DEVELOPING VISIONS – A SUMMARY OF IDEAS FROM ICTMT 14 PARTICIPANTS ON FUTURE DEVELOPMENTS REGARDING THE USE OF TECHNOLOGY IN MATHEMATICS EDUCATION

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One of four main conference themes – developing visions – addresses the challenge for mathematics educators to integrate new technologies meaningfully into the practice and research of teaching and learning mathematics. It aims to encourage visualizations of new possibilities for using affordances of technology to, e.g., foster sense-making of mathematical contents, create successful learning environments and teaching scenarios, innovate new ways to obtain empirical findings, or develop research-based theories. In this article, we summarize ideas of ICTMT 14 participants on possible future developments regarding the use of technology in mathematics education. These ideas have emerged as a product of a cooperative discussion session during the conference, in which groups of participants addressed three main questions together: What might the future of 1) teaching and learning mathematics, 2) technological innovation, and 3) mathematics education research look like?

Keywords: developing visions, technology, mathematics education

INTRODUCTION

With the development of new technologies (e.g., augmented or virtual reality), mathematics educators face the challenge of examining whether these technologies support mathematics learning and – if so – how they can be integrated into the mathematics classroom. Furthermore, there is a need to develop visions of how technology can be used to support teaching and learning mathematics as well as educational research in the future. This is why one of four main conference themes of ICTMT 14 was called “developing visions”.

In order to elaborate, discuss and share ideas on possible future developments regarding the use of technology in mathematics education, we held a plenary session in form of a collaborative “open space” during ICTMT 14. While for each of the remaining conference themes – enhancing assessment, inspiring learning and teaching as well as networking of theories – plenary talks were given by two keynote speakers, the theme of “developing visions” demanded a more inclusive approach. Instead of having keynote speakers present insights into this conference theme from their perspectives, the aim was to utilize the expertise and creativity of all participants and spark cooperation among them. During the plenary session, a short input was given by Bärbel Barzel and Florian Schacht (University of Duisburg-Essen, GER) together with Paul Drijvers (Utrecht University, NL). They raised three questions which were then discussed in groups of 4–10 conference participants. The groups documented their discussion outcomes using an online Google document which was shared among participants. Their notes build the foundation for this article, in which we interpret and summarize the ideas that emerged regarding the following questions:

1) What will future technology-rich mathematics teaching and learning look like?
2) What will technology look like and how will it impact future learning goals and curricula?
3) How will mathematics education research be affected by technological innovation in terms of agenda, theories, tools and practices?

VISIONS FOR TEACHING AND LEARNING MATHEMATICS

Summary of the session input by Barzel, Drijvers, and Schacht

High-quality education can be viewed as one of humans' most valuable possessions. This can be made apparent, for example, by considering the Millennium Development Goals [MDGs] formulated by the United Nations [UN] at the beginning of this millennium to reduce extreme poverty. One of the eight goals is to "Achieve universal primary education" (UN, 2015). However, the question of what constitutes high-quality education is not easy to answer. Praetorius, Klieme, Herbert, and Pinger (2018) suggest that quality teaching is formed by three basic dimensions: *classroom management, student support,* and *cognitive activation* (Praetorius et al., 2018). Integrating technology into the mathematics classroom can influence all three of these dimensions. This is why currently the challenge for mathematics educators lies in rethinking curriculum goals and teaching styles in regard to the use of digital tools. Teachers as well as students need to accumulate knowledge about a) technologies themselves, e.g., how to use them, and b) learning mathematics with technology. This presents a challenge as a large diversity of digital tools are already available for mathematics education.

On the one hand, current technologies can be categorized into more general digital tools. These can either focus more on work or everyday life, for example, technologies that foster communication or the presentation of content, or aim specifically at the use in education. Examples of such tools are student response systems, digital textbooks, or learning platforms. On the other hand, mathematics-specific technologies are available, which again are either more suitable to be used in economical or everyday contexts (e.g., computer algebra systems or statistics tools) or aim at the implementation in education such as specific-purposed tools (e.g., applets) or general-purposed tools (e.g., dynamic geometry systems).

Besides the question of how to best implement different types of technology into the mathematics classroom, visions for teaching and learning mathematics in a digital era might include ideas about new theories used to describe learning processes. While social constructivism is currently the most commonly used learning theory among mathematics educators, it might not be able to explain technology-based learning processes fully. Other theories, such as humanism or connectivism, are starting to emerge and might be more suitable to describe the process of gaining knowledge in the digital era. Connectivism, for example, was developed in regard to e-learning environments and, thus, "views learning as a network phenomenon influenced by technology and socialization" (Goldie, 2016, p. 1064). It is based on the idea that people process information by forming connections between different fields, ideas and concepts. In addition, it emphasizes learning as taking place in connected learning communities, in which thoughts can be shared via words, images or other multimedia formats. Thus, abilities to access and filter relevant information, being aware of the diversity of different peoples' opinions as well as making decisions about one's own learning process are essential in such e-learning environments (Goldie, 2016).

Barzel, Drijvers, and Schacht (2019) summarize their vision by stating that in the future students should be:

- aware of the role of mathematics in the world,
- able to use mathematics flexibly and confidently,
- creative and open-minded problem-solvers,
- engaged and willing to solve the problems of the future.

Thus, they see the need for the teaching and learning of mathematics to:

- be engaging, exploratory and focused on sense-making,
- be supportive, focused on individual needs,
- integrate technology to support a) students' understanding of mathematical concepts, b) the integration of real-life situations in classrooms (e.g., using simulations or augmented reality), and c) both individual and cooperative work.

First group's discussion outcomes

The first group of ICTMT 14 participants to discuss the question of how a technologically-rich teaching and learning of mathematics might look in the future consisted of: Gulay Bozkurt (Eskisehir Osmangazi University, TR), Maxim Brnic (Westfälische Wilhelms Universität Münster, GER), Patrick Ebers (Universität Duisburg-Essen, GER), Maria Fahlgren (Karlstad University, SE), Christos Itsios (University of Duisburg-Essen, GER), Bea Kristinsdóttir (University of Iceland, IS), Matthias Müller (TU Braunschweig, GER), Hana Ruchniewicz (University of Duisburg-Essen, GER), and Stefanie Schallert (Johannes Kepler University Linz, AT).

This group's discussion mainly focused on collecting ideas about how to scale-up research results and examples of "good practice" to make use of technological affordances in many mathematics classrooms. Furthermore, some challenges of trying to introduce technologies into more classrooms were addressed. Changes were seen to be necessary in the following areas:

Teachers' professional development (PD):

- The increasing availability of online PD courses offers the possibility for teachers to repeat content and improve their skills and knowledge about using technology to teach and learn mathematics independent of time and location.
- As new innovations are mostly brought into practice via PD courses, there is a need to show their efficacy, especially in regards to students' learning outcomes.
- There is a need to identify quality criteria of "good" tasks and materials for teachers to select useful digital learning environments or tasks for their classrooms.
- Web 2.0 technology offers new opportunities for teachers to work collaboratively, e.g., in online professional learning groups (PLGs).
- Video-recordings of real classrooms using digital technologies can be used to demonstrate examples of "good practice" to other teachers.
- Technology offers teachers the opportunity to share own experiences of using digital tools in their mathematics classrooms (e.g., via self-reports or video-recordings) and receive peer- or expert-feedback on their teaching. Thus, the element of micro-teaching could be implemented into PD courses.
- A challenge of integrating new technologies into mathematics classrooms is the speed of technological development: while researchers and teacher educators are addressing how to integrate digital tools, new technologies emerge.
- Online PD can help to reduce the number of required (mathematics) teachers. In order to reach the goal to provide quality education and promote lifelong learning to all children, the UNESCO Institute of Statistics [UIS] predicts that, globally, we need 9.8 million additional primary and 22.3 million additional secondary school teachers by 2020 (UIS, 2016).

Textbooks:

- Textbooks still represent one of the main influences for school teachers in planning mathematics lessons. Therefore, the use of technology needs to become apparent in textbooks or there even needs to be a shift towards using digital textbooks.

- A “choose your own textbook” approach might be easy to handle for teachers. The idea is that teachers can choose a number of applets, tasks, etc. for a specific topic to create their own digital textbook for a mathematics lesson.
- PD courses or workshops are needed on “How to use a digital textbook?”.
- Not enough research results are available on “How to use digital textbooks?”. Especially, as their structure and elements differ from pen-and-paper textbooks. For example, if digital textbooks are only composed of applets, will the diversity of classroom activities be reduced and, thus, student engagement be reduced?
- Quality criteria and typologies on digital textbooks are needed to describe their potentials and restraints.

Interdisciplinary and out-of-school Learning:

- The use of technology offers new possibilities to engage students in doing and learning mathematics outside the typical classroom setting. For example, they might play a game on their mobile phones at home, which teaches them mathematical contents.
- More opportunities for seamless learning: currently there is a disconnection between the students’ use of digital technologies in their everyday life (digital natives) and at school (use of own mobile devices is often forbidden).
- Technologies might change the way we arrange or design classrooms. The vision of the discussion group is that there will be mixture of “open spaces” and “quiet areas” in schools rather than the traditional “desks facing the front of the classroom” set up.

Subject matter – choosing what to teach:

- As future teachers are taught at universities, one has to focus more on how to implement the use of technology in higher education. Many institutions still focus on using paper-and-pencil methods in undergraduate courses.
- Guidelines on “What to teach students?” in regards to learning mathematics through and with technologies are needed and the mathematics education research community should take part in that discussion.

Second group’s discussion outcomes

The second group discussing visions for the use of technology in teaching and learning mathematics consisted of: Lynda Ball (University of Melbourne, AU), Bärbel Barzel (University of Duisburg-Essen, GER), Giulia Bernardi (Politecnico di Milano, IT), Paula Beukers (University of Groningen, NL), Domenico Brunetto (Politecnico di Milan, IT), Mats Brunström (Karlstad University, SE), Zeger-Jan Kock (Eindhoven University, NL), Maria A. Lepellere (university of Udine, IT), and Marios Pittalis (University of Cyprus, CY).

Aim of mathematics education:

This group started their discussion by thinking about the goal of quality mathematics education. Their idea was that the aim of education – as well as the way of reaching it – is to find balance between each of the following six pairs: conceptual and procedural knowledge; paper-and-pencil and computer technologies; modelling real-life situations and acting within mathematical contexts; guided and open teaching approaches; individuality and cooperation; lower-order and higher-order skills. Teachers should be able to view mathematics education from a “meta-perspective”. This means, for example, that they are able to create opportunities for students to relate new to previously learned knowledge, make connections, stress sense-making over the accumulation of skills and are able to re-balance their teaching according to specific situations and lesson goals.

More cooperation between teachers and researchers is needed:
A main idea, that emerged from this group's discussion, was that teachers and researchers should work together more cooperatively to implement technology in the teaching and learning of mathematics and in return learn more about learning processes that take place when new technologies are used in the mathematics classroom. The group stressed that we need critical analyses of *how* technology is used to do teach and learn mathematics with and through digital tools.

Out-of-school learning:
Similar to the first group, the second also saw a potential of new technologies in offering new learning opportunities, especially in out-of-school or informal settings.

New contents are needed for 21st century skills:
One point of this group's discussion was that new technologies will demand different skills. Gravemeijer (2012) addresses the need for mathematics education to change in order to teach students 21st century skills. He explains that changes in education tend to happen slowly while both our society and work environment change rapidly. Thus, there is "an 'achievement gap' between what schools […] are teaching and what students will need to succeed in today's global knowledge economy" (Gravemeijer, 2012, p. 30). Mathematical contents, such as modelling real-life situations using variables and using statistics to overlook an increasing amount of data, will be more important in the future (Gravemeijer, 2012).

Affordances of using technology in teaching and learning mathematics:
- Digital technologies allow for students to explore mathematical contents (e.g., by interactive or linked representations of mathematical objects).
- Digital technologies allow for students to gain higher-order skills by activating them cognitively.

Chance to change attitudes:
A problem for mathematics educators to engage students is that a bad attitude towards the subject is oftentimes accepted socially. It seems positively connoted to be "bad at maths". Using technology to learn mathematics might change students' attitudes towards the subject. It offers opportunities to highlight the "beauty of mathematics".

Teacher Education:
If the aim is to use technology to support the teaching and learning of mathematics, the pre- and in-service education of teachers becomes crucial. They need to learn, for example, how to select digital tools, how to create tasks and activities with these tools, how to make use of their affordances, which constraints might become apparent, how to structure lessons, etc. The goal for teachers must be to be critical and know when and how to use digital tools.

VISIONS FOR TECHNOLOGICAL INNOVATIONS AND THEIR IMPACT

Summary of the session input by Barzel, Drijvers, and Schacht

When the task is to predict how technologies of the future might look like, history has shown that it is often far from accurate. Examples can be found plentiful in motion pictures and literature. One example is the cover art by Robert Tinney for the 1981 April issue of Byte Magazine (Morgan, 1981), which depicts a future smartwatch including a tiny keyboard and floppy disk drive. Even though it is stated to be a humorous in regard to the design and functionalities of modern smartwatches, it shows the way people think of future technologies. McCracken (2014) summarizes in a TIME magazine article: “We tend to think that new products will be a lot like the ones we know. […] Oftentimes, we don’t dream big enough” (McCracken, 2014). Thus, the 1981 envisioned wrist computer included floppy disks as it was the known way for data storage. However, some historical predictions envision future technologies partly correctly. One example given by Barzel et al. (2019) is from the movie franchise *Back to the future.* In the second movie (Canton & Gale, 1989), technological innovations such as smart glasses and face recognition were predicted, which are nowadays available (Barzel et al., 2019; Hill, 2015).

These examples of more or less accurate predictions show that it is not so easy to plan for the future. Nevertheless, envisioning future technologies can produce exciting ideas. Emerging technologies which are also used in education are, for example, augmented and virtual reality (Swidan et al., 2020). But with the emerging technologies used for education, one has to address the difficult relationships between private and public companies as well as academia. Funding through external companies is beneficial, but the research should be independent and objective regardless for whom the research is conducted (Barzel et al., 2019).

Barzel et al. (2019) concluded their input regarding the visions for technological innovations and their impact with the question: “Which technology supports the teaching of mathematics in the future?” and asked for criteria, examples and connections. To support the discussion, the following starting points were given:

- What will future technology look like?
- What should future technology look like?
- How will we use future technology in the (mathematics) classroom?
- How does/will technology impact on learning goals and curricula?
- Cooperation between private and public companies and academia?

First group’s discussion outcomes

The first group discussing future technologies and their impact consisted of Peter Boon (Numworx, NL), Alison Clark-Wilson (UCL Institute of Education, UK), Florian Schacht (University of Duisburg-Essen, GER), and Anna Shvarts (Utrecht University, NL):

They focused on the question “*What are the (future) mathematical practices which technology can support?”* in their discussion and noted the following six practices.

- The first practice concerns **shared collaborations** – which could lead to collaborative productions and even to fundamentally new goals.
- But new media also necessarily leads to the question of how conceptualizations of mathematical content beyond mathematics changes through them.
- The possibility that conceptualization changes through the media connects to research that **cognitive functions** (attention, perception, writing abilities) are changing together with technology. For the use of smartphones, it has even been shown that the sensory representation in the brain is influenced by the usage (Gindrat et al., 2015).
- As the usage of smartphones and other media increases and many high school students have access to them (for Germany 99 % of 15-year old students have access to smartphones; Feierabend et al., 2018), something called **open schools** could emerge. These are schools where everyday reality and the classroom are not separated opposed to school where the use of smartphones is forbidden. If the future technology is used in a beneficial way it might help in making mathematics knowledge meaningful.

- **Goals** are changing with technology-supported transformations in society. Thus, it has to be asked and reflected if the way how mathematics is **used** is still current or what aspects of mathematics need to be outsourced to technology completely.
- Digital technologies have already changed and might continue to change **social norms** by connecting a diversity of people worldwide in real-time. Thus, the appropriate way of interacting, communicating, cooperating, or learning together will be influenced.

Second group's discussion outcomes

The second group to discuss the questions raised by Barzel et al. (2019) regarding future technologies consisted of Rotem Abdu (Haifa University, IL), Ana Donevska-Todorova (Humboldt-University of Berlin, GER), Eirini Geraniou (UCL Institute of Education, UK), Miriam Romberg (University of Duisburg-Essen, GER), Andreas Trappmair (Johannes Kepler University Linz, AT), Sylvia van Borkulo (Utrecht University, NL), and Johanna Zöchbauer (Johannes Kepler University Linz, AT).

The second group's discussion revolved around four parts that have to be considered and discussed in the frame of future technology used in mathematics education. These four parts concern the transformation of already existing technologies, new technologies, ethical issues and delaying factors for implementation in the mathematics classroom:

1. The aim is to **transform already existing technologies** and find ways to integrate them in the mathematics classroom in different ways regarding different technologies.
    - Intelligent tutoring systems should be transformed and adaptive as well as personalized features included, so that they may become more efficient in mathematics education practices.
    - But one also has to convince the teachers of the effectiveness of these "new" technologies, so that they are actually introduced into their classrooms. This has been difficult in the past and so the effective technologies are not implemented in real classrooms.
    - One also has to catch up with updates of existing software and teachers need to develop digital competences to manage the changes in their classrooms.
    - Augmented and virtual reality should be included in the mathematics classroom, as they become more widely available (Swidan et al., 2020).

2. Apart from transforming the already existing technologies, there are also **new technologies appearing in industry**, these might lead to new ways of teaching and learning mathematics.

- For this goal interdisciplinary approaches in development of appropriate software for school mathematics should be undertaken to bridge the gap between industry and education more quickly and effectively.
- One example of this technology is artificial intelligence, which might offer more adaptive technology for education such as intelligent tutoring systems that give explicit feedback to the learners (e.g., ASSISTments; Roschelle et al., 2016).
- But one also has to question, if all new and emerging technology can and should be used for education and in particular in mathematics. Do they all match or meet our educational needs?

3. An important point to consider, when using technology in school are **ethical issues**:

- There are controversial issues, e.g., microchips utilized to enhance connectivity between a learner and the world, but they also raise data privacy issues, when using the software in class.
- Ethical issues can also emerge, when cooperating with big technology players. The question remains, if we can afford to ignore them, because of their global reach and the resulting possibility to scale access to education globally. How do we ensure an ethical, equitable, high quality mathematics education "offer" that is accessible to all?

4. While discussing how to implement existing technology and emerging technology in the mathematics classrooms around the world, there are a number of **delaying factors** that have to be considered.

- One of the biggest problems is non-existent school infrastructure for the implementation of technology. For the use of software, there is often a need for internet access to work properly, which might not be available. Also hardware in sufficient numbers, so students can use it effectively, is often expensive and public schools might not have the funds to acquire the necessary hardware.
- Often there is a perceived need of “big” scientific data to evidence impact and successful implementation, before stakeholders advocate the use of new technologies in education.
- Depending on the country, more or less strict educational policies are in place. If these include or exclude technology use specifically, it can lead to significantly delayed introduction of beneficial technology into classrooms.
- Even if the infrastructure and education policies enable the use of technology in a timely manner, teachers need to be educated through PD programs on how to use the new technology meaningful. As these PD programs first have to be developed themselves, this leads to a delay in using the technology meaningful.

VISIONS FOR MATHEMATICS EDUCATION RESEARCH

Summary of the session input by Barzel, Drijvers, and Schacht

The third theme for discussion focused on the impact of technological innovations on the mathematics education research in terms of agendas, theoretical frameworks, and tools. In terms of research agendas, several questions show the considerable variety of research directions focusing on technology:

- Can and should mathematics education research focus on students’ higher-order thinking skills and/or on mathematical literacy, both regarding the significance of technology for the learning processes?
- Can and should embodiment with and through digital technology as well as embodied instrumentation be a fruitful research investigation?
- How can student-student- and/or student-teacher-collaboration be affected while using digital technology? How does the language change due to the use of technology?
- What are the relationships between mathematical and computational thinking? How does computational transposition during the design and implementation of computer learning environments transform the knowledge to be taught through these technological means?

In order to investigate in these research agendas, there is a growing need for adequate theoretical frameworks. Accordingly, one must look at available theoretical approaches that will help us to understand and exploit the affordances of digital technology for mathematics education. To do so, a look at theories from mathematics education but also from cognitive science, cognitive ergonomics, game research, human-machine interaction, etc. is auspicious. For the purpose of dealing with the diversity of theories, exploring the possibilities of interactions between theories, such as contrasting, coordinating, and locally integrating them, serves as a beneficial method (Bikner-Ahsbahs & Prediger, 2014).

In addition to looking at the variety of theories, the technological devices at disposal – for researches and for learners – have increased largely in the last couple of years. Focusing on the perspective of researches, there are more than a handful of different tools that (are likely to) affect researchers' work and research outcomes, such as:

- Eye tracking and finger tracking devices,
- Automated transcription,
- Automated coding in software for qualitative data analysis,
- Multiple camera recording and motion tracking,
- Data logging and learning analytics,
- Hand writing recognition.

Overall, technological progress will influence mathematics education research. The question is how will and should mathematics education research be affected by technological innovation, in terms of agendas, theories and tools. This issue was discussed by two groups.

First group's discussion outcomes

The first group discussing future technologies and their impact on mathematics education research consisted of Angelika Bikner-Ahsbahs (University of Bremen, GER), Florian Berens (University of Goettingen, GER), Eleonora Faggiano (University of Bari Aldo Moro, IT), Federica Mennuni (University of Bari Aldo Moro, IT), Laura Ostsieker (University of Applied Sciences Cologne, GER), Benjamin Rott (University of Cologne, GER), and Moritz L. Sümmermann (University of Cologne, GER).

They raised a variety of sub-questions regarding the main impact of technology on research agendas, frameworks, and tools ranging from the issue of technological enhancements (e.g., smartwatches, smart glasses, artificial legs, etc.) – leading to an alternative view on our body – to treating technology as an opportunity for changes instead of a black box in order to realize the innovation technology brings to learning mathematics.

Along these lines, one of the main points of discussion was put on the change in mathematics learning through technology and, therefore, on artificial intelligence as well as new tools helping researchers to manage and process data collection, coding, etc. Although the benefits of such tools were not argued but seen as a real chance, the group agreed on concentrating on better planning what and how data is collected in terms of being careful not to trust automated analyses too much. With that being said, the group saw the starting point of research projects in problems and research questions and in particular not in the collected data. This is due to the fact that a lot of data can be collected during on-going research, which is easy to collect and to analyze, but possibly leading to negligible data as well. Therefore, the focus in mathematics education research should be theory-driven.

Furthermore, the group discussed issues of ethics as well as conference standards which might change due to technological innovations and, therefore, affect research principles. One aspect was the influence of platforms like *arXiv* to discuss manuscripts before publication: How can we, as a research community, ensure that experienced reviewers look at articles? But also: Will new technologies be developed that will change international conferences? Will we still travel around the world? At the moment, technologies like Skype cannot replace personal meetings.

Finally, issues of computational thinking, statistical thinking, dealing with big data, and data literacy were considered as they will also address teaching issues:

- New topics will have to be researched (machine learning, networks, graph theory, statistics, big data, etc.).
- New technologies will have an impact on curriculum design and research.

Second group's discussion outcomes

The second group to discuss the questions raised by Barzel et al. (2019) regarding the influence of technology on mathematics education research consisted of Cecilie Carlsen Bach (Aarhus University, DK), Christine Bescherer (University of Education Ludwigsburg, GER), Paul Drijvers (Utrecht University,NL), Rikke M. Gregersen (Aarhus University, DK), Karina Höveler (Westfälische Wilhelms Universität Münster, GER), Mathilde K. Pedersen (Aarhus University, DK), and Marianne Thomsen (Aarhus University, DK).

This group discussed four different facets regarding the impact of technological innovations on the mathematics education research in terms of agendas:

- Bridge the gap between using physical manipulatives and digital technology and investigate on how to exploit the potentiality of their synergic use.
- Address mathematics education and societal issues (e.g., equity or diversity).
- Teacher professional development, for example, How to use technology to better monitor and understand all students' learning in the classroom?
- How to prevent researchers from re-inventing the wheel?

Regarding theoretical frameworks, the following aspects were addressed:

- Input from neuroscience research?
- How to benefit from the different theoretical lenses to look at the same set of data?

Focusing on research tools, it was concluded that researchers will need their own professional development; for research practices, ethical issues and the community's responsibility towards the society was addressed.

CONCLUDING REMARKS

The collaborative "open space" on developing visions at ICTMT 14 showed many current and possible future developments regarding the use of technology in mathematics education practice and research. The points addressed in the discussion groups reveal that many issues raised are not country specific but common themes around the world. The three main questions of the discussion regarding future teaching and learning, technological developments and research practices encouraged the conference participants to share a wide variety of goals, challenges and visions.

Regarding the teaching and learning of mathematics, it became apparent that the use of technology affords the exploration of new content areas (e.g., modelling, statistics, big data), materials (e.g., digital textbooks, interactive learning environments) and ways to connect mathematical contents, disciplines, learners, teachers and researchers. However, the difficulty for teachers to select beneficial technology and to orchestrate their use in the classroom remains. This is why there is not only a need for guidelines and quality criteria on what and how to teach with and through digital tools, but also for PD courses and pre-service teacher education. Such teacher trainings could be supported by the use of technology in that it connects teachers (e.g., online PD courses, online PLGs) and gives them the opportunity to share examples of good practice or receive peer- or expert-feedback on their teaching. Finally, both discussion groups stressed a need for more cooperation between teachers and mathematics education researchers in the future.

Regarding future developments in technology, both groups discussed how this might change the conceptualization of mathematical contents. Educators must consider emerging technologies (currently e.g., virtual or augmented reality, adaptive features, artificial intelligence) in light of their affordances and constraints for learning mathematics. Technological developments are seen to support, for example, collaborations, cognitive functions, or the exploration of new mathematical contents. However, challenges of using technology in mathematics education include scaling-up scientific findings, providing schools with the necessary infrastructure (e.g., WIFI access), or ethical issues in regards to data privacy as well as cooperation between educators and commercial companies. Nevertheless, such collaborations could have great impact in terms of new technological developments for mathematics education.

Regarding the use of technology in mathematics education research, groups addressed that not only research agendas, but also research frameworks and tools might change. With technological advances new mathematical contents (e.g., machine learning, statistics) will become more important and, thus, need to be investigated as they become part of curriculums. In addition, the question of how to best educate teachers on the use of technology needs to be researched further. Concerning research methodologies technology can support the collection, managing, processing, coding and analyzation of data. Moreover, approaches from other disciplines such as neuroscience might be introduced to mathematics education. Finally, technology might impact the work process of researchers as it offers, for example, platforms to share and review scientific work, or change the way researchers exchange ideas in the future. However, both groups stressed that technological developments come with ethical issues that need to be addressed.

Over all three topics it became evident that ICTMT 14 participants stress the importance of using technology to explore and connect mathematical ideas and actively engage in mathematical activities. In particular, possibilities to connect and collaborate through technology were addressed. Furthermore, the education of mathematics teachers (pre- and in-service) on how to use digital tools efficiently in their classrooms was a common discussion point. Besides these opportunities to support mathematics education, groups identified ethical issues that need to be considered by mathematics educators. Finally, the discussions showed a demand for more collaboration between teachers, teachers and researchers as well as researchers and commercial companies. The local organizing team of ICTMT 14 hopes that the "open space" and discussion on developing visions serves as a starting point for this purpose.

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