' point of view, this task requires conceptual understanding (Kilpatrick and al. (2001)) of functions and their representations (table of values, graph). Functions have two different characters as Sfard (1991) or Drijvers (2012) showed: "*In lower secondary grades, functions mainly have an operational character and are seen as an input–output 'machine' that process input values into output values. In higher grades, functions have a more structural character with various properties (Sfard, 1991). They become mathematical objects that are represented in different ways, are ordered into different types according to their properties, and are submitted to higher-order processes such as differentiation and integration. We argue that the transition from functions as calculation operations to functions as objects is fundamental for conceptual understanding in this domain.*" (Drijvers (2012)). Students can adapt the problem adding intermediary information using the calculation and graphing tool. On one hand, they can then opt for a trial and error method. This would consist in entering a number of months, comparing the results returned either in the numerical or graphical representation, deciding to enter another number of months until the solution (390) is found. Alternating tries around the target value or aiming at it from below or above could improve the trial and error process. These students essentially show good understanding of the concept of functions in their operational character. This method could imply a relatively large number of tries. On the other hand, students having a good understanding of both functions variations, notably from studying them in the graphical representation, can quickly aim at the target number of months. The pencil can for example be used to draw lines and introduce a continuous representation of the functions. The inversion of tree heights between 300 and 500 months can also be noticed. These students comprehend functions as objects with properties, in their structural character.

The following digital tools are at students' disposal within the item:

- A keyboard (with or without number pad) and mouse.
- A "calculation and graph" tool: specific to the item.

- A pencil (common to any item on the platform). Usage: click the starting point, move the mouse to trace, click to stop writing.
- An eraser only allowing to erase pencil traces or measurement tool traces. Usage: clicking erases all pencil traces together.
- A compass
- A calculator

The “calculation and graph tool” does not require complex usage schemes. Two usage schemes are identified for this tool: enter a number of months within the domain [0 ; 600] via an input box and a popup number pad and understand that the tool returns unique heights (outputs) for both trees (numerical and graphical representations) when the button “calculate and graph” is clicked. No tutorial or tool training is proposed to students. Usage schemes are pretty close to relatively usual tools such as a currency converter. Nevertheless, we can imagine some students feeling the need to appropriate the tool by first using it for testing purposes, for example entering extreme values or values not directly connected to the primary task. Using this tool is compulsory to succeed the item. In reference to the Instrumental Approach, we describe next how the tool can be instrumented in this situation, once assumed that responding students will build an instrument from the “calculation and graph” tool, within the item environment, in order to solve the task (Trouche, 2005, p 272). Instrumented action schemes are organized around the core elements that follow:

1. Knowing the difference between input and output in a contextual use of a function as a model.
2. Understanding that the tool returns unique heights (outputs) for both trees (numerical and graphical representations) when choosing a number of month (inputs).
3. Entering a number of months within the domain [0 ; 600].
4. Comparing outputs either in the numerical or graphical representation. Validating by linking to the real life situation.
5. Decide on the next number of month to enter considering the comparison to the previous one.
6. Iterating the process.

This type of instrumentation is linked to an operational approach of the concept of functions.

As mentioned earlier, the pencil can also be used in order to get a continuous model on the domain or part of it. Students can link points together using it. This step is an intermediate towards the primary task. The following core elements participate to instrumented action schemes using the pencil as well as the “calculate and graph” tool and are characteristic of a structural understanding of the concept of function.

1. Understanding the growth phenomenon is a continuous one. Hence, the functions modelling it are continuous.
2. Assuming both functions will be strictly increasing.
3. Using the pencil to link consecutive points together.
4. Decide on the next number of month to enter considering line intersection
5. Going back to numerical values to aim at accuracy

Of course one might operate composite instrumented action schemes, mixing the use of both principal tools. For example, one can use the pencil to sketch continuous graphs (potentially after trying 100 and or 200 months to get a more complete view of the graphs shapes), or rely on points’ colours and choose 400 months in the first tries and then use a trial and error strategy to aim precisely at the target with the “calculate and graph” tool.

Interactions at stake within the item are principally addressing the two different representations of the functions: numerical and graphical. When students use the “calculation and graph” tool, the feedback is given in both representations as new numbers in the table of values and two points on the graph. Students are consequently relieved from having to convert one representation into another. The colours of numbers and points related to the same function match so students can more easily interpret the feedback. This important interaction participate to students’ reflection towards formulating the problem into both representations and then compare results (for example using colours’ inversions in the graphical display) to either conclude or decide on other tries to make. Besides it can also participate to validate or invalidate tries that are outside the domain or very far from target.

LOGDATA ANALYSIS

CEDRE’s interactive items, “tree growth” among them, have been experimented in a pilot test in May 2017 with a sample of 3 000 Grade 9 students per item. Students’ digital traces have been recorded in log data files. As log data files contain a very large amount of data and in order to aim at interpretable results as well as to avoid noisy signals, variables of interest were defined, as a result of the *a priori* didactical analysis. They could potentially lead to build a model able to explain success or failure to the task considering either the operational or structural character of functions used by students. The main variables used in the analytical models are the following: Month list length, First input between 200 and 600, Number of alternating within the month list, Time spent on the item, Distance between first input and target value, Distance between the second input and the target, Distance between the last input and the target, Standard deviation of the month list, Target value is in the month list, Pencil use.

Classical machine learning analysis was then applied to the logdata. Unsupervised learning first aimed at clustering the sample. DBSCAN and k-means algorithms have principally been used at this stage. Supervised learning was also implemented to determine the predictive power of students’ achievement of the model and estimate the weight of the variable in the prediction. Mostly logistic regression and random forest algorithms were used in the supervised learning stage. Mean values of the most important variables were then calculated for each of the clusters issued from the unsupervised learning.

RESULTS AND DISCUSSION

Clustering analysis distinguishes 4 clusters corresponding to 4 different students’ profiles. Each cluster’s size represents 25 % of the responding students. Non responding students (33 % of the sample) have been excluded from analysis as they did not use any of the available tools and did not input any number in the response box. It is to be noted that if the proportion of non-responding students is high, it is not higher than in other LSA such as PISA (OCDE, 2014). Figure 2 illustrates DBSCAN clustering results as well as important variables values for each cluster.

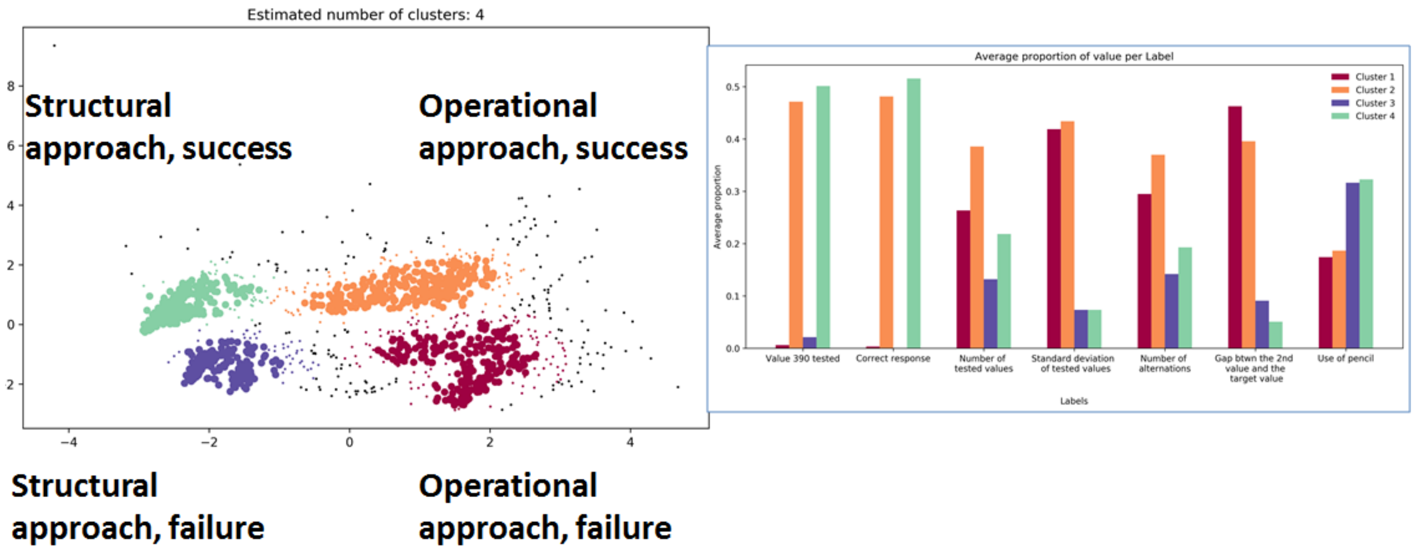

Figure 2: dbscan results and explanatory variables

Two profiles (green and orange in the graph) achieved the task. The other two (blue and red) correspond to students who failed. Apart from the obvious variable "value 390 tested", no other variable allows to discriminate between success and failure. This result is disappointing in the sense that the model cannot explain students' achievement to the item, which is one of our research questions. However, variables of interest in the *a priori* analysis allow describing profiles along another dimension than achievement. Two clusters (orange and red on the figure 2) show a large number of inputs, a large number of alternating inputs, a first input away from target and a large distribution of inputs. These characteristics allow us to interpret that students from these groups preferred a "trial and error" solving strategy, approaching the underlying concept of function in its operational aspect. Half of them achieved the task successfully, the other half did not. On the other hand, the other two groups share a different and opposite profile description according to the same variables: a small number of inputs, a small number of alternating inputs, a first input close from target, a narrow distribution of inputs. Moreover this second category of students used the pencil more often, altogether identifying solving strategies related to the structural aspect of functions. Alike the first two groups, the structural approach led to failing the task for half the students favouring it.

Hence, if the didactical and analytical models used in this study could not help us explain grade 9 students' achievement to an interactive mathematical task in a computer based summative assessment, they could nevertheless help us identify two solving strategies well known in didactics literature. The potential usage of such result in diagnostic or formative assessment is obvious, this type of information being very valuable from teachers' perspective in order to address students' specific needs.

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Part 4: Inspiring Learning and Teaching

TEEN-IMMIGRANTS EXPLORE A MATH MOBILE APP

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We present the pilot phase of the project "Teenagers Experience Empowerment by Numbers" (TEEN), which is funded by Politecnico di Milano through the Polisocial Award 2018 and concerns the development of a mobile app to teach essential mathematics to young immigrants. The project aims at preparing them for living in a conscious, autonomous way in a Western country, increasing their ability to deal with everyday tasks that require some mathematical understanding. We present the app, some materials and an activity with the learners who have interacted with that. The set of tasks, tested in small groups, is rooted in daily activities, such as shopping at the supermarket, choosing a mobile internet plan, planning a trip. Our theoretical background is related to existing research findings on teaching to immigrants, Rabardel's instrumental orchestration and feedback.

Keywords: Immigrants; Intelligent Tutoring Systems; Out-of-school mathematics.

INTRODUCTION

The research project TEEN (Teenagers Experience Empowerment by Numbers http://www.teen.polimi.it) deals with the phenomenon of young immigration, from Africa to Europe, which has received increasing attention in the last years, given the very large number of young immigrants who leave their country (without parents). Once they arrive in Italy, the teen-immigrants are accommodated in communities for minors. Institutional protection plans provide teen-immigrants for their basic needs (accommodation, food, health services and a language course). This turns out to be insufficient to deal with the requirements of the "real world" that they need to face early, considering that the protection guaranteed by the Italian legislation to foreign unaccompanied minors ends on the day of their 18th birthday. The TEEN project aims at promoting basic mathematical literacy as another fundamental right that may significantly increase the level of autonomy of teen-immigrants, helping them to deal with daily needs, such as to manage their monthly budget, to buy a train ticket, to read the pay slip, and even to manage their time. All these activities have a common root, that is: the understanding of elementary mathematics.

The idea that mathematics can play a role in social integration of disadvantaged students has been proven successful in other contexts. Civil (2008) reports a growing attention towards mathematics taught to immigrant students at school, with a special focus on both developing educational methodologies which improve the immigrant students' learning; and exploring and pointing out the social and ethical implications of teaching mathematics to a "minority". Powell and Brantlinger (2008) shed a light on the actual tension between "academic" and "everyday" mathematics, underlying the centrality, for mathematics teachers, to create alternative curricula with the purpose of showing mathematics as an accessible (and useful) discipline. It is crucial that teachers promote a new idea of mathematics as valuable skills and social integration. In Greece, Stathopoulou and Kalabasis (2007) and Chronaki (2005) have provided evidence that Romani students learn better when mathematics is related to their identity and their attitudes. In U.S., Gutstein (2003) shows that if teachers consider the students' language and the way they interact when they design the lesson, the students turn out to be more aware of the role of mathematics, and tend to appreciate the discipline.

Drawing on these research findings, the TEEN project aims at developing a mobile app that can be used by teen-immigrants, outside the context of the classroom and without the interaction with a teacher, to develop their basic mathematical skills. In order to make the discipline more accessible and attractive, the app is designed to deal with everyday mathematics, with non-academic language and, to be inclusive, with the least possible amount of written words. The activities proposed by the app address situations that are familiar for teen-immigrants. In designing the app for teaching mathematics to teen-immigrants, we consider the app both as an artefact and as an instrument, in Rabardel's (1995) sense. In the sequel, we first recall the theoretical framework that informed the design of the app, then we present the results from the pilot phase of the project, which consists of activities carried on with small groups of immigrants organized in the communities where they live.

THEORETICAL FRAMEWORK

Artefact is an object designed with the purpose to achieve a particular goal and embedding a specific knowledge (Rabardel, 1995). Within this view, artefacts can be seen as facilitators of knowledge acquisition, or more generally as extensions of the individual. However, Radford (2012) stresses that artefacts can play a deep cognitive role: artefacts "become part of the way in which we come to think and know" (p.285): in a Vygotskyan perspective, in fact, human cognition is affected by its relationship with social and cultural settings, which are grounded in the use of artefacts. Artefacts are cultural devices that shape, affect and change our cognitive functioning (Radford, 2012). Since artefacts are artificial devices, Radford (2012) argues that "investigating the proper conditions of artifact use in educational settings constitutes an important research problem" (p.283).

Instrument is the artefact jointly with the utilization schemes developed while the individual uses it. This notion of instrument allows us to take into account both the artefact itself and its actual use, namely to distinguish between the intention of the designer, who produces the artefact, and its use. In case of a learner, through a cognitive artefact she is supposed to internalize knowledge embedded in it. In particular, we are interested in the instrumental genesis, a process that underlies the construction and evolution of the instrument (Beguin and Rabardel, 2000) and encompasses two dimensions related to the subject: instrumentalization and instrumentation. Instrumentalization concerns the recognition of the features of the artefact, for instance its potentialities and constraints. In this perspective, the artifact evolves as the learner's activity unfolds. Instrumentation concerns the way(s) the subject interacts with the artefact: utilization schemes are built and enhanced by the learner. The "utilization scheme" (Rabardel, 1995) is intended as an active structure into which past experiences are incorporated and organized. The schemes have a history, they may change and are characterized by private and social dimensions. The social one can be thought of as the transmission of the schemes, in particular those that are embedded by the designers of the artifact, including various type of user aids, for instance manuals and instructions (Béguin & Rabardel, 2000).

An important research imperative, thus, concerns not only a general focus on the conditions under which artifacts are effective in educational settings, as Radford (2012) put it, but in particular on the social dimension of utilization schemes, and even more specifically on the way instruments provide effective feedback to the users. This holds true especially for educational software that offers guided learning support to students engaged in activities like problem solving. The kind of support may range from identifying and correcting errors to promoting mastery learning (Baker, D'Mello, Rodrigo & Graesser, 2009). Feedback is information provided by an agent (e.g., teacher, peer) regarding aspects of one's performance or understanding, and it is seen as "consequence" of performance (Hattie & Timperley, 2007). Also Intelligent Tutoring Systems (ITS) can provide valuable feedback, according to Baker et al. (2009). Hattie and Timperley (2007) argue that feedback should address the goal, the progress and the quality of the performance of a learner, helping her answering three questions: Where am I going? How am I doing? and Where to next? Moreover, these questions, which correspond to notions of “feed up”, “feed back”, and “feed forward”, work at four levels (Hattie & Timperley, 2007): the one of the task, if they provide information about how well tasks are understood and/or performed; the level of the process; the level of self-regulation, if they concern self-monitoring, directing and regulating of actions; and the level of self, if they point to personal evaluations and positively affect learner’s identities. Hattie and Timperley (2007) also point out that feedback is part of a range of practices that aid the learners’ understanding. Moreover, feedback and instruction become intertwined until the provided aid is not a sole check of correctness (Hattie & Timperley, 2007). In the process of instrumental genesis, thus, the nature of feedback provided by the artefact can be considered as part of its features.

METHODOLOGY

We aim at answering the following research question: which features of the app emerge from immigrant’s interaction with it, that can help us design a proper feedback system? In order to address this question, we report data from an activity that took place on November 2018 in one community for minors that hosts teen-immigrants. It involved two teenagers who come from Africa, which are fictitiously named Drissa and Sedou. Drissa comes from Mali, where he attended elementary school; he has been in Italy for two years and he is attending middle school (grade 6). Sedou comes from Ivory Coast and has been in Italy for 1 year. Sedou has never attended school in Africa and he is attending a language course (level A1). Both of them do not work and live in a large community of 20 people.

This activity is part of a three-months long pilot phase of the project TEEN. The research project follows a design-based research methodology, which focuses on examining a particular intervention by continuous iteration of design, enactment, analysis, and redesign (Cobb *et al.*, 2003). The pilot phase of TEEN consists of meetings with groups of teen-immigrants in different communities, which provided us with important information about the functioning of the app. The goal of the research is, in fact, to produce an app for smartphone that can be used by teenagers outside school, without the aid of any tutor or teacher. Following Powell e Brantlinger (2008), we aim at designing an app that proposes "everyday" mathematical problems, given the importance of showing mathematics as an accessible discipline. Following Stathopoulou & Kalabasis (2007) and Chronaki (2005), the problems presented in the app relate to teen-immigrants’ identities and attitudes. Along this line, following also Gutstein (2003), we consider the teen-immigrants’ language, and in particular we tried to reduced the verbosity of the tasks given their difficulties with the Italian language, supporting the problem statements with graphical content. Four meetings in three different communities took place in the pilot phase. Each meeting lasted between 60 and 90 minutes. We selected the case of Drissa and Sedou, because it illuminates important aspects of the interaction with the app. In some meetings, we observed difficulties mostly related to the Italian language, that were overcame talking in English and Bambara language thank the mediation of a peer who speaks I. In other meetings, the emotional burden provoked by our presence was overwhelming. We agree with Civil (2008) that the difficulties related to data collection within this special sample of teenagers are part of the ethical issues concerning working with a minority.

The app is structured as follows (Figure 1a): it proposes some practice problems, which involve basic mathematical facts, to allow the user to become confident with the instrument. For example, a calculator is embedded in the app and it is activated once the user clicks on the blank space to submit her answer (Figure 1c). After the practice section, the users are engaged in a budgeting activity, in which they have to make their lifestyle compatible with a given salary (as a 18-years-old boy in Italy). Then, more complex scenarios are proposed and problem solving tasks are shown.

a) The dashboard b) Task 1 c) The calculator d) Task 4

Figure 1. Screenshot from the app used for the pilot phase

The practice section represents the most challenging part of the app and it is the focus of this paper. Being the entrance door of the app, it should be engaging, easy to be understood even by teenagers who are not fluent in Italian, intriguing, not boring, and self-explaining given that in the experimental phase of the project TEEN the users would no longer interact with a tutor. In order to monitor all these features of the practice section, we employed Rabardel's notions of intrumentation and instrumentalisation to examine how the app is experienced by teenagers and the utilisation schemes that are prompted. At the same time, we examine the nature of feedback that is provided by the tutor (who was present for the pilot phase).

DATA ANALYSIS

To recall, we now present an excerpt from a meeting with Drissa and Sedou. They are presented for the first time with the app and they have been invited to explore it. The first task they encounter concerns the choice of a fare for the monthly internet plan (Figure 1b). From previous meetings with the educators who work in the community, we know that these boys take into consideration two important features of a monthly fare for mobile phone: one is the possibility to surf the internet the most possible, and the other one is the possibility to call their country of origin (that is even more important), spending the least possible money. The former need is addressed in the first task.

For a long while, Drissa and Sedou remain silent, reading the question on the screen of the smartphone: "You have finished your giga for this month. If you buy the SOS-giga, how much do you spend this month?". The tutor noticed that they are struggling with its meaning and that they are also reading the text embedded in the image (Figure 1b), where it is shown that the monthly fee for internet mobile is 8 euros (which gives 5 giga), and the SOS-giga (which gives 2 extra giga) costs 3.50 euros. The tutor, after some minutes of silence, intervenes: "this is about your mobile phone, do you use it to surf on the web? Do you have 'Giga'? Have you ever finished your giga before the end of the month?" While Drissa replies that he has an internet plan, and adds "but I never ended the giga", Sedou says "I do not have internet". The tutor addresses Drissa, and asks "Really? Have you ever received an SMS proposing SOS internet?" [when the monthly data are going to finish, the company texts customers to offer extra internet data]. And Drissa replies "Oh! Yes, I have! but I wait for the next month, and I use only wifi connection". Then the tutor asks Drissa and Sedou to figure out the problem in this way: "But if you pay 8€ monthly, and you need more data, so you pay further 3.5€ to have extra data, how much do you pay?". Drissa answers: "11.50".

We recall that the app shows a blank space under the question, which is meant to be used as a calculator. However, Drissa makes the computation by heart. We notice that, in terms of instrumentalisation, Drissa is not fully exploiting the potentiality of the app. In terms of the learning trajectory designed for the use of the app, since this task is the simplest one, Drissa is missing the opportunity to practice with the calculator add-on (see Figure 1.c), which becomes essential in more complex tasks (later). In other words, we notice that the process of instrumentation vanishes, and Drissa is not acquiring its basic utilization schemes. We further comment that the nature of the question, and the computations required, are too simple to require the (proper) use of the app as a calculator, and this hinders Drissa's instrumentalisation process. The tutor intervenes two times: the first one, to clarify the task and to connect it to Drissa's and Sedou's sense-making. The second time, given that Drissa and Sedou do not buy extra giga, the tutor invites them to answer to the question anyway.

The interaction among Drissa, Sedou and the tutor goes on. Once Drissa has found the answer, the tutor invites him to write this number in the blank space, and to record his answer on the app. Drissa has difficulties to write the number 11.50 due to the point that indicates the decimal number. Drissa searches for the comma, which in his understanding signals the decimal number. We comment that this difficulty with the writing is mostly related to the representation of decimal numbers (in Italy, differently from English speaking countries, the comma separates the decimals). Another difficulty is related to a feature of the app, that is: once Drissa has written the answer "11.50", he shows difficulties in figuring out how to submit the answer. The command is an equal sign, which does not make sense if one does not insert a computation to be carried out, e.g. 8+3.5. In this case, Drissa aims at writing only the result of the sum: namely, 11.5. In terms of the process of instrumentalisation, we notice that this is a constraint of the app.

Drissa submits the answer and the app provides a positive feedback, that is "right answer", then Drissa and Sedou are exposed to the second task, which concerns the buying of milk at the superstore: "A bottle of milk costs 1.50 euros. How do 12 bottles cost?" They read the task and suddenly they activate the calculator embedded in the app to compute the multiplication. We observe that the time required to make sense of the task is shorter with respect the previous one and, in terms of instrumentation, the utilization scheme concerning the activation of the calculator is surfacing. At this point of the episode a new difficulty is observed: Drissa inserts 1.50 and searches for the multiplication sign, that is expected to be "x", but the calculator presents the symbol "*". Like in the first task, we see a difficulty related to the representation of mathematical objects, which can be read as a constraint in terms of the process of instrumentalization. The tutor has to intervene and shows the symbol for multiplication. Drissa performs the multiplication and clicks on the equal sign. The app provides the results, which is 18, and a positive feedback. A this point the third task shows up, Drissa and Sedou deal with it quickly and not showing difficulties related to the process of instrumental genesis.

Then, a task concerning time to arrive at the hospital with public transport is presented: "You have to go to the hospital for vaccination. There is a bus stop in front of your home. You have to take a bus that stops there, then you have to take a tram which stops in front of the entrance of the hospital. Which time should you take the bus to arrive before 9.30?" Above the text of the problem there is an image (Figure 1d).

Sedou: I have to take the bus at 8:30, because if I take the bus at 9:30 I arrive late.

Drissa: Yes, you have to arrive before 9:30.

Tutor: If you leave at 8:30, which time do you arrive?

Drissa: The bus takes one hour.

The first interactions around the task show that Sedou is making sense of the situation: in the past, someone might have explained that, in order to be punctual for a meeting, one has not to get out from home at the time he has to be there. Drissa believes that the bus takes one hour to bring him to the hospital. Both boys are not taking into consideration the information provided in the image, where the bus and the tram schedules are given. The image contains relevant information to perform the task. The tutor intervenes and, referring to the time schedule, he asks: “Have you ever seen this table?”. Drissa and Sedou reply negatively and the tutor explains the meaning of it. At a certain point, Drissa says “I should take the bus at 8:20”, and the tutor asks if this time is present on the bus schedule. Drissa replies “We should compute the one that we have to take before we arrive. If you take…” and Sedou “Let’s leave at 8:19”.

Tutor: If you leave at 8:19, which time do you arrive at the tram stop?

Drissa: 23 minutes

Tutor: To arrive ...where?

Drissa: At the tram stop

Tutor: So, how long does the journey take?

Instead of answering to the tutor’s question, Drissa proposes “I should take 8:28 to arrive at 9:30”. The tutor follows his line of reasoning: “Is there 8:28 on the bus schedule?”, and Drissa replies affirmatively. Then the tutor proposes again “If I take 8:28, which time do I arrive at the tram stop?” This time, Drissa replies: “8:41 no… 8:51”, so the tutor asks: “If I arrive at 8:51, I can take the tram at 8:52. Is it clear?”. Instead of answering, Drissa tries another time on the bus schedule (8:03). The tutor asks to check the time they arrive at the tram stop in case they take the bus at 8:03, and so which tram they can take. Drissa replies that at 8:42 they arrive at the hospital.

Tutor: 40 minutes behind schedule. Too early. So, what do we do? Do we leave later?

Drissa: Yes, this one [8:38]

We notice that Drissa acts as if he can click on the touchscreen, on the time schedule. He does not only point to the time, but he acts as if an action is expected to be taken by the app after his choice. However, the answer should be written in the blank space, not selected on the touchscreen. In terms of instrumentalisation, this feature of the app can be seen as a constraint on the range of possible actions that can be undertaken.

Drissa: Early. Too early. 9:20 is fine.

Sedou: However, the earlier the better.

Drissa: Let’s try with 8:46

Sedou: 8:46 + 23

Drissa: Yes, 9:09

Tutor: So, which tram do you take?

Drissa: 9:13

Tutor: 9:13 + 14, which time do you arrive at the hospital?

Drissa: 9:27. This is ok!

Drissa writes the answer on the app. Drissa and Sedou have proceeded by trial and error, with a significant amount of feedback provided by the tutor. As the episode unfolds, we notice that the students gradually acquire the utilization schemes regarding the bus and tram schedule: they select a time from the bus schedule, then they compute the time they need to arrive at the tram stop, then the time they need to arrive at the hospital.

Few minutes later, the tutor asks Drissa and Sedou if they have found the activity interesting. They comment that all the tasks were interesting and useful for them, in particular Drissa says: "I think that the time schedule for the public transportation is very useful. Very good. We all arrive at the train station 40 minutes, 30 minutes in advance, because we do not know how to read the time schedule. The Italian people arrive 2 minutes before departure. We need to know how to read the time schedule, and not to arrive so early at the station". This comment arises another interesting theme concerning the app use, that is: the perceived usefulness of the task, and the consequent motivation that may derive. Affective issues related to the use of an app of this sort have been proven important by Andrà, Parolini and Verani (2016), however they are not the focus of this work. Further investigation is, thus, necessary. We can see that one of the features of the app that is related to the process of instrumentalisation is the possibility to start over again, trying different times and see which one is best. Being able to catch teen-immigrants attention for an extended period of time in order to solve a task is one of the features of the app that we aim at developing.

DISCUSSION AND CONCLUSIONS

The goal of our work is to understand the process of instrumental genesis related to the app designed for teen-immigrants, in order to inform a second step of design of the app. To this end, we want to figure out the features of the app that either promote or obstruct the instrumental genesis that supports the acquisition of basic mathematical skills. The episodes relate to different contexts of everyday life: the internet monthly fee for mobile phone, the price of items at the superstore, public transport. In the first and the third cases, Drissa and Sedou show difficulties in understanding the task: in the case of extra giga, neither Drissa and Sedou have used them, so the tutor has to intervene and asks them to solve the problem arithmetically; in the case of the trip to the hospital, they have experience of it, to the point that they say they are frustrated by the time they need to read a bus schedule, and they struggle for completing the task. The reported episodes further allow us to observe that difficulties of different kinds emerge: a difference in the representation of the mathematical objects (e.g., decimal numbers and multiplication), the use of the calculator embedded in the app, the ambiguous meaning of the equal sign, and reading a bus time schedule.

As we observe how teen-immigrants overcome these difficulties, the process of instrumental genesis unfolds. We notice that the process of instrumentation goes on quite straightforwardly, since Drissa and Sedou progressively acquire the utilization schemes that are necessary. In fact, the first time they make a mistake, or they do not know how to proceed, they learn how to deal with an unfamiliar situation and became able to effectively apply what they have learnt in the subsequent tasks. For example, in the first task Drissa faces difficulties with the point for the decimals and equal sign for submitting the answer, but in the second task he immediately writes 1.50 and uses the equal sign appropriately.

With respect to the urgency of redesigning the app so that it promotes teen-immigrants' learning without the support of a tutor, we can conclude that it is necessary to find a way to avoid the difficulties related to differences in the representation of mathematical objects. For example, a tutorial that pops up in the first tasks, or when appropriate, and shows how to proceed. The bus schedule task deserves special attention, given the importance it has for teenagers like Drissa and Sedou. The structure of the feedback to support the understanding of the bus schedule needs to be more complex, and the single task may be subdivided into more tasks. The bus schedule task further shows that immigrants persist on it, since it is relevant for them.

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INTERACTIVE LEARNING ON DEMAND: A CHAT-BASED APP FOR STATISTICS EDUCATION

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In addition to course meetings, the learning process of students includes many components of self-study. While communication with lecturers about individual questions is difficult in large classes, access to help is even harder to get in the self-study phases. This project provides an app and tests it in a statistics course that supports all learning steps and learning locations. In form of a chat that answers automatically via artificial intelligence, the student is supervised and challenged at all times.

Keywords: artificial intelligence, individualization, pedagogical conversational agent, self-study, supervision

CURRENT SITUATION AND CHALLENGES

At universities many courses are held in the form of a large-scale lecture. Students have the role of listeners and recipients in this format. Taking up students´ prior knowledge is only possible to a limited extent and the individuality of the learners has hardly any room. The individual questions of the learners usually have little space in a lecture. The frontal format as well as the often extremely poor teacher-learner ratio result in learners receiving little direct support (De Paola, Ponzo, & Scoppa, 2013).

In many cases, these problems will be addressed by tutorials or exercise formats in smaller group sizes. However, also in tutorials due to resource constraints there is no possibility to supervise each student individually. Furthermore, interindividual differences remain poorly considered and personal prior knowledge can only be used insufficiently. The content-related and organisational questions of the individual therefore find only limited space in this form of present teaching.

While such a learning environment represents a challenge for all students, it is a problem especially for students in the introductory phase of their studies. First-year students are not yet familiar with the university´s learning culture and are not accustomed to the high level of personal responsibility. In particular, the high proportion of self-study in learning is unusual for these learners and difficult to organise. This is made more difficult by the poor availability of personal support in such phases. Questions regarding content or organisation often have to be put back in this phase and will only be answered in the next class. The learning process of the students is thus interrupted and often broken off, so that the low level of supervision becomes an obstacle to continuous learning.

For many students in these situations there is also the problem that they do not have the confidence to ask their questions in front of an audience (Hanna, Shevlin, & Dempster, 2008). In this case, social and subject-specific fears prevent learners from continuing the actual learning process. Learners then do not have the opportunity to realise their full potential.

Introductions to statistics for students outside the STEM area typically are a subject encountered by many students with negative attitudes and anxieties. They are very diverse in their mathematical skills and transfer negative experiences to statistics (Berens, 2018). The frustration tolerance can often be regarded as very low and learners tend to ask questions in small steps when problems arise. Since such introductions to statistics are usually organised as large lectures with practice sessions, the problems described above arise in a special way.

TEACHING CONCEPT

In order to better address the individual problems of learners and, in particular, to reach and help anxious and unmotivated learners, the project complements the existing present teaching with additional support. For all phases of the course, especially the self-study phases, a system is created that provides on-demand support. Based on the widespread use of messenger services (such as WhatsApp, Facebook Messenger, Telegram or Threema), students are provided with a smartphone app that answers student questions in a chat immediately and automatically. In this way, questions about the content of lectures or the organisational framework can be answered at any time to meet individual needs. The constant availability of the artificial tutor supports the continuous learning process of students, which no longer has to be interrupted until the next contact with the tutor or lecturer if problems arise.

In addition, the added value for lecturers is that an aggregated, anonymous overview of frequently requested learning content is made available. Teachers can use this information as feedback and starting point for the further preparation of subsequent courses in order to pick up problematic learning content in the lecture or tutorials and to deepen fields of interest. In addition, the app will be able to answer frequently asked organizational questions automatically, so that there will be less demand for lecturers.

The project schedule envisages that the pedagogical conversational agent will be technically implemented by mid-March 2019. Laboratory experiments for the first tests will evaluate the functionality of the technical system. As a field experiment starting in April 2019 with the lectures of the summer term the agent and the underlying teaching concept will be tested in an introductory statistics course for social scientists.

In addition, both the underlying teaching concept and the pedagogical conversational agent are to be researched using the example course within the framework of university teaching with qualitative and quantitative panel interviews. Thus, we hope to evaluate the potential of continuous, close supervision in large courses and to derive statements on how student-friendly supervision should be organised in order to enable continuous learning. Beyond our concrete teaching context, we would also like to make a contribution to the testing of artificial intelligence in university teaching.

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USING A MATHEMATICAL FORUM IN A GRADUATE COURSE: THE NATURE OF RICK'S AND JOHN'S PARTICIPATION

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Our work focuses on a traditional graduate course paired with online asynchronous forum that involved engineering students. The number of students in the course was larger than expected and the forum was suggested to all students as a way to interact between students and with the teacher. We use network analysis to look for central students in the network of the interactions within the forum and then interview two of the most active students to better understand their use of this technological tool. In this paper, we present our findings with the network analysis and the feedback the students gave us to understand the reason why they use the forum.

Keywords: network analysis, online math forums, students' interaction.

INTRODUCTION AND THEORETICAL FRAMEWORK

In the last years, many universities adopt their own educational online platform, which allows instructors to share materials and provide homework, and allows students to interact with each other and with the teacher to discuss the topics of the course. Indeed, Byman, Järvelä, and Häkkinen (2005) point out that on-line discussions encourage the collaborative process to negotiate meaning and construct knowledge, which is a fundamental aspect of the learning process (Vygotsky, 1980).

According to Kontorovich (2018), we distinguish between general *online asynchronous forums* (OAFs) and *course accompanying online asynchronous forums* (CAOAFs) associated to an undergraduate courses. The main differences of a CAOAF, with respect to an OAF is that the participants are students of the course, not anonymous registered members and the online discussions are usually relevant to all students in the course (ibidem). Another difference is that in CAOAFs the lecturer of the course can participate in the discussion and, in this case, she assumes a special role. In OAFs, all participants are at the same level. The work of Perkins and Murphy (2006) concerns the categorization of participants' posts in an OAF, finding that the majority of students post to ask clarification and/or to evaluate some aspects of the discussion.

The theoretical background of our research regards the results in literature that point to the design of mathematical forums, where students acquire mathematical knowledge by construction, not only by transmission (Engelbrecht & Harding, 2005). Van de Sande (2011) describes the interactions in an OAF as a mathematical help exchange between seekers and providers, since there is a voluntary participation that is not restricted to any particular course and theme. Moreover, van de Sande (2011) distinguishes users of the forum between core and peripheral users, according to their post frequency. However, students' online interaction in math forums does not have to be confused with 'simple' interaction, such as posting and commenting: in a forum, indeed, the students participate to satisfy needs, to accomplish a desire, to negotiate identities, to learn and to discuss (Andrà, Brunetto &

Repossi, 2018). Recent research, exploiting network analysis to investigate the potential of tasks, identifies three kinds of students' behaviour in mathematical CAOAFs. These three kinds are the following: specialised students who tend to comment a lot on few selected tasks; students that comment a few on many tasks; and very active students, who comment a lot on some specific task, but who also comment on the others (Andrà *et al.*, 2018). Kontorovich (2018) argues that an online dialogue between students seems to indicate more engagement within the course material, moreover in order to seek and provide help on online forums students need to recall and reorganize the learned material. This helps not only the help-seeker but also the help-provider to better understand the topics of the course. A special focus of these researches is the intertwining of cognitive and affective aspects in students' interaction and their learning mathematics in a forum.

The aim of this work is to address the open problem, summarized by Kontorovich (2018) with the following question: *which elements bring some students answer this online call for help?* In particular, we refer to the previous problem considering the following research questions: (1) how can we identify active students? (2) Which are the reasons that push students to actively participate in a CAOAF? In order to answer these research questions, firstly it is necessary to identify the most active students, and then to investigate the reasons for their behaviour: the methodology describes the context of our research and the method of analysis.

METHODOLOGY

The sample of students we consider in this work is taken from the graduate course in Game Theory, delivered in English to the students of Computer Science and Management Engineering, at Politecnico di Milano. The course lasted 8 weeks and its schedule was 5-hours per week for theory (a full professor lectured this part) and 3-hours a week for exercises (a tutor, who has a post-doc fellowship taught this part). The number of students enrolled in the course doubled with respected to the previous years, reaching about 400 students, but the number of students who attended classes was less than 200.

At Politecnico di Milano there is an educational platform, called BeeP and hosted on the website beep.metid.polimi.it, where teachers share materials and news with the enrolled students in each course. The platform BeeP allows a lecturer to open a CAOAF in which both teachers and students can post. In the Game Theory course under our analysis, the forum was opened at the beginning of the semester to allow students to share doubts and questions among them and with the teachers. We highlight that since the beginning of the course, the tutor gave high importance to the use of the forum and prompted students to use it instead of asking her via email. The tutor was aware that an open public discussion is more helpful to students than a private one.

The data collected is the set of messages posted on the forum by students. The total number of students involved in activities on the CAOAF is 36. Considering that only 200 students were attending classes, this means that less than 20% of the students was active on the forum. The CAOAF was available for all students enrolled in the course, what about the other 80% who never posted on the forum? On one hand, in online communities there are 'lurking' users (Preece, Nonnecke & Andrews, 2004), namely students reading the post but who never write something. On the other hand, in the CAOAF we are analysing, it was not possible to track these users and distinguish them from students who never accessed the forum at all.

In the forum there are more than 200 messages, mainly written in English and organized in 53 threads. Threads, all started by students, mainly regard questions about homework and exercises, done in class by the tutor, Table 1 reports some examples. A small number (4) of threads is about theory, while some others (6) are about organizational aspects of the course. We notice that the subject of the course is very specialized, thus there are not so many online resources students can refer to, in comparison with other topics such as linear algebra and calculus. Despite the specialized content, the written communication is easier than an average mathematical course since specific symbols such as integrals, limits, graphs are not required for the content of the course. In Table 1 there is an example showing the level of complexity of the symbols used by students.

Solutions of exercises 49 and 51 Did anyone else have found different solutions of exercises 49 and 51? As for exercise 49 my solution it's swapped with respect to the one on the pdf. I obtained for the first player (1-2/a, 0, 2/a) for a>4 and (1/2, 0, 1/2) for 3<a<4. As for exercise 51, I obtained for the first player (2/(1+x), 1 -2/(1+x)) for x>2 and (2/3, 1-2/3) for x<=2.
Non empty core super additive Is it true the game is non-empty if and only if the game is super additive? To check if the game is empty, we just have to check if it's super additive? Do we also have to check v(S) + v(T) <= 2v(N)?

Table 1. Examples of starting messages of threads

The tutor was an active user of the forum, usually she made only general remarks, for instance providing some external links, and directly answered the questions only when it was strictly necessary, for instance because students explicitly asked her intervention, or to reassure students that they were doing correctly (see also Swan *et al.*, 2000).

To study the interaction among students and identify different kinds of active students we build a network in the following way: each student is a node in the network and there exists a link between two students if they posted something in the same thread. The network is weighted because the more students "meet" in different threads, the stronger the link between their nodes is. The network is also undirected, because of the symmetry of the link. We underline that the tutor is not a node in our network, since we are interested only in student-student interactions.

The analysis of this network allows us to identify the students that are most central, by looking at their degree, at their weighted degree and at their betweenness (Newman, 2010). The degree of a node is the number of links of each node, in our network it represents the number of students each student discussed with in the forum. The weighted degree of a node is defined as the sum of the weights of the links of each node: this allows us to detect not only the number of students each node discussed with, but how much they posted. The betweenness of a node *i* is related to the concept of distance among nodes and it measures how much "longer" would be to pass from node *j* to node *k* in case node *i* is removed from the network. Roughly speaking it measures how much a node connects the network: removing a node with high betweenness may split the network into two or more components.

Moreover, we are interested in establishing if students are core or peripheral users, as described in van de Sande (2011). To that end, we adopt the concept of *k*-core from network analysis (Newman, 2010). The *k*-core is the set of at least *k* nodes such that each of them has at least degree *k*. We argue that active students, as identified in van de Sande (2011), are the students corresponding to the node in the maximal *k*-core of the network.

We integrate the quantitative analysis done with the network analysis with a qualitative analysis on the students, based on the answers to a set of questions (see Table 2), designed by the teacher to better understand the use of the forum within her course.

This set of questions allows us to distinguish students' participation in forum as help-seeker or help-provider (Kontorovich 2018), and the reasons why some students are active members of the forum. We select two students from the 5-core of the network, one of them, who is fictitiously called John, passed the exam with the maximum grade at the first attempt, whilst the other, Rick, failed the first exam.

q1. Was the online forum a useful tool? Why?
q2. What were the main reasons that led you use the forum?
q3. Which was mainly the goal of your messages in the forum? Why did you keep using it and participating actively?
q4. What would you suggest to next year students about the use of the forum for this course?

Table 2. Questions for active students in the forum

DATA ANALYSIS

The network is depicted in Figure 1, where all the 36 students who wrote at least once on the forum are shown. The dimension of the nodes is proportional to their degree: nodes with a higher degree have a larger radius. The links between nodes have different thickness, which is proportional to the weight of the link. The seven nodes in blue correspond to students who wrote only in one threads and did not receive an answer from another student; the eighteen green nodes are the peripheral students, which do not belong to the 5-core of the network. All other nodes are in the 5-core and correspond to the more active students: we highlight in red the node with degree 15 that corresponds to John and in yellow the node with degree 17 associated to Rick.

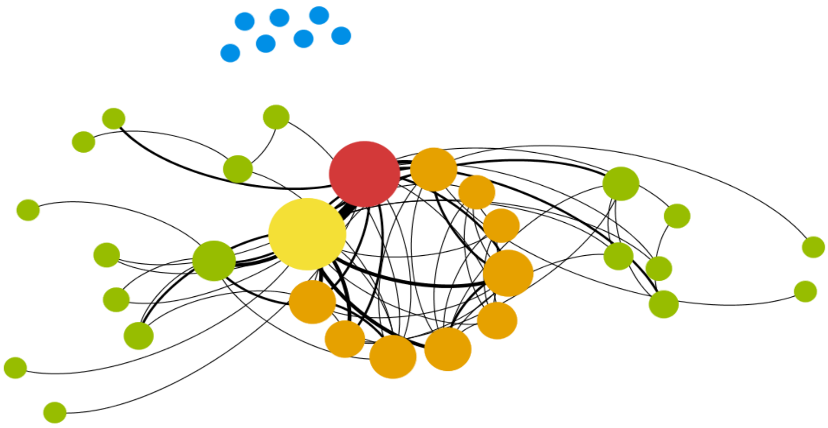

Figure 1. The network associated to students on the forum

The network analysis allows identifying the core, which is composed by 11 students. Among them Rick and John stand out since their degree and weighted degree are almost the double with respect to the average of all students in the core (see Table 3). This means that Rick and John wrote in threads interacting 36 times with 17 peers and 29 times with 15 peers, respectively. Moreover, Rick and John's betweenness is around 0.2, while the betweenness of other students in the core is in a range from 0 to 0.13. This information confirms that Rick and John are the most active users in the forum, both because they posted a lot and because they are well-connected with all the other nodes, that is they took part in many threads and engaged in online discussions with different peers.

	Degree	**Weighted Degree**	**Betweenness**
Rick	17	36	0.208
John	15	29	0.219
Average of the values of the students in the 5-core	8.63	13.54	0.065

Table 3. Properties of the nodes associated to Rick and John

Nevertheless, this set of properties does not provide insights about the reasons why they are active, and their attitudes towards the use of the CAOAF. To that end, we need to take under examination the other set of data composed by the answers to the questions in Table 2. In the following, we summarize their answers. We recall that John passed the exam right at the end of the course with the maximum grade, whilst Rick failed the first attempt and had to take again the exam after a couple of months. Both students were attending the first year of a graduate course in Computer Science Engineering.

First of all, John makes a general comment: he frequently uses other forum online, such as Stack overflow, for his personal and academic projects; and the platform hosting the forum under analysis is not so friendly, because threads are not well-organized and it is a bit difficult to search for other people's replies. He underlines that using a CAOAF can help all the students' community, because they frequently have the same doubts and can benefit from a general discussion on some topics. From his words, we can argue that John is expert of (CA)OAFs, and that he is aware that learning is also a social process, not only a private one.

Regarding questions q1 and q2, John says that his main goal in using this forum was to find some difficult exercises to practice, since he did not have much time to prepare the exam.

We infer that John is a motivated student, since he is looking for difficult exercises but he is also a pragmatic student since he wants to exploit his time in studying for the exam. It is well known that the management of time is one of the crucial aspects for university students to succeed in their career.

John, looking at what people asked and discussed on the forum, was a bit disappointed by his mates: he writes that there were mainly trivial questions and other students did not exploit entirely the potential of this tool. We recall that John wanted challenging tasks that his peers did not provide; this could had led him to quit the forum. However, his answer to question q3 sheds light on why John kept using it: "It was more funny to answer the questions on the forum than only doing the exercises. Answering to other students' questions usually helps me to communicate ideas and concepts in a better and more clear way". From his words, we can infer his positive attitude towards the use of the forum and the learning process: John, by using the word ‘funny’, shows that there is an affective dimension in his approach; moreover from this answer it emerges that he is aware of the importance of communication, in terms of socio-cultural dimension, in the learning process.

Finally, answering question q4, John suggests to next year students to use the forum actively, since it can be a good place to test themselves before the exam and to find difficult and particular exercises. What is relevant in this answer is that John stresses about an active use of the forum by all users to share difficult tasks and to improve their own study. We can notice that not only affective reasons took John inside the forum, but he also evaluated cognitive ones: he is saying that learning is more effective in this way.

We now analyse Rick’s answers. First of all, he replies to question q1, saying that discussing on the forum is more helpful than a short concise answer from the tutor, since in a forum there is a more open conversation that everyone can join. We notice that Rick is aware that open discussion among peers can help the learning process, as well as John points out. Rick adds a further element: he believes that teachers provide solutions, without giving space to open discussions with students. We claim that Rick considers the tutor as an authority who delivers the knowledge and does not help the construction of that, while forums are places for trial and error, and this improves a learner’s experience.

Answering to questions q2 and q3, Rick says that he started using the forum because the tutor had said to do that if students had some doubts, and he actually had some questions. He adds that CAOAFs were not very common among other courses, in his experience as a student. He kept using it to ask and answer questions, because he noticed that it was a useful tool and many other students were participating. We note that Rick is less experienced than John with the use of (CA)OAFs; and he is a dutiful student, reinforcing the fact that tutors are authorities that should be listened to. Actually, Rick followed the instruction of the tutor, asserting that he used the forum mainly to ask doubts. Rick provided also answers to his peers’ questions when he could, becoming an active member of the community. This indicates that Rick wants to be part of the community, not only as help-seeker but also as help-provider.

Rick’s answer to question q4 is that he definitely suggests it to next year students, because it is a useful way to prepare for the exam. Rick writes that he understood many things from the online conversations, either asking or trying to provide some answers. Rick says that, in order to use the forum, he had to write clear explanations, and it was possible to do that only when he was mastering the topic, otherwise other questions and doubts were coming up. We note that Rick has something in common with John: they are both aware of the importance of communication in the learning process. However, there is a difference between the two students: John was not satisfied with the level of the discussions, he was looking for challenging exercises and deep contents; while Rick was satisfied because on the forum he found what he was looking for, namely answers to his doubts and the opportunity to discuss with his peers.

DISCUSSION AND CONCLUSIONS

This work aims at addressing the issues regarding students’ motivations behind their participation in a CAOAF, problem that has been raised by Kontorovich (2018) studying the interaction of students in the CAOAF linked to a linear algebra course. In particular, we address these two research questions: (1) How can we identify different kinds of active students in a math forum? (2) Which are the reasons that prompt students to participate in a CAOAF actively? In order to do that, we adopt a quantitative approach using network analysis, and a qualitative approach analysing the answers to a survey.

From the network analysis we address question (1) identifying active students as the ones corresponding to the nodes in the *k*-core. This information is enriched by indices such as degree, weighted degree and betweenness. Many scholars, e.g. van de Sande (2011), detect core and peripheral students looking at the frequency of posts. Network analysis allows us to further enrich the characterization of active students, providing insights on the interactions among users, which would not be possible otherwise. However, there is still the issue of how to identify, in the context of a mathematical CAOAF, active students in terms of help-seekers and help-providers, since in our setting the two kinds of students are not a priori labelled as in van de Sande (2011). A more accurate design of the network could address this problem, for instance considering a directed graph, in which help-seekers could be characterized by a large number of links towards them.

Network analysis identifies the students who are the most active in terms of posting and interacting. As a pilot study, we use this information to select two of the most active students to interview in order to address question (2). From the analysis of their answers, we infer that not all active students use the forum with the same purpose. Even if John and Rick were both answering other people's questions, they were doing that for different reasons. John was looking for more deep discussions and difficult exercises to prove himself, while Rick was asking questions and answering in order to get answers for his own doubts, be part of the community and provide his own contribution to the discussion.

John is an help-provider, who seems to be a wise user, expert with online forums and who wants to find extra material with respect to what is discussed in class, his posts are in the category "assessment" (Perkins & Murphy, 2006). On the other hand, Rick is both help-seeker and help-provider, asking for clarification (ibidem): at the beginning he was motivated only by his trust in the tutor's instruction but then he found that the forum was a helpful tool and decided to use it to be part of the online discussions. Despite the different attitudes of the students, after this experience, they both are aware of the various benefits they can get by using a CAOAF. They recognize that it is helpful to share and discuss with peers the doubts, and that, to answer other students' questions, is necessary to have a more complete and deep knowledge of the topic, confirming the findings in the literature (e.g., Engelbrecht & Harding, 2005), namely mathematical knowledge is constructed and not merely acquired.

Our answer to question (2) is that some students might be looking at the forum to challenge themselves, while other students might resort the forum as a safe place for discussions among peers. Of course, a more deep analysis is needed to explore some of the aspects that are hidden behind Rick and John's answers, such as other affective dimensions, rewarding aspects, status among the community. Moreover, another interesting direction to develop this work is to interview all the 36 students who were active on the forum and use their answers to enrich the analysis. Therefore our answer has to be considered a partial one and the issue posed in Kontorovich (2018) is still open.

To conclude, we notice that the tutor plays an important role in the participation of a CAOAF, too. John and Rick's answers are particularly interesting also to address the problem of the small number of participants in the CAOAF with respect to the total numbers of students enrolled. Indeed, as John's answers suggest, the tutor may prompt students to use the forum posting more difficult exercises or specific tasks to improve online discussion. On the other hand, looking at Rick's experience, the instructors could warmly invite students to use forums, in order to make them aware of the importance of online conversations in the learning process.

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PREPARING PROSPECTIVE MATHEMATICS TEACHERS TO DESIGN AND TEACH TECHNOLOGY-BASED LESSONS

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This study is an ongoing research aiming to develop four prospective secondary mathematics teachers' skills in designing and teaching technology-based lesson plans within the scope of 14-week Teaching Practice course. The Dynamic Geometry Task Analysis and Instrumental Orchestration frameworks have been chosen as the conceptual basis of the study. In this study, qualitative research paradigm has been adopted and action research methodology aiming to plan a cyclical process of designing technology-based lesson plans through modifying, implementing and reflecting has been used. Data mainly consisted of individual/group interviews, lesson observations/field notes and multiple versions of lesson plans. Data analysis process is in progress, however, preliminary results focusing on one task of one participant indicated that the participant developed, evaluated and implemented her technology-based task in the light of adopted frameworks.

Keywords: mathematical task design, technology-based lesson plan, instrumental orchestration, prospective mathematics teachers, classroom practices.

INTRODUCTION

In many countries, using digital technologies has been considered as an effective way to improve teaching and learning in mathematics, which in the last few decades has given a direction to the research on the integration of technology into pedagogical practice. However, research show that the incorporation of technology into mathematics education in ordinary classrooms has been slow and that teachers' failure to change their practices with the use of digital technologies contrary to their optimism that technology would support a greater focus on conceptual understanding (Drijvers et al. 2013; Laborde, 2001; Monaghan, 2004). In other words, it has been indicated that digital technologies have not still been utilised as a means of transforming mathematics in ordinary classrooms as proposed and studied by the researchers in the field. Therefore, it is important for teachers to develop mathematical tasks that incorporate digital technologies in a more high-level cognitive demand as described by Smith and Stein (1998). Furthermore, other studies (Lagrange & Ozdemir Erdogan, 2009; Ruthven, 2009) have pointed out that teachers' plans might sometimes be overly optimistic regarding the potentials of technological tools. There is the additional chance that they might encounter unforeseen problems once in the classroom. These are issues that are particularly crucial in teacher education and need to be taken into consideration when working with prospective teachers. In line with this idea, the main objective of this study is to focus on and develop the skills of prospective teachers as future teachers of today in designing and teaching technology-based lesson plans in secondary mathematics education context.

Along this direction, two research questions guide this study: "How do prospective secondary mathematics teachers develop technology-based lesson plans?" and "How do they reflect these lesson plans into their teaching in ordinary classrooms?"

CONCEPTUAL FRAMEWORK

Two frameworks are used to reveal prospective teachers' skills in preparing and teaching technology-based lessons. By using the Dynamic Geometry Task Analysis framework (Trocki & Hollebrands, 2018), we aim to examine and improve the content of technology-based mathematical tasks particularly focusing on mathematical depth and technological action. In addition, by using Instrumental Orchestration framework (Drijvers et al., 2010; Trouche, 2004), we aim to illuminate prospective teachers' classroom practices with the use of digital technologies. The detail information regarding the frameworks is given in the following part.

The Dynamic Geometry Task Analysis Framework

For the aims of this study, in order to conceptualise the task design of prospective teachers, we utilise The Dynamic Geometry Task Analysis framework (see Table 1), which is recently developed by Trocki and Hollebrands (2018). By considering the research-based recommendations for using dynamic software in mathematics teaching and learning, Trocki and Hollebrands call our attention to two components: mathematical depth and technological action. They based the first component, mathematical depth, mainly on Smith and Stein's (1998) mathematical task analysis guide through assimilating the characteristics of lower level (memorisation, procedures without connections) and higher-level tasks (procedures with connections, doing mathematics) for the tasks designed in a dynamic geometry environment. For example, while Smith and Stein (1998) described doing mathematics as "require students to explore and understand the nature of mathematical relationships", Trocki and Hollebrands (2018) associate this for mathematical depth of Level 4 code. The component of technological action was also developed by considering the studies related to the use and importance of dynamic software in mathematics education (Arzarello, Olivero, Paola, & Robutti, 2002; Baccaglini-Frank & Mariotti 2010; Christou et al., 2004; Hollebrands 2007; Hölzl, 2001; Sinclair, 2003). For instance, Sinclair's (2003) research related to promoting students' exploration with pre-constructed dynamic geometry sketches shaped the technological affordance G in Trocki and Hollebrands' (2018) framework.

Table 1: The Dynamic Geometry Task Analysis Framework

Allowance for Mathematical Depth	
Levels	**Hierarchical Levels and Descriptions**
N/A	Prompt requires a technology task with no focus on mathematics.
0	Sketch does not have mathematical fidelity required to respond to prompt.
1	Prompt requires student to recall a math fact, rule, formula, or definition.
2	Prompt requires student to report information from the construction. The student is not expected to provide an explanation.
3	Prompt requires student to consider the mathematical concepts, processes, or relationships in the current sketch.
4	Prompt requires student to explain the mathematical concepts, processes, or relationships in the current sketch.
5	Prompt requires student to go beyond the current construction and generalise mathematical concepts, processes, or relationships.
Types of Technological Action	
N/A	Prompt requires no drawing, construction, measurement, or manipulation of current sketch.

A	Prompt requires drawing within current sketch.
B	Prompt requires measurement within current sketch.
C	Prompt requires construction within current sketch.
D	Prompt requires dragging or use of other dynamic aspects of the sketch.
E	Prompt requires creation/consideration of multiple examples from which one can generalise.
F	Prompt requires a manipulation of the sketch that allows for recognition of emergent invariant relationship(s) or pattern(s) among or within geometrical object(s).
G	Prompt requires manipulation of the sketch that may surprise one exploring the relationships represented or cause one to refine thinking based on themes within the surprise (adapted from Sinclair (2003), p. 312).

By conceptualising hierarchical levels of mathematical depth of a task and technological action used for such mathematical depth, we aim to examine how prospective mathematics teachers' task design evolve focusing on these components and to provide the tools of language to guide them in technology-based task design.

Instrumental Orchestration

After designing the technology-based tasks for a lesson plan, prospective mathematics teachers implemented these plans in their school placements. The instrumental orchestration framework (Drijvers et al. 2010; Trouche, 2004) is used to describe observed teaching practices aiming to conceptualise the structures of their designed activities. Instrumental Orchestration points out the necessity for teachers to guide their students' instrumental genesis by systematic organisation of their tasks and use of the various artefacts available. Guin and Trouche (2002) define instrumental orchestration underlining four components: "a set of individuals; a set of objectives (related to the achievement of a type of task or the arrangement of a work- environment); a didactic configuration (that is to say a general structure for the plan of action); a set of exploitations of this configuration" (p. 208).

Research on Instrumental Orchestration (e.g. Drijvers et al., 2010; Drijvers, 2012; Trouche, 2004) has provided operational descriptions, based on a combination of data-driven and theory-driven analysis, for several orchestration types relating to particular didactical configurations and exploitation modes (i.e. Technical-demo, Explain-the-screen, Discuss-the-screen, Spot-and-show, Sherpa-at-work). In this study, while preparing the lesson plans, prospective teachers were encouraged to consider the possible orchestration types relating to particular didactical configurations and exploitation modes in which they could achieve their planned objectives. These parts of instrumental orchestration are related to decisions, which teachers make before teaching in advance. However, Drijvers et al. (2010) introduced an additional third component, a didactical performance, in order to emphasise teachers' ad hoc strategies when unexpected aspect of the mathematical task or the technological tool occurs in classroom teaching with regard to chosen didactic configuration and exploitation mode. Hence, during school placements of prospective teachers, we focused on the parts that may emerge spontaneously during a lesson and the strategies they developed when unexpected issues occured in the classroom as a result of applying techniques instrumented by the available technological tools.

By focusing on already defined orchestration types in the literature (Drijvers et al. 2010; Drijvers, 2012), we aimed to picture what type of orchestrations occured in prospective teachers' classroom practices, particularly how they structured their technological actions for supporting pupils' mathematical depth by using their designed tasks.

RESEARCH METHODOLOGY

The aim of this study is to plan a process, where prospective mathematics teachers design and teach technology-based lessons in actual classroom environments followed by reflection on the planning and teaching experiences. In this sense, we studied actions that prospective mathematics teachers took to develop and improve their skills in planning and teaching technology-based lessons through the collaborative work with researchers. This was done through multiple cycles of action, data collection and analysis, and reflection. Hence, an action research methodology, one of the qualitative research methods, was adopted in the study as the enquiry at stake was to understand, evaluate, change and improve educational practice (Bassey, 1998). By using an action research, one aims to generate new knowledge by pursuing action and learning through action leading to personal or professional development through a cyclical process consisting of planning, acting, observing and reflecting (Koshy, 2005; O'Leary, 2004). In other words, researchers initially plan about what they want to change, then they act and observe the process and consequences of the change. In the following step, they reflect on the process and consequences, accordingly they re-planned the process (Kemmis & McTaggart, 2000).

Participants

Participants of the study were four prospective secondary mathematics teachers, who were enrolled in a secondary mathematics education program at a state university in Turkey. The participants were in their final year of the program during which they experienced classroom teaching in schools as a requirement for the 'Teaching Practice' course. The participants of the study were chosen based on the criterion sampling method (Patton, 2002). Their prior knowledge regarding the use of technological tools used in mathematics education was considered as a criterion. The chosen prospective teachers took almost all of the theoretical mathematics and mathematics education courses in their teacher training program, particularly, related to the use of technology in mathematics education such as Mathematics Software and Using Information Technologies in Mathematics Education courses. In addition, they had the opportunity to observe the real classroom environments for an academic semester while taking the School Experience course.

Research Plan

After identifying the focus of the study, an initial research plan including three cycles (see Figure 1) was developed by the researchers (the authors of this paper) within the scope of the 14-week Teaching Practice course. The initial research plan was revised based on participants' responses and actions throughout the process.

The first cycle was designed to educate prospective mathematics teachers regarding designing technology-based tasks and lesson plans in the light of two frameworks of the study. Based on this, prospective teachers developed lesson plans on the topic they chose. At the end of this cycle, the developed lesson plans were examined by the researchers and then each of prospective teachers was given individual feedback. The second cycle consisted of micro-teaching process, in which each prospective teacher taught their lessons hypothetically to fellow teachers and the researchers. In the following step, the whole group initially discussed each prospective teacher's micro-teaching process considering the classroom context and teaching with technology.

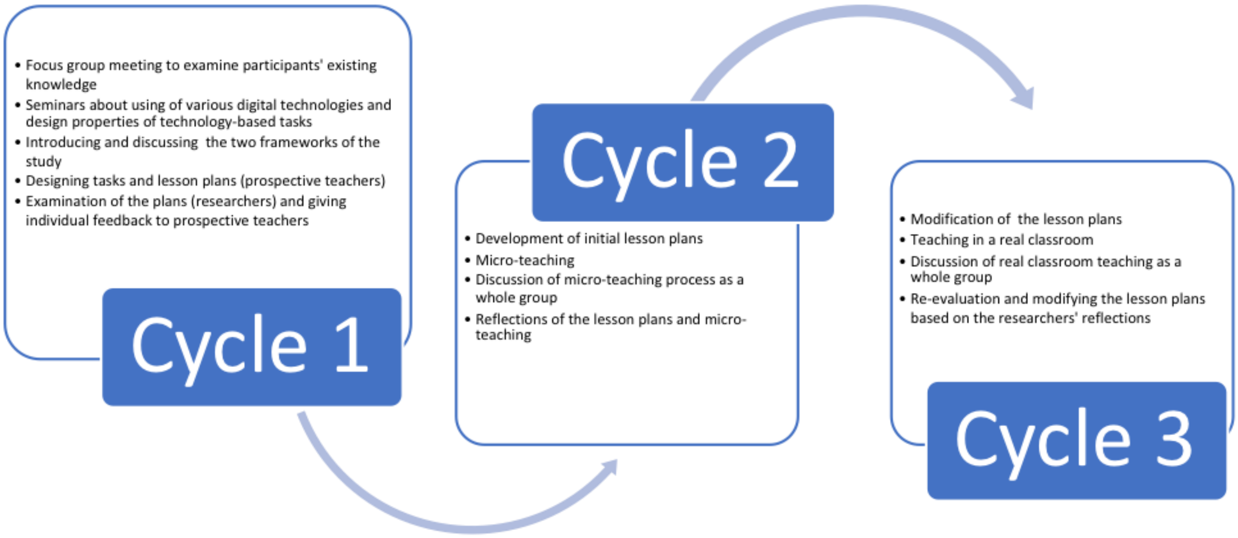

Figure 1: The three cycles of the research plan

Then, the revised lesson plans and video records of micro teaching were examined by the researchers and then each of them was given individual feedback. During the last cycle, as a first step, prospective teachers finalised the lesson plans based on their micro-teaching experiences and feedback given by the researchers and fellow participants. Then, prospective teachers implemented their revised lesson plans in real classrooms. Finally, they re-evaluated and modified lesson plans based on the classroom implementation experiences and feedback by the researchers as well as by the post-lesson discussion conducted with the whole group.

Data Collection

Within the scope of the action research, multiple data sources were collected to increase the validity of the research. In addition, the written and visual materials, which are important in analysing the findings, were included in the research (Flick, 2018). The data collection process, the period to be collected, and the goals of collecting the particular data source are detailed in the table below (see Table 2).

Table 2. The data collection process of the study

Data	The Period	Goal
The First Focus Group Meeting	First week of the research	to identify participants' thoughts, perceptions and existing knowledge regarding the use of technology in mathematics education.
Lesson Plans	-After training program (Week 6) -After feedback given based on first design process (Week 8) -After implementation in real classroom and Focus Group (Week 14)	to examine how prospective teachers develop/change/evolve technology-based lesson plans.

Microteaching Video Records	The participants taught their lesson plans to the fellow participants and the researchers before implementing them in real classrooms (Week 8).	to evaluate the participants' micro-teaching of their lesson plans and provide feedback on their micro-teaching.
Real Classroom Video Records	The participants implemented their developed lesson plans in real classrooms (Week 10-12).	to examine participants' implementations of the developed lesson plans in school placements.
Observation Notes	While participants implemented the lesson plans in real classrooms, researchers took observation notes (Week 10-12).	to examine participants' implementations of the developed lesson plans in school placements.
Video Records of Focus Group Interview	After all participants implemented the developed lesson plans, a focus group discussion took place (Week 13).	to examine each participant's classroom practices in a group discussion with the researchers and all the participants.

Data Analysis

All three cycles of the research were completed; however, data analysis is in progress and still ongoing. Therefore, the analysis process of only one participant of one technology-based task has been shared in this section. The designed technology-based task has been analysed according to Task Analysis framework considering the hierarchical levels of mathematical depth of the task and technological action used for such mathematical depth. The prospective teacher's own accounts during individual interviews about what mathematical depth she aimed in this task and what kind of technological action she aimed to use in order to achieve her intended depth has been identified. To characterise how this prospective teacher practiced this task in the classroom, we made use of the already identified orchestration types in the literature (Drijvers et al., 2010). In particular, a set of descriptors for teacher-led whole class teaching has been used as codes in order to analyse the structural interaction in teacher's classroom practices. Therefore, video records of the task implementations and the prospective teacher's instructions in micro teaching as well as in real classroom environment have been evaluated combining together with observer field notes and post-lesson interviews. Additionally, looking at the different cycles and multiple stages of her development in the process, changes in the designed task in terms of mathematical depth, technological action used and orchestration types chosen have been examined in detail. The triangulation aimed to be used in order to ensure the validity of data. Specifically, the plan of the designed task, the video records of the implementation of this task, observation notes, and the video records of focus group interviews allowed us to validate and verify the data.

PRELIMINARY RESULTS

As mentioned above, in this section, we share the preliminary results of one technology-based task designed by one participant. Particularly, we examine and describe the mathematical depth and the technological action of her designed task as well as the instrumental orchestrations while teaching this task in the classroom. Her chosen topic was Trigonometry and this specific task consisted of Unit Circle investigation with the use of GeoGebra. For the task, she prepared a dynamic GeoGebra file and developed 10 sequential prompts (see Figure 2) involving steps for students to explore trigonometric ratios. By designing those questions, she aimed to show how the lengths of the ratios change according to the angle, and how changes in angle affect measures of side length, and also wanted her students to discover while x axes represent the cosine, y axes represent the sine through dragging the movable point around the unit circle.

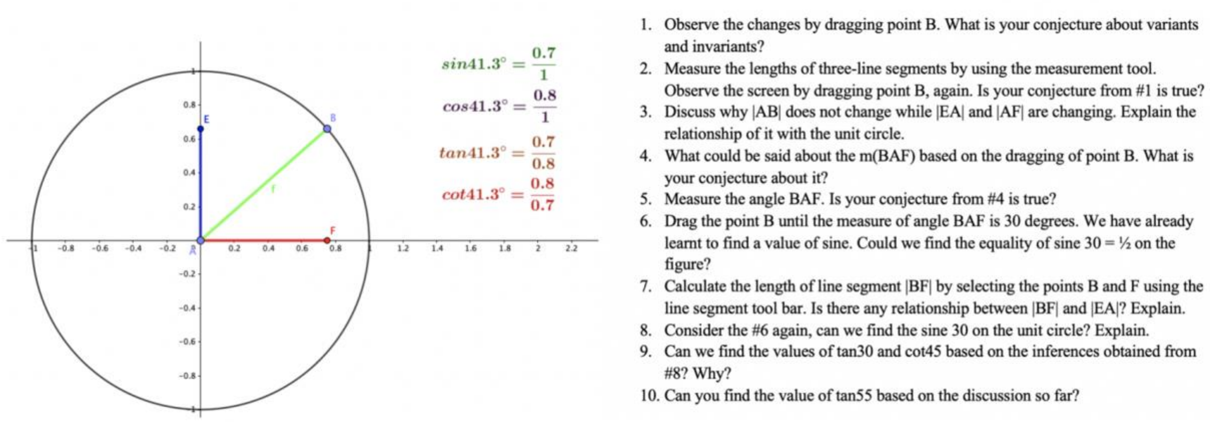

Figure 2. A screenshot from the GeoGebra file and 10 sequential questions guiding the task

At the end of the first cycle during the individual interviews with the researchers, she realised that the task would not reach the generalisation level that she aimed in terms of mathematical depth because of missing steps for students to reach generalisation. Hence, she revised and changed some questions in the sequence of the task. After micro teaching the designed activity, she stated that the main thing that she thought about was the meaning and importance of the instrumental orchestration framework in classroom teaching since she failed to manage her activity as she imagined and planned to practice. In this sense, micro teaching cycle of the process was of critical importance in enabling her to reconsider and improve the instrumental orchestrations of her task, specifically discuss-the-screen, link-screen-board and predict-and-test. In the last cycle, starting to implement of her plan in a real classroom also led her to change and improve the instrumental orchestrations used in this specific task. Additionally, she noticed that she could use a slider and animate it for the point around the unit circle instead of dragging it by hand, which affected her management of the classroom discussion and also helped her to achieve the aimed mathematical depth better. Particularly, adding a slider eliminated angle measuring related issues experienced during micro teaching and also prevented the limited dragging. The preliminary results show that she developed, evaluated and implemented her technology-based task in the light of adopted frameworks. The next step in the analysis process is to develop a more general model by comparing and contrasting parts of changes, evolution and/or development of four prospective teachers' technology-based lesson plans.

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TECHNOLOGY AS A RESOURCE TO PROMOTE MATHEMATICS TEACHING

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This paper aims to show how a range of technologies may be used as a resource to promote mathematics teaching. This study occurred in the framework of a STEM Continuing Professional Development Programme targeted to in-service primary school teachers. In this study we use a qualitative methodology and an interpretative approach by means of a case study. We conclude that technological resources enable the development of mathematical tasks appropriate for the primary school syllabus, that motivate students to learn this subject matter.

Keywords: Technology, mathematics, hands-on, mobile technologies, primary school.

INTRODUCTION

The importance of developing research on technology in mathematics teaching and learning is recognized by the mathematics education community. This is evident since the first Congress of European Research in Mathematics Education (CERME) in 1999, that included a Thematic Working Group (TWG) on technology among the seven proposed themes (Trgalová, Clark-Wilson, Weigand, 2018). The same authors conclude their reflection by referring the need to focus on emerging technologies in future European Researchers in Mathematics Education (ERME) congresses.

Advances in technology have the potential to enhance the implementation of integrative approaches related to STEM (Science, Technology, Engineering and Mathematics) by highlighting mathematics (Stohlmann, 2018). This author states that, although there is evidence of the potential of this approach, “there is a need for additional curriculum development and research on the effectiveness of various approaches” (p. 3)

This research is situated within a broader STEAMH (Science, Technology, Engineering, Arts, Mathematics and Heritage) project in Portugal, which began in 2013 and is coordinated by the first author of this paper (Costa & Domingos, 2018). Since 2015, in collaboration with a training centre and elementary schools, the project promotes in-service primary school teachers’ professional development. The team members are university teachers who design hands-on experiments and prototypes to promote STEAMH learning. Examples are “Sonicpaper” (Ferreira, Neves, Costa, & Teramo, 2017) and “SolarSystemGO” (Costa, Patrício, Carrança, & Farropo, 2018).

The focus of this study is to discuss technologies, namely videos, internet and mobile technologies, as a resource to design mathematical tasks related to the topic of sound. In this regard, our research question is: How may technologies be used as a resource to promote mathematics teaching related to the context of sound? We answer this question by presenting an empirical study that occurred in the framework of a STEM Continuing Professional Development Programme (CPDP) targeted to primary school teachers. In particular, we present a case study of a teacher who participated in the CPDP for two school years and used technology to design and implement mathematical tasks related to sound.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

The great lack of professionals in STEM subjects must be countered with an early intervention at the first grades of primary school (DeJarnette, 2012; Rocard et al, 2007). In this regard, it is recommended the incorporation of hands-on experiments in class, since it leads to significant improvements in performance and produce positive attitudes towards science (Mody, 2015; Johnston, 2005).

Kim e Bolger (2017) support the integration of STEM by involving teachers into interdisciplinarity lessons adequate to this approach. Kermani and Aldemir (2015) defend STEM integration in the first years of school, through teachers' professional development, being necessary to design resources to implement hands-on experimental activities.

STEM education can be both a form of innovation for teaching mathematics (Fitzallen, 2015) and can increase mathematical performance (Stohlmann, 2018). Also, technology has potential to integrate mathematics and to promote students' motivation and meaningful learning (Costley, 2014). In fact, technology resources, in particular mobile technologies, are efficient to catch children's attention and can engage students to learn mathematics and science, according to primary school syllabus (Costa & Domingos, 2017).

According to Pepin (2016) "A resource can be of material (e.g. texts) or human (e.g. colleagues) nature" (p.11). In particular, resources can be science and mathematical tasks, or the work developed by academics, to be used in the classroom or in sessions to promote teachers' professional development (PRIMAS, 2010). Resources are crucial for teachers who shape them according to their idiosyncrasies, preferences and personality during the process of their interpretation and design (Pepin, Gueudet, & Trouche, 2013). Tasks are teaching and learning activities, carried out in school environments, being inserted in the curriculum in action (Gimeno, 2000). But, according to Ball (2003), "No curriculum teaches by itself and content does not act independently of the interpretation of the professionals who convey it" (p. 1). In fact, it is the teacher's job to analyse curriculum contents in order to stimulate the students to learn them (Gimeno, 2000). In this regard, the teacher becomes "an active agent in curricular development, a modeler of the transmitted contents (…) conditioning the way students learn" (Gimeno, 2000, p166). This leads to the need of thinking and designing adequate teachers' Professional Development models and to select the contents for the Professional Development Program (PDP) (Gimeno, 2000, p166). A PDP will only be successful if teachers can apply in their classrooms what they learned and experienced during training (Buczynski & Hansen, 2010); and only achieve real effects if innovation is appropriated by the teachers and transformed into their own practice (Zehetmeier, Andreitz, Erlacher, & Rauch, 2015).

METHODOLOGY

In this research, we use a qualitative methodology and an interpretative approach by means of a case study. According to Yin (2005), a case study is an empirical investigation that looks at a contemporary phenomenon within its real-life context, allowing a generalization of the obtained results. Data collection includes participant observation (first author of the paper is a participant observer) and portfolios compiled by the teachers (Cohen, Lawrence, & Keith, 2007). Participant observation takes place in the workshops with the teachers (to learn and practise what they are expected to implement) and in their classrooms (to observe them in action). During participant observation notes and photos are taken for future analysis. Also, document analysis is performed on the teachers' portfolios (presented by the end of each school year), alongside their critical account of their CPDP and their proposals and implementation of innovative practices.

In the CPDP, teachers choose a science theme to develop in the classroom with their students. In this paper, we selected a teacher who used technology to develop several mathematical tasks related to sound. Teacher Marina (fictitious name) started the CPDP at 48 years old with 27 years of service and she was responsible for a 2nd grade class with 16 students. She developed STEM tasks related to sound in the school years 2016/2017 and 2017/2018. At the workshops of the CPDP, she learned about sound and how to perform STEM hands on experiments. These workshops were conducted by electrical engineering lecturers who designed prototypes to reproduce and measure the sound, amongst other tasks. They also used internet resources to provide the teachers with tools that they could use with their students such as videos and free software. An example is the Sound Meter application that allows sound intensity measurements (in decibels) and can be used on smartphones.

DATA ANALYSIS AND DISCUSSION

In this section, we begin by analysing the case study of teacher Marina to show her development of mathematical tasks with technology resources. The following information, namely tables and pictures result from participant observation and the teacher's portfolio.

Marina's case study

In class, with her students, Marina started by introducing sound by inquiry. In this regard, questions such as "What is sound?" were asked of her students. The teacher wanted them to reflect on this subject and to discuss possible answers. She also asked the students to make draws related to sound in order to understand better their perceptions about sound (Figure 1).

Figure 1. Students' perceptions about sound

After this discussion, Marina encouraged her students to find more information about sound using Wikipedia. She also showed videos related to this subject to engage the students to learn about the sound properties. After debate, students built a prototype to "visualize" sound (similar to the prototype teachers learned about during the sound workshop) and simulated "wave propagation" (Figures 2 and 3).

Figure 2. Prototype to "visualize" the sound of human voice

With the purpose of exploring mathematics, several sound measurements were taken, such as frequency (measured in hertz) and intensity (measured in decibels). For example, with the Sound Meter application in a mobile phone (Figure 3), the teachers asked each student to measure and record the intensity (in decibels) of several sounds in a table (Table 1).

Figure 3. **Wave propagation and sound intensity measurement**

Action / Decibels	Whisper	Speak	Laugh	Cry	Scream	Sing	Clap hands
40	X						
50							
60		X					
70							X
80			X	X	X	X	

Table 1. Each student registered the intensity of their sound in decibels

After each student had collected their data, they shared it with the whole class. Next, with the help of the teacher, another table with the information from the whole class students was built (Table 2).

	30	40	50	60	70	80	90
Whisper	0	12	4	0	0	0	0
Speak	0	0	0	4	9	3	0
Laugh	0	0	0	0	0	16	0
Cry	0	0	3	13	0	0	0
Scream	0	0	0	3	8	5	0
Sing	0	0	0	4	8	4	0
Clap hands	0	0	0	0	16	0	0

Table 2. Intensity measurements of the sound in decibels of all the students

With the collected data, several tables were built to promote the organization and processing of the data obtained from the measurement results (Figure 4), in accordance with the mathematics' syllabus.

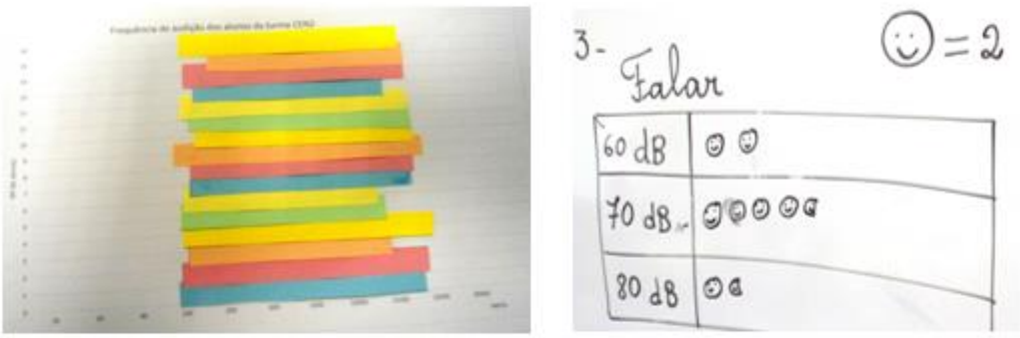

Figure 4. **Organization and processing of data obtained from sound measurements**

In figure 4, the first picture shows a graphic where the frequencies (in hertz) are represented on the horizontal axis and the students' names on the vertical axis. Each colourful rectangle represents the amplitude of the sound frequencies measured by each student. The second image is a pictogram, where each "face" represents two students who recorded the corresponding sound intensity measurement (in decibels) while they were talking.

In the 2017/2018 school year, Marina decided to conduct more tasks related to sound. She asked the students to look at the technical labels of home appliances to investigate the noise that they produce and bring the collected information to share with their peers. In class, all of the information was organized by the students with the teachers' supervision (Table 3).

HOME APPLIANCE	MODEL	NOISE LEVEL
Clothes dryer	Indesit IS41V	66 dB
	Orima	69 dB
	Indesit IDV75	69 dB
Refrigerator-freezer	Bosh	41 dB
	Candy	43 dB
	LG	37 dB
Washing machine	AEG	39 dB
	Balay	50 dB
	Candy	43 dB
Vacuum cleaner	AEG	76 dB
	Balay	64 dB
	Siemens	81 dB
Extractor hood	Candy – CBT6240X	64 dB
	Candy – CBG 640X	67 dB
	Meireles	65 dB
Refrigerator	Hotpoint/Ariston	35 dB
	Indesit	45 dB
	Samsung	41 dB

Table 3. Noise level of some home appliances according to the respective model

The teacher used this collected information to create worksheets for the students to solve several problems and exercises. Marina used technology to develop exploratory and investigative mathematical tasks. The technology resources included computer, smartphone, internet, video, Wikipedia and software such as the Sound Meter application. Mathematical tasks included problems and exercises mainly related to organization and processing data. Figure 5 gives an example of the work developed with the students.

In her final report, Marina argues that "it's possible to implement a transversal approach of contents, relating mathematics to science and technology". Regarding her students, she verifies that they "motivate and engage much more easily in these types of tasks". Below follows an excerpt of the reflections made by the teacher in her final report:

(…) the training action has contributed to the acquisition of new knowledge that will allow me to improve my professional performance and to have a positive impact in class, providing the students with diversified experiences of learning and development of scientific skills. (Marina's final report, June 2017)

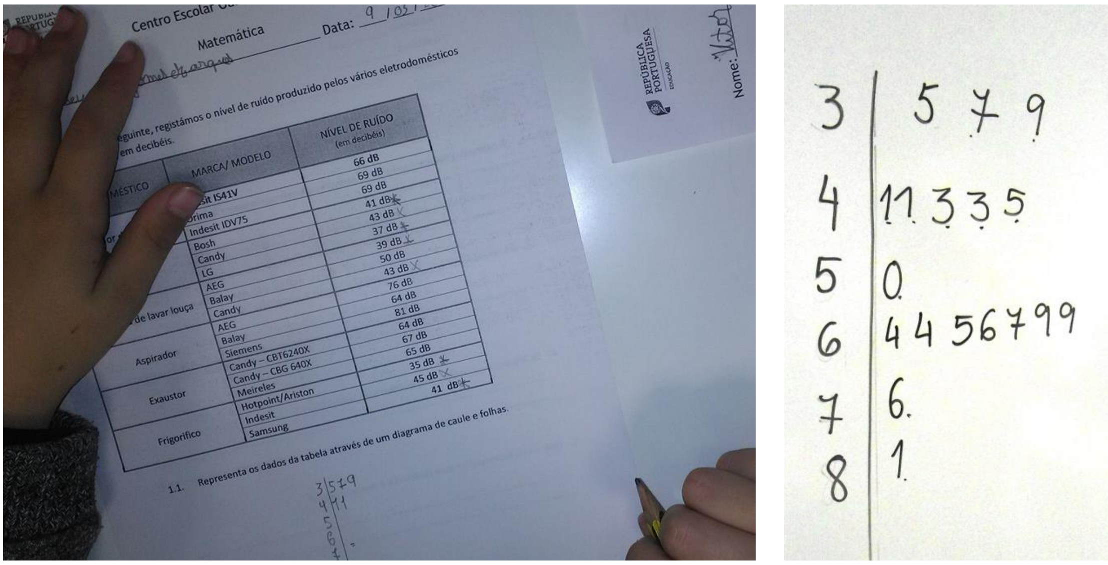

Figure 5. **Representation of the data from the table through a stem-and-leaf diagram**

Marina's perceptions show that participating in the CPDP was a positive experience that impacted her knowledge and practices. She also recognized impact on her students and that these practices promoted their interest to learn the subject matters. In summary, Marina used technology resources to design and implement mathematical interdisciplinary tasks related to sound and recognized that this kind of approach promoted the interest of her students to learn other subject matter such as mathematics. As exemplified above, mathematical tasks proposed by the teacher and performed by the students included problems and exercises related to several topics of the Portuguese curriculum such as "Numbers and Operations" and "Data Organisation and Processing", including tables, graphics and diagrams

FINAL CONSIDERATIONS AND CONCLUSIONS

This paper aims to contribute to existing literature by presenting an empirical study about the development of mathematical tasks using technological resources. In this regard, we presented a case study of a teacher who participated in a CPDP for two school years. In the context of her participation in the CPDP, she designed and implemented interdisciplinary tasks in her class. Indeed, teacher Marina designed and developed several mathematical tasks using appropriate technology, becoming "an active agent in curricular development" (Gimeno, 2000, p166).

Marina's example shows the development of exploratory and investigative mathematical tasks using technological resources such as computer, smartphone or the Sound Meter application. It appears that the CPDP was successful in that Marina applied in her class her learnings from her training (Buczynski & Hansen, 2010). Also, innovations were appropriated by the teacher and transformed into her own practice, which according to Zehetmeier et al. (2015) evidences that the PDP achieved real effects.

In the framework of a STEM CPDP, we verified that it is possible to engage teachers with technological resources that enable the development of mathematical tasks, appropriate to the primary school syllabus. In this regard, technology resources may innovate the teaching of mathematics (Fitzallen, 2015) and motivate students to learn this subject matter (Costa & Domingos,2017). In fact, teacher Marina recognized that students become more engaged with this kind of integrative approach. Finally, we conclude that technology may be used as a resource to promote mathematics teaching as exemplified by teacher Marina’s case study.

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STUDENTS' USE OF DIGITAL SCAFFOLDING AT UNIVERSITY LEVEL: EMERGENCE OF UTILIZATION SCHEMES

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This paper is focused on a pilot study involving a group of first year engineering students and concerning the design and implementation of two digital tasks on multiple representations of subsets of the plane. The tasks were engineered in order to provide university students with hints and feedback aimed at scaffolding their work. The analysis of the video-recordings of the students' screens while interacting with the tasks, supported by the reflections developed by students during audio-recorded interviews after the activity, enabled us to highlight utilization schemes that characterize students' use of digital tools for scaffolding their learning. We will also discuss how this analysis gave us suggestions for a future re-design of the tasks.

Keywords: teaching at university level, digital scaffolding, feedback

INTRODUCTION AND BACKGROUND

The teaching experiment reported in this paper is part of a wider study aimed at identifying effective tools and methods for the design and implementation of teaching-learning paths at university level (Alessio, Demeio & Telloni, in press). In particular, in tune with recent research (Descamps et al. 2006, Albano & Ferrari 2008, Calvani 2005), we focus on the role of digital tools and e-learning environments in supporting the teaching-learning processes at this level, because of the strong impact they could have on cognitive, metacognitive and affective levels of learning (Albano & Ferrari, 2008).

Here we present two digital tasks that we designed with the aim of providing students with hints and feedback to scaffold their work on the tasks. These tasks, focused on representations of subsets of the plane, are part of a set of online activities addressed to first year engineering students of the Università Politecnica delle Marche (Italy) attending to Calculus courses [1]. We will analyse students' interaction with the digital tasks, with the aim of identifying the instrument-mediated action schemes (Rabardel, 2002) that emerge and of collecting evidence to inform the future re-design of the tasks.

FEEDBACK AND SCAFFOLDING WITHIN A DIGITAL ENVIRONMENT

In the last years, the role of scaffolding in technology-enhanced environments has become a focus of interest for research on the use of digital tools in mathematics education. We refer to Holton and Clarke's (2007) definition of scaffolding, as an "act of teaching that (i) supports the immediate construction of knowledge by the learner; and (ii) provides the basis for the future independent learning of the individual" (p.131). Holton and Clarke (2007) stress on the fact that metacognition is an essential element in students' use of the provided scaffolding and subsequent development of awareness. When scaffolding is realized within technology-enhanced learning environments, the feedback provided by digital tools plays a central role.

In her review of literature on task-level formative feedback, Shute (2008) identifies some characteristics of effective feedback: it should be provided after learners have attempted a solution, it should be presented in small enough pieces so that it is not overwhelming and discarded, it should be written or via computer so that it could be perceived as unbiased and objective by learners, it should include elements of both verification (judgment of whether an answer is correct) and elaboration (relevant cues to guide the learner toward a correct answer).

In this paper, we interpret, in terms of *co-action*, the students' work with specific tools within a digital environment, to scaffold their learning. The term co-action is introduced by Hegedus and Moreno-Armella (2009) to interpret the nature of the relationship between a user and an environment (in particular, a software environment) *as actors and re-actors* in performing actions. They extend Rabardel's (2002) notion of *instrumentation processes*, that is those relative to the emergence and evolution of *utilization schemes* and instrument-mediated actions. Rabardel distinguishes two types of utilization schemes: the *usage schemes*, which are orientated towards tasks corresponding to the specific actions and activities directly related to the artifact; and the *instrument-mediated action schemes*, which incorporate usage schemes as constituents and "make up what Vygotsky called instrumental acts, which, due to the introduction of the instrument, involve a restructuring of the activity directed towards the subject's main goal" (Rabardel 2002, p. 83).

Hegedus and Moreno-Armella (2009) stress that the dynamic for co-action is possible thanks to what they call *border objects*, that is digital-dynamic embodiments of mathematical objects, initially defined within a paper-and-pencil environment, which can be meaningfully explored within the digital environment.

DESIGN OF DIGITAL TASKS

In this paper we study how first year engineering students, attending to a course on Multivariable Calculus (MC), interact with two digital tasks focused on different representations of subsets of the plane. MC is a pervasive topic in Mathematics for Engineering, since it is a prerequisite course for specialized subjects. According to the literature (Kashefi et al, 2012), many of the critical issues in MC are related with the coordination between multiple procedures and different representation registers. These difficulties arise in many fields of application, such as the calculation of a double integral, in which often students are not able to visualize the integration domain provided analytically, or in typical Mechanics problem, in which students need to find a parametrization, that is an analytic description, for a set given graphically. Martínez-Planell and Trigueros Gaisman (2012), focusing on two-variable functions, highlighted that, in order to help students to encapsulate this notion, they should be given opportunities to identify domain and range of functions in different representation registers and carry out the necessary transformations between registers to be able to relate information across them. The potentialities of digital technologies in supporting the learner's handling of multiple representations, widely recognised by research (Drijvers, 2013), make them effective tools to foster the understanding of Calculus topics (see, for example, Tall, Smith & Piez, 2001), in particular in the context of MC (Kashefi et al, 2012).

Following these suggestions, we designed and implemented online tasks with the software Geogebra, focused on conversions between different representation registers and designed with the aim of providing students with the necessary scaffolding to carry out these conversions. Here we focus on two tasks that require to describe a region in the plane (represented graphically) in polar coordinates (ρ,θ), by inserting the minimum and maximum values of the parameters ρ and θ in input fields (specific descriptions of the two tasks are in Table 1 and Table 2, together with screenshots that highlight the hints that are provided). The two tasks have the same structure: (a) a brief summary of the transformation between Cartesian and polar coordinates is provided on the screen; (b) within the graphical representation, some information is given (for example, coordinates of points), but itis not sufficient to perform the task; (c) students are asked to select one further piece of information among three alternative options (only one option is sufficient to describe analytically the region, another one is useful and one is useless); (d) students can change the selected information while they are working on the task; (e) students can submit their answers even if they do not select the further piece of information; (f) if the students submit wrong range for ρ and θ, a warning message appears on the screen together with two sliders for ρ and θ; (g) as the students move the sliders between the minimum and the maximum values previously inserted, the program dynamically shades the corresponding set in the Euclidean plane.

Task 1: The subset of the plane is a sector of a circle centred in the origin of the axes. To describe it analytically, students have to identify a further information (the ordinate of P) needed to determine the minimum value of the parameter θ (π/6). Here, the student has tried to answer without selecting the needed information.

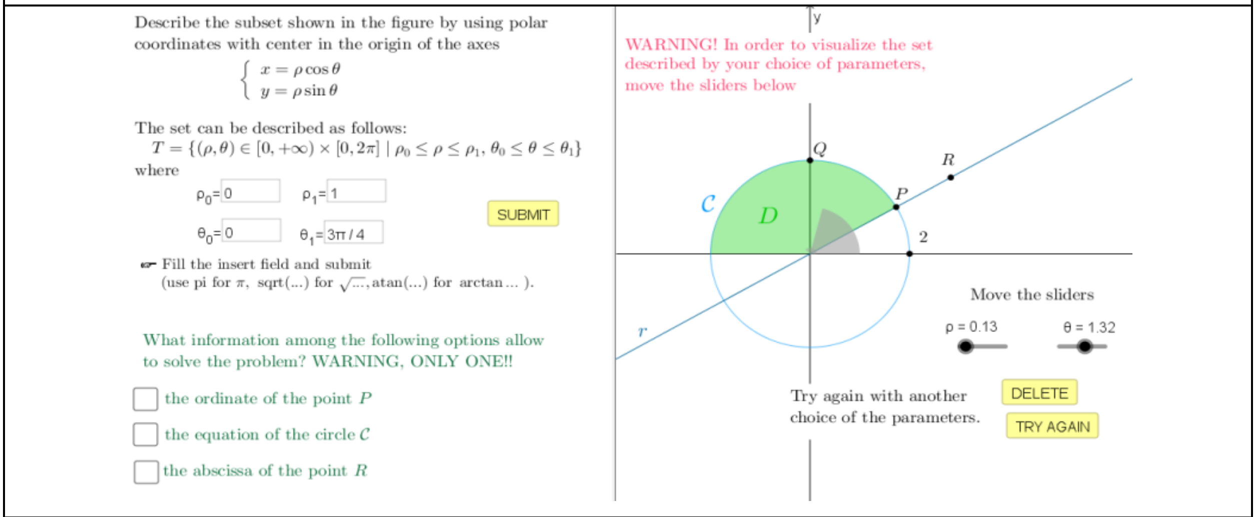

Table 1. Screenshot and description of task 1

Task 2: The subset of the plane is a sector of an annulus. The information provided in the graphical representation are enough to determine the centre of the polar coordinates system, the radii of the annulus and the maximum value of the angle θ. A further information (equation of s) is needed to determine the minimum value of θ (arctan2). Here, the student has not chosen this information.

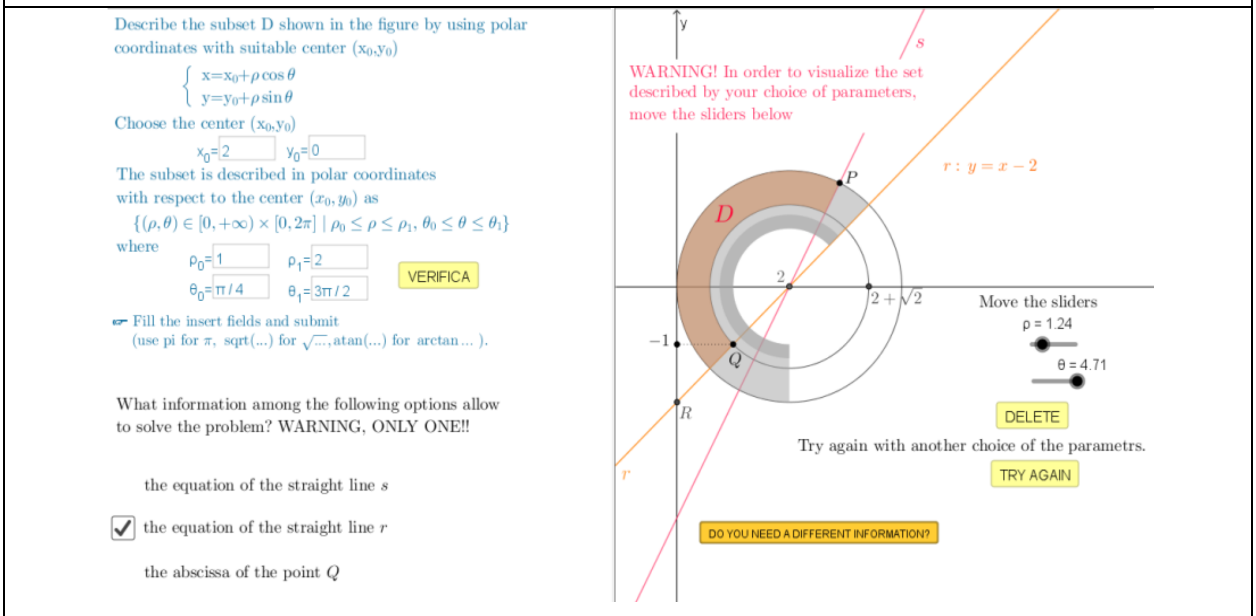

Table 2. Screenshot and description of task 2

This design of feedback and scaffolding is in tune with what Narciss & Huth (2004, in Shute, 2008) call informative tutoring, that is elaborated feedback that present verification (without supplying the correct answer) and strategic hints on how to proceed; moreover, it automatically appears after students have submitted a solution and it is presented in small and specific pieces (Shute, 2008). In particular, in this paper we focus on sliders as part of the scaffold within the environment, since they have been engineered in order to allow a direct and dynamic comparison between the representation in polar coordinates and the graph of the region. In order to examine how students use sliders to identify where the mistakes are, we decided not to provide information about the exact location of mistakes.

RESEARCH AIMS AND METHODOLOGY

The exploratory study documented in this paper has both a didactical and a research aim. The didactical aim is to test the effectiveness of the design of digital tasks (and, in particular, of specific hints and feedback) to scaffold students' learning, focusing on evidences that could inform the future re-design of the tasks.

The research aim is to analyse students' *instrumentation processes*, referring to Hegedus and Moreno-Armella's (2009) extended definition. When the students perform specific *actions* on the digital tools provided within the tasks, they have to interpret the *environment's re-actions* (that is the feedback from the environment) to activate the effective strategies to perform the task. In this paper, we focus, in particular, on two re-actions of the environment: the message that appears when the students submit their answers and the *border objects* (Hegendus & Moreno-Armella, 2009) which appear together with this message, that is the shaded figures that can be dynamically transformed through the use of the sliders. The main aim of our analysis will be to study the emergence of *instrument-mediated action schemes* (Rabardel, 2002) throughout the activity in the students' use of the sliders.

To analyse the co-actions between the students and the digital environments, we observed the students' interactions with the digital tasks and video-recorded them by using the software CamStudio. For each student, we developed an analytic description of his/her use of sliders, aimed at identifying categories of *instrument-mediated action schemes* that students adopt in using the sliders to scaffold their work on the tasks.

To collect further evidence about our re-construction of students' schemes (and, in general, about students' strategic approaches in exploiting the provided scaffolding), we designed a set of questions that we asked to students, during an audio-recorded interview, after their completion of the two tasks. For each task, students were asked to reflect about: (1) their selection and use of the needed further piece of information; (2) their use of sliders to perform the task; (3) the influence of their experience with task 1 when they faced task 2; (4) the usefulness of the feedback and tools provided to face the two tasks.

DATA ANALYSIS

In this section we discuss the main results from a pilot study that involved 15 first year engineering students attending to a MC course [2]. The students, who volunteered to participate in the study, individually faced, in a computer room, the two tasks described in the previous paragraph. Here we present the main results of our analysis of the data (video-recordings of the students' interaction with the task and audio-recordings of the interviews). We propose three paradigmatic examples, which enable to highlight categories of *instrument-mediated action schemes* related to the use of sliders to scaffold the students' work on the tasks.

When we analysed the video-recordings of students' screens in order to develop an analytic description of each student's interaction with the tool 'sliders', we focused on the following aspects: What sliders does the student use and in what order? How quickly does he/she move each slider and in what ways? How could the sequences of actions performed by the student be characterised?

Our analysis enabled us to identify three main *instrument-mediated action schemes* developed by students when they use the sliders to scaffold their work on the tasks. Each scheme could be associated with specific functions of the sliders, according to students' strategic use of them. We named these functions as *replacement* function, *diagnostic* function and *elaboration* function.

The *scheme related to the replacement function* of a tool is activated by those students who rely completely on the information that can be provided by digital tools, using them to find out the answers, without referring to theoretical knowledge.

The *scheme related to the diagnostic function* of a tool emerges when students use the tool to control the correctness of their answers, often before submitting their response, and to detect where the mistake is and, possibly, even how it could be corrected. This scheme is characterised by an alternation of the use of the tool and the reference to theoretical knowledge to find out the answers.

The *scheme related to the elaboration function* of a tool arises when the students refer to the tool to deepen their understanding of the theoretical knowledge subtended to the task and their interpretation of the representations involved in it. Also, in this scheme the use of the tool is alternated to the reference to theoretical knowledge.

Often, students adopt strategies that highlight combinations of more than one of these schemes.

Table 3 summarises our analytic description of three students' interaction with the tool 'slider': in the second column we summarise the description of the interaction with the sliders referring to the questions previously listed, while, in column 3, we summarise our re-construction of students' strategic approaches to the use of sliders, highlighting the instrument-mediated action schemes that emerge. To perform this re-construction, we referred also to the audio-recorded interviews. We stress that this re-construction is the result of our interpretation of what we observed in the videos and of what students declare in the interviews.

Student	Description of the interaction with the sliders	Strategic approach
Mario	He repeatedly uses the sliders in task 2. After having tried to substitute "known" values ($\pi/6$, $\pi/3$…) in the θ_0 input field and received a warning message, he works outside the digital environment, then he uses the slider tool. He alternatively moves the sliders very quickly. Each slider is moved throughout the whole interval (from the minimum to the maximum value of the parameter or vice-versa).	In the interview he declares that in both tasks he initially relied on perception, referring to 'known' values of angles and that in task 2 he used the slider to identify where he made a mistake (the values of θ_0): "when someone makes a mistake, as the $\pi/6$ I wrote, through the slider he can see (where) $\pi/6$ (is) and says 'it is clear! I made a mistake!'". Thanks to this, he identifies the further information needed to determine the exact value of this angle. He developed, therefore, the *scheme related to the diagnostic function* of the sliders.
Federico	He uses the sliders in both the tasks, with pauses between one use and the following one. The movements are almost slow and localized within the interval of variability of each parameter (he focuses on small sub-intervals containing the extreme value he has to determine). The sliders are moved one at a time, depending	He initially tries to insert a value in each input field to immediately display, on the screen, the effects of his choice (the environment's *re-action*), then he works separately at the identification of the different values to be written in the input fields, decomposing the problem in sub-problems. Since the movements of the sliders are slow and localised during the whole activity, it seems that he aims at understanding how each slider works and what information it provides to highlight the different roles played by ρ_0, ρ_1, θ_0, θ_1.

	on the parameter (ρ or θ) on which he wants to focus. He inserts one value at a time in the input fields (or two values that limit the same interval), and he clicks on "verify" to highlight the effects of his choice.	In this way, he is activating a local control of his work to deepen his understanding of the meanings subtended to the representation in polar coordinates. The main *scheme* connected to this approach is, therefore, the one *related to the elaboration function* of the sliders.
Giulia	In both the tasks, initially she inserts values corresponding to empty sets (for example $\theta_1 < \theta_0$) or to sets outside the visible window (ρ_0=10 and ρ_1=20), so she is not able to visualize the described set nor to use the sliders. Then, when this tool becomes available, she repeatedly uses the sliders, moving them throughout the whole interval, by means of fast movements. The sliders are alternatively moved very quickly.	In the second phase of her exploration, she uses the sliders not only to highlight where she made mistakes (*scheme related to the diagnostic function* of the sliders), but also to identify possible values of θ and ρ without performing calculations (in the interview she says that she used the sliders to understand "how the spaces were filled", "how the angle was moved"). In fact, she moves the θ and ρ sliders until the shaded region coincides with the one to be represented and insert the values she reads in the sliders. Therefore, she activates also a *scheme related to the replacement function* of sliders. At the beginning she only slightly modifies the values (for example, she writes ρ_0=1,42 instead of ρ_0=1,41[3]), as if she is not aware that most of them are only approximate values of θ_0, θ_1, ρ_0 and ρ_1. Then she works outside the digital environment to determine the correct values. We hypothesise that she realised the need of referring to theoretical knowledge, using the information she did not use in the other phases of her exploration.

Table 3. Analytic description and strategic approach to the use of the sliders

As in the three paradigmatic examples that we presented, our analysis enabled us to highlight that most of the students used sliders activating *the scheme related to their diagnostic function* or a combination of this scheme with others. Specifically, four students activated only the scheme related to the *diagnostic* function; other four students activated, alternatively, the schemes related to both the *diagnostic and elaboration* function; and one student activated, alternatively, the schemes related *to the three functions*. Finally, two students activated only the scheme related to *replacement* function. The remaining four students did not activate any of these schemes because they gave the correct answers without using the sliders.

CONCLUSIONS

In this paper we analysed university students' interaction with digital tasks that require to perform conversions from the graphical to the symbolic registers in the representation of subsets of the plane. The focus of our analysis was on students' development of *instrument-mediated action schemes* in the use of a digital tool provided within the tasks – the sliders - to scaffold their work.

We analysed the video-recordings of students' screens during the activity, performing an analytical description of students' use of sliders. This first step of analysis highlighted different behaviours in relation to the parameters to which we referred to code the video-recordings of students' screens (frequency of the use of the sliders, speed of the movements on the slider, extension of the movement, order in the use of the sliders). We, then, looked at this analytic description referring to students' audio-interviews to re-construct the typical strategic approaches in their use of sliders to scaffold their work on the tasks. This re-construction enabled us to highlight three main *instrument-mediated action* *schemes* associated with three specific functions of the sliders: *replacement* function, *diagnostic* function and *elaboration* function. Our analysis highlighted that most of the students adopted approaches that could be characterised by a combination of these schemes. In particular, in our data, the scheme connected to the *elaboration function* seems to be always activated to support the one related to the *diagnostic function*.

Our design was mainly aimed at creating an environment that could scaffold students' work on the tasks, fostering their activation of the scheme connected to the diagnostic function of the tool 'slider'. Although the analysis of the data showed the effectiveness of the design in this sense, the emergence of other schemes induces us to develop further reflections.

First of all, the emergence of the scheme connected to the *replacement function* displays a widespread pitfall of digital technologies, that is the risk that students thoughtlessly rely to digital tools without activating a metacognitive control on their use. This lack of metacognitive control is also connected to other "problematic approaches" that we observed. For example, some students did not use the offered information (necessary to perform the task) and mainly relied on perception. These students seem to implicitly have assumed that further information is optional and that all the angles involved in the required analytical description are "known". This is an example of implicature (Boero et al., 2008), that is an additional assumption with respect to the actual content of an information. We have deliberately chosen to give the possibility to submit the range of variation of the parameters even if the additional information was not used by students to highlight these implicit assumptions. Many students, in their interviews, displayed to have understood the need of balancing between perception, use of digital tools and reference to theoretical knowledge. Therefore, this aspect of the design was effective in making them reflect on their behaviour.

The identification of students who activated only the scheme connected to the *replacement function* suggests us that the explicit recommendation of using sliders when students send an incorrect answer is not enough. The intertwining of explicit (directive) scaffolding and tacit (less directive) scaffolding is therefore not well balanced. To better enable students to activate themselves at the metacognitive level, we will perform a re-design of these tasks to realize what Pea (2004) calls meta-scaffolding, offering feedback that support students in understanding how to use these digital tools to scaffold their work.

On the other hand, the occurrence of the scheme related to the *elaboration function* suggests us to better exploit this potentiality of digital tools in our future re-design, to guide students' interpretation of mathematical representations and deepen their understanding. Our idea is to insert the re-design of these tasks in a longer sequence of tasks with similar characteristics to foster an evolution of the schemes activated by students toward those connected to the *elaboration function*. Our expectation is that, in this way, students' metacognitive control of their work could develop, and the scaffolding could be spontaneously faded. The re-design and implementation of this sequence of tasks will be the focus of our future research, through which we will test the aforementioned hypothesis and also investigate the possible evolution of the schemes activated by students through the whole sequence.

NOTES

1. The online activities were developed and implemented within the University project "Didattica Multimediale della Matematica" (multi-media didactic of mathematics).

2. The topic introduced in the MC course are: description of subsets of the plane in Cartesian, polar and elliptic coordinates, theory of curves, calculus of functions of several variables (limits, derivative, differentiability), constrained maxima and minima, multiple integrals, integrals over curves and surfaces, vector fields, conservativity, work and flow of a vector field, ordinary differential equations.

3. According to our implementation, Geogebra displays 1.41 when the user writes sqrt(2).

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DESIGNING AND DISSEMINATING REVIEW CRITERIA FOR QUALITY OF TABLET APPS IN PRIMARY SCHOOL MATHEMATICS

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The last two decades of creating digital vis-à-vis analogue resources for learning mathematics have led to a hyper-production that is not sufficiently verified as being fully effective yet. Therefore, further design of novel resources along to sustainability and dissemination of existing ones through re-design appear as crucial issues that need to be tackled further. In particularly, this relates applications for touch devices in primary school mathematics. The scientific significance of this paper is to deepen the understanding of the quality of the tablet apps and design criteria for analysis of their potentials, e.g., related to contents about space and shape, through a design-based research approach as a convenient methodology for their dissemination along to their design. This is principally relevant for pre- and in- service teachers' informed decisions about implementation of apps in own teaching practices, aspects that we empirically investigate and qualitatively analyse through a DBR cycle.

Keywords: criteria for reviewing quality of tablet apps, potentials of touchpad apps, geometry, dissemination, design-based research

INTRODUCTION AND RELEVANCE OF THE THEME

Years of design research have facilitated identification and documentation of potentials of educational materials and digital tools for learning school mathematics. Yet, inspiring learning and teaching approaches involving tablet apps in mathematics classrooms seem still challenging for both research and practice. Researchers have already addressed the issue of sustainability of results of national projects referring mathematics didactical aspects of technology-enhanced learning environments, e.g. Krauthausen, (2012) in Germany. We witness interrupted projects or finished projects without any updates after completion both in national and international contexts. One reason therefore may be the appearance of apps that is exposed to many factors that cannot be controlled (Krawehl, 2010), e.g., the dynamic feature of the on-line market changing on a daily basis. Preventing bringing good practices up the rear might be a stimulating mission for tangible research actions. Sustainability and scale up of already established good practices remain thought provoking and require long-term active engagement of interdisciplinary research and practitioner teams. Research findings that suggest and study examples of the teaching and learning of mathematics with digital technology as well as innovative solutions of scaling and sustaining impacts were addressed in one of the four themes in the Call for papers at the 14th International Conference on Technology in Mathematics Teaching. The aims of this paper are to show a means for designing review measures for quality of apps that could assist teachers in deciding about the usefulness and efficiency of the apps in their own teaching practices whilst to embrace an approach for sub-sequent dissemination of partial outcomes during and after a complete design-based research process. Consequently, it suggests an innovative elucidation of scaling impacts about quality of touchpad apps, e.g. for geometry through pre-service teacher education and in-service teacher continuous professional development program. An empirical part of the study is also offered to serve as an exemplary for the proposed approach.

REVIEW ON LITERATURE

Defining quality of tablet apps for primary school mathematics

The imbalance between quantity and quality or commercial and educational purpose of existing apps on the market has already been registered in literature (e.g. Callaghan & Reich, 2018; Larkin, 2013). What are ‘good’ learning environments (LE) for mathematics in primary schools in general (Krauthausen, 2012) and which are main ideas for design of quality LE (Wollring, 2009)? How do these criteria for ‘good’ LE and crucial ideas for their design relate to environments involving tablet apps (Donevska-Todorova & Eilerts, 2019)? In order to make ourselves clear about what do we understand under the term *quality* of tablet app based learning environments for primary school mathematics we refer firstly to mathematics and its didactics, but also to pedagogical and technological concerns. In particular, we relate prominence of the apps primarily to presentation of the mathematical content and clarity/ simplicity of the goals related to curricula, but also to richness of ways for communication, collaboration and cooperation; structure of challenge; quality of feedback, guidance and rewards; connections of different nature; sense-based interactions and functionality. Since attempts for evaluation of the quality of apps are not new in the research of mathematics education and not only that their number, but also their rigor, tends to increase lately, we have used the above *operational definition* in order to take a closer look at theoretically based frameworks and empirical evidence for its analysis.

Reviewing quality of app based learning environments for mathematics

Measures for quality of technological tools for mathematics education according to three types of fidelity: pedagogical, mathematical and cognitive have been suggested by Dick (2008). However, criteria for quality of different kinds of technology enhanced resources for teaching and learning mathematics, e.g. for dynamic geometry software (e.g., by Kimeswenger, 2017; Trgalova, Soury-Lavergne & Jahn, 2011; Trouche et al., 2013) or e-textbooks and digital curriculum resources (Pepin, Choppin, Ruthven & Sinclair, 2017) are not straightforward transferable to other types of resources e.g., apps for touch pads. In particularly, it is worth mentioning that appraisal measures in generative reviews need to be clarified for corresponding specific types of technological tools, e.g., tablet apps that may facilitate the learning of mathematics at a certain level of education. In this regard, there are already studies that analyse quality of apps for primary school mathematics. For instance, numerous apps for developing differing forms of mathematics knowledge: declarative, procedural and conceptual have been reviewed by Larkin (2013). Further evaluation of apps targeted at geometry used a variety of modified evaluative frameworks, e.g., cluster analysis as “a highly versatile and useful methodology that can assist classroom teachers in their initial selection of Geometry apps, as well as providing additional information on pedagogical approaches to support teachers’ classroom practices” (Larkin & Milford, 2018a, p. 102). The authors have argued that “this approach provides more specific information regarding how teachers can coordinate the use of various elements of different apps to support mathematical learning beyond what can be achieved using individual apps” (Larkin & Milford, 2018b, p. 14) than the designed instrument by Namukasa, Gadanidis, Sarina, Scucuglia, and Aryee (2016). The instrument for evaluation of apps for upper primary and junior secondary mathematics by Namukasa et al., (2016) bases on four dimensions: the nature of the curriculum addressed in the app, the degree of actions and interactions afforded by the app, the level of interactivity and the quality of the design and graphic features. It uses a different three-level scale in each of the dimensions to guide teachers when selecting apps. We argue that such instruments struggle to assure validity and reliability of the measures. Therefore, our intention in the overall project is not to create a checklist or an isolated and static online questionnaire for assessment of apps, but rather to offer a virtual place where teachers, designers and researchers can meet through sharing own professional expertise in order advancement of the efficiency of the app designs for specific purposes and improving instructional practices.

THEORETICAL BACKGROUND

Mathematical tasks are the 'driving engine' in mathematics classroom. They have potentials to activate students cognitively and stimulate their engaged participation. Therefore, their design is the core of planning activities and arrangements in particular when they are wealthy in media of use. For these reasons, drawing on the above literature review and previous work of the first author, this paper bases on the theoretical model about mathematical tasks, task-formats and tablet app rich teaching and learning environments (the 'golden' rings in Figure 1), and their essential characteristics (the blue puzzles in Figure 1) (Donevska-Todorova, 2019). More precisely, the crucial characteristics are framed in six categories according to the suggested operational definition for the quality of tablet apps for primary school mathematics (in the previous section): (1) mathematical and didactical meaning, (2) communication, cooperation and collaboration, (3) differentiation, (4) feedback and assessment, (5) connections and networking and (6) logistic and technical support (Figure 1). These categories ground a base for establishing and structuring criteria for didactical quality of app-based tasks and learning environments. The accent is firstly set on the mathematical and didactical matters, and hence, this category is emphasised in the model. The next four categories relate to pedagogical aspects whilst the last one to technical features of the apps.

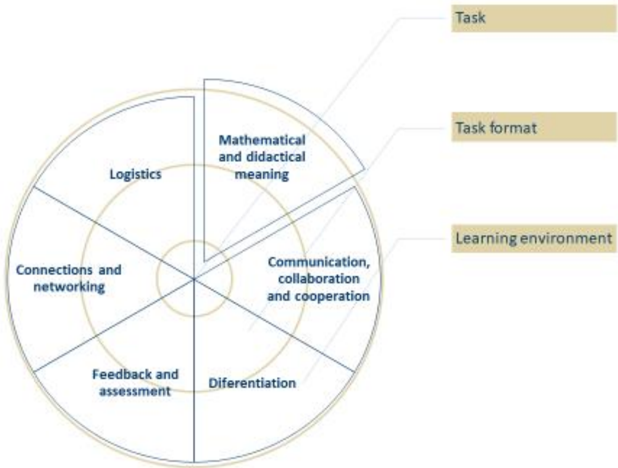

Figure 1. Theoretical model for categories of review criteria for quality of tablet app assisted LE (Donevska-Todorova, 2019, p. 124)

The first category *mathematical and didactical meaning* relates to a structured representation of mathematical concepts and their correctness but also relevance according to content interrelated and process oriented competences defined in national educational standards and curricula. In the empirical part of our study, we have referred to German national standards and curricula for geometry in primary schools in Berlin-Brandenburg. The second category of characteristics *communication, cooperation and collaboration*, tidily refers to ways of interactions between peers (students or teachers), a teacher and students and also devices and users in forms of pair or group activities. The category *differentiation* denotes open possibilities for supporting and providing means for all students, e.g., by offering hints, guidance and adaptive aids along individuals' learning processes. It is closely connected to the category *feedback and assessment*, which aims to show the importance of personalized in-time response and evaluation on given answers or task solutions in regard to both summative and formative forms of assessment. *Networking* means relations between contents in and of the scope of mathematics and natural sciences, between educational haptic and digital materials and networking users. The last category *logistic and technical support* relates user-friendliness of the interface, intuitive handling of the tools, etc. Each of the categories may further be structured into sub-categories by further specification and they are all closely linked to another rather than strictly separated from another.

These categories may also be organized as the three sub-dimensions suggested by Larkin and Milford (2018b), e.g., categories (2), (3) and (4) would be corresponding to the sub-dimension Child-Centred, the categories (1) and (5) to the Learning Design and the category (6) to the Technical Design. Yet, the model (Figure 1) is rather Student-Centred, than Artefact-Cantered, and therefore promoting this feature as three different categories was of importance.

To summarize, the diversity of theoretical frameworks and methodological approaches for identifying reviewing measures for the quality of apps, involve fidelity, cluster analysis, creation of instruments or categorization. These frames and approaches significantly influence the design, re-design and dissemination of apps along to the usage and the evaluation of their quality.

RESEARCH QUESTION AND RESEARCH METHODOLOGY

The overview on literature about the current research state and the theoretical framings have led to a synthesis of our thoughts into a concrete research investigation on the following questions.

What could review criteria for quality of tablet apps for mathematics be, in accordance to the proposed theoretical model (in Figure 1)? How can they be designed and disseminated through teacher training and further professional development?

The relevance of the posed question is two-sided:

(1) helping teachers making informative decisions about usability and efficiency of quality apps and
(2) contributing to sustaining valuable apps through re-design by experts in the research field of mathematics education.

Due to the complexity of the research problem addressing not only potentials of tablet apps based learning environment but also teachers’ awareness of their usefulness for practical implementation in classrooms, a single research method does not seem to be appropriate for collecting and analysing data. Moreover, designs of tablet apps change on a daily base. A design-based research (DBR) approach involving different methods at certain of its parts seems to be more suitable. Thus, the DBR intended to design a virtual tool for analysing apps and tasks for apps, enlighten teachers and scale criteria for good app based environments within the first DBR cycle.

Design-based research approaches appear in different forms and under diverse labels through its own historical development (e.g., design experiments, development research, developmental research, design science, design research and design-based research). In the field of mathematics education, there exist DBR models in which there is a clear separation of all phases in two parts, a scientifically one and a practically one (e.g. Prediger, Schnell, & Rösike, 2016). The six phases in the DBR cycle used in this study, an adaptation of the Complete Cycle of Design Research by Kelly, Lesh, & Baek (2008, p. 32) on Figure 2 continuously exchange, beginning with a research phase followed by an applied phase taking place at university. The first phase involved grounded theory approach and qualitative data for creating the theoretical frame (Figure 1) which served as a foundation for the initial design of the review instrument. The second, the fourth and the fifth phases are design experiments, while the third phase tends to measure the validity and reliability of the designed criteria. The last phase aims at arising with a final design that may further initiate a new DBR cycle.

RESULTS AND DISCUSSION

Three paths are relevant for the design of the measures and their dissemination in the suggested approach. Two of them are through the both inner semi-cycles: the first one including phases 1 to 3 and the second one including phases 4 to 6 (Figure 2). The third one is through the complete DBR cycle. They are explained in detail below. An empirical study offering insights about the design and the dissemination of the reviewing measures in praxis through the first inner semi-cycle follows afterwards.

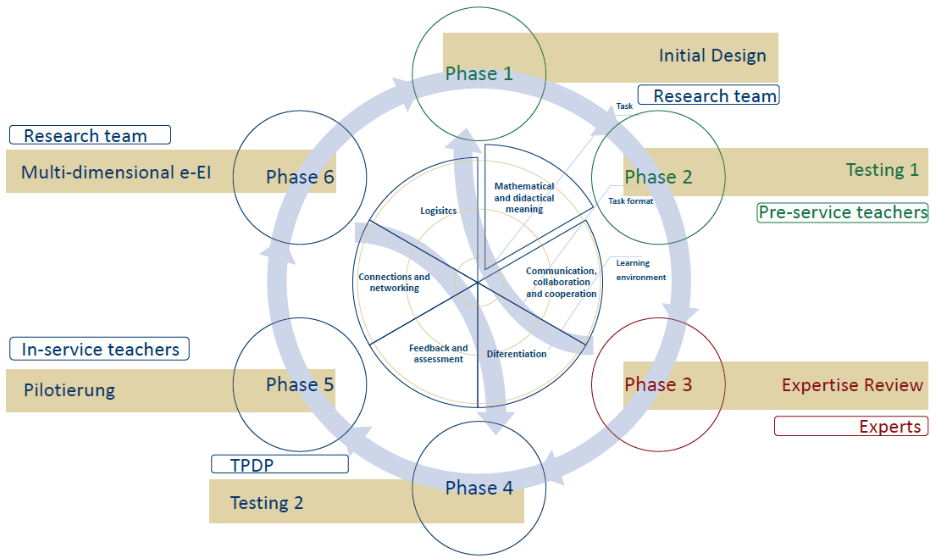

Figure 2. Design-based research cycle of developing reviewing measures evolving from the theoretical model in Figure 1

1. Design and disseminating review measures through the first inner DBR semi-cycle

The first inner DBR cycle begins with Phase 1 that includes the initial design of the reviewing criteria evolving from the categories in the theoretical model situated in the centre of the DBR cycle (Figure 1). It then continues with Phase 2, which comprises a university course for pre-service teachers. This course is a way for dissemination of the reviewing measures along to evaluation of a particular set of apps (empirical part of the study below). The third phase is envisioned as an expertise review of the created initial criteria. After it, certain modification of the design may be required. Therefore, an inner cycle continuing with the beginning phase after phase 3 instead of direct transfer into phase 4 seems much reasonable (therefore made explicitly visible in Figure 2).

2. Design and disseminating review measures through the second inner DBR semi-cycle

Once an expertise examination of the initial criteria is completed and the initial design has being adapted, it may further enter Phase 4. This phase takes account of a larger teacher professional development program. The participants form a smaller sample of in-service teachers than the one involved in the next phase - the pilot trial for the functionality of the reviewing criteria. These two phases allow further re-design and dissemination of the criteria through revising apps in which in-service teachers may play a significant role. This may contribute to development of teacher design capacities due to possibility for reflexion and repetition of this inner cycle.

3. **Design and disseminating review measures through the complete DBR cycle**

After undergoing the whole DBR cycle the design of the measures should be completed. A whole new cycle may begin with a purpose of scaling by adapting the designed instrument for apps that may support the learning of other mathematical contents, e.g. arithmetic. Another reason therefore may be setting a new specific focus on one of the dimensions from the theoretical model (Donevska-Todorova, 2019), e.g., feedback and assessment in tablet app environments for the learning of geometry.

Empirical evidence for the practical implementation

The development of the reviewing criteria for the quality of apps for mathematics education in primary schools has undergone the first two phases of the DBR cycle by now. On the base of the theoretical model (Figure 1) an initial design, containing criteria for apps for geometry organized in the six categories suggested in the model was designed by using the software Lime Survey. However, the criteria do not represent a closed checklist but are created as questions, part of which are open questions asking teachers for own opinion or best practice examples. Further, they are planned as a part of a comprehensive model for valuation having the characteristics of:

- accessibility (without having time and space limitations),
- interactivity (engaged participation by users),
- structuralism (consisting elements are organized in a particular hierarchy with a clear goal),
- multi-disciplinarily (interconnected domains in mathematics, its didactics, pedagogy and media education) and
- sustainability (maintaining changes in a balanced manner to meet needs and aspirations of future users).

Additionally, accompanying elements to the criteria include a repository of apps[1] that were reviewed and documentation like instructions for teachers and students or scientific literature.

The empirical evidence in Phase 2 of the DBR cycle comprises of an undertaken course for pre-service primary school teachers' education. The course was created and implemented by the first author of this paper and took place at the Humboldt-Universität zu Berlin in the winter semester 2018/19. Forty-eighth participants were working in two groups in duration of 90 minutes seminars for 15 weeks. The Moodle platform was used to maintain the organisation of the disseminating process of the designed criteria. The course intended to enlighten teacher students about the rapidly growing amount of apps that do not necessarily possess high quality for mathematics education, offer a possibility for direct investigation of their potentials and limitations and develop sophisticated opinion about an appropriate and meaningful practise of valuable apps. They also aimed to develop teachers' positive attitudes towards usage and needs for further developments. Moreover, in the practical phase of the course, students visited in-service teachers in schools in Berlin and Brandenburg, shared and discussed the criteria for 20 apps for geometry that were suggested in the repository along to their evaluation. The analysis of the results shows the difference between the early criteria identified by the students on the basis of three apps selected from the repository. The most of them were keywords or prompts as "training spatial abilities", "involving different types of tasks", "self-controlling", "intuitively feasible", "combinable", "flexibly applicable", etc. These prompts were later grouped and structured according to the six categories in the theoretical model (Figure 1) after which an increased engagement with the apps and the designed initial criteria took place. Students were working in groups of maximum three participants and the small groups met with at least one in-service teacher. These activities resulted with developed criteria by the students, which were no longer only keywords, but formulations that are more sensible, accurate and clearly fitted a corresponding category in the model.

CONCLUSIONS AND FURTHER RESEARCH

Based on literature review, this paper has arisen with an operational definition about quality of touchpad apps for primary school mathematics. It then presented an overview of existing approaches for studying apps and their values (in section 2) and theoretically framed imperative categories for review criteria in a hypothetical model (in section 3). The discussion gave rise to the question about possible approaches for dissemination of review measures through design and estimation of apps quality. It then argued that the DBR is an appropriate methodology therefore and possible through two of its inner semi-cycles and the complete initial cycle that were described in detail and lead to answering the tackled research question. This has been further justified with a qualitative analysis of empirical data collected during the project and presented in the fifth section.

Several questions arise for further research. Could the designed criteria for quality of tablet apps in elementary geometry be adapted for other areas of primary school mathematics as for example arithmetic? Could that be possible by re-design of some of the criteria and undergoing a semi-cycle or requires starting a new complete design based research cycle? Could the criteria for quality of tablet apps be transferred, disseminated and scaled in other educational areas different from mathematics education by substituting category 1 in the model? These questions may initiate future research concerning effectiveness of tablet apps, learning arrangements and new technology enhanced learning environments.

NOTES

1. Additional information about the repository may be provided by the authors.

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DEVELOPMENT OF VIDEO CASES REGARDING TECHNOLOGY USE FOR PROFESSIONAL DEVELOPMENT PROGRAMS

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Although an adequate use of digital tools in mathematics might raise the quality of learning, they are not common enough in classrooms. One reason is that many teachers do not know how to integrate them in everyday teaching. This is where professional development (PD) programs can help. To bridge the gap between theory and practice, video case-based learning, is used in the presented study.

Keywords: Digital tools, technology, video cases, professional development, in-service teachers

"We can no longer consider that teaching mathematics with technology is just an option" (Artigue, 2013, p. 2). The same applies for Germany, where using technology in mathematics classrooms is compulsory. Many positive effects can be observed while teaching with digital tools. They "allow students to focus more on conceptual issues, not just on the learning of techniques", connect different visualisations or facilitate working with real data (Artigue, 2013, p. 3). Taking these advantages into consideration, one would expect teachers to use technology frequently in their classrooms. Results of various studies are, however, contradictory to these expectations, since technology is not used as often as possible in (German) schools (Lorenz et al., 2017). Some teachers face substantial challenges in planning their lessons and teaching with digital tools. When technology is used, students can be given more autonomy, inquiry-based practices can be integrated and more open tasks can be implemented (Artigue, 2013). However, using open and inquiry-oriented tasks might lead to teachers being confronted with various solutions and unexpected situations. This could be perceived as obstacle and hinder their effort to plan lessons with technology. While teachers justify their rare use of technology with the lack of technical equipment in schools, their knowledge, beliefs and skills must be taken into account as well (Drijvers et al., 2016). Furthermore, teachers often do not know about potential ways to teach and learn mathematics with technology (Lorenz et al., 2017).

These are some reasons why PD programs are required. There are many different courses for learning how to use digital tools, what kind of tasks to give to students or how to teach with technology (Drijvers et al., 2016). Nevertheless, there is a gap between the theory and practice of PD programs. It has been argued, that using video cases might bridge this gap (Seidel et al., 2013). In addition, the method of video case-based learning possesses the advantage of allowing for a deeper analysis of lessons, helping to raise the quality of group discussions (Goeze, 2016) and grasping the intricacy of classroom interaction (Koellner et al., 2018). Videos are more economical, for example, than role-playing learning arrangements, because they can be used more than once, paused and replayed. They play an important role to assist teaching and learning of routines without time pressure (Goeze, 2016).

Using video cases in a PD program means more than just showing the video. They include background information to understand the context of the scene shown. Additional information regarding the content or comments from people in the video can be entailed. The integration into PD programs depends on the pursued aim. They can be used to reflect one's own teaching, foster group discussions or analyse students' learning processes (Goeze, 2016; Koellner et al., 2018). Sherin and van Es (2009) differentiate between three paradigms on how video can be integrated. The paradigms "bring teachers into the […] practice" (p. 21), "learn to notice" (p. 21) are adopted here.

In the presented project, video cases are developed for PD programs for in-service teachers. Thus, we recorded real classroom footage of mathematics lessons, in which digital tools are used. We focused on the use of open tasks that are suitable for students to explore their thinking using digital tools. The scenes selected for cases can be used as prototypes for similar situations, e.g. demonstrating how technology can be used, initiate a fruitful discussion or raise a cognitive conflict for students (Goeze, 2016). The combination of necessary background information, given tasks and comments from students and the teacher in the video make up a case. The research questions are:

1) Which processes of teacher professionalization can be initiated by the video cases?
2) Which impulses can prompt a beneficial use of the video cases in PD programs?
3) For which problematic situations in teaching mathematics with digital technology are video cases necessary?
4) How can the professional vision be trained in PD programs regarding to this topic?

To answer these questions, the cases were used in a PD program and university teacher education courses. Data in form of single interviews with experts, recorded group discussions from the PD sessions and questionnaires are used to inform the (re-)design process. By analysing the answers and discussion topics among participating teachers, we can identify the potential use of a video sequence for PD programs. It becomes apparent that one video case can address manifold issues regarding teaching mathematics with technology. For example, it can help teachers learn how to react to challenging situations, show them how students work with digital tools and which misconceptions or difficulties might arise.

The first results show that the cases can be used for PD programs regarding the topics *teacher (re-)action, tool usage* and *learning processes of the pupils*. These correspond to the levels of the research questions. Another outcome is that teachers have different views on the same video, which can be connected to the professional vision (Sherin & van Es, 2009). This will be analysed in more detail in the next step of the research project.

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ALIGNING EMBODIED AND INSTRUMENTAL EXPERIENCES TO FOSTER MATHEMATICS TEACHING AND LEARNING

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According to the "embodied instrumentation" approach, recently described by Paul Drijvers in his plenary at CERME11, the aim of this paper is to focus on the idea that embodied and instrumental experiences can be coordinated and aligned. Two examples are presented in an explorative way and the interplay between bodily and instrumented activities is analysed from students' dialogues to show how an integrative approach to tool use could foster mathematics teaching and learning. This first insight into earlier works calls for the need to deeply define the main features of the embodied instrumentation as a promising approach and attempts to sketch a possible direction for further investigations into this new, interesting and relevant topic.

Keywords: Resources and instruments, Digital technology, Embodied cognition, Instrumental approach, Embodied instrumentation

INTRODUCTION AND THEORETICAL BACKGROUND

Learning and teaching with technology still needs inspiration. Despite the spread of technological tools in everyday life and the growing interest of scholars toward the potentialities of digital technology in mathematics teaching, the effects of using these tools for learning seem overall to be modest.

Researchers in mathematics education are aware that technology alone does not solve any educational problems, especially if it is used only as an auxiliary tool, and as such it enhances students' actions without qualitatively transforming them. However, research and practice have shown that the use of technology can play an important role in helping students develop an appreciation and disposition to practice genuine mathematical inquiry, posing questions, searching for diverse types of representations, and presenting different arguments during their interaction with mathematical tasks (Monaghan, Trouche & Borwein, 2016; Faggiano, Ferrara & Montone, 2017). And the importance of these aspects in helping students to view mathematics as something more than a fixed static body of knowledge has already been recognized (see for instance the NCTM Principles and Standards of School Mathematics, 2000).

Moreover, assuming that any cognitive activity is a mediated activity (Rabardel, 1995), as researchers in math education we would like to try to better understand the nature of the mediational role of tools in the learning and teaching of mathematics. The research into the nature of tool mediation is a crucial goal that can also help teachers to understand and become aware of the affordances, the constraints, and the mediating role of technologies as educational resources. In particular, as far as the use of technological tools is concerned, we believe that the bodily engagement of the students needs to be prompted together with the conceptual development of the mathematical meaning which is the aim of the intervention. In the field of mathematics education, indeed, there have been many studies about the use of tools with regard to gestures, sensorimotor experiences and embodied cognition (e.g. Edwards, Radford & Arzarello, 2009; Nemirovsky, Kelton, & Rhodehamel, 2013; Radford, 2014; Sinclair & de Freitas, 2019). They are based on contemporary studies in cognitive science, which highlight that the body actively participates in learning processes and this is connected to the centrality of the action in knowledge processes (Caruana & Cucci, 2017): conceptual learning reveals to be a dynamic and body-centred phenomenon.

A further key view in the theoretical background of this paper is the instrumental approach to tool use (Drijvers & Trouche, 2008). It distinguishes between artefact and instrument (Vérillon & Rabardel, 1995) and focuses on the artefact becoming part of an instrument through the development of associated utilization schemes. The notion of instrumental genesis was coined to reflect the long and complex process (at the same time social and individual, connected to the limits and potential of the artefact and to the student's qualities) during which a student turns an artefact into an instrument, developing schemes that allow him to use the artefact for a well-defined purpose. In this view, on the one hand, the main educational potential of using an artefact in a given situation is firmly related to the schemes and the knowledge that may co-emerge (Artigue, 2002), thus deeply depending on the task. On the other, the instrumentation schemes that students develop depend on the tool use, hence tools are not neutral.

Motivated by the above considerations and inspired by Drijvers's plenary talk at CERME11 in Utrecht (Drijvers, in printing), I'm interested in focusing on possible coordination and alignments of embodied and instrumental experiences. For this purpose, some insights, coming from earlier activities carried out through different teaching experiments, will be presented and discussed. In particular, attention is drawn to highlight the ways in which the instrumental experience and the embodied experience can be intertwined and aligned in order foster mathematics teaching and learning activities.

Herein I will present two examples to reveal how the relationship between embodied and instrumental experiences allows students' reasoning to emerge through action and so mathematical meanings to be constructed. The first example draws from a grade four students' exploration of axial symmetry through the integrated use of manipulative and digital artefacts. It is analysed to show how the artefacts have acted as mediators between the embodied and instrumental experiences and the conceptualization. The second example involves a small group of secondary school students interacting with each other and with concrete objects to find a winning strategy for a combinatorial finite game. This example illustrates how the instrumental and the embodied experiences with the concrete objects could guide further actions when working with a technological tool and allow reasoning to emerge and meaning to be built.

The aim of this paper is not to present a research study explaining the way a certain question on embodied instrumentation can been answered. The discussion of the examples, rather, attempts to address the aim of suggesting insights into possible integrative approaches to technology use, aiming to foster the development of teaching and learning activities. It is also an opportunity to stress the need of a research agenda for further investigations aiming to shed some light and more deeply define the main features of the embodied instrumentation as an integrative approach to tool use.

EXAMPLES OF ALIGNMENT

The verb "to align" comes from the French "a", meaning "to", and "ligne", meaning "line", thus it literally means to bring something into line with something else. In this work, however, I mainly refer to its metaphorical significance. In a broader sense, indeed, the alignment can be seen as a way to change something so that it has a somehow correct or desirable relationship to something else. From this point of view, the examples of alignment I'm going to describe have to be seen as examples of situations by which I intend to show how students experiencing well designed instrumented activities can be engaged in embodied experiences that naturally tend to come into coordination with the instrumented ones, thus resulting in fostering the construction of meanings.

Combined use of different type of artefacts

The first example refers to a teaching experiment for the conceptualization of axial symmetry, conducted with a class of fourth grade students. The aim of the experiment was to investigate the didactic potentiality of a designed sequence of six activities based on the combined use of manipulative and digital artefacts (Montone, Faggiano & Mariotti, 2017). The analysis of some key episodes shows that the combined, intentional and controlled use of the two artefacts may develop a synergy, so that each activity within the sequence enhances the potential of the others (Faggiano, Montone & Mariotti, 2018). What I want here is to show how the construction of the mathematical meaning arose from the coordination and alignment of the instrumental and the embodied experiences.

The manipulative artefact consists of a sheet of paper, with a straight line drawn on it marking where to fold it, and a pin to be used to pierce the paper at the right points in order to construct their symmetrical points. The digital artefact appears as an interactive book with a sequence of activities. It is embedded in a Dynamic Geometry Environment (DGE) and includes tools that allow the construction of some geometric objects (point, straight line, segment, middle point, perpendicular line, intersection point), the "Symmetry", the "Compass" and the "Trace" tools. As in every DGE, a fundamental role is played by the drag function that, boosted by the tracing tool, allows to observe the invariance of the properties characterizing the figures.

For the purpose of this paper, I will briefly describe here only the first two activities of the sequence. The first activity involves the manipulative artefact and aims at introducing: the meaning of axial symmetry as punctual correspondence and the dependence of the symmetric figure on the folding line. Students are required: to construct a symmetric figure with respect to a given line by folding and making punctures with a pin; to construct the symmetric figure of the first figure with respect to a new given line; to explain what looks the same and what looks different about the two obtained symmetric figures. The second activity involves the digital artefact and aims at focusing on the dual dependence of a symmetric point from the point of origin and from the axis. Students are asked: to build the symmetric point of a point A with respect to a given line, using the button/tool "Symmetry" and call it C; to activate the "Trace" on point A and point C, drag A, drag C, and drag the line; to see, in each cases, what moves and what doesn't, and explain why.

In what follows I will present and discuss some excerpts taken from the discussion held with the class at the end of the second activity. The first excerpt refers to the very beginning of the discussion when G. was summarising what they have done with the digital artefact.

G. : we clicked on the point and the line and what comes out was the point symmetric to the point A and we called it C

The expression "comes out" seems to reveal that the result has been obtained as the product of the action students have done, namely clicking on the button "Symmetry". It is the starting point of a semiotic chain which will bring to the meaning of axial symmetry as a point-to-point correspondence and to the fact that the symmetric point depends on the point of origin. And it is followed by V.'s detailed description of what they asked to the artefact to do:

V. : clicking first on the point and then on the line you're telling the computer: make the symmetric point of this point with respect to this line

They have already experienced the construction of the axial symmetry directly acting on the folded paper through the use of the pin and here they are at the beginning of their instrumental experience with the technology.

During the development of the discussion it becomes clear that what students are performing is a combined embodied and instrumental experience. When the teacher focuses on why, when we move A, the symmetric point C moves too, M. refers to the dynamic process visualized with the digital artefact:

M. : If you move point A only, point C has to move with point A because they must be symmetrical [Fig. 1a]… like, if you move point A higher [Fig. 1b]... point C moves lower... so it is the same [Fig. 1c]… because there must be... the same space... between the two points [Fig. 1d]

1a	1b	1c	1d

Figure 1. Gestures which matches M. speech (Faggiano, Montone & Rossi, 2017)

The pupil's body learns while acting: when M. refers to the digital artefact she describes and simulates the actions that she performs with her own hands. She identifies herself with what she observed and, moving her arms as lines and her hands as points, simulates drawing in the air the movements of the objects as seen on the screen. She is mentally moving the objects: the implicit dynamism of thinking mathematical objects is made explicit thanks to the didactical functionality of the dragging function, together with the tracing. But it is when the teacher asks how they know that the distance is always the same that the alignment between the embodied and instrumental experiences become more visible: V. asks to and receives from the teacher a sheet of paper and a pin.

V. : If we have a paper that can be folded… we draw a point… I fold the paper and I pierce... I pierce on the point, I make a hole where it is the point and on the other side it comes out a hole… This hole is the symmetric figure of this point…

Teacher. : And what about the distance? The space, as we said before, is the same?

V. : Yes, it is the same!... there [on the screen] you can move the point so I can understand more easily that if I drag the point… the given figure… there is the same distance because by moving it's clear, especially when we move away a lot the point from the line, that also point C moves away… thus there is always the same distance. But I got it even on the paper.

And the role of the coordination and alignment of the instrumental and embodied experiences emerges again at the end when the teacher asks to figure out the reason why, if you move point A, point C moves also but the line does not move [1].

M.: It is the same as on the paper [Fig. 2a]… the folded paper… This for example is the line [Fig. 2b]. If I pierce [Fig. 2c] the paper on point A… [Fig. 2d] and eventually I move also the point A [Fig. 2e], the line is always there [Fig. 2f].

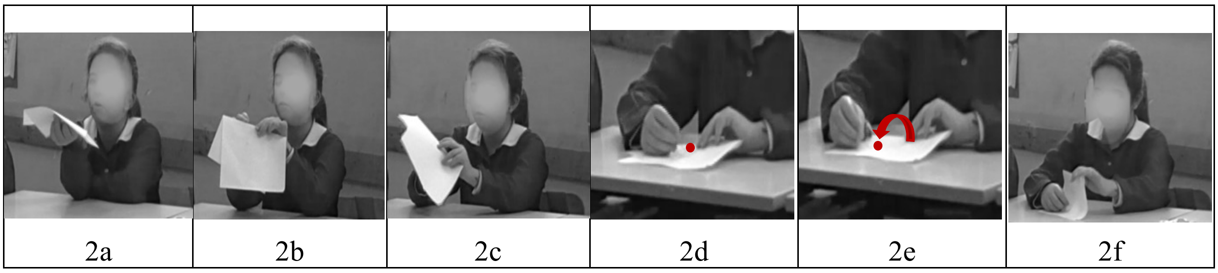


Figure 2. Gestures which matches M. speech

M. here is connecting the instrumental experience with the two artefacts: she underlines that the situation is the same; when she says "and eventually I move" she is linking the act of dragging to the consideration of another point and the act of doing another hole. The schemes related to the two different artefacts have been associated and the role of the embodied experience appears to be fundamental. The coordination and alignment of the experiences allow her to conclude that the line does not move in the digital artefact because it is the same on the paper.

Mediated investigations

The second example refers to a group of four tenth grade students attempting to find a winning strategy to the following combinatorial finite game, which is a version of the well known game called Nim:

> Start with 15 tokens. Two players take turns to remove a whole number from 1 to 3 to the running total tokens on the desk. The player who removes the last token wins the game.

The activity is part of a learning trajectory which aims at involving students in inquiry-based learning. The main idea is creating challenging situations by varying some aspects of a phenomenon while keeping the others invariant (Soldano, Luz, Arzarello & Yerushalmy, 2019). In this way students are guided to grasp the intended object of learning, drawing their attention to critical aspects and fostering inquiry processes: they are engaged in exploring various aspects of the same phenomena, asking questions, raising conjectures, carrying out experimentations and providing explanations.

Students were initially required to play the game with their classmate and to figure out a winning strategy. They were equipped with the 15 tokens and started to play some matches. They quite easily understood that if you leave to your opponent 4 tokens on the desk you win. At a certain point one of the students whispered to his friend:

> A.: We can leave them 4 if there are 5, 6 or 7 tokens when it is our turn

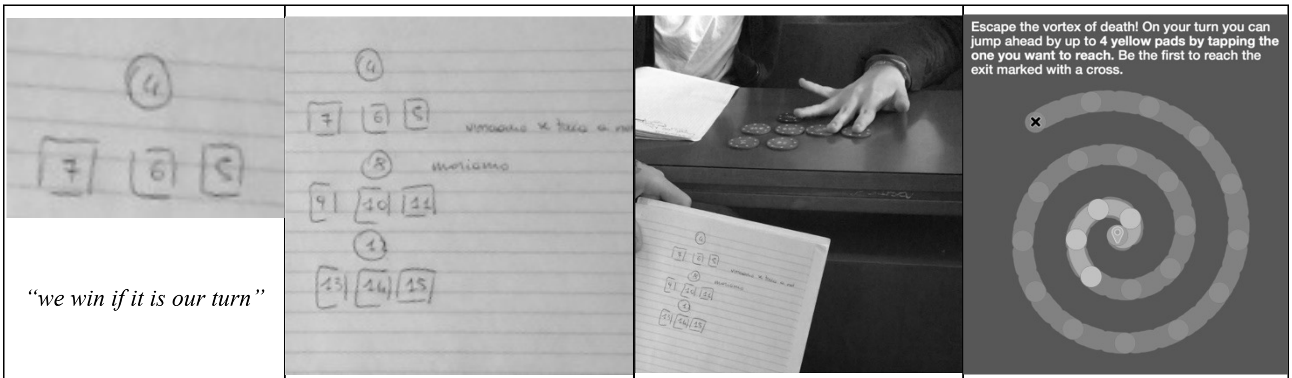
and they started to draw on their notebook a graphical representation of the winning stance and the possibilities to reach it (Fig. 3a). After a while the other student added:

> B.: If we leave them 8, as they can remove 1, 2 or 3, in any case there will be 5, 6 or 7 tokens when it is our turn, so we can leave them 4… and we win!

This scheme was than represented as in Figure 3b and became the instrument they used to get control of their moves during the next matches. At the same time, they started to arrange the tokens on the desk so that they were getting in line 4 in a row (Fig. 3c).

The students' behaviour turned out to be important in the next step of the trajectory when, according to the logic of inquiry (Hintikka, 1999), they were asked to explore if the winning strategy they found is still the same when changing the total number of tokens or the highest number of tokens that can be removed at each turn. They decided to play with 17 tokens, removing from 1 to 3 tokens each turn, and in a further step also to vary the highest catch (number of tokens that can be removed), rising it up to 4. The alignment between the embodied and the instrumental experience brought students to arrange the tokens getting in line a number in a row which is one more the highest catch.

But it is mainly during the class discussion orchestrated by the teacher that the alignment between the embodied and the instrumental experiences allows them to focus on the remaining tokens and to figure out that the general winning strategy for the first player consists of removing the number of tokens corresponding to the remainder of the division between the total number of tokens and the highest catch.



"we win if it is our turn"

3a 3b 3c 3d

Figure 3. Elements of a mediated investigation activity

The final step of the trajectory consists of getting students involved in another version of the game in which the aim is to reach the target of 23, taking turns to add a whole number from 1 to 4 to the running total (Fig. 3d). Students are required to play against the computer (https://nrich.maths.org/397) in order to find out a strategy for beating it. A final discussion, orchestrated by the teacher, aimed at the co-emergence of the entangled instrumental and embodied schemes for the related mathematical knowledge to be revealed. Students were able to tackle with this slightly different problem: winning the game became a matter of conceptual understanding. A detailed analysis of the students' video and transcript is still under study but for the purpose of this paper some interesting elements concerning the intertwinement between the instrumental and embodied experience have already emerged. Further analyses will be devoted to focus on the role of the digital tools in this trajectory: results would be important to inform a new way to design tools and tasks taking into account the opportunity to exploit the role of body in fostering mathematical conceptual development.

CONCLUDING REMARKS

Through the examples I discussed above, I aimed to highlight how the interplay between embodied and instrumental views can be coordinated and aligned in order to foster mathematics teaching and learning. Mathematical cognition could emerge throughout a bodily-based instrumental genesis grounded on embodied experience and on the related development of both cognitive and sensorimotor schemes.

The first example shown as the synergic use of the two different artefacts seems to have strengthened both the mathematical and didactical potentialities of the digital artefact. The instrumental experience with the use of the DGE was amplified by the embodied experience students had with the manipulative artefact. The second example highlighted the emergence and evolution of the alignment between instrumental and embodied schemes in tackling with investigative challenges.

In both the examples, however, it is important to stress that the alignment of the experiences was mainly fostered by the type of task (open and investigative, finalised to the discovery of the properties and the construction of the mathematical meaning) and by the teacher's behaviour during the class discussion. In this sense the discussion of this example aims at inspiring teaching in order to design and develop activities suited to foster meaningful learning trajectories.

Through an integration of many modalities including sensing, perceiving, acting and observing, we can say that higher cognitive structures emerged from recurrent patterns of perceptually guided actions. Although the arguments presented in this paper need to be further investigated, they seem to resonate well with the embodied instrumentation approach introduced by Drijvers and with the following thought:

> The language of things puts back together the physicality of the world through the impalpable but revealing net of gestures, utterances, and formulas (Arzarello's adaptation from I. Calvino, American Lectures, Exactness, 1988, in occasion of the Turin Workshop on Semiotic, April 2017).

Further investigations and empirical evidences are required to shed some light and to more deeply define the main features of the embodied instrumentation as an integrative approach to tool use.

NOTES

1. The data come from a research project presented in 2017 during the XXXIV AIRDM Italian Seminar on Research in Mathematics Education (https://www.airdm.org/sem_naz_2017_29.html) by E. Faggiano, A. Montone and P. G. Rossi and developed also in collaboration with M.A. Mariotti.

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A QUANTITATIVE STUDY ON THE USAGE OF DYNAMIC GEOMETRY ENVIRONMENTS IN DANISH LOWER SECONDARY SCHOOL

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The paper describes the development and analysis of a quantitative study investigating to what extent the potentials of dynamic geometry environments (DGE), in relation to mathematical reasoning competency, are utilized in lower secondary school in Denmark. The study entails a questionnaire, which was developed on the basis of an extensive review that uncovered four potentials of DGE in relation to reasoning competency: feedback; dragging; measuring; tracing. 220 Danish lower secondary mathematics teachers completed the questionnaire. Analysis indicates that the potentials of measuring and dragging are utilized to some degree, feedback to a lesser degree, while tracing is almost non-existent. Furthermore, there are signs that DGE is used as a substitute for the paper and pencil environment to solve tasks that were originally designed for paper and pencil. Possible improvements of praxis are discussed, and the integration of the results in further research is elaborated upon.

Keywords: DGE, GeoGebra, reasoning competency, survey

INTRODUCTION

Digital technologies are widely implemented in Danish primary and lower secondary school, in part due to heavy investments from the Ministry of Education over the last couple of decades (e.g. Undervisningsministeriet, 2015). Consequently, the availability and usage of digital technologies has become commonplace in mathematics education at all levels in Denmark. In primary and lower secondary school the dynamic geometry software GeoGebra is particularly popular. This can be considered a positive outcome, since many studies highlight the affordances of dynamic geometry environments (DGE hereafter), as potentials in supporting students' development of mathematical reasoning, conjecturing and in proving (e.g. Jones, 2000; Leung, 2015; Edwards et al., 2014). In the optic of the Danish competency based KOM-framework (Niss & Højgaard, 2011) these potentials are related to the reasoning competency. However, it seems that the manner in which the affordances are utilized (if they are utilized) is essential (Jones, 2005). Part of the research motivation stems from an underlying assumption that the potentials of DGE, such as GeoGebra, are not utilized in Danish lower secondary school, even if the software is indeed popular. The assumption is that DGEs are predominantly used only as a substitution for the paper and pencil environment.

No previous quantitative research exists regarding how DGE is actually used in Danish lower secondary school. The author of this paper could not find quantitative results on the matter internationally either. In order to gain insight into this area, the paper aims to investigate how DGE is actually used in lower secondary school in Denmark, with particular interest in the potentials of DGE in relation to reasoning competency, by posing the following research question:

To what extent do Danish lower secondary school mathematics teachers utilize the potentials of DGE, in relation to mathematical reasoning competency, in their teaching?

THEORETICAL CONSTRUCTS FOR DEVELOPING THE QUESTIONNAIRE AND ANALYSING THE RESULTS

The reasoning competency (RC hereafter) is one of eight mathematical competencies in the KOM framework (Niss & Højgaard, 2011), which has shaped the curriculum at all levels of mathematics education in Denmark. The framework has also influenced mathematics education in other parts of the world (Niss, Bruder, Planas, Turner & Villa-Ochoa, 2016).

The RC constitutes the ability to create and carry out informal and formal arguments and to follow and evaluate argumentation put forward by others. Additionally, to understand what a mathematical proof is, including the role of counter-examples. It also comprises the ability to discern proof from other forms of mathematical reasoning such as example-based explanations and being able to develop such arguments into formal proof. Furthermore, it is about creating and justifying mathematical claims in general, such as answers to questions and problems (Niss & Højgaard, 2011). As argued in Højsted (2019) the RC also entails the first steps of the proof process, exploration and conjecturing.

In order to formulate questions that investigate to what extent the potentials of DGE in relation to RC are being utilized, it is first necessary to describe what these potentials are. To this end, the results from an extensive review into the potentials of DGE in relation to supporting students' development of RC were used (Højsted, 2019). The meaning of "potentials" in this context are affordances of DGE, which are not available in other typical mathematics education tools such as paper and pencil. Four potentials were discovered: feedback; dragging; measuring; tracing. It is particularly in relation to conjecturing that they were deemed to be potentials. A short elaboration of the potentials is presented here.

Since DGEs mimic theoretical systems, typically Euclidean Geometry, they offer an environment in which only constructions that abide by the rules of the theoretical system can be constructed (Balacheff & Kaput, 1997). Therefore, the DGE inherently provides **feedback** to the user, e.g. by not allowing for imprecise measurements or constructions that contradict Euclidian theory. Figures constructed in the environment can also be manipulated dynamically by **dragging**. The elements of a dynamic figure are locked in a hierarchy of dependencies, which decide the outcome of a dragging action (Hölzl, Healy, Hoyles & Noss, 1994). The dependencies are in fact the theoretical properties of the figure, which are decided by the construction method and by the theory of Euclidean Geometry governing the system. These properties remain invariant during dragging, which allows for discovery of the theoretical properties of constructions in Euclidian Geometry. Therefore, dragging in DGE can link the perceptually mediated appearance of a figure to the theoretical properties of the figure, which Laborde (2005b) refers to as moves from the spatiographical level to the theoretical level. In "robust" constructions, the properties are conserved during dragging, whereas in "soft" constructions not all properties are conserved (Healy, 2000; Laborde, 2005a). Most DGEs, including GeoGebra, contain **measuring** tools, which allow the students to find the measures of constructions. When figures are manipulated dynamically, the measurements are updated instantly. Therefore, it is possible to observe invariant relationships between measures (Olivero & Robutti, 2007). Additionally, the possibility of **tracing** an object during dragging, offers the possibility of visualizing underlying invariant relationships of the construction, for instance in soft constructions in which a property is maintained when dragging is performed in a particular manner (Baccaglini-Frank & Mariotti, 2010).

Developing the questionnaire consisted of the careful back and forth process of formulating questions that would investigate whether these DGE potentials related to RC were being utilized, while at the same time being formulated concise and clearly enough to be understood by lower secondary school teachers.

METHOD

A web-based questionnaire was developed for lower secondary school mathematics teachers. The questionnaire consisted of 19 multiple-choice questions, in which a five-point Likert scale was used, as well as four open-ended questions and some background information questions. Results from nine multiple-choice questions are presented in this paper. The questionnaire was distributed through two platforms: (1) a link for self-enrolment to the questionnaire was sent through the email list of the Danish national maths counsellor network with participation from 97 of the 98 municipalities of Denmark. (2) a link for self-enrolment was posted on two popular Facebook groups for Danish lower secondary school teachers, “GeoGebra Hangouts” and “We who teach in lower secondary school”.

In order to combat the usual problem of low participation response and completion in web-based questionnaires, steps were taken with regards to design and language (Fan & Yan, 2010), and a monetary incentive was included (Göritz, 2010) in the shape of a lottery with a prize of DKK 4000,- to a single winner. Respondents were required to add their names and email in order to take part in the lottery. This also gave the opportunity to check for double entries.

GeoGebra is by far the most popular DGE in lower secondary school in Denmark. Consequently, it was decided, for the sake of clarity, to formulate the questionnaire directly towards GeoGebra usage instead DGE usage. However, GeoGebra is a system that includes not only the geometry environment, but also CAS, spreadsheet etc. Therefore, in every question it was explicitly specified that the question was related to the “geometry part” of GeoGebra.

RESULTS AND ANALYSIS

Ten questions are included in this paper. For the sake of clarity, the questions are given numbers 0-9, even though it was not their actual number in the questionnaire.

Albeit not directly related to the research question, it is relevant to mention the results from question 0, which was a background question, in order to shed some light on the population of the study.

N=220	0. How often do you and your students use the geometry part of GeoGebra in the mathematics class?
Every week	35.9%
Every other week	39.5%
Once a month	17.3%
Every other month	4.1%
Less	3.2%

Table 1. A background question on rate of GeoGebra usage

Based on the results from question 0, in combination with the questionnaire being distributed on the GeoGebra Facebook group, and that participation in the survey was through self-enrolment, it is reasonable to assume, that the respondents in the survey are teachers who actually use GeoGebra regularly in their mathematics classes. Perhaps it is even reasonable to assume that many of the respondents are teachers who are somewhat enthusiastic about GeoGebra.

Some questions were posed to find out which types of tasks the teachers give to their students. This was partly done in order to investigate the hypotheses that GeoGebra is merely used as a substitute for the paper and pencil environment. Partly, the results from these questions would also indirectly indicate whether the potentials are being utilized or not. For instance, if the students primarily work on tasks in GeoGebra, which were made for paper and pencil geometry, it is a strong indication that potentials such as dragging are not involved in the tasks, since dragging is not possible in paper and pencil geometry.

N=220	Always	Frequently	Occasionally	Rarely	Never	Don't know
1. Do students work on tasks in the geometry part of GeoGebra, which were originally made for paper and pencil geometry?	14.5%	38.6%	35.5%	9.1%	2.3%	0.0%
2. Do students work on tasks that were originally made for paper and pencil geometry, which you have adapted to be used in the geometry part of GeoGebra?	6.8%	34.1%	40.5%	11.4%	5.5%	1.8%

Table 2: Questions 1 and 2 regarding types of task

As can be seen from the results to question 1, a large part of the students seem to be working on such paper and pencil tasks, which indicates that the hypotheses holds some truth. Although, the results from question 2 show that many teachers adapt the paper and pencil tasks. Therefore, it is possible that some teachers adapt the paper and pencil tasks in such a way, that some of the four potentials, which are linked to the RC, are utilized.

A premise for investigating theoretical properties of figures by dragging is that the students hold some understanding of the difference between free and non-free objects. Question 3 aimed at finding out if students work on understanding this difference. It was expected that some teachers might not know what free and non-free objects meant. Therefore, a short video was shown prior to question 3 along with accompanying text to explain, in which it was illustrated how two free points could be dragged, while their constructed midpoint could not. The predominant answers were occasionally (35.9%) and rarely (30.5%), which indicates that it is not something a lot of time is spent on. Moves from the spatiographical level to the theoretical level (Laborde, 2005b) are unlikely to occur if the students do not realise that the perceptually mediated appearance of the figure during dragging in DGE, is linked to the theoretical properties of the figure.

N=220	Always	Frequently	Occasionally	Rarely	Never	Don't know
3. Do students work on understanding the difference between free objects and non-free objects in the geometry part of GeoGebra?	5.0%	13.2%	35.9%	30.5%	15.0%	0.5%
4. Do students work on constructing so-called “robust” figures, i.e. figures that retain certain properties, when the free objects of the figure are dragged, in the geometry part of GeoGebra?	1.4%	24.1%	46.8%	17.7%	10.0%	0.0%
5. Do students work on investigating figures, to see which properties are maintained during dragging in the geometry part of GeoGebra (e.g. that	5.0%	33.2%	47.3%	10.9%	2.7%	0.9%

the medians of a triangle meet at a point)?						

Table 3: Questions 3-5 concerning activities focusing on theoretical properties and dragging

Results from question 4 show that more time is used on constructing robust figures. Considering that understanding the difference between free and non-free objects is necessary to construct robust figures, this result is somewhat contradictory, but perhaps it is because the teachers discern between the implicit understanding needed to construct robust figures, and time spent explicitly focusing on the difference between free and non-free objects, as two distinct activities. Question 5 was aimed directly at finding out if **dragging** is being used as a means to investigate the theoretical properties of figures. An example was given of properties that can remain invariant under dragging (the medians of a triangle meeting at a point), because it was expected that some teachers might not understand the meaning of "investigating figures, to see which properties are maintained during dragging". The teachers predominantly responded that their students occasionally (47.3%) or frequently (33.2%) engage in such activity, which is a higher rate than in questions 3 and 4, and a higher rate than expected beforehand. It is also somewhat surprising in light of the answers to question 0, since tasks which were originally made for a paper and pencil environment would not include prompts requiring dragging to investigate theoretical properties of figures. Nevertheless, the result does suggest that the potential of dragging is used regularly by students to investigate the properties of figures. It indicates that potentials of DGE linked to the exploration and conjecturing part of the RC are utilized to some degree.

N=220	Always	Frequently	Occasionally	Rarely	Never	Don't know
6. Do students work on measuring of figures in the geometry part of GeoGebra?	12.7%	66.8%	18.6%	0.9%	0.5%	0.5%
7. Do students work on measuring of figures combined with dragging in order to investigate how the measures change in the geometry part of GeoGebra (e.g. that the sum of angles in a triangle remains 180°)?	5.9%	41.4%	43.6%	5.9%	3.2%	0.0%

Table 4: Questions 6 and 7 on measuring

Looking at results from question 6 and 7, we can see that **measuring** features at a relatively high rate compared to dragging. In fact, it was also the highest rate compared to all other questions, also those that are not included in this paper. In question 6, 66.8% of teachers report that their students frequently work with measuring of figures. Question 6 does not reveal what sort of measuring is done. For instance, if it is the sort of measurement of figures, which might as well be done with paper and pencil. However, question 7 gives more nuanced insight by asking more specifically regarding the measuring activity. The teachers report that their students frequently (41.4%) and occasionally (43.6%) work with measuring in combination with dragging in order to investigate invariant measurements.

N=220	Always	Frequently	Occasionally	Rarely	Never	Don't know

8. Do the students work on tasks in the geometry part of GeoGebra, where they are asked to try to construct figures, which cannot be constructed?	0.9%	10.5%	45.5%	32.7%	10.5%	0.0%
9. Do the students work with the trace command in GeoGebra?	0.5%	1.4%	16.4%	34.1%	45.0%	2.7%

Table 5. Questions 8 and 9 on non-constructable figures and tracing

One way of utilizing the **feedback** potential of DGEs is in tasks that instigate the students to construct non-constructable figures. Question 8 reveals that 45.5% of teachers reported that their students occasionally work on such tasks, while 32.5% responded that their students rarely do so. Of course, there are also other ways of utilizing the feedback potential.

The results from question 9 indicate that possibility of **tracing** is unfamiliar to the teachers. The teachers mainly responded that their students rarely (34.1%) and never (45%) work with the trace command. It can be concluded that the potential of tracing to visualize underlying invariant relationships of constructions in conjecturing activities that yield development of the RC, is yet to be utilized, for example in maintaining dragging tasks (Baccaglini-Frank & Mariotti, 2010).

CONCLUDING DISCUSSION

The results indicate that the potentials of DGE in relation to RC, in particular measuring and dragging, are to some degree utilized in Danish lower secondary school. The particular utilization of the feedback potential in non-constructable tasks seems also to present, although at a lower rate. It may be regarded as a positive result, since especially dragging is considered to be a key feature of DGE that affords a visual representation of invariant geometrical phenomenon allowing for generalization, reasoning and conjecturing (e.g. Arzarello, Olivero, Paola & Robutti, 2002; Laborde, 2001; Baccaglini & Mariotti, 2010; Edwards et al., 2014). However, the results from question 1 gives rise to further questions about the actual utilization of these potentials. The results from question 3 indicate that the understanding of locked and free objects is not a particular focal point. As mentioned previously, the locked and free objects are the manifestations of the theoretical properties of figures, which are mediated perceptually in DGE during dragging, thereby potentially linking the spatiographical and theoretical levels (Arzarello et al., 2002). Perhaps the lack of focus on this basic understanding can be linked to the students remaining at the spatiographical level in the dragging activities. If that is the case, the conjectures will not be anchored in the theoretical properties of the figures, but at the spatiographical level. Additionally, it seems that the tracing command is rarely used, which implies that dragging with trace activated in order to highlight underlying invariant relationships is not presently utilized.

In order to improve the utilization of the potentials in relation to RC, it is necessary, first and foremost, to increase the availability of tasks that are actually made for utilizing DGE potentials. Secondly, even if some teachers adapt paper and pencil tasks, it would at least be beneficial for them to use guidelines to this end. Several DG task quality models have been suggested (e.g. Trgalova, Jahn & Soury-Lavergne, 2009; Trocki & Hollebrands, 2018). It cannot be expected that teachers, without any guidance, will adapt paper and pencil tasks into somewhat specialized tasks that utilize the potentials of DGE in relation to RC, such as soft construction tasks that are solved by using the maintaining dragging model (Baccaglini-Frank & Mariotti, 2010). Additionally, it is necessary to highlight the mathematical meaning of free and locked objects in instruction and tasks, in order to support the students in linking the spatiographical and theoretical levels. This is important, since awareness of the theoretical relationship between elements of a figure, which is mediated perceptually by DGEs as invariants during dragging, is a premise for investigating figures in conjecturing and reasoning activities, which are activities that are characteristics of RC.

FURTHER RESEARCH: INTEGRATION OF FINDINGS INTO PRACTICE

The insights gained from this research are integrated into another ongoing related project, in which the aim is to develop principles for the design of didactic sequences that utilize the potentials of DGE in relation to students' development of the RC (Højsted, 2019). The survey results suggest that the didactic sequence should include initial instruction and tasks aimed at supporting the students in understanding the theoretical underpinnings of locked and free objects, so that they can interpret the theoretical aspects of figures, which decide how the figure reacts when elements of the figure are dragged. Implementing "construction tasks" as coined by Mariotti (2012) may support this process. It is also clear that the sequence must take into account the likely lack of teacher and student knowledge regarding the trace command and tasks instigating the construction of non-constructable figures.

Additional studies are needed, in order to further complement the results presented in this paper. By supplementing the quantitative approach used in this study with a qualitative approach, in the form of interviews with some of the teachers from the study, it is possible to get a more detailed and nuanced account of the way feedback, dragging, measuring and tracing are being used in Danish lower secondary school. It is also an opportunity to unveil some of the reasons that lie behind the utilization choices of the teachers. The next step of this ongoing research is to categorize teachers based on their answers in the survey, particularly in some of the open-ended questions, and to interview teachers from each category, in order to gain deeper understanding of how each category of the teachers actually use GeoGebra in Danish lower secondary school.

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REWARDS AND CHALLENGES IN USING WEBWORK IN A MULTIVARIABLE CALCULUS COURSE

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I will describe the rewards and challenges in using WebWorK in a Multivariable Calculus Course at Brown University during spring semester,2019. In addition, the results of a student survey about the ease and use of the software will be summarized

Keywords: WebWork, Multivariable Calculus

BACKGROUND

I was assigned to teach multivariable calculus less than 2 weeks before the beginning of spring, 2019 semester. I understood that the course was to use the website that was developed by the original instructor; this website used both Canvas and WebworK. I had used the Learning Management Systems of Blackboard and Sakai and thought that I would be able to transfer my knowledge base to Canvas. For computer generated problems, I had previous used commercial textbook software, so I was not concerned about my being able to learn how to use WebWorK.

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Description of WebWork

WebWorK, developed by the University of Rochester and supported by the MAA and the NSF is open source and includes a National Problem Library (NPL) of over 20,000 problems for the following courses: college algebra, discrete mathematics, probability and statistics, single and multivariable calculus, differential equations, linear algebra and complex analysis. To create a homework assignment or quiz, the user first selects "library browser;" the software then provides the user the choice of the subject (such as multivariable calculus), chapter (such as vector geometry) and section (dot product, length and unit vector). WebWorK can be integrated into various LMS systems such as **Blackboard, Moodle** and **Canv**as.

Rewards for using WebWorK:

The major advantage for the both the instructor and student is that the problems are automatically graded. The instructor saves time grading and the student gets immediate feedback. In addition, WebWorK provides useful statistics such as the percent who answered a question correctly, the median number of tries required to get the correct answer for each problem. WebWorK also provides the number of times that each student needed to solve a problem.

Challenges for Using WebWorK

The major challenge was that the grades earned on homework or quizzes in WebWorK were not automatically generated into the LMS. Since WebWorK has its own Learning Management System, the instructor must access the grade in WebWorK and then manually insert these values into their LMS.

Another major challenge was that WebWorK does not allow certain inputs; as a result, some problems with incorrect input were not marked correctly. For example, students had trouble when the answer involved special constants such as pi.

Finally, while WebWorK is a free software, the institution needs to find a server to host the software. MAA will host members for one semester only, but then charges $200/ course to host WebWorK.

<u>Description and Analysis of Student Survey of use and ease of software</u>

Ninety-two of the 110 registered students completed a survey about their opinion of the WebWorK Homework. There were five multiple choice questions and 2 open ended question. The following 5 multiple choice questions were asked:

1. <u>How many hours do you spend on the WebWorK homework</u>?

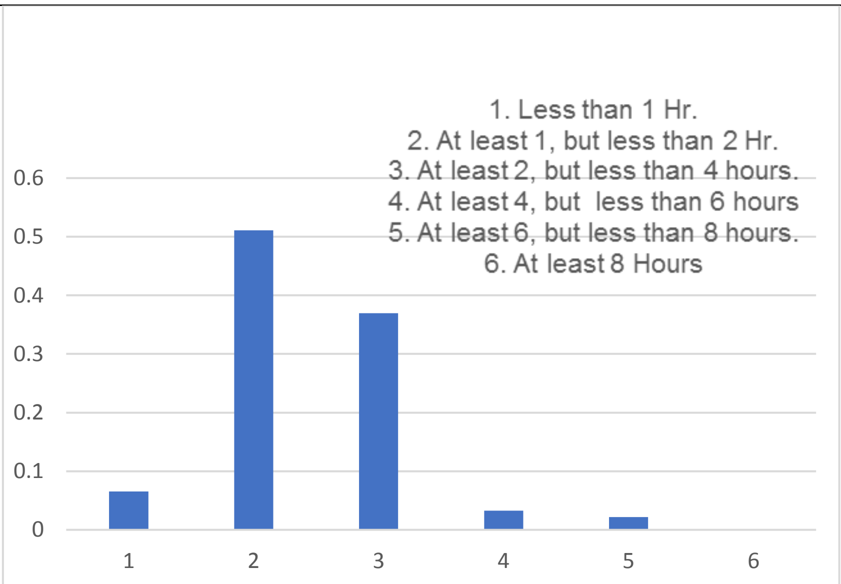


Table 1: Time Spent on WebWorK

Approximately eight WebWorK problems were assigned each week. The majority of the students (51%) spent between 1 to 2 hours on the WebWorK homework, while about 37% spent between 2 and 4 hours per week. All students spend less than 8 hours a week.

2. <u>Are WebworK assignments too short, somewhat short, about the right length, somewhat long or too long?</u>

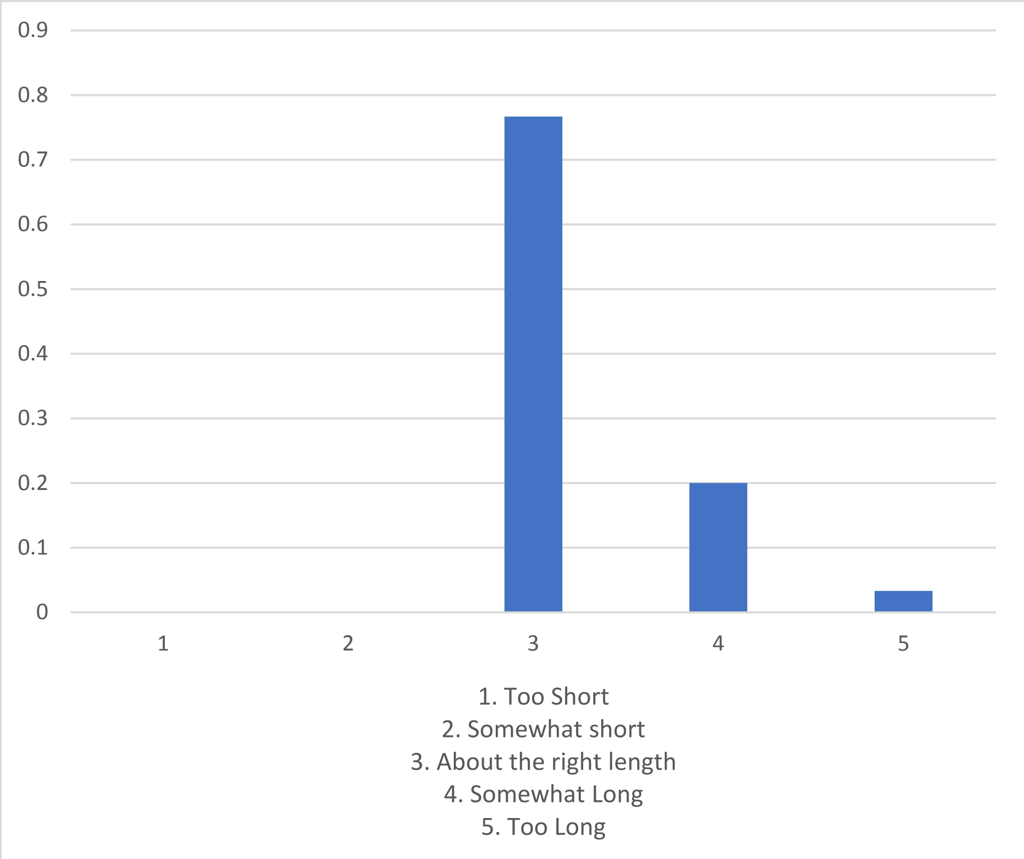


Table 2: WebWorK Assignments are:

The majority of students felt that the WebWorK assignments were about the right length.; No one felt that the assignments were short, and only 20% thought the assignments were somewhat long.

3) How was the process of entering the answer?

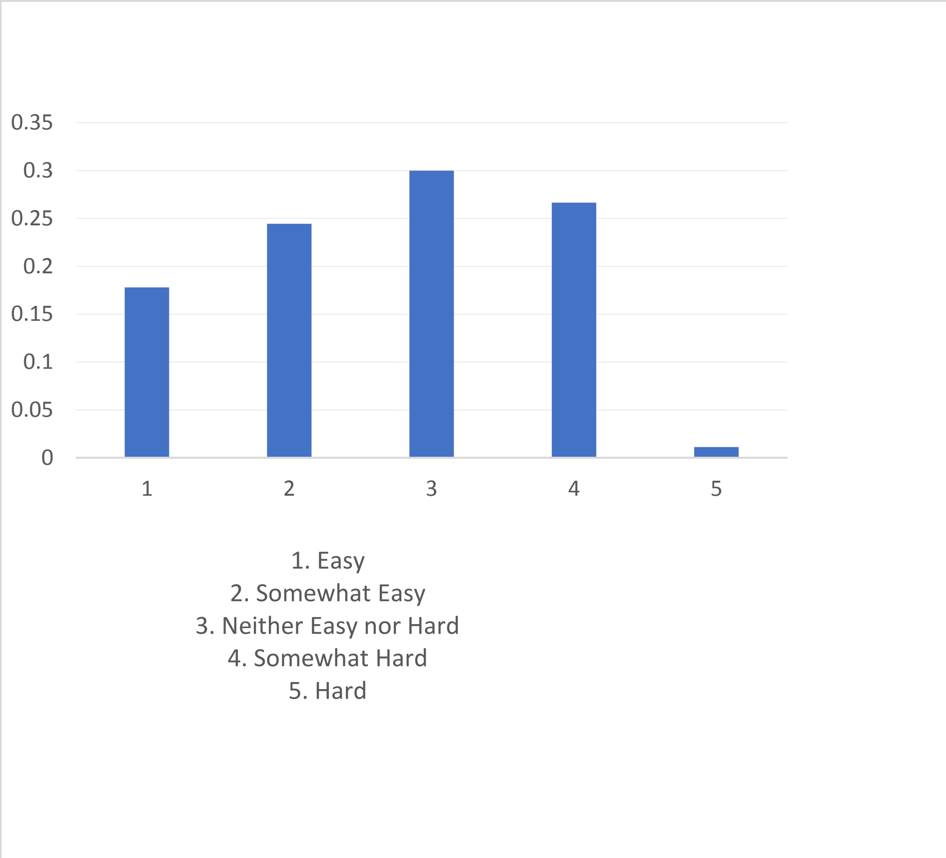
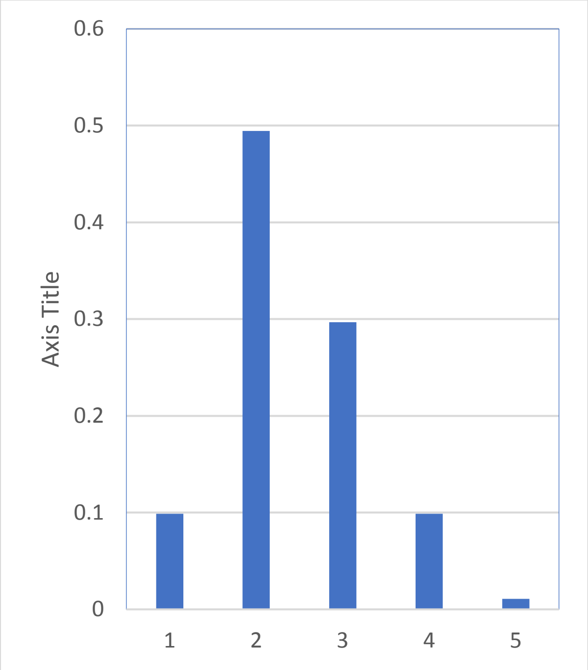


Table 3: Entering answers is

Students were divided in their opinion of the ease of entering the answer. While 30% thought entering the answer was neither easy nor hard, 27% thought it was somewhat hard and 24% thought it somewhat easy

4. <u>How useful was WebWorK in helping you understand the material</u>?



1: Extremely Useful

2: Very Useful

3: Moderately Useful

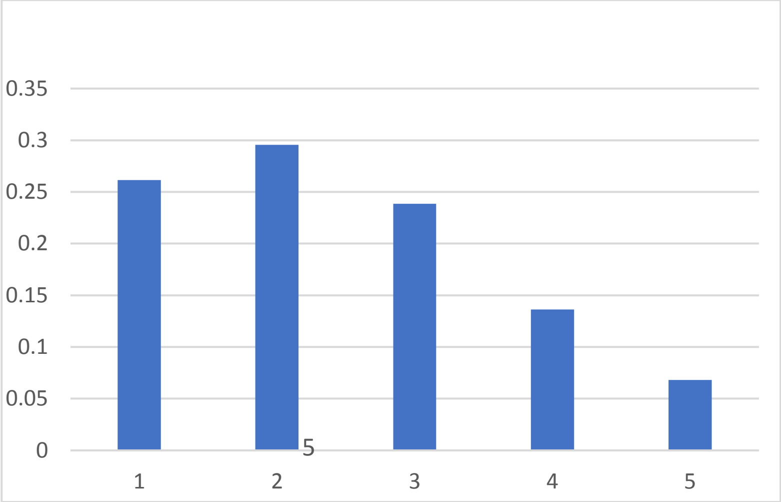
4. Slightly Useful

5. Not Useful

Table 4: How Useful was WebWorK in Helping you Understand Material

Most students felt WebWorK was extremely or very useful, 30% felt the tool was only moderately useful. Around 10% thought WebWorK was slightly useful and a mere 1% thought it was not useful at all.

5. <u>How would you compare the required WebworK HW to the required written homework</u>?



1: Strongly Prefer WebWorK
2. Somewhat Prefer WebWorK
3. No Preference
4. Somewhat Prefer Written Homework
5.Strongly Prefer Written Homework

Table 5: Compare WebWorK to Written Homework

The majority of students either strongly preferred or somewhat preferred WebworK to written homework. A surprising 24% didn't have any preference, and a total of 15% either strongly preferred or somewhat preferred written homework**.**

When asked what they liked about using WebWorK, most students answered: immediate feedback, ability to enter alternative answers without penalty if the initial answer is incorrect, and that there was no time limit to solve the problems. On the other hand, students response to what they disliked included the following: that there was both written and computer homework, that some math symbols were not recognized for answers and the number of decimals required for an answer was inconsistent and not provided

The survey asked students if they had used computer homework in any other course, and if so, whether they preferred WebWorK or another system. Most students replied that they had not used any other computer homework in other courses. A few mentioned Sapling, a computer homework in chemistry and WebAssign, a commercial homework system. Of those who used other systems, most preferred WebWorK, although a few preferred Sapling because it gave hints.

The University of British Columbia (http://www.math.ubc.ca) has been collecting survey data for students who use WebWorK in mathematics. In particular, the results of the survey for their multivariable calculus course (Math 253) found comparable results (http://www.math.ubc.ca/~cwsei) to what was found in Brown's Math 180. Dr Vicki Roth and others (Roth et al, 2008) at the University of Rochester, evaluated survey responses of 2387 students and found that students especially liked the immediate response that WebWorK provides; they disliked the fact that WebWorK could not provide them with partial credit and was unable to let them know if their answer is "almost correct."

Summary and Relevance to Other Studies

While learning WebWorK was challenging, the benefits far outweighed the disadvantages. In general, the students liked the online computer homework, and found it easy to use.

Many institutions now use WeBWorK and the literature describes the advantages and techniques for using this open source software successfully. For example, Dr. Grace Kennedy (Kennedy) from the University of California at Santa Barbara recommends that instructors require both WeBWorK and written work in order to encourage meaningful engagement; according to Dr. Kennedy, " In a differential equations class, the teacher can select four to five topics that are not well tested by WeBWorK and have students submit a written assignment on these topics."

Many WebWorK users haven written testimonials (http://webwork.maa.org) for WebWorK. For example, Tom Shemanske (Dartmouth College) lists many advantages of WebWorK including: instructors can use any textbook, more problems are continually added, and the product is extremely flexible to the needs of the individual instructor. In addition, the MAA webpage also cited many instructors (For example Lars Jensen, Truckee Meadows Community College and Mel Hochster and Gavin LaRose, University of Michigan) have found that WebWorK is superior to commercial computer systems that they have used.

In a 2010 AMS online survey of 1,230 U.S. mathematics and statistics departments (with a response of 467) asking for their experiences using homework software found that all software systems received praise (Kehoe, 2010). At this time, most software used was commercial: MyMathLab was used by 230,000 students; WeBWorK by a little over 100,000 students; and WebAssign by a little under 100,000 students. It is now possible to check the locations and schools that are using WebWorK. The map that the MAA (http://webwork.maa.org/wiki/WeBWorK_Sites#.XZzm41VKiUl) provides indicates that the majority of WebWorK users reside in the United States and Canada. This imbalance is probably because WebWork was designed by the educators at the University of Rochester and the rest of the world is not yet aware of this amazing free software.

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BILINGUAL MATH LESSONS WITH DIGITAL TOOLS – CHALLENGES CAN BE DOOR OPNER TO LANGUAGE AND TECHNOLGY

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Although, mathematics can be understood as universal, some authors take the view that there are differences in school mathematics among different countries. The more complex mathematical topics are, the more the linguistic or cultural differences disappear. In this text, three hypotheses are tested: In a school context, differences in mathematical education between Germany and the US exist. These differences are relevant to bilingual math lessons. Digital mathematical tools can be door openers and help to deal with these differences. Based on two examples of American-German bilingual learning environments, conclusions can be drawn. Math lessons were observed in the MISTI Global Teaching Lab program and the German International School Boston. A two-step mixed-method study was developed to identify and describe differences in the mathematical education and the role of digital mathematical tools.

Keywords: Bilingual math lessons, Differences between American and German mathematical education, digital mathematical tools, instrumental genesis

INTRODUCTION

In attempting to compare German and US American math education, generalizations are difficult, if not impossible, to make. First, one cannot clearly define what constitutes a "German mathematics education". The diversity among the German *Länder* (states) with their 16 educational systems is present and empirically proven, as the current INSM education monitor shows (Anger, Plünnecke, & Schüler, 2018). In addition, the language of instruction at Austrian schools and many Swiss schools is also German. Likewise, in the US there is no homogenous setting of "American" mathematics education. The 50 states differ in their educational systems. Comparisons in the form of ratings and rankings are common. For example, the educational system of the State of Massachusetts is considered the best in the US (Trimble, 2018). In spite of the difficulties with a delimiting definition, some observations on mathematics education in the USA can be formulated without claiming to be generalized.

As early as 1913, Jourdain (2007, p. 1) expressed his expectations to math teachers with the words: "[...] he was never satisfied with his knowledge of a mathematical theory until he could explain it to the next man in the street." There are two views: Everyone (learners) can expect mathematics to be explained in "simple words". However, a mathematical theory cannot be extremely simplified, until it only applies to special cases. That would be too heavy on the meaningfulness (Jourdain, 2007). Mathematics teachers in Germany as well as in the USA move between these two views. It is interesting to see where the priorities are set. In teacher education, it is important to prepare for the challenges of teaching. Breux & Whitaker (2015, p. xii) note the impossibility of general solutions: "We are highly aware that there are no two teachers exactly alike and that no one solution fits all circumstances." Nevertheless, they feel able to offer 60 simple answers to everyday problems. This pragmatic approach is beneficial for young teachers yet to gain their own experience in the classroom. American colleagues largely assume that mathematical performance can be well-grasped and measured in multiple-choice tests. The results in standardized final exams with thematically consistent, compact tasks ("items") determine the school career from Middle and High School through to College. Math lessons are often characterized by preparation for the standardized tests (Hyun, 2006; Kaplan, 2009). Geometry is present throughout the whole education and geometric problems are important at all levels (Balley, 2012; Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009).

These are interesting observations, which give an idea of the challenges of bilingual mathematical learning environments. For the current study, two projects are of particular interest: MISTI Global Teaching Lab and the German International School Boston.

MISTI GLOBAL TEACHING LAB AND GISB

In January 2019, 43 students from the Massachusetts Institute of Technology (MIT) visited Germany as part of the MISTI Global Teaching Lab exchange program to work with students in math and science classes at German schools. In doing so, they inspired the students with their enthusiasm for their fields of research and provided many important ideas for the learning process. Participation in the Global Teaching Lab program provided the basis for cooperation with the MIT Science and Technology Initiatives (MISTI). The goal of the project is to prepare STEM research results for learners in an educationally useful manner.

The German School, Boston was founded in 2001 by a group of parents and teachers from the German-speaking community. The opening was preceded by a four-year planning and organization phase. Since the school unites students of different nationalities and is committed to multiculturalism, the name has been extended to German International School, Boston. In recent years, the number of registrations has increased continuously and the campus has been expanded systematically. Currently, the GISB has more than 300 students from preschool to grade 12. Since 2013, successful graduates of the school have received both the Massachusetts High School Diploma and the German International High School Diploma. Graduates of the school can study in the US or in Germany and the EU.

THEORETICAL BACKGROUND

"Doing mathematics is different in different languages." This statement from Barwell (2003, p. 38) represents a point of view that directly links differences in the school mathematics of different countries with language. It is important that mathematics can be understood as universal (Rolka, 2004), but the examination of mathematics in the classroom, thus "doing mathematics", can differ in different cultures. Moreover, the consideration of cultural influences contributes to understanding students’ problems: "However, positioning mathematics as culture-free and neutral reinforces the belief that the problem lies with the students or their families as opposed to with the curriculum, pedagogical choices, or the educational system" (Felton-Koestler, & Koestler 2017, p. 68).

In terms of school mathematics, it is possible to say, the more complex mathematic topics are, the more the linguistic or cultural differences disappear (Novotná, & Moraová, 2005). In school contexts, differences in mathematical work exist. It is interesting how these differences are dealt with in a bilingual learning environment. When teachers’ backgrounds differ from their students, they may have difficulty recognizing the existing knowledge of students, thus, the knowledge that can be built on in the classroom (González, Andrade, Civil, & Moll, 2001). The MISTI GTL Germany project and the GISB are good examples of bilingual mathematical learning environments. In both environments, the classroom language (as a foreign language) differs from language used by most learners (the daily lived language). In both cases, teachers teach mathematics in their mother tongue. At the MISTI GTL American MIT students teach German high school students (in English) and at the GISB German teachers teach international students (in German).

In both learning environments, students work with digital mathematical tools, which have the potential to act as door openers in either direction (Müller, 2018). The implementation of digital tools in mathematics education has a long tradition in the US. This is not only about using such tools, but about actively shaping them and developing mathematical and informative content (Papert, 1993). Based on exploratory interview studies (Szücs, & Müller, 2013), the hypothesis that differences between the German and American cultural areas can be identified made regarding the level of school mathematics has been validated. These differences are relevant for bilingual education. Digital math tools can play a crucial role in this process. The educational theory of instrumental genesis can provide clues to the role of digital tools as a door opener to language and technology (Müller, 2018). Part of the instrumental genesis is the relation between cognition and artefacts (Verillon, & Rabardel, 1995), which act as linking points between communication and culture. The instrumental genesis is also covered by the Task-Technique-Theory (Drijvers, 2004; Kieran, & Drijvers, 2006), which is an anthropological theory in the first place without particular reference to digital mathematical tools. Another part of the instrumental genesis is the impact of digital tools on the construction of knowledge (Rieß, 2018), which also gives starting points for cultural aspects and communication.

RESEARCH AIMS

Considering the theoretical background and findings of explorative studies, three statements can be made and need to be proven. First, elementary school mathematics differ more between Germany and the US than do high school or university level mathematics. Second, the listed differences are important to bilingual mathematical learning environments and teachers need to be aware of them. Third, digital mathematical tools can be door openers to the foreign language and can promote deeper understandings of mathematical topics.

METHODOLOGICAL FRAMEWORK AND STUDY DESIGN

In order to identify the differences between American and German mathematics education which are relevant for bilingual education, a two-stage mixed-method design was chosen. Corresponding to the research aims, the use of digital mathematical tools attracted particular attention. The first stage had an exploratory character and essentially included qualitative instruments such as standardized interviews. German and American teachers were interviewed using a standardized interview schedule (Szücs, & Müller, 2013). The MISTI GTL participants were able to reflect on their own actions in class using video vignettes. The evaluation as part of an interpretive video analysis was based on Knoblauch, Schnettler, & Tuma (2010). At this stage, the aim was to formulate hypotheses based on the findings of the qualitative study. Five teachers of the GISB and four MIT teaching students participated in the interview study. The lessons of the MIT students were video recorded. The transcription guidelines for interviews and video recordings were standardized (Kuckartz, Dresing, Rädiker, & Stefer 2008). For the video recordings an inductive system of categories was develop (mathematical content, linguistic peculiarities, mathematical symbols …) Because of the standardized interview schedule, a deductive system of categories (mathematical working, notation, word meanings …) was used to analyse interview data.

Based on the first step, three hypotheses could be formulated (see section above). In a second quantitative step, teachers and teaching students were asked about the relevance of the differences that had been identified using a standardized online questionnaire. To date, 75 teachers and teaching students of the GISB and the MIT have participated in the online survey. All of them have experience with bilingual mathematical education either at the GISB or from the MITSTI GTL Germany project.

The online questionnaire includes 13 differences, which were identified in the first step (see first paragraph). All 13 items were followed by two additional items, each with aim of asking about relevance in the classroom and the use of digital mathematical tools (see Image1). In order to describe the panel more accurately, four motivational aspects were part of the questionnaire, too. These were Self-assessment (two items), Interest in mathematics education (three items), Interest in mathematics (three items), Self-efficacy (seven items). The items had been used and validated in earlier educational studies (Benölken, 2014). With respect to the scale level, non-parametric statistical tests such as Mann-Whitney-U-test were chosen for the data analysis. The data regarding the differences were collected in fourfold tables. Shifts in the tables were identified by using the exact Fischer test (for two sides). P-values were calculated by using the method of Agresti (1992) while using a program code similar to Langsrud, & Gesellensetter (n.d.). Regarding the number of statistical tests, a level of significance of 0.01 was chosen.

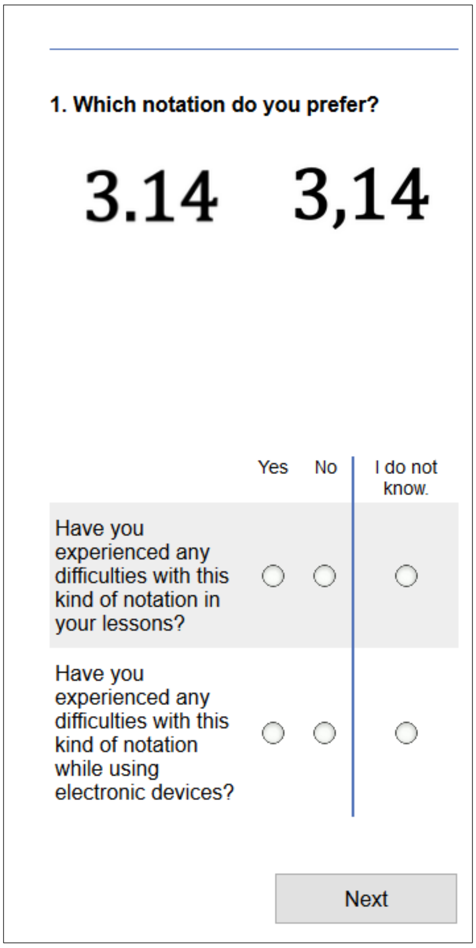

Image1: *Online questionnaire optimized for mobile devices. Shown is one of 13 differences with the two additional items about relevance for classroom and digital tools.*

RESULTS

In this article, we will present the results of the recent online survey. To compare the groups of teachers (and teaching students) four characteristic variables are of interest. Self-assessment, Interest in mathematics education, Interest in mathematics and Self-efficacy are important to the motivation of dealing with problems (Eccles et al., 1983). As shown in Tab. 1 there or no significant differences, neither between the groups of teachers (and teaching students) from Germany and the US nor between the two institutions. All teachers (and teaching students) are interested in mathematical topics and mathematics education. All participants score highly according to Self-assessment and Self-efficacy. Therefore, we can assume that all participants are highly motivated in what they do.

Variable	Cronbachs Alpha	ALL (75)	USA (65)	GER (10)	MIT (59)	GISB (16)
Self-assessment	0.755	3.15 (0.58)	3.15 (0.61)	3.15 (0.32)	3.15 (4.46)	2.88 (0.84)
Interest in mathematics education	0.766	3.21 (0.59)	3.22 (0.60)	3.2 (0.50)	3.21 (0.57)	3.04 (0.64)
Interest in mathematics	0.840	3.06 (0.67)	3.03 (0.69)	3.3 (0.43)	3.02 (0.54)	2.85 (0.99)
Self-efficacy	0.877	2.96 (0.55)	2.97 (0.58)	2.89 (0.33)	2.96 (0.47)	2.69 (0.75)

Tab. 1: *Aspects of motivation: Self-assessment (two items), Interest in mathematics education (three items), Interest in mathematics (three items), Self-efficacy (seven items). Shown are mean and standard deviation of scale from one to four.*

Four of the 13 differences included in the online questionnaire have been shown to be important from the teacher's point of view. German and US American teachers (and teaching students) use different symbols or use different mathematical terms and definitions. The following fourfold tables include highly significant values and show differences between German and US American teachers (and teaching students). Tab. 2 shows the different uses (and meanings) of Point and Comma. For 59% of all teachers (and teaching students) this causes problems in bilingual math lessons. Furthermore, 71% agreed that this makes a difference when using digital mathematical tools.

	GER	USA	
Point (3.14)	2	58	60
Comma (3,14)	8	7	15
	10	65	75

Tab. 2: *Example of differences between German und US American school mathematics in category notation (teachers view). Fourfold table with significant shift (p=0.0000141, exact Fischer test).*

According to Tab. 3, teachers use different symbols for the right angle in geometric drawings. It is an interesting fact, that US Americans have a more sophisticated way of naming (and ordering) basic geometric figures (triangle, quadrilateral, pentagon …). In German, it is common sense to call the figures *Dreieck, Viereck, Fünfeck* …. Especially, naming the quadrilateral can be problematic in bilingual math classes as far as teachers agree (55%). Teachers report of difficulties to find the right geometric figure while using digital mathematical tools (48%).

	GER	USA	
Right angle symbol square	2	56	58
Right angle symbol quarter circle plus dot	8	9	17
	10	65	75

Tab. 3: *Example of differences between German und US American school mathematics in category symbols (teachers view). Fourfold table with significant shift (p=0.0000502, exact Fischer test).*

As shown in Tab. 4 there is a difference in dealing with third roots of negative values. US American teachers (and teaching students) like to distinguish between odd and even roots. German teachers except only positive values for any roots. Most teachers (55%) have had problems with these definitions in bilingual math lessons. For 61% of all teachers this distinction is relevant for the use of digital mathematical tools.

	GER	USA	
$\sqrt[3]{-8}$ is defined	1	51	52
$\sqrt[3]{-8}$ is not defined	9	14	23
	10	65	75

Tab. 4: *Example of differences between German und US American school mathematics in category mathematical working (teachers view). Fourfold table with significant shift (p=0.0000526, exact Fischer test).*

Many teachers (47%) have experienced difficulties in using mathematical terms for the solutions of quadratic equations in their bilingual math lessons. German teachers prefer a sum of the final term where US American teachers (and teaching students) prefer a fraction (see Tab.5). This problem is relevant to at least 51% of all teachers regarding their use of digital mathematical tools.

	GER	USA	
Solutions of quadratic equations given as a fraction	1	54	55
Solutions of quadratic equations given as a sum	9	11	20
	10	65	75

Tab. 5: *Example of differences between German und US American school mathematics in category mathematical working (teacher's view). Fourfold table with significant shift (p=0.0000114, exact Fischer test).*

DISCUSSION

First, we have to admit that 75 questionnaire responses might be insufficient to allow us to draw meaningful conclusions. In particular, the data seems too limited to identify differences between German and American school mathematics with a degree of certainty. Nevertheless, such differences do seem to exist and the survey results give hints about where we may find them. It is necessary to increase the number of responses and invite more teachers from both Germany and the United States to participate. In the current sample, there are motivated and highly educated teachers and teaching students from two high-performing educational institutions. In order to obtain an even more multifaceted picture, it would be wise to invite teachers from different schools with different backgrounds to participate in the study. At this point, we believe the chosen statistical methods (non-parametric test, exact Fischer test) are suitable for the type of data and the numbers. Therefore, the results are accurate and provide a foundation for the next steps in an ongoing research process. More data will be collected and analyzed.

Early results show that the teachers are aware of differences in the school mathematics in the two countries. These differences are important for bilingual learning environments and they might present difficulties that teachers will have to deal with. It is possible to distinguish differences in the language (such as different words or false friends) and more culturally related differences. Symbols (see Tab. 3) and procedures can also differ. Sometimes, the different traditions open a door to a deeper understanding of the content, such as the definition of the third root of negative numbers (see Tab. 4). Another example is the mathematical representations for the solutions of quadratic equations (see Tab. 5).

For most teachers, these differences can be dealt with by using digital math tools. The tools show the differences and give feedback on them. They can be translators for the language and for the mathematics. A theoretical approach to this practical experience could be the instrumental genesis (Rabardel, 2002; Wygotski, 1985). Communication and cultural issues need to be addressed more clearly. In a bilingual learning environment, a digital tool can be a door opener between classroom language and first or commonly used language. The tool can take the role of a mediator. This is also related to cultural aspects. The digital tools can help to investigate the differences and obtain a deeper understanding of the mathematical concepts. This factor might be relevant even to regular math classes where digital mathematical tools are used, because of the design and programming of the digital tools, which refer to their cultural background.

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A FRAMEWORK DESCRIBING STUDENTS' MATHEMATICS LEARNING EXPERIENCE WITH A TABLET-BASED PEDAGOGICAL MEDIUM: THE CASE OF A GEOMETRY EXPLORATION

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In this study, we presented a theoretical framework that describes factors of student's learning experience in a tablet-based learning environment with math apps. The theoretical framework is based on a synthesis of the literature. We further illustrated through empirical data the underpinning factors of our proposed theoretical framework and provided data to illustrate the learning experience of 60 Grade 6 students exploring a triangle through the use of tablets and Geogebra apps. The findings suggested that students were actively engaged in the learning process since they were exploring independently the elements of a triangle. In addition, the affordances of the tablet and Geogebra apps, such as the haptic dragging, reshaped the learning trajectory and students' perspective on the mathematical content.

Keywords: mobile technology, learning experience, math apps

INTRODUCTION

Advances in digital technologies are altering the variety of tools available to teachers and students (Attard & Curry, 2012). Mobile technology, currently dominated by tablets, is prevalent in many school classrooms (Hilton, 2018). A growing number of research findings documented that the use of mobile technologies has the potential to transform and reshape students' learning experience (Attard, 2017; Calder & Campbell, 2016). The characteristics of mathematics learning with mobile technologies, the affordances of mobile technologies and math apps combined with appropriate inquiry-based pedagogical approaches may create an interactive and visual learning environment, different from that of traditional teaching. Calder and Murphy (2018) supported that mobile technological tools, such as tablets, if used appropriately, may help students investigate mathematical ideas in ways that promote mathematical thinking and concretize abstract mathematical concepts. In addition, a learning environment that used mobile technology and apps gives a new potential to students' mathematical understandings by repackaging the mathematical content and processes, differentiating learning experiences and enhancing students' engagement by exploring simultaneously visual, symbolic and numerical representations and fostering independent learning (Olive et al., 2009).

Due to the rapid pace of technological developments, there is little published research investigating the ways that students' learning experience is shaped in digital-based environments (Calder & Murphy, 2018). In addition, further research is needed to investigate the ways in which effective pedagogies may change the nature of a digital based mathematics classroom from a teacher-led environment to a student-centred one. Research-based evidence is needed regarding new ways for designing such learning environments that teachers could use as exemplars of effective learning and teaching and to enrich existing literature regarding the theoretical conceptualizations of learning within digital driven environments (Attard, 2017). The purpose of this study is twofold: First, to present a theoretical framework describing factors that relate to student's learning experience within a digital tablet-based learning environment with apps. We will discuss in what ways the learning experience might be reshaped compared to a traditional learning environment. Second, we will report on an exploratory study of sixty Grade 6 students that worked with geogebra apps to exemplify some parameters of the proposed framework.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Students' active participation in rich mathematical activities is critical for students' learning experience (Choon, Lam, & Berinderjeet, 2014). It is widely acknowledged that learning experiences must include opportunities where students discover mathematical ideas and participate actively in the learning process. Students should be given opportunities to develop collaborative and communication skills, set learning goals, obtain satisfaction from the learning process, build up their mathematical understandings and problem solving ability, while teachers should design engaging learning activities. Thus, the role of learning experiences in understanding mathematics is extremely important. Digital technologies have the potential to reshape students' learning experience by offering new kinds of authentic learning experience, ranging from experimentation to real world problem solving (Hilton, 2018). Research findings showed that students' learning at school is enhanced when using tablets (Clark & Luckin, 2013). The quality, depth and breadth of students' learning experience in a digital-based technological learning environment relates to students' engagement in the learning process, the adopted pedagogical approaches and students' learning trajectory (Calder & Campbell, 2016). Research findings suggest that digital technologies influence students' engagement and actual learning trajectory by reshaping the learning process and repackaging the potential of exploring the mathematical content. Moreover, digital environments have potentials for differentiating the learning activities that may facilitate the development of individual learning trajectories (Attard & Curry, 2012). By the term learning trajectory we refer to the hypothetical learning trajectory (planned based on curricula) and the actual learning trajectory that involves the actual pathway of the learner while his thinking evolves during working on activities (Sacristán, et al., 2009). Sacristán, et al. (2009) asserted that digital technologies may differentiate actual learning trajectories.

Learning engagement has been conceptualized by Fredricks, Blumenfeld and Paris (2004) as a multi-faceted construct that operates at the cognitive, affective and behavioural level. Specifically, Attard (2017) defined engagement in mathematics as the 'coming together' of cognitive, emotional, and behavioural engagement that leads to students' enjoyment and valuing of mathematics. Thus, learning engagement is related to the learning enjoyment, doing mathematics and viewing the learning and doing of mathematics as a valuable and useful task within and beyond the mathematics classroom (Attard, 2017). Cognitive engagement involves recognizing the value of learning and the willingness to go beyond the minimum requirements; affective engagement conceptualizes students' willingness to become involved in school work; and behavioural engagement encompasses students' active participation and involvement in academic and social activities. Research findings showed that integrating tablets in mathematics teaching has an effect on student engagement by promoting interactivity, motivation, immediate feedback, challenge and fun, while teacher's pedagogical approaches are a decisive factor on maximizing the potential of digital technologies to engage students in mathematics (Clark & Lucking, 2013).

The learning process and subsequently students' learning engagement is influenced by the affordances of the digital environment. Digital environments provide multiple representations that contribute to the enrichment of mathematical concepts. For instance, apps present the mathematical ideas in an investigative context and provide a visual and interactive learning environment. In addition, tablets provide a kinaesthetic orientation of learning while multiple senses are incorporated that create a friendly, creative and pleasant learning environment for students (Beschorner &

Hutchison, 2013; Judge et al., 2015). Moreover, research findings showed that the use of tablets contributes to an increased engagement of the students by providing the opportunity to work in groups and allowing the students to move around the room and get involved in a variety of activities. A well-organized lesson that integrates tablets, math apps and adopts explorative and investigative approaches builds a pedagogical medium that allows students to test informal conjectures, link different representations and explore the interactive affordances of the medium. Students' approach exhibit a more complex and nuanced way of engaging with the availability of different kinds of technologies, as well as making considered decisions about using the available tools in unexpected ways, take risks and employ investigative strategies. It should be noted that apps should match the curriculum and offer functionalities that enable applying productive pedagogies.

The potential of digital tools to facilitate the visualization of abstract mathematical concepts, conjecturing and testing of ideas is closely related to the applied pedagogies and the appropriateness of the used tasks. An inquiry based approach has been identified as being particularly appropriate for technology-enhanced mathematical activities (Attard, 2017), while research findings suggested that technology-based mathematics instruction involves the teacher's ability to make changes to pedagogy.

The learning experience is also related to the curriculum and the learning trajectory. Research findings suggested that the investigation of mathematical concepts through a pedagogical medium that integrates apps and tablets may transform students' learning experience by offering an alternative learning trajectory (Sacristán, et al., 2009). However, researchers call for the need to further explore the way in which a digital based learning environment may emerge new mathematical meaning and discourses and reveal new hierarchies of learning, by reshaping existing hypothetical learning trajectories (Sacristán, et al., 2009). In addition, a reshaped learning trajectory may allow students to explore mathematical content that may arise during digital-based investigations and as a result may enrich the depth and breadth of the mathematics content.

We suggest that a digital tablet-based learning environment that integrates math apps and explorative pedagogies transforms the apps and the tablet to a pedagogical medium. Based on the above, we propose a theoretical framework that describes factors that relate to student's mathematics learning experience with a tablet-based pedagogical medium (see Figure 1). Learning experience that involves students' active participation in the process of learning relates to three main factors: (a) student engagement, (b) the learning process and (c) the alternative perspectives offered by the pedagogical medium. Student engagement includes the cognitive, affective and behavioural dimensions of students' interactions and processes in the learning environment and depends on the affordances of the digital tools and the student's participation in the learning process. The learning process relates to the pedagogical approaches adopted by the teacher and the extent to which the teacher reshapes the learning process based on the affordances of the digital tablet-based learning environment and transforms the available digital tools to a pedagogical medium. Finally, the alternative perspectives that may arise during the learning process may lead to alternative learning trajectories that transform the quality of student's learning experience and the depth and the breadth of the examined mathematical content.

In the present study, we describe an exploratory study underpinning some factors of our proposed theoretical framework and provide empirical data illustrating the learning experience of 60 Grade 6 students exploring the elements of a triangle using tablets and Geogebra apps. In particular, we will address the following research question: In what ways the affordances of the pedagogical medium (a) facilitated student's active participation and engagement and (b) revealed alternative perspectives in respect to the mathematical content and the actual learning trajectory.

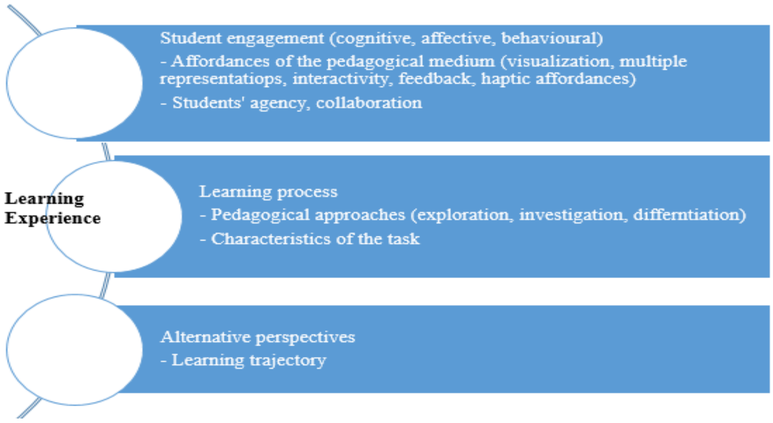

Figure 1. The proposed theoretical framework

METHODOLOGY

The participants were sixty Grade 6 students from three intact classes in one public primary school in Cyprus. The school's population came from a middle to high socioeconomic status. The students reflected a broad spectrum of academic achievement levels. The school was chosen as an appropriate site because it was recently equipped with tablets and the teachers of the three classes were helpful in the whole procedure. In addition, the students had used apps and tablets in mathematics several times before. The lesson was developed by the research team in collaboration with the three teachers. The development of the lesson took into consideration the fundamental parameters of the proposed model. The lesson is part of a Geometry Unit in Grade 6 and the relevant attainment targets included students' exploring and understanding the altitude, the bisector and the median of a triangle. The lesson was planned for 80 minutes. It was delivered by one of the members of the research team and consisted of the following three exploration activities that were developed based on the factors of the proposed model: In Exploration 1, students were given three ready-made Geogebra constructions. The constructions were presented in an applet form and included a triangle ABC and a segment AD. In the first construction the segment AD was a median, in the second one AD was a bisector and in the third one AD was an altitude. Students were asked to explore freely the three applets and explain the function of the segment AD in each construction and provide an operational definition of the segment AD in each case. Then, students were asked to present their work. The teacher introduced the formal definitions and asked to compare the three elements of the triangle. In Exploration 2, a ready-made applet was provided that presented a triangle, the measures of its three angles and an altitude. Students were asked to drag the vertices of the triangle to investigate the position of the altitude in the case of an acute-angled, obtuse-angled, and right-angled triangle. In Exploration 3, an applet presenting a triangle, its median, altitude and bisector was provided. Students were asked to investigate when the altitude, the bisector and the median of the triangle coincide.

This exploratory study used qualitative methods for collecting data. In each class, two researchers observed four students to capture their actions while working. In addition, researchers posed clarification questions to explore further possible hidden processes in students' work. To do so, each researcher used an observation protocol. The protocol consisted of a set of 10 questions examining the way in which each student utilized the affordances of the tablet and the app in respect to the specific exploration, such as the following: (a) Do they observe the available measures while dragging?, (b) Do they drag the triangle vertices flexibly and purposefully?, (c) Do they construct different kinds of triangles while dragging to examine the role of each segment? and (d) Do they explain the function of each segment based on the available measures? Further, we videotaped the screens of the tablets to capture student's actions while working, emphasizing on dragging. A qualitative interpretive framework was used for the analysis of the data (Miles & Huberman, 1994). A comparative analysis of the data collected by the two researchers was undertaken to ensure reliability and initial analysis was conducted immediately following each lesson of data collection.

RESULTS

In this study, analysis focused on the ways that the affordances of the pedagogical medium facilitated student's active participation and engagement and revealed alternative perspectives in respect to the mathematical content and the actual learning trajectory. To do so, we present the way in which students utilized the affordances of the tablet and the applet in the three explorations. Exploration 1. Eight out of the twelve students surfed between the three available applets and tried to compare the function of the segment AD. The remaining four students did not switch between the applets but they kept working in only one of them. Thus, the design of the activity and the way that the majority of the students worked showed that they did not follow a traditional learning trajectory (i.e., learn each of the three separately, one after the other) but they tried to understand the characteristics of each element of the triangle by making comparisons. To do so, they documented their explanations based on the measures of the applet. In the following excerpt, we present the discussion with one student that worked strategically and utilized the affordances of the software.

Researcher: Can you explain your work in order to explain the differences between the three segments?

Student 1: Yes, we first dragged the vertex A in the three applets. In the first applet, we observed that the segment AD divides the opposite side, because the segments BD and CD were equal (indicating the measures of the applet). Then, we examined if AD divides the opposite side in the second applet. We dragged A in the second applet, but nothing happened. Thus, we understood that AD does not function in the same way in the two applets.

Researcher: What did you do next?

Student 1: We moved to the third applet and we observed again that AD did not divide the opposite side, thus we decided to observe how the measures of the angles change, while dragging A.

We observed the way that students utilized the dragging capability of the software. Only two students did not drag the vertices of the triangle, but tried to figure out what was happening based on the one available measure, using the applet as a static image. Four students dragged the vertices of the triangle near to their existing positions (see the blue trace in Figure 2a), while the other six students dragged the vertices all around the screen and constructed different kinds of triangles (see the blue trace in Figure 2b). These six students, when asked to document their answers, utilized the dragging capability to explain that their conjecture is valid, even in extreme cases. The following excerpt shows the way that a student documented her reasoning based on the dragging:

Researcher: Are you sure that AD (in the second applet) bisects the angle?

Student 2: I was not sure either. But, I dragged the vertex A in many different places on the screen to see. If you make a very big or a very small triangle (she dragged vertices A and B to transform the existing triangle) or if you move the triangle up or down the screen, the two angles are always equal.

Researcher: Is this characteristic valid in the case that the angle A gets very small?

Student 1: Yes, I checked it. I dragged at the same time vertices B and C to make angle A as small (see the trace of the two vertices in Figure 2c) as possible and the result was the same (she used the multi-touch affordance of the tablet).

Exploration 2. In the second exploration students had to investigate the position of a triangle altitude in the case of an acute, obtuse and right-angled triangle. Four of the students dragged the vertices of the triangle randomly and consequently could not connect the position of the altitude with the kind of the triangle. Four students dragged vertex A to the right side of the screen to transform the triangle to an obtuse-angled one (see Figure 3a). Students explained that in the case of an obtuse-angled the altitude gets outside of the triangle, by displaying on the screen the size of the angle C. However, in the case of a right-angled triangle, they could not explain that the altitude coincides with the side of the triangle. When asked whether a right-angled triangle does not have an altitude, they dragged again vertex A to make a right-angled triangle and suggested that the two lines (side and altitude) are joined, thus we cannot see clearly the altitude. When prompted to think if we see the triangle side or the altitude, they observed the colour and deduced that we see the altitude because the colour of the segment is black. One of them clarified that the altitude lies on the segment AC. The other four students utilized the dragging capability to a greater extent and dragged vertex A in many different positions and transformed first angle C to an obtuse one and then angle B (see Figure 3b). At the crucial point of transforming the angle C from an acute to a right and then to an obtuse one, they repeated gradually the procedure several times (see Figure 3b). They repeated also the same procedure in the case of angle B. After this, they easily explained that in the case of a right-angled triangle the altitude coincides with the side of the triangle.

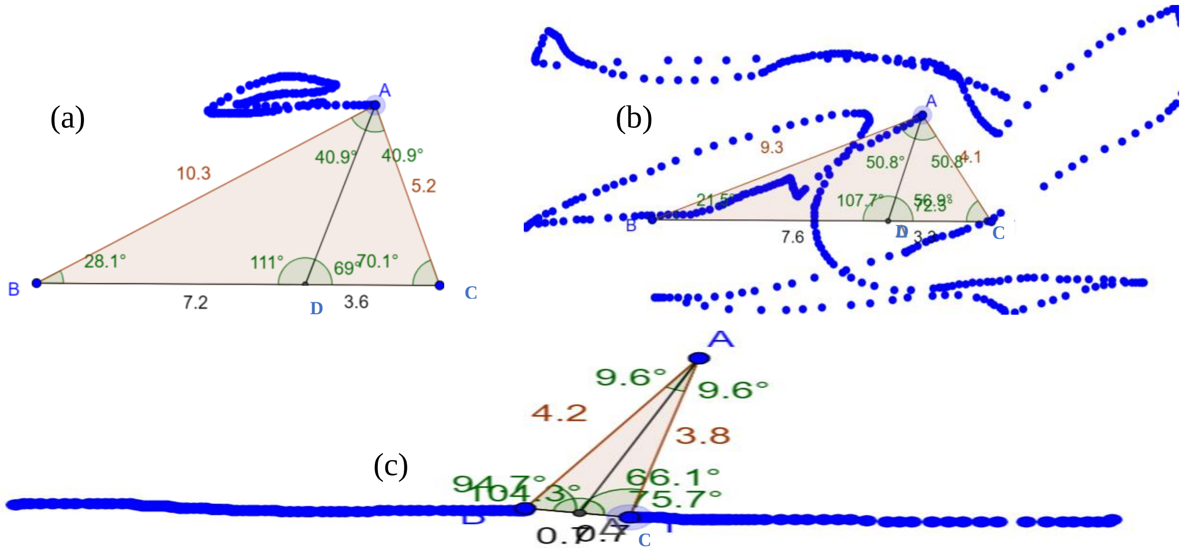

Figure 2. The trace of the triangle vertices during dragging

Exploration 3. Students were asked to find when the altitude, the bisector and the median of the triangle coincide using a ready-made applet presenting the three segments. The nature of the activity did not follow a traditional learning trajectory because students had to combine the three segments in a unified shape. All the students dragged easily vertex A, so the three segments coincided. In addition, all of them discovered that in the case of an isosceles triangle, the three segments coincide and they justified their answer based on the measures of the sides. The teacher asked them if they could find another kind of triangle. Six of them dragged vertex A to construct an isosceles triangle and then dragged along a virtual perpendicular bisector (from up to down) on BC to keep the three segments coinciding (see Figure 4a). Thus, they could not construct an equilateral triangle, because the two sides were getting smaller and the third one was constant. On the other hand, the rest of the students used dragging to reduce the size of the segments and after coinciding the three segments they dragged vertex A up and down along the perpendicular bisector on BC (see Figure 4b) and deduced that the three segments coincide also in the case of an equilateral triangle.

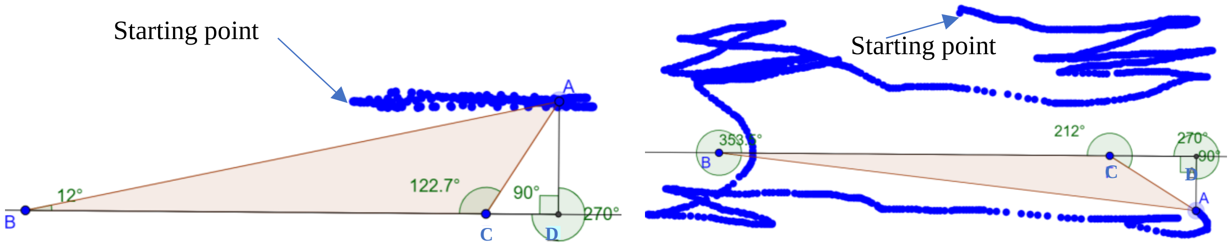

Figure 3. Exploring the position of the altitude of a triangle

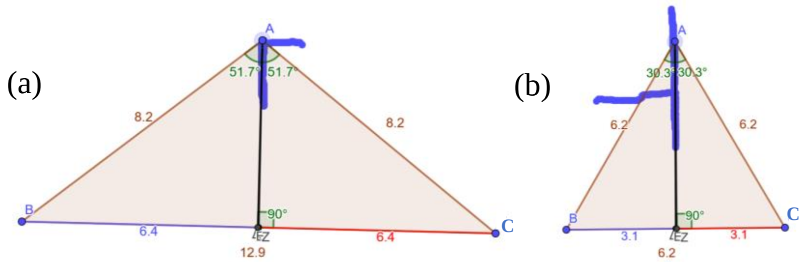

Figure 4. Exploring when the three segments coincide

DISCUSSION

We presented a theoretical framework describing factors that relate to the quality of student's learning experience in a tablet-based learning environment with math apps. The framework identified three key factors, namely: (a) student's engagement, which is mainly influenced by the affordances of the pedagogical medium, (b) the learning process that is related to the pedagogical approaches and the characteristics of the tasks and (c) the alternative perspectives of the mathematics content offered by the pedagogical medium. We further described an empirical study that aimed to provide further insight in the ways that the affordances of a pedagogical medium facilitated students' active participation and revealed alternative perspectives in respect to the mathematical content. Analysis showed that students were actively engaged in the learning process by self-exploring the elements of a triangle. Students utilized the interactivity of the tablet and the Geogebra applets, used the provided feedback to restructure their work, while many students exhibited flexibility to take into consideration all the available displays of the applets (geometric shape, measure of angles and sides). Analysis showed that an important factor that contributed to maximizing students' active involvement was the extent to which they used the affordances of the dragging tool. We could conclude that dragging in a tablet environment becomes more interactive and dynamic because of its haptic nature and this "haptic dragging" reshapes working in a dynamic geometry environment to a multi-sense and authentic interactive activity. An indicative example of the different nature of haptic dragging is the multi-touch dragging, where students dragged two points on the screen at the same time. This functionality facilitates resizing more than two segments or angles at the same time. In addition, analysis showed that students that used haptic dragging in a flexible and systematic way explored a bigger number of examples or extreme cases. Findings suggested that the affordances of the pedagogical medium reshaped the traditional learning trajectory by providing the opportunity to study the three elements of the triangle at the same time and to reach mathematical understandings that could not be conceptualized in a traditional learning environment. That is students (a) conceptualized the functionality of the elements of a triangle by making comparisons and validating their conjectures in a number of different examples, (b) self-explored how the position of the altitude of a triangle varies based on the type of the triangle and (c) self-guided discovered that the three elements of a triangle coincide in isosceles and equilateral triangles. This exploration could be achieved because of the visual and the measurement affordances of the app and the power of haptic dragging and enriched the mathematical content by providing alternative perspectives and deepen mathematical meanings.

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EVALUATING *CAS* AND *DGS* AT THE MATHS CLASSROOM: A PROPOSAL FOR AN UNBIASSED EXPERIMENTAL STUDY ABOUT THE IMPACT OF THE COMPUTATIONAL ROLE OF THE STUDENTS IN THEIR LEARNING

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There are didactic theories about teaching mathematics with Computer Algebra Systems (CAS), like the 'White-Box/Black-Box Principle' or the 'Scaffolding Method'. There are also many experiences on implementing specific mathematical topics with the aid of CAS or Dynamic Geometry Systems (DGS) in the maths classroom and even of courses proposals using these technologies. Nevertheless, we know of no proposal of evaluation of the impact of the computational role of the students in their learning. We therefore propose a design for such an evaluation at Secondary Education with three scenarios. The three scenarios should use the same methods (in order the results to be unbiased): oral presentation, interrogative method and enquiry based learning. The three scenarios incorporate an increasing computational role of the students. We plan to implement it along the second semester of the academic year 2019-2020 at different high schools.

Keywords: Active methodological strategies; computer algebra systems; dynamic geometry systems; educational evaluation.

1 INTRODUCTION: CAS AND DGS

In our opinion, Computer Algebra Systems (CAS) and Dynamic Geometry Systems (DGS) are the key software for mathematics teaching.

1.1 Some brief notes about CAS

CAS were initially developed for performing calculations in high energy physics and astronomy at the end of the '60s, and were later spread among mathematicians (Wester, 1999). While its adoption at university levels is widespread, its use at Secondary Education level is not very frequent (with exceptions, like Austria). For instance, they are not much more popular at this level in Spain than twenty years ago (Burrel, Cabezas, Roanes-Lozano and Roanes-Macías, 1997).

As CAS use exact arithmetic by default (Figure 1), the user can trust the numerical results obtained (what is very important when the students work without supervision). Moreover, CAS can handle non assigned variables, what allows these systems to deal with polynomial computations, symbolic differentiation and integration, symbolic matrices, etc. For instance, Sarrus' rule in its usual form can be obtained just asking the CAS for the determinant of a 3×3 matrix which elements are a_{ij}, i=1,…,3, j=1..3 (Figure 2).

```
> 200!;
78865786736479050355236321393218506229513597768717326329474253324435944996340334292030428401198462\
  390417721213891963883025764279024263710506192662495282993111346285727076331723739698894392244562145\
  166424025403329186413122742829485327752424240757390324032125740557956866022603190417032406235170085\
  879617892222278962370389737472000000000000000000000000000000000000000000000000
> evalf(%);
                                   0.7886578674 10^375
> 1/7+1/3;
                                          10
                                          --
                                          21
> evalf(%);
                                     0.4761904762
```

Figure 1: Two examples of computations in exact arithmetic in the CAS Maple[1] and the corresponding values in floating point arithmetic.

```
> with(LinearAlgebra):
> Matrix([seq([seq(x[i,j] , j=1..3)] , i=1..3)]);
```

$$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \end{bmatrix}$$

```
> Determinant(%);
```

$$x_{1,1}\,x_{2,2}\,x_{3,3} - x_{1,1}\,x_{2,3}\,x_{3,2} - x_{1,2}\,x_{2,1}\,x_{3,3} + x_{1,2}\,x_{2,3}\,x_{3,1} + x_{1,3}\,x_{2,1}\,x_{3,2} - x_{1,3}\,x_{2,2}\,x_{3,1}$$

Figure 2: Obtaining Sarrus' rule in Maple.

1.2 Some brief notes about DGS

Meanwhile, the first DGS were developed at the end of the '80s (Figure 3), but they have become much more popular at Secondary Education level after the release of GeoGebra[2] (that is now really a DGS with CAS capabilities, as the earlier Geometry Expressions[3] –see Figure 4). Nevertheless, its classroom use is many times restricted to function plotting and little more.

The main application of DGS is the exploration of geometry. The user can check results or make guesses just drawing the geometric construction with the mouse and dragging the initial objects (points). In case it holds when dragging, a formal proof is not obtained, but that it holds (at least) in most cases becomes a certainty.

1.3 Some brief notes about CAS and education

The relatively sparse use of CAS and DGS at Secondary Education is surprising, as they cover almost all its curricula.

[1] Maple is a trademark of Waterloo Maple Inc.
[2] GeoGebra is a trademark of GeoGebra Inc.
[3] Geometry Expressions is a registered trademark of Saltire Software Inc.

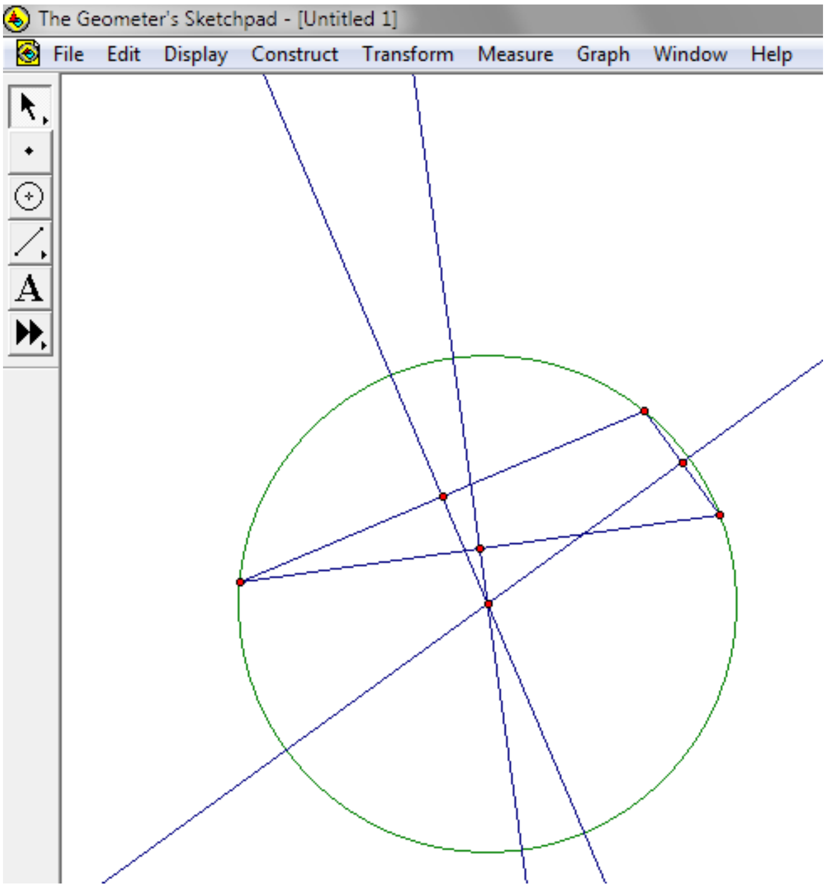

Figure 3: Exploring the circumcentre of a triangle with the DGS The Geometer's Sketchpad[4].

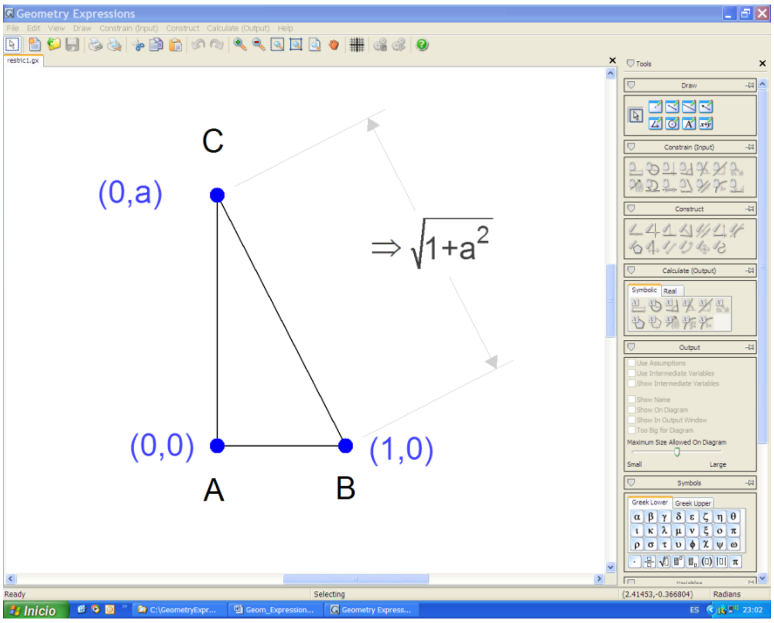

Figure 4: The DGS Geometry Expressions showing how it can handle non assigned variables.

[4] The Geometer's Sketchpad is a registered trademark of Key Curriculum Press.

There are specific didactic theories about teaching mathematics with computer algebra systems (CAS), like:

- the 'White-Box/Black-Box Principle' (Buchberger, 1990; Drijvers, 1995)
- the 'Scaffolding Method' (Kutzler, 1996; Kutzler, 1998),
- the 'Elevator Principle' (Cabezas & Roanes-Lozano, 2015).

There are also many experiences on implementing specific topics with the aid of CAS or DGS in the maths classroom and even of courses proposals using these technologies. Nevertheless, we know of no proposal of evaluation of the impact of the computational role of the students in their learning.

1.4 Some brief notes about DGS and education

The main application of DGS is the exploration of geometry. The user can check results or make guesses just drawing the geometric construction with the mouse and dragging the initial objects (points). In case it holds when dragging, a formal proof is not obtained, but that it holds (at least) in most cases becomes a certainty.

This allows to easily change the classic geometry master class into a class where the students explore possible results (as challenges).

1.5 The spark for this work

Due to the success of this previous research, we have considered that it would be interesting to implement a research that compared different levels of use of technology.

More precisely, we would like to implement a related study that evaluated the impact of the computational role of students of the last years of Secondary Education in their learning. It would be necessary to experiment it in the classroom and to evaluate the impact of applying the same methods with different uses of the technological utensils.

This new research proposal is focused on students of a different level (Secondary Education), with different educational needs and objectives.

2 DESIGN OF THE EXPERIMENTAL STUDY

2.1 General description

The authors have a long experience in educational software development, its implementation in the classroom and continuing education. After using different methodologies supported by technology, we would like to carry out a comparative analysis of the goodness of different levels of use of technology at a certain educational stage.

The idea is to carry out the experience in three scenarios, where the students have different computational roles. The chosen scenarios try to represent the possible levels of use of the technological utensils for mathematics learning, from inexistent to intensive:

- Scenario I: the students don't use technology.
- Scenario II: the students use some previously developed specific purpose resources (simulations). This approach has the 'advantage' that students wouldn't have to learn how to use the computational systems. The approach would be traditional, 'White-Box/Black-Box'.
- Scenario III: the students use the CAS and DGS as 'symbolic calculators' or 'geometric calculators' (what requires some knowledge about the computational tool) and sometimes

even develop small applications. For instance, if we suppose that f(a)=f(b), a Maple procedure that checks Rolle's Theorem for any given function, *f*, is one line of code long!:

```
rolle:=proc(f,a,b)
  fsolve(diff(f,x)=0,x,a,b);
 end:
```

(what can be implemented by students). On the other hand, complex implementations such as Roanes-Lozano (2017b) could be used as simulations. Again, the approach would be traditional, 'White-Box/Black-Box' (in all cases).

2.2 Pedagogical details

The key for developing an unbiased experiment lies, in our opinion, in using the same theory and methodological strategies (Viladot, 2002) in the three scenarios.

We believe that an active learning based on pure discovery, as proposed in Logo microworlds (Papert, 1980), without teacher intervention or clear curricular objectives, slows down the learning process and doesn't focus on optimized goals. A similar point of view can be found in the last lines of the first paragraph of Mayer (2004):

> *In each case, guided discovery was more effective than pure discovery in helping students learn and transfer. Overall, the constructivist view of learning may be best supported by methods of instruction that involve cognitive activity rather than behavioral activity, instructional guidance rather than pure discovery, and curricular focus rather than unstructured exploration.*

Therefore, we propose to choose:

- theory: constructivism, a pedagogic theory that intends the student to develop skills (Perrenoud, 1999),
- methodological strategies: both expository instruction and active teaching, with strong emphasis in the latter.

The expository instruction would use:

- oral presentations (technique: master class)
- interrogative methods (technique: open questions)

in all the three scenarios.

The active teaching would apply the techniques of the method:

- enquiry based learning (EBL)

to mathematical problems in all the three scenarios.

Regarding active teaching, we would also use:

- the simulation method (technique: computer simulations) in Scenarios II and III.

Concerning EBL:

- techniques borrowed from the EBL method would be applied to mathematical problems in Scenarios I, II and III.

Moreover:

– techniques borrowed from the EBL method would be applied to software development in Scenario III.

The situation is clarified in (the non-exhaustive) Tables 1, 2 and 3. These classifications are inspired by Bernal, Fernández-Salinero and Pineda (2019), Fernández-Salinero (2013), Fernández-Salinero (2004).

<table>
<tr><th>Scenario I
Theory</th><th>Methodological strategy</th><th>Method</th><th>Technique</th></tr>
<tr><td rowspan="7"><u>Constructivism</u></td><td rowspan="3"><u>Expository Instruction</u></td><td><u>Oral presentation</u></td><td><u>Master class</u>
Round table</td></tr>
<tr><td><u>Interrogative method</u></td><td><u>Open questions</u></td></tr>
<tr><td>Self learning</td><td>Individualized learning</td></tr>
<tr><td>Demonstrative Teaching</td><td>Demonstration</td><td>Mentoring
Training Within Industry (TWI)
Coaching
…</td></tr>
<tr><td rowspan="2"><u>Active Teaching</u></td><td>Simulation</td><td>Role playing
Computer simulation</td></tr>
<tr><td>Projects based learning
Challenge based learning
Problems based learning
<u>Enquiry based learning</u>
Guided discovery
…</td><td>Design thinking
Visual thinking
Phillips 66
SCAMPER
applied to:
<u>* Mathematical problems</u>
* Software development</td></tr>
</table>

Table 1. Classification of methodological strategies, methods and techniques (all related to constructivism) corresponding to Scenario I.

Banchi and Bell (2008) introduce a hierarchy of levels in EBL:

- Level 1: *Confirmation Inquiry*. The students confirm (check or prove, according to the case) a known result.
  Example: Draw function $f(x)=1/x$ and check that its one-sided limits at 0 are +/- infinity, respectively.
- Level 2: *Structured Inquiry*. The students investigate about a problem proposed by the teacher in a way proposed by the teacher.

Scenario II Theory	Methodological strategy	Method	Technique
Constructivism	**Expository Instruction**	**Oral presentation**	**Master class** Round table
		Interrogative method	**Open questions**
		Self learning	Individualized learning
	Demonstrative Teaching	Demonstration	Mentoring Training Within Industry (TWI) Coaching …
	Active Teaching	**Simulation**	Role playing **Computer simulation**
		Projects based learning Challenge based learning Problems based learning **Enquiry based learning** Guided discovery …	Design thinking Visual thinking Phillips 66 SCAMPER **applied to:** *** Mathematical problems** * Software development

Table 2. Classification of methodological strategies, methods and techniques (all related to constructivism) corresponding to Scenario II.

Example: Napoleon's theorem is presented as a research to be carried out using a DGS. The DGS has to be used to determine the kind of triangle the resulting one is. The teacher gives as clues to use GeoGebra's 'regular polygon tool' and to measure the sides or angles of the resulting triangle.

- Level 3: *Guided Inquiry.* The teacher only proposes the problem. The procedures to be followed in the research are designed or chosen by the students.
  Example: Napoleon's theorem presented as a research to be carried out using a DGS (with no clues).
- Level 4: *Open Inquiry.* The students investigate problems proposed by themselves (using procedures designed or chosen by themselves too).
- Example: the students try to discover (or, more probably, rediscover) geometric theorems with the help of a DGS.

In the experiment proposed, Level 2 (Structured Enquiry) and Level 3 (Guided Enquiry) would be normally used. Level 1 (Confirmation Enquiry) would be sparsely used.

Scenario III Theory	Methodological strategy	Method	Technique
Constructivism	**Expository Instruction**	**Oral presentation**	**Master class** Round table
		Interrogative method	**Open questions**
		Self learning	Individualized learning
	Demonstrative Teaching	Demonstration	Mentoring Training Within Industry (TWI) Coaching …
	Active Teaching	**Simulation**	Role playing **Computer simulation**
		Projects based learning Challenge based learning Problems based learning **Enquiry based learning** Guided discovery …	Design thinking Visual thinking Phillips 66 SCAMPER **applied to:** *** Mathematical problems** *** Software development**

Table 3. Classification of methodological strategies, methods and techniques (all related to constructivism) corresponding to Scenario III.

2.3 Computational details

Let us note that the educational digital resources (simulations) of Scenario II (and III) should be developed in advance, prior to the experimentation (as isolated applications of specific purpose, but making use of the power of CAS and DGS, not from a standard programming language).

The authors have a long experience in developing, implementing and using this kind of resources (Roanes-Lozano, 1987; Roanes-Lozano, 1993; Roanes-Lozano, 2017a; Roanes-Lozano, 2017b; Roanes-Macías & Roanes-Lozano, 1992; Roanes-Macías & Roanes-Lozano, 1994; Roanes-Macías & Roanes-Lozano, 2016;). Some of them can be adapted to the present proposal and others will be developed in collaboration with the Secondary School teachers. Moreover, simulations based on the use of CAS could benefit from the GUI detailed in Roanes-Lozano & Hernando (2014).

Which issues could be developed by the students with a CAS or a DGS (Scenario III) should be determined.

2.4 Evaluation of the experience

The last step prior to experimentation would be to design ad hoc evaluation tests on the issues addressed in the three ways.

Subsequently, the results obtained by the students in the math tests designed ad hoc would be evaluated and compared.

The assessment will take place in collaboration with the Secondary School teachers in the research project.

2.5 Implementation if the experience

We plan to implement the experiment during the second semester of the academic year 2019-2020 in Secondary Education centres.

There are Secondary School teachers that collaborate with our university. Among those most experimented, we shall choose some for this study. It will be applied to whole classes without selecting the students. The assessment will take place in collaboration with the Secondary School teachers.

A key issue is that exactly the same hours of face-to-face and homework should be dedicated by students in the three scenarios (what would imply that more time could be spent doing 'mathematical exercises' in Scenarios I and II than in Scenario III).

As the research is at a very early stage, the amount of hours in face-to-face and for homework haven't been detailed yet.

3 CONCLUSSIONS

This is so far only a theoretical development, but we believe that the hypothesis that Scenario III will produce the best learning will be confirmed.

In order to verify that hypothesis we plan to implement the experiment along the second semester of the academic year 2019-2020 at different Secondary Schools in Madrid area.

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REASONING COMPETENCY AND THE LINK BETWEEN GEOGEBRA AND ORIGINAL MATHEMATICAL SOURCES

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This poster presents an in-progress PhD study concerning how middle school students (10 to 12 years old) can combine the study of original mathematical sources with a geometrical content and work within dynamic geometry environments in ways that support the development of their mathematical reasoning competency. The study focuses on the students' mathematical argumentations and proofs from an epistemic point of view when using digital tools.

Keywords: GeoGebra, original mathematical sources, reasoning competency and epistemic purpose

The mathematics education literature is rich on studies addressing the use of digital technologies (DT) as well as the use of historical, original sources in the teaching and learning of mathematics. Yet, the intersection between the two has rarely been described (exceptions are e.g. Chorlay, 2015; Kidron & Tall, 2015; Zengin, 2018). The PhD study concerns this intersection and will mainly use the dynamic geometric software in GeoGebra (GG) and a selection of Euclid's propositions. It emphasizes how students work with GG to support their mathematical learning and understanding of the general mathematical reasoning and deductive proofs in the original sources. The aim of the study is to design didactical guidelines for the potential interplay between original sources and GG as a means to support the development of students' mathematical reasoning competency.

Mathematical reasoning competency is defined according to the Danish competency KOM-framework (Niss & Højgaard, 2011). It concerns "knowing and understanding what a mathematical proof is and how this differs from other forms of mathematical reasoning" and it also "consists of the ability to devise and carry out informal and formal arguments (on the basis of intuition) and hereby transform heuristic reasoning to actual (valid) proofs" (p. 60).

Artigue (2002) distinguishes between DT serving a pragmatic and an epistemic value for the students. The PhD study focuses on how students can learn about this distinction and acknowledge whether they are working with GG in an epistemic way that supports their mathematical understanding or when they are using DT in pragmatic ways as an effective tool to solve a given task. Students are often convinced by empirical examples (EMS, 2011). When teaching primarily focuses on empirical examples, it can weaken students' learning of how to understand and perform general reasoning and proofs (Harel & Sowder, 2007). A predominantly pragmatic use of DT makes this situation worse (Jankvist & Misfeldt, 2015). Working with original sources is known to potentially support the students' development of mathematical competencies (Jankvist & Kjeldsen, 2011). Working with the interplay between GG and original sources may support students' understanding and reading of original sources and in the same time contribute to the students' using DT in both a pragmatic and epistemic way (Olsen & Thomsen, 2017).

The PhD study consists of three different parts:

1) A review of empirical research and theoretical constructs within the subfields of technology and history in mathematics education focusing on the reasoning competency. The aim is to consider potentials and challenges of using different types of original mathematical sources and choose appropriate original mathematical sources for the interplay with GG to support the students' development of mathematical reasoning.

2) A quantitative analysis of selected Danish mathematical textbook systems and topic-oriented web portals for 4th to 6th grade addressing how they set the stage for the students' work with GG to support their development of reasoning competency. This will provide insight into how the interplay between original sources and GG may qualify existing teaching materials.

3) New teaching approaches are developed and the connection between these and students' competency development and use of GG as an epistemic purpose are explored. This part of the project will follow a Design-Based Research approach, based on outcomes of 1 and 2 above, and complete three iterations in two different lower secondary classes. This will be the take-off for development of new theory and didactical guidelines. These guidelines will e.g. focus on 1) how to support students understanding, devising and phrasing of mathematical argumentations and proofs while working with the intersection between original sources and GG and 2) how to support students getting insight in "the nature of mathematics as a subject area" (Niss & Højgaard 2011, p. 27).

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PEER INSTRUCTION IN ELEMENTARY MATHEMATICS WITH THE PYTHAGOREAN THEOREM

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Peer instruction is an active learning method which was primarily invented by Eric Mazur for the needs of university level physics. The effectiveness of this method stands primarily on the group discussion that was raised by the conceptual question of the so-called ConcepTest. In this paper we will introduce peer instruction and then we will see how to implement this strategy in the teaching of elementary level mathematics using the Pythagorean Theorem. The following text will be also supplemented by examples of specific ConcepTests and it will be accompanied by statements of pupils about peer instruction.

Keywords: peer instruction, Pythagorean Theorem, Socrative, mathematical concepts

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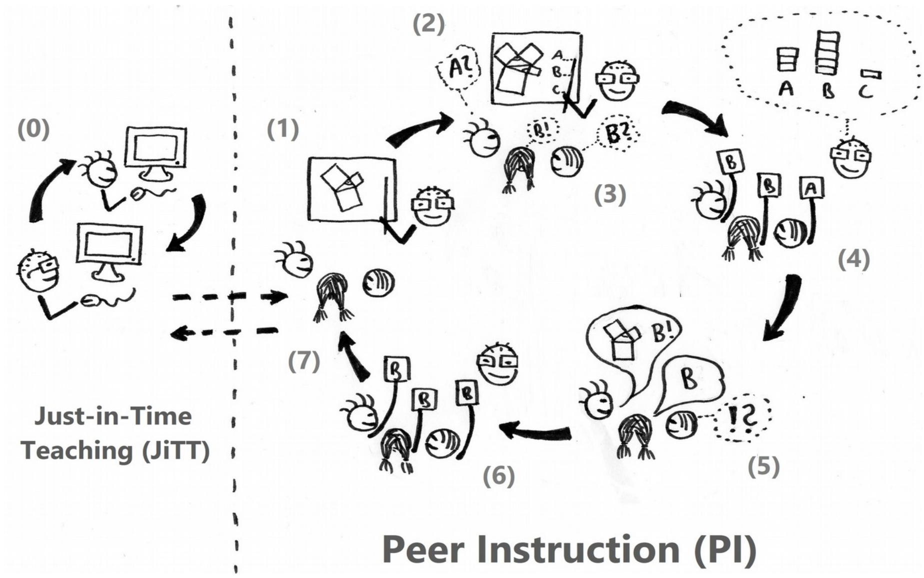


Figure 1. One block of peer instruction with just-in-time teaching

Please note in the introduction that most of the presented material and data was tested and collected in a class of 8th grade students at a Czech grammar school.

PEER INSTRUCTION

In 1984 Eric Mazur started teaching Harvard's introductory physics courses for medics. His lectures were evaluated quiete positively, and his students achieved very good results in terms of classic tests and exams. Based on these indications Mazur considered himself to be a very good lecturer. However, after about seven years of “successful teaching,” he read the article from Hestenes and Hallouin referring to introductory physics courses. The courses change practically nothing on students’ input misconceptions about Newtonian mechanics. Mazur's first response to the article was simply a statement: “Not my students – not Harvard's students!” However, as a scientist he knew he needed data to assert his claim. Therefore, students were given a simple test which was discussed in the mentioned article (this test is known as Force Concept Inventory Test – FCI). The test was aimed at the conceptual understanding of three of Newton's laws. The results he received shocked him completely. Some students succeeded little better than a gorilla randomly pressing the keys on a keyboard. This finding led Mazur to change his teaching approach completely and he developed peer instruction (Mazur, 1997).

The lessons taught by peer instruction are usually divided into several blocks. The schematic structure of one such block can be seen in *Figure 1*. Each block starts with a short presentation of the selected concept (1). In his presentation an instructor tries to avoid formulas or other mnemonics that mislead students from the true meaning. After the presentation, the instructor provides a ConcepTest aimed at deepening the understanding of the presented concept (2). Students are given a short amount of time to think individually. Subsequently they are called to vote by voting cards, clickers or smart devices (3). Based on the distribution of pupils' responses, the instructor briefly explains the correct answer (more than 70% for the correct answer), tries to explain the problem once more (less than 30% for the correct answer), or goes to a group discussion phase (between 30% and 70% for the correct answer). At the stage of group discussions, students try to persuade their colleagues about the correctness of their answers, and they are encouraged by the instructor to justify them – not just make mere statements (4).

Research shows that a student is often able to understand the concept more easily through his classmates' interpretations than from his instructor's interpretations. Students who have understood the discussed concept remember the obstacles they had to overcome and the steps they had to make. On the other hand, the instructor often suffers from the so-called "curse of knowledge" because he understands the discussed concept very well and he is no longer able to see students' difficulties. Group discussions end with a revised student vote (5) and a brief explanation of the correct answer (6). There will usually be a significant increase in votes in favour of the right answer (Mazur, 1997; Vickrey et al., 2015).

The described block takes approximately 10 to 15 minutes. We are able to discuss three to four concepts during the 45-minute lesson. Therefore, it is obvious that in order to achieve the same amount of curriculum in the classic classwork design, we have to place some work on the students. For example, we can do this by submitting preparatory self-study materials before the lessons. After the lessons, the students will have the necessary knowledge to master them (0). In his book, Eric Mazur (1997) recommends a Just-in-Time Teaching strategy. Just-in-time Teaching is a feedback strategy based on a feedback loop between the online preparation environment and follow-up in the classroom. In short, the instructor provides preparatory materials to the students via the Internet. Preparatory materials are accompanied by tasks and questions that students must work on and submit before the beginning of the next lesson. Based on the feedback provided by the students' answers, the instructor will appropriately adjust the content of the next lesson. The instructor will also adjust the content of the preparatory materials that have been adapted to the events of the previous lesson (Novak, 1999).

THEORETICAL FRAMEWORK

Although peer instruction is one of the most surveyed teaching methods (Vickrey, 2015) not many studies were aimed on peer instruction in mathematics neither in elementary school education.

In his study (2001) Scott Pilzer showed that we could teach calculus by peer instruction with similar learning gains (Hake, 1998) which are usually obtained in physics (Mazur, 1997; Vickerey 2015). Specifically, success rates for conceptual questions of Pilzers students taught by peer instruction were in average three times higher than success rates of his students taught classically (54% versus 17%). An experimental group also had a slightly better average of success rates for conventional problems than a control group (73% versus 63%). In addition to mentioned benefits Pilzer also pointed out several difficulties associated with the use of peer instruction in teaching of mathematics. Students usually do not have any preconceptions connected to concepts that are newly introduced to them. Mathematics is also more abstract than physics and included ideas are harder to imagine for students. In other words, it is more difficult for the instructor to prepare appropriate ConcepTests and it is more difficult for students to take their part in group discussions.

Similar results as Pilzers but in university algebra courses were reported by (Teixtera et. al., 2015).

Another study (Weurlander, 2016) showed an improvement in students' attitudes towards calculus due to the use of peer instruction.

A positive attitude of pre-service mathematics teachers towards peer instruction itself was shown in Turkish study (Olpak et. al., 2018). This study also showed significant increase of success rates between pre and post test which was aimed on understanding of statistics and probability.

Again by comparing pre and post test successes rates Kenyan study (Aurah & Ouko, 2015) showed that high school students taught by peer instruction achieved on average considerably higher understanding to vectors than their peers taught classically. In parallel to this was found a positive relationship of participants to peer instruction itself.

In this study (Yu-Fen Chen et. al., 2005) was tested an effectiveness of using peer instruction in teaching of elementary school physics. Parallel to typical benefits it was pointed out that pupils had troubles with visualisations of presented concepts and that they also had a pretty low social skills necessery for group discussions. In other words pupils needed to be helped by an instructor.

Based on the presented review we could clearly say that more studies of implementation of peer instruction in elementary school mathematics are needed.

TWO BASIC WAYS OF VOTING

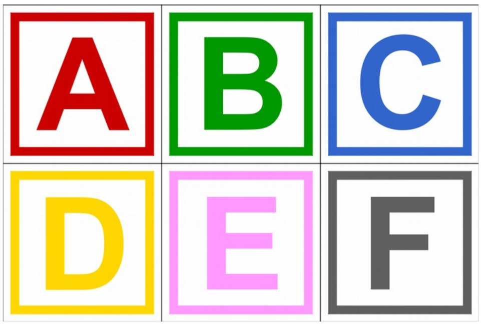


Figure 2. Flash cards

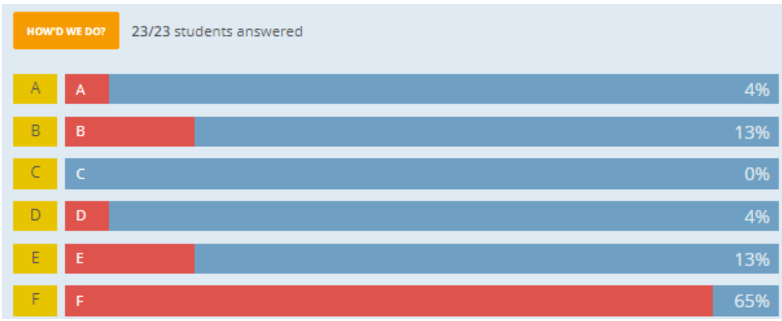


Figure 3. Socratevite - the instructor 's point of view

As we could see in the previous section, voting is really important for peer instruction and there are two ways we can deal with it. We can use simple flash cards (see *Figure 2*) or we can use wireless devices. Because almost everyone has a smartphone, we will choose the application Socrative (see *Figure 3*) as an example.

There were studies (Vickrey et al., 2015) which indicated that peer instruction can be effectively implemented with wireless devices or flash cards without statistical differences in learning gains. Therefore, I have asked my pupils if there is a difference between voting by phone (via Socrative) or by flash cards. My pupils have responded with these statements:

Feedback01: It seems to me that when I'm on the phone it's more anonymous.

Feedback02: We are not supposed to check answers of the others but everyone is so curious…

Feedback03: It is embarrassing for me if my answer isn't correct and anyone can see it because the cards are transparent and the others are always turning around.

Feedback04: It has an influence on me when I see different answers of the others.

Feedback05: The others usually turn to me and try to persuade me if they have a different answer from me.

Based on the pupils' statements, we could say that using a phone for voting is more comfortable because it is more anonymous and they are not affected by their peers’ answers. Using Socrative is also really comfortable for the instructor because we can see the distribution of pupils' responses in real time (see *Figure 3*). This application is also really useful for future statistics.

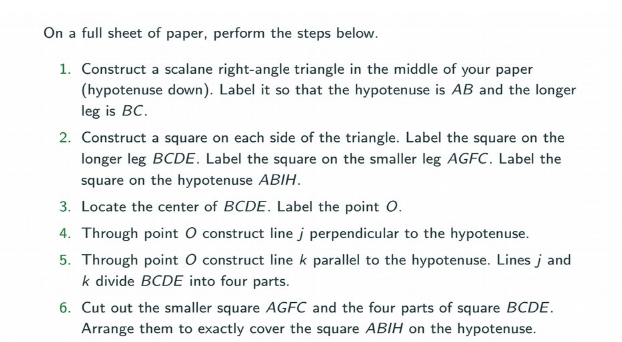
MATHEMATICAL CONCEPTS

Point (2) in *Figure 1* can be a viable according to David Tall's approach to mathematical concepts (Tall & Vinner, 1981). In this approach, there is both the definition of a mathematical concept and its image. Both of these components interact with each other. In a particular situation, a specific part of the concept’s image can be evoked. However, the image of the concept may be subject to misconceptions.

For example, if we are to determine the height of the staircase at the known length of the railing, the number of stairs and the known length of one stair will usually recall a right-angled triangle. In other words, we get an evoked image of a right-angle triangle, the Pythagorean Theorem and “the Pythagorean formula for computing lengths in a right-angle triangle: $c^2=a^2+b^2$”. Without a deeper understanding of the Pythagorean Theorem, it is possible that the formula could lead us to completely absurd results. We could calculate the length of the hypotenuse instead the length of the leg, or we could simple forget to calculate the root of the obtained result.

As for the ConcepTest, we can work in two planes. We can focus on understanding the particular definition of a mathematical concept or refine the relevant concept’s image. In the second of these cases, we purposefully selected or designed questions that could cause pupils to evoke the wrong image of the chosen concept in order to cause a cognitive conflict.

It is also possible to design our ConcepTests to target pupils’ ability to apply their knowledge in a non-traditional context.


On a full sheet of paper, perform the steps below.

1. Construct a scalane right-angle triangle in the middle of your paper (hypotenuse down). Label it so that the hypotenuse is AB and the longer leg is BC.
2. Construct a square on each side of the triangle. Label the square on the longer leg $BCDE$. Label the square on the smaller leg $AGFC$. Label the square on the hypotenuse $ABIH$.
3. Locate the center of $BCDE$. Label the point O.
4. Through point O construct line j perpendicular to the hypotenuse.
5. Through point O construct line k parallel to the hypotenuse. Lines j and k divide $BCDE$ into four parts.
6. Cut out the smaller square $AGFC$ and the four parts of square $BCDE$. Arrange them to exactly cover the square $ABIH$ on the hypotenuse.

Figure 4. Group activity

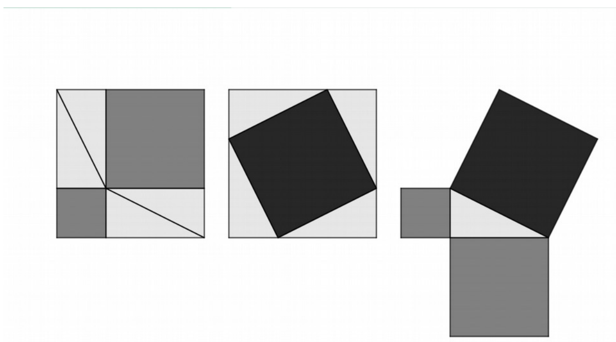


Figure 5. Picture for discussion

TEACHING THE PYTHAGOREAN THEOREM VIA PEER INSTRUCTION

As mentioned in the first section, we have to make a short presentation of a chosen mathematical concept without using formulas or other mnemonics. For the Pythagorean Theorem, we used an activity shown in *Figure 4*. It seems that for pupils who are not used to following a sequence of instructions on their own, it is really hard to accomplish this activity without any help. That is the reason why we usually let pupils work in groups of three or four. After each group is finished, we discuss the outcomes. Since there is just a small probability that two or more groups have drawn the same triangle, it leads us to the formulation of the Pythagorean hypothesis. After the Pythagorean hypothesis, we discuss the picture on *Figure 5* to turn the hypothesis into the Pythagorean Theorem.

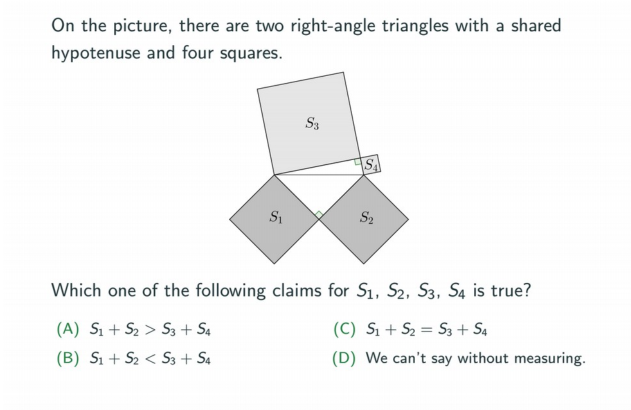


Figure 6. The first ConcepTest

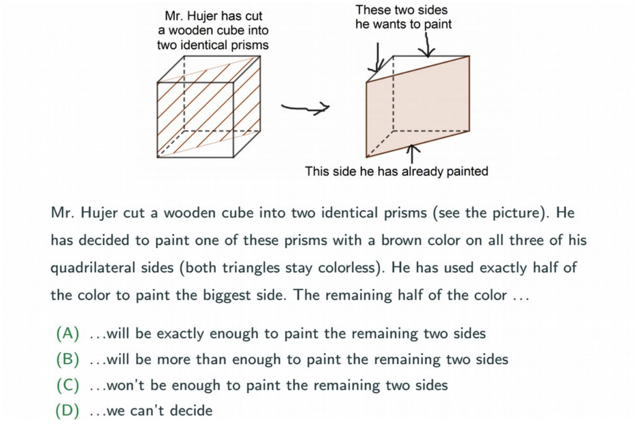


Figure 7. The second ConcepTest

After formulation of the Pythagorean Theorem, it is time for the first and second ConcepTests (see *Figures 6 and 7*). The first ConcepTest usually has really good results (almost 100% for the correct answer (C) at first voting). On the other hand, the second ConcepTest usually has a success rate around 40%–55% at first voting and around 80%–95% at second voting. Pupils know that they need to use the Pythagorean Theorem, but the biggest obstacle is the knowledge that the painted quadrilateral is a rectangle, not a square. There were pupils who used the voting card to demonstrate the fact that the diagonal of the square is longer than its side.

After the first two ConcepTests, there was time for an application question (see *Figure 8*). This question was independently submitted to three groups (see *Figure 9*). The first group S1 was the class of 30 eighth grade pupils. The second group S2 was formed by 18 future mathematics teachers in their first year of university. The third group S3 was formed by 28 university students majoring in mathematics or physics.

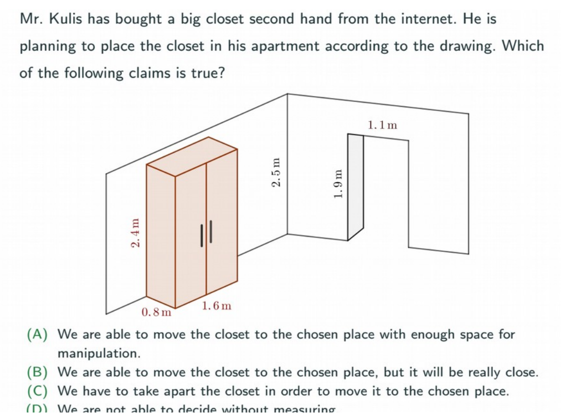


Figure 8. The third ConcepTest

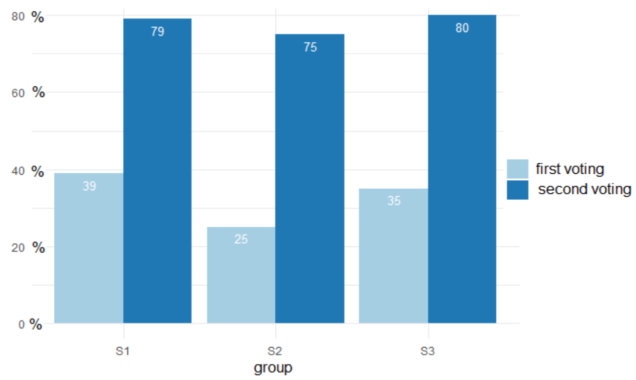


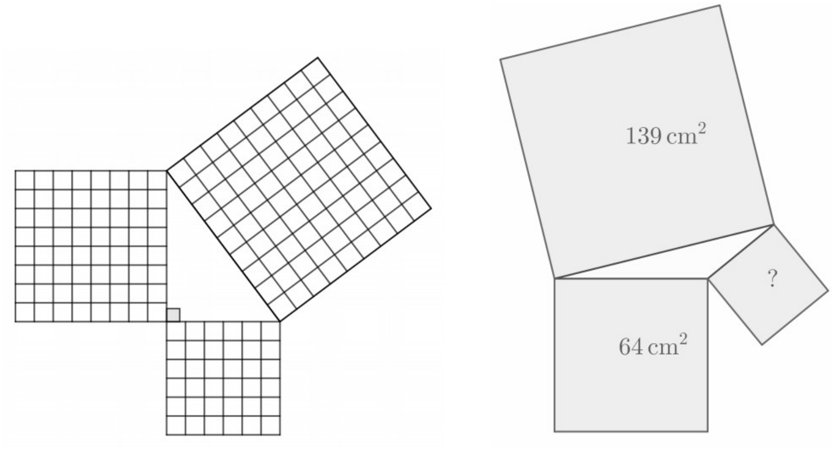
Figure 9. Results of the third ConcepTest

In *Figure 9* we can see different results after the first voting, but really similar results of revised voting following the group discussion. For this particular ConcepTest, the group discussions lead to similar learning gains regardless of the input levels of participating students. The right answer for this question is option (C). After the closet has been moved into the room through the door, we have to stand it up against the ceiling height, which is less than the length of the diagonal of the closet's marginal side. My pupils have given me following feedback about this questiondifficulties:

Feedback07: They kept focusing on that door. It was really hard to tell them it was not in that door, but it was in that ceiling.

Feedback08: As she was talking, she was drawing it. And as I looked at it and then I heard it I figured it out too.

Group discussions about this question have shown that although most of the students have realized it is necessary to lay the closet down in order to move it through the door, more than half of them have not realized it is necessary to stand it up again in the room – which is impossible.



(a) (b)

1. On figure (a) is a right-angle triangle, three big squares and many little squares. Are all the little squares identical? Briefly explain your answer.

2. Decide which of the following statements for $S_?$ on figure (b) is true. Briefly explain your answer.
(A) $S_? < 75\,\text{cm}^2$, (B) $S_? = 75\,\text{cm}^2$, (C) $S_? > 75\,\text{cm}^2$, (D) Can't decide.

3. For right-angle triangle ABC with a right angle at vertex C, calculate the length of side c if $a = 12\,\text{cm}$ and $b = 9\,\text{cm}$.

4. Grandpa David doesn't like pigeons. He lives on the third floor exactly above the entrance door. Pigeons are usually fed by pensioners from a bench that is located at a direct distance of 10 steps from the entrance door. Grandpa David has already decided to solve the pigeon problem with a crossbow. Determine the minimal effective range of the crossbow, which he needs in order to permanently solve this problem.

Figure 10. Post-test

Pupils' understanding of the Pythagorean Theorem was tested approximately one month after finishing the theme in *Figure 10*. The first question had a success rate of approximately 83% (25 out of 30 pupils were able to answer correctly and justify their answer). The second question had a success rate of approximately 67% (20 out of 30 pupils were able to answer correctly and justify their answer). The third question had a success rate of exactly 80% (24 out of 30 pupils were able to answer correctly). The fourth question had a success rate of approximately 73% (22 out of 30 pupils were able to answer correctly). The majority of incorrect solutions on the fourth question originated from an incorrect estimation of the necessary distances. Obtained results clearly imply that most pupils understand the formal part of the Pythagorean Theorem and they are able to adapt the corresponding formula accordingly. A typical misconception related to the Pythagorean Theorem is an inability to transfer the formula from a right-angle triangle ABC with a right angle at vertex C to another right-angle triangle without a right angle at vertex C.

PUPILS' ATTITUDES TOWARDS PEER INSTRUCTION

To find out what popularity and usefulness pupils assign to peer's instructions and other learning activities, a simple questionnaire was submitted. In this questionnaire pupils had to mark 10 (one for each 10 evaluated activities – see *Figure 11*) points on each two 147 millimetres length segments of each line. The first line with the two opposite measures of USELESS and USEFUL was for persuaded usefulness, and the second line with the two opposite measures of UNPOPULAR and POPULAR was for perceived popularity. The results of the questionnaire can be seen in *Figure 11*, where coordinates of each point are the arithmetic averages of the given scales.

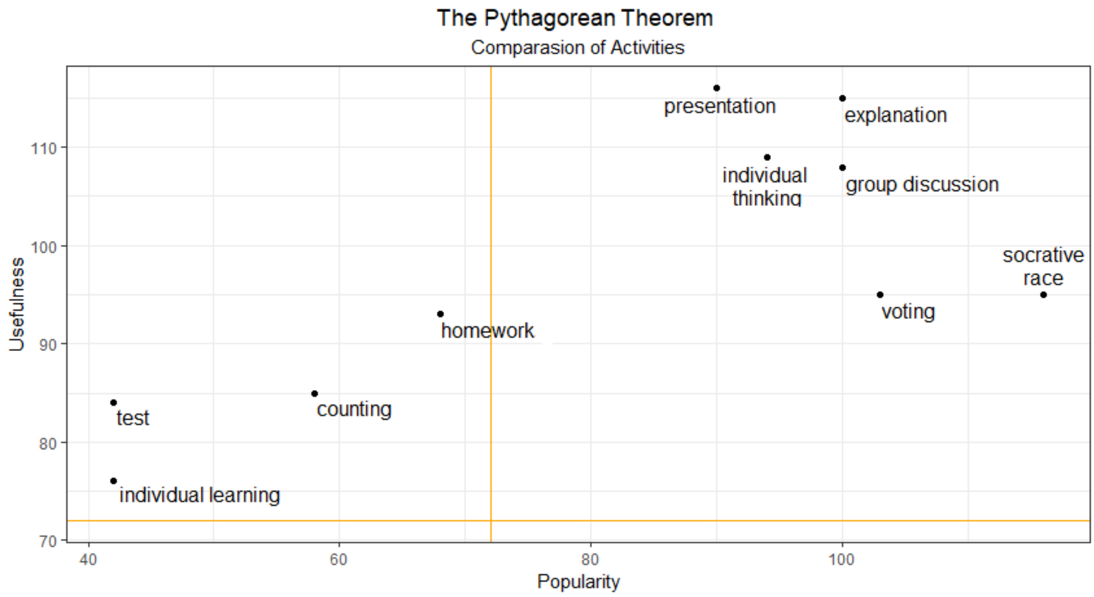


Figure 11. Results of the questionnaire

Based on the chart in *Figure 11*, it can be said that pupils enjoy peer's instruction activities and that they see them as useful in comparison to classic learning activities. Peer's instruction activities have also been discussed with a group of eight chosen pupils. This discussion was recorded on an audio-recorder.

A few pupils' interesting answers are listed below.

Question: What benefits have group discussions provided to you?

Feedback08: We have to think more carefully about what the others say than if it is explained by you because we don't know whether their arguments are true or not. You do it always well and so over your arguments we do not think so much.

Feedback09: I usually remember it longer when it was discussed.

Question: Does it make sense to discuss even easier questions with high success during the first voting?

Feedback08: Well, it makes sense. At least I can confirm that my ideas were good and that I understand it well.

Feedback09: It does because we can practise it on something easier so we can get ready for the harder ones. An athlete will also get warmer before he goes running.

Feedback10: It does make sense. We will not always solve it in the way we should, or we solve it as it happens and we don't know why our answer is right.

CONCLUSION

In the introduction, a peer instruction teaching strategy was presented. It was mentioned that this method is based mainly on conceptual questions, so-called ConcepTests, and group discussions. It has been shown that it is possible to implement peer instruction to teach mathematics in elementary school using the Pythagorean Theorem. It has also been shown that pupils prefer voting with wireless devices over flash cards because cards are less anonymous, leading to higher levels of discomfort. The paper was also guided by examples of particular ConcepTests that were used in real classwork. The last part of the paper was devoted to pupils' attitudes towards peer's instruction activities, especially group discussions. It was shown that these activities are perceived useful and popular, and that the group discussions are perceived as an opportunity to think and verify the truth of their understanding of discussed mathematical concepts. Finally, it is necessary to mention that collections of mathematical ConcepTests at the elementary or high school level are needed.

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Part 5: Networking of Theories

MAKING SENSE OF BIOLOGICAL PHENOMENA THROUGH INQUIRY OF MATHEMATICAL REPRESENTATION AND INTERACTIVE TECHNOLOGICAL TOOLS

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This study aims at exploring how interrogative processes may foster making sense of biological concepts that related to the phenomena protein synthesis such as positive transcription factors, enzyme, DNA, mRNA destroyer, as they are learned in a multiple representational environment that consists of biological simulation and mathematical representation. Five pairs of high school students which they 17 years old participated in this study, which was guided by the logic of inquiry approach and the variation theory. We analyzed the data by identifying the students' questions through their engagements with the digital tool and the students' transitions between mathematical representation and biological simulation. The results indicated that the digital tool supports the students in posing several kinds of questions, and the answers to these questions support the students in making sense of the biological knowledge embodied in the digital tool.

Keywords: digital simulation, logic of inquiry, mathematical representation, questioning, variation theory.

INTRODUCTION

Educational committees and STEM proponents suggest that knowledge should be accessible to students through a combination of several topics in context. Inquiry-based learning (IBL) is one of the strategies that STEM educators recommend implementing in educational settings. IBL is a vague concept and no consensus exists among scholars about its definition. Keselman (2003) defines IBL as a strategy in which students follow methods and practices like those of professional scientists in order to construct their knowledge. Pedaste, Mäeots, Leijen and Sarapuu (2012) define inquiry-based learning as a process of discovering, formulating and testing hypotheses by conducting experiments and/or making observations.

Artigue and Blomhøj (2013) observe that inquiry-based learning has only recently been applied to mathematics education. To conceptualize the term inquiry-based learning, they refer to Dewey's work, which conceives inquiry-based education as a way of supporting students' development of habits of minds that promotes learning and introduces a vision of inquiry as a process that incorporates the determination of the object or a problem to be inquired . In the inquiry-based learning introduced by Dewey, students create their own scientifically oriented questions, give priority to evidence in responding to questions, formulate explanations from evidence, connect explanations to scientific knowledge, communicate and justify explanations.

Even though no consensus exists among scholars about the definition of inquiry-based learning, it seems that questioning is one of the fundamental components of any definition of inquiry-based learning. Questioning and discussion began approximately 2,000 years ago with Socrates, who strove to engage intellectuals in solving the political, medical, religious and philosophical problems of the day (Gross, 2002).

Recent research studies also highlight the value of questioning as an important teaching and learning tool (e.g., Walsh & Sattes, 2016). Several studies focused on the questions asked by teachers and the role of these questions in stimulating thinking styles. For example, Joseph (2018) investigated the types of questions teachers ask to stimulate student discussion and critical thinking in elementary and secondary school classrooms. The findings reveal that teachers tend to ask low-level cognitive questions that require information, rather than higher-order questions that stimulate classroom discussion. Chen et al. (2017) investigated the various roles that early elementary school teachers adopt when questioning to scaffold dialogic interaction and students' cognitive responses for argumentative practices over time. Their findings indicate that teacher questioning serves different functions in promoting students' conceptual understanding.

In a study conducted by Chin and Chia (2004), they examined how students are inspired to ask questions, identified the types of questions students ask, and determined how asking questions helps junior high school students construct their knowledge. They found that the observations, curiosity and issues raised during previous lessons inspired the students to ask questions. The results show that 'what is' questions require basic information and basic data collection, 'how' questions seek explanations and tend to focus on cause-effect relationships, and 'what if' questions encourage students to raise hypotheses.

Although much is known about the questioning processes of students and teachers in traditional science classes (e.g., Chen et al., 2017; Joseph, 2018; Watson, 2018), less is known about the role of questions asked by the students in the process of making sense of scientific concepts when they are taught in digital-rich environments, environments that display simultaneous simulations of biological phenomena and mathematical representations of the biological processes involved in the phenomena. In this study, we aim at shedding light on questions asked by students and exploring how these questions prompt students to make sense of biological phenomena, following the theory logic of inquiry which considering the inquiry as a process of asking questions and answering them is the way which students construct their knowledge, more details about this approach is described in the theoretical background. Our working hypothesis is that the use of these kinds of digital learning environments may prompt students to ask questions; answering these questions and use of the digital tool may foster making sense of biological phenomena. the questions research are :

1. How the interactive environment simulates the students to investigate the biological phenomena ?
2. What types of questions raised by the students during the inquiry process ?
3. How the students interpret the biological phenomena by using mathematical concepts ?

THEORETICAL BACKGROUND

In this study, we used a hybrid model proposed by Arzarello (2016) that was created by networking two theoretical approaches: logic of inquiry (Hintikka, 1999) and the variation theory (Marton et al., 2004). The former defines inquiry as an interrogative process, while the latter frames and defines the concept of learning. Below we briefly outline the main ideas of the two approaches and elaborate on the hybrid model called MVI (Arzarello, 2016). The researchers (Swidan, Arzarello, Beltramino, 2017) use the MVI model to bridge the gap between the formal world of mathematics and the real-life situations. The researchers noticed that using MVI model increase learning opportunities that encouraged a deep understanding of pre-calculus ideas. In our recent research we took the MVI to a biological phenomena.

The main idea behind the logic of inquiry approach involves seeking rational knowledge by questioning (Hintikka, 1999). Hintikka conceived the process of seeking new knowledge as an interrogative process between two players. The first player (the inquirer) has the role of asking questions, and the second player has the role of answering and is called the verifier (or oracle). The former is the seeker of knowledge who tries to prove a conclusion to be reached from prior experiences or even from theoretical premises. The latter is considered the source of knowledge.

The variation theory (Marton et al., 2004) defines learning as a change in the way something is discerned, i.e., seen, experienced or understood. According to this theory, meanings emerge as the learner focuses his awareness on the object of learning. In this case, some aspects of the object appear at the forefront of his attention. Yet, not all aspects are discerned at the same time or in the same way. In order to understand an object of learning in a certain way, various specific critical aspects must be discerned by the learners. To facilitate the discerned object of learning, Marton et al. (2004) proposed four interrelated functions (or patterns) of variation to be taken into account when designing educational tasks: (a) Contrast: "...in order to experience something, a person must experience something else to compare it with"; (b) Generalization: "...in order to fully understand what 'three' is, we must also experience varying appearances of 'three'..."; (c) Separation: "In order to experience a certain aspect of something, and in order to separate this aspect from other aspects, it must vary while other aspects remain invariant"; and (d) Fusion: "If there are several critical aspects that the learner has to take into consideration at the same time, they must all be experienced simultaneously" (Marton et al. 2004, p. 16).

Integration of variation theory and logic of inquiry is done by Arzarello (2016) and what he called MVI model, to implement the MVI model we should design educational situations that may promote inquiry processes. This model proposes that drawing students' attention to critical aspects, asking to vary them and observing their effects on the phenomena may foster students' inquiry processes. The main idea of MVI is creating challenging situations by varying some aspects of the phenomena (real-world or mathematical) while keeping the others invariant. Exploring various aspects of the same phenomena may lead the students to grasp the intended object of learning.

We find the MVI model suitable for serving as a theoretical basis for our study since it attributes to questioning a central role in constructing knowledge and also assumes that questioning is catalyzed when students are exposed to situations that are varied, which is the main feature of the digital tool used in this study. In the following section, we will elaborate on the features of the digital tool used in this study and the study's methodological issues.

METHODOLOGY

The simulation used in this study

In this study, we used the simulation Gene Expression Essentials (Version 1.0.5; Dalton, 2018). this interactive simulation of protein synthesis (SPS) , which consists of graphical and numerical representations (mathematical representation) and a simulation of biological phenomenon (Fig. 1). SPS allows the students to create different situations by varying some aspects of the phenomenon while keeping the others invariant. For example, SPS enables students to vary parameters (concentration, affinity, degradation, etc.) that affect protein production (right side parameters, Fig. 1) and see how varying these parameters affect the level of protein production.

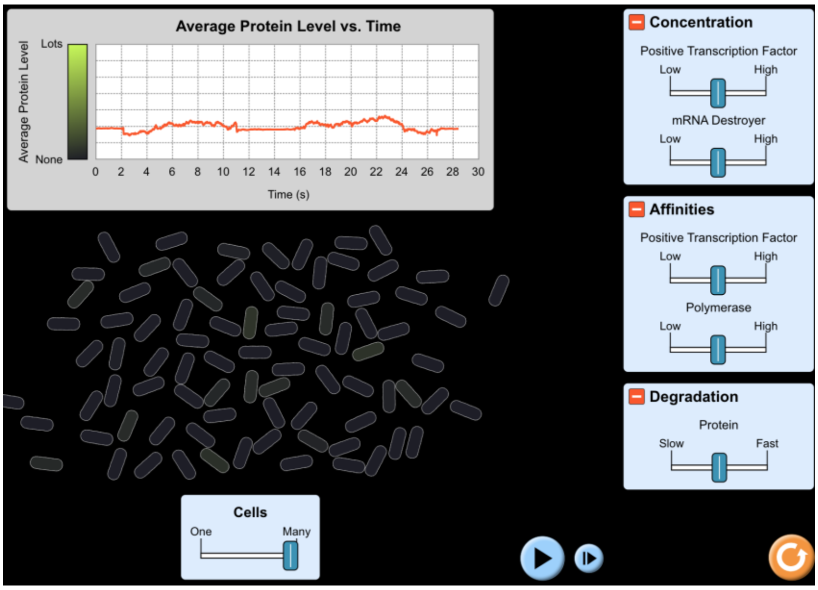

Figure 1. Mathematical representation (top left), cell simulation (middle left), factors affecting protein production (right).

Study design, data collection and analysis

We video-recorded the entire learning process of five pairs, 17-year-old students from an Arab high school in Israel who volunteered to participate in after-school meetings while working with SPS. To document the entire learning process, we used the task-based interview (Goldin, 2000) as the main tool for this study. The students learn in a science-oriented school and study biology at a high level, i.e., they were taught the subject from DNA to protein. In addition, these are 4 and 5 bagrut mathematics students which considered the highest level in mathematics learning in the Israeli curriculum.

The tasks given to the students were designed according to the theories that guided this study, at these tasks the student required to change some parameters and examine the influence of these parameters on the mathematical representation (graph) and the biological representation (color of the cell/s).

The procedure was conducted in a computer laboratory in the students' school whereby every two students shared one computer. The students were video-recorded and their corresponding computer screens were captured. After collecting the data, the task-based interviews were transcripted. We analyzed the data by divided the transcripts into episodes. We defined each episode as part of the transcript, which started with a question and ended with drawing a conclusion. Each episode was coded according to types of questions raised by the students, the biological knowledge the students constructed, the mathematical concepts the students used, and the transition directions between the mathematical and biological representations.

FINDINGS

The results show that a difference exists between the frequency of each question type: questions like 'what if' and 'why' have a higher frequency, 41 and 58, respectively; questions like 'when' and 'how' have a lesser frequency, 3 and 5, respectively (Fig. 2).

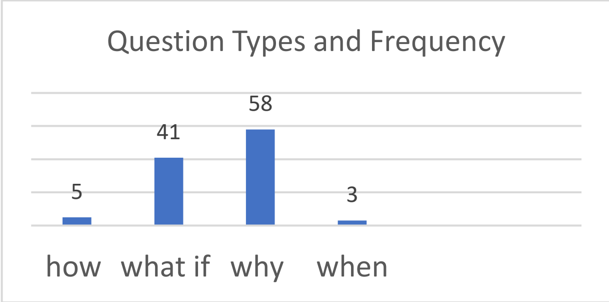

Figure 2. The frequency of each type of question.

Due to space limitations in this paper, we will concentrate on ‘why’ questions and how these questions foster the construction of knowledge. Sawsan and Hala is a pair of students who took a part in the research, the first episode illustrates how the students recognized factors affecting enzyme activity. We shed light on the relationship between the concentration of positive transcription factors with enzyme activity.

1 Sawsan: Why did these factors affect enzyme activity?

2 Hala: When the enzyme binds the positive transcription factor, these factors guide the enzyme from where it started; when the enzyme binds more positive transcription factors, the activity is greater.

3 Sawsan: Does it determine enzyme activity?

4 Hala: Yes.

5 Sawsan: How does enzyme activity affect DNA?

6 Hala: The DNA is like the substrate: when the substrate is higher, the enzyme activity is higher.

7 Sawsan: If we suppose that DNA is like a substrate, the concentration of positive transcription factors is high, and the mathematical graph is high until a certain limit.

The students examined the relationship between the concentration of positive transcription factors and enzyme activity. The students varied the concentration of the transcription factor parameter in the digital tool while keeping the other parameters invariant. In this way, the students separated this parameter from the others to observe how the concentration of positive transcription factors affects enzyme activity. The inquiry process was triggered thanks to the questions asked by Sawsan throughout the episode. Sawsan observed that the positive transcription factor parameter affects the graph. She observed the constant function graph when the parameter of the positive transcription factor was in the middle; the function increased when she set the parameter to high and decreased when she set it to low. Following these observations, Sawsan posed a ‘why’ question (line 1).

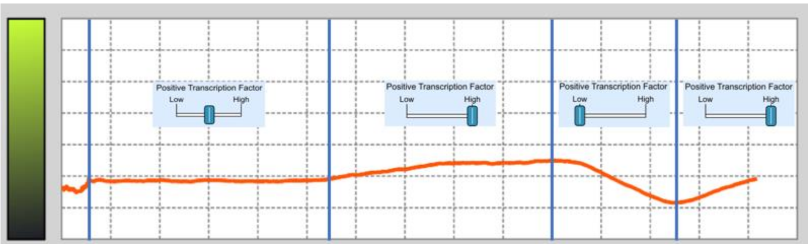

Figure 3. Graph reaction to the concentration of the positive transcription factor.

Following the ‘why’ question, the students drew a biological conclusion. They answered the ‘why’ question using the logic form of when-then twice. In this way, they recognized the proportional relationship between the concentration of the positive transcription factor with enzyme activity (line 2). In line 3, Sawsan, as an inquirer, continued questioning Hala. This time, she asked a clarification question (line 3). In line 5, Sawsan took the inquiry one step further by asking a ‘how’ question and introduced the DNA concepts into the students’ discourse. In line 6, Hala – the source of knowledge in this case – answered Sawsan’s ‘how’ question and elaborated on the relationship between DNA and enzyme activity. In line 7, Sawsan elaborated on Hala’s statement and connected between the DNA, the concentration of positive transcription factors and the graph. She described the relationship between the three elements qualitatively using the word ‘high.’

The second episode illustrates the ways the students recognize the effect of the concentration of positive transcription factors and molecules destroying mRNA on the amount of protein in the cell. By doing so, the students move between mathematical representations and simulation of the biological phenomenon.

18	Hala:	When the mRNA is low and the number of cells is low, the graph increases but the increment is not sharp. At first, it begins to rise and then it reaches equilibrium.
19	Sawsan:	Why is there an increase? I think that the concentration of molecules destroying a messenger RNA is like a substrate, it has a certain graph.
20	Hala:	Let’s try to set the mRNA to high when there are multiple cells. The graph decreases.
21	Sawsan:	Why? Because molecules destroy the transcript stand so the decrease occurs linearly.
22	Hala:	Let’s try to change the concentration of the transcription factors; the number of molecules destroying mRNA is low. When we have a single cell, the graph increases. Why does the graph increase when the concentration of transcription factors is low?
23	Sawsan:	Since the influence of molecules destroying mRNA is high, something is positive and the other is negative.
24	Sawsan+ Hala :	Let’s try to increase the number of cells to see if the number of cells affect the graph. The graph is straighter, the changes in the graph are not sharp, and when the cells are multiple, the line is completely straight.

Initially, the students separated the variables, varying the concentration of molecules destroying mRNA while keeping the other variables invariant. At this moment, the students discerned that the graph was increasing. This discrimination invited the ‘why’ question (line 18). Answering the question, the students came up with conjecture that the molecules destroying a messenger RNA were like a substrate. To verify their conjecture, they changed another parameter, the cell amount. After observing a decrease in the graph, they posed another ‘why’ question. Like in the first case, this time they also came up with a conjecture about the relationship between the mRNA destroyer and protein production (line 20), while again referring to the rate of change of the graph. They qualitatively described the rate of change of the graph: in line 18 they used word ‘sharp’ and in line 21 they used words ‘straight way.’ In line 22, the students fixed the number of cells and the mRNA destroyer, and varied only the transcription factor. Following this action, they posed, for the third time, a 'why' question in an attempt to explain the increment in the graph. To answer this question, they referred to the simulation of the cells. The light and dark cells caught their attention so they interpreted the increment in the graph by referring to the dark cells as negative and the light ones as positive (line 23). Generally, the students changed the number of cells and the concentration of molecules destroying the mRNA; these actions helped the students learn about how the mRNA destroyer affects the amount of proteins and make a comparison between the two cases. When the students changed the mRNA destroyer and concentration of positive transcription factors together, they fused the variables, i.e., they changed two aspects at the same time to explore their combined effects.

CONCLUDING REMARKS

This study aimed at exploring how high school students construct knowledge through an interrogative process while using digital and multiple representation simulation of biological phenomena. The results of the study showed that the digital tool has the potential of supporting students in posing questions and answering them. The micro analysis of the data attributes a central role to the linked mathematical representations and the biological simulation of the phenomenon in fostering the interrogative processes of the students. These interrogative processes were essentially triggered by the shape of the graphs and the parameters that affect the graph. In addition, the micro analysis highlights the dynamicity of the inquirer/oracle roles through the learning processes. This study showed how one student can play the role of the oracle (Hintikka, 1999) when being familiar with ideas that are discussed, and how the oracle becomes the inquirer when the same student encounters an unfamiliar situation.

The digital and multiple representation simulation tools foster the students to ask several kinds of questions. These questions help the students to explore by interpreting the mathematical representation of the biological phenomena and to recognize the biological knowledge that is simulated in the digital simulation. This insight is in accordance with Pedaste, Mäeots, Leijen and Sarapuu (2012), who argue that inquiry-based learning is a process of formulating hypotheses and testing them by conducting experiments and/or making observations.

According to Chin and Chia (2004), students raise questions as a result of observation and curiosity. In our research, we found that the students raised questions as a result of varying parameters in the simulation and after observing the graphical and biological representations. As shown in episode one, the interaction with the simulation results in asking a 'why' question, then the students draw a biology-related conclusion. Moreover, asking 'how' and 'when' questions help the students make sense of the biological concepts while drawing mathematical and biological conclusions. In addition, the design of the task, which takes into account the four principles of the variation theory, allows the students to raise questions. As shown in episode two, when the students vary some aspects while keeping the others invariant, they create a challenging situation that helps them ask 'why' questions, which leads them to explore the biological phenomena. For this reason, it seems that using the MVI model for designing tasks that prompt inquiry-based learning is a promising method. However, we recognize that the small number of students participating in this study and their background is one of the study limitations. To better understand the role of the MVI approach in fostering inquiry-based learning, further research is needed.

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MATHEMATICAL COMMUNICATION COMPETENCY IN INTERPLAY WITH DIGITAL TECHNOLOGIES

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An ongoing PhD-project examines the interplay between the mathematical communication competency and digital technologies. The project aims to describe the interplay theoretically using constructs from the field of mathematics education, including the Danish competency framework: KOM. To combine the theoretical constructs with mathematics education, networking of theories and design research are both applied.

Keywords: Networking of theories, communication competency, representation competency, design-based research, digital technologies.

This poster describes an ongoing PhD-project, which is a part of a larger study, including two other PhD-projects. The overall study aims to investigate the interplay between the use of digital technologies and mathematical competencies due to the increased focus of both digital technologies and mathematical competencies in mathematics education (Trouche, Drijvers, Gueudet & Sacristán, 201; Niss & Højgaard, 2019). The KOM-framework is an analytical tool describing mathematical activity and defines mathematical competency as “someone’s insightful readiness to act appropriately in response to *a specific sort* of mathematical *challenge* in given situations” (Niss & Højgaard, 2019, p. 14). KOM consists of eight mathematical competencies, among them: the communication competency. This PhD-project focuses mainly on the interplay between students’ communication competency and their use of GeoGebra in lower secondary schools in Denmark. Then, the research question for the ongoing PhD project is: *What is the interplay between students’ (aged 14-16) mathematical communication competency and their use of GeoGebra?*

The communication competency concerns the interpretation of others' mathematical expressions and one’s ability to express oneself. According to KOM, mathematical expressions include “written, oral, visual or gestural […], in different genres, styles, and registers, and at different levels of conceptual, theoretical and technical precision” (Niss & Højgaard, 2019, p. 17). Several perspectives on language and communication exist within the field of mathematics education. However, Sfard’s (2008) *commognition* seems to include more aspects of the communication competency than a construct that only focuses on language. In Sfard (2008), communication is defined as “a rule-driven activity in that discursants’ actions and re-actions arise from certain well-established repertoires of options and are matched with one another in a nonaccidental patterned way” (p. 146). Furthermore, it is word use, visual mediators, narratives, and routines that define a mathematical discourse. A mathematical discourse differentiates from colloquial discourses in its access to objects (Sfard, 2008). The objects in mathematics are only accessible by different representations in mathematics (Duval, 2017). As stated in KOM, being able to communicate mathematically involves using different representations and registers. The use of representations in communication means that a special relationship between the communication competency and the representation competency exists. The representation competency both concerns the ability to interpret, translate, and move between representations (Niss & Højgaard, 2019). Mathematical representations belong to different semiotic registers, e.g., natural language, symbolic systems, or graphic illustrations. Further, according to Duval (2017), it is not the use of the particular representations, which is essential but the transformation between them in mathematical activities. In this PhD project, commognition is utilised to analyse and assess students' communication competence and their ability to communicate within a mathematical discourse using different representations. However, Sfard (2008) does not concern the use of digital technologies and how it influences on students' mathematical communication. The project must also include perspectives on digital technologies because of the explicit focus on GeoGebra. A theoretical contribution to the use of digital technologies could be instrumental genesis, which describes how an artefact transforms into an instrument in a problem-solving activity. When an artefact is an instrument for a student, a reorganisation of a problem-solving activity happens (Guin & Trouche, 1998).

To investigate the interplay between GeoGebra and the communication competency in lower secondary schools, Networking of Theories and Design Research act together. Design research has both a practical and a theoretical aim (Barab & Squire). In three iterations, students' uses of GeoGebra and the communication competency are investigated. The content is functions, and teaching activities involve several problem-solving activities with multiple representations. These elements are fundamental in the preparation phase and the development of hypothetical learning trajectories (Bakker, 2018). A framework is developed based on existing research and theories from the field of mathematics education. Hopefully, the strategies from networking of theories are brought into play when theoretical perspectives are developed to describe the interplay. The goal of networking is to reach the strategy *integrating locally* in which a small number of theoretical perspectives are integrated and put together. In order to create a thorough framework, the theoretical contributions must be analysed with focus on compatibility and complementariness (Prediger, Bikner-Ahsbahs, & Arzarrello, 2008).

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MATHEMATICAL DISCOVERIES USING COMPUTATIONAL THINKING

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For about 40 years now programming is used in mathematics classrooms as a constructivist concept of learning mathematics. Preservice teachers at the University of Education Ludwigsburg showed great difficulties in implementing this concept in a third grade mathematics classroom. Therefore we propose a framework for mathematical discoveries using computational thinking based on frameworks for inquiry-based learning and learning of computational thinking.

Keywords: Inquiry-based mathematics learning, computational thinking, mathematical discoveries

INTRODUCTION

The 'hype' of introducing computational thinking in primary and secondary classrooms has reached Germany for some time now. In a design-based-research project 'Digital Learning in Primary Schools Stuttgart / Ludwigsburg' ('Digitales Lernen in der Grundschule Stuttgart / Ludwigsburg' dileg-SL) at the Ludwigsburg University of Education funded by Deutsche Telekom Stiftung we tried to combine the learning of mathematical concepts with computational thinking. The project as a whole aimed at the development of digital learning scenarios at primary schools. Beneath the productive and critical exposure to digital media in different contexts and subjects like German and English language, mathematics, biology, music, physical education, another important objective was the development of primary schoolchildren's basic competencies in computer science and algorithmic thinking.

In the mathematics sub-project, 25 preservice teachers learnt in two special mathematics education seminars about 'computational thinking' and the use of programming to understand mathematical concepts. Then they had to use this knowledge to develop learning scenarios for third grade classrooms fostering the understanding of mathematical concepts or the development of mathematical mental models. The learning scenarios covered topics like patterns with regular polygons or orientation in a plane. To support the school children developing these mental models they had to work with the programming language Scratch (https://scratch.mit.edu/). These scenarios were tested in two 90 min sessions in a 3rd grade classroom and afterwards reflected on by the whole group.

We found that all preservice teachers had a very big problem in common: They all tended to forget about the mathematics in their learning scenarios. Even the constant reminders 'that we are supposed to teach mathematics' and a planning structure where they had to describe the mathematical objectives and background extensively didn't help. The newness of programming in a mathematics classroom and the 'fun things' which are possible with Scratch made it very difficult for the preservice teachers not to ignore the mathematics.

Therefore, we want to do research on the topic: How to get preservice teachers to succeed in creating learning scenarios for inquiry-based mathematics learning using coding. To answer this question, we will follow the design-based research

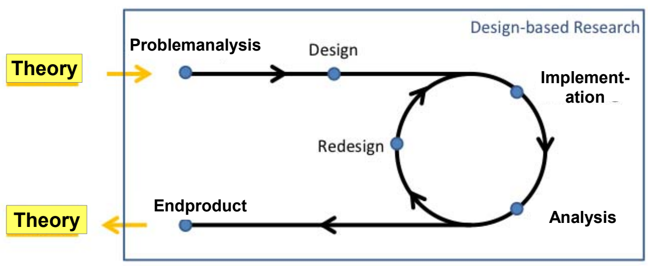

Figure 1: Design Based Research Approach

approach (cf. Easterday, Lewis, & Gerber, 2014). In the design-based research approach theory is an important input as well as an important output (s. Fig. 1). For our study we need a theoretical framework which guides students during the creation of the learning scenarios, works for the evaluation of the scenarios as well as can be used for measuring the competences the preservice teachers acquire during the seminar. The focus in this paper will be on the development of the theoretical framework, on which the study will be based.

THERETICAL BACKGROUND

First, we describe the learning concepts we based our project on and then we will introduce our framework.

Inquiry-based-learning in mathematics

Following the constructivist approach to mathematics learning, learners have to build their own network of knowledge and mathematical competences which include mathematical processes like mathematical problem solving or reasoning (NCTM, 2000). So teaching mathematics in a way that allows learners to develop their own mathematical competencies requires different pedagogical approaches than traditionally used in mathematics classrooms. 'Inquiry-based learning is a more student-centered way of learning and teaching, in which students learn to inquire and are introduced to mathematical and scientific ways of inquiry.' (Maaß & Artigue, 2011, p. 779)

Common to most definitions of inquiry-based-learning is that it is learner-centred and lets learners experience how real scientist in the field work. Pedaste et al. (2015) give a very good definition at the beginning of their paper:

> 'Inquiry-based learning is an educational strategy in which students follow methods and practices similar to those of professional scientists in order to construct knowledge. It can be defined as a process of discovering new causal relations, with the learner formulating hypotheses and testing them by conducting experiments and/or making. Often it is viewed as an approach to solving problems and involves the application of several problem solving skills. Inquiry-based learning emphasizes active participation and learner's responsibility for discovering knowledge that is new to the learner. In this process, students often carry out a self-directed, partly inductive and partly deductive learning process by doing experiments to investigate the relations for at least one set of dependent and independent variables.' (Pedaste et al., 2015, p. 48)

Inquiry-based learning is not the only constructivist, student-centred approaches in mathematics teaching and they all have a lot in common (Artigue & Blomhøj, 2013). Discovery learning i.e. 'occurs whenever the learner is not provided with the target information or conceptual understanding and must find it independently and with only the provided materials.' (Alfieri, Brooks, Aldrich, & Tenenbaum, 2011, p. 5). However, the 'pure' discovery without any guidance of the teacher has limited effects. It is suggested that …

> 'overall, the effects of unassisted-discovery tasks seem limited, whereas enhanced-discovery tasks requiring learners to be actively engaged and constructive seem optimal. Based on the current analyses, optimal approaches should include at least one of the following:
>
> 1) guided tasks that have scaffolding in place to assist learners,
>
> 2) tasks requiring learners to explain their own ideas and ensuring that these ideas are accurate by providing timely feedback, or
>
> 3) tasks that provide worked examples of how to succeed in the task.' (Alfieri et al, 2011, p. 35)

We want the mathematics learners to make mathematical discoveries using programming, but we choose the inquiry-based learning framework because it is closer to our purpose. Students in schools rather re-discover mathematical knowledge, which is already in the world, than make a 'free discovery'. Further the teacher decides on the pedagogical approach and he or she plans the aims, problems, supporting tools and materials for making the discoveries. Very often the teacher also decides on the questions which are to be investigated (Dobbler, Zwart, Tanis & van Oers, 2017). So, we decided to choose the inquiry-based-learning framework by Pedaste et al. (2015).

Pedaste et al. (2015) developed the framework from a systematic literature review of 32 papers on inquiry-based learning. Figure 2 gives an overview of the framework:

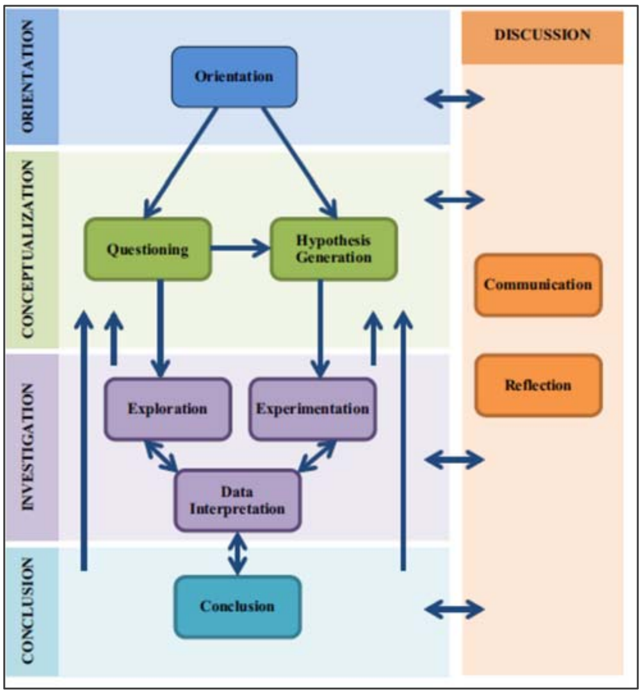

Fig. 2: Inquiry-based learning framework (general phases, sub-phases, and their relations), Pedaste et al., 2015, p. 56)

There are different pathways through this framework depending on the kind of questions and the approach (hypothesis-, question-, or data-driven).

Computational thinking

'Computational thinking' is a rather inflationary used concept, which is still defined in a wide variety. Wing (2006) describes 'conceptualization' and 'thinking on multiple levels of abstraction' as important aspects of computational thinking. Grover & Pea (2013) give in their review of the state of the field on computational thinking in K-12 a list with different elements of computational thinking i.e. 'abstractions and pattern generalizations (including models and simulations), systematic processing of information, symbol systems and representations, algorithmic notions of flow of control, conditional logic, … debugging and systematic error detection' (Grover & Pea, 2013, p. 39-40). Obviously, these elements cover very different levels of knowledge: some are factual knowledge (i.e. conditional logic), some are basic concepts of computer science thinking (i.e. algorithmic notions of flow of control) as well as typical procedures computer scientists use (like debugging and systematic error detection). Therefore, these elements are not a list to go through step by step to master computational thinking, rather the factual knowledge and basic concepts should be understood and deepened by working like a typical computer scientist.

Brennan & Resnick (2012) describe a framework of three dimensions '*computational thinking concepts* (the concepts designers engage with as they program' such as sequences, loops, events, iteration, parallelism, conditionals, operators, data), '*computational thinking practices* (the practices designers develop as they engage with the concepts' such as being incremental and iterative, testing and debugging, reusing and remixing, abstracting and modularizing), 'and *computational perspectives* (the perspectives designers form about the world around them and about themselves' such as expressing, connecting, questioning). (Brennan & Resnick, 2012, p.1).

Computational thinking concepts are similar to mathematical concepts (i.e. regular polygons, limits, functions, patterns,…) which have to be learned specifically and they also require specific methodological approaches. The computational thinking practices on the other hand are more like tools to do 'real' programming. This means that a programmer uses all these practices and very often different tools can be used to solve the same problem.

Specific pedagogical approaches to learn computational thinking concepts already start in primary school, i.e. Gadanidis, Hughes, Minniti, & White (2017) describe a concept for introducing computational thinking in primary mathematics classrooms where grade 1 pupils experience several aspects of computational thinking. Kotsopoulos, Floyd, Khan, Namukasa, Somanath, Weber, & Yiu, (2017) describe a *pedagogical framework for computational thinking* (CTPF) which

'includes four pedagogical experiences: (1) unplugged, (2) tinkering, (3) making, and (4) remixing. Unplugged experiences focus on activities implemented without the use of computers. Tinkering experiences primarily involve activities that take things apart and engaging in changes and/or modifications to existing objects. Making experiences involve activities where constructing new objects is the primary focus. Remixing refers to those experiences that involve the appropriation of objects or components of objects for use in other objects or for other purposes. Objects can be digital, tangible, or even conceptual.' (Kotsopoulos et al. 2017, p. 154)

The pedagogical framework for computational thinking (CTPF) as shown in fig. 3 was 'intended to provide a preliminary lens for structuring teaching and learning for students by considering how to teach computational thinking (Kotsopoulos et al. 2017, p. 168).'

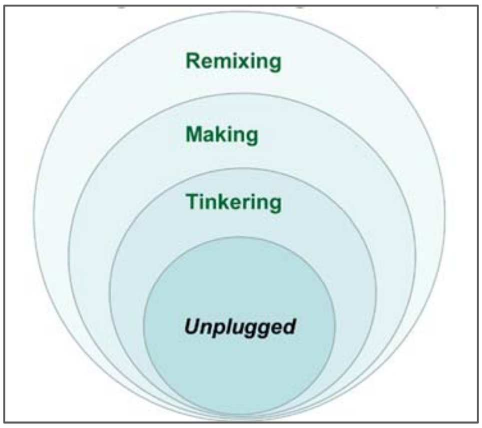

Figure 3: Pedagogical Framework for Computational Thinking (Kotsopoulos et al. 2017, p. 159)

Figure 3 also shows that these experiences follow a certain sequence. The *unplugged experience* is usually at the beginning of a teaching unit and no 'real' computer is used.[1] For this experience a little Play Mobil figure with an attached pen or even a Bee Bot (Highfield, Mulligan, & Hedberg, 2008) which can be 'programmed' using the arrows on his back to drive to certain spot (s. fig. 4) can be used.

Fig. 4: Tools for unplugged experiences

'*Tinkering experiences* primarily involve taking things apart and engaging in changes and/or modifications to existing objects.' (Kotsopoulos et al. 2017, p. 160). The tinkering experience is similar to the worked examples approach used in mathematics teaching (Renkl, 2002, Schermann, 2017) where example solutions of complex mathematical

[1] To use ‚unplugged' games, materials and situations for teaching computer science is a teaching concept used for quite some time, s. https://csunplugged.org/en/

problems are given, sometimes with mistakes, which the students have to identify, or as a cloze, which the students have to complete. This is also connected to the discovery learning (s. above). Both these experiences follow the rule that you should never start learning to program with an empty screen.

In the *making experience* student build new objects/programs. 'Making experiences, depending on the objects used, require deeper knowledge than tinkering where objects for the most part are already constructed or pre-existing. In making experiences, students are required to problem-solve, make plans, select tools, reflect, communicate, and make connections across concepts.' (Kotsopoulos et al. 2017, p. 162).

'*Remixing experiences* involve sharing (intentionally or through hacking) an object and modifying or adapting it in some way and/or embedding it within another object to use it for substantially different purposes. Remixing requires a significant level of proficiency to identify a useable object and then adapt and modify it to suit new purposes.' (Kotsopoulos et al. 2017, p. 165)

For the ScratchMath project (https://www.ucl.ac.uk/ioe/research/projects/scratchmaths) a 'framework for action' to bring together primary programming and mathematics was developed by Benton, Hoyles, Kalas & Noss (2017). It consists of the '5Es' – 'explore', 'explain' 'envisage', 'exchange' and 'bridgE' which scaffold the design of learning scenarios combining programming and mathematics learning. These '5E' are described as follows:

> '*Explore*: Opportunities should be provided for pupils to investigate ideas by trying things themselves and debugging errors.
>
> *Envisage*: Pupils should be encouraged to predict outcomes before running scripts and then reflect on the actual outcome.
>
> *Explain:* Opportunities should be provided for whole-class discussions led by teachers as well as with peers through the inclusion of reflective questioning.
>
> *Exchange*: Meaningful opportunities to share and build on others' ideas should be included.
>
> *bridgE:* The links with the primary mathematics curriculum as a powerful idea should be made explicit. (Benton et al., 2017, p. 122-123)

Using these pedagogical concepts for learning mathematics respectively learning computational thinking we propose the new framework for *mathematical discoveries using computational thinking*.

FRAMEWORK FOR MATHEMATICAL DISCOVERIES USING COMPUTATIONAL THINKING.

The framework as shown in fig. 5 will be explained using the example of discovering the properties of regular polygons. The start of a mathematical discovery is a mathematical object (i.e. a square as one example of a regular polygon). Using the unplugged experience (CTPF) the children explore what are the defining parts to draw a square. (i.e. Repeat 4 times: Go forward a distance of 50, turn right 90°). Then the question is given by the teacher: Will it be the same with all regular polygons? So now the children have to inquire about the mathematical properties of regular polygons. The goal is to find the minimal defining information for a plane figure. (In the case of the regular polygons this would be the number of edges and the length of one side.). To understand these properties of the regular polygons the children tinker with given programs or make their own (CTPF), but they always have to predict the expected outcome of their program (envisage, 5 E).

The exploring (5E) and experimentation to find the mathematical properties is accompanied by explanations (5E) and exchanges (5E) among the children and/or with the teacher. When the properties of the mathematical object are verified the bridge (5E) to the results of the mathematical discoveries – the mathematical concepts or mental models – will be built.

The remixing experience (CTPF) is not included in the framework because it requires high level abstraction. So remixing is not forbidden, but the question arises whether the cognitive effort of remixing programs is adequate to the mathematical discovery.

Using the inquiry-based learning framework we added the specific experiences and aspects of the two pedagogical frameworks for computational thinking. The main objective is always the mathematical one and the computational thinking parts are chosen and used to support the discovery of the mathematical concepts/ development of the mental models. To do it the other way around – using mathematical concepts to support the understanding of computational thinking – could be also a fascinating research question – for another time.

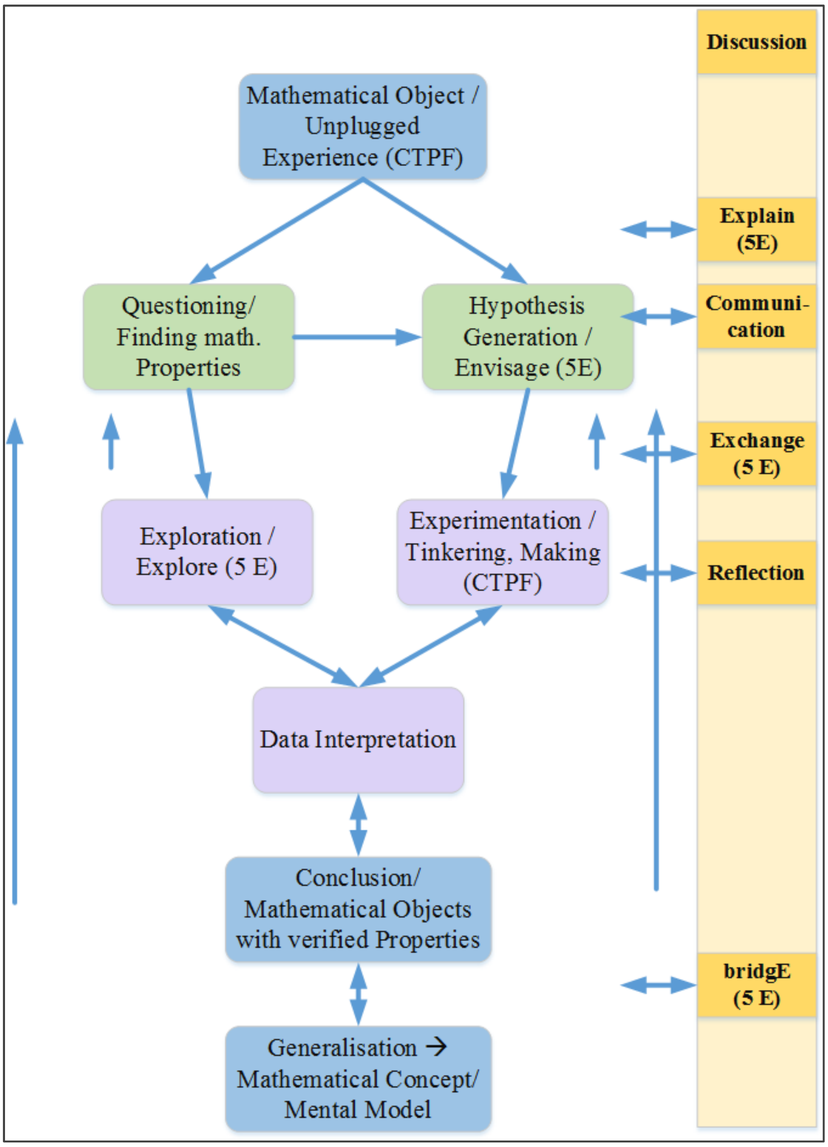

Fig. 5. Framework for Mathematical Discoveries Using Computational Thinking

EVALUATION USING A DESIGN BASED RESEARCH APPROACH

This is a framework which originated from the observation of preservice teachers' difficulties to develop learning scenarios for mathematical concepts using programming. Our overall objective is to develop seminars at university level, which allows teacher students to master the development of learning scenarios for inquiry-based mathematics learning using programming. For that we follow the design-based research (Anderson & Shattuck, 2012) approach in several cycles (s. fig. 1).

The Framework for Mathematical Discoveries Using Computational Thinking (MaDUCT) will on three levels: As guidelines for the development of the learning scenarios, the evaluation of the learning scenarios as well as for the description/measuring of the competencies the teacher students are supposed to acquire in the university seminar.

The next steps of our study will be:

- Development of best practice examples: The authors will create several learning scenarios for inquiry-based mathematics learning using programming for primary and secondary classes using the MaDUCT-framework. These scenarios will be implemented and evaluated in schools by different preservice teachers in their master thesis.
- Development of an evaluation rubric on the quality of learning scenarios for inquiry based mathematics learning using programming.
- Selection, adaption and development of questionnaires und interview guidelines for the teacher students to document their understanding of the framework and its application in mathematics classrooms.
- Design of the teaching concepts for the university seminar on inquiry-based mathematics learning using technology. Conducting a pilot seminar at the Ludwigsburg University of Education and analysis of the students increase in competencies and of the quality of the learning scenarios developed by the teacher students.
- Redesign of the seminar according to the results of the evaluation.
- Implementation of the seminar at the Ludwigsburg University of Education and dissemination of the concept.
- Publishing the evidence-based theoretical framework on learning scenarios for inquiry based mathematics learning using programming.

CONCLUSION AND OUTLOOK

The process from identifying a ‘problem’ to developing an intervention which could solve the problem in a university context takes several semesters. But using a design based research approach also allows us to development evaluation instruments, best practice examples and new theories, which makes the effort worth it.

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“MATHEMATICAL DIGITAL COMPETENCIES FOR TEACHING” FROM A NETWORKING OF THEORIES PERSPECTIVE

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Often mathematics teachers are reluctant to use even free digital tools, such as GeoGebra and Scratch, due to their lack of insight and conviction about how tools impact learning, but also other contextual and personal factors, involving their own digital competencies and confidence in their technological pedagogical knowledge. In this paper, we take a theoretical look at four frameworks regarding teachers' mathematical, pedagogical and technological knowledge and competencies and discuss the potentials of networking these four theoretical frameworks, namely KOM, MKT, TPACK and instrumental orchestration. We use this networking of theories as a basis for coining the notion of teachers' mathematical digital competencies when teaching mathematics.

Keywords: Mathematical competencies; digital competencies; mathematical digital competencies.

INTRODUCTION

Despite rapid advances in technology and evidence of its positive impact on education, the application of digital technologies (DT) in schools lags behind educators' and researchers' expectations (e.g. Survey of Schools Report, 2013). Even though there is significant international research on how best to design digital tools that address students' difficulties with mathematical concepts and have great potential for mathematical learning (e.g. Noss et al., 2012), such tools are not widely used. Equally, past and current research carried out in various projects [1] suggests that students often fail to ‘see’ the mathematics in their interactions with digital tools and rarely use ideas, concepts or strategies they have acquired through their interactions with such tools in their lessons (Geraniou & Mavrikis, 2015).

Teachers are often not convinced of a digital tool's value for mathematical learning and are reluctant to use them in their practice due to their perceptions, attitudes, professional development experiences and technical or pedagogical support networks (e.g. Clark-Wilson, Robutti & Sinclair, 2014). Teachers' lack of confidence in DT and the need for a significant amount of time to effectively integrate them into classrooms was evidenced in a recent project on the implications of DT for teachers' knowledge and practice (see Clark-Wilson & Hoyles, 2017).

The importance of teachers' own confidence, knowledge and skills and how these may influence and impact the nature of the potential use of DT in their classroom cannot be denied. Considering a number of theoretical frameworks involving teachers' knowledge and competencies – such as the Mathematical Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008; Shulman, 1986), the Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2009), the KOM framework's pedagogical and didactic competencies (Niss & Højgaard, 2011), and the theory of Instrumental Orchestration (TIO) for looking into how DT may be integrated in the mathematics classroom (Drijvers et al., 2014; Trouche, 2004) – we argue that there is a need for a combination of mathematical and digital competencies for teaching. This could be referred to as *Mathematical Digital Competencies for Teaching* (MDCT). In doing so, we build upon past work on a framework for students' Mathematical Digital Competencies (MDC), which we shall return to later. Furthermore, even though we consider four theoretical frameworks, the presentation and discussion is purely theoretical and thus does not include any empirical evidence. Collecting and analysing empirical evidence is a future research agenda for us.

PEDAGOGICAL AND DIDACTIC COMPETENCIES FOR TEACHING MATHEMATICS

The Danish KOM framework (Niss & Højgaard, 2011) defines eight mathematical competencies encompassing the mastery of mathematics. These are the competencies of mathematical thinking; modelling; problem handling; representing; symbols and formalism; reasoning; communication; and finally the aids and tools competency. Furthermore, the Danish KOM framework also describes six didactic and pedagogical competencies for teaching, which a good teacher of mathematics is to possess in addition to the eight mathematical competencies. We briefly describe these below.

The *curriculum competency* firstly consists in being able to study, analyse and relate to current and future mathematics curricula at a given educational level, and being able to evaluate the associated plans and the impact on one's teaching tasks. Secondly, it also involves being able to draw up and implement different types of curricula and course plans, while taking into account overarching frameworks and terms of reference which may exist under current as well as future conditions.

Second, the *teaching competency* is about being able, either alone or in collaboration with students, to devise, plan and carry out concrete mathematics teaching sequences. This involves the creation of a rich spectrum of teaching and learning situations for different students and student groups, including the ability to find, judge, select, and produce a variety of means and materials for teaching. It is also about the selection and presentation of tasks and assignments for students. Finally, it involves being able to discuss, with the students, the content, forms and perspectives of mathematics teaching, while motivating and inspiring them to engage in mathematical activities.

The *competency of revealing learning* is about being able to reveal and interpret the actual mathematical learning of students and the extent of their mastery of the eight mathematical competencies, as well as their conceptions, beliefs about and attitudes towards mathematics, including the identification of development of these over time. Hence, it concerns getting behind the facade of the ways an individual's mathematics learning and understanding is expressed in concrete situations and contexts, with the intention of grasping and interpreting their cognitive and affective sources.

The *assessment competency* comprises being able to choose or construct a broad spectrum of instruments for revealing and evaluating students' learning outcomes and competencies, both in relation to specific courses and in more global – absolute or relative – terms. In addition, the competency involves being able to critically relate to the validity and extension of conclusions reached by using given assessment instruments. Finally, the competency involves the ability to characterise an individual student's learning outcome and mathematical competencies, as well as the ability to communicate with the student about these matters and assist that student to correct, improve, and further develop his or her mathematical competencies.

The *cooperation competency* is firstly about being able to cooperate with colleagues, both in the subject of mathematics and in other subjects, regarding matters relevant to teaching. The competency involves the ability to bring the above-mentioned four pedagogical and didactic competencies into play. Secondly, it includes the ability to cooperate with non-colleagues, e.g. students' parents, administrative agencies, education authorities, etc. about teaching and its boundary conditions.

The *professional development competency* concerns the development of one's own competency as a mathematics teacher, in other words it is a kind of meta-competency. More precisely, it involves being able to enter and relate to activities which can serve the development of one's mathematical, didactic and pedagogical competencies, taking into consideration changing conditions, circumstances and possibilities. It is about being able to reflect on one's own teaching and discuss it with mathematics colleagues, being able to identify developmental needs, and being able to select or organise and assess activities, which can promote the desired development. In addition, it is also about keeping oneself up-to-date with the latest trends, new materials and new literature in one's field, thus benefiting from research and development contributions, and maybe even about writing articles or books of a mathematical, didactic or pedagogical nature.

MATHEMATICAL KNOWLEDGE FOR TEACHING

Another, and better-known, perspective on what good teachers need to know in relation to the teaching of mathematics is the framework of MKT (e.g., Ball & Bass, 2009; Ball, Thames & Phelps, 2008). Hill, Rowan, and Ball (2005) defined MKT as "... the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). MKT is of course a further development of Shulman's (1986) conceptions of teacher knowledge. In his seminal paper, Shulman distinguished between "Pedagogical Content Knowledge" (PCK), "Subject Matter Knowledge" (SMK) and "Curricular Knowledge". Ball and colleagues (2008) renamed the latter "Knowledge of Content and Curriculum" (KCC), and divided PCK and SMK into three subdomains each.

PCK consists firstly of "Knowledge of Content and Students" (KCS), which is focused on how students think about, know and learn mathematics (Hill, Ball & Schilling 2008). One example is knowledge about typical student (mis)conceptions and errors. Next, "Knowledge of Content and Teaching" (KCT) combines mathematical knowledge and design of instruction; the sequence of examples used to introduce a new concept is one example. The last subdomain of PCK is "Knowledge of Content and Curriculum" (KCC). Shulman (1987) suggested that this domain includes, at minimum, a "particular grasp of the materials and programs that serve as 'tools of the trade' for teachers" (p. 8). According to Shulman (1986), curricular knowledge is related to knowledge of alternative curriculum materials within a grade. However, he also distinguished between two additional aspects of curriculum knowledge: lateral curriculum knowledge and vertical curriculum knowledge. Lateral curriculum knowledge relates to teachers' knowledge of how the mathematical content relates to the content in other classes at the same grade level. Vertical curriculum knowledge relates to how the mathematical content of a particular grade level relates to topics and issues that have been taught in earlier years – or topics that will be taught in later years.

SMK firstly consists of "Common Content Knowledge" (CCK), which is mathematical knowledge used in the work of teaching, in ways that correspond with how it is used in settings other than teaching. CCK is what Shulman likely meant by his concept of subject matter knowledge: knowledge teachers hold in common with professionals in other fields. It thus refers to a mathematical knowledge that is not unique to teaching. Next, "Specialized Content Knowledge" (SCK) is mathematical knowledge unique to the work of teaching mathematics. Hill and colleagues (2008), state that it "allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual methods to problems" (p. 378). It relates to subject matter knowledge since it is about the content and not about the students. As for the final subdomain, "Horizon Content Knowledge" (HCK), Ball and Bass (2009) remark, "that teaching can be more skillful when teachers have mathematical perspective on what lies in all directions, behind as well as ahead, for their pupils, that can serve to orient their navigation of the territory" (p. 11). Ball and Bass (2009) argue that HCK can support teachers in hearing students' mathematical insights, orienting instruction to the discipline, and in making judgments about what is mathematically important. Jakobsen, Thames, Ribeiro and Delaney (2012) proposed a practice-based definition of HCK, stating that it is: “an orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory” (p. 4642). They continue: “HCK enables teachers to “hear” students, to make judgments about the importance of particular ideas or questions, and to treat the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field” (ibid., p. 4642).

NETWORKING OF THE KOM AND THE MKT FRAMEWORKS

Although KOM’s six pedagogical and didactic competencies and MKT’s six subdomains address the same issues of mathematics teaching and mathematics teachers, they do so from different perspectives, i.e. one of competency and one of knowledge. Still, it appears that the two frameworks may complement each other. We are only aware of one study that so far has attempted to do so. Sloth and Højsted (2017) compare (network) the two frameworks around an empirical study of what preservice teachers learn as part of their teacher education:

> “MKT and KOM give different perspectives on mathematics teacher knowledge, that there are overlaps and differences when applied to practical situations, but also that the frameworks themselves may benefit from the perspective of each other. Using both frameworks on our case, we find that they can complement each other and describe a greater range of mathematics teacher knowledge. Furthermore, we suggest that using or combining concepts from both frameworks can result in a new understanding of the knowledge and ability needed by mathematics teachers.” (Sloth & Højsted, 2017, p. 3411)

Yet, neither KOM’s six pedagogical and didactic competencies for mathematics teaching nor the MKT framework address explicitly what is needed of a mathematics teacher to successfully use DT in his or her teaching of mathematics. Does this require a certain type of pedagogical and didactic competencies? Does it require a certain kind of mathematical knowledge, not already embedded in the six subdomains of MKT? Some frameworks seem to suggest so.

TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE: THE TPACK

The TPACK framework concerns the body of knowledge and skills required for the implementation of DT in teaching (Koehler & Mishra, 2009). It was developed to address the need for technological knowledge and in particular, to analyse teachers’ pedagogical digital technology competence and the associated skills required of teachers (Law, 2010). As such, it is an extension of the PCK concept (Shulman, 1986). Koehler’s and Mishra’s (2009) diagrammatic representation of TPACK (http://www.tpack.org/) shows the PCK as the intersection of two circles representing general pedagogic knowledge and content knowledge. Using a Venn diagram with three overlapping circles, they include technology knowledge as a third domain of teacher knowledge to identify the skills or knowledge needed to successfully operate DT – also referred to as technical competence (Law, 2010). This inclusion introduces two other dyads, the “Technological Pedagogical Knowledge” (TPK), i.e. the intersection with pedagogic knowledge and the “Technological Content Knowledge” (TCK), i.e. the intersection with content knowledge. The triad intersection (Technological Pedagogical Content Knowledge) characterises the knowledge by teachers for technology integration in their teaching around a specific subject matter that “is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies […] and how technology can help redress some of the problems that students face […] and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones” (Koehler & Mishra, 2009).

INSTRUMENTAL ORCHESTRATION

Research (e.g. Kieran and Drijvers, 2006) has shown the benefits of the instrumental approach and how crucial the role of the teacher is in the effective integration of DT in the classroom and its benefits to students' learning. To describe how a teacher manages the use of DT and orchestrates mathematical situations, the theory of Instrumental Orchestration was derived by Trouche (2004). It involves "the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment –in this case a computerised environment– in a given mathematical task situation, in order to guide students' instrumental genesis" (Drijvers et al., 2014, p.191).

There are three elements within an instrumental orchestration (Drijvers et al., 2014). First, a *didactic configuration*, which is the arrangement of artefacts in the teaching environment. Second, an *exploitation mode*, which is the approach a teacher chooses to exploit a didactical configuration to assist their didactical intentions. Third, a *didactical performance*, which is the decisions a teacher needs to make on the fly, while teaching to accommodate the chosen didactic configuration and exploitation mode. Six orchestrations have been identified for whole class teaching in up-to-date research studies and one for students working alone or in pairs with technology (Drijvers et al., 2014).

The *Technical-demo* orchestration involves the teacher demonstrating tool techniques. The teacher may use student work or a new task to demonstrate a technique of using the tool and encourage students to copy their actions. The *Link-screen-board* orchestration, where the teacher draws students' attention to the relationship between anything that happens in the digital resource and the conventional mathematics representations on book, paper and board. The teacher may use a student's work as a starting point for that class discussion or set a new task or problem. The *Discuss-the-screen* orchestration involves the teacher running a class discussion focused on what happens on the computer screen. The teacher may choose a student work, a task, a problem or a strategy to initiate the class discussion and welcome student reactions and input. The *Explain-the-screen* orchestration involves the teacher running a class discussion to explain what happens on the computer screen. The teachers are expected to focus on the mathematical content and consider using a student's work or model themselves a solution to a task for example. The *Spot-and-show* orchestration involves the teacher 'spotting' a student's piece of work, which is worth sharing with the rest of the class and then runs a class discussion to allow the student to justify their work and invite comments and reactions from peers as well as feedback from the teacher themselves. The *Sherpa-at-work* orchestration involves a 'Sherpa' student (as used by Trouche, 2004) using the digital resource to present their work or follow the teacher's instructions and showcase something in the digital resource. The *Work-and-walk-by* orchestration involves students working independently or collaboratively and the teacher circulating the room to monitor students' progress and help those in need.

All the above orchestrations involve whole-class teaching (Drijvers et al., 2014) and are used to describe the role of the teacher in guiding students during their interactions with a digital resource and supporting them in mastering this resource usage and learning the mathematics involved.

NETWORKING OF THE TPACK AND THE TIO FRAMEWORKS

TPACK has the benefit of simplicity and accessibility (Drijvers et al., 2014), as by using the Venn-diagrammatic representation it showcases the various intersections between knowledge of mathematics, knowledge of technology and knowledge of pedagogy. But it has also been criticised because of its ambiguities, and especially the weak theoretical definitions of its constructs (ibid.; Voogt et al, 2012; Ruthven, 2014). In fact, Ruthven (2014) suggested that the TPACK framework provides "a rather coarse-grained tool" for analysing teachers' knowledge and therefore, it might need complementing by other frameworks to achieve an adequate depth of analysis. He explored how the

TPACK could be combined with the Instrumental Orchestration framework, but also one of his own frameworks, that of "Structuring Features of Classroom Practice", in order to analyse deeper teachers' skills and knowledge involved in their practices.

TPACK refers to a number of constructs that involve either knowledge, understanding or competencies or combinations of these, showcasing how all these are necessary for the teaching practice and the potentially successful integration of DT in the mathematics classroom. TIO, on the other hand, focuses on factors and strategies that influence and can direct and organise teachers' usage of DT for mathematics teaching and learning. A number of orchestrations, as described earlier, seem to be linked mostly to students' development of TCK, e.g. Technical-demo, Discuss-the-screen or Sherpa-at-work, but also students' development of mathematical content knowledge, e.g. Explain-the-screen, Link-screen-board or Spot-and-show. Both frameworks take into account teachers' pedagogies and assume their importance in managing students' development of instrumental genesis. There seems to be an assumption though that teachers' TPACK may prescribe or influence their instrumental orchestration of a certain digital resource in their teaching. As Ruthven (2014) proposed there is a need "for fuller and more systematic investigation of the phenomenon of technology integration into subject teaching" (p.391) and he argued for the need for combining theoretical frameworks to supplement their ideas and generate illuminating findings.

MATHEMATICAL DIGITAL COMPETENCIES FOR TEACHING

Geraniou and Jankvist (2018) argue that in the modern-day mathematics classroom, a distinction between students' mathematical competencies and students' digital competencies may occasionally appear somewhat artificial, since DT may be such an integral part of the mathematics teaching and learning that is going on. Hence, they coin and discuss the term mathematical digital competencies (MDC), and briefly outline a tentative framework for such by combining KOM's eight mathematical competencies with the digital competencies of the DigComp framework (Ferrari, 2013). The notion of MDC has been further discussed and deepened by networking with the theoretical frameworks of the instrumental approach (e.g. Guin & Trouche, 1999) and conceptual fields (Vergnaud, 2009) in Geraniou and Jankvist (2019). Hence, it seems apparent that if a teacher is to assist students in developing their MDC, she will – besides possessing MDC to some extent herself – need mathematical digital competencies for teaching, i.e. MDCT.

In this paper, we have reviewed four theoretical frameworks related to teaching of mathematics, and discussed the networking of the KOM and the MKT frameworks, as well as the networking of the TPACK and TIO frameworks. Of course, there is a need for further investigations of the adaptation of the TPACK model for mathematical knowledge and therefore its link to the MKT framework. However, we also firmly believe that networking of the KOM and the TIO frameworks holds a large potential – in a similar way as the networking of KOM's eight mathematical competencies and the instrumental approach does (Geraniou & Jankvist, 2019; Jankvist, Geraniou & Misfeldt, 2018). Yet, we argue for the need of networking all four theoretical frameworks to lead to a combined theoretical frame, i.e. that of mathematical digital competencies for teaching (MDCT). This we plan to do in our future endeavours.

NOTES

1. For example, MiGen, 2008-2010, www.migen.org; Metafora, 2010-2013, www.metafora-project.org; iTalk2learn, 2012-2015, www.italk2learn.eu; Mathematical Creativity Squared, 2013-2016, mc2-project.eu.

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NETWORKING THEORIES – COMPETENCIES, MATHEMATICS EDUCATION RESEARCH AND DIGITAL TECHNOLOGIES

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The poster presents an ongoing PhD study that network theories to inform the use of digital technologies in mathematics education. Through networking strategies, the reasoning and the problem handling competencies from the Danish competence framework (KOM) is to be networked with theoretical contributions from mathematics education research to form a new framework, which will be developed and refined through Design-Based Research (DBR).

Keywords: Networking of Theories, Design-Based Research, Reasoning competency, Problem handling competency, GeoGebra

This PhD study is part of a larger research project at the Danish School of Education that includes three PhD studies in total. The overall project addresses the potential interplay between digital technologies and the introduction of mathematical competencies the KOM (Niss & Højgaard, 2011). To examine and inform this interplay the larger research project aim to further develop well-established theoretical contributions in the field of mathematics educations. KOM is an analytical tool to describe cognitive processes of mathematical activity across mathematical topics. The framework consists of eight distinctive, yet mutually related competencies and defines a mathematical competency as the individual's "*…well informed readiness to act appropriately in situations involving a certain type of mathematical challenge*" (Niss & Højgaard, 2011, p. 49). This PhD study aims to network well-established theoretical contributions to inform how GeoGebra can support lower secondary school student's development in the interrelated aspect of the reasoning competency and problem handling competency. This is further limited to the topic of variables as a general number. The interrelation of the two competencies concerns students' ability to justify solutions to subjectively non-routine problems (Niss & Højgaard, 2011). This leads to the following research question: *How can well-established theoretical contributions and the KOM-framework be integrated through networking strategies, to inform how GeoGebra may support student's justifications of solutions with and about variables as a general number in problem-solving processes?*

This will be answered through utilizing strategies from the method Networking of Theory (NT) (Prediger, Bikner-Ahsbahs, & Arzarello, 2008) in interplay with three DBR cycles (Bakker, 2018). The ambition is to reach *integrating locally* where a small number of frameworks and constructs are integrated to produce a new framework (Prediger et al., 2008). To obtain this stage of integration, other networking strategies can support the process of selecting theoretical constructs and the integration, at different stages of the study. DBR involve preparation of and designing teaching experiments and theoretical developments (Bakker, 2018). Particular in the retrospective analysis of teaching experiments DBR can contribute to the integration and refinement of the new framework. In networking, the theory development also entails serving attention to the compatibility and complementariness of theoretical constructs (Prediger et al., 2008). E.g KOM has a cognitive perspective. For theoretical constructs to be compatible, they must align with this perspective on a principle level. This analysis can be assisted by the strategies *contrasting* and *comparing*. In the preparation of teaching experiment in the DBR cycles (Bakker, 2018), the *coordinating* strategy can produce insights on how different analytical tools can bring about different information from the empirical data. Currently, the project is in the first preparation phase and thus focus on the development of a well-informed task sequence considering theoretical perspectives, as well as empirical results derived within these perspectives. Following, initial decisions in terms of the design of the task sequence are discussed.

Lithner (2008) argues that students' reasoning in problem solving processes should be derived from their mathematical knowledge, in contrast to outer authorities as formulas and prototypical solutions in textbook or solutions guided by the teacher. It follows that for students to be able to form mathematical anchored justifications, the task must relate to student prior knowledge. Variable as a general number covers a range of situations that students in early secondary school are not necessarily familiar with (Ursini & Trigueros, 2001), hence the use of variables in the tasks sequence should relate to topics that are more familiar for the students. Throughout primary school, students learn about the coordinate plane as well as basic geometrical shapes and properties. Relating variables in the task sequence to these topics can more likely present the possibility for students to justify solutions mathematically. At the same time, the tasks should present a non-trivial problem for the students, to allow students to build chains of arguments (Niss & Højgaard, 2011). To support students' arguments in GeoGebra it should be carefully considered what tools are used and which representations are mediated to the students. In GeoGebra, variables are constructed as objects in the geometric plane or as letters whose value is changed using a slider. As the use of multiple representations is a great tool to support argumentation (Noss et al., 2009), the latter is used in the task sequence, as students gets access to and hence can translate between geometric, numeric and symbolic representations.

Actual theoretical contributions that are considered for integration in the new framework, how DBR influence the development of the new framework and an example from the task sequence will be presented in the poster.

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STUDENTS LEARNING WITH DIGITAL MATHEMATICAL TOOLS – THREE LEVELS OF INSTRUMENTAL GENESIS

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There is a widespread aphorism that if your only tool is a hammer every problem looks like a nail. The solution to mathematical problems does not always follow the same pattern but may require the use of multiple tools or tool components. It is therefore important as a learner to be aware of the variety of possibilities that are available and to be able to select and use the optimal tool in a problem situation. However, a previously unknown tool is not directly operational for a learner. For the solution of existing problems, only tools whose operation is known can be used. This text examines the ways in which differences in the progress of instrumental genesis are expressed and when digital tools support learners' thinking. This raises the following research question: How far must instrumental genesis be advanced until it can be used with confidence. Three different mathematical problems and a think-aloud-study have been used to identify different steps of the instrumental genesis.

Keywords: instrumental genesis, math lessons with computers, think aloud method

INTRODUCTION

As technology advances in society, digital tools are becoming more commonplace in school life, especially in mathematics education. Since 2011 the use of the CAS calculator in the 9th and 10th grades has been required in Thuringia and since 2014 its use has been an integral part of the final exams. It is therefore important for teachers to understand how the process of learners acquiring knowledge about digital tools takes place. Digital tools have many advantages, but the use of a tool can be quite difficult in the beginning, especially if the user interface is complicated (Barzel, 2011).

The instrumental genesis of teachers has recently become a topic of discussion in educational research, for example, in conjunction with dynamic geometry software (Alqahtani & Powell, 2017), and with reference to the teacher's influence on the instrumental genesis of learners in instrumental orchestration (Trouche, 2004). In addition, the instrumental genesis of non-digital tools has been examined in contexts such as historical drawing instruments (van Randenborgh, 2015) or the textbook as a tool for learners (Rezat, 2010). In both digital and non-digital contexts, writers also included the social influence on the respective instrumental genesis. In this text, only the process of the instrumental genesis of digital tools is of interest; social influences are not considered.

In 2018, Rieß brought underlined the importance of the instrumental genesis from Rabardel (2002) associated with the research of concept acquisition of students and intensively explored the connections between a tool and the formed concepts. He juxtaposed and linked together several relevant theories of the last decades and therefore represents the main source of the present work. The theoretical introduction is followed by the presentation of the three mathematical problems used in the study. Afterwards, the research question will be examined and answered with the help of the thinking aloud Method (Lewis, 1982/ Düsing, 2014).

THEORETICAL BACKGROUND

The main basis of this study is formed by theories about the instrumental genesis between learners and digital tools. Particular interest is given to the theories of Rabardel (2002), Verillon & Rabardel (1995) and Béguin & Rabardel (2000), as well as Rieß (2018).

Instrumental genesis consists of two complementary aspects. The instrumentation works from the instrument towards the subject, as the subject evolves already existing schema and acquires new ones during the learning activity while dealing with the artifact. There are usage schemes, which affect the handling of the artifact, and instrument-mediated-action schemes, which help to solve problems. The instrumentalization process runs from the subject towards the instrument as the subject changes/ adapts to/ develops the instrument or parts of it to make it work for the situation (Rabardel, 2002).

To create a category system for the qualitative analysis the terms artifact, instrument and tool were defined, because they have different uses in the language. *Artifacts* are (not necessarily material) objects that have been man-made or altered for purposeful, completed actions, primarily with the aim of solving a problem. In the hands of a subject, artifacts become an *instrument* in the course of instrumental genesis, in that the subject assigns to or changes the properties of the artifact (instrumentalization) and acquires and develops schema in parallel (instrumentation) (Verillon & Rabardel, 1995). *Tools* are permanent instruments, which means they are artifacts that are permanently linked to specific properties and schemes (Rieß, 2018).

The second important theory is the didactic tetrahedron - which is based on Chevallard's 1982 didactic triangle - with the corners being pupil, teacher, instrument and mathematics. In this theory, all connections between these corners are significant for the learning process. The triangle of pupil-instrument-mathematics is of the highest interest because it includes the process of instrumental genesis. (Rieß, 2018).

PRESENTATION OF THE EXEMPLARY PROBLEMS

Three example problems from different mathematical fields have been chosen for this research. Each of them can be solved with or without technology. For each problem, an Excel- or GeoGebra-file was created to use in the study.

Task 1 - Geometry: Tetrahedral cross-section with the minimal circumference

Given: You have a regular tetrahedron with the edge length 1 unit of length in front of you, which is composed of 2 cardboard parts. There is now a rubber band stretched so that it touches each side of the tetrahedron. Initially, the band lies on the centers of the 4 edges that it spans.

• A: Specify the length of the rubber band in this position.

• B: Determine the shortest distance around the tetrahedron, so that the band goes over all surfaces. Is that the only one? Explain your decision.

• C[1]: Describe what all rubber band positions with the length of 2 units have in common.

Figure 1: Tetrahedron with a rubber band.

[1] Task 1C was only given as help if the student can`t figure out the solution of task 1B.

The length of the rubber band in this position is 2 units of length, which is the shortest distance. This can be explained using intercept theorems. To solve the problem with technology, GeoGebra can be used to construct a tetrahedron or its net (see Figure 3). A scrollbar can conclude both forms.

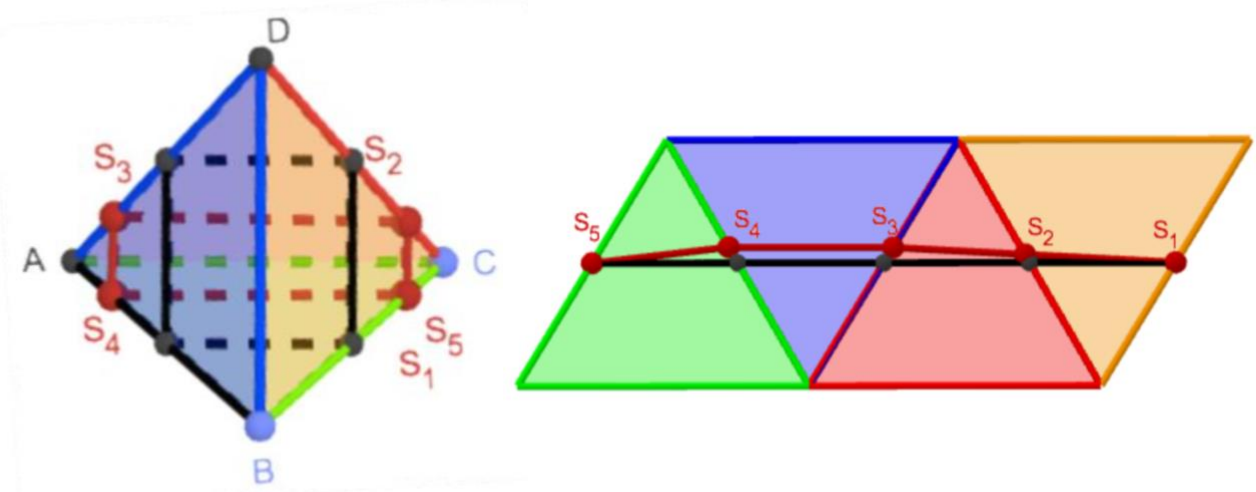

Figure 3: GeoGebra Screenshot of Tetrahedron with different lines on the surface and the unfolded Tetrahedron net in the plane.

Task 2 - Stochastic: Double dice

Given: A double dice (or two different colored dice) is thrown 100 times. Each time the sum of the two numbers on the dice is written down.

Figure 2: Double dice.

- A: Indicate all possible events.
- B: Determine the probability of the events.
- C: Determine the distribution of the events.

The sum of two dice can be anything between 2 and 12. Sum 7 has the highest probability, 2 and 12 have the lowest. Excel can help to calculate probabilities or even create a simulation.

Task 3 – Algebra / Number theory: Unequal products

Given:

x	3	4	5	6	7
y	7	6	5	4	3
x + y					
x · y					

- A: Fill in the table.
- B: What do you notice?
- C: Is that always the case? Explain your decision. Use equations or drawings.

There are different things to note in the table. For example, the product from a + 1 and a – 1 (with a ϵ ℕ) is always less than the square number of a. In fact, the difference is always 1. That can be justified with binomial formula. Technology can help in different ways. A spreadsheet can be used to get a large number of examples very quickly. A computer algebra system, such as GeoGebra, can solve equations and GeoGebra can also be used to create a geometrical solution.

METHODOLOGICAL FRAMEWORK AND STUDY DESIGN

To obtain information about the participants' thoughts during an exercise the thinking aloud method was chosen. The study included ten interviews with five students age 14 to 16. Each of them worked on two of the example problems. While they were working on the task, participants shared their thoughts with the researcher. All the interviews were audio-recorded. The interview guide is based on the theories of Düsing (2014) and is structured in five sections: 1. personal data and introduction of the method, 2. exercise and assurance of understanding, 3. presentation of the task, 4. working on the task, 5. debriefing.

During the study, the computer programs Excel and GeoGebra were approved as aids. To obtain information about how far the instrumental genesis between the students and the instruments (in this case either of the programs) was advanced, each "working on the task" section was split. At first, participants were asked to work on the task and had a free choice of whether they used the technology or not. Secondly, if they had not done so far, they were asked to try and solve the problem by using the computer. Finally, a GeoGebra-file or Excel-file created by the authors was given to them. They should try to use or understand it and figure out how it works.

Category	Code	Definition	Example (Anker)
(F) Use as an instrument	1 Yes, mostly successful	Use of various functions; File brings new insights to the task; Planning and reflection of the procedure	"(...) everything is calculated and that is why there are a 100. And here are several values (student indicates sums in the right column of the simulation) and if this value could be higher (student indicates formula in cell H17), this would finally run."
	2 Yes, sometimes successful	The attempt brings little or no new knowledge; Plan can only be implemented in stages	"I just wanted to find out how long the black one is, but I do not remember if that is already stored somewhere. If you shift red line, that is what is shown here, and named length of the line."
	3 No, never successful	No idea how the task with digital tools can be implemented or plan exists, but cannot be implemented	"As if someone had just diced and on the left, we have the number, and the dices, dice 1 and dice 2, and finally the sum of the eyes. And again the table including all events."

Table 1: Category (F) Use as an instrument. Shown are Code with manifestations, definitions and examples (anker).

All interviews were transcribed using the system developed by Dresing & Pehl (2011). The transcripts include observations by the interviewer. In the fifth section, the students were asked if they felt influenced or distracted by the think-aloud-method (A) and how they assessed their skills in using digital tools in general (B). Besides these two categories for the qualitative content analysis, five categories with three expressions each were generated from the theoretical background of the components of the instrumental genesis: (C) mathematical understanding, (D) instrumentation – usage schemes, (E) instrumentation – instrument-mediated-action schemes, (F) use as an instrument (see Table 1), (G) application – analysis of the given files. After the recorded audio sequences had been transcribed, the text parts, as well as the observations, were allocated to the categories.

FINDINGS & DISCUSSION

The expressions of the instrument-mediated-action schemes were at least as high as those of the usage schemes. Also, the students were more likely to be able to discuss a given file than to create one on their own. Pupils with low expressions in both types of schema were unable to solve the problem with the computer. The data analysis revealed differences in the process progress of the instrumental genesis. From this, three levels were derived, which need to be discussed.

The first level includes all learners who previously had little knowledge of using digital tools. (low expressions in categories D - F; low or middle expression in category G) On the second level are all learners who could reproduce the best-known solutions. In new situations or when dealing with unknown problems, the digital tool is still an artefact. They have insufficient instrument-mediated-action and usage schema to use it as an instrument (middle or low expressions in categories D - G; high expression in category F only if the problem is known; see Figure 4). In the third stage, learners have access to more schema and usage patterns. Only at this stage can learners use the digital tool as an instrument (middle or high expressions in categories D - G; at least one high expression). An overview of all three identified levels within the group of students provides Figure 5

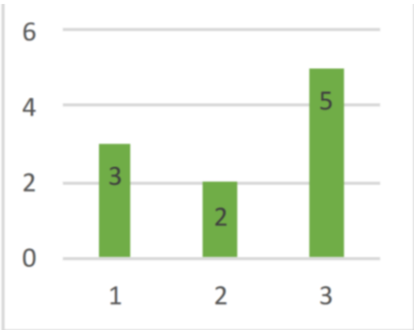

Figure 4: Shown are absolute values of all three manifestations (codes) of category F.

In order to examine the extent to which this gradation satisfies the characteristics of all learners and how the progress of the process changes over the years in school, it would be useful to carry out further longitudinal studies in qualitative terms and supplement these with quantitative methods.

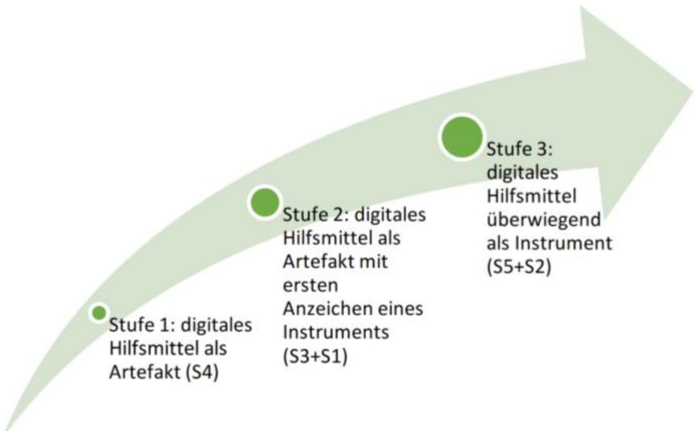

Figure 5: Three levels of instrumental genesis. Level 1: digital tool (device) is used like an artefact, Level 2: first attempts to use a digital tool (device) as an instrument, Level 3: digital tool (device) is mainly used like an instrument.

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COMPUTATIONAL THINKING AND MATHEMATICAL THINKING: DIGITAL LITERACY IN MATHEMATICS CURRICULA

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In technology-rich mathematics education, teachers nowadays experience the challenge to foster both mathematical thinking and computational thinking. To address this issue, the main research question is: How can a teaching-learning strategy, focusing on the use of digital tools, support 16-17 years old pre-university students in developing computational thinking skills related to mathematical thinking in pure and applied mathematics courses?

Keywords: computational thinking, mathematical thinking, digital technology

INTRODUCTION

Mathematics teachers in the Netherlands are facing two different, though related, challenges: how to foster mathematical thinking (e.g., Drijvers, 2015), central in the new mathematics curricula, in their teaching, and how to include computational thinking (Wing, 2006), stressed in the informatics curricula, but also apparent in national educational policies such as the ongoing curriculum.nu reform. These challenges emerge in the context of the growing use of digital technology in mathematics education that goes beyond the regular graphing calculators and includes for example software for statistics and the very popular software GeoGebra for graphs, geometry and (computer) algebra.

When exploring the notions of computational and mathematical thinking in the frame of using digital technology, for example coding, several questions arise. First, how can the concepts involved be better defined and delimited? Previous research has investigated different facets of embedding computational thinking in mathematics education (Benakli, Kostadinov, Satyanarayana, & Singh, 2017; Barcelos, Munoz, Villarroel, Merino, & Silveira, 2018; Grover & Pea, 2013; Weintrop et al., 2016). However, the common core concepts involved in computational and mathematical thinking are still to be explored. Second, while interesting initiatives are undertaken to use programming to address the interface of informatics and mathematics at university level, theoretically and empirically validated practice-oriented approaches for secondary education are lacking. Consequently, the knowledge gap addressed in this study concerns the need to identify aspects of computational thinking that match with the notion of mathematical thinking and the need for theory-based concrete technology-rich learning activities that involve coding in an effective way in secondary mathematics education.

The main research question of our study is: How can a teaching-learning strategy, focusing on the use of digital tools, support 16-17 years old pre-university students in developing computational thinking skills related to mathematical thinking in pure and applied mathematics courses?

RESEARCH PLAN

As the research topic is innovative, and teaching materials are not available, we use a theory-informed design-based research setup to answer the research questions (Bakker & van Eerde, 2015). The study is carried out by a consortium, consisting of five schools, two universities and a curriculum development institute. The study's design includes four phases: (1) an inventory phase, (2) a first design cycle phase, (3) a second design cycle phase, and (4) a concluding phase. In the inventory phase, we aim to identify the common core aspects of computational thinking and mathematical thinking based on a literature study and a Delphi interview study. A pilot study is conducted this spring with a first trial of a learning activity about root finding algorithms. In the first design cycle, we will design and field test learning activities using digital tools in pure and applied mathematics courses. The second design cycle resembles the first one, but focuses more on the learning gains and will be applied in a larger scale. In the concluding phase, the results from the partial studies will be summarized into an answer to the main research question and into guidelines for teaching practice.

AIMED OUTCOMES

The project's results will include:

1. A literature-based, but practice-oriented identification of key elements of computational thinking that relate to mathematical thinking and are suitable to be addressed in technology-rich mathematics teaching to upper secondary pre-university education students;
2. A set of empirically validated learning activities for upper secondary pre-university education students in pure and applied mathematics courses that involve the use of sophisticated digital tools and address key aspects of computational and mathematical thinking;
3. A set of assessment instruments to assess the learning outcomes of the learning activities;
4. A guide for mathematics teachers who design or carry out learning activities targeting computational thinking and mathematical thinking using digital tools.

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Appendix:
Conference Agenda

ICTMT 14
Conference Agenda

Session Overview

Date: Monday, 22/Jul/2019

10:00am - 12:00pm	**Registration** Location: **S06 S00 (Foyer)**			
12:00pm - 1:00pm	**(Light) Lunch** Location: **S06 S00 (Foyer)**			
1:00pm - 2:00pm	**Opening Ceremony** Location: **S06 S00 B29** Chair: **Bärbel Barzel** Chair: **Florian Schacht**			
2:00pm - 3:00pm	**Keynotes 1: Inspiring Learning and Teaching** Location: **S06 S00 B29** Chair: **Jürgen Roth** Chair: **Lynda Ball** The two keynote speakers will give insights into the same conference theme from different perspectives.			
3:00pm - 3:15pm	**Coffee Break** Location: **S06 S00 (Foyer)**			
3:15pm - 4:15pm	**Discussion Groups 1: Inspiring Learning and Teaching** Location: **S06 building various rooms** After the keynotes, we will split into small groups to exchange ideas about the main conference themes.			
4:15pm - 4:30pm	**Coffee Break** Location: **S06 S00 (Foyer)**			
4:30pm - 5:15pm	**Presentations 1.1: TOWARDS AUTOMATED GROUPING: UNRAVELING MATHEMATICS TEACHERS' CONSIDERATIONS** Location: **S06 S01 A16** Chair: **Rotem Abdu**	**Presentations 2.1: MAKING SENSE OF BIOLOGICAL PHENOMENA THROUGH INQUIRY OF MATHEMATICAL REPRESENTATION AND INTERACTIVE TECHNOLOGICAL TOOLS** Location: **S06 S01 B06** Chair: **Nareman Amal Agbaria**	**Presentations 3.1: USABILITY IMPROVEMENT OF A MOBILE GRAPHING CALCULATOR APPLICATION** Location: **S06 S01 B29** Chair: **Melanie Tomaschko**	**Presentations 4.1: ORCHESTRATING WHOLE-CLASS DISCUSSIONS IN MATHEMATICS USING CONNECTED CLASSROOM TECHNOLOGY** Location: **S06 S01 B35** Chair: **Maria Fahlgren**
5:15pm - 6:00pm	**Presentations**	**Presentations 2.2: MATHEMATICAL DISCOVERIES USING COMPUTATIONAL THINKING** Location: **S06 S01 B06** Chair: **Christine Bescherer**	**Presentations 3.2: SUPPORTING ALGORITHMIC APPROACH TO BASIC SCHOOL MATHEMATICS BY PROGRAMMING TASKS** Location: **S06 S01 B29** Chair: **Rein Prank**	**Presentations 4.2: AN EYE-TRACKING LANGUAGE-MATHEMATICS COMPARATIVE STUDY ON ARGUMENTATION** Location: **S06 S01 B35** Chair: **Camilla Spagnolo**
6:00pm - 7:00pm	**An Eye into the Future** Location: **S06 S00 B29** Chair: **Alison Clark-Wilson** Chair: **Daniel Thurm** ICTMT offers an innovative forum for a research conversation between researchers, teachers and emerging technology developers, focused on aspects of mathematics education. By bringing these different communities together, we aim to: - provide a space for teachers and researchers to see and try out emerging technology - enable technology developers...			
7:00pm - 8:00pm	**Eye into the Future / Tech Exhibition** Location: **S06 S00 B32 & S06 S00 B41** You will have time to visit the Exhibition of EdTech Companies & Start-Ups.		**Gallery Walk / Poster Session** Location: **S06 S00 (Foyer)** You will have time to visit poster contributions and speak to the authors. We wish you inspiring discussions!	

Date: Tuesday, 23/Jul/2019

9:00am - 10:00am	**Keynotes 2: Networking of Theories** Location: **S06 S00 B29** Chair: **Angelika Bikner-Ahsbahs** Chair: **Arthur Bakker** The two keynote speakers will give insights into the same conference theme from different perspectives.			
10:00am - 10:15am	**Coffee Break** Location: **S06 S00 (Foyer)**			
10:15am - 11:15am	**Discussion Groups 2: Networking of Theories** Location: **S06 building various rooms** After the keynotes, we will split into small groups to exchange ideas about the main conference themes.			
11:15am - 11:30am	**Coffee Break** Location: **S06 S00 (Foyer)**			
11:30am - 12:15pm	**Presentations 1.3: TEEN-IMMIGRANTS EXPLORE A MATH MOBILE APP** Location: **S06 S01 A16** Chair: **Domenico Brunetto**	**Presentations 2.3: STUDENTS' USE OF DIGITAL SCAFFOLDING AT UNIVERSITY LEVEL: EMERGENCE OF UTILIZATION SCHEMES** Location: **S06 S01 B06** Chair: **Annalisa Cusi**	**Presentations 3.3: Comparing Digital and Classical Approaches - The Case of Tessellation in Primary School** Location: **S06 S01 B29** Chair: **Frederik Dilling**	**Presentations 4.3: DESIGNING ONLINE FORMATIVE ASSESSMENT THAT PROMOTES STUDENTS' REASONING PROCESSES** Location: **S06 S01 B35** Chair: **Raz Harel**
12:15pm - 1:00pm	**Presentations 1.4: TECHNOLOGY AS A RESOURCE TO PROMOTE MATHEMATICS TEACHING** Location: **S06 S01 A16** Chair: **Maria Cristina Costa**	**Presentations 2.4: "MATHEMATICAL DIGITAL COMPETENCIES FOR TEACHING" FROM A NETWORKING OF THEORIES PERSPECTIVE** Location: **S06 S01 B06** Chair: **Eirini Geraniou**	**Presentations 3.4: Cancelled** Location: **S06 S01 B29**	**Presentations 4.4: Digital dynamic formative assessment** Location: **S06 S01 B35** Chair: **Morten Elkjær**

Date: Wednesday, 24/Jul/2019

9:00am - 10:00am	**Keynotes 3: Enhancing Assessment** Location: **S06 S00 B29** Chair: **Michal Yerushalmy** Chair: **Bastiaan Heeren** The two keynote speakers will give insights into the same conference theme from different perspectives.			
10:00am - 10:15am	**Coffee Break** Location: **S06 S00 (Foyer)**			
10:15am - 11:15am	**Discussion Groups 3: Enhancing Assessment** Location: **S06 building various rooms** After the keynotes, we will split into small groups to exchange ideas about the main conference themes.			
11:15am - 11:30am	**Coffee Break** Location: **S06 S00 (Foyer)**			
11:30am - 12:15pm	**Presentations 1.5: A QUANTITATIVE STUDY ON THE USAGE OF DYNAMIC GEOMETRY ENVIRONMENTS IN DANISH LOWER SECONDARY SCHOOL** Location: **S06 S01 A16** Chair: **Ingi Heinesen Højsted**	**Presentations 2.5: AN AUGMENTED REALITY APPLICATION TO ENGAGE STUDENTS IN STEM EDUCATION** Location: **S06 S01 B06** Chair: **Maria Cristina Costa**	**Presentations 3.5: STUDENTS CHOICE AND PERCEIVED IMPORTANCE OF RESOURCES IN FIRST-YEAR UNIVERSITY CALCULUS AND LINEAR ALGEBRA** Location: **S06 S01 B29** Chair: **Zeger-Jan Kock**	**Presentations 4.5: CONSTRAINED SKETCHING ON A GRID: A LENS FOR ONLINE ASSESSMENT OF DERIVATIVE SKETCHING** Location: **S06 S01 B35** Chair: **Galit Nagari-Haddif**
12:15pm - 1:00pm	**Presentations 1.6: A FRAMEWORK DESCRIBING STUDENTS' MATHEMATICS LEARNING EXPERIENCE WITH A TABLET-BASED PEDAGOGICAL MEDIUM: THE CASE OF A GEOMETRY EXPLORATION** Location: **S06 S01 A16** Chair: **Marios Pittalis**	**Presentations 2.6: THE USE OF QUIZZES ON MOODLE FOR TEACHING DIFFERENTIAL EQUATIONS TO ENGINEERING STUDENTS** Location: **S06 S01 B06** Chair: **Maria Antonietta Lepellere**	**Presentations 3.6: Implementing augmented reality in flipped mathematic classrooms to enable inquiry-based learning** Location: **S06 S01 B29** Chair: **Stefanie Schallert**	**Presentations 4.6: Using silent video tasks for formative assessment** Location: **S06 S01 B35** Chair: **Bjarnheiður (Bea) Kristinsdóttir**
1:00pm - 2:00pm	**Lunch** Location: **Mensa**			
2:00pm - 2:45pm	**Presentations 1.7: USING A MATHEMATICAL FORUM IN A GRADUATE COURSE: THE NATURE OF RICK'S AND JOHN'S PARTICIPATION** Location: **S06 S01 A16** Chair: **Giulia Bernardi**	**Presentations 2.7: ENHANCING FORMATIVE ASSESSMENT PRACTICES IN UNDERGRADUATE COURSES BY MEANS OF ONLINE WORKSHOPS** Location: **S06 S01 B06** Chair: **Giovannina Albano**	**Presentations 3.7: DRIVING AUGMENTED REALITY: GEOGEBRA'S NEW AR FEATURES IN TEACHING MATHEMATICS** Location: **S06 S01 B29** Chair: **Andreas Trappmair**	**Presentations 4.7: TEACHER'S ATTENTION TO CHARACTERISTICS OF PARABOLA SKETCHES: DIFFERENCES BETWEEN USE OF MANUAL AND AUTOMATED ANALYSIS** Location: **S06 S01 B35** Chair: **Kholod Abu Raya**
2:45pm - 3:30pm	**Presentations 1.8: DESIGNING AND DISSEMINATING REVIEW CRITERIA FOR QUALITY OF TABLET APPS IN PRIMARY SCHOOL MATHEMATICS** Location: **S06 S01 A16** Chair: **Ana Donevska-Todorova**	**Presentations 2.8: REWARDS AND CHALLENGES OF USING CANVAS, WEBWORKS AND GEOGEBRA IN A MULTIVARIABE CALCULUS COURSE** Location: **S06 S01 B06** Chair: **Ann Moskol**	**Presentations 3.8: BILINGUAL MATH LESSONS WITH DIGITAL TOOLS – CHALLENGES CAN BE DOOR OPENER TO LANGUAGE AND TECHNOLGY** Location: **S06 S01 B29** Chair: **Matthias Müller**	**Presentations 4.8: Automated Feedback At Task Level: Error analysis or Worked out examples – which type is more effective?** Location: **S06 S01 B35** Chair: **Guido Pinkernell**
3:30pm - 3:45pm	**Coffee Break** Location: **S06 S00 (Foyer)**			
3:45pm - 4:30pm	**Presentations 1.9: Aligning embodied and instrumented experiences to foster powerful mathematical activities** Location: **S06 S01 A16** Chair: **Eleonora Faggiano**	**Presentations 2.9: EVALUATING CAS AND DGS AT THE MATHS CLASSROOM: A PROPOSAL FOR AN UNBIASED EXPERIMENTAL STUDY OF THE IMPACT OF THE COMPUTATIONAL ROLE OF THE STUDENTS IN THE MEANINGFUL OF THEIR LEARNING** Location: **S06 S01 B06** Chair: **Eugenio Roanes-Lozano**	**Presentations 3.9: PREPARING PROSPECTIVE MATHEMATICS TEACHERS TO DESIGN AND TEACH TECHNOLOGY-BASED LESSONS** Location: **S06 S01 B29** Chair: **Gulay Bozkurt**	

Date: Thursday, 25/Jul/2019

9:00am - 10:00am	**Open Space: Developing Visions** Location: **S06 S00 B29** Chair: **Paul Drijvers** Chair: **Bärbel Barzel** Chair: **Florian Schacht** In regards to the conference's third main theme "Developing Visions" we will have an interactive format for a planery session with much room for discussions and exchange.		
10:00am - 10:15am	**Coffee Break** Location: **S06 S00 (Foyer)**		
10:15am - 11:15am	**Discussion Groups 4: Developing Visions** Location: **S06 building various rooms** After the keynotes, we will split into small groups to exchange ideas about the main conference themes.		
11:15am - 11:30am	**Coffee Break** Location: **S06 S00 (Foyer)**		
11:30am - 12:15pm	**Presentations 1.10: Peer Instruction in Elementary Mathematics With the Pythagorean Theorem** Location: **S06 S01 A16** Chair: **Tomáš Zadražil**	**Presentations 2.10: STUDENTS LEARNING WITH DIGITAL MATHEMATICAL TOOLS – THREE LEVELS OF INSTRUMENTAL GENESIS** Location: **S06 S01 B06** Chair: **Swantje Schmidt**	**Presentations 3.10: WHEN DIDACTICS MEETS DATA SCIENCE** Location: **S06 S01 B29** Chair: **Franck Salles**
12:15pm - 12:30pm	**Coffee Break** Location: **S06 S00 (Foyer)**		
12:30pm - 1:00pm	**Closing Ceremony** Location: **S06 S00 B29** Chair: **Bärbel Barzel** Chair: **Florian Schacht**		

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