# A QUANTITATIVE STUDY ON THE USAGE OF DYNAMIC GEOMETRY ENVIRONMENTS IN DANISH LOWER SECONDARY SCHOOL

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*The paper describes the development and analysis of a quantitative study investigating to what extent the potentials of dynamic geometry environments (DGE), in relation to mathematical reasoning competency, are utilized in lower secondary school in Denmark. The study entails a questionnaire, which was developed on the basis of an extensive review that uncovered four potentials of DGE in relation to reasoning competency: feedback; dragging; measuring; tracing. 220 Danish lower secondary mathematics teachers completed the questionnaire. Analysis indicates that the potentials of measuring and dragging are utilized to some degree, feedback to a lesser degree, while tracing is almost non-existent. Furthermore, there are signs that DGE is used as a substitute for the paper and pencil environment to solve tasks that were originally designed for paper and pencil. Possible improvements of praxis are discussed, and the integration of the results in further research is elaborated upon.*

*Keywords: DGE, GeoGebra, reasoning competency, survey*

## INTRODUCTION

Digital technologies are widely implemented in Danish primary and lower secondary school, in part due to heavy investments from the Ministry of Education over the last couple of decades (e.g. Undervisningsministeriet, 2015). Consequently, the availability and usage of digital technologies has become commonplace in mathematics education at all levels in Denmark. In primary and lower secondary school the dynamic geometry software GeoGebra is particularly popular. This can be considered a positive outcome, since many studies highlight the affordances of dynamic geometry environments (DGE hereafter), as potentials in supporting students' development of mathematical reasoning, conjecturing and in proving (e.g. Jones, 2000; Leung, 2015; Edwards et al., 2014). In the optic of the Danish competency based KOM-framework (Niss & Højgaard, 2011) these potentials are related to the reasoning competency. However, it seems that the manner in which the affordances are utilized (if they are utilized) is essential (Jones, 2005). Part of the research motivation stems from an underlying assumption that the potentials of DGE, such as GeoGebra, are not utilized in Danish lower secondary school, even if the software is indeed popular. The assumption is that DGEs are predominantly used only as a substitution for the paper and pencil environment.

No previous quantitative research exists regarding how DGE is actually used in Danish lower secondary school. The author of this paper could not find quantitative results on the matter internationally either. In order to gain insight into this area, the paper aims to investigate how DGE is actually used in lower secondary school in Denmark, with particular interest in the potentials of DGE in relation to reasoning competency, by posing the following research question:

*To what extent do Danish lower secondary school mathematics teachers utilize the potentials of DGE, in relation to mathematical reasoning competency, in their teaching?*

## THEORETICAL CONSTRUCTS FOR DEVELOPING THE QUESTIONNAIRE AND ANALYSING THE RESULTS

The reasoning competency (RC hereafter) is one of eight mathematical competencies in the KOM framework (Niss & Højgaard, 2011), which has shaped the curriculum at all levels of mathematics education in Denmark. The framework has also influenced mathematics education in other parts of the world (Niss, Bruder, Planas, Turner & Villa-Ochoa, 2016).

The RC constitutes the ability to create and carry out informal and formal arguments and to follow and evaluate argumentation put forward by others. Additionally, to understand what a mathematical proof is, including the role of counter-examples. It also comprises the ability to discern proof from other forms of mathematical reasoning such as example-based explanations and being able to develop such arguments into formal proof. Furthermore, it is about creating and justifying mathematical claims in general, such as answers to questions and problems (Niss & Højgaard, 2011). As argued in Højsted (2019) the RC also entails the first steps of the proof process, exploration and conjecturing.

In order to formulate questions that investigate to what extent the potentials of DGE in relation to RC are being utilized, it is first necessary to describe what these potentials are. To this end, the results from an extensive review into the potentials of DGE in relation to supporting students' development of RC were used (Højsted, 2019). The meaning of "potentials" in this context are affordances of DGE, which are not available in other typical mathematics education tools such as paper and pencil. Four potentials were discovered: feedback; dragging; measuring; tracing. It is particularly in relation to conjecturing that they were deemed to be potentials. A short elaboration of the potentials is presented here.

Since DGEs mimic theoretical systems, typically Euclidean Geometry, they offer an environment in which only constructions that abide by the rules of the theoretical system can be constructed (Balacheff & Kaput, 1997). Therefore, the DGE inherently provides **feedback** to the user, e.g. by not allowing for imprecise measurements or constructions that contradict Euclidian theory. Figures constructed in the environment can also be manipulated dynamically by **dragging**. The elements of a dynamic figure are locked in a hierarchy of dependencies, which decide the outcome of a dragging action (Hölzl, Healy, Hoyles & Noss, 1994). The dependencies are in fact the theoretical properties of the figure, which are decided by the construction method and by the theory of Euclidean Geometry governing the system. These properties remain invariant during dragging, which allows for discovery of the theoretical properties of constructions in Euclidian Geometry. Therefore, dragging in DGE can link the perceptually mediated appearance of a figure to the theoretical properties of the figure, which Laborde (2005b) refers to as moves from the spatiographical level to the theoretical level. In "robust" constructions, the properties are conserved during dragging, whereas in "soft" constructions not all properties are conserved (Healy, 2000; Laborde, 2005a). Most DGEs, including GeoGebra, contain **measuring** tools, which allow the students to find the measures of constructions. When figures are manipulated dynamically, the measurements are updated instantly. Therefore, it is possible to observe invariant relationships between measures (Olivero & Robutti, 2007). Additionally, the possibility of **tracing** an object during dragging, offers the possibility of visualizing underlying invariant relationships of the construction, for instance in soft constructions in which a property is maintained when dragging is performed in a particular manner (Baccaglini-Frank & Mariotti, 2010).

Developing the questionnaire consisted of the careful back and forth process of formulating questions that would investigate whether these DGE potentials related to RC were being utilized, while at the same time being formulated concise and clearly enough to be understood by lower secondary school teachers.

## METHOD

A web-based questionnaire was developed for lower secondary school mathematics teachers. The questionnaire consisted of 19 multiple-choice questions, in which a five-point Likert scale was used, as well as four open-ended questions and some background information questions. Results from nine multiple-choice questions are presented in this paper. The questionnaire was distributed through two platforms: (1) a link for self-enrolment to the questionnaire was sent through the email list of the Danish national maths counsellor network with participation from 97 of the 98 municipalities of Denmark. (2) a link for self-enrolment was posted on two popular Facebook groups for Danish lower secondary school teachers, "GeoGebra Hangouts" and "We who teach in lower secondary school".

In order to combat the usual problem of low participation response and completion in web-based questionnaires, steps were taken with regards to design and language (Fan & Yan, 2010), and a monetary incentive was included (Göritz, 2010) in the shape of a lottery with a prize of DKK 4000,- to a single winner. Respondents were required to add their names and email in order to take part in the lottery. This also gave the opportunity to check for double entries.

GeoGebra is by far the most popular DGE in lower secondary school in Denmark. Consequently, it was decided, for the sake of clarity, to formulate the questionnaire directly towards GeoGebra usage instead DGE usage. However, GeoGebra is a system that includes not only the geometry environment, but also CAS, spreadsheet etc. Therefore, in every question it was explicitly specified that the question was related to the "geometry part" of GeoGebra.

## RESULTS AND ANALYSIS

Ten questions are included in this paper. For the sake of clarity, the questions are given numbers 0-9, even though it was not their actual number in the questionnaire.

Albeit not directly related to the research question, it is relevant to mention the results from question 0, which was a background question, in order to shed some light on the population of the study.

| N=220 | 0. How often do you and your students use the geometry part of GeoGebra in the mathematics class? |
|---|---:|
| Every week | 35.9% |
| Every other week | 39.5% |
| Once a month | 17.3% |
| Every other month | 4.1% |
| Less | 3.2% |

**Table 1. A background question on rate of GeoGebra usage**

Based on the results from question 0, in combination with the questionnaire being distributed on the GeoGebra Facebook group, and that participation in the survey was through self-enrolment, it is reasonable to assume, that the respondents in the survey are teachers who actually use GeoGebra regularly in their mathematics classes. Perhaps it is even reasonable to assume that many of the respondents are teachers who are somewhat enthusiastic about GeoGebra.

Some questions were posed to find out which types of tasks the teachers give to their students. This was partly done in order to investigate the hypotheses that GeoGebra is merely used as a substitute for the paper and pencil environment. Partly, the results from these questions would also indirectly indicate whether the potentials are being utilized or not. For instance, if the students primarily work on tasks in GeoGebra, which were made for paper and pencil geometry, it is a strong indication that potentials such as dragging are not involved in the tasks, since dragging is not possible in paper and pencil geometry.

| N=220 | Always | Frequently | Occasionally | Rarely | Never | Don't know |
|---|---|---|---|---|---|---|
| 1. Do students work on tasks in the geometry part of GeoGebra, which were originally made for paper and pencil geometry? | 14.5% | 38.6% | 35.5% | 9.1% | 2.3% | 0.0% |
| 2. Do students work on tasks that were originally made for paper and pencil geometry, which you have adapted to be used in the geometry part of GeoGebra? | 6.8% | 34.1% | 40.5% | 11.4% | 5.5% | 1.8% |

**Table 2: Questions 1 and 2 regarding types of task**

As can be seen from the results to question 1, a large part of the students seem to be working on such paper and pencil tasks, which indicates that the hypotheses holds some truth. Although, the results from question 2 show that many teachers adapt the paper and pencil tasks. Therefore, it is possible that some teachers adapt the paper and pencil tasks in such a way, that some of the four potentials, which are linked to the RC, are utilized.

A premise for investigating theoretical properties of figures by dragging is that the students hold some understanding of the difference between free and non-free objects. Question 3 aimed at finding out if students work on understanding this difference. It was expected that some teachers might not know what free and non-free objects meant. Therefore, a short video was shown prior to question 3 along with accompanying text to explain, in which it was illustrated how two free points could be dragged, while their constructed midpoint could not. The predominant answers were occasionally (35.9%) and rarely (30.5%), which indicates that it is not something a lot of time is spent on. Moves from the spatiographical level to the theoretical level (Laborde, 2005b) are unlikely to occur if the students do not realise that the perceptually mediated appearance of the figure during dragging in DGE, is linked to the theoretical properties of the figure.

| N=220 | Always | Frequently | Occasionally | Rarely | Never | Don't know |
|---|---|---|---|---|---|---|
| 3. Do students work on understanding the difference between free objects and non-free objects in the geometry part of GeoGebra? | 5.0% | 13.2% | 35.9% | 30.5% | 15.0% | 0.5% |
| 4. Do students work on constructing so-called "robust" figures, i.e. figures that retain certain properties, when the free objects of the figure are dragged, in the geometry part of GeoGebra? | 1.4% | 24.1% | 46.8% | 17.7% | 10.0% | 0.0% |
| 5. Do students work on investigating figures, to see which properties are maintained during dragging in the geometry part of GeoGebra (e.g. that | 5.0% | 33.2% | 47.3% | 10.9% | 2.7% | 0.9% |

| | | | | | | |
|---|---|---|---|---|---|---|
| the medians of a triangle meet at a point)? | | | | | | |

**Table 3: Questions 3-5 concerning activities focusing on theoretical properties and dragging**

Results from question 4 show that more time is used on constructing robust figures. Considering that understanding the difference between free and non-free objects is necessary to construct robust figures, this result is somewhat contradictory, but perhaps it is because the teachers discern between the implicit understanding needed to construct robust figures, and time spent explicitly focusing on the difference between free and non-free objects, as two distinct activities. Question 5 was aimed directly at finding out if **dragging** is being used as a means to investigate the theoretical properties of figures. An example was given of properties that can remain invariant under dragging (the medians of a triangle meeting at a point), because it was expected that some teachers might not understand the meaning of "investigating figures, to see which properties are maintained during dragging". The teachers predominantly responded that their students occasionally (47.3%) or frequently (33.2%) engage in such activity, which is a higher rate than in questions 3 and 4, and a higher rate than expected beforehand. It is also somewhat surprising in light of the answers to question 0, since tasks which were originally made for a paper and pencil environment would not include prompts requiring dragging to investigate theoretical properties of figures. Nevertheless, the result does suggest that the potential of dragging is used regularly by students to investigate the properties of figures. It indicates that potentials of DGE linked to the exploration and conjecturing part of the RC are utilized to some degree.

| N=220 | Always | Frequently | Occasionally | Rarely | Never | Don't know |
|---|---|---|---|---|---|---|
| 6. Do students work on measuring of figures in the geometry part of GeoGebra? | 12.7% | 66.8% | 18.6% | 0.9% | 0.5% | 0.5% |
| 7. Do students work on measuring of figures combined with dragging in order to investigate how the measures change in the geometry part of GeoGebra (e.g. that the sum of angles in a triangle remains 180°)? | 5.9% | 41.4% | 43.6% | 5.9% | 3.2% | 0.0% |

**Table 4: Questions 6 and 7 on measuring**

Looking at results from question 6 and 7, we can see that **measuring** features at a relatively high rate compared to dragging. In fact, it was also the highest rate compared to all other questions, also those that are not included in this paper. In question 6, 66.8% of teachers report that their students frequently work with measuring of figures. Question 6 does not reveal what sort of measuring is done. For instance, if it is the sort of measurement of figures, which might as well be done with paper and pencil. However, question 7 gives more nuanced insight by asking more specifically regarding the measuring activity. The teachers report that their students frequently (41.4%) and occasionally (43.6%) work with measuring in combination with dragging in order to investigate invariant measurements.

| N=220 | Always | Frequently | Occasionally | Rarely | Never | Don't know |
|---|---|---|---|---|---|---|

| | | | | | | |
|---|---|---|---|---|---|---|
| 8. Do the students work on tasks in the geometry part of GeoGebra, where they are asked to try to construct figures, which cannot be constructed? | 0.9% | 10.5% | 45.5% | 32.7% | 10.5% | 0.0% |
| 9. Do the students work with the trace command in GeoGebra? | 0.5% | 1.4% | 16.4% | 34.1% | 45.0% | 2.7% |

**Table 5. Questions 8 and 9 on non-constructable figures and tracing**

One way of utilizing the **feedback** potential of DGEs is in tasks that instigate the students to construct non-constructable figures. Question 8 reveals that 45.5% of teachers reported that their students occasionally work on such tasks, while 32.5% responded that their students rarely do so. Of course, there are also other ways of utilizing the feedback potential.

The results from question 9 indicate that possibility of **tracing** is unfamiliar to the teachers. The teachers mainly responded that their students rarely (34.1%) and never (45%) work with the trace command. It can be concluded that the potential of tracing to visualize underlying invariant relationships of constructions in conjecturing activities that yield development of the RC, is yet to be utilized, for example in maintaining dragging tasks (Baccaglini-Frank & Mariotti, 2010).

## CONCLUDING DISCUSSION

The results indicate that the potentials of DGE in relation to RC, in particular measuring and dragging, are to some degree utilized in Danish lower secondary school. The particular utilization of the feedback potential in non-constructable tasks seems also to present, although at a lower rate. It may be regarded as a positive result, since especially dragging is considered to be a key feature of DGE that affords a visual representation of invariant geometrical phenomenon allowing for generalization, reasoning and conjecturing (e.g. Arzarello, Olivero, Paola & Robutti, 2002; Laborde, 2001; Baccaglini & Mariotti, 2010; Edwards et al., 2014). However, the results from question 1 gives rise to further questions about the actual utilization of these potentials. The results from question 3 indicate that the understanding of locked and free objects is not a particular focal point. As mentioned previously, the locked and free objects are the manifestations of the theoretical properties of figures, which are mediated perceptually in DGE during dragging, thereby potentially linking the spatiographical and theoretical levels (Arzarello et al., 2002). Perhaps the lack of focus on this basic understanding can be linked to the students remaining at the spatiographical level in the dragging activities. If that is the case, the conjectures will not be anchored in the theoretical properties of the figures, but at the spatiographical level. Additionally, it seems that the tracing command is rarely used, which implies that dragging with trace activated in order to highlight underlying invariant relationships is not presently utilized.

In order to improve the utilization of the potentials in relation to RC, it is necessary, first and foremost, to increase the availability of tasks that are actually made for utilizing DGE potentials. Secondly, even if some teachers adapt paper and pencil tasks, it would at least be beneficial for them to use guidelines to this end. Several DG task quality models have been suggested (e.g. Trgalova, Jahn & Soury-Lavergne, 2009; Trocki & Hollebrands, 2018). It cannot be expected that teachers, without any guidance, will adapt paper and pencil tasks into somewhat specialized tasks that utilize the potentials of DGE in relation to RC, such as soft construction tasks that are solved by using the maintaining dragging model (Baccaglini-Frank & Mariotti, 2010). Additionally, it is necessary to highlight the mathematical meaning of free and locked objects in instruction and tasks, in order to support the students in linking the spatiographical and theoretical levels. This is important, since awareness of the theoretical relationship between elements of a figure, which is mediated perceptually by DGEs as invariants during dragging, is a premise for investigating figures in conjecturing and reasoning activities, which are activities that are characteristics of RC.

## FURTHER RESEARCH: INTEGRATION OF FINDINGS INTO PRACTICE

The insights gained from this research are integrated into another ongoing related project, in which the aim is to develop principles for the design of didactic sequences that utilize the potentials of DGE in relation to students' development of the RC (Højsted, 2019). The survey results suggest that the didactic sequence should include initial instruction and tasks aimed at supporting the students in understanding the theoretical underpinnings of locked and free objects, so that they can interpret the theoretical aspects of figures, which decide how the figure reacts when elements of the figure are dragged. Implementing "construction tasks" as coined by Mariotti (2012) may support this process. It is also clear that the sequence must take into account the likely lack of teacher and student knowledge regarding the trace command and tasks instigating the construction of non-constructable figures.

Additional studies are needed, in order to further complement the results presented in this paper. By supplementing the quantitative approach used in this study with a qualitative approach, in the form of interviews with some of the teachers from the study, it is possible to get a more detailed and nuanced account of the way feedback, dragging, measuring and tracing are being used in Danish lower secondary school. It is also an opportunity to unveil some of the reasons that lie behind the utilization choices of the teachers. The next step of this ongoing research is to categorize teachers based on their answers in the survey, particularly in some of the open-ended questions, and to interview teachers from each category, in order to gain deeper understanding of how each category of the teachers actually use GeoGebra in Danish lower secondary school.

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Published in: 14th International Conference on Technology in Mathematics Teaching 2019



**DOI:** 10.17185/duepublico/70789
**URN:** urn:nbn:de:hbz:464-20191122-113232-9