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Abstract

Empirical studies show that years of schooling are positively correlated with good health, and that education is better correlated with health than with variables like occupation and income. This can be explained in different ways as the implication may go from education to health, from health to education, and there may be variables that influence health and education in the same direction. The effect of different policy instruments to reduce the social gradient in health will depend on the strength of these causalities. In this paper we formalize a model that simultaneously determines an individual’s demand for knowledge and health based on the mentioned causal effects. We study the impacts on both health and education of different policy instruments such as subsidies on medical care, subsidizing schooling, income tax reduction, lump sum transfers and improving health at young age. Our results indicate that income transfers such as distributional policies may be the best instrument to improve welfare, while subsidies to medical care is the best instrument for longevity. However, subsidies to medical care or education would require large imperfections in the markets for health and education to be more welfare improving than distributional policies. Finally, our simulations suggest that underlying factors that impact both health and education is the main explanation for the correlation shown empirically.

JEL Classifications: C61; D91; I12; I21

Keywords: Demand for health; Demand for education; Human capital; Numerical modeling; Causality.

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1. Introduction

People with high levels of education generally have good health. There exists a social gradient in health as empirical studies show that years of schooling are positively correlated with good health (see, e.g., Huisman et al., 2005, for European countries, and Cutler et al., 2011, for the US). Examples from the research literature show that people with high education do more exercise than people with low education, that people with low education have more sick leaves than people with high education, or that the expected lifetime varies within different socio-economic groups, which are again linked to education. Studies also conclude that education is better correlated with health than with variables like occupation and income. Why do people with high education also have good health?

The positive correlation may be explained in three ways, see, e.g., Grossman (2000) and Cutler and Lleras-Muney (2007). First, higher education may cause better health; a high level of education could mean that people are more efficient in taking care of their health and that people with higher education are more likely to use new technologies. Both effects are due to more knowledge or information. A reinforcing mechanism is that high education usually gives a high income, and the individual is, therefore, in a position to spend more money on health care. Thus, there may be both productive and allocative effects of education on health, where productive efficiency refers to the fact that education leads to a larger health output from a given set of health inputs, while allocative efficiency suggests that a more educated person selects more efficient inputs to produce health. However, there may also be other effects at force. A high education usually means that one has a job with less health risk. It has also been proposed that education is important for social status, and that being low on the status range, is stressful and damaging for the health (see, e.g., Marmot, 2004). Also, if education makes the future look better, it may give an incentive to invest in health to increase the probability of enjoying it. In addition, people with high education may be given a higher priority in the health service, due to for instance health insurance paid by the employer. Finally a high education can give you access to larger social networks that may provide support and, therefore, have causal effects on health (Berkman, 1995).

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1 See our discussion of the evidence in Section 2 of this paper.
Second, better health may cause individuals to attain higher levels of education. Students with good health may be more willing to take a long education, or they may be more efficient in producing knowledge. As an illustration of the first point, Grossman (2003) suggests that good health causes schooling because a lower mortality increases the number of years over which the returns from investments in knowledge can be collected. The latter point suggests that good health increases the ability to learn, given equal effort. Good health also increases the possibility to work during studies, and therefore reduces the costs of studying, which makes the student less dependent on her parents or a governmental loan.

A third explanation is that there is another factor that influences both health and education in the same direction. People may possess certain characteristics, which give a high or low preference or possibility for investing in health and education, see, e.g., Fuchs (1982). Some examples of such variables could be time preferences, initial resources, traumatic experiences or socio-economic background. For instance, putting less weight on future outcomes (high time preference rate) would mean that you would do fewer investments if the costs come today and the benefits arrive in the future. Both health and education investments would fall into this category. Your socio-economic background may define the expectations of your peers when it comes to health behavior and educational attainment. Finally, individuals with higher innate cognitive abilities may be more efficient at producing higher levels of health and education.

Both health and education are top public policy concerns. If health equality is a political goal, the policy implications may vary depending on the implications described above, see Grossman (2000) and Cutler and Lleras-Muney (2007). If the causal effect of education on health is strong, there may be reasons to subsidize education and to give people a better opportunity to take higher education. Further, if the causal effect of health on education is strong, it may justify a policy emphasis on improving health – especially for children and teenagers. If background and socio-economic characteristics are important for both health and education, this may be another argument for general distributional policy. Some policy recommendations are found in the literature. In a series of papers, James Heckman argues for early child intervention (see, e.g., Heckman, 2007) instead of adult investments for later health outcomes. Auld and Sidhu (2005) concludes that subsidies to college education are unlikely to increase population health as
cognitive ability accounts for roughly one quarter of the association between schooling and health.

In this paper, we model an individual who chooses how much to invest in education and health, as well as how much to work, consume and save throughout adult life. This allows us to generate predictions on how individuals may react to different policies based on the different causal effects. In particular we study the effects of subsidies on medical care, subsidies on schooling, a general income tax reduction, a lump sum wealth transfer and improvements in health at young age.

From a normative view, a simple neoclassical model of consumer choice (such as the one we construct) will naturally predict that a lump-sum wealth transfer will welfare-dominate policy interventions targeted specifically at investments in education or health for private households. Nevertheless, a number of important market failures are relevant. For example, positive spillover effects from education may be large, so that the one who makes the investments does not take into account its full social benefits. Restrictions on behavior such as social expectations, culture, addiction, bounded rationality, uncertainty and risk aversion as well as lack of information may also lead individuals to make suboptimal choices and, as a result, justify corrective policy measures.

From a positive view, the strength of the causalities and the interactions between health and education choices in the consumer’s problem will be important for the effects of the policy instruments. For example, policies promoting higher education may have a complementary effect of raising an individual’s health with potentially important consequences for the cost of public health programs.

There are several empirical studies on the relationship between health and education (see Section 2 below) but theoretical contributions are few. The incentives for a consumer to invest in knowledge have been studied in human capital models (see, e.g., Becker, 1993; Ben-Porath 1967; Mincer, 1974). The pioneering model for the demand for health and health services (Grossman, 1972) also build on the human capital tradition, but considers education as an exogenous variable;
higher education makes the consumer more efficient in producing health (productive efficiency). Muurinen (1982) assumes that higher education reduces the depreciation of health capital (use-related depreciation) leading to allocative efficiency of education. Further, Becker and Mulligan (1997) endogenize the time preference rate by assuming that individuals can invest in goods or activities, such as schooling, to reduce this rate. In their model, health differences cause differences in time preferences because better health reduces mortality and raises future utility levels.

Numerical simulation models have become more important in studying individual health behavior and well-being the last few years. Some early studies were Gjerde et al. (2005), Carbone et al. (2005) and Murphy and Topel (2006). These papers have a Grossman-model structure, but do not include human capital accumulation. Carbone et al. (2005) does, however, include investments in both health capital and addiction capital. New numerical models studying choices over the life-cycle include Scholz and Seshadri (2010) and Halliday et al. (2010). They study the interplay between consumption choices and investments in health and the motives underlying health investments, but again they do not include investments in education.

While there are theoretic models for the determination of an optimal health stock and for an optimal human capital, we are not aware of models determining both stocks endogenously, and study theoretically the interplay between health and education.

The advantage of doing this is to see how policies aimed at increasing education affect health behavior as well as the other way around. In this paper we will, therefore, develop a simple model to study this. A simulation model is also presented that will give a much more detailed analyses of the interplay and the effects of policy instruments. We also demonstrate a new method for calibrating this model by matching key moments in data on health and education investment decisions.

The paper is organized in the following way. In section 2, we survey the empirical evidence on the relationship between education and health. To illustrate the effects of different policy instrument, we analyze a two-period, analytical model in Section 3. In Section 4, we describe a more detailed, numerical model which is calibrated to match empirical data for the US in Section

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2 See also the plea for development of comprehensive theoretical models in which the stock of health and knowledge are determined simultaneously in Grossman (2000, 2003).
5. Section 6 describes the policy experiments while the simulation results are given in Section 7. The final section concludes.

2. Empirical evidence

There is a large empirical literature trying to estimate causal effects between education and health, for surveys, see, e.g., Grossman and Kaestner (1997); Grossman (2000, 2008); Cutler and Lleras-Muney (2007); Cutler et al. (2011); Mazumder (2012).

To test the causal effect from health to education, there have been several studies on birth weight and the implications for education and labor market outcomes. Birth weight is an indicator of initial health, and when studying birth weight, data for twins are often used to correct for genes and socioeconomic factors. Almond et al. (2005) used American twin data and conclude that the short-term effects of low birth weight are rather small, while Behrman and Rosenzweig (2004) find significant long-term effects of low birth weight on, e.g., education and wages, also using American data. Based on Norwegian data, Black et al. (2007) find that those with low birth weight do significantly worse in the short term (mortality rate at age one) and long term when it comes to education and income. Another study on Norwegian data (Kristensen et al., 2004) confirm the results and find that low birth weight reduce the probability of being employed at age 29. High birth weight as a result of increasing obesity rates, can also be harmful for cognitive outcomes, see Cesur and Kelly (2010), indicating that the relationship between birth weight and such outcomes is nonlinear. Currie (2011) shows that low birth weight may partly be explained by prenatal exposure to pollution, and children born to less educated and minority mothers are more likely to be exposed. Other measures of health are also used. Sick children are more likely to miss school, to learn less while in school, obtain fewer years of learning and have a lower socioeconomic status as adults (Case et al., 2005; Madsen, 2012), and they are also more likely to become sick adults (Case et al., 2002). Based on data from the Whitehall II study, Case and Paxson (2011) find that childhood circumstances predict current health status. Poor mental health inn early childhood has also a large impact on years of schooling completed (Fletcher and Lehrer, 2009). However, even if there is a significant causal effect running from health to education, it is

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3 Birth weight can be an indicator of the development of the brain and the central nerve system before the baby is born.
not likely to be the entire explanation of the correlation between health and education, at least not in a developed country where all children attend school.

There is also evidence that at least part of the correlation between health and education outcomes stems from the causal link from education to health. One way of studying this effect is to utilize the natural experiments of extensions to compulsory schooling. While some early studies found quite large effect of changes in compulsory schooling laws (Lleras-Muney, 2005, 2006), recent studies using different methodologies find no effect (e.g., Clark and Royer, 2010; Meghir et al., 2012; Jürges et al., 2012), with the exemption of Fonesca and Zheng (2011) who use data for people aged 50 and over from thirteen OECD countries. However, by using data on identical twins, Lundborg (2013) finds that completing high school improves health measured as self-reported health, chronic conditions and exercise behavior, but that additional schooling does not lead to additional health gains. Similar results using twin data are found in Fujiwara and Kawachi (2009), but most of their results are statistically insignificant. Edwards (2010) also finds positive health returns to education, but they are monotonically diminishing in age, suggesting that the effects of education on later-life health is due to accumulation of, e.g., knowledge of healthy practices, human capital and health earlier in life, and not through income and wealth achieved later in life.

Several studies have tried to estimate the causal effect of income on health. Some panel data studies that do not support this causal effect are Adams et al. (2003), Contoyannis et al. (2004) and Smith (2007), but the effect is supported by several other studies. Frijters et al. (2005) use data for the reunification of Germany to control for other effects. The reunification resulted in a large income transfer to East Germany, while the health programs remained more or less unchanged. The study concluded that there was a significant positive, but small effect of the income transfer on health. Lindahl (2005) used lottery prizes as an exogenous shift in income, and find a significant causal link between income and health, while Apouey and Clark (2009) found positive effects on mental health, but negative effects on physical health due to more risky

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4 For a recent survey on this see Mazumder (2012).
5 There may still be income effects from education. Autor et al. (2008) find that labor returns to education were increasing in the 1980s and 1990s. Studies on compulsory schooling laws also support the notion that more education leads to higher earnings, even though the increase in schooling is compulsory (see, e.g., Clark and Royer, 2010; Meghir et al., 2012).
behavior in the short run. In general, it is assumed that there are decreasing returns from income on health for higher levels of income (Grossman, 1972). This is also confirmed by several studies such as Chapman and Hariharan (1996), which finds that the causal effect from income on mortality is larger for the poor than for the rich.

However, Cutler and Lleras-Muney (2007) show that after controlling for income and health insurance, education is still a significant determinant of health status in the US. Not only the length of education matters, but also the quality (see, e.g., Johnson, 2011). Further, the better educated may have less risky jobs, but Lahelma et al. (2004) and Cutler and Lleras-Muney (2007) find small effects of the labor environment. Education may provide social status and possibilities to be higher on the social hierarchy, and the health effects of social status have been documented in a series of papers, see, e.g., Marmot (2004). One reason for this may be that individuals with low social status live less self-determined lives, causing stress and stress-related diseases.

Goldman and Smith (2002) find that more highly-educated people are better able to manage disease, and there is also evidence supporting the idea that more education leads to lower rates of smoking (de Walque, 2007a). The effect of health behavior including smoking is actually found to contribute to the effect from education on health significantly (Brunello et al., 2012), meaning that education affects health behavior. Diffusion of health information through education also reduces smoking (Aizer and Stroud, 2010), which shows that knowledge is important and higher education is a way to get more knowledge and better access to information that impacts health (see, e.g., de Walque 2004, 2007b; Avitabile et al., 2008). Better educated people are also faster to exploit technological advances in medicine (Glied and Lleras-Muney, 2008). Knowledge also has a strong effect on self-assessed health (Karlsson, 2007). On the other hand, Altindag et al. (2010) find only weak evidence for an improvement in health knowledge for those who attend college, and conclude that the allocative hypothesis may not be the primary reason for why schooling impacts health outcomes. Studies on developing countries find that health affects schooling through its effect on morbidity or through cognitive development (Cutler et al., 2011).

The effect of a third factor has also to some extent been tested. Fuchs (1982) and Leigh (1990) find that only a small portion of the education gradient is explained by differences in time preferences. This is supported by Cutler and Lleras-Muney (2010) who test the effects of patience
and forward looking behavior and find that this explains very little of the education gradient in health. On the other hand, Chiteji (2010) concludes that non-cognitive skills such as the degree to which an individual is future oriented and self-efficacy, are associated with good health behavior, and van der Pol (2011) finds that the effects of education on health is reduced (but does not disappear) when controlling for individuals’ time preferences, indicating a positive effect from such preferences. While Auld and Sidhu (2005) and Cutler and Lleras-Muney (2010) emphasize the importance of cognitive ability on the gradient, Conti and Heckman (2010) conclude differently. They study the role of child development or early life endowment on education and health, and find that early cognitive factors have a larger impact on educational attainment than on later life health, while early endowments in non-cognitive skills and health affect both.

In summary, there appears to be empirical evidence supporting all three causal mechanisms between health and education, but a striking result is that studies trying to identify causal effects often find relatively weak impacts. However, there may be some indications that the strongest mechanism – at least in the developed world – is the causal effect from education to health (Cutler and Lleras-Muney, 2007). Cutler and Lleras-Muney (2010) find that the total returns from education may increase by 15 to 55 percent if they include their estimates of the health benefits from education.

3. A two-period model

To get an intuition on how different policy measures may affect an individual’s choices between consumption and investments in education and health, we set up a simple two-period model that captures the main features of the numerical model we present in the next section.

In this model, a representative consumer maximizes the present value of utility over the two periods, where the first period is the present and the second period represents the future, i.e.,

\begin{equation}
U = u(C_1, E_1, H_1) + S(H_1) \cdot u(C_2, E_2, H_2).
\end{equation}

Utility at time \( t, t = 1, 2 \), is increasing in consumption \( (C_t) \), education level \( (E_t) \) and health \( (H_t) \), and we assume that \( u_t > 0, u_{tt} < 0, i = C_t, E_t, H_t, t = 1, 2 \), where a subscript means the partial
derivative. The introduction of education in the utility function represents the non-material benefits of education inspired by Michael (1973).

For simplicity, we ignore labor-leisure choice in the two-period model. Without loss of generality, we also set the time preference rate equal to zero here. Note, however, that the utility in period two is discounted with a factor $0 < S < 1$ that represents the probability of surviving the first period. This probability is increasing in the health level in the first period, but the returns from better health are decreasing with a higher health level, i.e., $S_{H_1} > 0$, $S_{H_1,H_1} < 0$.

The health stock develops in the following way based on Grossman (1972),

$$
H_1 = h_0 + I(IH_1, TH_1, E_1) \\
H_2 = H_1(1 - \delta) + I(IH_2, TH_2, E_2)
$$

where the health stock increases in health investments ($I$), but falls as the stock depreciates with time at a fixed depreciation rate, $\delta$. $h_0$ is the initial health, i.e., the health given at birth. Investments in health are positively dependent on buying health services or medical care ($IH$), spending time on healthy activities ($TH$) and the education level, i.e., $I_j > 0$, $I_{ij} < 0$, $I_{jk} = 0$, $j = IH_1, TH_1, E_1$, $k = IH_1, TH_1, E_1$, $j \neq k$, $t = 1, 2$. Note that we assume the cross derivatives of inputs in the investment function to be zero.\(^6\)

Education increases by spending time on schooling ($TE$) and by monetary spending ($IE$), but in a similar way as for health investments, the returns from time and monetary spending is falling with the level of these input factors, i.e., $J_j > 0$, $J_{jj} < 0$, $J_{ij} = 0$, $l = TE_1, IE_1$, $v = TE_1, IE_1$, $l \neq v$, $t = 1, 2$.\(^7\) As for health investments, we assume the cross derivatives to be zero. Finally, the depreciation of the education stock is set to zero as well.

\(^6\) This is a simplification. Whether it is positive or negative depends on whether time and money are substitutes or complements in producing health.

\(^7\) A simplified version of this would be to assume that the monetary investments are linked to time such as a tuition fee per year of schooling. This means that time and monetary investments are used in a fixed proportion.
As seen, we have chosen to explicitly model time investments in education and health separately from money investments. There is empirical evidence to suggest that the levels of time investments in these goods are considerable for the average individual (U.S. Bureau of Labor Statistics (BLS), 2012b). More importantly, we consider a reduction in income tax rates as a policy-relevant counterfactual experiment in the analysis that follows. Time and money investments will respond differently under such a policy if the opportunity cost of time is the after-tax wage rate the individual faces. Time investments should fall as the opportunity cost rises, while money investments should rise as the value of an hour of work rises, resulting in more dollars to spend on all monetized activities.

Note that we ignore the impacts from health to education in equation (3). Having a causality effect from health to education complicates the analysis as education becomes a function of health, which is again a function of education, which is a function of health, etc. Including this causality would have made the relevant functions more elastic as the impacts of investments on the stocks and the utility function will be larger, but the qualitative conclusions from the policy analysis below will not change. This motivates a numerical model where we can see the impacts of this causality on the quantitative results.

Finally, the wealth of the consumer increases in income, where income is a function of time spent at work and the net-of-tax wage rate, $W$. We assume for the time being that the wage rate is constant and exogenous to the model, but relax this assumption in an extension of the basic analytical model as well as in the numerical model. The consumer spends her wealth on consumption goods (the numeraire good with a price normalized to unity), health services to a price $P$, and she also has to pay for education where $Q$ is the price per unit of the educational inputs. Based on this, we introduce an intertemporal budget constraint, where future money flows are discounted with the survival rate, and the interest rate is set to zero for simplicity. Initial wealth is set to $n_0$ and $\omega$ is time available at each time period. Time can be spent on work, health

\[
\begin{align*}
E_t &= J(TE_t, IE_t) \\
E_{t+1} &= E_t + J(TE_{t+1}, IE_{t+1})
\end{align*}
\]
investments and education. Thus, this simple set up ignores the effects of health on the available time budget.

\[(4) n_0 + (\alpha - TE_1 - TH_1) W - C_1 - P \cdot IH_1 - Q \cdot IE_1 + S(H_1) \cdot \left[ (\alpha - TE_2 - TH_2) W - C_2 - P \cdot IH_2 - Q \cdot IE_2 \right] = 0\]

The consumer chooses \(IH_t, IE_t, TH_t, TE_t\) and \(C_t\) to maximize (1) with respect to the constraints (2) - (4). This yield the following first order conditions (see Appendix 1 for details), where

\[A = (\alpha - TE_2 - TH_2) W - C_2 - P \cdot IH_2 - Q \cdot IE_2, i.e., \text{wealth addition in period 2:}\]

\[\left( u_{C_t} \cdot J_{TE} + u_{H_t} \cdot J_{TE} \right) + \left[ S \left( u_{E_t} \cdot J_{TE_t} + u_{H_t} \cdot \left( I_{E_t} \cdot J_{TE_t} (1 - \delta) + I_{E_t} \cdot J_{TE_t} \right) \right) + S_{H_t} \cdot I_{E_t} \cdot J_{TE_t} \cdot u \left( C_2, E_2, H_2 \right) \right] \]

\[u_{C_t} = W \]

\[\left( u_{E_t} \cdot J_{TE} + u_{H_t} \cdot I_{E_t} \cdot J_{TE_2} \right) = W \]

\[S_{H_t} \cdot I_{E_t} \cdot J_{IE_t} \cdot A = Q \]

\[u_{C_t} = Q \]

\[\left( u_{H_t} \cdot I_{IH_t} + S \cdot \left( u_{H_t} \cdot I_{IH_t} (1 - \delta) + S_{H_t} \cdot I_{IH_t} \cdot u \left( C_2, E_2, H_2 \right) \right) + S_{H_t} \cdot I_{IH_t} \cdot A = W \]

\[u_{C_t} = W \]

\[u_{H_t} \cdot I_{IH_t} = W \]

\[\left[ S \cdot (u_{H_t} \cdot I_{IH_t} (1 - \delta) + S_{H_t} \cdot I_{IH_t} \cdot u \left( C_2, E_2, H_2 \right) \right] + S_{H_t} \cdot I_{IH_t} \cdot A = P \]

\[u_{C_t} = P \]
The first terms on the left hand sides of equations (5)-(8) show the marginal benefits of increasing time and expenditures on education relative to the marginal benefits of increasing consumption in period one and two respectively, thus giving the substitution effect between education investments and consumption expenditures. However, increasing education in period one also has an income effect as it increases the expected wealth in the second period due to increased expected lifetime, represented by the second term on the left hand side of equations (5) and (7). In a similar way, equations (9)-(12) show the substitution effects between health investments and consumption and the income effect of increasing health investments. Finally, equation (13) shows that the marginal utility of consumption should be the same in both time periods. In addition to these equations, (14) that gives the intertemporal budget constraint, also indicates the income effects.

To understand the interplay of education and health we can use the equations above to get an intuition of the different effects of initial conditions and public policy. In particular we want to study the effects of:

a. Increase in initial wealth \((d n_0 > 0)\)
b. Increase in initial health \((d h_0 > 0)\)
c. Subsidies to medical care \((d P < 0)\)
d. Subsidies to schooling \((d Q < 0)\)
e. Lower income taxation \((d W > 0)\)

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8 We do not do the full formal analysis in this paper as the model is quite complex, but instead give the intuition that is useful to understand the numerical model in Section 4. However, the equations for a total differentiation of the first order conditions are available from the authors.
An increase in initial wealth \( (n_0) \) (or, equivalently, a lump-sum income transfer) has only an income effect, as it affects only the budget constraint, see (14). Thus, if health and education are normal goods, investments in both goods will increase.

How would people born with good health adapt when it comes to education and health compared to people less lucky at birth? To study this we consider an increase in initial health \( (h_0) \). In this model, this will have an impact on the budget via the increase in expected lifetime, thus giving a positive income effect on both health and education. However, the substitution effects depend on what we assume about cross derivatives. Assume that \( u_{EH} > 0 \) and \( u_{CH} > 0 \), i.e., that the pair education and health, and the pair consumption and health are both complements in utility. (This is the assumption used in the simulation model in Section 4 below as well.) From equation (5) and (6), we see that the substitution effect on education investments is ambiguous. The reason is that a higher health level not only increases the benefits from education, but also from consumption. However, if \( u_{EH} \leq 0 \) and \( u_{CH} > 0 \), the substitution effects go in the direction of lower investments in education. In either case, this implies that the effects on education of a higher initial health are ambiguous and depends on magnitudes of the substitution and income effects.

Further, from (9)-(12) we see that the substitution effects go in direction of lower health investments. This is due to the fall in the marginal benefits of these investments when the health stock is higher. Once again, the total effect on health investments is ambiguous for a higher initial health stock.

Let us now turn to policy measures and start with the effects of subsidizing medical care (health services). We see from (11) and (12) that a fall in the price of medical care, \( P \), will increase health investments relative to consumption. From (12), the substitution effect goes in the direction of higher health investments in the second period. For period one, a lower \( P \) means that the left hand side of equation (11) has to decrease, and health investments will, therefore, go up. A fall in \( P \) also gives a positive wealth effect. Thus the income and substitution effects go in the same directions meaning that subsidizing health care will increase consumption of medical care. The effect of time spent on healthy activities is ambiguous. On one hand we get a substitution
towards medical care as the relative prices change, but we also get an income effect that will work in the other direction.

Effects on education from subsidizing medical care will depend on cross derivatives. As above, if $u_{EH} > 0$ and $u_{CH} > 0$, we see from equations (5) and (6) that the effects on education investments are ambiguous. The reason is again that the benefits of both education and consumption will increase for better health with these assumptions.

*Subsidizing schooling*, a lower $Q$, gives a substitution effect where consumption is reduced relative to educational expenditures, see (7) and (8). In addition to this, we see from (7) that this also gives a positive income effect as health in period 2 will increase and, therefore, expected income in period 2, in addition to a direct budget effect via reduced price on expenditures, see (14). Thus, both the substitution and income effects go in the direction of more investments in education expenditures. The effect of time spent on education is ambiguous as the substitution effect and income effect work in different directions.

Subsidizing schooling also has an effect on health investments, but again this depends on the cross derivatives. If $u_{HE} > 0$ and $u_{CE} > 0$, the effects on health investments are ambiguous, see (9)-(12).

There is also another cost component of schooling, namely the alternative use of time spent on education, measured by the wage rate, $W$. Thus, *lowering income taxation* (increasing $W$) will increase the price on time spent on schooling, and will discourage education relative to consumption, see (5) and (6). Note however that there is also an income effect of lowering income taxation (increasing wages) that goes in the direction of spending more money on education investments, see (14). So the total effect on time spent on education is ambiguous. However, for education expenditures we only have the income effect, so they will increase.

Further, increasing $W$ has a similar effect on time spent on healthy activities as on time spent on education, see (9) and (10), i.e., the income and substitution effects go in different directions,
giving an ambiguous result. For medical care we have a positive income effect that gives a higher consumption.

So far we have assumed wages to be exogenous. However, wages normally increase in education. Let \( W_t = B(E_t) + \tau, t = 1,2 \), where \( \tau \) is an income subsidy (negative \( \tau \) is an income tax), and \( B_E > 0 \) and \( B_{EE} < 0 \). With this endogenous wage formulation, the benefits of education will increase compared to the model above. We get a positive income effect from increased education, but the alternative cost of spending time on education and health investments will also increase. Most conclusions would not change qualitatively, see Appendix 1, but an initial wealth increase will not only give a positive income effect. It also gives an incentive to lower investments in education as the individual does not have to invest in a higher wage rate to be able to buy the same quantity of goods as before.

Table 1 summarizes the results from the analysis of the main model (exogenous wage rate):

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<thead>
<tr>
<th>Endogenous variables</th>
<th>Exogenous variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_t )</td>
<td>( d_n &gt; 0 )</td>
</tr>
<tr>
<td>( TE_t )</td>
<td>( d_h &gt; 0 )</td>
</tr>
<tr>
<td>( IE_t )</td>
<td>( d_P &lt; 0 )</td>
</tr>
<tr>
<td>( TH_t )</td>
<td>( d_Q &lt; 0 )</td>
</tr>
<tr>
<td>( IH_t )</td>
<td>( dW &gt; 0 )</td>
</tr>
</tbody>
</table>

Table 1: Behavioral implications from changes in exogenous variables.

### 4. The numerical model

In the numerical model, the consumer maximizes expected lifetime utility subject to balancing her intertemporal budget and equations that describe the probability of survival over time \( S_t \) and the dynamics of the health \( H_t \) and education \( E_t \) stocks in the model. The choice variables in the model are consumption \( C_t \), leisure \( L_t \), monetary investments in health \( IH_t \) and education \( IE_t \) and time investments in health \( TH_t \) and education \( TE_t \) at each point in time \( t \).
(or \(tt\)). In the numerical implementation, we solve the model over a 110-year time horizon using a five-year time step, starting at age 20.

\[
\max_{c_t, l_t, h_t, e_t, \theta_t} U = \sum_{t=0}^{T} \left( (1 + \rho)^{tT} \left[ \tau_c (E_t / \bar{c}_t)^{\kappa} + \tau_h h_t^{1/\kappa} + (1 - \tau_c - \tau_h)(Z_t^c - Z_0^c) / \xi \right] \right)^{\nu/\kappa} 
\]

subject to the sub-utility function,

\[
Z_t = ((1 - sl)(C_t / c_0)^{\gamma} + sl(L_t / l_0)^{\gamma})^{1/\gamma}
\]

the survival probability,

\[
S_t = 1 - e^{-\phi t \beta}
\]

the wealth constraint,

\[
n_0 + \sum \left\{ [\omega - L_t - pte_{t_0}(TE_t / te_{t_0} - 1) - ph_{t_0}(TH_t / th_{t_0} - 1) + (H_t / h_{t_0})^{\beta} - 1]W_t + \right. \\
\left. b_t - C_t - ph_{t_0}(IH_t / ih_{t_0} - 1) - pce_{t_0}(IE_t / ie_{t_0} - 1) \right\} S_t / (1 + r)^{-t} = 0
\]

the health stock transition,

\[
H_t = h_{t_0} + \sum_{i,j} h_{t_0} \left( (E_{t_0} / \bar{c}_t)^{\mu} ((IH_{t_0} / ih_{t_0})^{\nu} (TH_{t_0} / th_{t_0})^{(1 - \gamma)/\nu}) \right)^{(1 + \tau_t / T)^{\delta}} (1 - \delta_h)^{\tau_t}
\]

and the wage/education transition:

\[
E_t = e_{t_0} + \sum_{i,j} e_{t_0} \left( (H_{t_0} / \bar{h}_t)^{\mu} ((IE_{t_0} / ie_{t_0})^{\xi} (TE_{t_0} / te_{t_0})^{(1 - \eta)/\xi}) \right) - 1) (1 + \tau_t / T)^{\phi} (1 - \delta_e)^{\tau_t}.
\]

Finally, we assume that the wage evolves over time solely as a function of the education stock, thus: \(W_t = E_t\).
In equation (15), lifetime utility is a CES function that depends on the level of the education stock, the level of the health stock and the level of the full consumption good \((Z_t)\) above a subsistence level \((z_0)\). Period utility at time \(t\) is discounted at the pure rate of time preference \((\rho)\) as well as probability of survival to time \(t\). The form the period utility function implies that the cross derivatives on \(E_t, H_t\) and \(Z_t\) are all assumed to be positive, as discussed in the two-period model in Section 3. Full consumption in each time period \((Z_t)\) is produced by combining leisure and consumption goods as described in equation (16), and the probability of survival to year \(t, S_t\), is functionally related to the individual’s level of health stock in (17).

The individual must maintain an intertemporally-balanced budget over her lifetime see (18), where income comes from existing initial assets at the beginning of life \((n_0)\), wage income, and any transfers to households \((b_t)\). Wage income is expressed as the amount of the individual’s benchmark total time endowment \((\varnothing)\) that is not devoted to leisure. In counterfactual experiments, the individual’s effective time endowment will also depend on how levels of time investments in health and education vary. It will also depend on how the individual’s health status varies, affecting the number of sick days required. In equation (18), the three terms corresponding to these effects appear within the square brackets on the left-hand side of the equation.\(^9\) Time is valued at the individual’s wage rate \((W_t)\).

Equation (19) describes the transition of the health stock over the life cycle. Health depends on the levels of past investments in time and money dedicated to health production as well as the individual’s education stock. Finally, equation (20) describes how wages and the education stock evolve over time. The variables depend on the levels of past investments in time and money dedicated to education, but also on the health stock. Both the health and education stock equations contain time-trend terms \((1+tt/T)^\varnothing\) and \((1+tt/T)^\rho\) that allow the productivity of

\(^9\) In each case, the relevant endogenous variable responsible for producing the effect \((TW_t, TH_t, H_t)\) is divided by the levels these variables take on in the benchmark equilibrium in the calibrated model \((tw_0, th_0, h_0)\). Thus, these terms take on a value of unity in the benchmark equilibrium. When unity is subtracted from these terms, as it is in these expressions, they make no contribution to the individual’s budget.
investments in these stocks to vary with age. For example, the decline in health or human capital may be more difficult to abate as one becomes older.

A full listing of the model variables and parameters used in the numerical model is included in Appendix 2.

There are a few important differences in the structure of the numerical model and the analytical model presented in the previous section of the paper. There is a leisure activity in the numerical model that competes for use of the individual’s time endowment with labor supply and time investments on health and education in the model. In addition sick time is introduced and reduces the total time endowment. The level of the education stock also depends on the level of the health stock in the numerical model so there is potential for feedback effects moving from health to education as well as from education to health. These features of the numerical model may be important to the quantitative significance of the results of our simulation experiments but should not influence the qualitative predictions relative to those produced by the analytical model.

The model is summarized in Figure 1 below.

Figure 1: A flow diagram of the model.
The linkages from education to health and from health to education as described above are illustrated in the figure by means of solid arrows. In addition, there are also indirect effects from education and health to leisure and consumption goods, as well as from background characteristics to health, education and consumption as are shown in Figure 1 as dashed arrows:

- Individual specific background variables such as the time preference rate and initial wealth give indirect effects on health, education and total consumption.
- Good health reduces the time being sick, which means that there is more time available for schooling, as well as for working, leisure activities and healthy activities.
- More education increases the wage level and therefore affects consumption.

5. Calibration Procedure

The calibration procedure builds off of the one developed in Murphy and Topel (2006). That study did not attempt to model endogenous investment in health or education as we do here. Rather, they calibrated a life cycle consumption model to exogenous trajectories of $H$ and $S$. In the Murphy-Topel procedure, $S$ is chosen to reproduce data on mortality rates. $H$, which is not observed directly, is calibrated to fit consumption and earnings data for an average individual in the United States given the structural assumptions in the model. The other key parameters in the model are calibrated to imply a specific value for the consumer's willingness to pay for marginal reductions in the probability of death (their value of a statistical life or VSL). We calibrate the model based on data covering residents of the United States from the study of Murphy and Topel (2006), from the U.S. Bureau of Labor Statistics (BLS) (2012a,b;2013) and U.S. life tables (Arias, 2012).

We follow this procedure to calibrate the model to exogenous trajectories for $S$ and $H$ and then go on to describe a new method for calibrating the features of the model related to the endogenous health and education stocks. Specifically, using the calibrated version of the model with exogenous levels of these stocks (based on the Murphy and Topel calibration described above), we calculate the shadow prices associated with a marginal increase in the levels of the health and education investment goods ($IH$, $IE$, $TH$ and $TE$). These are, by definition, the
effective prices of the investment goods (\( ph_{0t} \), \( pe_{0t} \), \( pth_{0t} \), and \( pte_{0t} \) respectively) required to replicate the benchmark trajectories in the model. We normalize the benchmark levels of the investment goods in the model to unity and calibrate key parameters influencing the effectiveness of these investments with the objective of producing the best fit between the predictions on expenditures on these goods generated by the model and data on monetary and time expenditures on these goods as well as consumption expenditures. The details of the calibration procedure are described in Appendix 3, while the calibrated parameter values are shown in Table 2.

5.1 Calibration Results

Figure 2 depicts the benchmark trajectories for consumption \((C)\), the individual’s health stock \((H)\) and full income, i.e., total time endowment valued at the wage rate \((W)\), over the life cycle. The consumption path is chosen to match BLS data on expenditures over the life cycle – rising in early life, peaking and then falling through later years.\(^{10}\) The individual’s health stock remains roughly constant through early and mid-life and then declines as the individual approaches old age. Full income – including both monetary income sources as well as the value of leisure time – rises until retirement (assumed to be age 65) – after which point it is assumed that retirement benefits replace half of projected wages based on the calibrated wage profile.

Figure 3 depicts the calibrated trajectories for expenditures on the various health and education investment goods in the model – monetary expenditures on health (ph0), time expenditures on health (pth0), monetary expenditures on education (pe0) and time expenditures on education (pte0). Investments in health peak around age 60 and then remain roughly constant until the end of life. Investments in education are weighted toward the beginning of life and then decline after age 35. Monetary and time investments on health and education are calibrated to data. Empirically, the average individual spends far more on health investments than on education and education investments are weighted toward the beginning of life. The calibration of the model aims to capture both of these features of the data.

![Figure 3: Benchmark Expenditures on Health and Education Investments](image)

Our calibration of the education investments is based only on data for expenditures of time and money on education from ages 20-25 because measurable expenditures of formal education after this age are quite small for a representative individual in the data. Similarly, the model agent is restricted to choosing the levels of these investments from ages 20-25 in the counterfactual experiments. The reason for this is that, conceptually, the measures of investments in education in our model should be interpreted as all investments that enhance human capital – including formal and informal education as well as job training and skills acquisition. While it is reasonable to assume that the majority of these investments take the form of formal education (for which we
have data to calibrate the model to) early in life, this is not a reasonable assumption later in life. This is the rationale for restricting the calibration to years 20-25. It also explains the significant investments in education after this age predicted by the model – these are informal sources of education.

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ slope term in survivorship function</td>
<td>3.6649</td>
</tr>
<tr>
<td>$\theta$ exponent term in survivorship function</td>
<td>1.2104</td>
</tr>
<tr>
<td>$\beta$ elasticity of health in sick days</td>
<td>0.0131</td>
</tr>
<tr>
<td>$\gamma$ elasticity parameter for health-related inputs to production of health investments</td>
<td>0.9904</td>
</tr>
<tr>
<td>$\eta$ elasticity parameter for education-related inputs in production of education investments</td>
<td>0.8006</td>
</tr>
<tr>
<td>$\tau_c$ share parameter of direct effects of education on utility</td>
<td>0.0110</td>
</tr>
<tr>
<td>$\tau_h$ share parameter of direct effects of health on utility</td>
<td>0.0136</td>
</tr>
<tr>
<td>$\alpha$ elasticity parameter for education stock in health production</td>
<td>0.0076</td>
</tr>
<tr>
<td>$\nu$ elasticity of new health investments on $H_t$</td>
<td>0.0345</td>
</tr>
<tr>
<td>$\mu$ elasticity parameter for health stock effect in education production</td>
<td>0.0208</td>
</tr>
<tr>
<td>$\xi$ elasticity of education investments on $E_t$</td>
<td>0.0252</td>
</tr>
<tr>
<td>$\sigma$ elasticity of time trend in productivity of health investments</td>
<td>-1.8416</td>
</tr>
<tr>
<td>$\psi$ elasticity of time trend in productivity of education investments</td>
<td>-1.5037</td>
</tr>
<tr>
<td>$\rho_{hz}$ elasticity of substitution between health, education and full consumption in utility</td>
<td>-1.3434</td>
</tr>
<tr>
<td>$c_0$ benchmark consumption at year 50</td>
<td>6</td>
</tr>
<tr>
<td>$z_0$ subsistence level of full consumption</td>
<td>0.1</td>
</tr>
<tr>
<td>$r$ market interest rate</td>
<td>0.04</td>
</tr>
<tr>
<td>$\rho$ pure time rate of preference</td>
<td>0.02</td>
</tr>
<tr>
<td>$\sigma$ intertemporal elasticity of substitution</td>
<td>0.8</td>
</tr>
<tr>
<td>$\rho_l$ Value share of leisure in full consumption</td>
<td>0.5</td>
</tr>
<tr>
<td>$\omega$ Time endowment</td>
<td>1</td>
</tr>
<tr>
<td>$\delta_h$ depreciation rate of new investments in $H_t$</td>
<td>0.05</td>
</tr>
<tr>
<td>$\delta_e$ depreciation rate of new investments in $E_t$</td>
<td>0.01</td>
</tr>
<tr>
<td>$\zeta$ $(\sigma - 1) / \sigma$ where $\sigma_z$ is the elasticity of substitution between leisure and consumption in full-consumption bundle</td>
<td>-0.25</td>
</tr>
<tr>
<td>$\kappa$ $(\sigma_z - 1) / \sigma_z$</td>
<td>-1</td>
</tr>
</tbody>
</table>

Table 2: Calibrated Parameter Values

Among the parameter estimates, a few results deserve further discussion. The parameters describing the contribution of the different determinants of the education and health stocks show
that the interaction terms – how education level affects the health stock and how health level affects the education stock – are similar in magnitude to the parameters describing the direct effects of investments in these stocks. For example, the elasticity of direct inputs to health investments \((\nu)\) is calibrated to 0.035 while the elasticity of the education level on the health stock \((\alpha)\) is 0.008. The elasticity of direct inputs to education \((\xi)\) is calibrated to 0.025 while the interaction of the health level in determining the education stock \((\mu)\) is 0.021. Thus a one percent increase in the health stock appears nearly as important as a one percent increase in direct investments in education. Nevertheless, it is important to note that the interaction effects are denominated in terms of changes in the stock level – not the change in the level of investments in the interacting stock. A one percent increase in investment in education will lead to an increase in the level of the education stock that is (considerably) less than one percent. As a result, the implied effect of a one percent increase in investments in education on health through the interaction is also considerably less than the effect of one percent increase in the education stock on health. Thus direct health investments are significantly more productive in producing higher health than are investments in education in our calibration despite the similar magnitude of the parameter values. The effect of education on health is not as strong in our calibration. Both health and education make similarly large contributions to utility directly. The productivity of health and education investments falls as the individual ages at a similar rate. The calibration implies an elasticity of substitution between the health, education and consumption arguments in the utility function of approximately 0.4. Thus, the three goods are significantly stronger complements than would be implied by a Cobb-Douglas preference function for example.

6. Policy Experiments

Our main interest is in understanding of how the different policy interventions related to health, education and general well-being affect individual choices regarding investments in health, education and overall welfare. In line with this, we have designed a number of policy scenarios similar to those used in the two-period model that we can analyze by solving for the optimal behavior using our calibrated numerical model.
6.1 Policy Scenarios\textsuperscript{11}

*Initial Wealth Transfer* - we increase the present-value of household wealth by 1% of the benchmark level by increasing the value of $n_0$ in equation (18).

*Medical Care Subsidy* - we introduce a proportionate reduction in the price of medical care at all time periods ($ph_0$, in equation (18))\textsuperscript{12}.

*Education (Tuition) Subsidy* - we introduce a proportionate reduction in the price of education at all time periods ($pe_0$, in equation (18))\textsuperscript{13}.

*Income Tax Reduction* - we introduce a proportionate increase in the after-tax wage at all time periods ($W_t(1 + ws)$ in equation (18) where $ws$ is a constant, positive subsidy rate or, equivalently, the negative of the income tax rate reduction).

The initial-wealth transfer serves as a benchmark against which to judge the results of the alternative policy interventions. The reduction in the prices and taxes are calculated to ensure that the lifetime present value of the subsidy payments and the tax reduction to the household at benchmark demands for medical care and education and the benchmark labor participation rates respectively are equal to the value of the initial-wealth-transfer policy.

However, we also study one policy measure which is not directly comparable to the other policy scenarios in monetary terms:

*Initial Health Increase* - we introduce a 1% increase in the size of the initial health stock from benchmark levels through a change to $h_{0t}$ in equation (19). Note that this means an increase in health at age 20.

\textsuperscript{11} Note that we do not study how the transfers are financed, i.e., we assume that the individual is unaffected by the public authorities budget constraints.

\textsuperscript{12} Similar to $P$ in the two-period model.

\textsuperscript{13} Similar to $Q$ in the two-period model.
In our model, the individual makes decisions with perfect foresight and information. As a result, she must obtain the highest level of lifetime utility from this policy, as she has complete flexibility in how to allocate these resources to the different activities described in the model and she chooses this mix of activities optimally. The three subsidy policies – which are scaled to deliver the same level of monetary transfer to the individual in present-value terms – all feature restrictions on how the individual may use the funds.

7. Simulation Results

7.1 Effects on investments
We now discuss the changes in the levels of investment in health and education under the different policies, described in Figures 4-8. The changes reported in the figures are percentages of the benchmark levels of the model variables in the initial calibration of the model. Each figure reports changes in time investments in health ($TH$) and education ($TE$) as well as monetary investments in these two stocks ($IH$ and $IE$, respectively). While the model is solved over a 110-year time horizon, the horizon reported in the figure is restricted to models years 20-80 as expected life length to which the model is calibrated is 78 years. Note, once again, that the choices of $IE$ and $TE$ are restricted to model years 20-25, thus percentage changes in these levels are only depicted for year 20 in the figures.
Figure 4 describes the results of the *initial-wealth transfer policy*. Time investments in health rise under the wealth transfer program. When wealth rises under the transfer, higher health and life extension is required to enjoy the higher level of consumption now possible. Thus time and money investments rise by comparable amounts under this policy. However, investments in education fall. The initial wealth transfer relieves some of the motivation to invest in education – to increase consumption through higher wages – because higher levels of consumption are possible at the same wages after the wealth transfer. Moreover, the increase in the health stock resulting from higher investments in health tends to increase equilibrium wages and education due to the feedback mechanism from health to education in the model, further reducing the incentive to invest directly in education. Thus, while there is a positive income effect in the model that works to increase education levels, these substitution effects appear to more than offset it at our calibration.

As discussed in the analysis of the analytical model, the wealth transfer represents a pure income effect. As a result, demand for health and education should both rise if both are normal goods. We find that *investments* in health rise while *investments* in education fall. The main reason is that the wage rate was exogenous in analytical model, but it is endogenous in the simulation model. While investments in education fall, the level of the education stock rises relative to benchmark levels due to the fact that the higher health stock feeds back to increase the productivity of investments in education.
Figure 5 shows the results of the medical care subsidy. Naturally, the subsidy stimulates demand for medical care, thus monetary investments in health rise under this policy. Time investments in health fall due to the substitution effect between money and time investments in health. Investments in education rise slightly as the overall cost of living falls with the medical care subsidy, increasing the returns to working and investing in higher wages. While the analytical model demonstrates that all investments goods – with the exception of medical care – exhibit negative substitution effects and positive income effects. The simulation results suggest that the income effects dominate for the education investments, but not for time investments in health.

The results of the education (or tuition) subsidy are shown in Figure 6. The subsidy stimulates the demand for money investments in education. Other investments – time and money in health as well as time in education – are little changed from benchmark levels under this policy. While there are negative substitution effects and positive income effects for all investments goods except for money investments in education, the simulations suggest that these effects roughly cancel out for all three other investment goods.

The income tax reduction increases monetary investments in both health and education, see Figure 7. Time investments in health rise modestly while time investments in education fall modestly, both changes found to be ambiguous in the analytical model. The increase in the wage
makes monetary investments more affordable and time investments less affordable because the opportunity cost of time has risen. Greater income that comes with higher wages means that better health and longer life expectancy is required to take advantage of the higher level of consumption possible. This may explain why slightly more new investment is directed at health than at education. Finally, the analytical model predicts that money investments in both health and education should rise – with both positive substitution and income effects – under the increase in wage while time investments may rise or fall. The simulations are consistent with these predictions – with money investments rising substantially while time investments change very little.
Figure 8 describes the results of the initial-health shock policy. Investments in both education and health fall under this scenario. Nevertheless, both the education and health stocks (not depicted in the figure) rise over the individual’s lifetime. The reason the health stock rises is straightforward. If direct investments in education fall, then the fact that the education stock rises must be due to the indirect effect that the higher health stock has on education. Consumption rises as well. Thus, the individual uses the natural advantage accorded to it by the larger endowment of health to draw down investments and consume more. While we found ambiguous predictions on the sign of the changes in the investments levels for both health and education in the analytical model, the simulation results suggest that the negative substitution effects dominate at our calibration.

7.2 Utility and life extension

Figures 9 and 10 report the impacts of the policy scenarios on lifetime utility levels and life length respectively. Note that the initial health increase is not included in these figures as this policy is not comparable in magnitude to the other policy measures. Once again, quantities are reported as percentage changes from benchmark levels in the calibrated model. Naturally, all policies – which imply transfers to the individual – result in welfare gains. However, the gains vary substantially by policy. The effects on utility are to a large extent a result of higher consumption, and a wealth transfer and an income tax reduction have larger effects on consumption than subsidies on education and medical care. By definition, the initial wealth transfer leads to the largest increase in welfare. The income tax reduction yields slightly lower – though nearly identical – benefits to the individual. In contrast, both the medical care subsidy and the education subsidy are dramatically less effective in welfare terms. The relatively ranking of the policies is consistent with intuition. As discussed, the wealth transfer allows full flexibility to the individual in how to use these additional resources. All of the other policies considered imply some restriction on use or, equivalently, distort the relative prices faced by the individual which leads to a higher cost of producing private well-being. Moreover, while the income tax reduction distorts only the choice between leisure and consumption, both the medical care and education subsidies additionally distort the choice between these goods and other forms of consumption. It follows that these policies would be expected to perform less well than the income tax reduction.
It is worth re-emphasizing at this point that we ignore any external, social benefits associated with investment in health or education. In practice, many of these benefits are thought to be quite large. Thus subsidies to these activities may be justified on those grounds. Nevertheless, it is instructive to see the magnitude of the differences between the different policies as a measure of their relative costs.

![Figure 9: % Change in Utility by Policy](image1)

Life is extended under all policies. Note that the effect is dependent on the health path. As health is little affected least under the education subsidy (see discussion on Figure 6) the expected

![Figure 10: % Change in Life Length by Policy](image2)
lifetime increases very little as well. Not surprisingly, the largest effect is from subsidizing health directly, i.e., by introducing a medical care subsidy. As for lifetime utility, the effect of an increase in initial wealth is not very different from that of an income tax reduction. Both of these policies yield noticeably less life extension than the medical care subsidy, however. Thus, our simulation results show that the policies that effectively increase welfare may differ from policies that have significant life extension effects.

Under the initial wealth transfer, a one percent increase in lifetime wealth leads to approximately a 0.03% increase in the expected length of life. At a benchmark life length of 78 years, this increase amounts to a life that is longer by approximately 9 days. Extrapolated linearly, this would mean that a 50% increase in wealth would increase expected life length by approximately one and a quarter years.\textsuperscript{14} Taken another way, a one percent of present-value lifetime wealth for the calibrated income level in our model translates approximately into an annual payment of 200 present-value dollars in every year of life. Extrapolated linearly, an annual present-value wealth transfer of approximate $7200 would be required to extend life by a year in our model.\textsuperscript{15}

7.3. Sensitivity Analysis

To explore the degree to which the interactions between health and education investments shape the results of our policy experiments, we conducted sensitivity analysis with respect the key parameters in the model that govern these interactions. Recall that the parameter $\alpha$ controls how the level of the education stock influences the productivity of investments in health. In our calibration of the benchmark model, this parameter takes on a value of approximately 0.008. Similarly, the parameter $\mu$ controls how the level of the health stock influences the productivity of investments in education in the model. In the benchmark calibration, it takes on a value of approximately 0.02. We conduct simulations in which we set one or both of these parameters equal to zero and then run the same set of policy experiments as described in section 6.1. When,

\textsuperscript{14} By comparison, Statistics Canada reports that moving from the third after-tax income quintile (approximately $40,000 in 2009) to the top income quintile (approximately $80,000 in 2009) corresponds to an increase in life expectancy from 83.3 years to 84 years amongst females and from 78.7 years to 80.3 years amongst males. See http://www.statcan.gc.ca/pub/75-202-x/2009000/analysis-analyses-eng.htm for the report of income quintiles and http://www.statcan.gc.ca/pub/82-624-x/2011001/article/chart/11427-06-chart5-eng.htm for the report on life expectancy by income quintile.

\textsuperscript{15} To compare this to data for the U.S., Tengs et al. (1995) found that prices of life-saving interventions varied a lot when comparing more than 500 interventions, but the median was about $42,000 (1993-dollars) per life-year saved.
for example, $\alpha$ is set equal to zero, the marginal effect of an increase in the education stock on the productivity of investments in health in the model is also zero. Thus, the incentive for the individual to invest in education to get the co-benefits in health that were present in our core model no longer exists. Similarly, when $\mu$ is set to zero, there no longer exists an incentive to invest in health to get co-benefits in education.

We find that the effect of changing these assumptions on behavior is small. This is consistent of the magnitude calibrated parameter values discussion in section 5.1. Removing the causal link from education to health has an almost imperceptible effect on the optimal pattern of investments the individual chooses. Removing the causal link from health the education has a small but perceptible effect. Intuitively, the individual invests somewhat less in their health when these investments no longer have a beneficial effect on their education stock. The change in the welfare and longevity gains due to the policy interventions we consider are also minimal in these sensitivity runs.

This suggests that neither of the interaction terms – from education to health or from health to education – is a likely candidate to explain much of the correlation between levels of education and health observed in the population. It also leaves other underlying factors that impact both outcomes as the remaining explanation for the pattern provided one accepts the calibration of our model. As we discussed at the outset, there is empirical evidence in the existing literature to support this view; there is evidence that cognitive ability (Auld and Sidhu, 2005; Cutler and Lleras-Muney, 2010), non-cognitive skills (Chiteji, 2010; van der Pol, 2011) and early childhood development factors (Conti and Heckman, 2010) may all play important roles in shaping the social gradient.

Our result that neither of the interaction terms appears to significantly affect the gains from investments in health and education is at odds with evidence that higher levels of education causes large gains in health (Cutler and Lleras-Muney, 2007). One possible explanation for this disparity is the absence of some behavioral effects in our model. That is, our model is deterministic and the agent exhibits perfect foresight in making consumption, saving and investment decisions. If households fail to fully optimize as we suppose in our model – either
because of bounded rationality, imperfect information or because of uncertainty and risk aversion – then our model may overestimate the degree to which individuals can appropriate the gains from education-health interactions in the absence of government intervention. In that case, exogenous shocks that cause individual to obtain higher levels of education (such as the ones that lead to the identification strategies in the empirical literature) may lead to much larger gains than would be predicted by our model. Moreover, our calibration strategy relies on matching empirical observations on health and education investments to the predicted behavior of our model agent. Observing low levels of investments in these goods leads the model to attribute small utility gains from further investment in these activities (as opposed to attributing importance to any of the behavior effects discussed), leading to small parameter estimates and the modest effects that we find in our sensitivity analysis. Thus, building these types of behavioral complications into a model like ours seems like a natural direction for future research.

8. Conclusions

In this study, we have produced analytical and numerical models of lifecycle investments in health and education. Both health and education have the potential to affect individual well-being through a number of distinct channels as well as to produce feedback effects between the two outcomes that past research on the nexus of health and education suggests are likely to be important.

Because of the close connections and feedbacks between health and education outcomes and the important policy implications of understanding the causal mechanism at work, researchers have devoted considerable effort to disentangling these effects. We contribute to this literature by proposing a new structural model of lifecycle health and education choices and analyzing which interpretation of the data a calibrated version of the model best supports.

Our analytical model identifies the key substitution and income effects that drive the changes in equilibrium investments expected in response to exogenous changes in wealth, health and related prices in the model. The perhaps unsurprisingly conclusion given the number of different ways health and education interact with well-being in the model, is that many of the net effects of the policy interventions we consider are ambiguous – with offsetting substitution and income effects.
We then calibrate an expanded, numerical model using data on wages, consumption, life expectancy and expenditure levels on education and health. The numerical model allows us to quantify the effects of the policy responses and to determine which channels are likely to be most important in driving behavior.

In the policy scenarios we examine, we find that health and education investments are sometimes substitutes and at other times complements. A lump-sum wealth transfer and an income tax reduction are the most welfare-enhancing policies we consider. In a forward-looking model with perfect information, the lump-sum wealth transfer is destined to top the welfare rankings of different policy interventions, meaning that distributional policies could be an effective policy measure if we care about welfare. If we instead care for life extension, policies that are directed against health, such as medical care subsidy, are the most efficient. Even not directly comparable to the other policy measures studied in the model, increasing health when young is an effective measure for both improving health and increasing the education level, and lowers the need for time investments in health for large parts of the life. It should be noted, however, that our findings suggest only a private ranking of the policies. That is, they ignore any external benefits from subsidizing education or health. They also ignore any possible failures of the individual to optimize in the face of uncertainty, limits to information or rationality. However, based on sensitivity analyses we conclude that underlying factors that impact both health and education may be the main explanation for the correlation between these two variables that is shown in the empirical literature.
Appendix 1: Solving the two-period model

Solving the model

The Lagrangian is as follows:

\[
L = u(C_1, J(TE_1, IE_1), h_0) + I(TH_1, J(TE_1, IE_1)) \\
+ S(h_0 + I(TH_1, J(TE_1, IE_1))) \\
+ u(C_2, J(TE_2, IE_2)) + J(TE_2, IE_2) \left[ h_0 + I(TH_1, J(TE_2, IE_2)) \right] \left( 1 - \delta \right) + I(TH_2, J(TE_2, IE_2)) + J(TE_2, IE_2)) \\
+ \lambda \left[ n_0 + (\omega - TE_1 - TH_1)W - C_1 - P \cdot IH_1 - Q \cdot IE_1 \\
+ S(h_0 + I(TH_1, J(TE_1, IE_1))) \cdot \left( (\omega - TE_2 - TH_2)W - C_2 - P \cdot IH_2 - Q \cdot IE_2 \right) \right].
\]

Maximizing \( L \) with respect to \( TE_1, TE_2, IE_1, IE_2, IH_1, IH_2, TH_1, TH_2, C_1 \) and \( C_2 \) gives the following 1. order conditions, where \( A = (\omega - TE_2 - TH_2)W - C_2 - P \cdot IH_2 - Q \cdot IE_2 \), i.e., wealth addition in period 2:

\[
\frac{\partial L}{\partial TE_1} = u_{E_1} \cdot J_{TE_1} + u_{H_1} \cdot I_{E_1} \cdot J_{TE_1} + S \left( u_{E_2} \cdot J_{TE_2} + u_{H_2} \cdot \left( I_{E_1} \cdot J_{TE_1} \left( 1 - \delta \right) + I_{E_2} \cdot J_{TE_2} \right) \right) \\
+ S_{H_1} \cdot I_{E_1} \cdot J_{TE_1} \cdot u \left( C_2, E_2, H_2 \right) - \lambda W + \lambda S_{H_1} \cdot I_{E_1} \cdot J_{TE_1} \cdot A = 0
\]

\[
\frac{\partial L}{\partial IE_1} = S \left[ u_{E_2} \cdot J_{IE_1} + u_{H_2} \cdot I_{E_1} \cdot J_{IE_1} \right] - \lambda \cdot S \cdot W = 0
\]

\[
\frac{\partial L}{\partial IE_2} = S \left[ u_{E_2} \cdot I_{IE_2} + u_{H_2} \cdot I_{E_1} \cdot J_{IE_2} \right] - \lambda \cdot S \cdot Q = 0
\]

\[
\frac{\partial L}{\partial TH_1} = u_{H_1} \cdot I_{TH_1} + S_{H_1} \cdot I_{TH_1} \cdot u \left( C_2, E_2, H_2 \right) + S \cdot u_{H_2} \cdot I_{TH_1} \left( 1 - \delta \right) - \lambda \left( W - S_{H_1} \cdot I_{TH_1} \cdot A \right) = 0
\]

\[
\frac{\partial L}{\partial TH_2} = S \cdot u_{H_2} \cdot I_{TH_2} - \lambda SW = 0
\]

\[
\frac{\partial L}{\partial IH_1} = u_{H_1} \cdot I_{IH_1} + S_{H_1} \cdot I_{IH_1} \cdot u \left( C_2, E_2, H_2 \right) + S \cdot u_{H_2} \cdot I_{IH_1} \left( 1 - \delta \right) - \lambda \left( P - S_{H_1} \cdot I_{IH_1} \cdot A \right) = 0
\]

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\[ \frac{\partial L}{\partial H_2} = S \cdot u_{H_2} \cdot 1_{TH_2} - \lambda SP = 0 \]

\[ \frac{\partial L}{\partial C_1} = u_{C_1} - \lambda = 0 \]

\[ \frac{\partial L}{\partial C_2} = S \cdot u_{C_2} - \lambda \cdot S = 0 \]

\[ \frac{\partial L}{\partial \lambda} = n_0 + (\omega - TE_1 - TH_1)W - C_1 - P \cdot IH_1 - Q \cdot IE_1 + S(H_1) \cdot A = 0 \]

This gives 11 equations to determine \( TE_1, TE_2, TH_1, TH_2, IE_1, IE_2, IH_1, IH_2, C_1, C_2 \) and \( \lambda \).

We find from (9) and (10):

\[ \lambda = u_{C_1} = u_{C_2} > 0 \]

Substituting for \( \lambda \) gives us a system of 10 equations that can be written in the following way:

\[ \left( u_{E_2} \cdot J_{E_2} + u_{H_2} \cdot J_{H_2} \right) + \left[ S \left( u_{E_2} \cdot J_{E_2} + u_{H_2} \cdot \left( I_{E_2} \cdot J_{E_2} \left( 1 - \delta \right) + I_{E_2} \cdot J_{H_2} \right) \right) \right] + S_{H_1} \cdot I_{E_1} \cdot u \left( C_2, E_2, H_2 \right) \]

\[ \frac{u_{E_2} \cdot J_{E_2}}{u_{C_1}} + \frac{u_{H_2} \cdot J_{H_2}}{u_{C_1}} = W \]

\[ \left( u_{E_2} \cdot J_{E_2} + u_{H_2} \cdot J_{H_2} \right) + \left[ S \left( u_{E_2} \cdot J_{E_2} + u_{H_2} \cdot \left( I_{E_2} \cdot J_{E_2} \left( 1 - \delta \right) + I_{E_2} \cdot J_{H_2} \right) \right) \right] + S_{H_1} \cdot I_{E_1} \cdot u \left( C_2, E_2, H_2 \right) \]

\[ \frac{u_{E_2} \cdot J_{E_2}}{u_{C_1}} + \frac{u_{H_2} \cdot J_{H_2}}{u_{C_1}} = Q \]

\[ \frac{u_{E_2} \cdot J_{E_2}}{u_{C_2}} + \frac{u_{H_2} \cdot J_{H_2}}{u_{C_2}} = Q \]

\[ \frac{u_{H_2} \cdot I_{TH_2}}{u_{C_1}} + \left[ S \cdot (u_{H_2} \cdot I_{TH_2} \left( 1 - \delta \right) ) + S_{H_1} \cdot I_{TH_1} \cdot u(C_2, E_2, H_2) \right] + S_{H_1} \cdot I_{TH_1} \cdot A = W \]

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Endogenous wage formation

If \( W_t = B(E_t) + \tau \), \( t = 1,2 \), equation (13) would change to

\[
\left( u_{c_1} J_{T_{E_t}} + u_{h_1} I_{E_t} J_{T_{E_t}} \right) + \left[ S \left( u_{e_1} J_{T_{E_t}} + u_{h_1} \left( I_{E_t} J_{T_{E_t}} (1 - \delta) + I_{E_t} J_{T_{E_t}} \right) \right) + S H_{E_t} I_{T_{E_t}} \cdot u(C_2, E_2, H_2) \right] + (\theta - TE_1 - TH_1) B_{E_t} J_{T_{E_t}} + S H_{E_t} I_{T_{E_t}} \cdot A + S \cdot (\theta - TE_1 - TH_1) B_{E_t} J_{T_{E_t}} - B(E_t) = \tau.
\]

First, reduced taxation will increase the right hand side of equation (23). This will increase the costs of schooling and thus give a substitution effect going in the direction of less education, i.e., the fraction on the left hand side of (23) will also increase for a lower education level. The income effects represented by the other terms on the left hand side of (23) also increase as the marginal benefits from education increases in lower education level. However, we still have the effect that a higher income (lower income taxation) increases the demand for normal goods, the total budget increases, see (22), which means that, as before, we get ambiguous effects on schooling of reduced income taxation (increasing \( \tau \)). Most conclusions will not change qualitatively with an endogenous wage rate, but an initial wealth increase will not only give a positive income effect. It also gives an incentive to lower investments in education as the individual does not have to invest in a higher wage rate to be able to buy the same quantity of goods as before.
Appendix 2: Symbol definitions

Variables

- $Z_t$: full consumption
- $C_t$: market consumption
- $L_t$: leisure
- $S_t$: survivorship probability
- $H_t$: quality-of-life health stock
- $W_t$: wage rate
- $E_t$: education stock
- $IH_t$: investment in quality of life health
- $IE_t$: investment in education
- $TH_t$: investment in time on health
- $TE_t$: investment in time on education

Parameters

- $\rho$: discount rate
- $\rho_h$: $1/(1-\rho_c)$ - elasticity of substitution between health, education and full consumption in utility
- $\zeta$: calibrated to imply a specific value for the intertemporal elasticity of substitution
- $z_0$: subsistence level of full consumption
- $\kappa$: calibrated to imply a specific value for the elasticity of substitution between consumption and leisure
- $sl$: value share of leisure in full consumption at year 50
- $c_0$: benchmark consumption at year 50
- $l_0$: benchmark leisure demand at year 50
- $n_0$: initial non-wage wealth
\( \omega \) - time endowment

\( b_t \) - other non-wage assets

\( ph_{it} \) - price of effective financial health investments

\( pe_{it} \) - price of effective financial education investments

\( pth_{it} \) - price of effective time health investments

\( pte_{it} \) - price of effective time education investments

\( \bar{h}_i \) (and \( h_{it} \)) - benchmark trajectory of \( H_i \)

\( \bar{e}_i \) (and \( e_{it} \)) - benchmark trajectory of \( E_i \)

\( ih_{it} \) - benchmark trajectory of \( IH_i \)

\( ie_{it} \) - benchmark trajectory of \( IE_i \)

\( th_{it} \) - benchmark trajectory of \( TH_i \)

\( te_{it} \) - benchmark trajectory of \( TE_i \)

\( r \) - market interest rate

\( \delta_h \) - depreciation rate of new investments in \( H_i \)

\( \delta_e \) - depreciation rate of new investments in \( E_i \)

\( \nu \) - elasticity of new health investments on \( H_i \)

\( \alpha \) - elasticity parameter for education stock in health production

\( \beta \) - elasticity of health in sick days

\( \gamma \) - elasticity parameter for health-related inputs to production of health investments

\( \mu \) - elasticity parameter for health stock effect in education production

\( \xi \) - elasticity of education investments on \( E_i \)

\( \eta \) - elasticity parameter for education-related inputs in production of education investments

\( \tau_e \) - share parameter of direct effects of education on utility

\( \tau_h \) - share parameter of direct effects of health on utility

\( \dot{\delta} \) - elasticity of time trend in productivity of health investments

\( \psi \) - elasticity of time trend in productivity of education investments

\( \phi \) - slope term in survivorship function
\( \theta \) - exponent term in survivorship function.
Appendix 3: Description of the Numerical Model Calibration Procedure

1st Stage Calibration

Formally, the model calibration proceeds in the following way. The first stage of the calibration follows the procedure described in Murphy and Topel (2006). Parameters $s_l, c_0, l_0$ are calibrated to match consumption in midlife for the average individual based on expenditures and earnings data from the U.S. Bureau of Labor Statistics (2012a). In midlife, benchmark consumption and wage income are assumed to both be equal to approximately $60,000. We also assume that households split their time evenly between work and leisure at midlife in the benchmark. $\kappa$ is chosen to imply representative estimates from the literature on the elasticity of substitution between consumption and leisure. In our benchmark model, this elasticity (defined at $1-1/\kappa$) is equal to 0.5. Similarly, $\zeta$ is chosen to ensure that the individual’s survivorship-weighted average of willingness to pay for marginal reductions in the probability of death (their VSL) is equal to $6.3$ million (a number commonly used in benefit-cost assessments of policies designed to reduce mortality rates) between the ages of 25 and 55 given the values of the other parameters chosen.\footnote{See Viscusi and Aldy (2003) for a review of VSL estimates.}

$z_0$, interpreted as the subsistence level of full consumption, is calibrated at 10% of the benchmark full consumption levels. $r$, the exogenous interest rate, is set equal to an annual rate of 4%. $\rho$, the household's rate of time preference, is set equal to an annual rate of 2%.

At this stage of the calibration procedure the survival probabilities, $S_t$, the health stock, $H_t$, and the education stock, $E_t$, are taken to be exogenous. $S_t$ values are chosen to match U.S. mortality data for the average individual, producing a benchmark expected life length of approximately 78 years. The mortality data are taken from Center for Disease Control and Prevention (2013). The trajectory of $H_t$ is chosen to reproduce the lifecycle consumption pattern represented in the BLS data. The education stock is set equal to benchmark levels ($\bar{e}_t$) which is assumed to follow the benchmark trajectory of wages generated by the BLS earning data (U.S. Bureau of Labor Statistics, 2012a). Candidate values are chosen for $\tau_h$ and $\tau_e$, which describe the relative
importance of health, education and full consumption in generating household utility, and $\rho_{we}$, which governs the degree of substitutability between these arguments. These values are updated in the final stage of the calibration.

This procedure represents a complete calibration of the model down to exogenous survivorship, health and education stocks. Thus it produces a benchmark sufficient to solve the reduced model:

$$\begin{align*}
\max_{c_t, l_t} \sum_{t=1}^{T} & (1 + \rho)^t \left[ \tau_o + \tau_h H_t^\rho_{we} + (1 - \tau_o - \tau_h)(Z_t^\rho - Z_o^\rho) / Z_o^\rho \right]^{1/\rho_{we}} \\
\text{subject to} & \\
\sum_{t=1}^{T} (\omega - L_t) W_t + b_t - C_t S_t / (1 + r)^{t-1} = 0 \\
H_t = H_t' \\
E_t = E_t'
\end{align*}$$

plus equations (16) and (17). $H_t'$ and $E_t'$ represent exogenous levels of the health and education stock trajectories. The remaining tasks are to link survivorship to the level of health (2nd stage) and link health and education to the levels of financial and time investments in these stocks (3rd stage).

2nd Stage Calibration

The second stage of the calibration links the survivorship probability, $S_t$, to health status, $H_t$, by way of the function described in equation (17). The idea is to choose parameters in (17) to reproduce the survival data. However, the fit between the model function and the data will not be exact. As a result, the benchmark consumption and earning paths described by the first stage calibration will no longer be optimal for a given trajectory of $H_t'$. Thus, we update the $H_t'$ values to once again match the BLS consumption and earnings profiles – as we did in the first stage calibration – now at the new survival rate estimates. Formally, the algorithm is:
1. If this is the first iteration of the algorithm, let $\bar{H}_t$ represent the trajectory implied by the outcome of the first stage calibration. If this is a subsequent iteration, let it represent the outcome from step 4 of the algorithm.

2. Choose the parameters of the survivorship function ($\phi$ and $\theta$) to minimize the sum of squared differences between the survivorship rates implied by the mortality data and the prediction from (17).

$$\min_{\phi, \theta} D = \sum_t (1 - e^{-\phi \rho_t} - S_t)^2$$

where $S_t$ represents the observed survivorship rates from the mortality data and $\bar{H}_t$ is the exogenous trajectory of the health stock.

3. Choose $\bar{H}_t$ such that the solution to (16)-(17) and (24)-(25) reproduces the observed consumption and earnings trajectories from the BLS data.

4. If there is no change in $\bar{H}_t$ from the previous iteration of the algorithm, then we are done. If not, return to step 1.

In our application, this algorithm converges within 5 iterations.

3rd Stage Calibration

The third stage of the calibration chooses parameter values to match the model’s prediction on the trajectories of benchmark health and education expenditure levels to data. The parameters calibrated here are: $\phi, \theta, \beta, \gamma, \eta, \tau, \tau_h, \alpha, \nu, \mu, \xi, \Delta, \psi$ and $\rho_{he}$. We match data on monetary expenditures on health (medical expenditures), monetary expenditures on education and time expenditures on health and education. The data on medical expenditures come from the National Health Expenditures Data (2004). The data on monetary expenditures on education come from National Center for Education Statistics (2008) and the U.S. Census (2008). The data on time
expenditures on education and health\textsuperscript{17} come from the American Time Use Survey (U.S. Bureau of Labor Statistics (BLS), 2012b).

To do this, we use gradient descent method that proceeds in the following way.

1. Solve the optimization problem described by equations (15)-(20) holding the levels of the investment goods in model \((IH_t, TH_t, IE_t, TE_t)\) fixed equal to unity (a convenient normalization).

2. Extract the shadow values associated with the investment variables in the model and install effective prices for the investment goods that rationalize the levels of the investments as optimal levels. Thus if \(V = U(C_t^*, L_t^*, IH_t = 1, TH_t = 1, IE_t = 1, TE_t = 1)\) represents the maximand evaluated at the solution to the problem from step 1, where the “\(*\)” superscript indicates an optimal value for a model variable, then the effective shadow prices can be written as:

\[
\begin{align*}
ph_{0t} &= \frac{1}{\lambda} \frac{\partial V}{\partial IH_t} \frac{(1+r)^t}{S_t} \\
pe_{0t} &= \frac{1}{\lambda} \frac{\partial V}{\partial IE_t} \frac{(1+r)^t}{S_t} \\
pth_{0t} &= \frac{1}{\lambda} \frac{\partial V}{\partial TH_t} \frac{(1+r)^t}{S_t} \frac{1}{E_t} \\
pte_{0t} &= \frac{1}{\lambda} \frac{\partial V}{\partial TE_t} \frac{(1+r)^t}{S_t} \frac{1}{E_t}
\end{align*}
\]

where \(\lambda\) is the marginal utility of wealth.

\textsuperscript{17}We do not include time spent on sports in this dataset as they are biased to the early part of life, and the model predictions make a better match with data without including sports.
4. Compute the value of a loss function that measures the quadratic distance between data on the model investment goods and model predictions based on the realization from step 2. The loss function is defined as

\[
L = \sum_i (p_{h0i} - \bar{p}_{h0i})^2 + (p_{e0i} - \bar{p}_{e0i})^2 + (p_{th0i} - \bar{p}_{th0i})^2 + (p_{te0i} - \bar{p}_{te0i})^2
\]

where the “barred” values represent the calibration data.

5. Perturb the value of one parameter in the set of those to be calibrated in this stage. For example, set \( \beta' = \beta + \delta \) where \( \delta \) is a small number.

6. Re-calculate the optimal shadow prices and loss function (as before in steps 1-3 for the perturbed parameter value. Call the value of the loss function that results from perturbing the level of parameter \( i \), \( L' \).

7. Repeat steps 1-5 for all parameters to be calibrated.

8. Approximate the elements of the gradient of the loss function with respect to each parameter \( i \), \( l_i \), as:

\[
l_i = \frac{L_i - L}{\delta}
\]

9. If the elements of the gradient, \( l_i \), are approximately equal to zero, then we are done.

10. If the gradient elements are not close to zero, update the vector of parameters, \( P = [\beta, \gamma, \ldots, \rho] \), to a new set of candidate values, \( P' \), based on the evaluation of the gradient:

\[
P' = P - \varpi \Lambda
\]

where \( \Lambda = [l_{\beta}, l_{\gamma}, \ldots, l_{\rho}] \) and \( \varpi \) is a speed of adjustment parameter.

11. Return to step 1.
In practice, we set $\sigma = 5e^{-7}$ and run the third procedure for approximately 5000 iterations.
References


