STUDENTS LEARNING WITH DIGITAL MATHEMATICAL TOOLS – THREE LEVELS OF INSTRUMENTAL GENESIS

Swantje Schmidt

Friedrich-Schiller-Universität Jena; swantje.schmidt@uni-jena.de

Matthias Müller

Technische Universität Braunschweig, Friedrich-Schiller-Universität Jena; mat.mueller@tu-braunschweig.de

There is a widespread aphorism that if your only tool is a hammer every problem looks like a nail. The solution to mathematical problems does not always follow the same pattern but may require the use of multiple tools or tool components. It is therefore important as a learner to be aware of the variety of possibilities that are available and to be able to select and use the optimal tool in a problem situation. However, a previously unknown tool is not directly operational for a learner. For the solution of existing problems, only tools whose operation is known can be used. This text examines the ways in which differences in the progress of instrumental genesis are expressed and when digital tools support learners' thinking. This raises the following research question: How far must instrumental genesis be advanced until it can be used with confidence. Three different mathematical problems and a think-aloud-study have been used to identify different steps of the instrumental genesis.

Keywords: instrumental genesis, math lessons with computers, think aloud method

INTRODUCTION

As technology advances in society, digital tools are becoming more commonplace in school life, especially in mathematics education. Since 2011 the use of the CAS calculator in the 9th and 10th grades has been required in Thuringia and since 2014 its use has been an integral part of the final exams. It is therefore important for teachers to understand how the process of learners acquiring knowledge about digital tools takes place. Digital tools have many advantages, but the use of a tool can be quite difficult in the beginning, especially if the user interface is complicated (Barzel, 2011).

The instrumental genesis of teachers has recently become a topic of discussion in educational research, for example, in conjunction with dynamic geometry software (Alqahtani & Powell, 2017), and with reference to the teacher's influence on the instrumental genesis of learners in instrumental orchestration (Trouche, 2004). In addition, the instrumental genesis of non-digital tools has been examined in contexts such as historical drawing instruments (van Randenborgh, 2015) or the textbook as a tool for learners (Rezat, 2010). In both digital and non-digital contexts, writers also included the social influence on the respective instrumental genesis. In this text, only the process of the instrumental genesis of digital tools is of interest; social influences are not considered.

In 2018, Rieß brought underlined the importance of the instrumental genesis from Rabardel (2002) associated with the research of concept acquisition of students and intensively explored the connections between a tool and the formed concepts. He juxtaposed and linked together several relevant theories of the last decades and therefore represents the main source of the present work. The theoretical introduction is followed by the presentation of the three mathematical problems used in the study. Afterwards, the research question will be examined and answered with the help of the thinking aloud Method (Lewis, 1982/ Düsing, 2014).

THEORETICAL BACKGROUND

The main basis of this study is formed by theories about the instrumental genesis between learners and digital tools. Particular interest is given to the theories of Rabardel (2002), Verillon & Rabardel (1995) and Béguin & Rabardel (2000), as well as Rieß (2018).

Instrumental genesis consists of two complementary aspects. The instrumentation works from the instrument towards the subject, as the subject evolves already existing schema and acquires new ones during the learning activity while dealing with the artifact. There are usage schemes, which affect the handling of the artifact, and instrument-mediated-action schemes, which help to solve problems. The instrumentalization process runs from the subject towards the instrument as the subject changes/ adapts to/ develops the instrument or parts of it to make it work for the situation (Rabardel, 2002).

To create a category system for the qualitative analysis the terms artifact, instrument and tool were defined, because they have different uses in the language. *Artifacts* are (not necessarily material) objects that have been man-made or altered for purposeful, completed actions, primarily with the aim of solving a problem. In the hands of a subject, artifacts become an *instrument* in the course of instrumental genesis, in that the subject assigns to or changes the properties of the artifact (instrumentalization) and acquires and develops schema in parallel (instrumentation) (Verillon & Rabardel, 1995). *Tools* are permanent instruments, which means they are artifacts that are permanently linked to specific properties and schemes (Rieß, 2018).

The second important theory is the didactic tetrahedron - which is based on Chevallard's 1982 didactic triangle - with the corners being pupil, teacher, instrument and mathematics. In this theory, all connections between these corners are significant for the learning process. The triangle of pupil-instrument-mathematics is of the highest interest because it includes the process of instrumental genesis. (Rieß, 2018).

PRESENTATION OF THE EXEMPLARY PROBLEMS

Three example problems from different mathematical fields have been chosen for this research. Each of them can be solved with or without technology. For each problem, an Excel- or GeoGebra-file was created to use in the study.

Task 1 - Geometry: Tetrahedral cross-section with the minimal circumference

Given: You have a regular tetrahedron with the edge length 1 unit of length in front of you, which is composed of 2 cardboard parts. There is now a rubber band stretched so that it touches each side of the tetrahedron. Initially, the band lies on the centers of the 4 edges that it spans.

- A: Specify the length of the rubber band in this position.
- B: Determine the shortest distance around the tetrahedron, so that the band goes over all surfaces. Is that the only one? Explain your decision.
- C[1]: Describe what all rubber band positions with the length of 2 units have in common.

Figure 1: Tetrahedron with a rubber band.

[1] Task 1C was only given as help if the student can`t figure out the solution of task 1B.

The length of the rubber band in this position is 2 units of length, which is the shortest distance. This can be explained using intercept theorems. To solve the problem with technology, GeoGebra can be used to construct a tetrahedron or its net (see Figure 3). A scrollbar can conclude both forms.

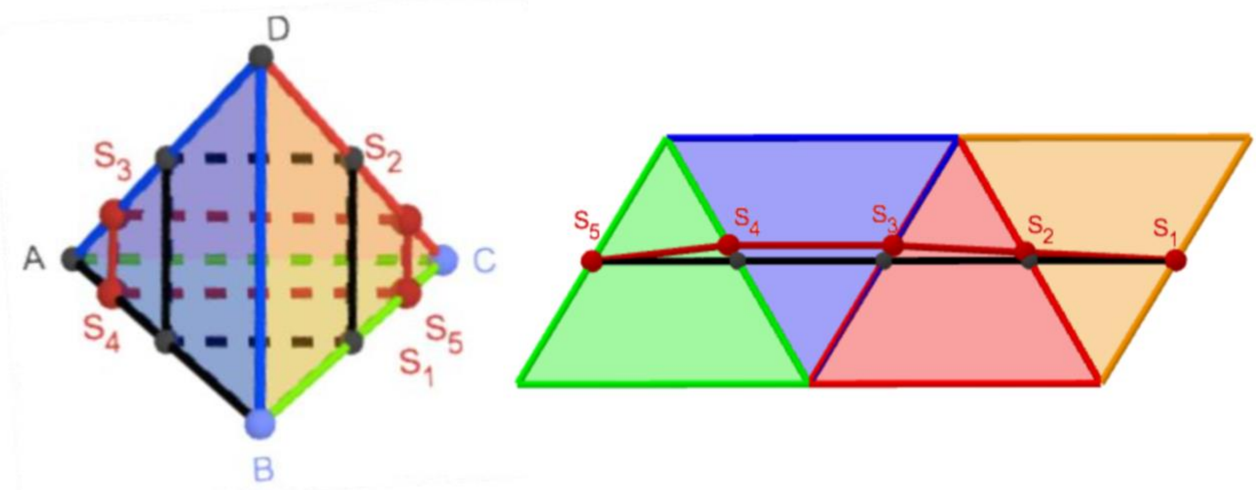

Figure 3: GeoGebra Screenshot of Tetrahedron with different lines on the surface and the unfolded Tetrahedron net in the plane.

Task 2 - Stochastic: Double dice

Given: A double dice (or two different colored dice) is thrown 100 times. Each time the sum of the two numbers on the dice is written down.

- A: Indicate all possible events.
- B: Determine the probability of the events.
- C: Determine the distribution of the events.

Figure 2: Double dice.

The sum of two dice can be anything between 2 and 12. Sum 7 has the highest probability, 2 and 12 have the lowest. Excel can help to calculate probabilities or even create a simulation.

Task 3 – Algebra / Number theory: Unequal products

Given:

x	3	4	5	6	7
y	7	6	5	4	3
x + y					
x · y					

- A: Fill in the table.
- B: What do you notice?
- C: Is that always the case? Explain your decision. Use equations or drawings.

There are different things to note in the table. For example, the product from a + 1 and a – 1 (with a ϵ ℕ) is always less than the square number of a. In fact, the difference is always 1. That can be justified with binomial formula. Technology can help in different ways. A spreadsheet can be used to get a large number of examples very quickly. A computer algebra system, such as GeoGebra, can solve equations and GeoGebra can also be used to create a geometrical solution.

METHODOLOGICAL FRAMEWORK AND STUDY DESIGN

To obtain information about the participants' thoughts during an exercise the thinking aloud method was chosen. The study included ten interviews with five students age 14 to 16. Each of them worked on two of the example problems. While they were working on the task, participants shared their thoughts with the researcher. All the interviews were audio-recorded. The interview guide is based on the theories of Düsing (2014) and is structured in five sections: 1. personal data and introduction of the method, 2. exercise and assurance of understanding, 3. presentation of the task, 4. working on the task, 5. debriefing.

During the study, the computer programs Excel and GeoGebra were approved as aids. To obtain information about how far the instrumental genesis between the students and the instruments (in this case either of the programs) was advanced, each "working on the task" section was split. At first, participants were asked to work on the task and had a free choice of whether they used the technology or not. Secondly, if they had not done so far, they were asked to try and solve the problem by using the computer. Finally, a GeoGebra-file or Excel-file created by the authors was given to them. They should try to use or understand it and figure out how it works.

Category	Code	Definition	Example (Anker)
(F) Use as an instrument	1 Yes, mostly successful	Use of various functions; File brings new insights to the task; Planning and reflection of the procedure	"(...) everything is calculated and that is why there are a 100. And here are several values (student indicates sums in the right column of the simulation) and if this value could be higher (student indicates formula in cell H17), this would finally run."
	2 Yes, sometimes successful	The attempt brings little or no new knowledge; Plan can only be implemented in stages	"I just wanted to find out how long the black one is, but I do not remember if that is already stored somewhere. If you shift red line, that is what is shown here, and named length of the line."
	3 No, never successful	No idea how the task with digital tools can be implemented or plan exists, but cannot be implemented	"As if someone had just diced and on the left, we have the number, and the dices, dice 1 and dice 2, and finally the sum of the eyes. And again the table including all events."

Table 1: Category (F) Use as an instrument. Shown are Code with manifestations, definitions and examples (anker).

All interviews were transcribed using the system developed by Dresing & Pehl (2011). The transcripts include observations by the interviewer. In the fifth section, the students were asked if they felt influenced or distracted by the think-aloud-method (A) and how they assessed their skills in using digital tools in general (B). Besides these two categories for the qualitative content analysis, five categories with three expressions each were generated from the theoretical background of the components of the instrumental genesis: (C) mathematical understanding, (D) instrumentation – usage schemes, (E) instrumentation – instrument-mediated-action schemes, (F) use as an instrument (see Table 1), (G) application – analysis of the given files. After the recorded audio sequences had been transcribed, the text parts, as well as the observations, were allocated to the categories.

FINDINGS & DISCUSSION

The expressions of the instrument-mediated-action schemes were at least as high as those of the usage schemes. Also, the students were more likely to be able to discuss a given file than to create one on their own. Pupils with low expressions in both types of schema were unable to solve the problem with the computer. The data analysis revealed differences in the process progress of the instrumental genesis. From this, three levels were derived, which need to be discussed.

The first level includes all learners who previously had little knowledge of using digital tools. (low expressions in categories D - F; low or middle expression in category G) On the second level are all learners who could reproduce the best-known solutions. In new situations or when dealing with unknown problems, the digital tool is still an artefact. They have insufficient instrument-mediated-action and usage schema to use it as an instrument (middle or low expressions in categories D - G; high expression in category F only if the problem is known; see Figure 4). In the third stage, learners have access to more schema and usage patterns. Only at this stage can learners use the digital tool as an instrument (middle or high expressions in categories D - G; at least one high expression). An overview of all three identified levels within the group of students provides Figure 5

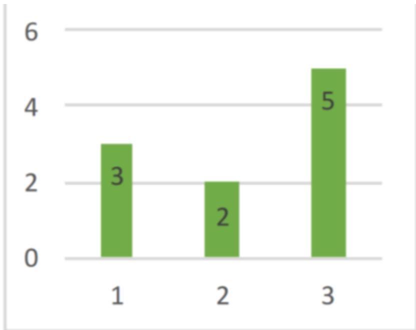

Figure 4: Shown are absolute values of all three manifestations (codes) of category F.

In order to examine the extent to which this gradation satisfies the characteristics of all learners and how the progress of the process changes over the years in school, it would be useful to carry out further longitudinal studies in qualitative terms and supplement these with quantitative methods.

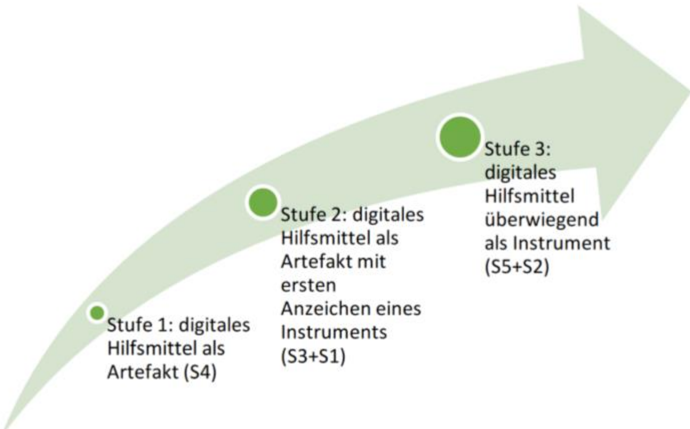

Figure 5: Three levels of instrumental genesis. Level 1: digital tool (device) is used like an artefact, Level 2: first attempts to use a digital tool (device) as an instrument, Level 3: digital tool (device) is mainly used like an instrument.

REFERENCES

Alqahtani, A. M., & Powell, A. B. (April 2017). Teachers' Instrumental Genesis and Their Geometrical Understanding in a Dynamic Geometry Environment. In: Digital Experiences in Mathematics Education (Vol. 3, Nr. 1), S. 9-38.

Barzel, B. (2011). Expertise zum Einsatz von Computeralgebra-Systemen (CAS) im Mathematikunterricht in Thüringen.

Béguin, P., & Rabardel, P. (2000). Designing for instrument-mediated activity. In Scandinavian Journal of Information Systems (Vol. 12), S. 173-190. http://iris.cs.aau.dk/tl_files/volumes/volume12/no1/7-PB_p173-190.pdf (02.06.2018).

Düsing, E. (26. November 2014). Lautes Denken. https://blogs.uni-paderborn.de/fips/2014/11/26/lautes-denken/ (23.04.2018)

Dresing, T., & Pehl, T. (2011). Praxisbuch Interview, Transkription & Analyse (3. Aufl.). Marburg: Eigenverlag.

Lewis, C. H. (1982). Using the "Thinking Aloud" Method In Cognitive Interface Design (Technical report). IBM. RC-9265.

Mayring, P. A. (2003). Qualitative Inhaltsanalyse. Grundlagen und Techniken. Weinheim: Beltz.

Rabardel, P. (2002). People and Technology. (authorized translation from "Les Hommes et les Technologies" (1995)).

Rezat, S. (2010). Das Mathematikbuch als Instrument des Schülers. Eine Studie zur Schulbuchnutzung in den Sekundarstufen. Vieweg Verlag.

Rieß, M. (2018). Zum Einfluss digitaler Werkzeuge auf die Konstruktion mathematischen Wissens. Studien zur theoretischen und empirischen Forschung in der Mathematikdidaktik. Springer Fachmedien Wiesbaden GmbH.

Trouche, L. (2004). Managing the complexity of human/ machine interactions in computerized learning enviroments: Guiding students command process through instrumental orchestration. In: International Journal for Computers for Mathematical Learning (Band 9) (S. 281-307).

van Randenborgh, C. (2015). Instrumente der Wissensvermittlung im Mathematikunterricht. der Prozess der instrumentellen Genese von historischen Zeichengeräten. Wiesbaden: Springer Spektrum.

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. In European Journal of Psychology of Education (Vol. X, No. 1), S. 77-101. https://slideheaven.com/cognition-and-artifacts-a-contribution-to-the-study-of-though-in-relation-to-ins.html (21.05.2018).

DOI: 10.17185/duepublico/70778
URN: urn:nbn:de:hbz:464-20191121-151702-6