COMPARING DIGITAL AND CLASSICAL APPROACHES - THE CASE OF TESSELLATION IN PRIMARY SCHOOL

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The relevance of the 3D-printing technology in mathematics education has largely increased in the last years. This triggers the discussion about the benefits of digital approaches and the relation of the digital and the classical approaches. This relation will be pursued in this article using the example of tessellation in primary school mathematics. The basis of the study is a three-day workshop at a primary school in North Rhine-Westphalia (Germany) in which students worked on the topic in a classical way and with 3D-printing technology. Reflection sheets, videographed group interviews and other data material are evaluated using the method of qualitative content analysis according to Mayring (2000) with a special focus on the theory of subjective domains of experience according to Bauersfeld (1983).

Keywords: 3D-Printing Technology, CAD-Software, Qualitative Content Analysis, Nibbling Technique, Tessellation

INTRODUCTION

In recent years, the 3D-printing technology has attracted increasing attention in mathematics education practice and research (c.f. Dilling, 2019, Ng, 2017, Panorkou & Pratt, 2016, Witzke & Hoffart, 2018). It can be used to develop many new and alternative approaches to mathematical topics. The following article shows how the 3D-printing technology can be used appropriately in primary school to teach the topic of tessellation. The comparison with a classical approach to this topic illustrates the opportunities and challenges of the use of digital media in mathematics class.

3D-PRINTING IN MATHEMATICS EDUCATION

The use of 3D printing technology offers many opportunities to enrich teaching. A particular focus can be placed on the qualitative concept building. Sustainable basic ideas can be developed by working with 3D-printing-technology and 3D-printed models. This results in interesting connections to the approach of subject matter didactics. Three options for using the 3D-printing technology in the classroom can be distinguished (cf. Witzke & Hoffart, 2018):

- The technology is used to reproduce existing material.
- The teacher develops individual material for the use in mathematics lessons and the students retrace the developmental process.
- The students develop 3D-printed objects on their own in the mathematics classroom.

This article focuses on the students working individually with CAD software in class. Thus, the role of CAD software as a digital mathematics tool with its parallels to Dynamic Geometry Software is particularly relevant (cf. Dilling, 2019).

Concerning CAD applications, direct and parametric modeling methods can be differentiated. Direct modeling applications enable the user to compile three-dimensional objects of geometric basic bodies (cuboids, cylinders, etc.). Those can be moved on the work plane, directly modified by dragging parts of it (points, surfaces, edges) and finally connected by Boolean operators (union, subtract, etc.). Parametric modeling is based on two-dimensional sketches in a selected drawing plane. The elements of the sketches are fully defined by inserting dimensionings and relationships (perpendicular, parallel, concentric, etc.). The created two-dimensional sketch is extruded subsequently (developed into a volume body). The use of direct modeling applications is generally easier and more intuitive, but it is much simpler to create complex objects with parametric modeling software.

In the workshop on the topic of tessellation described in this article, the students used the parametric modeling software SketchUp. The results of Panorkou & Pratt (2016) show that this software is appropriate for working with primary school students. It enables them to draw the tessellation-unit in two dimensions (figure 1/a). Afterwards, this drawing can be extruded for 3D-printing (figure 1/b).

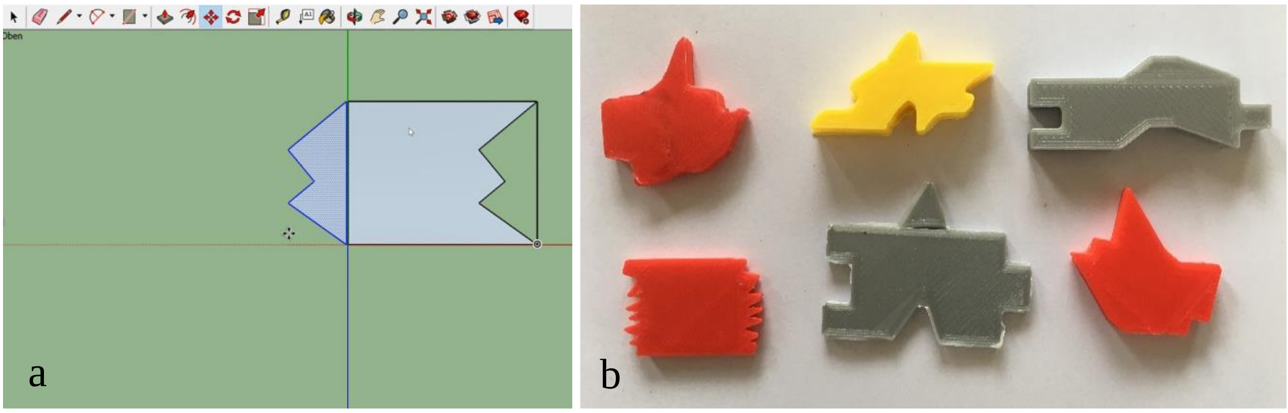

Figure 1: Creation of a tessellation-unit with SketchUp (a) and the 3D-printed objects of the students (b)

The various phases of the 3D printing process (design, construction with CAD, printing, use of objects) and the other parts of the lesson are often very different contexts for the students' development of knowledge. The theory of subjective domains of experience (SDE) is appropriate for the description of such processes of knowledge development and is used for the analysis in this study. According to Bauersfeld (1983) human experience is gained always linked to a context and can be described as separated concerning their situational link. The individual experiences are memorised in separate SDE. An SDE includes the cognitive dimension of an experience as well as motor skills, emotions and valuations. The "society of mind" forms the totality of the SDE of an individual. They are organized non-hierarchical, cumulative and compete for activation within this system. The repetition of a similar situation leads to a consolidation and thus to an effective activation of an SDE. A generalization of concepts occurs through the active attempt to recognize equalities in two different SDE, e.g. by establishment of an analogy. This forms a new SDE that enables a networking of the original SDE.

TESSELLATION IN PRIMARY SCHOOL

A tessellation is the tiling of a plane with no overlaps and no gaps using geometric shapes. Different activities are connected with this topic in the mathematics class in primary school. These include the description and analysis of tessellations, the classification of tessellations, the drawing of tessellations with stencils or on squared paper, the merging of prepared shapes and the production of individual tessellations with the nibbling technique (c.f. Franke & Reinhold, 2016). The study described in this article focuses on the creation of tessellations using the nibbling technique.

The nibbling technique can be used to develop complex elements for tessellation from basic shapes. To do this, a line is drawn between two adjacent corners within the shape. Depending on the basic shape (e.g. rectangle or triangle), the resulting piece can be attached to the opposite or adjacent side. The nibbling technique can be done by handicrafts with pen, scissors and paper as well as with digital media.

According to Eichler (2009), different competences can be promoted with the topic tessellation. On the one hand, this includes mathematical content (examining and classifying shapes, comparing and measuring lengths, angles and surfaces, naming and displaying reflections, rotations and displacements, etc.). On the other hand, different goals of personality development can be pursued (e.g. fantasy and creativity, accuracy and personal responsibility, etc.). Nevertheless, tessellations are not explicitly part of the North Rhine-Westphalian curricula (c.f. MoE NRW, 2008).

EMPIRICAL STUDY

Methodology and Conditions

The benefit of the use of 3D printing technology in the field of tessellation was to be investigated in an empirical study on the basis of a three-day workshop at a primary school in North Rhine-Westphalia. The 24 students of a fourth grade were already familiar with the topic since the previous academic year.

On the first day of the workshop, the term tessellation was repeated using various examples. A definition in the form of "A tessellation is the tiling of a plane with no overlaps and no gaps using geometric shapes." was developed together. Afterwards, the basic shapes suitable for tessellation were explored in groups. The students formed the initial shapes for the application of the nibbling technique. After the introduction, the students used pen, scissors and paper to create stencils with the nibbling technique (figure 2/a). The stencils were used to draw tessellations on a large sheet of paper (figure 2c).

The second day, the students worked in groups with the 3D-printing technology to create tessellation-units (figure 2/b). They drew those units with the CAD-software "SketchUp Make". The software was not introduced in advance, since most functions are intuitively understandable. Afterwards, the created virtual models were 3D-printed to enable the enactive work with them.

On the last day of the workshop, the students received their 3D-printed tessellation-units. These were merged to tessellate a section of a plane (figure 2/d). A written reflection brought together the different situations of the workshop and made them accessible for an empirical investigation. In addition, the students developed posters to present their work to their classmates.

The empirical study focused on the following two research questions:

- What are the characteristics of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp?
- What are the characteristics of drawing a tessellation with a handcrafted stencil and merging a tessellation with 3D-printed units?

A variety of different types of data was collected in order to investigate the research questions. These include the video recording of the workshop with three cameras, the recording of the group work on the computer with the screen and webcam recording function, the results of the students (reflection sheets, posters, tessellations, drawings) as well as the video recording of concluding group interviews. The analysis described in this paper focuses on the statements made by the students during the interviews.

The videotaped material was transcribed according to the rules of Dresing & Pehl (2015) and translated into English by the authors for a deeper analysis. The generated data was then categorized using the method of qualitative content analysis according to Mayring (2000). The summarizing content analysis is performed in four steps. The first step includes the detailed description of the data material. In the second step, the relevant parts of the text are summarized (paraphrasing). The paraphrases are generalized at a defined level of abstraction. The number of generalized statements is then reduced several times by increasing the level of abstraction and removing statements of the same meaning. The statements are compiled in a system of categories in the third step. This system is checked based on the material in the fourth step.

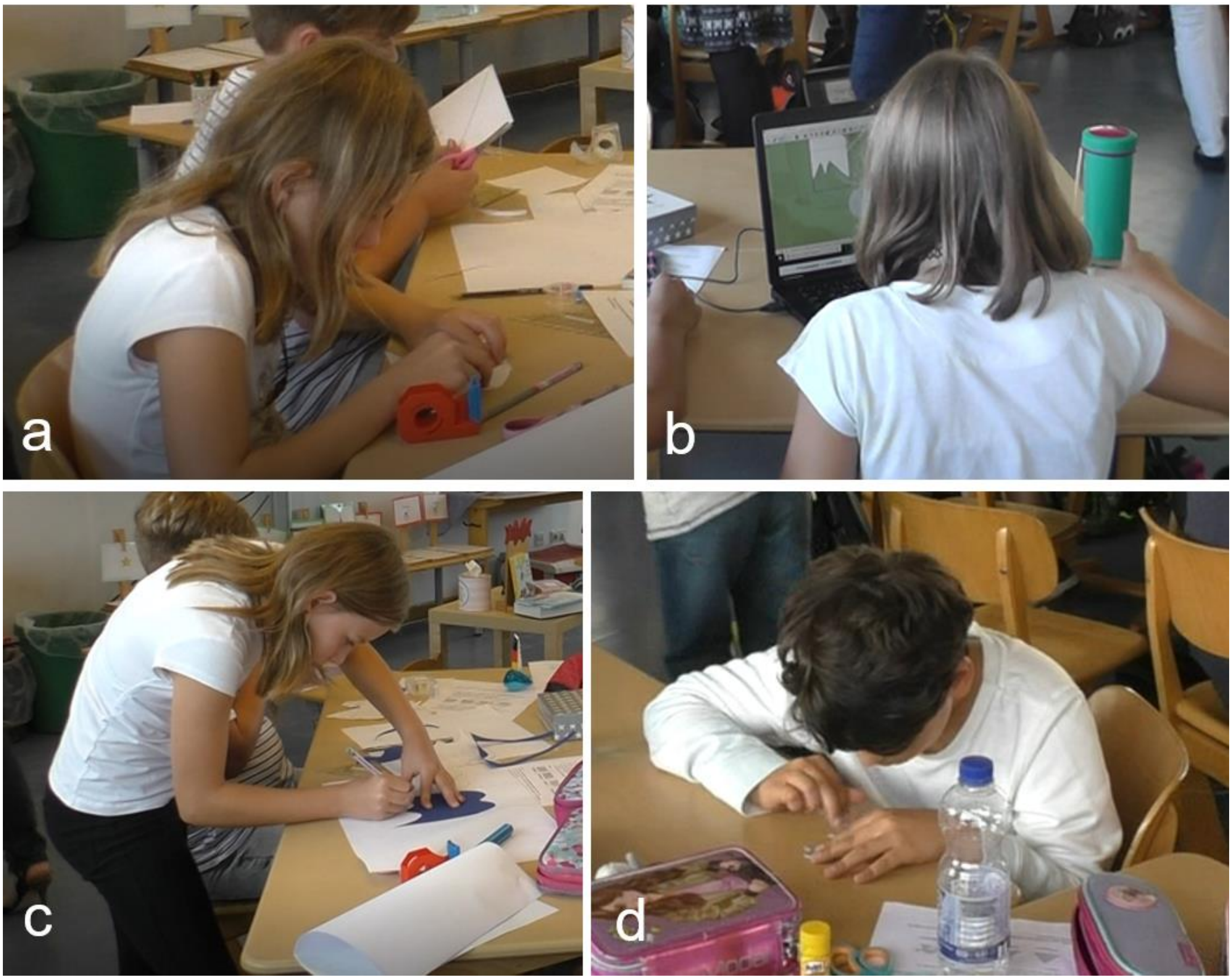

Figure 2: Students draw a tessellation stencil by hand (a), draw a tessellation-unit with SketchUp (b), draw a tessellation with a handcrafted stencil (c) and merge a tessellation with 3D-printed units (d)

Results and Interpretation

Eight categories could be formed inductively based on the data material in order to answer the first research question on the comparison of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp. The system of categories is presented in table 1.

Category (with a Computer and by hand)	*Paraphrase*
C1: Precise vs. not precise	Working with the computer is more precise and less error-prone.
C2: Fast vs. slow	Working with the computer is faster.
C3: Virtual-enactive vs enactive	Working with the computer is virtual-enactive while drawing by hand is actually enactive.
C4: Errors can be corrected vs. cannot be corrected	Errors can be corrected with the computer while errors cannot be corrected that easily by hand.
C5: Group work vs. individual work	Working with the computer takes place in groups while drawing by hand is individual.
C6: Aesthetic result	The results of both approaches are aesthetic.
C7: Motivation and concentration	Both approaches enable focussed and motivated learning.
C8: Problems and errors	There were problems and errors with both approaches.

Table 1: System of Categories for the comparison of drawing a tessellation stencil by hand and drawing a tessellation-unit with SketchUp

The statements of the students on the differences and similarities of the two approaches were very meaningful. Many of the statements concern the precision of both approaches (C1). The computer is described as more precisely in terms of drawing shapes and lines as well as in terms of joining shapes together. However, inaccuracies often occur when using pencil, scissors and paper.

Student 4: You could draw a bit more precisely.
Interviewer: Where was it possible to draw more precisely?
Student 4: You could make the rectangle more exact.
Interviewer: On the computer?
Student 2: Mhm. ((nods))

(Interview 1)

Student 4: Yes, because I think if you stick it together properly with your hand, it's not as neat as on a computer.

(Interview 1)

Other statements of the students concern the duration and extent of both approaches (C2). Working with a computer is much faster and less time-consuming because drawing is easier and the shapes do not have to be cut out. The 3D printer handles the production of the objects.

Student 10: Because you can draw better with it than with your hand.
Student 9: And then you can do a lot more things faster.

(Interview 3)

Student 14: And with the computer I thought it was cooler, because you didn't have to put the scissors in the corners and cut it out like that, but you could just press a button and then you could get it right out.

(Interview 5)

The relationship between the two forms of actions was also in the focus of the students' statements (C3). Working with pen, scissors and paper is perceived as a real action in which sensations play a role. In contrast, working with the computer is different because only the buttons are pressed and the computer performs the actions itself. These statements of the students can be described with the dimensions virtual-enactive and actually enactive described by Hartmann, Näf & Reichert (2007).

Student 13: There you could also have the sensation and see how to cut. There is no sensation on the computer. You just type things or just draw with your finger. You have to press this top row all the time to do something new or something like that. That was, I would say it sucks because with scissors, pencil and paper, there you could simply get the things out of the pencil case and draw along and cut and done.

(Interview 5)

The students consider the possibility of correcting errors with the undo function to be a significant advantage when working with the computer (C4). This is not that simple when doing things with a pen and scissors.

Student 7: Well, that will stay forever - you can't just erase it like on a computer.
Student 6: But you can erase it. ((looks at student 7))
Student 7: But not when you've cut it in.
Student 6: Yes.

(Interview 2)

One last difference between the two approaches is observed in the social form (C5). On the one hand, the work with pen, scissors and paper is done individually. On the other hand, computer work takes place in groups.

Student 2: And with the computer everybody has actually done something of it (...), so what she also said. We have/ So it isn't just created by me or by (...) her ((points to Student 4)) and that's where we did it together so I made the nibble (…) I don't know who made the rectangle anymore. ((looks at other students))
Student 1: That's what I did. ((answers))

(Interview 1)

In addition to the differences that the students notice in the two approaches, there are also some similarities. These include, among other things, the aesthetics of the results (C6). Individuality is particularly appreciated in the handcrafted products, while stability and accuracy are emphasized in the computer-assisted production.

Student 1: Somehow more beautiful because you know that you did it yourself. With the computer you can say "Yes, I did it myself" but only on the computer. Somebody else can do that as well, it's not just from you.

(Interview 1)

Student 14: But in the end it looks really nice.
Student 13: Yes, and it's more stable.
Student 14: As paper. ((looks at student 13))

(Interview 5)

According to their own statements, the students were motivated in both approaches and worked with concentration (C7). However, errors and problems occurred with both approaches (C8).

With reference to the theory of subjective domains of experience, two contexts can be identified. These are determined by the approaches with the computer and with pen, scissors and paper. The students' statements first show some differences in the approaches that emphasize different properties of the mathematical content tessellation. However, it can also be stated that the students were able to recognize a structural equality in the developed SDE and formed a comparative SDE.

Three categories were built in order to answer the second research question, which focuses on the comparison of the drawing of a tessellation with the handcrafted stencil and the merging of 3D-printed tessellation-units. The system of categories is presented in table 2.

Category	*Paraphrase*
C1: No difference between both approaches	The students do not see any difference between the two approaches.
C2: Different sizes	The stencil and the unit have different sizes.
C3: Merging is faster	Merging the units is faster than drawing with the stencil.

Table 2: System of Categories for the comparison of the drawing of a tessellation with the handcrafted stencil and the merging of 3D-printed tessellation-units

Almost all students do not see any difference between the two approaches (C1). The differences mentioned do not affect the structure of both approaches. They concern the different sizes of the stencil and the unit (C2) as well as the duration of both approaches (C3). The stencils are bigger and thinner, whereas the 3D-printed units are smaller and thicker. Merging units is generally faster than drawing with a stencil. Thus, the two approaches of drawing a tessellation by hand and merging 3D-printed units seem to result in the formation of a comparative subjective domain of experience.

Interviewer: Do you think it's something else or don't you think it matters?
Student 6: Um, actually I think it's the same and you? ((looks at student 5 and student 7))
Student 7: Yes.
Interviewer: Does it matter whether you draw or merge?
Student 5: No.

(Interview 2)

Student 13: So it takes a little longer to draw, but if you have them and you're sticking them together, it takes half a minute? ((shrugs his shoulders)) That's very fast.

(Interview 5)

Student 11: Different is of course that it/that the forms are thicker ((S2 nods approvingly)) but smaller, with the paper it is bigger but thinner. So there is no advantage.

(Interview 4)

SUMMARY AND OUTLOOK

The results of the empirical study show the opportunities of the 3D-printing-technology to support mathematics learning. In addition to the different chances of the approach to tessellation with digital media (i.e. precision, duration), various challenges arise (i.e. virtual enactivity, technical problems). Especially the parallel use of the digital and the classical approaches has led to an effect of synergy by the development of comparative subjective domains of experience. These enabled the students to get to know different facets of tessellation and combine the different advantages. The reflection on the characteristics of the two approaches led to the adoption of a meta-level and the linking of contents and methods.

It turns out that the use of 3D printing technology can be successful in primary school mathematics teaching. Many other contents of primary school mathematics also seem to be suitable for the use of 3D-printing technology. Further studies have to be conducted to investigate the use of this technology in greater depth.

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