NETWORKING OF THEORIES RECONSIDERED

Angelika Bikner-Ahsbahs

University of Bremen; bikner@math.uni-bremen.de

New directions of design for mathematics education may require insights from multiple resources including different theoretical perspectives. Rather than as a problem, the Networking of Theories conceptualizes this diversity as having potential for teaching and learning. Over the past decade more insight has been gained into how theories may work together and what this may offer. This paper provides an overview on research in which the Networking of Theories is used as a research practice including the notion of theory. By examples it highlights networking strategies and methodologies as ways to link theoretical perspectives and their potentials systematically. Reflecting on a recent design study that includes technology as part of a multimodal perspective, a case of networking theories across disciplinary boundaries to integrate technological as well as educational design is also considered as part of interdisciplinary design research.

Keywords: Networking of Theories, multimodality, notion of theories, design research, epistemological gap

A CASE OF NETWORKING OF THEORIES IN MATHEMATICS

The networking of theories seems to be a practice implicitly used in mathematics. To begin with, I would like to illustrate a case of local integration where an algebraic perspective uses geometrical objects to build an algebraic structure. Let us consider a square and its symmetry mappings, four rotations and four reflections across the symmetry axes of the square. Composing these mappings will shape symmetry mappings of the square, too. Hence, the set of these symmetry mappings establishes a dihedral group, an algebraic structure concretized by a set of geometric objects with an operation. This illustrates a Networking of Theory case of locally integrating two theoretical perspectives. Geometrical as well as algebraic features of the square beyond this specific situation are not in play.

NETWORKING OF THEORIES IN MATHEMATICS EDUCATION

The term Networking of Theories has emerged during the Thematic Working Group on theoretical perspectives at CERME 4 in 2005 as a reaction to the growing diversity of theories and paradigms in mathematics education and the problems it encounters (Artigue, Bartolini Bussi, Dreyfus, Gray, & Prediger, 2005). This diversity seems to be an intrinsic feature of the field because of the diverse philosophical traditions, the distinct cultural, institutional and social situations of the educational systems in the field. The problem was not the growing diversity itself, but the field's resulting fragmentation, which causes problems of communication between theory cultures, of how to integrate research results coming from different theory traditions, and of how to deal with the complex nature of the research objects such as teaching and learning in the classrooms, in research as well as design.

At CERME 4, the Networking Theories Group was founded to explore the potentials and pitfalls of the Networking of Theories (see Bikner-Ahsbahs & Prediger, 2014; Kidron, Bosch, Monaghan, & Palmér, 2018). This group used a common data source offered by the Turin research group to investigate how to connect the five theories present in the group by case studies, discussed results in the subsequent CERME theory groups and at PME conferences and published a book edited by Bikner-Ahsbahs and Prediger (2014). New concepts have emerged in this work, for instance a *common epistemological sensitivity* among theories made it possible to compare and contrast different kinds of conceptualizing context (Kidron, Artigue, Bosch, Dreyfus, & Haspekian, 2014). Networking of Theories has forerunners who did not explicitly use this term but were already engaged in research in a similar manner, for example Artigue (see Kidron & Bikner-Ahsbahs, 2016) and Bauersfeld (1992). However, Artigue and Bosch (2014) have also shown that the Networking of Theories as conducted in the Networking Theories Group was still at a stage of craft knowledge, which lacked a theoretically informed discourse on a meta-theoretical level. Additional case studies from outside this group added results, for example about different ways of conceptualizing mathematical objects (Font Moll, Trigueros, Badillo, & Rubio, 2016) or comparing the notion of practices with CAS from the instrumental and the Onto-semiotic approach (Drijvers, Godino, Font, & Trouche, 2013).

The Networking of Theories is also used in empirical research. For example Bikner-Ahsbahs and Kidron (2015) describe how the concept of General Epistemic Need (GEN) was built by a cross-methodology conducted from the two theoretical perspectives involved. Tabach, Rasmussen, Dreyfus and Hershkowitz (2017) have linked an individual and a collective perspective to investigate knowledge construction in an inquiry based classroom. The research coordinated key concepts of the two theories involved and showed their complementary nature for constructing knowledge.

Meanwhile, in design research the Networking of Theories is considered as a way of including different theoretical approaches into design cycles. For example Kouropatov and Dreyfus (2017) have used two theories, Proceptual Thinking and Abstraction in Context, for designing a task-based curriculum for teaching the integral as an accumulating function in three design stages, predesign, the initial design and re-design, and final design. Another interesting example on design research is shown in the project ReMath, where a common framework was established to link digital tools with different theoretical perspectives of design research studies. In ReMath a new networking methodology was implemented, *cross-experimentation* (Artigue, & Mariotti, 2014).

Previous definitions of the notion of Networking of Theories focussed on the connecting of theoretical perspectives (Bikner-Ahsbahs, & Prediger, 2010). But Kidron and Bikner-Ahsbahs (2015), Kidron et al. (2018) and also the book edited by Bikner-Ahsbahs and Prediger (2014) have worked out a more comprehensive understanding of this notion. After about fifteen years of research crucial elements of the Networking of Theories as a research practice have become visible. Based on this body of research I would like to propose five defining features: the Networking of Theories as a research practice

- investigates how specific theories work and may work together
- for a specific purpose
- by creating a dialogue, linking and building relations between (at least) parts of theoretical approaches in a methodological sound manner and
- reflect on this networking practice
- while respecting the identities of the theoretical approaches involved.

This definition builds on the assumption that the diversity of theoretical approaches is an intrinsic and enriching feature in the field of mathematics education. As any research practice, also this one may evolve by research, be further explored, and expanded to address new aims and provide new kinds of questions and methodologies.

Since theories are the objects of the Networking of Theories, the notion of theory should also be a topic of contemporary research.

NOTIONS OF 'THEORY'

Unfortunately, the field does not agree in what the term *theory* means (Assude, Boero, Herbst, Lerman, & Radford, 2008). But there are some commonalities. Mason and Waywood distinguish between background theory and foreground theory as relative categories (1996):

- A *background theory* "is a (mostly) consistent philosophical stance of or about mathematics education which plays an important role in discerning and defining what kind of objects are to be studied, …. " (p. 1058).
- *Foreground theories* are mostly local theories in mathematics education: "…. because [of] the foreground aim of most mathematics education ….. ." (p. 1056).

Constructivism as an individual learning perspective could be taken as a philosophical stance, which considers learning as individual construction allowing to discern what can be taken as research objects, for example mental models. Investigating how a mental model of a function concept is built would be a question addressing a foreground aim that may lead to a foreground theory on learning functions. However, this perspective does not allow for conceptualizing teaching and learning mathematics as a cultural activity like in activity theory. As activity theory considers teaching-learning as an irreducible entity (see Shvarts & Abrahamson, 2019), a foreground theory on functions within activity theory might be gained by exploring how the common labor in teaching and learning allows function concepts be built by actions and their goals being supported or mediated by tools.

In sum, any theory can be regarded as a lens with a language that consists of concepts and claims, but how these parts are structured depends on the specific notion of theory.

Radford (2008) for example proposes that „a theory can be seen as a way of producing understandings and ways of action based on:

- A system, *P, of basic principles,* which includes implicit views and explicit statements that delineate the frontier of what will be the universe of discourse and the adopted research perspective.
- A *methodology [methodologies], M*, which includes techniques of data collection and data-interpretation as supported by *P*.
- A set, *Q, of paradigmatic research* questions (templates or schemas that generate specific questions as new interpretations arise or as the principles are deepened, expanded or modified)" (Radford 2008, p. 320, emphasis in the original).

He uses the triplet (P, M, Q) as a representation of this notion of theory and enlarges it towards a dynamic understanding of theory: a theory develops by research results *R* that may inform the "way of producing understandings and ways of action". As a final shortcut of such a dynamic understanding of *theory* he introduces the quadruplet [(P, M, Q), R] (see Radford, 2012).

In Radford's description of theories Networking of Theories means building relations among their parts. What does it mean? Given two theories are used to investigate a common question by a common methodology that encompasses the methodologies of each of the two theories, this may lead to a new concept if a common principle can be included into the two theories. In the latter case we may have developed a local integration linking all the three parts (P, M, Q) of the theories. This kind of networking will now be illustrated by the first example described in Sabena, Arzarello, Bikner-Ahsbahs, & Schäfer (2014).

EXAMPLE 1: THE EPISTEMOLOGICAL GAP

During the first meetings of the Networking Theories Group a short episode was analyzed where the students explored the exponential function $y = a^x$ with Capri. The students observed the graph of this function (Fig. 1) with a secant when the teacher says: How does the exponential function grow for very big x?

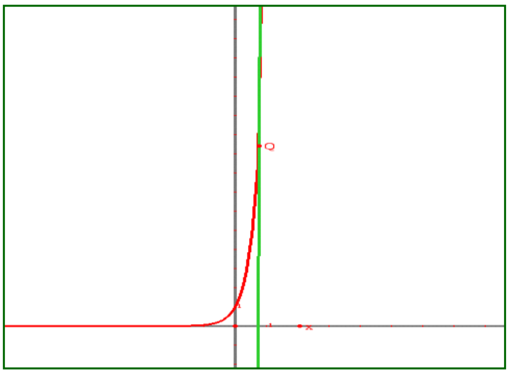

Fig. 1: Graph of $y = a^x$ (Sabena, Arzarello, Bikner-Ahsbahs & Schäfer, 2014, p. 181)

The first two utterances are:

(#1) G: … but always for *a* very big this straight line (pointing at the screen), when they meet each other, there it is again…that is it approximates the function very well

(#2) T: what straight line, sorry?

……

The following interaction between the teacher and the student G was investigated by Sabena et al. (2014), it happens in three phases (see Sabena et al. 2014, pp. 182-184).

Phase 1

G's view: the graph of the function can be approximated by a vertical line (#1-3)

Fig. 2 G shows that x is going up as part of the points on the graph

Phase 2

Teacher (#4): Will they meet each other?

Fig. 3 The teacher introduces with gestures how the graph and a vertical line should meet.

Phase 3

G (#5-7): That is, yes […] It makes so

Fig. 4 Dissonance between G's gestures with the gestures of the teacher

This episode was analyzed from two perspective: (1) the semiotic game the teacher played with the students in the teaching of the exponential function with Capri was analyzed by the semiotic bundle concept and (2) the epistemic process in the interaction was analyzed to explore if an interest-dense situation could emerge and why or why not.

The semiotic bundle consists of three semiotic sets and their developing relations as the bundle evolves, the sets of ”words, gestures and representations [of the exponential function $y = a^x$] in the Cabri file (Sabena, 2014, p. 186). In the semiotic game, the teacher tries to introduce new features of the content by tuning with the student's words and at the same time using gestures to address the new aspects he wants to be part pf the learning (see Fig. 2-4).

(#8)

Fig. 5 The teacher shows in line 8 how the graph should cross vertical lines

(#16)

Fig. 6 The teacher shows that for big x that the vertical line at x should be crossed by the graph

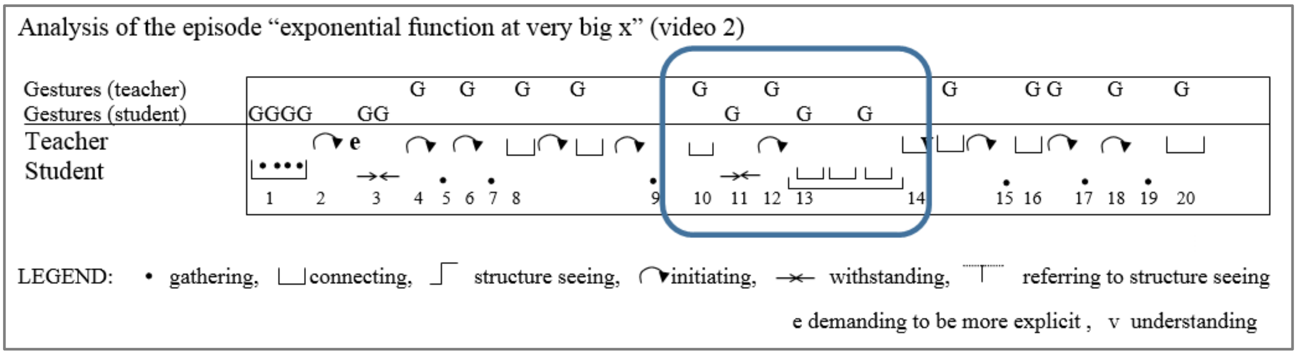

Fig. 7 The analysis diagram of the episode by the use of the epistemic action model (following line numbers). G indicates relevant gestures.

The second perspective investigates the interactions to characterize the epistemic process; and if and how interest-dense situations emerge. Interest-dense situation are specifically fruitful in that in the epistemic process the students achieve structure-seeing (Bikner-Ahsbahs & Halverscheid, 2014). The analysis uses an epistemic action model to reconstruct this epistemic process shaped by three epistemic action. It assumes that the epistemic actions *gathering* and *connecting* mathematical meanings must take place before *structure-seeing* is possible. If the students begin to deepen their engagement into the mathematical task by these actions an interest-dense situation emerges while catching and keeping students' interest for a while, at least until they see and explore mathematical structures.

Fig. 7 shows the analysis diagram of the epistemic process. Until line 9 the teacher tries to initiate his own view (see the arrows) while the student briefly gathers ideas (points in the diagram). At line 10 the situation changes because the student begins to withstand the teacher's view (double arrow) pointing to the screen in line 11. The big rectangle in the middle of the diagram highlights the part where an interest-dense situation emerges: an intense gesture exchange and a complex engagement by the student by *connecting* several mathematical ideas happens. This situation finishes with the following two utterances:

Student G: that is, at a certain point … that is if the function (image G-00:57) increases more and more, more and more (image G-00:59) then it also becomes almost a vertical straight line (image G-1:03)

Teacher: eh, this is what seems to you by looking at (pointing gesture)

The student G tries to describe his perceptive view in that he thinks that for very big x the graph of the exponential function grows with x and approximates finally a vertical line (this is not true but the screen gives this impression to the student). The teacher emphasizes that the screen gives a wrong impression: "this is what seems to you by looking at [the screen]" without taking into account how the student could achieve this evaluation himself. What the teacher just expresses is that what the student perceives is wrong trying then to describe what is true. At this moment, the student stops to engage himself and just gives short one-word answers. The interest-dense situation dries up.

Starting the process of Networking of Theories

The separate analyses of the two perspectives showed: Although the semiotic game was successful, the student did not profit from it, the emerging interest-dense situation dried up. The question why this happened could not be answered at the beginning. Both, Ferdinando Arzarello and Angelika Bikner-Ahsbahs, met to deeply become re-engaged with analysing the data, adding data, trying to make the theoretical views mutually explicit, and compare and contrast them. In the attempt to translate each interpretation into the other language both became aware that there was something missing in both approaches, an epistemological perspective which would explain what we finally were aware of, the *epistemological gap* between the teacher's and the student's epistemological views. While the student showed in his gestures that he associated big x with the top location of the points on the graph the teacher did not look at the graph, instead he argued on the basis of formal mathematics and its limit concept of calculus (Fig. 2-4, Fig. 5, 6). The two views, *perceptive fact seen on the screen* and *formal mathematics*, shape the epistemological gap that the student could not bridge by himself. How such a gap can be turned into a learning opportunity, would be a question for design research.

This case shows a local integration by adding a new concept at the boundary of the two theory cultures, the epistemological gap, which involved the three criteria of theory as proposed by Radford (2008) (see Sabena et al., 2014):

- A common research question: Why does the teacher-student interaction not work?
- A common theoretical principle: Adding an epistemological dimension by including the concept of the epistemological view into both approaches.
- A common methodology: Including both kinds of analysis into a common methodology.

NETWORKING STRATEGIES

Fig. 8 represents a revised version of the networking strategies landscape developed based on the contributions of the CERME theory groups (Prediger, Bikner-Ahsbahs, & Arzarello, 2008). The landscape does not consider the two poles, *ignoring others* and *unifying globally*, as belonging to networking practices. On the contrary, the diversity of theories is regarded as richness in the field and connecting theories as a way of how theory cultures may interact. Between the two poles, pairs of networking strategies are located according to their degree of integration. Integrating locally as described in the example above is one of the two strategies of integration. The other one is synthesizing which the Networking Theories Group has not yet observed.

The local integration of the two perspectives, *semiotic bundle* and *interest-dense situation*, did not happen by chance, it had been prepared by a process of mutual understanding, comparing and contrasting ideas and concepts of the two approaches. In this process, mutual understanding was more than just a first step. Radford conceptualized this strategy as an attempt to translate own theoretical aspects into the other language which probably is not possible and therefore results in unpacking implicit assumptions, blind spots or the lack of an important principle. By combining and coordination both approaches in the process of analysing the data, the experts arrived at producing a common question, a common additional principle and merged their methodologies into a common one, thus, yielding a case of local integration. This local integration was possible because the two perspectives already shared common assumptions, e.g., constructing knowledge is a micro process expressed semiotically. During the whole process, mutual understanding was improved by permanent reflections which Akkerman and Bakker (2011) call perspective taking and making, a learning mechanism of boundary crossing between the two theory cultures (Fig. 8).

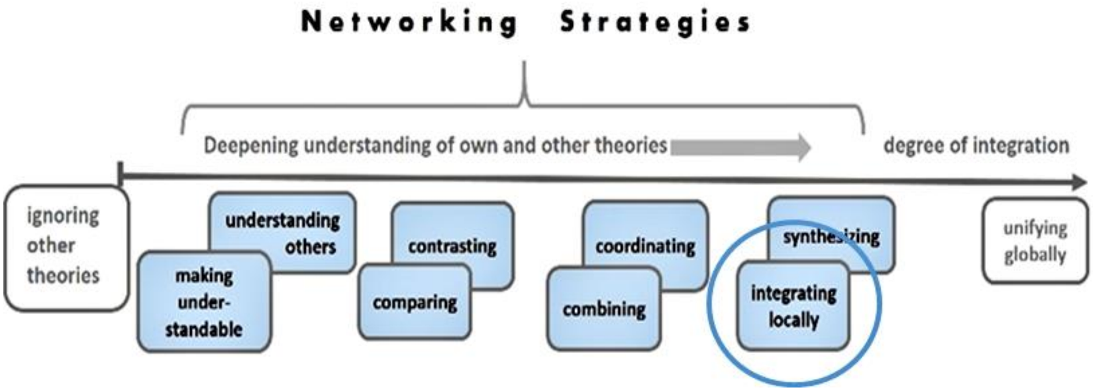

Fig. 8 Networking strategies (Bikner-Ahsbahs, 2016, p. 34, revised version from Prediger et al. 2008, p. 170)

EXAMPLE 2: DESIGN OF A TECHNOLOGICAL LEARNING SYSTEM

Algebra is a gatekeeper for mathematics and therefore should be accessible for all students. For that reason, tangible manipulatives and digital tools have been developed to support learning algebra. But many digital tools have shown conceptual weaknesses (see e.g. Janßen, Reid, & Bikner-Ahsbahs, 2019, in press). The MAL-project (MAL: Multimodal Algebra Learning) attempts to overcome these weaknesses by addressing learning algebra in a multimodal and conceptual way. With this aim in mind, computer scientists (from human computer interactions) and mathematics educators are working together from the beginning to design an algebra tiles system (MAL-system, MAL: Multimodal Algebra Learning) for the teaching and learning of linear equations in a multimodal way.

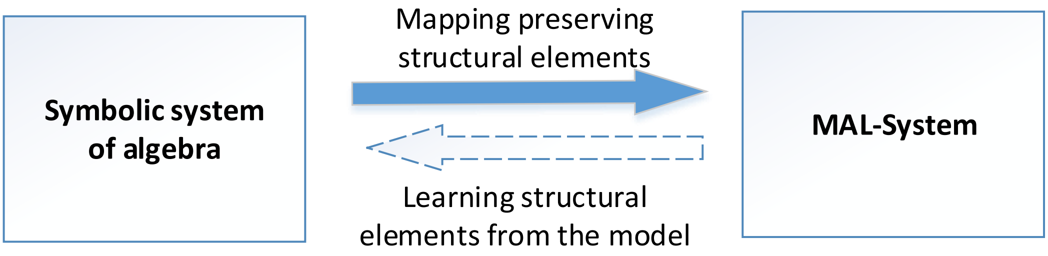

Fig. 9 Conceptualising the design process by mapping (see Janßen et al., 2019)

Based on the interface theory developed by Goguen (1999) the design of a didactic model may be regarded as a mapping from the symbolic system of algebra to an algebra tiles model (MAL-System) that preserves key structures (Fig. 9) (see Janßen et al. 2019).

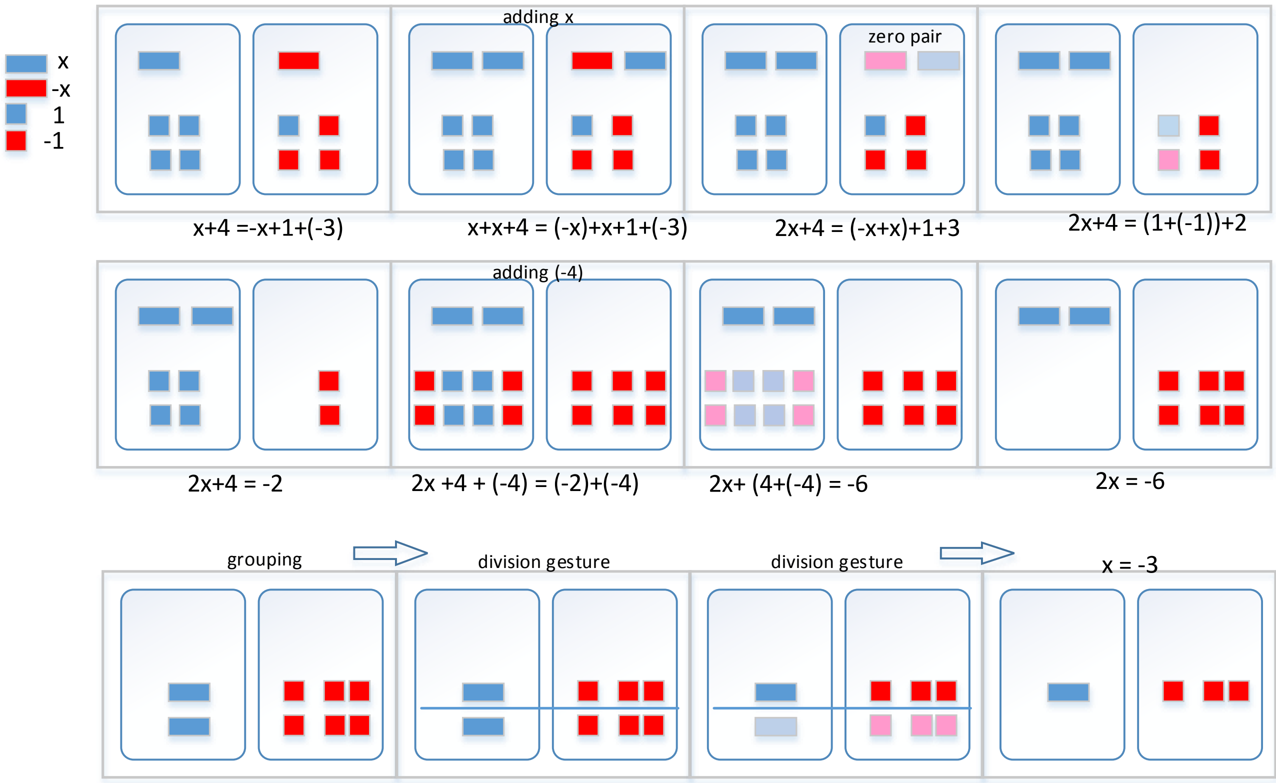

Fig. 10 How people may solve an equation with the MAL-system

These structures are extracted from epistemological analyses, and modelled in the technological design. Variables, numbers and algebraic expressions are represented by tiles, the two signs (+, -) are represented by colors and operations by actions with the tiles (Fig. 10). Variables and numbers are distinguished by the shapes of tiles, rectangles are used for variables and squares for unit tiles. Main features of the current version of the MAL-system are shown in Fig 10: addition, subtraction, zero pairs, division to solve linear equations.

While designing the didactic model is regarded as a mapping from the symbolic system of algebra to the target technological design learning goes into the opposite direction (Fig. 9): By working with the MAL-system students shall first learn the key structures, and then expand them beyond the restrictions the didactic model has. For this purpose, Activity theory (Leontjew, 1979; Shvarts & Abrahamson, 2019) is added to frame teaching-learning for designing and researching tasks for the MAL-system which are explored in teaching-learning situations to understand the functioning of the MAL-system and to inform revisions of technological as well as educational design in the next step.

Final remarks on design and research on the MAL-system

The current MAL-system as presented in Fig. 10 has been explored with four pairs of students of grade eight who already have learned how to solve linear equations. However, the use of the division gesture was new to them, hence, helped to understand the division of equations conceptually (see Janßen, Vallejo-Vargas, Bikner-Ahsbahs, & Reid, submitted). Fig. 11 shows one pair of students working on division tasks. The two students have grouped tiles on the two sides of the mat of the MAL-system in a way the MAL-system is not able to handle. Using the division gesture (see Fig. 10) for dividing by 2, that is drawing a line on the multi-touch display with a finger which separates the two groups on each side of the mat, would have required to split two tiles into halves. However, the MAL-system does not allow for splitting tiles although this should be theoretically possible. This situation indicates the students' need to go beyond the restrictions of the MAL-system, hence, to emancipate from it. This aspect points to one of four layers on which investigating the teaching-learning with the MAL system is conducted:

Layer 1: Learning the MAL-system
Layer 2: Learning with the MAL-System
Layer 3: Learning to link the MAL-symbolic expressions with the algebraic expressions
Layer 4: Learning to emancipate from the MAL-system

Fig. 11 Splitting tiles by dividing by 2 with the MAL-System (see Janßen et al., submitted)

Teaching and learning of algebra with the MAL-system is not yet investigated profoundly enough. But our pilot studies have pointed to four layers to be considered in research. Layer 1 and layer 2 address the instrumental genesis when using the MAL-system (Artigue, 2002; Drijvers et al. 2013). Layer 3 looks in more detail at forms of translations between the two semiotic systems (Duval, 2008) including feedback. Finally, layer 4 describes the transformation of the students' relations with the instrument when they emancipate from it.

The networking of theories may offer a way to link these four layers of learning through a common framework of different theoretical elements. Following this path would require an in-depth understanding of the theoretical approaches involved. This may be achieved by using the strategies of mutual understanding, comparing and contrasting before considering combining and coordinating them. In this process, principles and assumptions of the theoretical approaches should become visible, the way the perspectives behind shape what can be taken as research objects, what the researchable questions are and the way they address methodological affordances when the teaching and learning of algebra with the MAL-system is researched.

DISCUSSION

Looking at the topic of the Networking of Theories, Bakker's review (2016) has opened up the interesting perspective of considering it as boundary crossing between different theory cultures for which boundary objects are identified. If we look back on the development of the concept of epistemological gap, the video of the episode was our data source and the boundary object both groups could use. During research, we have revised the given transcript to make it a piece of data for both theoretical approaches, hence, data was made to serve for the local integration.

According to the MAL-project, the design of the MAL-system is our (dynamically changing) boundary object. For example, structural elements from an epistemological analysis of solving linear equations were offered to the computer scientists, but these elements had to be structured in a certain way, thus perspective taking and making was an important part of boundary crossing. Before the computer scientists could develop the product we conducted a paper prototyping study to investigate beforehand how our ideas would work together to avoid developing a product which would not be educationally useful, thus, perspective taking and making was systematically included.

The main challenge in the MAL-project was boundary crossing across the disciplines and this involved different notions of theory. The Networking of Theories was part of it. In the design process, the theory by Goguen (1999) and an epistemological theorizing of the mathematics informed the designing process, and Activity Theory assisted in researching the use of the design to understand and inform the next cycle of design research. Once, the MAL-system is ready for use in teaching and learning in school, research including several theoretical perspectives into a comprehensive research frame would be indispensable, hence, follow a Networking of Theories path.

Acknowledgement Some research reported on in this paper is part of the project "Multimodal Algebra Lernen" (MAL), which is funded by the German Federal Ministry for Education and Research in the grant program "Erfahrbares Lernen" (experienceable learning) (grant number: 16SV7550K).

REFERENCES

Akkerman, S. F., & Bakker, A. (2011). Boundary crossing and boundary objects. *Review of Educational Research, 93*(2), 132–169.

Artigue, M. (2002). Learning Mathematics in a CAS Environment: The Genesis of a Reflection about Instrumentation and the Dialectics between Technical and Conceptual Work. *International Journal of Computers for Mathematical Learning 7*(3), 245–274. *DOI: 10.1023/A:1022103903080*

Artigue, M., & Bosch, M. (2014). Reflection on Networking Through the Praxeological Lens. In Bikner-Ahsbahs & Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 249–279). New York: Springer

Artigue, M., & Mariotti, A. M. (2014). Networking theoretical frames: the ReMath enterprise. *Educational Studies in Mathematics 85*(3), 329–355. DOI 10.1007/s10649-013-9522-2

Artigue, A., Bartolini Bussi, M., Dreyfus, T., Gray, E., & Prediger, S. (2005). Different theoretical perspectives and approaches in research in mathematics education. In M. Bosch (Ed.), *Proceedings of the 4th Congress of the European Society for Research in Mathematics Education* (pp.1249–1243). Barcelona: Universitat Ramon Llull.

Assude, T., Boero, P., Herbst, P., Lerman, S., & Radford, L. (2008). *The notions and roles of theory in mathematics education research.* Paper presented by the survey Team at ICME 11, Mexico 2008. www.mathunion.org/fileadmin/ICMI/files/About_ICMI/Publications_about_ICMI/ICME_11/Assude.pdf (access 29.07.2019)

Bakker, A. (2016). Book Review: Networking theories as an example of boundary crossing. A. Bikner-Ahsbahs & S. Prediger (Eds.) (2014), Networking of theories as a research practice in mathematics education. *Educational Studies in Mathematics 93*(2), 265–27. doi.org/10.1007/s10649-016-9715-6

Bauersfeld, H. (1992). Activity theory and radical constructivism —What do they have in common and how do they differ? *Cybernetics and Human Knowing*, *1*(2/3), 15–25.

Bikner-Ahsbahs, A. (2016). Networking of theories in the tradition of TME. In A. Bikner-Ahsbahs, A. Vohns, O. Schmitt, R. Bruder, & W. Dörfler (Eds.), *Theories in and of Mathematics Education. Theory Strands in German-Speaking Countries* (pp. 33–42). Switzerland: Springer International Publishing AG.

Bikner-Ahsbahs, A., & Halverscheid, St. (2014). Introduction of the theory of interest-dense situations. In A. Bikner-Ahsbahs & S. Prediger (Eds.). *Networking of Theories as a Research Practice in Mathematics Education* (pp. 88–102). New York: Springer.

Bikner-Ahsbahs, A., & Kidron, I. (2015). A cross-methodology for the networking of theories: The general epistemic need (GEN) as a new concept at the boundary of two theories. In A. Bikner-Ahsbahs, Ch. Knipping, & N. Presmeg (Eds.), *Approaches to qualitative methods in mathematics education – examples of methodology and methods, Advances in Mathematics Education* (pp. 233–250). New York: Springer.

Bikner-Ahsbahs, A., & Prediger, S. (2010). Networking of Theories—An Approach for Exploiting the Diversity of Theoretical Approaches; with a preface by T. Dreyfus and a commentary by F. Arzarello. In B. Sriraman & L. English (Eds.), *Theories of mathematics education: seeking new frontiers* (pp. 479–512). New York: Springer.

Bikner-Ahsbahs, A., & Prediger, S. (Eds.), Networking Theories Group (2014). *Networking of Theories as a Research Practice in Mathematics Education.* New York: Springer.

Drijvers, P., Godino, J. D., Font, V., & Trouche, L. (2013) One episode, two lenses. *Educational Studies in Mathematics 82*(1), 23–49. DOI 10.1007/s10649-012-9416-8

Duval, R. (2008). Eight problems for a semiotic approach in mathematics education. In Luis Radford, Gert Schubring, & Falk Seeger (Eds.), *Semiotics in Mathematics Education. Epistemology, History, Classroom and Culture* (pp. 39–61). Rotterdam: Sense Publishers.

Font Moll, V., Trigueros, M., Badillo, E., & Rubio, N. (2016). Mathematical objects through the lens of two different theoretical perspectives: APOS and OSA. *Educational Studies in Mathematics 91*(1), 107–122. DOI 10.1007/s10649-015-9639-6

Goguen, J. (1999). An introduction to algebraic semiotics, with application to user interface design. In G. Goos, J. Hartmanis, J. van Leeuwen, & C. L. Nehaniv (Eds.), *Lecture Notes in Computer Science. Computation for Metaphors, Analogy, and Agents* (pp. 242–291). Berlin, Germany: Springer.

Janßen, T., Reid, D., & Bikner-Ahsbahs, A. (2019, in press). Issues in modelling terms involving subtraction in a manipulative environment for linear equations—and a possible solution. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education (xx-yy).* Utrecht, the Netherlands: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Janßen, T., Vallejo-Vargas, E., Bikner-Ahsbahs, A., & Reid, D. (submitted). Design and investigation of a touch gesture for dividing in a virtual manipulative model for equation-solving.

Kidron, I. & Bikner-Ahsbahs, A. (2016) Networking different theoretical perspectives. In B. R. Hodgson, A. Kuzniak, & J.-B. Lagrange (Eds.), *The Didactics of Mathematics: Approaches and Issues.* A Homage to Michèle Artigue. (pp. 43–57). New York: Springer, International Publishing Switzerland. DOI 10.1007/978-3-319-26047-1-3

Kidron, I., Artigue, M., Bosch, M., Dreyfus, T., & Haspekian, M. (2014). Context, Milieu, and Media-Milieus Dialectic: A Case Study on Networking of AiC, TDS, and ATD. In A. Bikner-Ahsbahs & S. Prediger (Eds.), *Networking of Theories as a Research Practice in Mathematics Education* (pp. 153–177). New York: Springer.

Kidron, I., Bosch, M., Monaghan, J., & *Palmér, H.* (2018). Theoretical perspectives and approaches in mathematics education research. In T. Dreyfus, M. Artigue, D. Potari, S. Prediger, & K. Ruthven (Eds.), *Developing Research in Mathematics Education: Twenty Years of Communication, Cooperation and Collaboration in Europe* (pp. 254–268). Oxon: Routledge.

Kouropatov, A., & Dreyfus, D. (2018). Combining two theories in the design for learning about integrals. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 2746–2753). Dublin: DCU Institute of Education and ERME.

Leontjew, A. (1979). *Tätigkeit. Bewusstsein. Persönlichkeit*. [Activity. Consciousness. Personality.] Berlin: Volk und Wissen.

Mason, J., & Waywood, A. (1996). The role of theory in mathematics education and research. In Bishop A. J., Clements K., Keitel C., Kilpatrick J., and Laborde C. (Eds.). *International Handbook of Mathematics Education* (pp. 1055–1089). Dordrecht: Kluwer.

Prediger, S., & Bikner-Ahsbahs, A. (2014). Introduction to Networking: Networking Strategies and Their Background. In Bikner-Ahsbahs & Prediger (Eds.), *Networking of theories as a research practice in mathematics education* (pp. 117–125). New York: Springer.

Prediger, S., Bikner-Ahsbahs, A., & Arzarello, F. (2008). Networking strategies and methods for connecting theoretical approaches: first steps towards a conceptual framework. *ZDM-International Journal on Mathematics Education, 40*(2), 165–178.

Radford, L. (2008). Connecting theories in mathematics education: challenges and possibilities. *ZDM - The International Journal on Mathematics Education 40*(2), 317–327.

Radford, L. (2012). *On the Growth and transformation on mathematics education*, Paper presented at the International Colloquium: The Didactics of Mathematics: Approaches and Issues. A Homage to Michèle Artigue. Université de Paris VII. May 31 to June 1, 2012. http://www.luisradford.ca/pub/Connecting%20theories%20-%20Paris%20Oral%20Presentation%20rev2014.pdf (access: 14.06.2015)

Sabena, C., Arzarello, A., Bikner-Ahsbahs, A., & Schäfer, I. (2014). The epistemological gap - A case study on networking of APC and IDS. In A. Bikner-Ahsbahs & S. Prediger (Eds.). *Networking of Theories as a Research Practice in Mathematics Education.* (pp. 165–183). New York: Springer.

Shvarts, A., & Abrahamson, D. (2019). Dual-eye-tracking Vygotsky: A microgenetic account of a teaching/ learning collaboration in an embodied-interaction technological tutorial for mathematics. *Learning Culture and Social Interaction 22,* 100316. DOI: 10.1016/j.lcsi.2019.05.003

Tabach, M., Rasmussen, C., Dreyfus, T., & Hershkowitz, R. (2017). Abstraction in Context and Documenting Collective Activity. In T. Dooley & G. Gueudet (Eds.). (2017). *Proceedings of the 10th Congress of the European Society for Research in Mathematics Education* (pp. 2692–2699). Dublin, Ireland: DCU Institute of Education and ERME.

DOI: 10.17185/duepublico/70738
URN: urn:nbn:de:hbz:464-20191119-150548-2