# TEACHER'S ATTENTION TO CHARACTERISTICS OF PARABOLA SKETCHES: DIFFERENCES BETWEEN USE OF MANUAL AND AUTOMATED ANALYSIS

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*This study compares the different characteristics of student answers addressed during classroom discussion by mathematics school teachers in various settings. The study examines 3 9th grade mathematics teachers from the same school in Israel, all of whom taught the topic of quadratic functions. We examined and analyzed the teachers' choices and sequencing of the different characteristics of parabola sketches in a classroom discussion about student answers. The teachers analyzed student answers either manually or using the Seeing the Entire Picture (STEP) automatic formative assessment platform. Findings suggest a difference between the characteristics that teachers addressed in the classroom discussions in the two conditions. In the manual setting, teachers focused on incorrect features of the example, whereas in the automatic setting they focused on characteristics that emphasize different mathematical dimensions.*

Keywords: *formative assessment, quadratic function, automatic assessment, classroom discussions, function sketches*

## INTRODUCTION

Conducting mathematics lessons based on the ideas of students and the analysis of their answers enables a meaningful dialog. To achieve this, teachers cannot settle for merely watching the students or reacting to correct answers (Black & William, 2014). Instead, they are required to provide feedback in the form of a class discussion that promotes formative assessment interactions. Under these conditions, teachers choose examples for the discussion without prior planning, or focus on the student's mistakes (Stacey, 2009). Many online platforms address this challenge by providing teachers with an immediate status of students' submissions and with further analysis of predefined characteristics of student answers. In this study, we examine the potential of a platform of this type, the Seeing the Entire Picture(STEP) platform (Olsher, Yerushalmy, & Chazan, 2016), when teaching the topic of quadratic functions. We examine the characteristics of student answers addressed by the teachers in the class discussion, and compare the characteristics addressed when the teachers analyzed the answers manually with the characteristics addressed when the answers were automatically analyzed and the teachers were able to access the results.

## THEORETICAL BACKGROUND

In the early stages of digital education, a great deal of attention was paid to technological attributes. By contrast, these days, most of the approaches to using technology adopt an integrated perspective, in which aspects such as social interactions, educational strategies, and the teachers' role are being assessed (Bottino, 2004). Many technological platforms for formative and summative assessment have been developed, and they are influenced by these approaches. Examples include TI-Nspire, ASSISTments, and DESMOS. These digital platforms inspired a shift in the teacher's role. In computer-mediated teaching, where student answers are automatically assessed (Koedinger, McLaughlin, & Heffernan,2010), teachers are released from the time-consuming task of evaluating student submissions, and can concentrate on their main function, which is guiding the class discussions in accordance with the students' needs (Yerushalmy &Elikan, 2010).

A key attribute of digital learning platforms, apart from enhancing classroom interaction, is the nature of the tasks. Mathematicians stress the importance of examples and of tasks that encourage example creation (Watson & Mason, 2002). The solutions to example-eliciting tasks (EET) can reflect the student's understanding, and suggest difficulties or misunderstandings of mathematical concepts (Zaskis & Leikin, 2007).

Quadratic functions are a central topic in school mathematics. They provide ample opportunities for EETs, and reveal many student difficulties (Dreyfus & Eisenberg, 1984). By providing students with rich EETs in a digital environment that supports automatic analysis of student answers and their classification according to different mathematical attributes (Olsher et al., 2016), we can reduce the teachers' workload, helping them to more thoroughly assess the characteristics of their students' work.

## METHODOLOGY

This study examines the pedagogical potential inherent in teaching the topic of quadratic functions using STEP, as well as its influences on class discussions and changes in their contents. The two research questions are: What are the characteristics of 9th grade student submissions in response to EETs on the topic of the intersection between a parabola and a point or a line, on which teachers chose to base class discussions? Are there any differences between the characteristics chosen by the teachers when they assess student answers manually and automatically?

### Sample

Three certified mathematics teachers from the same Israeli school, teaching 9th grade, participated in this study. Two of the teachers hold a bachelor's degree in mathematics and are currently studying for their master's degree: one studies mathematics, the other mathematics education. Both were in their second year of teaching. The third teacher holds a Bachelor's degree in statistics, and this was her fourth year of teaching.

### Research tools

The research tools included an online activity delivered on the STEP platform, which included two tasks, and the students' answers for these tasks.

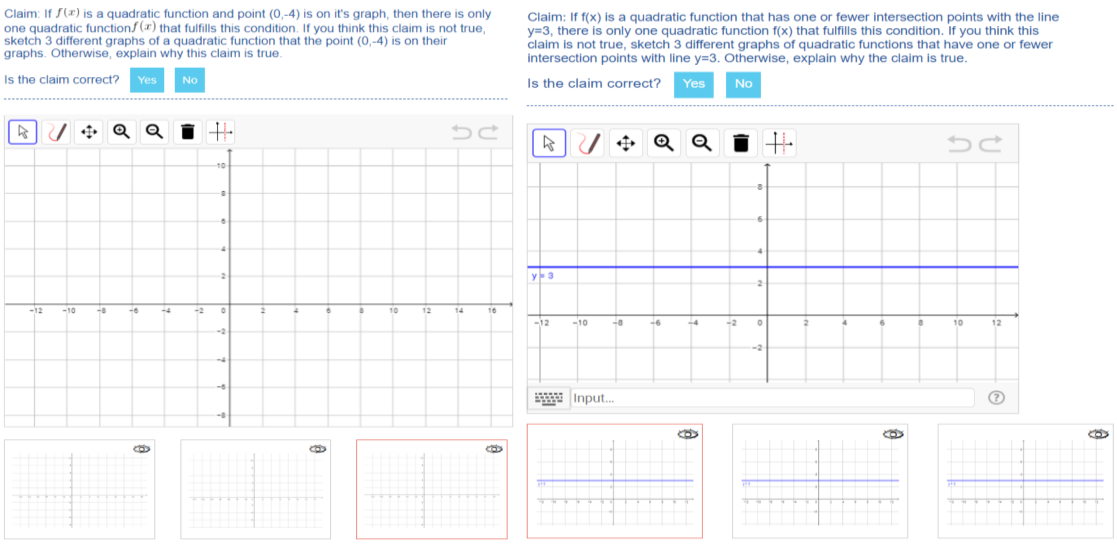


Figure 1. Tasks 1 and 2 as they appear on the student interface of the STEP platform

The students completed the tasks, which require sketching different parabolas with certain characteristics. For example, in the second task (Figure 1), the students were required to assess whether there is more than one quadratic function that intersects the line y=3, and to sketch 3 different examples for such functions, if any exist. The platform collected and automatically analyzed the students' answers (Figure 2), based on predefined characteristics. The characteristics may be categorized into three groups, as shown in Tables 1 and 2 for the two tasks.

| Incorrect example characteristics (1I) | Correct example characteristics (1C) | Cross-example characteristics (1E) |
|---|---|---|
| i sketch does not describe a function | i sketch has min at (0,-4) | i all 3 sketches have minimum points |
| ii sketch has a vertex at (-4,0) \|\| (4,0) \|\| (0,4) | ii sketch has max at (0,-4) | ii all 3 sketches have maximum points |
| iii sketch does not describe a quadratic function | iii sketch vertex is not (0,-4) | iii each of the 3 sketches has a different (1C) char. |
| iv sketch has a V-shape | | iv sketches have different parameter (where y=ax^2) |
| v asymmetric sketch | | |
| vi partial sketch | | |

**Table 1. Student answer's characteristics for task 1**

| Incorrect example characteristics (2I) | Correct example characteristics (2C) | Cross-example characteristics (2E) |
|---|---|---|
| i sketch intersects y=3 at 2 different points | i sketch has min at y=3 | i none of the sketches have a vertex at y=3 |
| ii sketch does not describe a function | ii sketch has max at y=3 | ii each sketch has a different number of intersection points with y=3 |
| iii sketch does not describe quadratic function | iii sketch has max below y=3 | iii all sketches have a vertex at y=3 |
| iv partial sketch | iv sketch has min above y=3 | |
| | v sketch has vertex at y=3 and on the y axis | |

**Table 2. Student answer's characteristics for task 2**

The first group includes various characteristics of incorrect examples: for example, sketches that do not represent functions (Figure 2, column 2Iii), or do not represent quadratic functions (Figure 2, column 2Iiii). The second group includes various characteristics of correct examples, such as sketches with a minimum at y=3 (Figure 2, column 2Ci), or sketches with a vertex at y=3 and on the y axis (Figure 2, column 2Cv). The third group includes characteristics that apply to all three submitted examples, for example all of the submitted sketches have a minimum point.

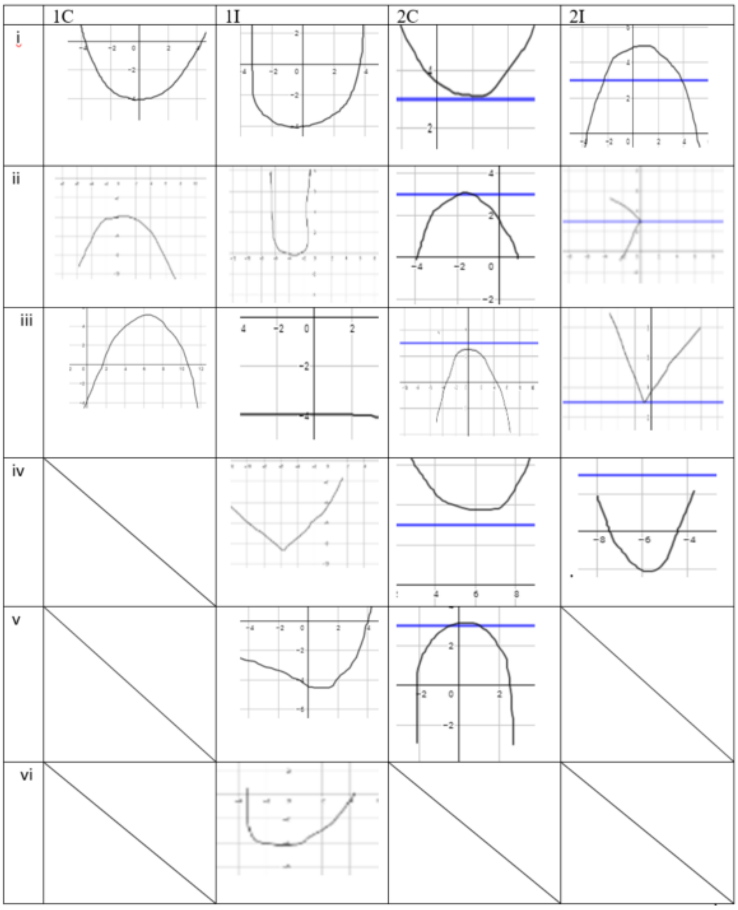


**Figure 2. Examples of student answers for tasks 1 and 2, grouped by characteristics**

The automatic analysis enables the teacher to filter the answers based on the characteristics and present only the answers that contain a given set of one or more characteristics to address during classroom discussion.

The student answers were presented to the teachers, who were asked to categorize them during an interview. After the teachers categorized the examples, they were shown the predefined categories within STEP. Next, the teachers chose the order in which they would address the characteristics of the answers in their classroom discussions.

Following the interview, teachers performed the activity in their classrooms, as part of their routine teaching sequence, and conducted a discussion after the students completed the activity. Then the teachers were interviewed once again to clarify various decisions and actions that were noted during the lesson.

## Data sources and analysis

Data sources include student answers to the tasks, two interviews with each of the teachers (before and after the class discussion following the students' submissions), and classroom observations conducted by the second author.

To describe our findings, we chose an analytical framework that regards professional teaching as incorporating unique knowledge. Therefore, we use the teaching episodes reported here as manifestations of various types of pedagogic content knowledge in mathematics, specifically common content knowledge (CCK), specialized content knowledge (SCK), knowledge of content and student (KCS), and knowledge of content and teaching (KCT) (Ball, Thames, & Phelps, 2008).

In our specific setting, the different types of knowledge are assessed using different methods. We asked teachers to solve the tasks assigned to the students, to assess their mathematical knowledge and skills in the topic at hand, which is described by Ball et al. (2008) as CCK. Next, we asked the teachers to estimate what types of answers their students will submit, which reflects their KCS (*ibid.*). To assess SCK (*ibid.*), we presented the teachers with student answers for each task, asking them to categorize and interpret the answers. To assess teachers KCT (*ibid.*), we asked them to choose various characteristics, to address in their classroom discussion to achieve efficient teaching, and later observed the actual discussions.

## FINDINGS

The findings show that there was a difference between the categories of the characteristics teachers chose to address during the classroom discussion on quadratic function sketches, and in the order in which these characteristics were discussed in the two different settings: when teachers analyzed the answers themselves and when their students' answers were automatically analyzed.

Below we describe the four types of teachers' pedagogical content knowledge (CCK, KCS, SCK, and KCT), as they were manifested at the different stages of the study. We begin by briefly discussing the teachers' CCK on the topic at hand when solving the tasks. Next, we describe and illustrate different aspects of KCS, as manifested in the teachers' expectations from student answers. We proceed to the various characteristics the teachers identified in the students' answers and compare them with the teachers' expectations, to shed light on the teachers' SCK. Finally, we report on the choices and sequencing of the various characteristics in the classroom discussion, and compare the manual and automatic analyses, to demonstrate the teachers' KCT.

### CCK

None of the three teachers had difficulty in solving the tasks of the activity. Teachers demonstrated CCK that is appropriate for the topic of quadratic functions, specifically sketching quadratic functions that meet the given conditions.

### KCS

Expected answers for the first task included all three correct example characteristics for the task (1Ci-iii, Table 1). The teachers expected three out of the total of 13 research-based predefined characteristics for task 1 (Table 1), all from the same category (correct example characteristics).

Expected answers for the second task included all five correct example characteristics for the task (2Ci-v, Table 2). In contrast to the first task, the teachers also expected answers with incorrect example characteristics (2Ii, 2Iiv, Table 2). All the teachers expressed concern about students not fully understanding the term "at most," which is implied by the description of "one or fewer." In general, the teacher's expectations were similar with respect to correct example characteristics, whereas in their expectations of incorrect example characteristics, one of the teachers expected only characteristic 2Ii, and the other two teachers also expected characteristic 2Iiv. In conclusion, the teachers expected 6 out of a total of 12 research-based predefined characteristics for task 2 (Table 2), from two different categories (correct and incorrect example characteristics).

### SCK

Analysis of actual student answers (as opposed to predicting the students' answers) led teachers to notice the expected characteristics in both tasks. Teachers discovered and categorized other characteristics that they did not initially expect.

In their analysis of student answers to the first task, teachers also noticed five incorrect example characteristics out of six predefined characteristics, such as sketches that do not represent a quadratic function (1Iiii, Table 1). They did not notice the incorrect example characteristic of an asymmetric sketch. Teachers also noticed two out of the four cross-example characteristics, such as the three sketches submitted with different distances from the symmetry line (1Eiv, Table 1). Each of the three teachers discovered additional characteristics (four, five, and six) to what they had expected before being exposed to the student answers.

In their analysis of student answers to the second task, teachers also noticed all 12 predefined characteristics, from all three categories. The teachers categorized a new characteristic in the form of the shape of the graph, and identified sketches of quadratic functions in the shape of the letter V. Similar to the analysis of answers to the first task, the teachers also addressed cross-example characteristics. Two of the three teachers discovered additional characteristics to what they had expected before seeing the student answers (seven and five additional).

**KCT**

When asked to sequence the characteristics that they would address in a classroom discussion after manually analyzing the students' submission for task 1, two teachers initially decided to attend incorrect example characteristics, seeking to address sketches that did not represent functions at all, or that did not represent quadratic functions. The teachers considered these mistakes to be severe because they indicated lack of understanding of the concept of functions in general, and of quadratic functions in particular, therefore the teachers intended to place these at the top of the list of topics to be discussed in class. Table 3 shows the first three characteristics that were chosen by the teachers in each of the settings for task 1. In contrast to her peers, the third teacher decided to initially address the correct example characteristic of having a vertex at the requested point, because she expected these to be the most common sketches, and intended to move on later to incorrect example characteristics from which her peers started (Table 3).

| | Manual analysis | | | Automatic analysis | | |
|---|---|---|---|---|---|---|
| | Teacher 1 | Teacher 2 | Teacher 3 | Teacher 1 | Teacher 2 | Teacher 3 |
| 1st characteristic | 1Ii | 1Ii | 1Ci | 1Iiii | 1Iiii | 1Iiii |
| 2nd characteristic | 1Iiii | 1Iiii | 1Ii | 1Ci | 1Ciii | 1Ciii |
| 3rd characteristic | 1Iii | 1Ci | 1Iiii | 1Cii | 1Ci | 1Ci |

**Table 3. Sequencing of characteristics for discussion by teachers for task 1**

When asked to sequence the various characteristics that they would address in a classroom discussion after manually analyzing the students' work on task 2, similarly to task 1, two of the teachers (1 and 3) chose to first discuss incorrect example characteristics. They wanted to begin by discussing sketches that did not represent quadratic functions, then quadratic functions that had two intersection points with line y=3, similarly to task 1, because in their opinion these mistakes indicate lack of understanding of the concept. The two teachers also stated that if they discussed the two tasks in the same lesson, they would skip the characteristic of sketches not representing functions in general. By contrast, teacher 2 chose to discuss correct example characteristics that are related to the mathematical content of the task. The teacher said:

> In my opinion, there will not be many examples of sketches above or under the line owing to the fact that students do not fully understand the concept. So it is important to start with this characteristic, and then proceed to answers that have a vertex on the line, which will be the common student answers.

Table 4 shows the first three characteristics that were chosen by the teachers in each of the settings for task 2.

| | Manual analysis | | | Automatic analysis | | |
|---|---|---|---|---|---|---|
| | Teacher 1 | Teacher 2 | Teacher 3 | Teacher 1 | Teacher 2 | Teacher 3 |
| 1st characteristic | 2Iiii | 2Civ | 2Iii | 2Civ | 2Cii | 2Ii |
| 2ndcharacteristic | 2Ii | 2Ciii | 2Iiii | 2Ciii | 2Ciii | 2Ci |
| 3rdcharacteristic | 2Ci | 2Ci | 2Ii | 2Cv | 2Civ | 2Cii |

**Table 4. Sequencing of characteristics for discussion by teachers for task 2**

When teachers used the automatic analysis by the STEP platform in their classrooms, the sequencing changed. They chose to discuss less incorrect example characteristics than they did in the case of manual analysis, and focused more on correct examples characteristics, as shown in Table 3 for task 1 and in Table 4 for task 2. All the teachers noted this difference, as shown in the description of task 1 by teacher 1:

> There was nothing critical in the student answers with sketches that are not functions, but there was more of a problem with functions that are not quadratic, which is why this characteristic [sketch does not describe a quadratic function] turned out to be less important than others.

The teachers started with what they found to be a critical mistake of the students, then moved on to sketches of quadratic functions that intersect the requested point, as shown in Table 3. Teachers 2 and 3 chose to start with functions that intersect the requested point but not in their vertex, then went on to functions where the requested point was in the vertex. Teacher 2 explained:

> I decided to address a characteristic that was less present in the students' work. I noticed that while the students were working, most of them thought that the point should be a vertex… This is why it was interesting to expose this characteristic to students who did not think of it.

Teacher 1 chose to begin with sketches that had a vertex at the requested point, although this characteristic was not one of her top three when analyzing the students' work manually.

In task 2, only teacher 3 discussed one incorrect example characteristic, in contrast to 3 examples she chose when analyzing the students' work manually. She said:

> I noticed that there were no submissions under or above the line… A lot of sketches intersecting the line in two points… This is why I decided to begin with this critical and common mistake revealed by the automatic analysis.

By contrast, teacher 1 did not discuss any incorrect example characteristics, as opposed to two such characteristics when analyzing the students' work manually. She explained her choice:

> When the students worked on the task, I noticed that most of them did not sketch a function that is above or under the line [without touching it], which is why I started with this characteristic. I checked all the characteristics and saw that there was only one answer with a sketch above the line… Then I asked the students: "What do you think? Is this sketch correct or incorrect? And a discussion arose around the meaning of "at most."

Teacher 2 changed only the sequence between the same characteristics she chose when manually analyzing the students' work, because of the commonality of the characteristics in the student answers.

## DISCUSSION

The findings of this study show a difference between the characteristics teachers address when conducting a discussion about student answers after having manually analyzed their work, and the characteristics they address when student answers are automatically analyzed. The literature suggests that teachers select examples for discussion in an unsystematic way or cling on to student mistakes (Stacey et al., 2009). This claim finds support in our findings when teachers manually analyzed student answers. But when teachers had access to automatic analysis of student submissions, they changed their sequencing of characteristics. Teachers moved from characteristics that focus on incorrect features of the examples, to those that emphasize different mathematical dimensions of the topic being taught, in this case, quadratic functions.

This study introduces the novel notion of topic-centered learning analytics: mathematical characteristics of student answers that are not limited to student mistakes. The findings suggest that teachers use these insights in classroom discussion when they are available, expanding the range of student work-based discussions in the mathematics classroom.

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