"MATHEMATICAL DIGITAL COMPETENCIES FOR TEACHING" FROM A NETWORKING OF THEORIES PERSPECTIVE

Eirini Geraniou and Uffe Thomas Jankvist

UCL Institute of Education, University College London; e.geraniou@ucl.ac.uk

Danish School of Education, Aarhus University; utj@edu.au.dk

Often mathematics teachers are reluctant to use even free digital tools, such as GeoGebra and Scratch, due to their lack of insight and conviction about how tools impact learning, but also other contextual and personal factors, involving their own digital competencies and confidence in their technological pedagogical knowledge. In this paper, we take a theoretical look at four frameworks regarding teachers' mathematical, pedagogical and technological knowledge and competencies and discuss the potentials of networking these four theoretical frameworks, namely KOM, MKT, TPACK and instrumental orchestration. We use this networking of theories as a basis for coining the notion of teachers' mathematical digital competencies when teaching mathematics.

Keywords: Mathematical competencies; digital competencies; mathematical digital competencies.

INTRODUCTION

Despite rapid advances in technology and evidence of its positive impact on education, the application of digital technologies (DT) in schools lags behind educators' and researchers' expectations (e.g. Survey of Schools Report, 2013). Even though there is significant international research on how best to design digital tools that address students' difficulties with mathematical concepts and have great potential for mathematical learning (e.g. Noss et al., 2012), such tools are not widely used. Equally, past and current research carried out in various projects [1] suggests that students often fail to 'see' the mathematics in their interactions with digital tools and rarely use ideas, concepts or strategies they have acquired through their interactions with such tools in their lessons (Geraniou & Mavrikis, 2015).

Teachers are often not convinced of a digital tool's value for mathematical learning and are reluctant to use them in their practice due to their perceptions, attitudes, professional development experiences and technical or pedagogical support networks (e.g. Clark-Wilson, Robutti & Sinclair, 2014). Teachers' lack of confidence in DT and the need for a significant amount of time to effectively integrate them into classrooms was evidenced in a recent project on the implications of DT for teachers' knowledge and practice (see Clark-Wilson & Hoyles, 2017).

The importance of teachers' own confidence, knowledge and skills and how these may influence and impact the nature of the potential use of DT in their classroom cannot be denied. Considering a number of theoretical frameworks involving teachers' knowledge and competencies – such as the Mathematical Knowledge for Teaching (MKT) (Ball, Thames, & Phelps, 2008; Shulman, 1986), the Technological Pedagogical Content Knowledge (TPACK) (Koehler & Mishra, 2009), the KOM framework's pedagogical and didactic competencies (Niss & Højgaard, 2011), and the theory of Instrumental Orchestration (TIO) for looking into how DT may be integrated in the mathematics classroom (Drijvers et al., 2014; Trouche, 2004) – we argue that there is a need for a combination of mathematical and digital competencies for teaching. This could be referred to as *Mathematical Digital Competencies for Teaching* (MDCT). In doing so, we build upon past work on a framework for students' Mathematical Digital Competencies (MDC), which we shall return to later. Furthermore, even though we consider four theoretical frameworks, the presentation and discussion is purely theoretical and thus does not include any empirical evidence. Collecting and analysing empirical evidence is a future research agenda for us.

PEDAGOGICAL AND DIDACTIC COMPETENCIES FOR TEACHING MATHEMATICS

The Danish KOM framework (Niss & Højgaard, 2011) defines eight mathematical competencies encompassing the mastery of mathematics. These are the competencies of mathematical thinking; modelling; problem handling; representing; symbols and formalism; reasoning; communication; and finally the aids and tools competency. Furthermore, the Danish KOM framework also describes six didactic and pedagogical competencies for teaching, which a good teacher of mathematics is to possess in addition to the eight mathematical competencies. We briefly describe these below.

The *curriculum competency* firstly consists in being able to study, analyse and relate to current and future mathematics curricula at a given educational level, and being able to evaluate the associated plans and the impact on one's teaching tasks. Secondly, it also involves being able to draw up and implement different types of curricula and course plans, while taking into account overarching frameworks and terms of reference which may exist under current as well as future conditions.

Second, the *teaching competency* is about being able, either alone or in collaboration with students, to devise, plan and carry out concrete mathematics teaching sequences. This involves the creation of a rich spectrum of teaching and learning situations for different students and student groups, including the ability to find, judge, select, and produce a variety of means and materials for teaching. It is also about the selection and presentation of tasks and assignments for students. Finally, it involves being able to discuss, with the students, the content, forms and perspectives of mathematics teaching, while motivating and inspiring them to engage in mathematical activities.

The *competency of revealing learning* is about being able to reveal and interpret the actual mathematical learning of students and the extent of their mastery of the eight mathematical competencies, as well as their conceptions, beliefs about and attitudes towards mathematics, including the identification of development of these over time. Hence, it concerns getting behind the facade of the ways an individual's mathematics learning and understanding is expressed in concrete situations and contexts, with the intention of grasping and interpreting their cognitive and affective sources.

The *assessment competency* comprises being able to choose or construct a broad spectrum of instruments for revealing and evaluating students' learning outcomes and competencies, both in relation to specific courses and in more global – absolute or relative – terms. In addition, the competency involves being able to critically relate to the validity and extension of conclusions reached by using given assessment instruments. Finally, the competency involves the ability to characterise an individual student's learning outcome and mathematical competencies, as well as the ability to communicate with the student about these matters and assist that student to correct, improve, and further develop his or her mathematical competencies.

The *cooperation competency* is firstly about being able to cooperate with colleagues, both in the subject of mathematics and in other subjects, regarding matters relevant to teaching. The competency involves the ability to bring the above-mentioned four pedagogical and didactic competencies into play. Secondly, it includes the ability to cooperate with non-colleagues, e.g. students' parents, administrative agencies, education authorities, etc. about teaching and its boundary conditions.

The *professional development competency* concerns the development of one's own competency as a mathematics teacher, in other words it is a kind of meta-competency. More precisely, it involves being able to enter and relate to activities which can serve the development of one's mathematical, didactic and pedagogical competencies, taking into consideration changing conditions, circumstances and possibilities. It is about being able to reflect on one's own teaching and discuss it with mathematics colleagues, being able to identify developmental needs, and being able to select or organise and assess activities, which can promote the desired development. In addition, it is also about keeping oneself up-to-date with the latest trends, new materials and new literature in one's field, thus benefiting from research and development contributions, and maybe even about writing articles or books of a mathematical, didactic or pedagogical nature.

MATHEMATICAL KNOWLEDGE FOR TEACHING

Another, and better-known, perspective on what good teachers need to know in relation to the teaching of mathematics is the framework of MKT (e.g., Ball & Bass, 2009; Ball, Thames & Phelps, 2008). Hill, Rowan, and Ball (2005) defined MKT as ". . . the mathematical knowledge used to carry out the work of teaching mathematics" (p. 373). MKT is of course a further development of Shulman's (1986) conceptions of teacher knowledge. In his seminal paper, Shulman distinguished between "Pedagogical Content Knowledge" (PCK), "Subject Matter Knowledge" (SMK) and "Curricular Knowledge". Ball and colleagues (2008) renamed the latter "Knowledge of Content and Curriculum" (KCC), and divided PCK and SMK into three subdomains each.

PCK consists firstly of "Knowledge of Content and Students" (KCS), which is focused on how students think about, know and learn mathematics (Hill, Ball & Schilling 2008). One example is knowledge about typical student (mis)conceptions and errors. Next, "Knowledge of Content and Teaching" (KCT) combines mathematical knowledge and design of instruction; the sequence of examples used to introduce a new concept is one example. The last subdomain of PCK is "Knowledge of Content and Curriculum" (KCC). Shulman (1987) suggested that this domain includes, at minimum, a "particular grasp of the materials and programs that serve as 'tools of the trade' for teachers" (p. 8). According to Shulman (1986), curricular knowledge is related to knowledge of alternative curriculum materials within a grade. However, he also distinguished between two additional aspects of curriculum knowledge: lateral curriculum knowledge and vertical curriculum knowledge. Lateral curriculum knowledge relates to teachers' knowledge of how the mathematical content relates to the content in other classes at the same grade level. Vertical curriculum knowledge relates to how the mathematical content of a particular grade level relates to topics and issues that have been taught in earlier years – or topics that will be taught in later years.

SMK firstly consists of "Common Content Knowledge" (CCK), which is mathematical knowledge used in the work of teaching, in ways that correspond with how it is used in settings other than teaching. CCK is what Shulman likely meant by his concept of subject matter knowledge: knowledge teachers hold in common with professionals in other fields. It thus refers to a mathematical knowledge that is not unique to teaching. Next, "Specialized Content Knowledge" (SCK) is mathematical knowledge unique to the work of teaching mathematics. Hill and colleagues (2008), state that it "allows teachers to engage in particular teaching tasks, including how to accurately represent mathematical ideas, provide mathematical explanations for common rules and procedures, and examine and understand unusual methods to problems" (p. 378). It relates to subject matter knowledge since it is about the content and not about the students. As for the final subdomain, "Horizon Content Knowledge" (HCK), Ball and Bass (2009) remark, "that teaching can be more skillful when teachers have mathematical perspective on what lies in all directions, behind as well as ahead, for their pupils, that can serve to orient their navigation of the territory" (p. 11). Ball and Bass (2009) argue that HCK can support teachers in hearing students' mathematical insights, orienting instruction to the discipline, and in making judgments about what is mathematically important. Jakobsen, Thames, Ribeiro and Delaney (2012) proposed a practice-based definition of HCK, stating that it is: "an orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory" (p. 4642). They continue: "HCK enables teachers to "hear" students, to make judgments about the importance of particular ideas or questions, and to treat the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field" (ibid., p. 4642).

NETWORKING OF THE KOM AND THE MKT FRAMEWORKS

Although KOM's six pedagogical and didactic competencies and MKT's six subdomains address the same issues of mathematics teaching and mathematics teachers, they do so from different perspectives, i.e. one of competency and one of knowledge. Still, it appears that the two frameworks may complement each other. We are only aware of one study that so far has attempted to do so. Sloth and Højsted (2017) compare (network) the two frameworks around an empirical study of what preservice teachers learn as part of their teacher education:

> "MKT and KOM give different perspectives on mathematics teacher knowledge, that there are overlaps and differences when applied to practical situations, but also that the frameworks themselves may benefit from the perspective of each other. Using both frameworks on our case, we find that they can complement each other and describe a greater range of mathematics teacher knowledge. Furthermore, we suggest that using or combining concepts from both frameworks can result in a new understanding of the knowledge and ability needed by mathematics teachers." (Sloth & Højsted, 2017, p. 3411)

Yet, neither KOM's six pedagogical and didactic competencies for mathematics teaching nor the MKT framework address explicitly what is needed of a mathematics teacher to successfully use DT in his or her teaching of mathematics. Does this require a certain type of pedagogical and didactic competencies? Does it require a certain kind of mathematical knowledge, not already embedded in the six subdomains of MKT? Some frameworks seem to suggest so.

TECHNOLOGICAL PEDAGOGICAL CONTENT KNOWLEDGE: THE TPACK

The TPACK framework concerns the body of knowledge and skills required for the implementation of DT in teaching (Koehler & Mishra, 2009). It was developed to address the need for technological knowledge and in particular, to analyse teachers' pedagogical digital technology competence and the associated skills required of teachers (Law, 2010). As such, it is an extension of the PCK concept (Shulman, 1986). Koehler's and Mishra's (2009) diagrammatic representation of TPACK (http://www.tpack.org/) shows the PCK as the intersection of two circles representing general pedagogic knowledge and content knowledge. Using a Venn diagram with three overlapping circles, they include technology knowledge as a third domain of teacher knowledge to identify the skills or knowledge needed to successfully operate DT – also referred to as technical competence (Law, 2010). This inclusion introduces two other dyads, the "Technological Pedagogical Knowledge" (TPK), i.e. the intersection with pedagogic knowledge and the "Technological Content Knowledge" (TCK), i.e. the intersection with content knowledge. The triad intersection (Technological Pedagogical Content Knowledge) characterises the knowledge by teachers for technology integration in their teaching around a specific subject matter that "is the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies […] and how technology can help redress some of the problems that students face […] and knowledge of how technologies can be used to build on existing knowledge to develop new epistemologies or strengthen old ones" (Koehler & Mishra, 2009).

INSTRUMENTAL ORCHESTRATION

Research (e.g. Kieran and Drijvers, 2006) has shown the benefits of the instrumental approach and how crucial the role of the teacher is in the effective integration of DT in the classroom and its benefits to students' learning. To describe how a teacher manages the use of DT and orchestrates mathematical situations, the theory of Instrumental Orchestration was derived by Trouche (2004). It involves "the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment –in this case a computerised environment– in a given mathematical task situation, in order to guide students' instrumental genesis" (Drijvers et al., 2014, p.191).

There are three elements within an instrumental orchestration (Drijvers et al., 2014). First, a *didactic configuration*, which is the arrangement of artefacts in the teaching environment. Second, an *exploitation mode*, which is the approach a teacher chooses to exploit a didactical configuration to assist their didactical intentions. Third, a *didactical performance*, which is the decisions a teacher needs to make on the fly, while teaching to accommodate the chosen didactic configuration and exploitation mode. Six orchestrations have been identified for whole class teaching in up-to-date research studies and one for students working alone or in pairs with technology (Drijvers et al., 2014).

The *Technical-demo* orchestration involves the teacher demonstrating tool techniques. The teacher may use student work or a new task to demonstrate a technique of using the tool and encourage students to copy their actions. The *Link-screen-board* orchestration, where the teacher draws students' attention to the relationship between anything that happens in the digital resource and the conventional mathematics representations on book, paper and board. The teacher may use a student's work as a starting point for that class discussion or set a new task or problem. The *Discuss-the-screen* orchestration involves the teacher running a class discussion focused on what happens on the computer screen. The teacher may choose a student work, a task, a problem or a strategy to initiate the class discussion and welcome student reactions and input. The *Explain-the-screen* orchestration involves the teacher running a class discussion to explain what happens on the computer screen. The teachers are expected to focus on the mathematical content and consider using a student's work or model themselves a solution to a task for example. The *Spot-and-show* orchestration involves the teacher 'spotting' a student's piece of work, which is worth sharing with the rest of the class and then runs a class discussion to allow the student to justify their work and invite comments and reactions from peers as well as feedback from the teacher themselves. The *Sherpa-at-work* orchestration involves a 'Sherpa' student (as used by Trouche, 2004) using the digital resource to present their work or follow the teacher's instructions and showcase something in the digital resource. The *Work-and-walk-by* orchestration involves students working independently or collaboratively and the teacher circulating the room to monitor students' progress and help those in need.

All the above orchestrations involve whole-class teaching (Drijvers et al., 2014) and are used to describe the role of the teacher in guiding students during their interactions with a digital resource and supporting them in mastering this resource usage and learning the mathematics involved.

NETWORKING OF THE TPACK AND THE TIO FRAMEWORKS

TPACK has the benefit of simplicity and accessibility (Drijvers et al., 2014), as by using the Venn-diagrammatic representation it showcases the various intersections between knowledge of mathematics, knowledge of technology and knowledge of pedagogy. But it has also been criticised because of its ambiguities, and especially the weak theoretical definitions of its constructs (ibid.; Voogt et al, 2012; Ruthven, 2014). In fact, Ruthven (2014) suggested that the TPACK framework provides "a rather coarse-grained tool" for analysing teachers' knowledge and therefore, it might need complementing by other frameworks to achieve an adequate depth of analysis. He explored how the

TPACK could be combined with the Instrumental Orchestration framework, but also one of his own frameworks, that of "Structuring Features of Classroom Practice", in order to analyse deeper teachers' skills and knowledge involved in their practices.

TPACK refers to a number of constructs that involve either knowledge, understanding or competencies or combinations of these, showcasing how all these are necessary for the teaching practice and the potentially successful integration of DT in the mathematics classroom. TIO, on the other hand, focuses on factors and strategies that influence and can direct and organise teachers' usage of DT for mathematics teaching and learning. A number of orchestrations, as described earlier, seem to be linked mostly to students' development of TCK, e.g. Technical-demo, Discuss-the-screen or Sherpa-at-work, but also students' development of mathematical content knowledge, e.g. Explain-the-screen, Link-screen-board or Spot-and-show. Both frameworks take into account teachers' pedagogies and assume their importance in managing students' development of instrumental genesis. There seems to be an assumption though that teachers' TPACK may prescribe or influence their instrumental orchestration of a certain digital resource in their teaching. As Ruthven (2014) proposed there is a need "for fuller and more systematic investigation of the phenomenon of technology integration into subject teaching" (p.391) and he argued for the need for combining theoretical frameworks to supplement their ideas and generate illuminating findings.

MATHEMATICAL DIGITAL COMPETENCIES FOR TEACHING

Geraniou and Jankvist (2018) argue that in the modern-day mathematics classroom, a distinction between students' mathematical competencies and students' digital competencies may occasionally appear somewhat artificial, since DT may be such an integral part of the mathematics teaching and learning that is going on. Hence, they coin and discuss the term mathematical digital competencies (MDC), and briefly outline a tentative framework for such by combining KOM's eight mathematical competencies with the digital competencies of the DigComp framework (Ferrari, 2013). The notion of MDC has been further discussed and deepened by networking with the theoretical frameworks of the instrumental approach (e.g. Guin & Trouche, 1999) and conceptual fields (Vergnaud, 2009) in Geraniou and Jankvist (2019). Hence, it seems apparent that if a teacher is to assist students in developing their MDC, she will – besides possessing MDC to some extent herself – need mathematical digital competencies for teaching, i.e. MDCT.

In this paper, we have reviewed four theoretical frameworks related to teaching of mathematics, and discussed the networking of the KOM and the MKT frameworks, as well as the networking of the TPACK and TIO frameworks. Of course, there is a need for further investigations of the adaptation of the TPACK model for mathematical knowledge and therefore its link to the MKT framework. However, we also firmly believe that networking of the KOM and the TIO frameworks holds a large potential – in a similar way as the networking of KOM's eight mathematical competencies and the instrumental approach does (Geraniou & Jankvist, 2019; Jankvist, Geraniou & Misfeldt, 2018). Yet, we argue for the need of networking all four theoretical frameworks to lead to a combined theoretical frame, i.e. that of mathematical digital competencies for teaching (MDCT). This we plan to do in our future endeavours.

NOTES

1. For example, MiGen, 2008-2010, www.migen.org; Metafora, 2010-2013, www.metafora-project.org; iTalk2learn, 2012-2015, www.italk2learn.eu; Mathematical Creativity Squared, 2013-2016, mc2-project.eu.

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