A FRAMEWORK DESCRIBING STUDENTS' MATHEMATICS LEARNING EXPERIENCE WITH A TABLET-BASED PEDAGOGICAL MEDIUM: THE CASE OF A GEOMETRY EXPLORATION

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In this study, we presented a theoretical framework that describes factors of student's learning experience in a tablet-based learning environment with math apps. The theoretical framework is based on a synthesis of the literature. We further illustrated through empirical data the underpinning factors of our proposed theoretical framework and provided data to illustrate the learning experience of 60 Grade 6 students exploring a triangle through the use of tablets and Geogebra apps. The findings suggested that students were actively engaged in the learning process since they were exploring independently the elements of a triangle. In addition, the affordances of the tablet and Geogebra apps, such as the haptic dragging, reshaped the learning trajectory and students' perspective on the mathematical content.

Keywords: mobile technology, learning experience, math apps

INTRODUCTION

Advances in digital technologies are altering the variety of tools available to teachers and students (Attard & Curry, 2012). Mobile technology, currently dominated by tablets, is prevalent in many school classrooms (Hilton, 2018). A growing number of research findings documented that the use of mobile technologies has the potential to transform and reshape students' learning experience (Attard, 2017; Calder & Campbell, 2016). The characteristics of mathematics learning with mobile technologies, the affordances of mobile technologies and math apps combined with appropriate inquiry-based pedagogical approaches may create an interactive and visual learning environment, different from that of traditional teaching. Calder and Murphy (2018) supported that mobile technological tools, such as tablets, if used appropriately, may help students investigate mathematical ideas in ways that promote mathematical thinking and concretize abstract mathematical concepts. In addition, a learning environment that used mobile technology and apps gives a new potential to students' mathematical understandings by repackaging the mathematical content and processes, differentiating learning experiences and enhancing students' engagement by exploring simultaneously visual, symbolic and numerical representations and fostering independent learning (Olive et al., 2009).

Due to the rapid pace of technological developments, there is little published research investigating the ways that students' learning experience is shaped in digital-based environments (Calder & Murphy, 2018). In addition, further research is needed to investigate the ways in which effective pedagogies may change the nature of a digital based mathematics classroom from a teacher-led environment to a student-centred one. Research-based evidence is needed regarding new ways for designing such learning environments that teachers could use as exemplars of effective learning and teaching and to enrich existing literature regarding the theoretical conceptualizations of learning within digital driven environments (Attard, 2017). The purpose of this study is twofold: First, to present a theoretical framework describing factors that relate to student's learning experience within a digital tablet-based learning environment with apps. We will discuss in what ways the learning experience might be reshaped compared to a traditional learning environment. Second, we will report on an exploratory study of sixty Grade 6 students that worked with geogebra apps to exemplify some parameters of the proposed framework.

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

Students' active participation in rich mathematical activities is critical for students' learning experience (Choon, Lam, & Berinderjeet, 2014). It is widely acknowledged that learning experiences must include opportunities where students discover mathematical ideas and participate actively in the learning process. Students should be given opportunities to develop collaborative and communication skills, set learning goals, obtain satisfaction from the learning process, build up their mathematical understandings and problem solving ability, while teachers should design engaging learning activities. Thus, the role of learning experiences in understanding mathematics is extremely important. Digital technologies have the potential to reshape students' learning experience by offering new kinds of authentic learning experience, ranging from experimentation to real world problem solving (Hilton, 2018). Research findings showed that students' learning at school is enhanced when using tablets (Clark & Luckin, 2013). The quality, depth and breadth of students' learning experience in a digital-based technological learning environment relates to students' engagement in the learning process, the adopted pedagogical approaches and students' learning trajectory (Calder & Campbell, 2016). Research findings suggest that digital technologies influence students' engagement and actual learning trajectory by reshaping the learning process and repackaging the potential of exploring the mathematical content. Moreover, digital environments have potentials for differentiating the learning activities that may facilitate the development of individual learning trajectories (Attard & Curry, 2012). By the term learning trajectory we refer to the hypothetical learning trajectory (planned based on curricula) and the actual learning trajectory that involves the actual pathway of the learner while his thinking evolves during working on activities (Sacristán, et al., 2009). Sacristán, et al. (2009) asserted that digital technologies may differentiate actual learning trajectories.

Learning engagement has been conceptualized by Fredricks, Blumenfeld and Paris (2004) as a multi-faceted construct that operates at the cognitive, affective and behavioural level. Specifically, Attard (2017) defined engagement in mathematics as the 'coming together' of cognitive, emotional, and behavioural engagement that leads to students' enjoyment and valuing of mathematics. Thus, learning engagement is related to the learning enjoyment, doing mathematics and viewing the learning and doing of mathematics as a valuable and useful task within and beyond the mathematics classroom (Attard, 2017). Cognitive engagement involves recognizing the value of learning and the willingness to go beyond the minimum requirements; affective engagement conceptualizes students' willingness to become involved in school work; and behavioural engagement encompasses students' active participation and involvement in academic and social activities. Research findings showed that integrating tablets in mathematics teaching has an effect on student engagement by promoting interactivity, motivation, immediate feedback, challenge and fun, while teacher's pedagogical approaches are a decisive factor on maximizing the potential of digital technologies to engage students in mathematics (Clark & Lucking, 2013).

The learning process and subsequently students' learning engagement is influenced by the affordances of the digital environment. Digital environments provide multiple representations that contribute to the enrichment of mathematical concepts. For instance, apps present the mathematical ideas in an investigative context and provide a visual and interactive learning environment. In addition, tablets provide a kinaesthetic orientation of learning while multiple senses are incorporated that create a friendly, creative and pleasant learning environment for students (Beschorner &

Hutchison, 2013; Judge et al., 2015). Moreover, research findings showed that the use of tablets contributes to an increased engagement of the students by providing the opportunity to work in groups and allowing the students to move around the room and get involved in a variety of activities. A well-organized lesson that integrates tablets, math apps and adopts explorative and investigative approaches builds a pedagogical medium that allows students to test informal conjectures, link different representations and explore the interactive affordances of the medium. Students' approach exhibit a more complex and nuanced way of engaging with the availability of different kinds of technologies, as well as making considered decisions about using the available tools in unexpected ways, take risks and employ investigative strategies. It should be noted that apps should match the curriculum and offer functionalities that enable applying productive pedagogies.

The potential of digital tools to facilitate the visualization of abstract mathematical concepts, conjecturing and testing of ideas is closely related to the applied pedagogies and the appropriateness of the used tasks. An inquiry based approach has been identified as being particularly appropriate for technology-enhanced mathematical activities (Attard, 2017), while research findings suggested that technology-based mathematics instruction involves the teacher's ability to make changes to pedagogy.

The learning experience is also related to the curriculum and the learning trajectory. Research findings suggested that the investigation of mathematical concepts through a pedagogical medium that integrates apps and tablets may transform students' learning experience by offering an alternative learning trajectory (Sacristán, et al., 2009). However, researchers call for the need to further explore the way in which a digital based learning environment may emerge new mathematical meaning and discourses and reveal new hierarchies of learning, by reshaping existing hypothetical learning trajectories (Sacristán, et al., 2009). In addition, a reshaped learning trajectory may allow students to explore mathematical content that may arise during digital-based investigations and as a result may enrich the depth and breadth of the mathematics content.

We suggest that a digital tablet-based learning environment that integrates math apps and explorative pedagogies transforms the apps and the tablet to a pedagogical medium. Based on the above, we propose a theoretical framework that describes factors that relate to student's mathematics learning experience with a tablet-based pedagogical medium (see Figure 1). Learning experience that involves students' active participation in the process of learning relates to three main factors: (a) student engagement, (b) the learning process and (c) the alternative perspectives offered by the pedagogical medium. Student engagement includes the cognitive, affective and behavioural dimensions of students' interactions and processes in the learning environment and depends on the affordances of the digital tools and the student's participation in the learning process. The learning process relates to the pedagogical approaches adopted by the teacher and the extent to which the teacher reshapes the learning process based on the affordances of the digital tablet-based learning environment and transforms the available digital tools to a pedagogical medium. Finally, the alternative perspectives that may arise during the learning process may lead to alternative learning trajectories that transform the quality of student's learning experience and the depth and the breadth of the examined mathematical content.

In the present study, we describe an exploratory study underpinning some factors of our proposed theoretical framework and provide empirical data illustrating the learning experience of 60 Grade 6 students exploring the elements of a triangle using tablets and Geogebra apps. In particular, we will address the following research question: In what ways the affordances of the pedagogical medium (a) facilitated student's active participation and engagement and (b) revealed alternative perspectives in respect to the mathematical content and the actual learning trajectory.

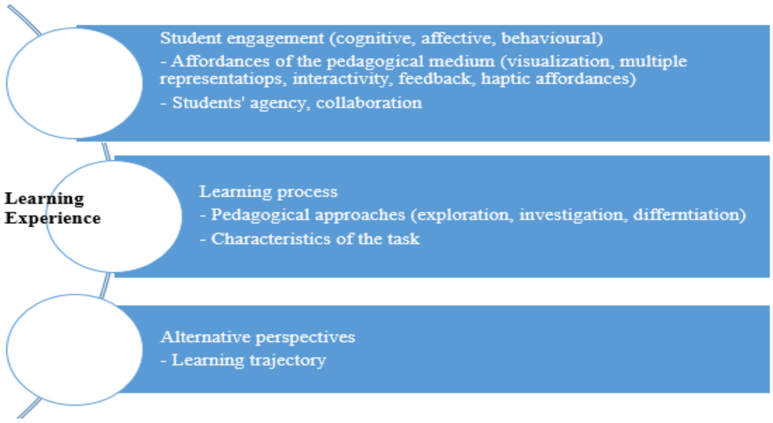

Figure 1. The proposed theoretical framework

METHODOLOGY

The participants were sixty Grade 6 students from three intact classes in one public primary school in Cyprus. The school's population came from a middle to high socioeconomic status. The students reflected a broad spectrum of academic achievement levels. The school was chosen as an appropriate site because it was recently equipped with tablets and the teachers of the three classes were helpful in the whole procedure. In addition, the students had used apps and tablets in mathematics several times before. The lesson was developed by the research team in collaboration with the three teachers. The development of the lesson took into consideration the fundamental parameters of the proposed model. The lesson is part of a Geometry Unit in Grade 6 and the relevant attainment targets included students' exploring and understanding the altitude, the bisector and the median of a triangle. The lesson was planned for 80 minutes. It was delivered by one of the members of the research team and consisted of the following three exploration activities that were developed based on the factors of the proposed model: In Exploration 1, students were given three ready-made Geogebra constructions. The constructions were presented in an applet form and included a triangle ABC and a segment AD. In the first construction the segment AD was a median, in the second one AD was a bisector and in the third one AD was an altitude. Students were asked to explore freely the three applets and explain the function of the segment AD in each construction and provide an operational definition of the segment AD in each case. Then, students were asked to present their work. The teacher introduced the formal definitions and asked to compare the three elements of the triangle. In Exploration 2, a ready-made applet was provided that presented a triangle, the measures of its three angles and an altitude. Students were asked to drag the vertices of the triangle to investigate the position of the altitude in the case of an acute-angled, obtuse-angled, and right-angled triangle. In Exploration 3, an applet presenting a triangle, its median, altitude and bisector was provided. Students were asked to investigate when the altitude, the bisector and the median of the triangle coincide.

This exploratory study used qualitative methods for collecting data. In each class, two researchers observed four students to capture their actions while working. In addition, researchers posed clarification questions to explore further possible hidden processes in students' work. To do so, each researcher used an observation protocol. The protocol consisted of a set of 10 questions examining the way in which each student utilized the affordances of the tablet and the app in respect to the specific exploration, such as the following: (a) Do they observe the available measures while dragging?, (b) Do they drag the triangle vertices flexibly and purposefully?, (c) Do they construct different kinds of triangles while dragging to examine the role of each segment? and (d) Do they explain the function of each segment based on the available measures? Further, we videotaped the screens of the tablets to capture student's actions while working, emphasizing on dragging. A qualitative interpretive framework was used for the analysis of the data (Miles & Huberman, 1994). A comparative analysis of the data collected by the two researchers was undertaken to ensure reliability and initial analysis was conducted immediately following each lesson of data collection.

RESULTS

In this study, analysis focused on the ways that the affordances of the pedagogical medium facilitated student's active participation and engagement and revealed alternative perspectives in respect to the mathematical content and the actual learning trajectory. To do so, we present the way in which students utilized the affordances of the tablet and the applet in the three explorations. Exploration 1. Eight out of the twelve students surfed between the three available applets and tried to compare the function of the segment AD. The remaining four students did not switch between the applets but they kept working in only one of them. Thus, the design of the activity and the way that the majority of the students worked showed that they did not follow a traditional learning trajectory (i.e., learn each of the three separately, one after the other) but they tried to understand the characteristics of each element of the triangle by making comparisons. To do so, they documented their explanations based on the measures of the applet. In the following excerpt, we present the discussion with one student that worked strategically and utilized the affordances of the software.

Researcher: Can you explain your work in order to explain the differences between the three segments?

Student 1: Yes, we first dragged the vertex A in the three applets. In the first applet, we observed that the segment AD divides the opposite side, because the segments BD and CD were equal (indicating the measures of the applet). Then, we examined if AD divides the opposite side in the second applet. We dragged A in the second applet, but nothing happened. Thus, we understood that AD does not function in the same way in the two applets.

Researcher: What did you do next?

Student 1: We moved to the third applet and we observed again that AD did not divide the opposite side, thus we decided to observe how the measures of the angles change, while dragging A.

We observed the way that students utilized the dragging capability of the software. Only two students did not drag the vertices of the triangle, but tried to figure out what was happening based on the one available measure, using the applet as a static image. Four students dragged the vertices of the triangle near to their existing positions (see the blue trace in Figure 2a), while the other six students dragged the vertices all around the screen and constructed different kinds of triangles (see the blue trace in Figure 2b). These six students, when asked to document their answers, utilized the dragging capability to explain that their conjecture is valid, even in extreme cases. The following excerpt shows the way that a student documented her reasoning based on the dragging:

Researcher: Are you sure that AD (in the second applet) bisects the angle?

Student 2: I was not sure either. But, I dragged the vertex A in many different places on the screen to see. If you make a very big or a very small triangle (she dragged vertices A and B to transform the existing triangle) or if you move the triangle up or down the screen, the two angles are always equal.

Researcher: Is this characteristic valid in the case that the angle A gets very small?

Student 1: Yes, I checked it. I dragged at the same time vertices B and C to make angle A as small (see the trace of the two vertices in Figure 2c) as possible and the result was the same (she used the multi-touch affordance of the tablet).

Exploration 2. In the second exploration students had to investigate the position of a triangle altitude in the case of an acute, obtuse and right-angled triangle. Four of the students dragged the vertices of the triangle randomly and consequently could not connect the position of the altitude with the kind of the triangle. Four students dragged vertex A to the right side of the screen to transform the triangle to an obtuse-angled one (see Figure 3a). Students explained that in the case of an obtuse-angled the altitude gets outside of the triangle, by displaying on the screen the size of the angle C. However, in the case of a right-angled triangle, they could not explain that the altitude coincides with the side of the triangle. When asked whether a right-angled triangle does not have an altitude, they dragged again vertex A to make a right-angled triangle and suggested that the two lines (side and altitude) are joined, thus we cannot see clearly the altitude. When prompted to think if we see the triangle side or the altitude, they observed the colour and deduced that we see the altitude because the colour of the segment is black. One of them clarified that the altitude lies on the segment AC. The other four students utilized the dragging capability to a greater extent and dragged vertex A in many different positions and transformed first angle C to an obtuse one and then angle B (see Figure 3b). At the crucial point of transforming the angle C from an acute to a right and then to an obtuse one, they repeated gradually the procedure several times (see Figure 3b). They repeated also the same procedure in the case of angle B. After this, they easily explained that in the case of a right-angled triangle the altitude coincides with the side of the triangle.

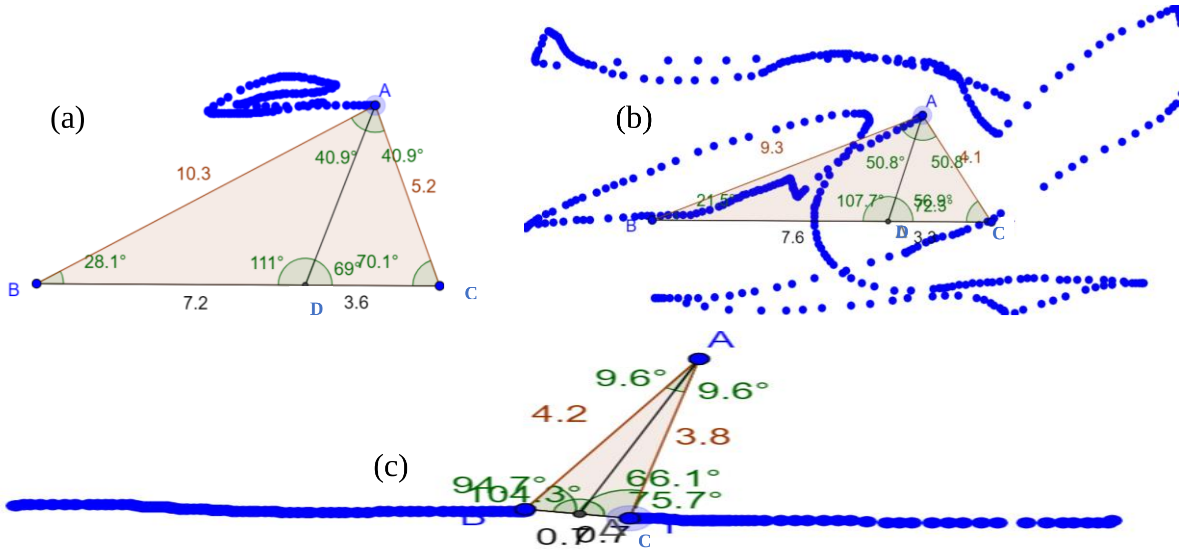

Figure 2. The trace of the triangle vertices during dragging

Exploration 3. Students were asked to find when the altitude, the bisector and the median of the triangle coincide using a ready-made applet presenting the three segments. The nature of the activity did not follow a traditional learning trajectory because students had to combine the three segments in a unified shape. All the students dragged easily vertex A, so the three segments coincided. In addition, all of them discovered that in the case of an isosceles triangle, the three segments coincide and they justified their answer based on the measures of the sides. The teacher asked them if they could find another kind of triangle. Six of them dragged vertex A to construct an isosceles triangle and then dragged along a virtual perpendicular bisector (from up to down) on BC to keep the three segments coinciding (see Figure 4a). Thus, they could not construct an equilateral triangle, because the two sides were getting smaller and the third one was constant. On the other hand, the rest of the students used dragging to reduce the size of the segments and after coinciding the three segments they dragged vertex A up and down along the perpendicular bisector on BC (see Figure 4b) and deduced that the three segments coincide also in the case of an equilateral triangle.

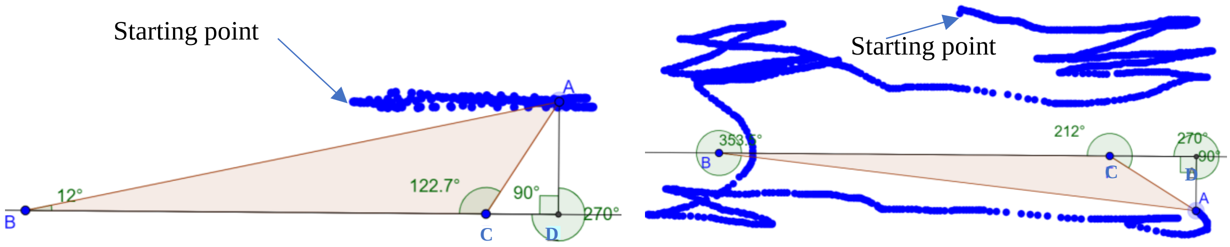

Figure 3. Exploring the position of the altitude of a triangle

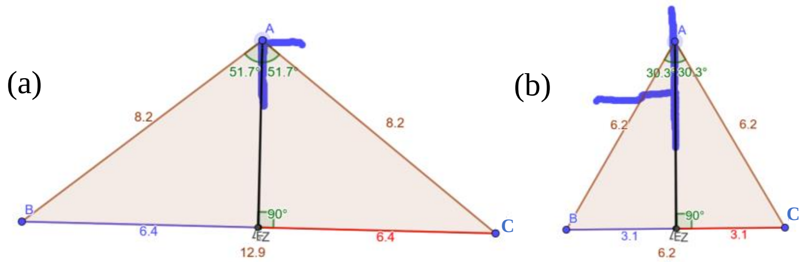

Figure 4. Exploring when the three segments coincide

DISCUSSION

We presented a theoretical framework describing factors that relate to the quality of student's learning experience in a tablet-based learning environment with math apps. The framework identified three key factors, namely: (a) student's engagement, which is mainly influenced by the affordances of the pedagogical medium, (b) the learning process that is related to the pedagogical approaches and the characteristics of the tasks and (c) the alternative perspectives of the mathematics content offered by the pedagogical medium. We further described an empirical study that aimed to provide further insight in the ways that the affordances of a pedagogical medium facilitated students' active participation and revealed alternative perspectives in respect to the mathematical content. Analysis showed that students were actively engaged in the learning process by self-exploring the elements of a triangle. Students utilized the interactivity of the tablet and the Geogebra applets, used the provided feedback to restructure their work, while many students exhibited flexibility to take into consideration all the available displays of the applets (geometric shape, measure of angles and sides). Analysis showed that an important factor that contributed to maximizing students' active involvement was the extent to which they used the affordances of the dragging tool. We could conclude that dragging in a tablet environment becomes more interactive and dynamic because of its haptic nature and this "haptic dragging" reshapes working in a dynamic geometry environment to a multi-sense and authentic interactive activity. An indicative example of the different nature of haptic dragging is the multi-touch dragging, where students dragged two points on the screen at the same time. This functionality facilitates resizing more than two segments or angles at the same time. In addition, analysis showed that students that used haptic dragging in a flexible and systematic way explored a bigger number of examples or extreme cases. Findings suggested that the affordances of the pedagogical medium reshaped the traditional learning trajectory by providing the opportunity to study the three elements of the triangle at the same time and to reach mathematical understandings that could not be conceptualized in a traditional learning environment. That is students (a) conceptualized the functionality of the elements of a triangle by making comparisons and validating their conjectures in a number of different examples, (b) self-explored how the position of the altitude of a triangle varies based on the type of the triangle and (c) self-guided discovered that the three elements of a triangle coincide in isosceles and equilateral triangles. This exploration could be achieved because of the visual and the measurement affordances of the app and the power of haptic dragging and enriched the mathematical content by providing alternative perspectives and deepen mathematical meanings.

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