BILINGUAL MATH LESSONS WITH DIGITAL TOOLS – CHALLENGES CAN BE DOOR OPNER TO LANGUAGE AND TECHNOLGY

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Although, mathematics can be understood as universal, some authors take the view that there are differences in school mathematics among different countries. The more complex mathematical topics are, the more the linguistic or cultural differences disappear. In this text, three hypotheses are tested: In a school context, differences in mathematical education between Germany and the US exist. These differences are relevant to bilingual math lessons. Digital mathematical tools can be door openers and help to deal with these differences. Based on two examples of American-German bilingual learning environments, conclusions can be drawn. Math lessons were observed in the MISTI Global Teaching Lab program and the German International School Boston. A two-step mixed-method study was developed to identify and describe differences in the mathematical education and the role of digital mathematical tools.

Keywords: Bilingual math lessons, Differences between American and German mathematical education, digital mathematical tools, instrumental genesis

INTRODUCTION

In attempting to compare German and US American math education, generalizations are difficult, if not impossible, to make. First, one cannot clearly define what constitutes a "German mathematics education". The diversity among the German *Länder* (states) with their 16 educational systems is present and empirically proven, as the current INSM education monitor shows (Anger, Plünnecke, & Schüler, 2018). In addition, the language of instruction at Austrian schools and many Swiss schools is also German. Likewise, in the US there is no homogenous setting of "American" mathematics education. The 50 states differ in their educational systems. Comparisons in the form of ratings and rankings are common. For example, the educational system of the State of Massachusetts is considered the best in the US (Trimble, 2018). In spite of the difficulties with a delimiting definition, some observations on mathematics education in the USA can be formulated without claiming to be generalized.

As early as 1913, Jourdain (2007, p. 1) expressed his expectations to math teachers with the words: "[...] he was never satisfied with his knowledge of a mathematical theory until he could explain it to the next man in the street." There are two views: Everyone (learners) can expect mathematics to be explained in "simple words". However, a mathematical theory cannot be extremely simplified, until it only applies to special cases. That would be too heavy on the meaningfulness (Jourdain, 2007). Mathematics teachers in Germany as well as in the USA move between these two views. It is interesting to see where the priorities are set. In teacher education, it is important to prepare for the challenges of teaching. Breux & Whitaker (2015, p. xii) note the impossibility of general solutions: "We are highly aware that there are no two teachers exactly alike and that no one solution fits all circumstances." Nevertheless, they feel able to offer 60 simple answers to everyday problems. This pragmatic approach is beneficial for young teachers yet to gain their own experience in the classroom. American colleagues largely assume that mathematical performance can be well-grasped and measured in multiple-choice tests. The results in standardized final exams with thematically consistent, compact tasks ("items") determine the school career from Middle and High School through to College. Math lessons are often characterized by preparation for the standardized tests (Hyun, 2006; Kaplan, 2009). Geometry is present throughout the whole education and geometric problems are important at all levels (Balley, 2012; Lappan, Fey, Fitzgerald, Friel, & Phillips, 2009).

These are interesting observations, which give an idea of the challenges of bilingual mathematical learning environments. For the current study, two projects are of particular interest: MISTI Global Teaching Lab and the German International School Boston.

MISTI GLOBAL TEACHING LAB AND GISB

In January 2019, 43 students from the Massachusetts Institute of Technology (MIT) visited Germany as part of the MISTI Global Teaching Lab exchange program to work with students in math and science classes at German schools. In doing so, they inspired the students with their enthusiasm for their fields of research and provided many important ideas for the learning process. Participation in the Global Teaching Lab program provided the basis for cooperation with the MIT Science and Technology Initiatives (MISTI). The goal of the project is to prepare STEM research results for learners in an educationally useful manner.

The German School, Boston was founded in 2001 by a group of parents and teachers from the German-speaking community. The opening was preceded by a four-year planning and organization phase. Since the school unites students of different nationalities and is committed to multiculturalism, the name has been extended to German International School, Boston. In recent years, the number of registrations has increased continuously and the campus has been expanded systematically. Currently, the GISB has more than 300 students from preschool to grade 12. Since 2013, successful graduates of the school have received both the Massachusetts High School Diploma and the German International High School Diploma. Graduates of the school can study in the US or in Germany and the EU.

THEORETICAL BACKGROUND

"Doing mathematics is different in different languages." This statement from Barwell (2003, p. 38) represents a point of view that directly links differences in the school mathematics of different countries with language. It is important that mathematics can be understood as universal (Rolka, 2004), but the examination of mathematics in the classroom, thus "doing mathematics", can differ in different cultures. Moreover, the consideration of cultural influences contributes to understanding students’ problems: "However, positioning mathematics as culture-free and neutral reinforces the belief that the problem lies with the students or their families as opposed to with the curriculum, pedagogical choices, or the educational system" (Felton-Koestler, & Koestler 2017, p. 68).

In terms of school mathematics, it is possible to say, the more complex mathematic topics are, the more the linguistic or cultural differences disappear (Novotná, & Moraová, 2005). In school contexts, differences in mathematical work exist. It is interesting how these differences are dealt with in a bilingual learning environment. When teachers’ backgrounds differ from their students, they may have difficulty recognizing the existing knowledge of students, thus, the knowledge that can be built on in the classroom (González, Andrade, Civil, & Moll, 2001). The MISTI GTL Germany project and the GISB are good examples of bilingual mathematical learning environments. In both environments, the classroom language (as a foreign language) differs from language used by most learners (the daily lived language). In both cases, teachers teach mathematics in their mother tongue. At the MISTI GTL American MIT students teach German high school students (in English) and at the GISB German teachers teach international students (in German).

In both learning environments, students work with digital mathematical tools, which have the potential to act as door openers in either direction (Müller, 2018). The implementation of digital tools in mathematics education has a long tradition in the US. This is not only about using such tools, but about actively shaping them and developing mathematical and informative content (Papert, 1993). Based on exploratory interview studies (Szücs, & Müller, 2013), the hypothesis that differences between the German and American cultural areas can be identified made regarding the level of school mathematics has been validated. These differences are relevant for bilingual education. Digital math tools can play a crucial role in this process. The educational theory of instrumental genesis can provide clues to the role of digital tools as a door opener to language and technology (Müller, 2018). Part of the instrumental genesis is the relation between cognition and artefacts (Verillon, & Rabardel, 1995), which act as linking points between communication and culture. The instrumental genesis is also covered by the Task-Technique-Theory (Drijvers, 2004; Kieran, & Drijvers, 2006), which is an anthropological theory in the first place without particular reference to digital mathematical tools. Another part of the instrumental genesis is the impact of digital tools on the construction of knowledge (Rieß, 2018), which also gives starting points for cultural aspects and communication.

RESEARCH AIMS

Considering the theoretical background and findings of explorative studies, three statements can be made and need to be proven. First, elementary school mathematics differ more between Germany and the US than do high school or university level mathematics. Second, the listed differences are important to bilingual mathematical learning environments and teachers need to be aware of them. Third, digital mathematical tools can be door openers to the foreign language and can promote deeper understandings of mathematical topics.

METHODOLOGICAL FRAMEWORK AND STUDY DESIGN

In order to identify the differences between American and German mathematics education which are relevant for bilingual education, a two-stage mixed-method design was chosen. Corresponding to the research aims, the use of digital mathematical tools attracted particular attention. The first stage had an exploratory character and essentially included qualitative instruments such as standardized interviews. German and American teachers were interviewed using a standardized interview schedule (Szücs, & Müller, 2013). The MISTI GTL participants were able to reflect on their own actions in class using video vignettes. The evaluation as part of an interpretive video analysis was based on Knoblauch, Schnettler, & Tuma (2010). At this stage, the aim was to formulate hypotheses based on the findings of the qualitative study. Five teachers of the GISB and four MIT teaching students participated in the interview study. The lessons of the MIT students were video recorded. The transcription guidelines for interviews and video recordings were standardized (Kuckartz, Dresing, Rädiker, & Stefer 2008). For the video recordings an inductive system of categories was develop (mathematical content, linguistic peculiarities, mathematical symbols …) Because of the standardized interview schedule, a deductive system of categories (mathematical working, notation, word meanings …) was used to analyse interview data.

Based on the first step, three hypotheses could be formulated (see section above). In a second quantitative step, teachers and teaching students were asked about the relevance of the differences that had been identified using a standardized online questionnaire. To date, 75 teachers and teaching students of the GISB and the MIT have participated in the online survey. All of them have experience with bilingual mathematical education either at the GISB or from the MITSTI GTL Germany project.

The online questionnaire includes 13 differences, which were identified in the first step (see first paragraph). All 13 items were followed by two additional items, each with aim of asking about relevance in the classroom and the use of digital mathematical tools (see Image1). In order to describe the panel more accurately, four motivational aspects were part of the questionnaire, too. These were Self-assessment (two items), Interest in mathematics education (three items), Interest in mathematics (three items), Self-efficacy (seven items). The items had been used and validated in earlier educational studies (Benölken, 2014). With respect to the scale level, non-parametric statistical tests such as Mann-Whitney-U-test were chosen for the data analysis. The data regarding the differences were collected in fourfold tables. Shifts in the tables were identified by using the exact Fischer test (for two sides). P-values were calculated by using the method of Agresti (1992) while using a program code similar to Langsrud, & Gesellensetter (n.d.). Regarding the number of statistical tests, a level of significance of 0.01 was chosen.

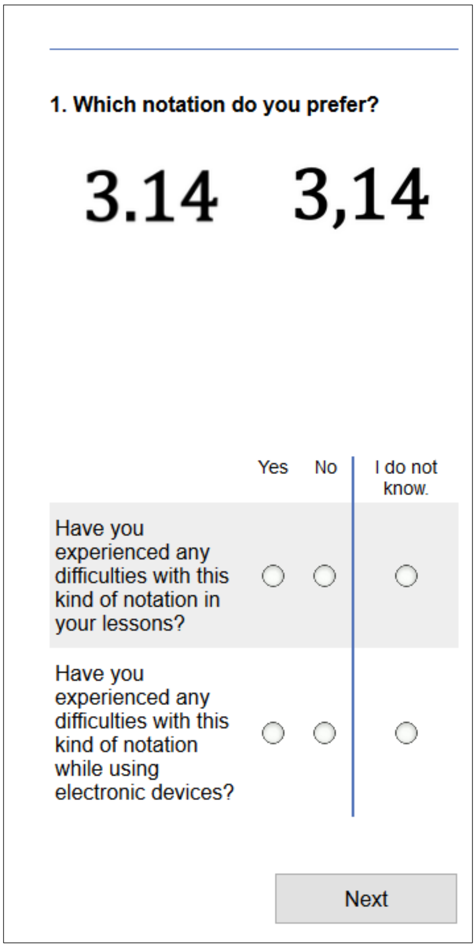

Image1: *Online questionnaire optimized for mobile devices. Shown is one of 13 differences with the two additional items about relevance for classroom and digital tools.*

RESULTS

In this article, we will present the results of the recent online survey. To compare the groups of teachers (and teaching students) four characteristic variables are of interest. Self-assessment, Interest in mathematics education, Interest in mathematics and Self-efficacy are important to the motivation of dealing with problems (Eccles et al., 1983). As shown in Tab. 1 there or no significant differences, neither between the groups of teachers (and teaching students) from Germany and the US nor between the two institutions. All teachers (and teaching students) are interested in mathematical topics and mathematics education. All participants score highly according to Self-assessment and Self-efficacy. Therefore, we can assume that all participants are highly motivated in what they do.

Variable	Cronbachs Alpha	ALL (75)	USA (65)	GER (10)	MIT (59)	GISB (16)
Self-assessment	0.755	3.15 (0.58)	3.15 (0.61)	3.15 (0.32)	3.15 (4.46)	2.88 (0.84)
Interest in mathematics education	0.766	3.21 (0.59)	3.22 (0.60)	3.2 (0.50)	3.21 (0.57)	3.04 (0.64)
Interest in mathematics	0.840	3.06 (0.67)	3.03 (0.69)	3.3 (0.43)	3.02 (0.54)	2.85 (0.99)
Self-efficacy	0.877	2.96 (0.55)	2.97 (0.58)	2.89 (0.33)	2.96 (0.47)	2.69 (0.75)

Tab. 1: *Aspects of motivation: Self-assessment (two items), Interest in mathematics education (three items), Interest in mathematics (three items), Self-efficacy (seven items). Shown are mean and standard deviation of scale from one to four.*

Four of the 13 differences included in the online questionnaire have been shown to be important from the teacher's point of view. German and US American teachers (and teaching students) use different symbols or use different mathematical terms and definitions. The following fourfold tables include highly significant values and show differences between German and US American teachers (and teaching students). Tab. 2 shows the different uses (and meanings) of Point and Comma. For 59% of all teachers (and teaching students) this causes problems in bilingual math lessons. Furthermore, 71% agreed that this makes a difference when using digital mathematical tools.

	GER	USA	
Point (3.14)	2	58	60
Comma (3,14)	8	7	15
	10	65	75

Tab. 2: *Example of differences between German und US American school mathematics in category notation (teachers view). Fourfold table with significant shift (p=0.0000141, exact Fischer test).*

According to Tab. 3, teachers use different symbols for the right angle in geometric drawings. It is an interesting fact, that US Americans have a more sophisticated way of naming (and ordering) basic geometric figures (triangle, quadrilateral, pentagon …). In German, it is common sense to call the figures *Dreieck, Viereck, Fünfeck* …. Especially, naming the quadrilateral can be problematic in bilingual math classes as far as teachers agree (55%). Teachers report of difficulties to find the right geometric figure while using digital mathematical tools (48%).

	GER	USA	
Right angle symbol square	2	56	58
Right angle symbol quarter circle plus dot	8	9	17
	10	65	75

Tab. 3: *Example of differences between German und US American school mathematics in category symbols (teachers view). Fourfold table with significant shift (p=0.0000502, exact Fischer test).*

As shown in Tab. 4 there is a difference in dealing with third roots of negative values. US American teachers (and teaching students) like to distinguish between odd and even roots. German teachers except only positive values for any roots. Most teachers (55%) have had problems with these definitions in bilingual math lessons. For 61% of all teachers this distinction is relevant for the use of digital mathematical tools.

	GER	USA	
$\sqrt[3]{-8}$ is defined	1	51	52
$\sqrt[3]{-8}$ is not defined	9	14	23
	10	65	75

Tab. 4: *Example of differences between German und US American school mathematics in category mathematical working (teachers view). Fourfold table with significant shift (p=0.0000526, exact Fischer test).*

Many teachers (47%) have experienced difficulties in using mathematical terms for the solutions of quadratic equations in their bilingual math lessons. German teachers prefer a sum of the final term where US American teachers (and teaching students) prefer a fraction (see Tab.5). This problem is relevant to at least 51% of all teachers regarding their use of digital mathematical tools.

	GER	USA	
Solutions of quadratic equations given as a fraction	1	54	55
Solutions of quadratic equations given as a sum	9	11	20
	10	65	75

Tab. 5: *Example of differences between German und US American school mathematics in category mathematical working (teacher's view). Fourfold table with significant shift (p=0.0000114, exact Fischer test).*

DISCUSSION

First, we have to admit that 75 questionnaire responses might be insufficient to allow us to draw meaningful conclusions. In particular, the data seems too limited to identify differences between German and American school mathematics with a degree of certainty. Nevertheless, such differences do seem to exist and the survey results give hints about where we may find them. It is necessary to increase the number of responses and invite more teachers from both Germany and the United States to participate. In the current sample, there are motivated and highly educated teachers and teaching students from two high-performing educational institutions. In order to obtain an even more multifaceted picture, it would be wise to invite teachers from different schools with different backgrounds to participate in the study. At this point, we believe the chosen statistical methods (non-parametric test, exact Fischer test) are suitable for the type of data and the numbers. Therefore, the results are accurate and provide a foundation for the next steps in an ongoing research process. More data will be collected and analyzed.

Early results show that the teachers are aware of differences in the school mathematics in the two countries. These differences are important for bilingual learning environments and they might present difficulties that teachers will have to deal with. It is possible to distinguish differences in the language (such as different words or false friends) and more culturally related differences. Symbols (see Tab. 3) and procedures can also differ. Sometimes, the different traditions open a door to a deeper understanding of the content, such as the definition of the third root of negative numbers (see Tab. 4). Another example is the mathematical representations for the solutions of quadratic equations (see Tab. 5).

For most teachers, these differences can be dealt with by using digital math tools. The tools show the differences and give feedback on them. They can be translators for the language and for the mathematics. A theoretical approach to this practical experience could be the instrumental genesis (Rabardel, 2002; Wygotski, 1985). Communication and cultural issues need to be addressed more clearly. In a bilingual learning environment, a digital tool can be a door opener between classroom language and first or commonly used language. The tool can take the role of a mediator. This is also related to cultural aspects. The digital tools can help to investigate the differences and obtain a deeper understanding of the mathematical concepts. This factor might be relevant even to regular math classes where digital mathematical tools are used, because of the design and programming of the digital tools, which refer to their cultural background.

REFERENCES

Agresti, A. (1992). A Survey of Exact Inference for Contingency Tables, *Statistical Science*, 7, 131-153.

Anger, C., Plünnecke, A., & Schüler, M. (2018). *INSM-Bildungsmonitor 2018. Teilhabe, Wohlstand und Digitalisierung*. Köln: Institut der deutschen Wirtschaft.

Balley, S. (2012). *Doodle Yourself Smart… Geometry. Over 100 Doodles and Problems to solve.* San Diego: Thunder Bay Press.

Barwell, R. (2003). Linguistic Discrimination: An Issue for Research in Mathematics Education. *For the Learning of Mathematics*, 23 (2), 37-43.

Benölken, R. (2014). Begabung, Geschlecht und Motivation. *Journal für Mathematik-Didaktik*, 35, 129-158.

Breaux, A., & Whitaker, T. (2015). *Quick Answers for Busy Teachers. Solutions to 60 Common Challenges.* San Francisco: Jossey-Bass.

Drijvers, P. (2004). Learning algebra in a computer algebra environment. *The International Journal of Computer Algebra in Mathematics Education*, 11 (3), 77-89.

Eccles, J., Adler, T. F., Futterman, R., Goff, S. B., Kaczala, C. M., Meece, J., & Midgley, C. (1983). Expectancies, values and academic behaviors. In J. T. Spence (Eds.), *Achievement and achievement motives* (pp. 26-43). San Francisco: Freeman.

Felton-Koestler, M. D., & Koestler, C. (2017). Should Mathematics Teacher Education Be Politically Neutral? *Mathematics Teacher Educator*, 6 (1), 67-72.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: Creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk*, 6 (1 & 2), 115-132.

Hyun, T. (2006). *Acing the SAT Subject Tests in Math. Level 1 and Level 2.* Thousand Oaks: Greenhall Publishing.

Jourdain, P. E. B. (2007). *The Nature of Mathematics.* New York: Dover Publications.

Kaplan, S. (2009). *SAT Raise Your Score (Even More) Handbook*. Fort Lauderdale: Kaplan Inc.

Kieran, C, & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning*, 11 (2), 205-263.

Knoblauch, H., Schnettler, B., & Tuma, R. (2010). Interpretative Videoanalysen in der Sozialforschung. In S. Maschke, L. Stecher (Eds.), *Enzyklopädie Erziehungswissenschaft Online (EEO), Fachgebiet Methoden der empirischen erziehungswissenschaftlichen Forschung*. Weinheim: Juventa.

Kuckartz, U., Dresing, T., Rädiker, S., & Stefer, C. (2008). *Qualitative Evaluation – Der Einstieg in die Praxis*. Wiesbaden: VS.

Lappan, G., Fey, J. T., Fitzgerald, W. M., Friel, S. N., & Phillips, E. D. (2009). *Connected Mathematics 2. Filling and Wrapping. Three-Dimensional Measurement*. New Jersey: Pearson.

Langsrud, O., Gesellensetter, R. (n. d.). *Fishers´s Exact Test*. Available online: https://www.langsrud.com/stat/Fishertest.htm (June 30 2019).

Müller, M. (2018). Digitale Werkzeuge als (Sprach-)Brücke im bilingualen Mathematikunterricht – Erste Ergebnisse der videogestützten Evaluation des Projektes MIT Global Teaching Lab am SFZJ. In Fachgruppe Didaktik der Mathematik der Universität Paderborn (Eds.), *Beiträge zum Mathematikunterricht 2018* (pp. 1279 – 1282). Münster: WTM.

Novotná, J., & Moraová, H. (2005). Cultural and linguisticproblems in the use of authentic textbooks when teaching mathematics in a foreign language. *Zeitschrift für Didaktik der Mathematik*, 37 (2), 109-115.

Papert, S. (1993). *Mindstorms Children, Computers and Powerful Ideas*. New York: Basic Books.

Rabardel, P. (2002). *People and Technology*. (Author-approved translation by

"Les Hommes et les Technologies" (1995)).

Rolka, K. (2004). Barrieren für den Einsatz einer Fremdsprache im Mathematikunterricht. In A. Heinze, S. Kuntze (Eds.), *Beiträge zum Mathematikunterricht 2004* (pp. 473-476). Hildesheim: Franzbecker.

Rieß, M. (2018). *Zum Einfluss digitaler Werkzeuge auf die Konstruktion mathematischen Wissens. Studien zur theoretischen und empirischen Forschung in der Mathematikdidaktik*. Wiesbaden: Springer.

Szücs, K., & Müller, M. (2013). Schwierigkeiten beim Einsatz digitaler Werkzeuge als Reaktion auf bilinguale Unterschiede. In G. Greefrath; F. Käpnick; M. Stein (Eds.), *Beiträge zum Mathematikunterricht 2013* (pp. 994-997). Münster: WTM.

Trimble, M. (2018). *The 10 Best U.S. States for Education. U.S. News & World Report*. Available online: https://www.usnews.com/news/best-states/slideshows/10-best-states-for-education?int=undefined-rec&slide=11 (February 27 2018)

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. European Journal of Psychology of Education, 10 (1), 77-101.

Wygotski, L. S. (1985). Die instrumentelle Methode in der Psychologie. In J. Lompscher, L. Wygotski (Eds.), *Ausgewählte Schriften* (Band 1, pp. 309-317). Köln: Paul-Rugenstein.

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