ORCHESTRATING WHOLE-CLASS DISCUSSIONS IN MATHEMATICS USING CONNECTED CLASSROOM TECHNOLOGY

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This paper reports on the planning of a pilot study where the aim is to develop and investigate teaching practices, using connected classroom technology (CCT), to support formative classroom practices in mathematics. The focus is on the design of a teaching unit including a whole-class discussion drawing on students' computer-based work. The paper outlines both generic and topic-specific theories underpinning the design. Moreover, findings from a previous study, in terms of student responses to a task developed for a dynamic mathematics software environment, are used in the planning. One important issue addressed in this paper is the sequencing of student responses to display and use as a basis for whole-class discussions.

Keywords: connected classroom technology, mathematics education, whole-class discussion

INTRODUCTION

In a concluding paragraph of a survey of the research on technology use in upper secondary mathematics education, Hegedus et al. (2017) present some practically oriented questions to consider in the future. For example,

> … is there, or could there be, a taxonomy for orchestrating student digital work? How can the teacher make best use of student created contributions? What new opportunities of interaction are there between the teacher and the students and what is the role of the teacher within these new forms of interactions? (p. 32)

These types of question have been addressed by studies investigating the educational use of the type of technology referred to as Connected Classroom Technology (CCT). Irving (2006) defines CCT as "… a networked system of personal computers or handheld devices specifically designed to be used in a classroom for interactive teaching and learning." (p. 16). Technological development in this area has led to more sophisticated teaching tools to support richer classroom discourse. For example, Clark-Wilson (2010) found how teachers who used a CCT system over a period of a few months developed their teaching towards new types of formative assessment practices. When describing the most 'desirable' feature of CCT, the teachers emphasised the enhanced opportunity of monitoring students' computer-based work during the lesson. For example this helped the teachers to identify 'interesting screens' to share with the whole class to promote a rich classroom discourse. Although the literature identifies promising opportunities provided by CCT, there is still a challenge for teachers in orchestrating (in the sense used by Hegedus) these types of teaching practice.

This paper reports on preparation for a project, located in a Swedish upper secondary school, which aims to develop methods and procedures for using CCT to support formative classroom practices in mathematics, and so to develop design principles to guide teaching activities using CCT. In this paper, we focus on the design, for a pilot study, of a teaching unit using CCT to support a formative classroom practice in mathematics. The design is guided by both generic and topic-specific theories.

GENERIC THEORIES GUIDING THE DESIGN

In the pilot study, a teaching unit will be trialled. The design of this teaching unit is guided by three generic theories: first, the theoretical framework for formative assessment (FA) by Black and William (2009); second, the three technological functionalities identified by the European project *Formative Assessment in Science and Mathematics Education* (FaSMEd) (e.g. Cusi, Morselli, & Sabena, 2017); and finally the model of five practices as a tool for "orchestrating productive mathematical discussions" by Stein, Engle, Smith, and Hughes (2008). The framework for formative assessment (Black & Wiliam, 2009) consists of five key strategies and three agents (teacher, learner, peer). The FA strategies are

(1) clarifying and sharing learning intentions and criteria for success;
(2) engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding;
(3) providing feedback that moves learners forward;
(4) activating students as instructional resources for one another; and
(5) activating students as the owners of their own learning (p. 8).

While the teacher takes the initiative in the first strategy, it is also important that students are actively engaged, e.g. by discussing the learning intensions. The second strategy aims to identify current student understandings and misunderstandings by making their thinking visible. To do this, the teacher can use various tactics, e.g. using carefully designed tasks that explicitly require students to explain their thinking. However, it is not enough to make students' thinking visible; as the third strategy indicates, the teacher also needs to use this information to provide feedback that guides learners towards the learning goals. The fifth strategy involves helping students to become aware of their own learning process so as to be able to perform self-assessment and self-regulation. Finally, in the fourth strategy, to act as learning resources for one another, students need to learn to collaborate, e.g. to provide as well as receive peer feedback (Black & Wiliam, 2009).

The literature on formative assessment has emphasized the potential of technology to enhance FA strategies in mathematics education (Clark-Wilson, 2010; Irving et al., 2016). The FaSMEd project investigated different aspects of the use of digital technology to promote formative assessment and developed a three dimensional theoretical framework. Besides the five key strategies and the three agents introduced above, the framework identifies a third dimension consisting of three technological functionalities: (a) sending and displaying, (b) processing and analysing, and (c) providing an interactive environment (Cusi et al., 2017).

The first functionality facilitates teacher – student and student – student communication. The second functionality concerns different kinds of management and analysis of data. Finally, the third functionality concerns interactive environments in two ways; technology that enables students to explore mathematical relations individually (or in small groups) and technology that provides a shared platform for whole-class collaboration.

Cusi et al. (2017) emphasize the challenge for teachers to plan whole-class discussions based on students' computer-based work. Accordingly, they complemented the FaSMEd framework with the five practices proposed by Stein et al. (2008):

(1) anticipating likely student responses to cognitively demanding mathematical tasks,
(2) monitoring students' responses to the tasks during the explore phase,
(3) selecting particular students to present their mathematical responses during the discuss-and-summarize phase,
(4) purposefully sequencing the student responses that will be displayed, and

(5) helping the class make mathematical connections between different students' responses and between students' responses and the key ideas. (p. 321)

Although this model focuses on how to follow up students' previous work on problem solving tasks, results from a previous study (Fahlgren & Brunström, 2016) indicate that the model might be useful when following up students' computer-based work as well.

To summarize, the generic theoretical frames that will underpin the design of a teaching unit are the five key strategies for formative assessment (FA-1 to FA-5), the three technological functionalities (TF-1 to TF-3), and the five practices for orchestrating whole-class discussions (TP-1 to TP-5).

GENERIC DESIGN PRINCIPLES FOR A TEACHING UNIT

The planned teaching unit consists of three stages: *introduction*, *pair work*, and *whole-class discussion*. A description of the stages and how elements from each theory have been taken into account is presented below.

Introduction. The teacher introduces the activity by clarifying the purpose (FA-1) and providing necessary instructions for performing the activity. The material, primarily in terms of e-worksheets, is delivered (TF-1) to pairs (or small groups) of students.

Pair work. While the students are working on the computer-based activity (TF-3), they are prompted to send (TF-1) their responses to each subtask as they proceed. Students will be encouraged to agree on a common response to each subtask. The reason for this is twofold. Primarily, this obliges the students to communicate their mathematical reasoning to each other. Moreover, in this way the number of responses for the teacher to monitor is reduced.

The teacher's role during this stage is to monitor (exploiting TF-1 to carry out TP-2 as a basis for subsequent FA-2) the students' work in two ways; on one hand the teacher follows all the students' progression across the whole activity, and on the other hand, the teacher monitors all students' responses to a particular item.

The most critical issue for the teacher during this stage is to identify and select appropriate student responses (exploiting TF1 to carry out TP-3) to display and use as a basis for the whole-class discussion. Moreover, the teacher has to decide the sequencing of these responses (TP-4).

Whole-class discussion. During this stage, the teacher purposefully displays the selected student responses so as to provide an instructive basis for the whole-class discussion (exploiting TF-1 to support FA-2, FA-3, FA-4, FA-5 in carrying out TP-5). Preferably, the teacher has had the opportunity to think about possible student responses in advance (TP-1), and hence, has prepared some useful questions to discuss in relation to different types of student responses. To conclude the activity, we suggest asking some evaluative (e.g. multiple-choice) questions (exploiting TF-1, TF-2 to support FA-2). In this way, the students are provided a further opportunity to clarify or consolidate their mathematical thinking. For the teacher, on the other hand, this supplies information regarding the students' current understanding, which will be useful in planning the subsequent teaching.

TOPIC-SPECIFIC CONSIDERATIONS AND THEORIES GUIDING THE DESIGN

In planning the teaching unit, careful attention needs to be given to the handling of student responses to the mathematical task in play. In this respect, we draw on a previous study (Fahlgren & Brunström, 2018) which investigated how small but potentially significant changes in wording might influence students' explanatory responses in a dynamic mathematics software (DMS) environment.

These student responses were collected in a study involving 229 tenth grade students, from eight different classes, at an upper secondary school in Sweden. The students worked on a sequence of tasks designed for a DMS environment. In this particular task sequence, students were introduced to graphical representations of quadratic functions written in the standard form $f(x) = ax^2 + bx + c$, and encouraged to examine the visual effect on the graph when changing the value of a parameter by using a slider tool. The students had previously worked with linear functions, linear equations and on solving quadratic equations algebraically.

The task sequence included 'explanation tasks' where students were asked to explain their empirical observations made in the DMS environment. As a basis for planning the teaching unit to be used in the pilot study, we used findings relating to Task 1c (see Figure 1).

Task 1

(a) Investigate, by dragging the slider *c*, in what way the value of *c* alters the graph. Describe in your own words.

(b) The value of the constant *c* can be found in the coordinate system. How?

(c) Give a mathematical explanation why the value of *c* can be found in this way.

Figure 1. The first task including a request for an explanation (subtask c)

Briefly, the analysis of students' responses to Task 1c identified the following elements of explanation (see Table 1) which could feature in a particular response.

Code	Explanation element
A	$x = 0$ gives $y = c$
B	*c* can be found where the graph intersects the *y*-axis (i.e. repeats the answer to the previous subtask)
C	Comparing with the standard linear equation, $y = kx + m$
D	*c* behaves like/corresponds to *m*
E	*c* is the constant term
F	*c* is independent of *x*
G	*c* is independent of *a* and/or *b*
H	solves for *c*
I	Providing example
J	Referring to the DMS feedback

Table 1. The categories of explanation element in Task 1c

These findings provide useful information about what kind of student responses to expect during this particular activity and within a similar context. To simplify the findings appropriately for the pilot study, we have dropped the categories G and H because there were very few answers in these categories. We have also merged the categories C and D, E and F, and I and J into the categories C/D, E/F, and I/J respectively.

Based on these findings, and the assumption that all categories are represented among the student responses in a class, we suggest the sequencing in Table 2 when displaying different student responses as a basis for the whole-class discussion.

Order	Category	Comments
1	B	Since this is a repetition of the answer to the previous subtask, it might form a base for a whole-class discussion concerning the distinction between a description and an explanation in mathematics.
2	I/J	These responses encourage discussions about the suitability of using examples as mathematical explanations.
3	C/D	This links to a topic that the students' have previously studied, and so are already familiar with, and provides a base for discussions about similarities and differences between the linear function and the quadratic function.
4	E/F	That *c* is the constant term and independent of *x*, naturally follows from the foregoing discussions.
5	A	Finally, the valid mathematical explanation is introduced and discussed

Table 2. The suggested sequencing for displaying student responses.

To investigate the presence of the different categories in the eight different classes, a further analysis was made. In this analysis, each single student response was categorized according to an order of precedence, based on the sequencing above. So, for example, a student response that only include the explanation element B, is categorized as B, while a student response that include both explanation element B and C or D, is categorized as C/D. Table 3 provides an overview of the presence of student responses across the eight classes, organized in the sequence suggested above.

Class	B (only)	I/J	C/D	E/F	A	Total
1	3	8	12	1	3	**27**
2	5	3	13	2	1	**24**
3	2	8	5	6	4	**25**
4	1	1	18	3	3	**26**
5	0	5	7	6	6	**24**
6	4	3	12	3	2	**24**
7	2	5	17	5	0	**29**
8	2	2	13	12	2	**31**
Total	**19 (9%)**	**35 (17%)**	**97 (46%)**	**38 (18%)**	**21 (10%)**	**210**

Table 3. The presence of the categories of student responses in the eight classes.

Table 3 indicates that all categories are represented in six out of eight classes. In particular, while the first response (B only) is relatively rarely offered (by around 10% of students overall) it is present in 7 of the 8 classes; the next response (I/J) is offered (by nearly 20% of students overall) in every class; the following response (C/D) is offered (by between roughly 20% and 70% of students) in every class, indicating the prevalence of the linear analogy; the penultimate response (E/F) is offered (by nearly 20% of students overall) in every class; and while the final 'desirable' response (A) is relatively rarely offered (by 10% of students overall) it is present in 7 of the 8 classes.

Functions and graphs

The literature points out various aspects to consider in the teaching of functions and graphs. One important aspect is the distinction between a 'local' and a 'global' view of functions (Leinhardt, Zaslavsky, & Stein, 1990). The local view is characterized by a point by point attention where the focus is on specific values of a function. A global view, on the other hand, draws attention to global features of a function such as the shape of its graph or the structure of its closed form equation. In Task 1a (see Figure 1), the focus is on how *c* alters the graph, which might promote a global view. In contrast, Task 1b encourages a local view by directing students' focus on a specific point in the coordinate system.

PLANNING FOR A WHOLE-CLASS DISCUSSION

The main aims of the whole-class discussion are to (a) discuss what constitutes an appropriate mathematical explanation, (b) to clarify that $x = 0$ when a graph intersects the y-axis (local view), and (c) to clarify how changing the constant term translates the whole graph vertically (global view).

Following the sequencing outlined in Table 2, and provided that all the categories of student responses (in Table 3) are present, we suggest that the categories are displayed one by one (by one or several examples of student responses). Of course, the teacher must be flexible in the orchestration of the discussion, however, we propose the following step-by-step guidance including some questions to discuss.

(1) Category B
What is the distinction between Task 1b and Task 1c? (i.e. what is the distinction between a description and an explanation in mathematics?)

(2) Categories I and/or J
Could 'providing an example' (Category I) be regarded as an explanation?
Could referring to the DMS feedback (Category J) be regarded as an explanation?

(3) Categories C and/or D
What do m in $f(x) = kx + m$ *and c in* $f(x)=ax^2 + bx + c$ *have in common?*

This discussion might provide a natural link to the next category.

(4) Categories E and/or F
Could the explanation be strengthened? Why does this mean that the graph intersects the y-axis when $y=c$*?*

Preferably, this discussion leads into the final (and most desirable) response.

(5) Category A
These discussions should lead to a class agreement on what constitutes an approprate explanation in this particular case (Task 1c).

To conclude the activity, we suggest using some follow-up questions. First, to consolidate students' understanding that $x = 0$ gives the *y*-intercept (local view), we recommend the following question:

What are the coordinates of the point where the function $f(x) = (x - 2)^2$ *intersects the y-axis?*

As a second follow-up question, we suggest the multiple-choice question in Figure 2.

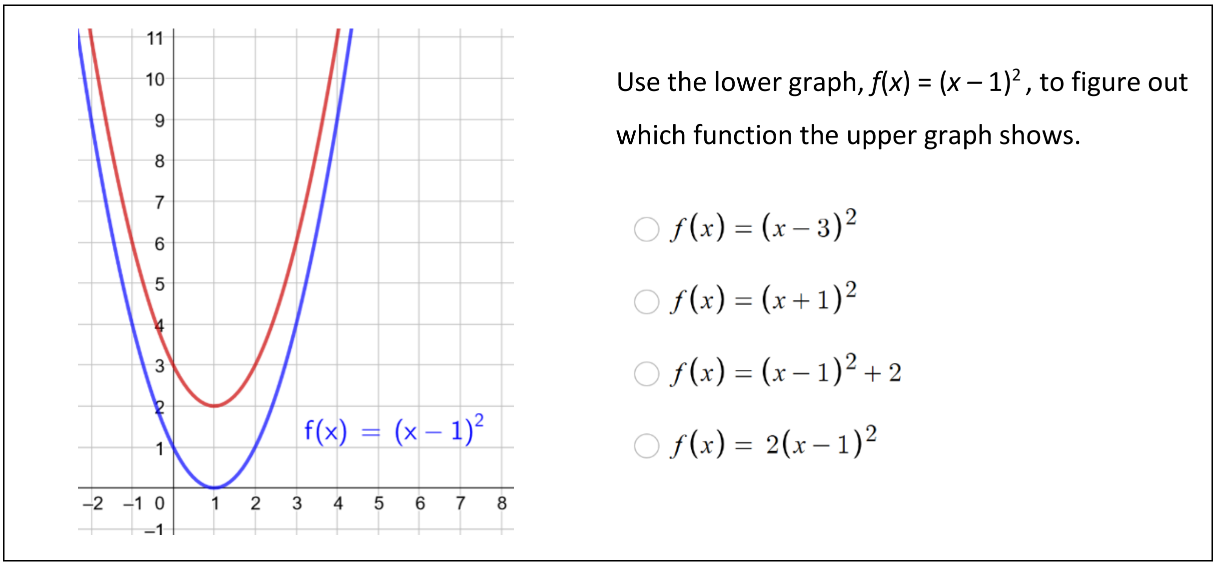

Figure 2. The second follow-up question.

The intention behind the latter question is to introduce the more general feature that changing the constant term translates the whole graph vertically (global view).

CONCLUDING REMARKS

The planned teaching unit will be trialled in four classes during the spring 2019. Preferably, the students would have access to both the DMS environment and the instructions and questions on the same screen display. However, we have not yet found any technical solutions that fulfill these pedagogical needs. Therefore, the students will use two computers; one displaying an e-worksheet and one displaying the DMS environment, in this case *GeoGebra*. After testing different available software systems, we decided to use the *Desmos Classroom Activity* as an e-worksheet where students can submit their responses.

Data in terms of field notes from classroom observations, audio recordings from the classroom and from teacher interviews, and screen recordings of the teachers' computer will be collected. In the analytical process, both generic and topic-specific theories will be used.

Reconsidering the potentials suggested in the literature on technology-enhanced strategies for developing formative classroom practices in mathematics (Clark-Wilson, 2010; Cusi et al., 2017; Irving et al., 2016), several questions appear. For example, what particular CCT features are useful? And how could these features be utilized to provide the teacher with appropriate information about their students' mathematical thinking during the lesson, and to support the whole-class discussion? These and related questions will guide the analysis. At the time of the conference, we will be able to report and discuss some results from the pilot study.

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