DESIGNING ONLINE FORMATIVE ASSESSMENT THAT PROMOTES STUDENTS' REASONING PROCESSES

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Automated online formative assessment of students' work in a rich digital environment has the potential to support and develop students' reasoning process. Previous studies have presented the challenge of designing e-tasks. Here we focus on the challenge of designing a personal online formative assessment that supports the students' reasoning process. A common type of online formative assessment is elaborated feedback. We provide a design principle for elaborated online feedback of students' work on an online example-eliciting task (EET) using the Seeing the Entire Picture (STEP) platform. We demonstrate two cases of attribute isolation elaborated feedback (AIEF) design, and the case of a pair of students who used the AIEF to support their reasoning process.

Keywords: automatic personal feedback, formative assessment, formative assessment design

INTRODUCTION

Online Mathematics Formative Assessment

Technological developments offer important tools for the assessment of students working in rich digital environments. We use the term "formative assessment" in the sense described by Black and Wiliam, (1998). Automated online formative assessment has the potential to promote reasoning processes by improving the learners' mathematics thinking and skills (Shute, 2008). We are interested in exploring the main design principles of an online formative assessment tool that supports students' reasoning processes. Researchers distinguish between two main types of online formative assessment feedback: verification and elaborated feedback. Verification feedback provides simple information about whether or not the student's answer is correct. Elaborated feedback provides an explanation of why a response is or is not correct. According to a meta-analysis by Van der Kleij, Feskens, and Eggen (2015), elaborated feedback is more effective than verification feedback for higher-order learning outcomes in mathematics. Shute (2008) identified the following six different types of online elaborated feedback: Attribute isolation, Topic contingent, Response contingent, Hint/cues/prompts, Bugs/misconceptions, Informative tutoring. In this study we used: "attribute isolation elaborated feedback" (AIEF), which consists of observations on the requirements of the task and on its mathematical characteristics, including the nature of mathematical objects and actions involved, and the mathematical reasoning processes entailed.

The Challenge in Designing Elaborated Feedback

The challenge in designing AIEF lies in presenting the information to the students in a rich digital environment in a way that supports their reasoning process. To design such elaborated feedback, we needed an environment that supports mathematical inquiry and reasoning, as well as automatic online elaborated feedback. For this purpose, we used the Seeing the Entire Picture (STEP) platform (Olsher, Yerushalmy, and Chazan, 2016). STEP is an environment that supports example-eliciting tasks, where students are asked to construct examples in a multiple linked representations (MLR) environment that supports their answer. Example eliciting is a vital element in the reasoning processes, and may also be indicative of the students' mathematical reasoning (Zaslavsky and Zodik, 2014). The STEP platform supports exemplification generated by work with interactive diagrams.

Yerushalmy, Nagari-Haddif, and Olsher, (2017) stressed the importance of example eliciting as an e-task design principle. According to Yerushalmy et al. (ibid.), asking students to submit as different as possible examples encourages them to develop a rich and varied example space. The example space can be automatically analyzed to provide feedback to students. The STEP platform, therefore, constitutes a novel pedagogical tool that supports reasoning processes as well as rich feedback. Yerushalmy et al. (2017) formulated the principles of e-task design, but to support the students' reasoning processes it is necessary to also design the feedback students receive.

In the present study we focused on conjectures, which are a key component of mathematical reasoning. Conjecturing is the process by which students raise conjectures, refute or dismiss some of the conjectures, and choose the conjectures they want to justify. During conjecturing, students enhance their ability to prove (Boero, Garuti, and Lemut, 2007). Here we present a design principle for AIEF that promotes the students' conjecturing process. The study consists of three parts. In the first part, we exhibit the design principle for AIEF. In the second part, we demonstrate the AIEF design principle in two different contexts. In the third part, we describe the case of a pair of students who used the elaborated feedback to support their reasoning process.

AIEF DESIGN PRINCIPLE

Before designing the AIEF, we must design the EET. The aim of the EET design was to support the students in raising conjectures, which is the beginning stage of the conjecturing processes. To this end, we designed the EET following Yerushalmy et al. (2017). Specifically, for each task, students were asked to submit three representative examples, as different as possible. Additionally, we asked the students to formulate a conjecture as general as possible.

The STEP platform provides various formats of personal feedback. For this study, we used AIEF, which consists of a list of mathematical characteristics of the task (e.g., the figure is a rectangle, the figure has equal sides). The list of characteristics was prepared in advance, as part of the AIEF design. STEP can analyze the submitted work, and mark the identified characteristics of the submitted example in blue (here presented by circles) and the non-identified ones in gray, automatically producing the AIEF (see Figures 3 and 4 for student submissions and AIEF). We hypothesized that giving the students elaborated feedback that consists of three lists of characteristics (one for each submitted example) that differ in the indication whether or not the characteristics of each example were identified, enables students to analyze the differences and similarities between the submitted examples, and in this way support the conjecturing process.

Our goal for the AIEF design was to support students in shifting from special cases to a general conjecture. In other words, we sought to support the process in which students observe special cases of conjectures (from among the conjectures they raised when working on the EET), choose relevant information to formulate the general conjecture, and finally choose or formulate a general conjecture. To this end, the guiding principle was to design a rich and varied list of characteristics that would enable STEP to identify characteristics of the students' submissions; these include special cases of the conjecture as well as relevant and extraneous information regarding more general cases. Below we demonstrate such a design.

DEMONSTRATIONS OF AIEF DESIGN

We present two cases of AIEF design in different contexts, based on different types of information regarding the general conjecture. Each case begins with an explanation of the EET design and continues with an explanation of the AIEF design.

First EET Design Example

The first EET example was formulated as follows: "A, B, C, and D form the quadrilateral ABCD. They are all dynamic and can be dragged. If possible, create 3 examples that are as different as possible from each other, in which the perpendicular bisectors to the sides of ABCD meet in a single point. In the dialogue box formulate a conjecture as general as possible of the conditions in which this happens." Figure 1 shows a screenshot of the task applet.

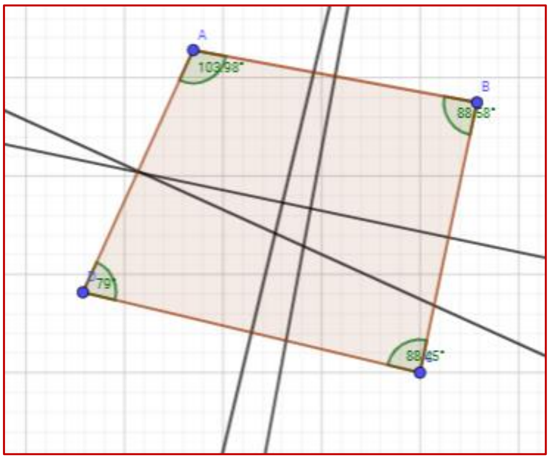

Figure 1. First task applet

The EET has been found effective for raising conjectures (Olsher, forthcoming). We used the Geogebra software to design the task applet. To enable students to focus on building quadrilaterals, the features of Geogebra were not available for them, and only points A-D were draggable, while maintaining the perpendicular bisectors. To support eliciting examples of various types of quadrilaterals, the applet provided measurements of squares and angles. Asking the students to create three examples as different as possible encouraged the construction of a personal example space that could be assessed and analyzed with STEP to produce elaborated feedback for each example submitted.

First AIEF Design Example

To design a rich and varied list of characteristics that would enable STEP to identify the characteristics of the students' submissions, first we analyzed the general conjecture: The perpendicular bisectors to the sides meet in a single point if and only if the opposite angles are supplementary. We observed that the formulation of the general conjecture contained the attribute: "opposite angles are supplementary."

Assuming that the students would also submit special cases of the conjecture, we listed all the types and attributes of quadrilaterals in which the perpendicular bisectors to the sides meet in a single point. To include extraneous information, we also predefined types and attributes of quadrilaterals that have a hierarchical relationship with the special cases. Thus, we choose to include the following types and attributes of quadrilaterals: "All angles are equal," "Square," "Rhombus," "Trapezoid," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," "Sum of opposite angles is 180º," and "All sides have equal length." For example, suppose the students submitted a square and STEP would identified the following characteristics: "All angles are equal," "Square," "Rhombus," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," "Sum of opposite angles is 180º," and "All sides have equal length." By doing so, STEP identified a list of special cases of the conjecture, as well as relevant and extraneous characteristics.

We hypothesize that this kind of AIEF can support the students' inquiry and conjecturing process. As explained in the EET design, the students were asked to submit three examples. The STEP platform can analyze each example and produce AIEF that consists of three lists of characteristics that differ from each other by the indication whether the characteristics of each example were or were not identified. The underlying principle in giving students such elaborated feedback is that they will further engage with the different mathematical characteristics of their examples, enabling them to analyze and communicate the differences and similarities between their submitted examples, which supports the conjecturing process.

Second EET Design Example

The second EET was formulated as follows: "f(x) (blue) and g(x) (green) are linear functions. h(x) is the product function of the two linear functions. Drag the blue and green points to create new functions. What kind of product functions appears when multiplying two linear functions? Create three examples that are as different as possible from each other. Formulate in the dialogue box a conjecture as general as possible of the product functions that appears." Figure 2 shows a screen of the task applet:

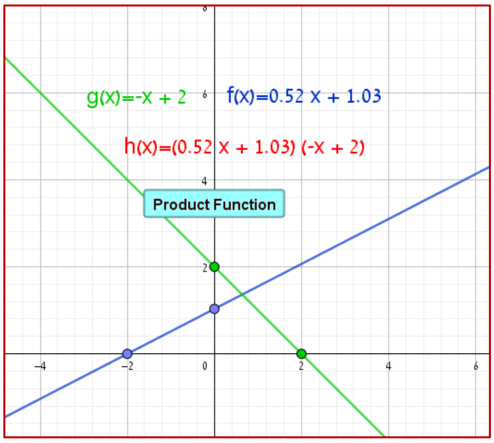

Figure 2: Second task applet

The task was chosen because the product function of two linear functions has a potentially varied and rich example space. Similarly to the previous task, the Geogebra features were limited to dragging the two linear functions and to the "Product Function" button, which supports the immediate construction of the product function. Similarly to the previous task, students were asked to submit three examples as different as possible.

Second AIEF Design Example

To design a rich and varied list of characteristics that would enable STEP to analyze the characteristics of the students' submissions, we considered the following possible general conjecture: The product function of two linear functions is either a linear function or a quadratic function that has at least one zero point. When deconstructing this statement, we noted that the formulation of the general conjecture contained, among others, several attributes that are relevant for this general conjecture. We included all the relevant attributes in the list of characteristics. For example, the information that the two linear functions and the product function share the zero points could be relevant for conjecturing that if the product function is quadratic, it has at least one zero point. To contain other mathematical characteristics that students may choose to address or take into account in their conjectures, we included other attributes of the functions, such as: "The two linear functions have the same slope tendency" and "The product function has a maximum value," which could serve in other conjectures, with different levels of generality for this given applet and setting.

As a consequence, we predefined the following types and attributes: "The product function has one zero point," "The two linear functions are rising," "The two linear functions have different slope tendencies," "The two linear functions are descending," "The two linear functions have the same slope tendency," "The product function has two zero points," "The product function is a quadratic function," "The product function has a maximum value," "The product function has a minimum value," "The product function has no zero points," "The two linear functions and the product function have the same zero points," "The product function is a linear function." For example, suppose the students submitted a product function of two linear functions that have the same slope tendency and different zero points. STEP would identify the following characteristics: "The two linear functions rising," "The two linear functions have the same slope tendency," "The product function has two zero points," "The product function is a quadratic function," "The product function has a minimum value," and "The two linear functions and the product function have the same zero points." In this case, the students are expected to observe the information and further engage with the different mathematical and relevant characteristics of their examples, in the process of formulating a general conjecture.

A SUPORTING EXAMPLE FOR THE AIEF DESIGN PRINCPLE

In the current section we demonstrate a supportive example for the AIEF design principle. To that end, we present a case of a pair of 9th grade students, Ella and Anna, chosen randomly from high schools in Israel. They performed the first activity, on perpendicular bisectors. First, they performed the online EET. To be able to check whether the students used the feedback to produce new conjectures, we gave the students the option to go back to the task, resubmit new examples, and formulate a new conjecture. The data were collected using the STEP platform. The students' work was recorded using the Camtasia screen recorder and video editing software, to triangulate with elements collected automatically. The three examples were analyzed by STEP to produce AIEF. Figure 3 shows a screenshot of the student submissions and the online AIEF they received:

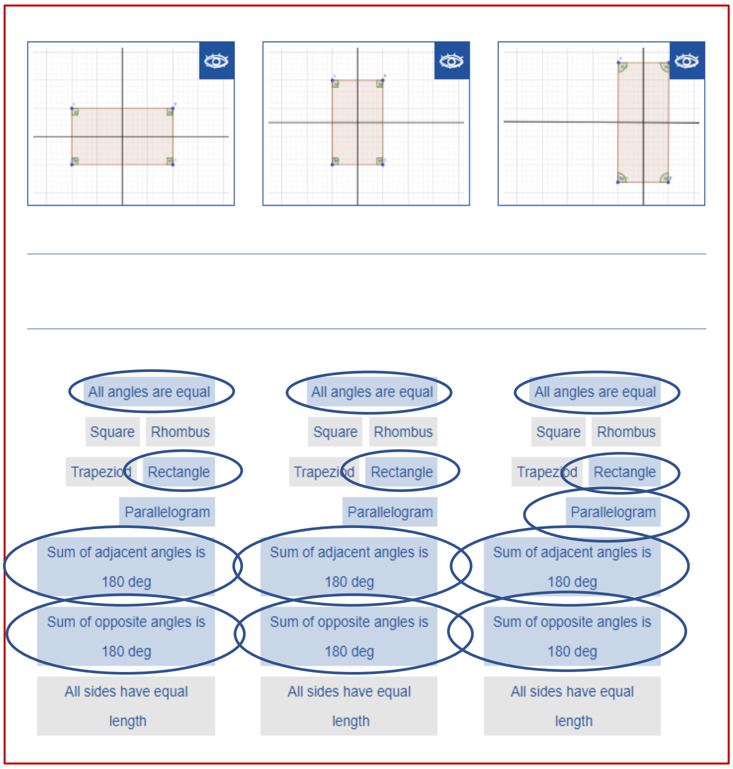

Figure 3. First set of submissions and feedback to students

As expected based on previous experiences and the literature, the students submitted a general conjecture that contains more conditions than necessary in the context provided. It is a special case consisting of alterations of a single shape: a rectangle. We sought to produce a rich list of characteristics that would enable STEP to identify various characteristics of the students' submissions. As can be seen in Figure 3, STEP identified the same five characteristics for each example: "All angles are equal," "Rectangle," "Parallelogram," "Sum of adjacent angles is 180º," and "Sum of opposite angles is 180º."

Below are some of the students' reactions to the feedback:

Anna: All the feedbacks are the same.

Ella: Yes. When you submit a rectangle.

Anna: No. But it doesn't have to be like that. It can also be a parallelogram [pointing at the attribute: "parallelogram" that appears in the feedbacks].

Ella: So it happens every time there is a pair of equal sides.

Anna: No. Whenever there is a pair of equal and congruent sides in the quadrilateral.

At this point, the students could use the characteristic "rectangle" to verify the conjecture they formulated (rectangles are shapes in which the perpendicular bisectors to the sides meet in a single point). Instead, they mentioned the type "parallelogram," and the attributes "equal sides," and "congruent sides," and used them to formulate a new conjecture. As explained in the AEIF design section, we predefined parallelogram as fitting only special cases of parallelograms, not providing a general claim that fits the required conditions. The students used this information to formulate a new conjecture.

The students had the option of returning to the task and resubmitting new examples, and they returned to the applet. First, they refuted the conjecture that parallelogram bisectors to the sides of ABCD meet in a single point. Based on their first action, we came to the conclusion that they returned to the applet to investigate the parallelogram shape, which led us to the conclusion that predefining the parallelogram supported the continuity of the conjecturing process.

Next, the students found several examples in which the perpendicular bisectors to the sides of ABCD meet in a single point. Confused by the variety, they decided to submit three examples and to get ideas from the feedback. In other words, they deliberately sought help from the feedback in formulating conjectures. Figure 4 shows a screenshot of the students' submissions and the elaborated feedback they received.

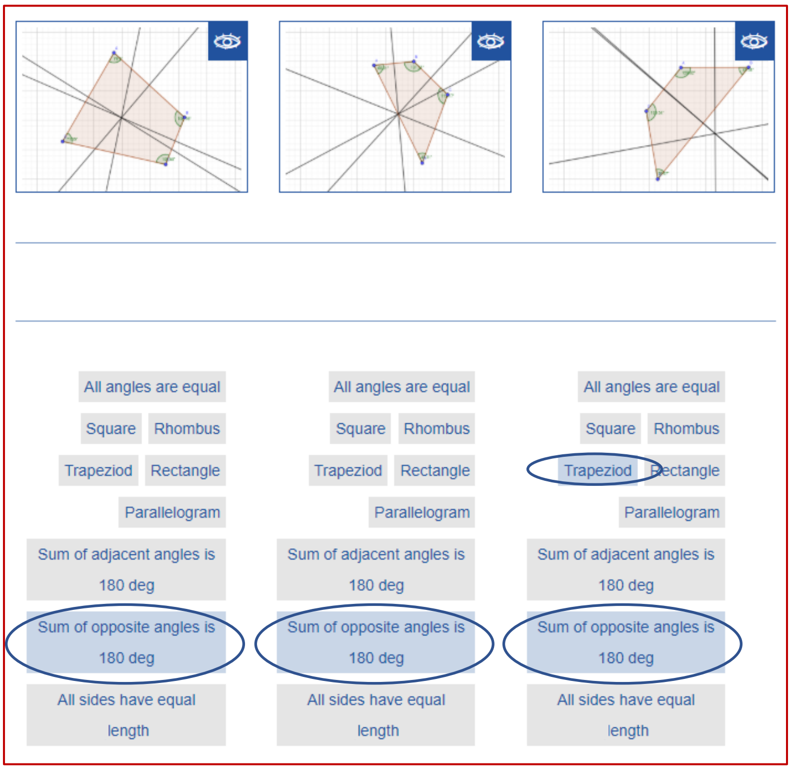

Figure 4. Second set of submissions and feedback to students

The students' reaction to the feedback was as follows:

Ella: Well, that explains it.

Anna: Opposite angles are supplementary.

Ella: This is our assumption. Opposite angles are supplementary.

The students used the feedback to formulate the general conjecture. Indeed, the AIEF dictated a single general conjecture. It is reasonable to assume that without this particular design of the AIEF, the students might not have proceeded beyond formulating different general conjectures that do not arrive to the most general level of conjecturing for this applet and setting.

DISCUSSION

The aim of our study was to offer a design principle for online formative assessment that supports the students' conjecturing process. The two design cases that were presented, provided different types of information regarding general conjectures. For example, in the first case, the students could generalize from one attribute appearing as part of the list of attributes of the AIEF. In the second case, the students could generalize from several characteristics appearing as part of the list of attributes of the AIEF. In this way, we seek to demonstrate the design principle of AIEF manifest in different contexts.

Our goal for AIEF design was to support students in moving from special cases to a general conjecture. The supporting example of the pair of students suggest that the AIEF supports the conjecturing process, which started with the students' specific conjecture and moved toward formulation of the general case. The students also succeeded in shifting from formulations that consist of shapes (rectangle, parallelogram) to those consisting of attributes (opposite angles are supplementary). We suggest that the AIEF design has the potential to support the reasoning process of the students. Shute (2008) argued that automated online formative assessment has the potential to improve the learners’ mathematical thinking and skills. We believe that AIEF, appearing as a well-designed component of an online formative assessment system, has the potential to improve the students' conjecturing skills.

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