The influence of blended learning on students' learning behavior with respect to the heterogeneity of students

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Preface

I have written this dissertation during my employment as a research assistant at the Chair of Microeconomics (Prof. Dr. Erwin Amann) at the University of Duisburg-Essen. Throughout these challenging years, I have developed my academic skills and met many people who I greatly esteem and to whom I am very grateful.

Special thanks go to my supervisor, Prof. Dr. Erwin Amann, who supported and guided me during these years. At any time he devoted time to fruitful discussions and suggestions and was open to new ideas. His support has always been helpful and enriching. In addition, I am very thankful to Prof. Dr. Thomas Retzmann who did not hesitate to be my second supervisor.

I am deeply grateful to my co-author, Marcel Braukhoff, who helped to create a productive and illuminating working atmosphere and who always empowered me. I also thank Ana Khazalashvili and Jan Schilling for proof-reading and being great colleagues. I wish to thank Ilona Braun and Eva Zielke who were there for me at any time and supported me with help and advice.

Most importantly, I am eternally grateful to my parents, family and friends who always believed in me and supported me through all ups and downs. Their encouragement made me strong and persistent.

Contents

Introduction

The current learning environment is characterized by an increasing heterogeneity of students and by a trend towards online learning.

In addition to a growing number of first year students, higher education has to cope with various dimensions of student heterogeneity. Traditional sources of heterogeneity like skills, professional experience, social and cultural background are extended by new dimensions (Hanft 2015). These new dimensions of heterogeneity are represented by a broader age span and by differences in organizing self-study among students (Schulmeister et al. 2012). For this purpose, higher education has to rethink homogeneous university structures and traditional teaching methods, which do not fit the current diversity. Until now, universities try to meet this increasing heterogeneity with receiving inspections and preparatory courses, concentrating mainly on the start of higher education studies (Hanft 2015). These measures try to transform heterogeneity into homogeneity instead of accepting the diversity of students. Hence, in order to accept heterogeneous students, higher education might be forced to adapt course structures (Biggs and Tang 2011; Wildt 2001). In this connection, the restructuring of higher education within the Bologna process represents a possibility to pay attention to heterogeneous students (EACEA 2012). The choice between bachelor's and master's degrees considers different cognitive abilities and levels of commitment.

Besides the heterogeneity of students, today's learning environment shows a new trend towards online learning. Since traditional teaching methods are taken up by this trend, blended learning is in the focus, being a mixture of face-to-face sessions and e-learning tools. Many studies show that e-learning tools have increased significantly in the current learning experience of students (Allen and Seaman 2006). Furthermore, students prefer to learn online and reduce their commitment in face-to-face education (Garrison and Kanuka 2004; Sebastianelli and Tamimi 2011). Making use of online learning tools, teachers who are confronted with a huge number of students benefit from economies of scale and scope (Twigg 2013; Morris 2008). In addition, feedback which is implemented in computer-supported learning has a positive influence on the students' motivation and problem-solving (Zumbach and Reimann 2003). Assuming courses with a considerable number of students, e-learning tools reduce the workload associated with individual and immediate feedback. Furthermore, feedback plays a crucial role in guiding the self-study and evaluating the learning progress. In this connection, Dunning et al. (2004) show that most students have a flawed self-assessment and especially overestimate their abilities. Furthermore, the decision to begin one's studies is driven by the self-perception of skills (Chevalier et al. 2009). Especially a high self-perception promotes the investment in higher education and the danger of dropping out. Thus, flawed self-perception justifies the need for feedback.

This dissertation examines the influence of blended learning on student's learning behavior with respect to student heterogeneity. The analysis is based on theoretical models. Since blended learning offers a variety of learning opportunities which are available around-the-clock, it is able to influence learning in a different way than traditional teaching methods. Especially online feedback helps students to self-regulate learning and to assess their abilities. Taking the growing heterogeneity of students into consideration, blended learning may be a measure to rethink homogeneous university structures.

In the following, I will describe the assumed blended learning setting. Then, I will give a brief overview of the theoretical concepts which are used in this dissertation. Afterwards, I will provide an outline of the studies which apply and extend the presented theoretical background.

Blended learning setting

The blended learning setting of all chapters is based on the courses of the Chair of Microeconomics at the University of Duisburg-Essen. They use an online tool called JACK, described in detail in Chapter 3. The e-learning tool is implemented in addition to traditional face-to-face sessions. While, weekly lectures and tutorial classes aim to teach basic knowledge and demonstrate examples, the offer of e-learning exercises represents a possibility of around-the-clock self-training and assessment of course contents. The combination of traditional face-to-face sessions and e-learning opportunities is called blended learning.

The described courses are characterized by a considerable number of students, placing new demands on teachers. In this context, e-learning opportunities are able to reduce the workload of teachers. The e-learning tool offers online exercises which are available around-the-clock and an online mid-term test with the possibility to improve the final exam mark.

Online exercises guide students' self-study, since they repeat every topic of the course. Students are able to solve online exercises anywhere and around-the-clock and receive an immediate feedback after handing in their solution. Variables are randomized in order to offer a variety of possibilities to retain course contents. In addition to online exercises, students have the possibility to take an online mid-term test and gain bonus points for the final exam. Thereby, a failed mid-term test is not credited. In contrast to this, a passed mid-term test only improves the final exam mark if the student passes the final exam. The key advantage of e-learning tools is the fairness and the immediate electronic correction. Since the correction of online exercises and mid-term tests is carried out automatically, the workload remains small and objectivity is guaranteed.

Theoretical background

This thesis is based on the concept of learning by doing (Arrow 1962; Göcke 2002) and on Spence's (1973) signaling model. Thereby, Chapter 1 and 2 apply learning by doing to university courses and Chapter 3 uses Spence's signaling model in order to analyze the higher education system.

Arrow (1962) was the first author who introduced learning by doing to the economic literature within the field of production. He describes learning as experience which is gained during the process of problem-solving and activity. Thus, performance improves over time based on intertemporal changes in production functions. As a result, knowledge is growing in time as a product of experience. Göcke (2002) takes up the idea of learning by doing and introduces the optimal allocation of time between working and leisure. He models learning as a by-product of working which is measured by aggregated output in time. Thus, human capital accumulation depends on working time which leads to learning and consumption of goods. Göcke assumes that utility consists of leisure and of the consumption of goods. Nevertheless, working time is limited due to the optimal time allocation to working and leisure. Therefore, growth of experience accumulation is limited because of consumption saturation effects in favor of the expansion of leisure. In contrast to this, the production side shows constant returns to scale on experience itself.

Chapter 1 and 2 apply this concept to university courses with the trade-off between learning and leisure. In this context, utility is based on the final exam mark and the disutility of learning. As a consequence, accumulation of skills is limited by passing the exam with the best mark and by the desire to consume leisure.

The second theoretical strain is represented by Spence's signaling model of 1973. In this paper he firstly introduced the effects of signals on the job market which is characterized by asymmetrical information. He assumes that the employer does not know the productivity of the individual before hiring him. With the aim to counteract this uncertainty, individuals can invest in education. Obtaining a degree serves as an observable signal for the employer, because it involves costs in terms of time and money. Spence distinguishes two groups of highly and less productive individuals who know their productivity. The employer believes that a critical level of education exists, distinguishing between high- and low-ability individuals. Therefore, he pays a wage which equals the individual productivity, since it is the contribution of the graduate to the firm. According to this wage schedule, individuals choose their investments in education with the aim to maximize their utility which is given by the difference between wage and signaling-costs.

Chapter 3 applies this model to the current higher education system comprising bachelor's and master's degrees. Since two possible investment in education exist, the model is extended by a third productivity level.

Overview of chapters

Chapter 1 and Chapter 2 follow Göcke (2002) by applying his model of learning by doing to the framework of university courses. These chapters analyze how learning behavior is influenced by online learning opportunities. Chapter 3 introduces a broader perspective, shedding light on the impact of blended learning on student heterogeneity which characterizes today's higher education system. In order to analyze the current higher education system, Spence's signaling model (1973) is extended by a third level of productivity. While students' learning behavior plays a central role in the first and second Chapter, student heterogeneity is the main focus of Chapter 3.

Chapter 1 was created in collaboration with Erwin Amann and Marcel Braukhoff. Chapter 2 is co-authored with Marcel Braukhoff and Chapter 3 is my own work.

The first chapter develops a simple model of learning by doing in the context of university courses which is based on Göcke (2002). Thereby, exam marks are a result of skills which are accumulated during the learning period. Students have the opportunity to repeat course contents whenever and how often they wish to, since blended learning creates around-the-clock learning opportunities. In this connection, e-learning exercises help to improve the retention of newly acquired skills. In addition, mid-term tests provide incentives for early learning, since they bear the possibility to improve the final exam mark. While solving online exercises, students accumulate skills. This accumulation of skills is limited by passing the exam with the best mark. Furthermore, they face a trade-off between learning and leisure, since learning leads to a reduction of leisure time. This trade-off curtails spending the total time for learning. Students have perfect knowledge about their skill level and differ by preferences for learning. While students with preferences for long-term learning prefer to learn constantly during the learning period, students with preferences for short-term learning tend to do last-minute learning. Utility is maximized for both types of students and the results for the case with and without mid-term test are compared. In addition, continuity of learning activity during the learning period is analyzed. Finally, Chapter 1 addresses the question how incentives and blended learning influence and improve learning behavior of the student.

Chapter 2 extends this model by assuming that students have an imperfect knowledge about their skill level. For this purpose, this chapter introduces the influence of feedback. Online exercises and a mid-term test serve as sources of feedback since they provide information about the individual skill level. Depending on the utilization of feedback implemented in blended learning, students may check their learning progress and individual skill level. As a first step, the influence of online exercises on the perceived learning progress is analyzed. Secondly, we look at the impact of the mid-term test as source of feedback on the perceived skill level. While students who do not use blended learning base their learning time on their perceived skill level, students using blended learning are able to compare their perceived skill level to the received feedback. This evaluation of accumulated skills enables students to update their further learning activity. For this purpose, dynamic utility maximization is analyzed in the case of average and low-ability students. In addition, this analysis sheds light on the influence of feedback on utility. Consequently, this chapter addresses the question how the feedback implemented in blended learning influences and improves learning behavior and utility.

Chapter 3 introduces a new perspective by looking at blended learning and the higher education system which is characterized by a growing heterogeneity of students. For this purpose I give an overview over dimensions of student heterogeneity. Thereafter, the higher education system which has been restructured within the framework of the Bologna process is compared to previous systems with only one possible degree. The choice between different degrees like the bachelor's and master's degree is a possibility to pay attention to student heterogeneity. In contrast to prior degrees, this differentiated system gives students the possibility to choose different levels of higher education degrees according to individual abilities and future plans. Since the completed degree serves as a signal for the employer and thereby reduces his uncertainty concerning the productivity of the graduate, I apply Spence's (1973) signaling model in order to compare the signaling power of different degrees. Nevertheless, Spence's signaling model assumes perfect knowledge about individual skills, although dropout rates in higher education indicate the opposite. Flawed self-assessment might be one reason for this dropout rate. With the aim to implement new teaching routines in higher education, Chapter 3 proposes blended learning as a response to the wrong self-assessment and increasing heterogeneity of students. As support to traditional face-to-face sessions, the online learning tool JACK is introduced and described in detail. With reference to the described dimensions of student heterogeneity and with the aim to correct flawed self-assessment, the gains of blended learning are depicted in detail. Thus, this chapter analyzes dimensions of student heterogeneity and shows how blended learning takes these dimensions into consideration and helps to correct flawed self-assessment.

Chapter 1

Impact of incentives and blended learning on students' learning behavior

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Abstract

This paper focuses on a blended learning approach implemented in university courses at the University of Duisburg-Essen with the aim to improve the performance of students in the exam and the course of study. In addition to the created online learning opportunities, an incentive-based approach aims to promote student engagement in courses and gives the opportunity to self-assess own skills. Due to this incentive-based blended learning concept, the learning process has been proven to become more effective and successful. Students tend to learn continuously and work off their backlog. Additionally, students' motivation to deal with the teaching material rises. Consequently, the lack of preparation which often results in poor performance decreases.

Exam outcomes are the result of training skills, which rise while solving exercises and spending learning time. Additional learning opportunities, automated feedback and hints help to reduce learning costs, to improve the organization of individual learning time and to increase the students' success. This paper uses a dynamic utility maximization approach considering the choice between learning time and leisure. Depending on individual skills, on preferences for leisure time and for discounting, the individual learning performance varies. Changing the opportunities and incentives has a specific impact on overall learning time as well as on partitioning learning time along the semester.

Hence, this paper introduces a specific model of accumulation of skills and then emphasizes the importance of the interplay of incentives to learn, e-learning opportunities and face-to-face sessions in regard to more efficient learning behavior.

1.1 Introduction

Since traditional teaching methods are taken up by the new trend towards online learning, blended learning becomes more important. Blended learning combines face-to-face sessions and e-learning opportunities and thus meets these growing expectations and needs for better learning opportunities and outcomes (Garrison and Kanuka 2004). This trend is confirmed by various surveys which show that online learning tools in students' learning experiences have increased significantly (Allen and Seaman 2006). In Germany, for example, nearly every University offers e-learning platforms (Henning 2015).

The significant growth rate of online learning strategies is driven by supply and demand side reasons. On the one hand, university teachers benefit from economies of scale and scope in teaching a large number of students (Twigg 2013; Morris 2008). On the other hand, students show a new attitude towards online learning (Sebastianelli and Tamimi 2011) and a reduced engagement with face-to-face education (Exetere et al. 2010).

Furthermore, students are characterized by an increasing heterogeneity these days (Hanft 2015). Besides social differences, students show diverging cognitive abilities and commitment levels. The homogeneous university structures and teaching methods are confronted with these heterogeneous student characteristics. Nevertheless, most initiated actions only affect the access to higher education studies like receiving inspections and take no account of teaching routines. In the future higher education has to cope with new demands concerning student heterogeneity.

In order to develop a simple model, this paper makes use of the concept of learning by doing to show the influence of blended learning on learning behavior. Until now, economic literature has only focused on learning by doing within the field of production. In this context, learning by doing was first introduced in 1962 by Arrow, who describes learning as a product of experiences which are gained during the process of problem-solving and activity. Thus, production functions underlie intertemporal changes in skills. As a result, skills are growing in time due to improvement in performance.

Göcke (2002) extends this model by analyzing the optimal allocation of time between work and leisure in the context of learning by doing. Furthermore, he models an experience curve which characterizes learning as a by-product of working. Thus, experience as human capital is accumulated via learning by doing and depends on working time and the consumption of goods. The author concludes that the rise in productivity is utilized for an expanding leisure time. Therefore, growth of experience accumulation is limited by utility side reasons, whereas the production side shows constant returns to scale on experience itself.

This paper introduces a simple model of learning by doing in the context of university courses. The exam outcome depends on the skill accumulation during the learning period. Thereby, blended learning creates additional learning opportunities which are available 24 hours per day during 7 days per week. E-learning exercises take student heterogeneity into account, since they are characterized by a high flexibility of time and space and since they offer many possibilities to improve the retention of newly-acquired skills. In addition, mid-term examinations provide incentives to continuous learning and consequently boost the commitment in blended learning with the help of rewards. While solving blended learning exercises, students acquire skills and human capital is accumulated, hereafter called skills in this paper. However, students have to decide upon the optimal time allocation to learning and leisure which differs in utility formulation. This model shows similarities to Göcke (2002), yet learning by doing is applied to university courses instead of production. Furthermore, the choice between working/learning and leisure is not the only concern of this paper. This trade-off just curtails the overall spending of time for learning.

Thus, we address the question how incentive and blended learning influence and improve learning behavior.

1.2 The model

This paper introduces the concept of learning by doing to university courses supported by incentive-oriented blended learning. Besides face-to-face sessions students have the possibility to solve online exercises during the semester in order to improve their skills with the overall aim to pass the exam. These online exercises take student heterogeneity into account, since they offer different levels of difficulty, individual hints and are characterized by a high flexibility of time and space and last but not least offer possibilities to repeat course contents.

1.2.1 The growth rate of skills

In this model the accumulation of skills via learning by doing is the result of solving exercises. In this first approach we assume that a student uses the blended learning platform to improve his skills. The relative learning time $q = q(t)$ describes the fraction of time a student spends using learning opportunities at time t during the semester. Overall time is split into learning time $q(t) \in [0, 1]$ and leisure $1 - q(t)$. Let $E(\xi, q)$ be the number of exercises solved by a student per time unit.

The number of exercises students can solve per time unit depends on their expertise and the time spent for learning. It is assumed that both increasing time and skills has a positive impact on E:

$$\frac{\partial E}{\partial \xi}(\xi, q) \geq 0, \quad \frac{\partial E}{\partial q}(\xi, q) > 0.$$

In the sequel, we work with a simplified model and suppose that E is bilinear in ξ and q, i.e. we have

$$E(\xi, q) = eq\xi \quad \text{with } e = const.$$

The learning parameter e differs between students but also can be positively affected by improving learning opportunities. Moreover, we assume that skills ξ are differentiable in time t and induced by the exercises solved fulfilling the ordinary differential equation

$$\frac{d}{dt}\xi = A(E, q, \xi),$$

for some differentiable function A depending on the learning activity of the student and his current skills. Assuming that A is linear in E and does not directly depend on ξ, $A(E, q, \xi) = a(q) \cdot E(\xi, q)$, we can rewrite

$$\frac{d}{dt}\xi = a(q) \cdot eq \cdot \xi \tag{1.1}$$

and interpret $a(q) \cdot eq$ as the growth rate of the skills. Keeping this in mind, a models the productivity. We suppose

$$\frac{\partial a}{\partial q}(q) \gtreqless 0 \text{ whenever } q \gtreqless q_0$$

for some $q_0 \in (0, 1)$. However, if a critical relative learning time q_0 is exceeded, the productivity decreases due to the absence of pauses. In the following, we apply a simple approach for the learning productivity by choosing

$$a(q) = q(1 - q).$$

Now, we can directly solve Equation (1.1) by

$$\xi(t) = \xi_0 \exp\left(e \int_0^t q(s)^2(1 - q(s))ds\right).$$

As we see, skills start with an individual initial skill level ξ_0 and are accumulated during the semester as a result of solving online exercises E.

1.2.2 The exam and the utility function

Utility is a result of accumulated skills and leisure as the residual of learning time. We assume quasi linear utility of a student writing the exam at time T

$$u := Xe^{-\delta T} - \gamma \int_0^T q(t)e^{-\delta t}dt$$

for some $\gamma, \delta \geq 0$ and $0 < t_0 < T$, where X is the result of the exam. We use the convention that $X = 1$ represents the best mark and $X = X_0 = \frac{1}{2}$ the minimal mark for a student to pass the exam. Therefore $X \in \{0\} \cup [\frac{1}{2}, 1]$, where $X = 0$ means failed.

Utility decreases with time spent on learning q, since a high learning activity results in less leisure time. Furthermore, the term $e^{-\delta t}$ models individual time preferences, where $\delta \geq 0$ specifies the individual discount rate. $\delta = 0$ implies no discounting of the future. Large values of delta implies a decrease of the valuation of the exam at the beginning of the semester. Students with high δ rather prefer intensive learning at the end of the semester instead of regular learning during the semester.

The result of the exam depends on the skills of the student at time T and thus, we interpret X as a function of ξ. However, this dependency is not perfectly given, since it is not possible for the student to exactly predict the result of the exam before actually writing it. Therefore, we introduce an additional parameter ϵ for the uncertainty modeling good luck or bad luck, concentration and further unpredictable influences.

Definition 1. The function

$$X(\xi, \epsilon) = \begin{cases} 0 & \text{if } \xi + \epsilon < \frac{1}{2}, \\ 1 & \text{if } \xi + \epsilon > 1, \\ \xi + \epsilon & \text{else} \end{cases} \tag{1.2}$$

describes the result function of the exam.

We can thus rewrite the utility function as

$$u(q,\epsilon) = X(\xi(T),\epsilon)e^{-\delta T} - \gamma \int_0^T q(t)e^{-\delta t}dt.$$

1.2.3 Utility maximization without mid-term examination

Since the only choice in this model is on the time spent on learning and leisure, utility is maximized by dynamic optimization in $q(t)$. As a baseline, we first determine the optimal choice of learning time q without taking a mid-term examination into account. Section 1.2.4 then examines dynamic optimization with mid-term examination in order to identify the specific incentives.

The resulting function of the exam $X(\xi,\epsilon)$ includes four cases, leading to different utility-maximizing time allocations q. We start with a student reaching in equilibrium a positive but not the best result with certainty. The second case describes students who fail the exam with certainty. The third case deals with students having a positive probability to fail and last, students who pass the exam with positive probability reaching the best score. We have chosen this order with the aim to start with the average and most common student.

Optimal time allocation for an average student

The first case deals with an average student. This case is given by $\frac{1}{2} < \xi(T) + \epsilon < 1$. Given that $X(\xi,\epsilon) = \xi + \epsilon$, we can derive the utility function as

$$u(q,\epsilon) = (\xi(T)+\epsilon)e^{-\delta T} - \gamma \int_0^T q(t)e^{-\delta t}dt. \tag{1.3}$$

The disutility of learning at time t given by $\gamma q(t)e^{-\delta t}$ decreases for increasing δ and t.

In order to maximize the utility in (1.3), we derive the first order condition

$$
\begin{aligned}
\frac{d}{ds}u(q+sv)\big|_{s=0} &= e^{-\delta T}\xi_0 e\int_0^T (2qv-3q^2v)dt\cdot\exp\left(e\int_0^T q^2(1-q)dt\right)-\gamma\int_0^T ve^{\delta t}dt\\
&= \int_0^T v\left(\xi_0 eq(2-3q)\exp\left(e\int_0^T q^2(1-q)dt'-\delta T\right)-\gamma e^{-\delta t}\right)dt\\
&\stackrel{!}{=} 0
\end{aligned}
$$

for all $v = v(t)$. Inserting the delta-function $v(t) = \delta_\tau(t)$ defined by

$$\int_0^T \delta_\tau f(t)dt = f(\tau),$$

it follows that

$$\xi_0 eq(t)(2-3q(t))\exp\left(e\int_0^T q^2(1-q)dt'-\delta T\right) = \gamma e^{-\delta t}.$$

Note that the factor $\exp e\int_0^T q^2(1-q)dt'$ is independent from time t. Defining

$$C := 3\frac{\gamma}{\xi_0 e}\exp\left(-e\int_0^T q^2(1-q)dt+\delta T\right) \tag{1.4}$$

entails

$$q(t)^2-\frac{2}{3}q(t)+\frac{1}{9}Ce^{-\delta t} = 0.$$

This equation only has a real-valued solution if $Ce^{-\delta t}\leq 1$. For $C\leq 1$, the optimal q is given by

$$q(t) = \frac{1}{3}\left(1+\sqrt{1-Ce^{-\delta t}}\right), \tag{1.5}$$

since the second order condition for a maximum,

$$0 \geq \frac{d^2}{ds^2} u(q+sv)\big|_{s=0} = \xi_0 \int_0^T 2v^2(1-3q)dt \exp\left(\int_0^T q^2(1-q)dt - \delta T\right) + \xi_0 \left(\int_0^T v(2q-3q^2)dt\right)^2 \exp\left(\int_0^T q^2(1-q)dt - \delta T\right),$$

ensures $q \geq \frac{1}{3}$: The second term is always non-negative such that the first term has to be ≤ 0 for a utility maximum leading to $q \geq \frac{1}{3}$. The constant C can be computed by solving Equation (1.4) after inserting Equation (1.5). For this, we need to evaluate the following integral

$$\int_0^t q^2(1-q)ds = \frac{2}{27\delta}\left[\frac{1}{3}(\sqrt{1-Ce^{-\delta t}}^3 - \sqrt{1-C}^3) - 2(\sqrt{1-Ce^{-\delta t}} - \sqrt{1-C}) - 2\log(\frac{1-\sqrt{1-Ce^{-\delta t}}}{1-\sqrt{1-C}})\right]$$

for $\delta > 0$ and

$$\int_0^t q^2(1-q)ds = \frac{t}{27}\left((2+C)\left(1+\sqrt{1-C}\right) - C\right)$$

for $\delta = 0$. Finally, Equation

$$C := 3\frac{\gamma}{\xi_0 e} \exp\left(-e\int_0^T q^2(1-q)dt + \delta T\right)$$

may be solved for given values of γ, ξ_0, e, T using numerics. The corresponding results are presented in the following figures.

In order to illustrate the derived optimal learning time q during the semester comprising four months, Figure 1.1 is introduced. Figure 1.1 shows the optimal learning time during the semester with T as date of the exam and three graphs characterizing different types of students. The first type is characterized by no discounting ($\delta = 0$). This type learns uniformly during the semester. The second and third type have time

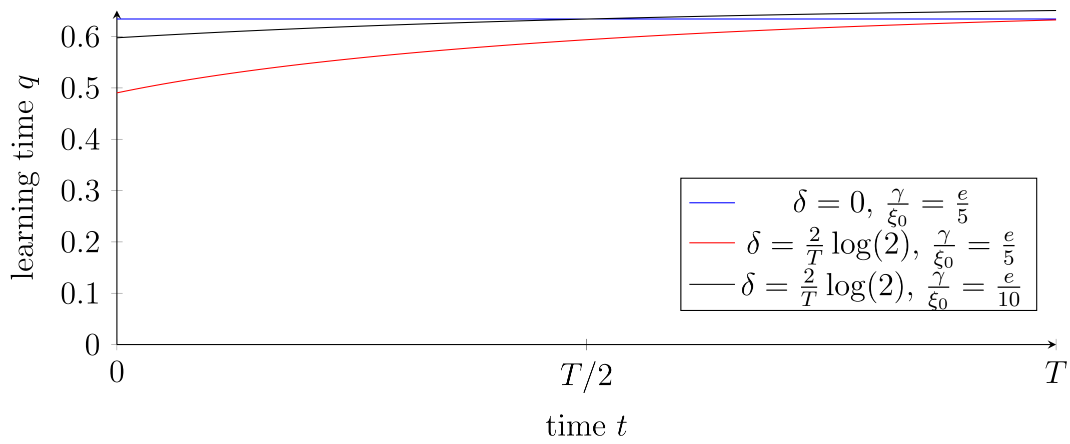

Figure 1.1: Graph of the optimal q for $e \cdot T = 8$.

preferences $\delta > 0$. Those types of students optimally show an increasing learning activity.

The student with $\gamma/\xi_0 = e/5$ and $\delta = 0$ is always more motivated than the student with preferences for short-term learning with the same γ/ξ_0. The third student is chosen such that his disutility of learning at half time is equal to the disutility at half time of the student preferring long-term learning. According to his time preferences, his average learning time is low at the beginning of the semester and continues to rise up to a higher level at the end of the semester. Nevertheless, it is always on a higher level than the average learning time of the other short-term learner, since he has a lower disutility for learning.

Finally, the third type prefers to learn short-term and his disutility of learning increases in the case of higher values of δ. According to his preferences for last-minute learning his learning time is very low at the beginning of the semester and continues to rise up to a high level at the end of the semester.

Parameter analysis

As we have seen above, the optimal time allocation $q(t)$ for a student can be computed using numerics. For this, we have to choose adequate parameters ξ_0, e, γ, δ and T. In

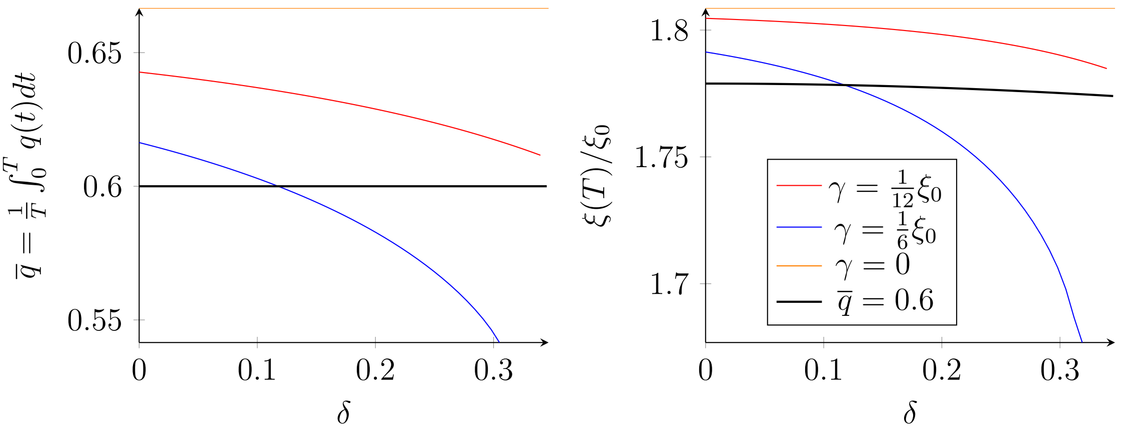

Figure 1.2: Graph of $\overline{q}$ and $\frac{\xi(T)}{\xi_0}$ as a function of δ for $T = 4$ and $e = 1$ for a utility optimising q for an average student.

this subsection, we denote $q(t) = q_1(\xi_0, e, \gamma, \delta, T, t)$ as the solution of (1.5) w.r.t. these parameters. However, there are some combinations of the parameters which entail the same optimal time allocation $q(t)$.

Lemma 2. *We have*

$$q_1(\xi_0, e, \gamma, \delta, T, t) = q_1\left(\frac{\xi_0}{\gamma}, e, 1, \delta, T, t\right) = q_1\left(\xi_0, 1, \frac{\gamma}{e}, \frac{\delta}{e}, e \cdot T, e \cdot t\right).$$

In particular, the optimal time allocation $q(t)$ is a function of three free parameters and time t. We therefore may write

$$q\left(t\right) = q_{opt}\left(\frac{\xi_0 e}{\gamma}, \frac{\delta}{e}, eT, et\right).$$

Proof. The first equation is a direct consequence of the fact that only the fraction of ξ_0 and γ contributes to Equation (1.4). For the second property, we readjust the time by substituting $s = et$. Thus, Equation (1.4) transforms to

$$C = 3\frac{\gamma}{\xi_0 e}\exp\left(-\int_0^{eT} q^2(1-q)ds - \frac{\delta}{e}eT\right),$$

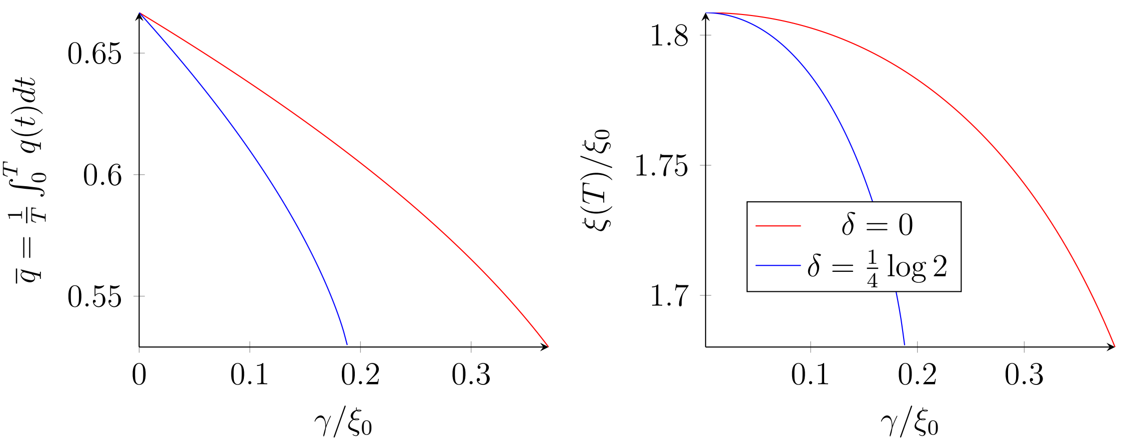

Figure 1.3: Graph of $\overline{q}$ and $\frac{\xi(T)}{\xi_0}$ as a function of γ/ξ_0 for $T = 4$ and $e = 1$.

where the equation for q reads

$$q(s) = \frac{1}{3}\left(1 + \sqrt{1 - Ce^{-\frac{\delta}{e}s}}\right). \qquad \square$$

In order to illustrate the analyzed parameters and to show their influence on learning, we introduce Figure 1.2, 1.3 and 1.4.

First of all, the influence of δ on learning is described. In this context, Figure 1.2 plots the average learning time $\overline{q}$ on the left-hand side and the learning progress $\xi(T)/\xi_0$ on the right-hand side against δ, the parameter describing preferences for learning time. The values of γ/ξ_0 are varied for the first three graphs, i.e. $\gamma/\xi_0 = \frac{1}{6}, \frac{1}{12}, 0$. The parameter γ/ξ_0 describes the trade-off between learning time and leisure time. If this parameter is high, leisure time rises because learning is more costly.

The average learning time $\overline{q}$ decreases when δ is rising, since the ratio between costs of learning and the positive part of the utility function increases in response to higher values of δ. According to the costs of learning, which vary according to the parameter γ/ξ_0, the average learning time $\overline{q}$ differs. The more costly learning gets, which is true in the case of increasing values of γ/ξ_0, the less students learn on average. On the right-hand side of Figure 1.2, we see that the learning progress is higher when γ/ξ_0 is

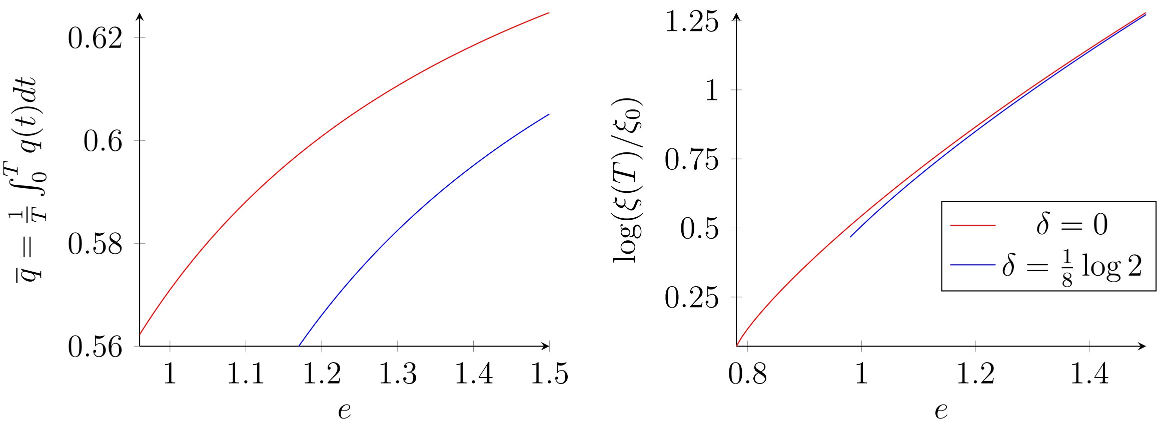

Figure 1.4: Graph of $\overline{q}$ as a function of e and a semilogarithmic plot of $\xi(T)/\xi_0$ as a function of e for $T = 4$ and $\frac{\gamma}{\xi_0} = 1$.

small. Furthermore, the dependency on δ is smaller in the case of decreasing values of γ/ξ_0.

Moreover, Figure 1.2 emphasizes that the learning progress decreases in δ and that simultaneously the average learning time falls. A fourth line shows the learning progress for a constant average learning time $\overline{q} = 0.6$. Here the graph indicates that higher values of δ lead to a less effective learning strategy.

We introduce Figure 1.3 with the aim to describe the influence of γ/ξ_0 on the average learning time and the learning progress. Thereby, Figure 1.3 plots the average learning time $\overline{q}$ on the left-hand side and the learning progress $\xi(T)/\xi_0$ on the right-hand side against γ/ξ_0. On the left-hand side of Figure 1.3, we see that the average learning time decreases as a response to higher values of γ/ξ_0, since learning is more costly. Thereby, the maximal average learning time is $\overline{q} = \frac{2}{3}$ and is reached in the case of a vanishing learning disutility ($\gamma = 0$). Furthermore, on the left-hand side of Figure 1.3 the learning progress is diminishing when values of $\xi(T)/\xi_0$ are expanded: this decrease is even stronger if $\delta = \frac{1}{4}\log 2$. The average learning time and the learning progress are smaller in the case of preferences for short-term learning.

Finally, we want to illustrate the impact of e on learning. For this purpose, Figure

1.4 illustrates the average learning time $\overline{q}$ and the learning progress $\xi(T)/\xi_0$ as a function of e. Thereby, e describes the effectiveness of solving exercises. The greater e is, the more exercises the student is able to solve per time unit. Note that the graph on the right-hand side is actually semi-logarithmic. Consequently, the learning progress is increasing as response to higher values of e in an almost exponential manner. In addition, the average learning time is also increasing with respect to e. This entails that increasing the effectiveness of learning has the biggest effect on individual learning behavior. Note that the average learning time and the learning progress are on a higher level in case of lower discounting.

Optimal time allocation for low-ability or high-ability students

Next, we take a closer look at the second case given by $\xi(T) + \epsilon_0 < \frac{1}{2}$, indicating that the student is not able to pass the exam. Given that $X(\xi, \epsilon) = 0$, we can rewrite the utility function as

$$u(q, \epsilon) = 0 - \gamma \int_0^T q(t) e^{-\delta t} dt.$$

As we have shown, skills at time T are maximized by a constant $q = \frac{2}{3}$. Therefore, the student is not able to pass the exam if $\frac{1}{2} - \epsilon > \xi_{max}(T) := \xi_0 \exp \frac{4}{27} T$. Here, $\xi_{max}(T)$ describes the maximal possible skill level the student is able to achieve until the date of the exam. This inequality depends on the uncertainty parameter ϵ with the range $[-\epsilon_0, \epsilon_0]$. Consequently, $q = 0$ maximizes expected utility of the student for $\frac{1}{2} - \epsilon_0 > \xi_{max}(T)$ being independent of ξ. This student cannot be activated by any incentives. The only way is to give him longer learning periods, increase his initial skills or improve learning productivity.

The third case deals with students whose time allocation derived as in Section 1.2.3 leads to a critical value of knowledge at time T by means of $\xi(T) \in [\frac{1}{2} - \epsilon_0, \frac{1}{2} + \epsilon_0)$. Hence, using the strategy from Section 1.2.3, success of the student also depends on luck.

Now let us assume that the student is aware of the probability distribution of $\epsilon \in [-\epsilon_0, \epsilon_0]$. Therefore, the student modifies his learning time allocation in order to maximize the expected utility

$$u(q) = e^{-\delta T} \int_{\frac{1}{2}-\xi(T)}^{\epsilon_0} P(\epsilon)(\xi(T) + \epsilon) d\epsilon - \gamma \int_0^T q(t) e^{-\delta t} dt \tag{1.6}$$

by requiring $\xi(T) \in [\frac{1}{2} - \epsilon_0, \frac{1}{2} + \epsilon_0)$.

Let us fix $X \in [\frac{1}{2} - \epsilon_0, \frac{1}{2} + \epsilon_0)$. In order to compute the utility-maximizing q for this case, we apply the method of Lagrange multipliers.

$$\begin{aligned} L(q, \lambda) &= \Xi(X) - \gamma \int_0^T e^{-\delta t} q dt + \lambda(\xi(T) - X) \\ &= \lambda \xi_0 \exp\left(e \int_0^T q^2(1-q) dt \right) - \gamma \int_0^T e^{-\delta t} q dt \\ &\quad + \Xi(X) - \lambda X, \end{aligned}$$

where $\Xi(X) := e^{-\delta T} \int_{\frac{1}{2}-X}^{\epsilon_0} P(\epsilon)(X + \epsilon) d\epsilon$. If we assume the existence of a utility-maximizing q, we suppose these two conditions:

$$\frac{d}{ds} L(q + sv, \lambda)\big|_{s=0} \overset{!}{=} 0 \qquad \text{for all } v = v(t), \tag{1.7}$$

$$\frac{d}{d\lambda} L(q, \lambda) = \xi(T) - X \overset{!}{=} 0. \tag{1.8}$$

The first condition can be written as

$$\begin{aligned} \frac{d}{ds} L(q + sv, \lambda)\big|_{s=0} &= \lambda \xi_0 e \int_0^T (2qv - 3q^2 v) dt \exp\left(e \int_0^T q^2(1-q) dt \right) - \gamma \int_0^T v e^{\delta t} dt \\ &= \int_0^T v \left(\lambda e q (2 - 3q) \xi(T) - \gamma e^{-\delta t} \right) dt \overset{!}{=} 0 \end{aligned}$$

for all $v = v(t)$. Inserting the delta-function $v(t) = \delta_\tau(t)$ and applying Condition (1.7) it follows that $\lambda e q(t)(2-3q(t))(\frac{1}{2}-\epsilon)=\gamma e^{-\delta t}$, which can be solved by

$$q(t)=\frac{1}{3}+\sqrt{\frac{1}{9}-\frac{1}{3}\frac{\gamma e^{-\delta t}}{\lambda e X}}.$$

Note that the student's learning capability is low such that the student needs to learn close to the skills maximizing $q=\frac{2}{3}$. For reasons of simplicity we now define

$$C_\lambda=\frac{3\gamma}{\lambda e X}.$$

In order to compute λ, we have to make use of Condition (1.8), which we recall as

$$\xi(T)=\xi_0\exp\left(e\int_0^T q^2(1-q)dt\right)=X$$

with $q(t)=\frac{1}{3}(1+\sqrt{1-C_\lambda e^{-\delta t}})$ being equivalent to

$$\int_0^t q^2(1-q)dt=\log\left(\frac{X}{\xi_0}\right).$$

Hence, C_λ can be computed by solving the previous equation using

$$\begin{aligned}\int_0^t q^2(1-q)ds=\frac{2}{27\delta}\Bigg[\frac{1}{3}\left(\sqrt{1-Ce^{-\delta t}}^3-\sqrt{1-C}^3\right)-2\big(\sqrt{1-Ce^{-\delta t}}-\sqrt{1-C}\big)\\ -2\log\left(\frac{1-\sqrt{1-Ce^{-\delta t}}}{1-\sqrt{1-C}}\right)\Bigg]\end{aligned}$$

for $\delta>0$ and

$$\int_0^t q^2(1-q)ds=\frac{t}{27}\Big((2+C)\left(1+\sqrt{1-C}\right)-C\Big)$$

for $\delta=0$. For the second step, let $q(t,X)$ denote the optimal time allocation for given parameter ϵ at time t. Now the optimal learning strategy can be achieved by the

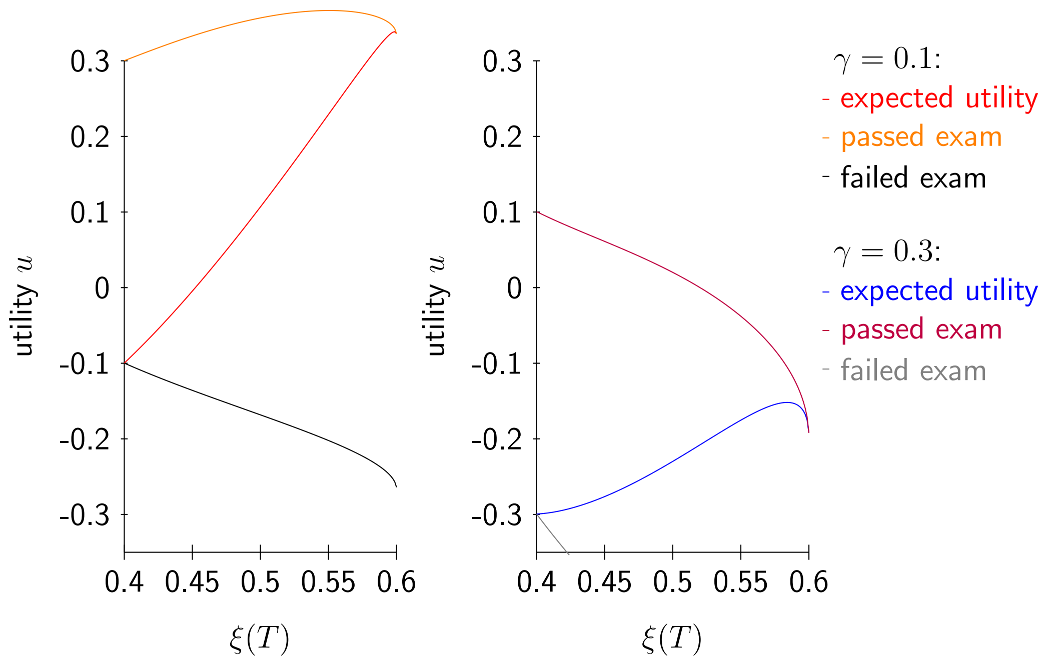

Figure 1.5: Graph of expected utility $\epsilon_0 = 0.1$, $\xi_0 = 0.3$, $\delta = 0$, $T = 4$, $e = 1$.

following program

$$\max_{-\epsilon_0 \leq X - \frac{1}{2} < \epsilon_0} \left(\Xi(X) - \gamma \int_0^T q(t, X) e^{-\delta t} dt \right),$$

which can be solved by deriving the first order condition or directly by a numerical solver.

In order to illustrate the low-ability student who is uncertain whether he will pass the exam, we introduce Figure 1.5. Thereby, Figure 1.5 plots individual utility against skill level at the date of the exam T for two outcomes of this low-ability student. Under the assumption of equipartition and preferences for long-term learning ($\delta = 0$), the left-hand side of Figure 1.5 shows a student with a low disutility of learning ($\gamma = 0.1$), in contrast to a student with a high disutility ($\gamma = 0.3$) on the right-hand side. At the beginning of the semester, both types face a trade-off concerning the choice between learning and leisure time. On the one hand, a high learning time increases the probability to pass the exam, on the other hand it reduces leisure time. For each student the intermediate graph shows the expected utility taking into account the probability of failure.

As a result, the student on the left-hand side of Figure 1.5 maximizes his utility by learning at the maximum learning time of $q = \frac{2}{3}$, the right-hand side type will not learn at all. Intermediate students might even fail with positive probability although they do not learn with maximum intensity.

Finally, we have a look at the last case which is given by a high-ability and ambitious student who aims to pass the exam with the best mark and is able to do so in the optimum. This case is characterized by $\xi + \epsilon > 1$, consequently the student passes the exam achieving the best mark with positive probability. If $X(\xi, \epsilon) = 1$ holds, the utility is given by

$$u(q, \epsilon) = e^{-\delta T} - \gamma \int_0^T q(t) e^{-\delta t} dt.$$

This case is very similar to the third case, because we can again apply the method of Lagrange multipliers. The incentives to learn decrease since the best grade is reached. Depending on his discounting, he will eventually not learn at all at the beginning of the semester or decrease his learning activity along the semester.

1.2.4 Utility maximization with mid-term examination

In order to illustrate the impact of a mid-term test on learning behavior, the result of the mid-term test X_t is added to the utility function. With the aim to reduce complexity, we consider only one mid-term test. The mid-term test takes place in the half of the semester in order to provide incentives for continuous learning and feedback. If a student passes the exam, the result of the test improves the exam mark. However, a successful test is not credited if the student fails the exam.

In this context utility changes to

$$u(x) = e^{-\delta T} X - \gamma \int_0^T e^{-\delta t} q dt,$$

where the result function of the exam is determined by the initial level of skills ξ_0, the uncertainty parameter ϵ and the growth of skills till time T:

$$\begin{aligned} X &= \xi(T) + \epsilon + \alpha\xi(\tau) \\ &= \epsilon + \xi_0 \exp\left(e \int_0^T q^2(1-q)ds\right) + \alpha\epsilon_t + \alpha\xi_0 \exp\left(e \int_0^\tau q^2(1-q)ds\right). \end{aligned}$$

The parameter α captures the fact, that mid-term examination only counts for a small proportion of the final grade.

As described in Section 1.2.3, the result function of the exam includes four cases, leading to different utility-maximizing time allocations q. Since the first case represents the average student, this case is computed in detail hereafter. The remaining three cases do not need exact calculation, as the influence of the test can be derived causally.

Likewise, the utility maximization without test, the first case describes a student, whose optimal behavior leads to an interior solution, not receiving the best mark but also not failing the exam. Thus, following the computation in Section 1.2.3, the first order condition is given by

$$\begin{aligned} &\frac{d}{ds} u(q+vs)\big|_{s=0} \\ &\quad = e^{-\delta T} \xi_0 e \int_0^T qv(2-3q)dt \exp\left(e \int_0^T q^2(1-q)dt\right) \\ &\qquad + e^{-\delta\tau} \alpha\xi_0 e \int_0^\tau qv(2-3q)dt \exp\left(e \int_0^\tau q^2(1-q)dt\right) - \gamma \int_0^T v e^{-\delta t} dt \\ &\quad \stackrel{!}{=} 0. \end{aligned}$$

Inserting the Delta-function $v = \delta_s(t)$, it follows

$$\xi_0 eq(t)(2-3q(t))\Bigg(\exp\left(e\int_0^T q^2(1-q)ds - \delta T\right) \\ + \alpha \exp\left(e\int_0^\tau q^2(1-q)ds - \delta\tau\right)\Bigg) = \gamma e^{-\delta t}$$

for $t \leq \tau$ and

$$\xi_0 eq(t)(2-3q(t)) \exp\left(e\int_0^T q^2(1-q)ds - \delta T\right) = \gamma e^{-\delta t}$$

for $t > \tau$. Analogously to the derivation in Section 1.2.3, we define

$$C_1 := 3\frac{\gamma}{\xi_0 e}\left(\exp\left(e\int_0^T q^2(1-q)ds + \delta T\right) + \alpha \exp\left(e\int_0^\tau q^2(1-q)ds + \delta\tau\right)\right)^{-1},$$
$$C_2 := 3\frac{\gamma}{\xi_0 e} \exp\left(-e\int_0^T q^2(1-q)ds + \delta T\right).$$

Thus, derivation leads to utility-maximizing learning time

$$q(t) = \frac{1}{3}\left(1 + \sqrt{1 - C_K e^{-\delta t}}\right)$$

for $K = 1$, if $t \leq \tau$ and $K = 2$, if $t > \tau$.

The second case is given by a student who is not able to pass the exam. This case does not differ from utility maximization without test, since the result of the test only improves the mark and does not influence the passing of the exam. Thus, again $q = 0$ represents the utility-maximizing choice of learning time.

The student in the third case will eventually shift his learning time to the start of the semester for two reasons: In case he passes the exam he achieves a better mark. The probability of passing the exam increases due to higher skills and this improves the learning success close to the exam. This again increases his incentives to achieve a better grade also in the final exam.

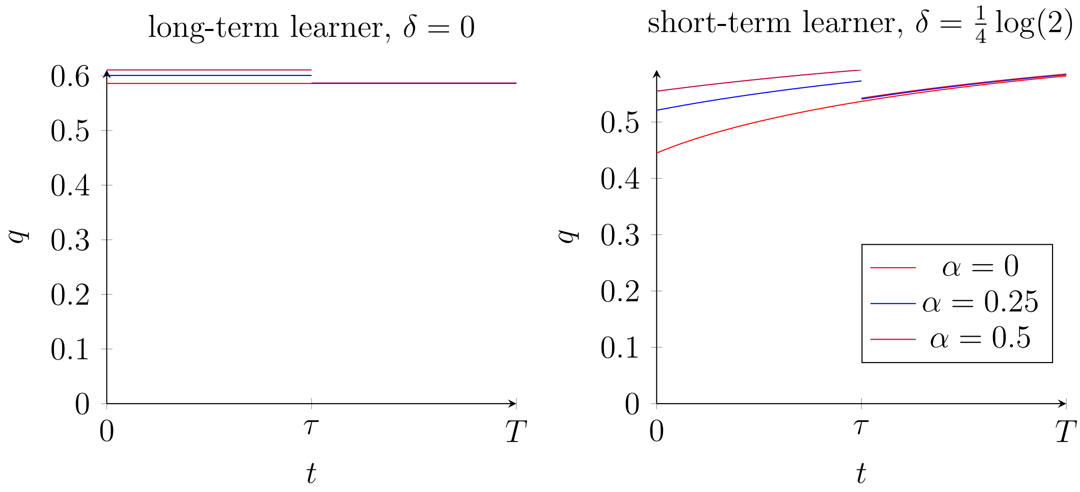

Figure 1.6: Graph of the optimal q for $\xi_0/\gamma = 4$, $\tau = 2$, $T = 4$, $e = 1$.

The fourth case deals with a high-ability student. The result of the test improves his exam mark. Therefore, thanks to the test he is able to learn less in order to achieve the best mark. His learning behavior will only stay high, if he is intrinsically motivated.

Consequently, the introduction of a mid-term examination for perfectly rational and completely informed students leads to an improvement of learning performance and outcomes only in the first and third case. In the third case an increase in the probability of passing the exam can eventually occur.

1.2.5 Results

Computation of the utility-maximizing q without test in Chapter 2.3 leads to

$$q(t) = \frac{1}{3}\left(1 + \sqrt{1 - Ce^{-\delta t}}\right).$$

In Section 1.2.4, the derivation of the utility-maximizing q with test results in

$$q(t) = \frac{1}{3}\left(1 + \sqrt{1 - C_K e^{-\delta t}}\right)$$

for $K = 1$ if $t \leq \tau$ and $K = 2$ if $t > \tau$.

The utility-maximizing q-functions with and without test for both types of students against time are plotted in Figure 1.6. Furthermore, each figure shows one graph in the case of no test, a second graph for $\alpha = \frac{1}{4}$ and a third graph for $\alpha = \frac{1}{2}$ varying the value of the mid-term test. As time frame we assume one semester comprising four months, with a mid-term test in the half of the semester after two months.

In the case of $\delta = 0$ denoting no discounting, the utility-maximizing q-functions are illustrated on the left-hand side of Figure 1.6. As a result, utility is maximized by a constant learning time q per unit during the time period without test. Introducing a test, the fraction of learning remains constant in time before and after the test. Nevertheless, it is characterized by a higher level before the test. The level of learning time before the test is increasing in α.

The utility-maximizing q-functions in the case of $\delta > 0$ denoting discounting are illustrated on the right-hand side of Figure 1.6. In contrast to the left-hand side of Figure 1.6, they are characterized by a minor increase of q in t during the semester. Moreover, assuming a mid-term test, there is a step at the time of the test with a higher overall level of q before the test. Furthermore, the overall level of q before the test increases with the value of the test. Although the choice of learning time q is not constant in time and despite preferences for short-term learning, last-minute learning is avoided with and without test.

Consequently, Figure 1.6 shows that blended learning results in a more continuous learning activity over time and the mid-term test leads to a higher level of learning time per unit at the beginning of the learning period. Thus, student engagement in constant and early learning is promoted independent of individual time preferences.

Next we introduce Figure 1.7 in order to analyze the dependency of the average learning time $\overline{q}$ on the learning progress $\xi(T)/\xi_0$ which can be interpreted as the efficiency of learning. Figure 1.7 comprises three graphs illustrating three types of students. The first student prefers to learn continuously with $\delta = 0$. He is the most

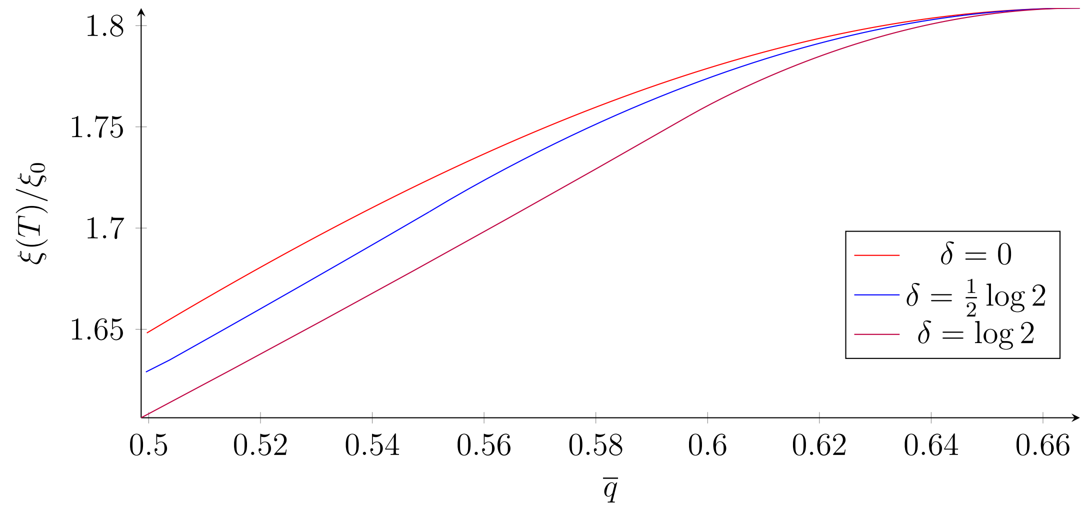

Figure 1.7: Graph of $\xi(T)/\xi_0$ as a function of $\overline{q}$ for $T = 4$ and $e = 1$.

effective learner, since his progress in learning shows the greatest rise in the case of increasing average learning time. The second and the third types of students are characterized by discounting. The third type has the highest discounting δ. Their graphs emphasize that learning gets less productive if values of δ are high. This effect becomes canceled out if $\overline{q} = \frac{2}{3}$ which characterizes the maximum learning time. In the case of $\overline{q} = \frac{2}{3}$, learning time is totally constant for every type and thus no improvement of learning can be realized. Consequently, this figure emphasizes the significance of a constant learning activity, since the learning progress is higher if students learn constantly.

Because of the importance of constant learning and in order to show the influence of a mid-term test on this continuity, we introduce Figure 1.8. As a measure for the continuity of learning, we define the variance

$$\sigma^2 = \frac{1}{T}\int_0^T (q - \bar{q})^2 dt$$

with $\bar{q} = \frac{1}{T}\int_0^T q dt$ being the mean value of q.

Figure 1.8 plots the measure of continuity $\sigma/\bar{q}$ against δ. Furthermore, the figure shows one graph in the case of no test ($\alpha = 0$), a second graph for $\alpha = \frac{1}{4}$ and a third graph for $\alpha = \frac{1}{2}$. Thus, we vary the value of the mid-term test, similar to Figure 1.6.

The learning activity of students with preferences for long-term learning with $\delta = 0$ is totally constant without a test. In contrast to this, a mid-term test encourages students to learn more before the test. This effect is enhanced with increasing values for α.

In the case of preferences for short-term learning and without a mid-term test, the continuity of learning time decreases with increasing δ. In contrast to this fact, the existence of a mid-term test leads to a rise of the continuity of learning behavior with increasing values for α. Thus, the more important the test becomes, the more constant the learning activity becomes (beyond a critical value of δ). This effect is even stronger in case of higher preferences for short-term learning.

Consequently, the continuity of $q(t)$ rises with increasing valuation of the test. This effect is even enhanced with increasing discounting. Nevertheless, students with preferences for long-term learning learn more constantly without a mid-term test, since a test intensifies a shift towards early learning.

If we combine these results with the results of Figure 1.7, we see that the learning progress of students with preferences for long-term learning is greater without mid-term test, because in this case they learn more constantly. In contrast to this, the learning productivity of students with preferences for short-term learning increases in the case of a mid-term test, leading to a more constant learning time. Therefore, short-term learners are able to increase their learning progress if a mid-term test exists.

These results support our concern to emphasize the importance of blended learning and mid-term tests leading to effective learning behavior and better learning outcomes. Consequently, the resulting utility-maximizing learning time of both types of students leads to better learning performance and the blended learning opportunities fully meet the demands of an optimal learning environment.

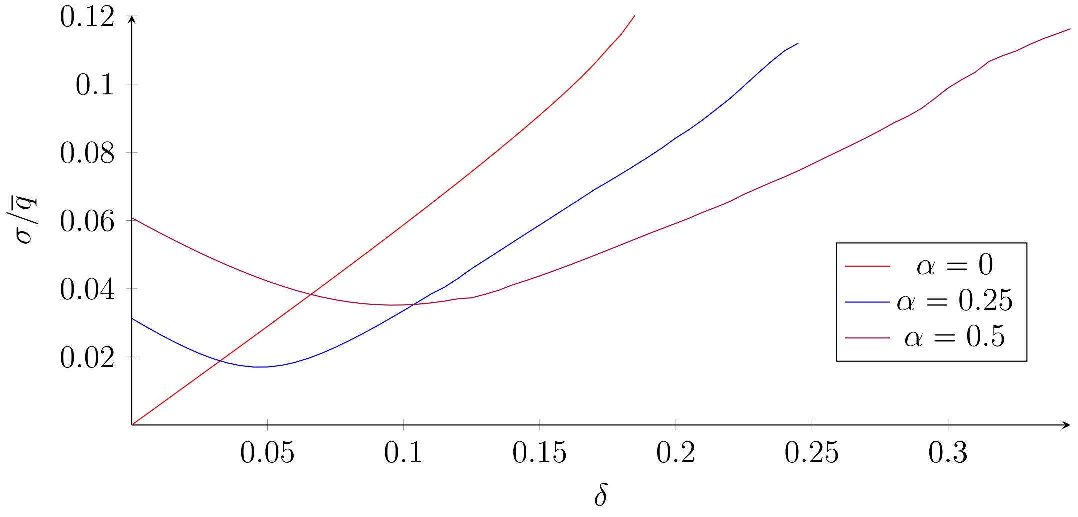

Figure 1.8: Graph of the measure of continuity $\sigma/\overline{q}$ as a function of δ for $\overline{q} = 0.55$, $e = 1$, $\tau = 2$ and $T = 4$.

1.3 Conclusion

This paper has analyzed a simple model based on learning by doing in the context of university courses. Exam outcomes depend on accumulated skills through learning during the semester. Thereby, the offered range of blended learning allows flexible and around-the-clock learning opportunities. On the one hand this improves effectiveness of learning behavior by increasing the productivity of learning e. On the other hand this range increases the incentives to learn and the probability to pass the exam.

In addition, a mid-term test entails the possibility to improve the exam outcome and to get an early feedback on individual skills. This increases incentives and skills at an early stage in the semester. Together with the rising efficiency of skills, students reach a better mark with lower learning intensity and lower aggregate learning time.

Considering dynamic utility maximization, overall invested time is lower in the case of a continuous learning strategy. Referring to the continuity of learning, we also show that a constant learning activity increases the learning progress.

Offering a mid-term test induces students to increase learning time preceding the mid-term examination. Therefore, blended learning combined with a mid-term test enhances the effect of constant and early learning and leads to more effective learning. In addition, all students who pass the mid-term test experience an improvement of the exam outcome and the probability of passing the exam increases.

In conclusion, we are able to show that learning behavior becomes more continuous and successful thanks to incentivizing tests and around-the-clock online learning opportunities.

Chapter 2

Influence of online feedback on students' learning behavior

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Abstract

This paper focuses on the feedback-function of blended learning implemented in university courses with the aim to improve learning performance. Since surveys show that students misconceive their own learning progress and especially overestimate their own ability, the feedback-mechanism of online learning tools aims to support students' self-assessment. Thereby, online exercises and online mid-term tests serve as sources of feedback.

For this purpose, the concept of learning by doing is introduced to blended learning and university courses. Exam outcomes are mainly the result of skills which rise in solving exercises and spending learning time. Dynamic maximization of utility is performed, considering the choice between learning time and leisure time. Furthermore, the self-assessment of skills depends on the utilization of feedback implemented in blended learning. While the learning behavior of students who do not use blended learning is based on their expected skill level, students using blended learning compare their expected skill level to the received feedback.

As a result, we are able to show that online exercises and a mid-term test improve self-assessment of skills and the failure rate of the exam if students persistently make use of these online learning opportunities. Thus, this paper emphasizes the importance of online feedback in regard to a better learning behavior.

2.1 Introduction

Self-perception of skills plays a crucial role in the decision to invest in higher education and has a significant influence on a successfull graduation.

A wide range of literature examines the self-perception of students and its impact on their studies. In this context, Chevalier et al. (2009), who investigated the impact of students' self-perception on participation rates in higher education, found out that students misconceive their own performance and especially over-estimate their own ability. Thereby, a high self-perception supports the decision to attend higher education.

Furthermore, by examining self-assessment in real-world settings, Dunning et al. (2004) show that most students are not able to assess themselves accurately and even tend to be overconfident concerning acquired skills. Moreover, students are often not able to assess whether they have understood newly acquired skills because they focus on the quick acquisition of contents and neglect and underestimate the retention and the transfer of course contents. For this reason, authors propose feedback as an opportunity to improve pupils' self-assessment and check newly acquired contents, since feedback helps students to evaluate their own performance. Attending to feedback, students self-regulate their learning activity and check their self-assessment. Thereby, self-regulated students are able to estimate their own skills and to update and organize their further learning activity (Butler and Winne 1995). Nevertheless, considering courses with a huge number of students, individual feedback places great demands on teachers. In this connection, e-learning tools might be a solution since they are able to offer immediate feedback, independent of the course size. Investigating computer-supported learning, Zumbach and Reimann (2003) confirm the positive influence of external feedback on students motivation and problem-solving.

Moreover, the current learning environment is characterized by a trend towards online learning. For this reason, blended learning catches up with traditional teaching methods. Various surveys show that online learning tools have increased significantly in students' learning experiences (Allen and Seaman 2006) and that students show a new attitude towards online learning (Sebastianelli and Tamimi 2011). In Germany, for example, nearly every university offers an e-learning platform (Henning 2015). The virtual learning environment places new demands on educators (Harasim et al. 1995). Besides, traditional requirements guiding and helping students through formative feedback becomes more important.

In order to show the impact of feedback and blended learning on learning behavior, this paper develops a simple model of learning by doing.

Until now, economic literature has only focused on learning by doing within the field of production. In this context, Arrow (1962) was the first who introduced learning as a product of experience gained during the process of problem-solving and activity. Göcke (2002) extends this model by analyzing the optimal allocation of time between working and leisure in the context of learning by doing.

This paper is based on the model of Chapter 1 and introduces the influence of feedback. The exam outcome depends on accumulated skills during the learning period. Thereby, blended learning creates learning opportunities which are available 24 hours per day during 7 days per week. E-learning exercises take student heterogeneity into account, since they are characterized by a high flexibility of time and space. While solving blended learning exercises, students acquire skills and human capital is accumulated. However, students have to decide upon the optimal time allocation to learning and leisure, leading to different utility formulations. This model shows similarities to Göcke (2002), yet we apply learning by doing to university courses. We apply this model and introduce the self-assessment of skills which depends on the utilization of feedback implemented in blended learning. In this context, online exercises and mid-term tests provide feedback. While students who do not use blended learning base their learning behavior on their expected skill level, students using blended learning are able to compare their expected skill level to the received feedback. For this purpose, we analyze the impact of online exercises and of the mid-term test as sources of feedback.

These sources of online feedback evaluate accumulated skills, so that further learning activity can be updated according to this evaluation.

Consequently, we address the question how feedback implemented in blended learning influences and improves learning behavior.

2.2 Feedback setting

Hattie (2013) describes feedback as an information about the individual performance or comprehension which is delivered by a teacher. Feedback aims to reduce the discrepancy between the current ability and the learning objective. He distinguishes between three types of feedback: “Feed Up”, “Feed Back” and “Feed Forward”. “Feed Up” focuses on the learning objective, “Feed Back” on the learning progress and “Feed Forward” on the future perspective. We assume that students have the opportunity to regulate their learning activity supported by online feedback. Thereby, online exercises which are available around the clock and a mid-term test offer feedback and thus information about the individual skill level and progress. Since these sources focus on the learning progress and the learning objective, they represent “Feed Up” and “Feed Back”. The utilization of the described online feedback is voluntary. While students who choose the traditional way of learning without online feedback rely on uncertain assumptions about their skills, students who make use of blended learning and participate in the test are able to estimate their skills and as a consequence to optimize their learning activity.

On the one hand, we analyze the influence of mid-term tests as feedback source. On the other hand, we examine the influence of online exercises as source of feedback.

In order to model feedback offered by the mid-term test, we divide the learning period into a pre- and a post-monitoring phase (Doerr 2013), separated by the date of the mid-term test when monitoring takes place. Since the mid-term test gives a mark and therefore a detailed evaluation of individual skills, students are able to connect skills and mark. In contrast to this, feedback on online exercises does not create this connection. Consequently, accumulated skills are evaluated at the date of the mid-term test and the learning activity of the post-monitoring phase may be updated based on the gained information.

Although working with online exercises does not create a connection between skills and mark, they offer many feedback possibilities. Students who use the online learning platform to solve exercises receive hints about their current abilities during the whole term. Consequently, they are able to check their learning progress on a regular basis, even though feedback without grades does not offer detailed information.

2.3 The model

This paper introduces the concept of learning by doing to university courses supported by blended learning and feedback. Beside traditional face-to-face sessions, students have the possibility to solve online exercises during the semester and participate in a mid-term test in order to estimate their skills with the aim to pass the exam. The offered online exercises take student heterogeneity into account, since students can solve exercises according to their individual learning progress and may repeat them around-the-clock.

2.3.1 Optimal learning behavior

Let us assume that skills ξ are accumulated during the process of learning by doing. The student solves exercises using the blended learning platform in order to improve his skill level. In Chapter 1, ξ is interpreted as a differentiable function of time t fulfilling an ordinary differential equation. However, a unique dependency between learning and skill improvement seems to be unrealistic. Therefore, we rather assume that the improvement of the skill level is normally distributed at each point in time with standard deviation proportional to the skill level. Moreover, every improvement shall be independent from another. Combining these consideration with the model from Chapter 1 motivates the stochastic differential equation

$$d\xi_t = a(q) \cdot eq \cdot \xi_t dt + \sigma \xi_t dB_t, \tag{2.1}$$

where B_t denotes a Brownian motion and σ is a parameter modeling the uncertainty. The growth rate of skills, $a(q)eq = eq^2(1-q)$, depends on the learning activity of the student. As in Chapter 1, the relative learning time $q = q(t)$ describes the fraction of time the student spends using the online learning opportunities at time t. The values of q belong to the interval $[0, 1]$, where $q(t) = 0$ describes no learning and $q(t) = 1$ learning without breaks, respectively. The parameter e models the number of exercises solved by the student per time unit. Thus, the slope of skills equals the number of solved exercises E times the factor $q(1-q)$ modeling the students productivity, which is increasing until a critical learning time q_0 and decreasing afterwards because of the absence of pauses. Note that $1 - q$ represents the students leisure time. Likewise to the model of Chapter 1, we can directly solve Equation (2.1) by

$$\xi(t) = \xi_0 \exp\left(e \int_0^t \left(q^2(1-q) - \frac{1}{2}\sigma^2 \right) dt + \sigma B_t \right).$$

Note that $q(t)$ is yet unknown. In contrast to Chapter 1, the skill level of a student is a random variable. Therefore, the student's optimal learning time maximizes his expected utility, which is determined by uncertainty with respect to the skill level as well as considering the outcome of the exam.

As suggested in Chapter 1, the utility functional u is given by

$$u(q) := e^{-\delta T} X(T) - \gamma \int_0^T q(t) e^{-\delta t} dt \tag{2.2}$$

for some $\gamma, \delta \geq 0$. Here, $X(T)$ denotes the result of the exam at time T and the second term, $\gamma \int_0^T q(t)e^{-\delta t}dt$, describes the disutility caused by the lack of leisure time.

Note that the parameter δ distinguishes between different types of students: While students with higer discounting prefer to shift their learning activity towards the end of the semester when the exam takes place, students with lower or no discounting prefer to learn continuously from the beginning of the semester until the exam and thus spend less time in aggregate for learning. In this article we want to focus on an average student, who will in his optimum pass the exam with a mark strictly between $X(T) = 1$ and $X(T) = \frac{1}{2}$, for which one still passes in the exam. Thus, for simplicity, we suppose that his result of the exam is given by

$$X = \xi(T) + \epsilon \quad \text{for some } \epsilon \in [\underline{\epsilon}, \overline{\epsilon}], \tag{2.3}$$

which is in accord with Chapter 1, where ϵ models the uncertainty of the result. We do this in order to focus on the individual incentives to learn in order to improve the grade. By this, we assume that the probability of failing the exam or obtaining the best result are neglectable.

The student's expected utility is thus given by

$$\mathbb{E}[u(q)] = e^{-\delta T}(\mathbb{E}\xi(T) + \mathbb{E}\epsilon) - \gamma \int_0^T q(t)e^{-\delta t}dt.$$

Note that the expected skill level solves the ordinary differential equation

$$\frac{d}{dt}\mathbb{E}\xi = eq^2(1-q)\mathbb{E}\xi, \tag{2.4}$$

which coincides with the one in Chapter 1. Hence, we can apply the following proposition to our scenario.

Proposition 3 (Chapter 1)**.** *The optimal learning strategy q for an average student is implicitly given by*

$$eq(2-3q)\xi_0 \exp\left(e\int_0^T q^2(1-q)dt - \delta T\right) = \gamma e^{-\delta t}. \tag{2.5}$$

Due to Eq. (2.4) and (2.5), a rational student maximizes his skills at time T based on his expected initial skill level ξ_0 at $t = 0$. This is also true the other way round, which can be seen by the following Lemma in conjunction with rewriting Eq. (2.5) to

$$eq(2-3q)e^{-\delta T}\mathbb{E}\xi(T) = \gamma e^{-\delta t}, \tag{2.6}$$

where we have inserted (2.1).

2.3.2 Feedback on solving online exercises

The optimal learning behavior is determined by Eq. (2.6). However, this equation contains several parameters: the expected skill level at the date of the exam $\mathbb{E}\xi(T)$, the parameter e connecting solved exercises to the growth rate of knowledge and the disutility parameters γ and δ. We assume that γ and δ may differ across students but are known by the students, since they represent individual learning preferences and attitudes. Therefore, the student's main purpose is to specify $\mathbb{E}\xi(T)$ and e.

As mentioned above, we assume that the student solves exercises using the blended learning platform. After having submitted his result of an exercise to the platform, the student obtains feedback about how well he has solved the exercise. However, an online exercise and a written exam use different methods to evaluate the student's skill level. A graded exam offers detailed information connecting skills and mark. In contrast to this, the blended learning platform provides feedback rather on the learning improvement than on the actual skill level which is relevant for the exam. Let us suppose that the time the student requires for solving an exercise is Δt. Then the learning improvement for solving an exercise at time t_i is given by the fraction $\frac{\xi(t_i+\Delta t)}{\xi(t_i)}$.

Nevertheless, the improvement in skills is normally distributed and therefore not directly given. As a consequence, the student needs to solve several exercises in order to gain knowledge about his personal growth rate of skills and in particular about the parameter e. Thus, the average relative learning improvement is more significant than each single learning improvement. We assume that the online learning platform provides a vast number of exercises so that the student is able to determine his average relative learning improvement $\frac{1}{N}\sum_{i=1}^{N}\frac{\xi(t_i+\Delta t)}{\xi(t_i)}$ for a large number of solved exercises. We are interested in the limit

$$I := \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\frac{\xi(t_i+\Delta t)}{\xi(t_i)}.$$

If the student solves a reasonable number of exercises, he is able to calculate I up to a certain degree.

The relative learning improvement after solving one exercise satisfies

$$\frac{\xi(t_i+\Delta t)}{\xi(t_i)} = \exp\left(\Delta t\left(eq^2(1-q)-\frac{1}{2}\sigma^2\right)+\sigma(B_{t_i+\Delta t}-B_{t_i})\right).$$

Since $B_{t_i+\Delta t}-B_{t_i}$ is normally distributed with mean 0 and variance Δt and independent, the relative learning improvements $\frac{\xi(t+\Delta t)}{\xi(t)}$ are independent, identically-distributed random variables with mean

$$\mathbb{E}\frac{\xi(t_i+\Delta t)}{\xi(t_i)} = \exp\left(e\Delta t q^2(1-q)\right).$$

According to the weak law of large numbers, we encounter that

$$I := \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\frac{\xi(t_i+\Delta t)}{\xi(t_i)} = \exp\left(e\Delta t q^2(1-q)\right).$$

Finally, if the student gets information about I, he is able to guess the parameter e connecting solved exercises to the growth rate of skills. As we see by Condition (2.6), the parameter is crucial for computing the optimal learning strategy. In addition, Chapter 1 exemplarily shows the huge influence of e to the utility function. The opportunity of blended learning allows the student to determine his effective growth rate of skills by using constant feedback.

2.3.3 Predicting optimal learning behavior

In the foregoing section, we have seen that the student can use the feedback of the solved exercises to compute the parameter e. However, this feedback does not provide a direct information on the actual skill level since a written exam and the online exercise use different methods to evaluate the skill level.

In contrast to Chapter 1, we assume that students are not able to estimate their skill level with precision, i.e. only a prediction of the individual ξ_0 can be made. Estimating ξ_0 is equivalent to predicting $\mathbb{E}\xi(T)$ for a rational student.

For this reason, we assume in the following that the student predicts his skill level $\xi(T)$ at time T by $\xi_{\text{pre}}(T)$. Thus, he computes his predicted optimal learning time q_{pre} by solving

$$eq_{\text{pre}}(2-3q_{\text{pre}})e^{-\delta T}\xi_{\text{pre}}(T) = \gamma e^{-\delta t}. \tag{2.7}$$

Here, we have replaced $\mathbb{E}\xi(T)$ in Eq. (2.6) by $\xi_{\text{pre}}(T)$. Because of his experience, the student supposes that the probability p that the optimal achievable skill level falls within a range

$$(1-p)\xi_{\text{pre}}(T) \le \xi(T) \le (1+p)\xi_{\text{pre}}(T) \tag{2.8}$$

close to 100%, where $p \in (0,1)$.

Definition 4. We say that a student is overconfident or under-confident if his presumed range given by p is too small and the probability $\mathcal{P}$ for (2.8) is reasonably small, i.e.

$$\mathcal{P}\big((1-p)\xi_{\text{pre}}(T) \le \xi(T) \le (1+p)\xi_{\text{pre}}(T)\big) \le 5\%,$$

for $\xi(T) = \xi_0 \exp\left(e\int_0^T q_{\text{pre}}^2(1-q_{\text{pre}})dt\right)$, where q_{pre} is given by (2.7). Of course the bound 5% is arbitrary. If the uncertainty parameter σ vanishes, this condition is reduced to

$$\xi(T) \notin [(1-p)\xi_{\text{pre}}(T), (1+p)\xi_{\text{pre}}(T)].$$

Otherwise, we call the student confident, because he has a reasonably well-calibrated self-perception.

Since the predicted skill level ξ_{pre} directly influences the computation of the learning time q_{pre} by means of Eq. (2.7), $\xi(T) \neq \xi_{\text{pre}}$ does not characterize a utility maximum:

In fact, utility rises with a good exam mark and decreases with time spent on learning as the residual of learning time. Since the exam mark depends on accumulated skills, under- and overestimation lead to disutility. While students who overestimate their skill level might not learn enough, students who underestimate their skill level might spent too much time on learning. In addition to this faulty optimiziation of learning time, an additional utility loss might result due to the surprise because of the gap between expected and actual exam performance. For high values of p, the difference in utility concerning optimal and real learning behavior might be significant.

2.3.4 Mid-term test

Let us extend our model by a mid-term test which aims to improve the student's knowledge about his skill level. A mid-term test with direct benefits was already introduced in Chapter 1. In contrast to that, we assume that the result of the mid-term test has no (direct) influence on the utility. In order to facilitate the computations, we assume in the following that σ vanishes and therefore ξ and $\mathbb{E}\xi$ coincides. Consequently, the utility for an average student is given by

$$u(q, \epsilon) = e^{-\delta T}(\xi(T) + \epsilon) - \gamma \int_0^T q(t) e^{-\delta t} dt$$

(compare with Subsection 2.3.1). Recall that $\delta, \gamma \geq 0$ are parameters modeling the disutility of spending time for learning and $\epsilon \in [\underline{\epsilon}, \overline{\epsilon}]$ describes the uncertainty during the exam, modeling unpredictable influences like luck, concentration and so on.

The result of the mid-term test has no direct influence on the utility, since it does not change the exam mark. Nevertheless, the test is marked and thus gives information about the true skill level. Consequently, the student's imperfect knowledge about his true skill level decreases if he participates in the mid-term test. We assume that the grading of the mid-term test equals the final exam grading according to the following definition.

Definition 5.

$$X_{\text{real}}(\tau) = \begin{cases} X_{t,\max} & \text{if } \xi + \epsilon_t \geq X_{t,\max}, \\ \xi(\tau) + \epsilon_\tau & \text{else.} \end{cases} \tag{2.9}$$

Since the mark of the test has no influence on the mark of the final exam, the student cannot fail the test. In addition, $X_{t,max}$ which denotes the test's maximal score is less than the exam mark, since it takes place during the semester and the lecture is not advanced yet. Thus, it applies that $X_{t,max} < 1$. We use ϵ_τ as uncertainty parameter, modeling unpredictable factors involved during the test situation like luck and concentration.

2.3.5 Updating learning behavior

Participating in the mid-term test, the student is able to reduce his uncertainty about his true skill level. This result helps to update his learning strategy given by q. As in the previous section, we assume that σ vanishes in order to simplify the calculations.

A student writing the mid-term test at time $\tau = \frac{T}{2}$ expects the mark

$$X_{\text{pre}}(\tau) = \xi_{\text{pre}}(T) \exp\left(-e \int_\tau^T q_{\text{pre}}^2 (1 - q_{\text{pre}}) dt\right)$$

with $q_{\text{pre}}(t)$ being the solution of Eq. (2.7). Here, $q_{\text{pre}}(t)$ describes the optimal learning behavior based on the assumption that $\xi_{\text{pre}}(T)$ is the skill level at the date of the exam. Despite this computation, the real mark is given by

$$X_{\text{real}}(\tau) = \xi_0 \exp\left(e \int_0^\tau q_{\text{pre}}^2 (1 - q_{\text{pre}}) dt\right) + \epsilon_\tau$$

for some $\epsilon_t \in [\underline{\epsilon}, \overline{\epsilon}]$. For simplicity, we assume that the range of uncertainty ϵ_τ for the mid-term test has the same size as the range of the uncertainty of the final exam. Note that the real mark might not coincide with its prediction $X_{\text{pre}}(\tau)$.

In order to optimize his learning behavior, the student compares the predicted and the observed value. First, he knows that the result of the test does not exactly describe his actual skill. Nevertheless, it holds

$$\xi(\tau) \in A := [X_{\text{real}}(\tau) - \overline{\epsilon}, X_{\text{real}}(\tau) + \underline{\epsilon}].$$

Second, the student is aware of the misjudgment of his skill level at the date of the exam $\xi_{\text{pre}}(T)$ according to (2.8). Therefore, he expects

$$\xi(\tau) \in B := [(1-p)X_{\text{pre}}(\tau), (1+p)X_{\text{pre}}(\tau)]$$

for some given $p \in (0,1)$, where $X_{\text{pre}}(\tau) = \xi_{\text{pre}}(T) \exp\left(-e \int_\tau^T q^2(1-q)dt\right)$.

The student is now able to improve and update his current learning strategy using the test result. On the one hand, the test result may serve as a signal changing his learning strategy. On the other hand, it may confirm the student's expectation. In order to analyze these situations, we have to distinguish four cases:

1. $A \cap B = \emptyset$. In this case the ranges of the real and the predicted skill level do not intersect, so that the student is over- or under-confident. Therefore, the student becomes aware of his over- or under-confidence after the test, which yields a strong signal.

2. $A \subseteq B$. Here, the range of the predicted ability is large, whereas the range of the real value is small. Therefore, the student is not able to calibrate his skill level well.

3. $A \nsubseteq B$ and $A \nsupseteq B$. Since in this case $A \cap B$ represents a real subinterval of A and of B, a strong signaling effect is the result. On the one hand, this situation occurs in the case of an overconfident student who scores badly in the test and

on the other hand, in the case of an under-confident student who scores better than predicted. In both cases the student discovers his flawed self-assessment.

However, note that this case may also include an over- or under-confident student, who does not discover his flawed self-assessment. If the predicted value falls within the range of the real value, it indicates that the range of the predicted ability is small, whereas the range of the real value is large. Depending on the range of the uncertainty parameter ϵ, the student might not find out his misjudgment because of being lucky or unlucky during the mid-term test.

4. $A \supseteq B$. In this case, the test result shows a high fluctuation and thus gives no specific information to update students self-assessment.

   As already described in the third case, this case may also include an over- or under-confident student, who does not discover his flawed self-assessment.

   Nevertheless, this case is more or less already taken into account in Chapter 1 since the prediction about the individual skill level is less uncertain than the outcome of the exam in consideration.

With the aim to optimize future learning behavior, the student updates his prediction of $\xi(\tau)$ from $\xi_{\text{pre}}(\tau)$ to a new

$$\xi_{\text{new}}(\tau) \in I := \begin{cases} A \cap B & \text{if } A \cap B \neq 0, \\ A & \text{else.} \end{cases}$$

Without knowing the distribution of uncertainty, we assume that a student aims to keep the percentage deviation p of skill level $\xi(T)$ at the date of the final exam as low as possible.

Therefore, we compute p_{new} and $\xi_{\text{new}}(\tau)$ using the following system of equations:

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 - p_{\text{new}} \\ 1 + p_{\text{new}} \end{pmatrix} \xi_{\text{new}}(\tau), \tag{2.10}$$

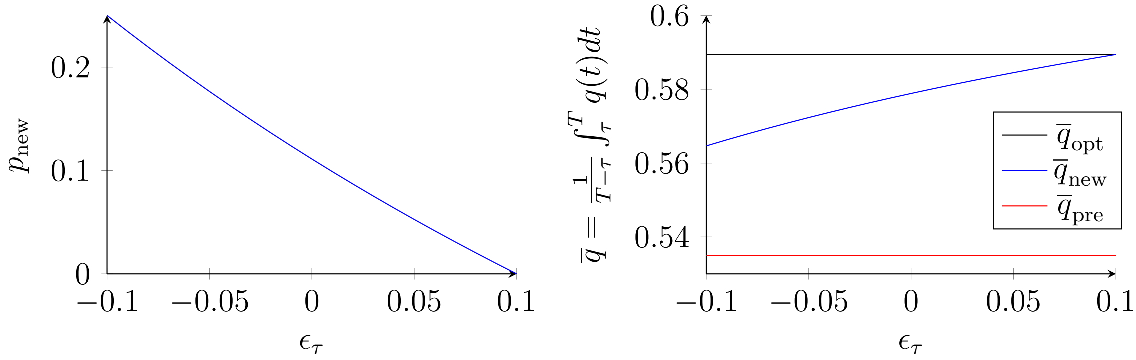

Figure 2.1: Graph of p_{new} and $\overline{q}$ as a function of $\epsilon_\tau \in [-\underline{\epsilon}, \overline{\epsilon}]$ for $T - \tau = 2$, $e = 1$, $\delta = \frac{1}{2}\log 2$, $\gamma = \frac{1}{8}e^{-\delta T}$, $\xi(\tau) = \frac{1}{2}$, $\underline{\epsilon} = \overline{\epsilon} = 0.1$ and $X_{\text{pre}}(\tau) = \frac{1}{3}$, $p = \frac{1}{2}$.

where a, b are given by $I = [a, b]$. In particular, we have

$$(1 + p_{\text{new}})a = (1 - p_{\text{new}})b \quad \Leftrightarrow \quad p_{\text{new}} = \frac{b-a}{a+b}.$$

Example 6 ($A \subseteq B$)**.** Assume that $\underline{\epsilon} = \overline{\epsilon} := \epsilon_0$ for $\epsilon_0 > 0$ as well as $I = [a, b]$ for $a = X_{\text{real}}(\tau) - \epsilon_0$ and $b = X_{\text{real}}(\tau) + \epsilon_0$. Thus,

$$p_{\text{new}} = \frac{\epsilon_0}{X_{\text{real}}(\tau)} \quad \text{and} \quad \xi_{\text{new}}(\tau) = X_{\text{real}}(\tau).$$

Having changed his prediction of the skill level at time $\tau = \frac{1}{2}$, the student updates his learning strategy q_{pre} after the mid-term test to q_{new}, implicitly given by

$$eq_{\text{new}}(t)(2 - 3q_{\text{new}}(t))\xi_{\text{new}}(\tau)\exp\left(\int_\tau^T q_{\text{new}}^2(1 - q_{\text{new}})ds - \delta T\right) = \gamma e^{-\delta t}$$

for $\tau < t \leq T$.

Figure 2.1 describes an under-confident student, since his predicted skill level of $X_{\text{pre}}(\tau) = \frac{1}{3}$ is less than his real skill level of $\xi(\tau) = \frac{1}{2}$. In addition, his expected range is large with $p = \frac{1}{2}$.

The left hand side of this figure shows the influence of the uncertainty parameter

ϵ_τ on the size of his new range with p_{new}. Participating in the mid-term test, the student is able to reduce p_{new}, therefore to narrow down the range of his predicted skill level. Furthermore, p_{new} decreases for higher values of ϵ_τ, representing a more successfull mid-term test. In this case, the result of the mid-term test is far away from the predicted value and thus, has a high signaling effect for the student.

On the right hand side we see the optimal, the predicted and the new average learning time as a function of the uncertainty parameter ϵ_τ. The predicted average learning time is lower than the optimal average learning time, because before participating in the mid-term test the student thinks that his ability is low and thus, he learns less. If he succeeds in the mid-term test - which is more probable for high values of ϵ_τ - his new average learning time approaches the optimal average learning time because he discovers that his ability is higher than expected. In contrast to this, he feels confirmed about his predicted low ability if he has a bad result in the mid-term test and therefore his new average learning time remains low.

Our first expectation was that students who participate in the mid-term test update their learning activity according to the gained knowledge about their true ability. More precisely, students who discover that they are under-confident reduce their learning activity because their ability is higher then expected. Besides, students who discover that they are overconfident increase their learning activity since their skills are lower than expected.

In contrast to this, utility maximization shows that overconfident students spend substantial time on learning before the mid-term test, but reduce their learning activity when they discover their overconfidence. In contrast to this, under-confident students learn less before the mid-term test, but increase their learning time when they discover their under-confidence.

This phenomenon may be explained by motivation as a key determinant of the individual commitment to learning and endurance in learning activity (Klein et al. 2006). Thereby, intrinsic motivation is understood as the engagement in an activity for pleasure without being reinforced (Davis, Bagozzi and Warshaw 1992). Chan and Ahern (1999) show that intrinsically motivated students are willing to learn more and achieve better results.

In contrast to this, perceived barriers to learn affect motivation and therefore, commitment to learning. If students are frustrated, they reduce their learning activity because they do not believe that additional workload will be worth the effort. Although these barriers to learn may be perceived, they directly influence motivation and they negatively affect learning performance and learning outcomes (Mathieu et al. 1992).

Therefore, high-ability students who enjoy learning are motivated to learn more, although they do not have to do so because of their high real or perceived skill level. On the other hand, low-ability students who should learn more because of their low skill level, learn less, because of a barrier to learn or demotivation.

In the following, we derive hypotheses for further empirical research, since our research focuses on a theoretical analysis. This empirical future research may confirm our theoretically derived results. The courses of the Chair of Microeconomics at the University of Duisburg-Essen use online learning tools in addition to traditional face-to-face sessions. In this connection, the chair has selected a lot of data about the students' learning behavior and their use of online learning tools. This data can be consulted in order to test the hypotheses of this chapter.

The results of this section can be summarized by the following two hypotheses:

Hypothesis 1. If an average student discovers his under-confidence, then he will increase his learning time, as long as he is not among the best students.

Hypothesis 2. If an average student discovers his overconfidence, he will reduce his learning activity or drop out.

2.3.6 Low-ability student

Most of the students who we call average students want to pass the exam with certainty. Nevertheless, there exist low-ability students with different learning strategies. They are characterized by a low ability and thus, they have lower skills and are less likely to pass the exam even if they learn with maximal learning intensity. Throughout this section we assume that $\sigma = 0$. For a not vanishing σ we need to replace every strict inequality by a condition involving the probability function. However, we leave this to the reader and use the simplified model without σ.

Let $\xi(T)$ denote the skill level at the date of the exam T. The result of the exam is given by

$$X_{\text{real}}(T) = \begin{cases} 0 & \text{if } \xi + \epsilon \leq \frac{1}{2}, \\ \xi(T) + \epsilon & \text{if } \frac{1}{2} \leq \xi(T) + \epsilon \leq 1, \\ 1 & \text{else,} \end{cases}$$

where ϵ is a parameter representing the uncertainty of the result. We assume that $\epsilon \in [-\underline{\epsilon}, \overline{\epsilon}]$ is uniformly distributed.

Definition 7. We call a student low-ability if his learning strategy q given by (2.5) leads to a skill level $\xi(T) = \xi_0 \exp \int_0^T q^2(1-q)dt$ which does not guarantee the passing of the exam, i.e. $\xi(T) - \underline{\epsilon} < \frac{1}{2}$.

Let us consider a low-ability student who wants to pass the exam in any case. He chooses a learning strategy q such that - based on his self-perception - he will just pass the exam in the worst case scenario, i.e.

$$\xi_{\text{pre}}(0) \exp\left(e \int_0^T q^2(1-q)dt\right)(1-p) - \underline{\epsilon} = \frac{1}{2}. \tag{2.11}$$

This assumption is motivated by Chapter 1 providing examples, where the utility of a low-ability student is optimised for a learning strategy, almost fulfilling the condition above.

Example 8. If the student learns constantly, his learning strategy is given by

$$q(t) := q_{\text{pre}} \quad \text{solving} \quad \xi_{\text{pre}}(0) \exp\left(eTq_{\text{pre}}^2(1-q_{\text{pre}})\right) = \frac{2\underline{\epsilon}+1}{2(1-p)}.$$

Clearly, the learning strategy has to be monotone, increasing in p. Thus, for smaller p, the student needs to learn less in order to be certain about passing the exam. However, for $\xi_0 < \xi_{\text{pre}}(0)(1-p)$ implying that the student is overconfident, the probability for the student to fail the exam is positive. In this case, we define $[\epsilon_0, \overline{\epsilon}]$ as the minimal interval for the student to pass the exam having $\epsilon \in [\epsilon_0, \overline{\epsilon}]$. Thus, the probability for the student to fail the exam is given by

$$P := \frac{\epsilon_0 + \underline{\epsilon}}{\overline{\epsilon} + \underline{\epsilon}} \quad \text{for} \quad -\underline{\epsilon} < \epsilon_0 < \overline{\epsilon}.$$

Note that in fact ϵ_0 depends on $\xi_{\text{pre}}(0)$ and p. More precisely, we observe that

$$\epsilon_0 = \frac{1}{2} - \xi_0 \exp\left(e \int_0^T q^2(1-q)dt\right),$$

where q solves (2.11), entailing

$$\epsilon_0 = \frac{1}{2}\left(1 - \frac{\xi_0}{\xi_{\text{pre}}(0)} \frac{2\underline{\epsilon}+1}{(1-p)}\right).$$

Mid-term test to discover individual ability

Introducing a mid-term test as in Definition 5 allows the student to gather more precise information about his current skill level. Since the student has low skills, we suppose that his test result is below its maximum. We have already seen that an overconfident student may be confronted with his overconfidence and an under-confident student with his under-confidence after the mid-term test. In this case, he is able to update his learning behavior in order to pass the exam. As before, let $X_{\text{real}}(\tau)$ be the result of the test, where the skill level lies within a range of $\epsilon_\tau \in [-\underline{\epsilon}, \overline{\epsilon}]$. We assume that the student overestimates his skill level, i.e. $\xi(\tau) < \xi_{\text{pre}}(\tau)(1-p)$. Thus, he realizes his overconfidence if his test result is worse than predicted even in the most optimistic view:

$$X_{\text{real}}(\tau) + \underline{\epsilon} < (1-p)X_{\text{pre}}(\tau).$$

The probability that the student faces his own overconfidence with no doubt is therefore given by $\frac{\epsilon_1+\underline{\epsilon}}{\overline{\epsilon}+\underline{\epsilon}}$ with

$$\xi(\tau) + \epsilon_1 + \underline{\epsilon} = (1-p)X_{\text{pre}}(\tau).$$

Consequently, a mid-term test helps the low-ability student to update his true skill level and to reflect his learning strategy. The knowledge about his current skills has two effects. On one hand, his skills are lower than predicted and thus his learning productivity is lower. This leads to decreasing incentives. If, on the other hand, this leads to a positive probability of failing the test, the incentives could either increase or lead to a complete dropout.

Finally, the probability to fail the exam is calculated by

$$\begin{aligned} P_{\text{new}} &:= \frac{(\epsilon_0 + \underline{\epsilon})(\overline{\epsilon} - \epsilon_1)}{(\overline{\epsilon} + \underline{\epsilon})^2} \\ &= \frac{1+2\underline{\epsilon}}{2(\overline{\epsilon}+\underline{\epsilon})^2}\left(1 - \frac{\xi_0}{\xi_{\text{pre}}(0)}\frac{1}{(1-p)}\right)^2 (1-p)\xi_{\text{pre}}(\tau) \end{aligned}$$

whenever it belongs to $[0,1]$, where

$$(1-p)\xi_{\text{pre}}(\tau) = \frac{1+2\underline{\epsilon}}{2}\exp\left(-e\int_\tau^T q^2(1-q)dt\right) \leq \frac{1+2\underline{\epsilon}}{2}.$$

according to (2.11). Comparing this to the case without mid-term test shows

$$P_{\text{new}} = P^2 \exp\left(-e\int_\tau^T q^2(1-q)dt\right) \leq P^2$$

since $P = \frac{1+2\underline{\epsilon}}{2(\overline{\epsilon}+\underline{\epsilon})}\left(1 - \frac{\xi_0}{\xi_{\text{pre}}(0)}\frac{2\underline{\epsilon}+1}{(1-p)}\right)$.

This comparison emphasizes the reduction of the probability to fail the exam as a result of participating in the mid-term test. If we assume an initial probability to fail the exam of 50%, taking the mid-term test decreases this probability to less than 25%. Consequently, a mid-term test is able to reduce the failure rate.

Dropout rate

Since low-ability students face a risk of failing the exam, dropping out of the exam can be a better choice than learning in vain. In the following we will have a closer look at the trade-off between learning and giving up.

For this, let us consider a low-ability student who is highly overconfident. Thus, his real skill level is quite low compared to his individually estimated skill level:

$$\xi_0 \ll \xi_{pre}(0).$$

Thereby, $\xi_{pre}(0)$ is his estimated skill level at $t = 0$. We consider a student who, according to his prediction, chooses a learning strategy with the aim to pass the exam in the worst case scenario. However, he learns too little since he is highly overconfident. Therefore, it is possible that the student has already no chance to pass the final exam with certainty at the date of the mid-term test even if he continues to learn at maximal intensity until the end of the semester. In this case, the optimal strategy for the student might be, to stop learning and drop out of the course. Here, the mid-term test may serve as a signal for the student to give up.

Let $X_{\text{pre}}(\tau)$ denote the predicted mid-term test result and $p > 0$ such that the student assumes that his true skill level at time τ lies within the range $[(1-p)X_{\text{pre}}(\tau), (1+p)X_{\text{pre}}(\tau)]$. The student therefore chooses the learning strategy q_{pre} so that his learning progress satisfies

$$\frac{\xi(T)}{\xi(\tau)} = \frac{0.5 + \underline{\epsilon}}{(1-p)X_{\text{pre}}(\tau)}.$$

Note that here $\xi(T)$ depends on his actual learning strategy $q = q_{\text{pre}}$. However, the actual skill level of the student at time τ is given by

$$\xi(\tau) = \xi_0 \exp\left(e \int_0^\tau q_{\text{pre}}(t)^2(1 - q_{\text{pre}}(t))dt\right), \tag{2.12}$$

which leads him unavoidably to fail the exam if

$$\xi_{\max}(T, \overline{\xi}) = \overline{\xi} \exp\left(\frac{4}{27} e(T - \tau)\right) < \frac{1}{2} - \overline{\epsilon}. \tag{2.13}$$

for $\overline{\xi} = \xi(\tau)$. Here, $\xi_{\max}(T, \overline{\xi})$ denotes the maximal possible skill level at the time of the exam under the condition that the student's skill level at time τ is given by $\overline{\xi}$. It can be computed using the maximal learning strategy $q = \frac{2}{3}$. This highly overconfident student discovers his inability to pass the exam after writing the mid-term test if the condition in (2.13) is fulfilled for every

$$\overline{\xi} \in [X_{\text{real}}(\tau) - \overline{\epsilon}, X_{\text{real}}(\tau) + \underline{\epsilon}].$$

Thus, the probability that this student learns about his hopeless situation can be computed by

$$P = \max\left\{\frac{\epsilon_0 + \underline{\epsilon}}{\underline{\epsilon} + \overline{\epsilon}}, 1\right\}$$

for ϵ_0 given by

$$(\xi(\tau) + \epsilon_0 + \underline{\epsilon}) \exp\left(\frac{4}{27} e(T - \tau)\right) = \frac{1}{2} - \overline{\epsilon}.$$

where $\xi(\tau)$ is given by (2.12). Consequently, the highly overconfident student gives up and stops learning if this case occurs.

Within this section dealing with the decision-making of low-ability students, four hypotheses summarize the main findings:

Hypothesis 3. If a low-ability student passes the mid-term test with an average result, he will continue to learn unchanged.

Hypothesis 4. If a low-ability student passes the mid-term test with a better result,

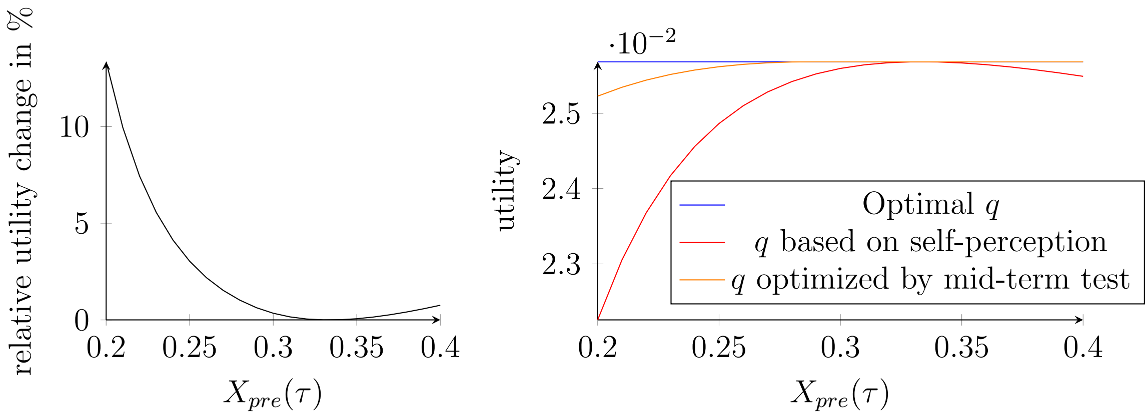

Figure 2.2: Graph of the expected utility as a function of $X_{pre}(\tau) \in [0.2, 0.4]$ in the case of ungraded exams. The figure shows the optimal case that the mid-term test result coincided with the actual skill level with parameters given by $T - \tau = 2$, $e = 1$, $\delta = \frac{1}{2}\log 2$, $\gamma = 1/6e^{-\delta T}$, $\xi(\tau) = \frac{1}{3}$, $\underline{\epsilon} = \overline{\epsilon} = 0.1$, $p = \frac{1}{2}$ and $\epsilon_\tau = 0$.

he will increase learning activity.

Hypothesis 5. If a low-ability student performs badly in the mid-term test, he will learn more if and only if he has to recover to pass the exam.

Hypothesis 6. If a low-ability student who is highly overconfident performs very badly in the mid-term test, he will give up and stop learning.

2.3.7 Ungraded exams

Ungraded exams can be found in many real-world settings, like in teacher-training courses. Therefore, students face two possible outcomes: passing or failing the exam. In order to analyze the impact on utility maximization for this case, we introduce a different model of examination where final exams are not graded. As before, mid-term tests remain graded. Thus, we assume that the results are given by

$$X(T) = \begin{cases} 1 & \text{if } \xi(T) + \epsilon \geq \frac{1}{2}, \\ 0 & \text{else.} \end{cases}$$

Similarly as above, $\epsilon \in [-\underline{\epsilon}, \overline{\epsilon}]$ is a parameter modeling unpredictable influences, which is assumed to be uniformly distributed. The utility function is given as in (2.2). Without knowing the parameter ϵ exactly, the student is rather interested in the expected utility. For $-\underline{\epsilon} \leq \xi(T) - \frac{1}{2} \leq \overline{\epsilon}$, it is approximately given by

$$\mathbb{E}u = e^{-\delta T} \frac{2\mathbb{E}\xi(T) - 1 + 2\underline{\epsilon}}{2(\overline{\epsilon} + \underline{\epsilon})} - \gamma \int_0^T q e^{-\delta t} dt.$$

In case of an ungraded exam, we assume that the range $[-\underline{\epsilon}, \overline{\epsilon}]$ is rather big such that $-\underline{\epsilon} \leq \xi(T) - \frac{1}{2} \leq \overline{\epsilon}$ is probably satisfied for average students. Thus, throughout this section, we will assume that the probability for the counter case is neglectable.

Likewise to Section 2.3.3, we suppose that the student has an imperfect knowledge about his true skill level and is only able to predict it within the range $[(1-p)\xi_{\text{pre}}(0), (1+p)\xi_{\text{pre}}(0)]$. Observing that the expected utility function is an affine transformation of the one in Section 2.3.3, we can similarly compute the student's optimal learning strategy.

Moreover, introducing a mid-term test at time $\tau = \frac{T}{2}$, the student is able to adapt his learning strategy to q_{new} being the solution of

$$q_{\text{new}}(t)(2 - 3q_{\text{new}}(t)) \frac{\xi_{\text{new}}(\tau)}{\overline{\epsilon} + \underline{\epsilon}} \exp\left(\int_\tau^T q_{\text{new}}^2 (1 - q_{\text{new}}) ds - \delta T \right) = \gamma e^{-\delta t} \quad \text{for } \tau < t \leq T,$$

where $\xi_{\text{new}}(\tau)$ is given by (2.10). Finally, we are able to derive the students' new expected utility by

$$\begin{aligned} e^{\delta T} \mathbb{E}u_{\text{new}} = & \frac{\xi_0}{\overline{\epsilon} + \underline{\epsilon}} \exp\left(e \int_0^\tau q_{\text{pre}}^2 (1 - q_{\text{pre}}) dt + e \int_\tau^T q_{\text{new}}^2 (1 - q_{\text{new}}) dt \right) \\ & + \frac{2\underline{\epsilon} - 1}{2(\overline{\epsilon} + \underline{\epsilon})} - \gamma \left(\int_0^\tau e^{-\delta t} q_{\text{pre}} dt + \int_\tau^T e^{-\delta t} q_{\text{new}} dt \right). \end{aligned}$$

In order to find out whether the new strategy dominates the previous one, we need

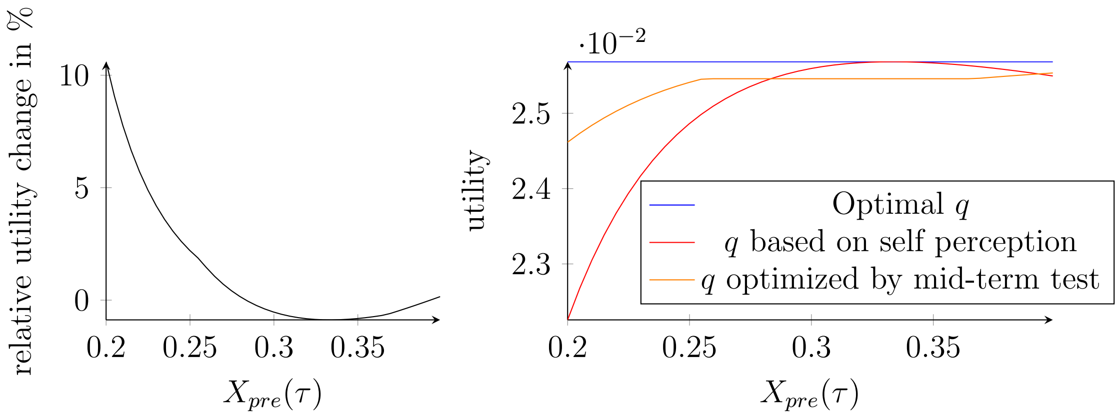

Figure 2.3: Graph of the expected utility as a function of $X_{pre}(\tau) \in [0.2, 0.4]$ in the case of ungraded exams. The figure shows the optimal case that the mid-term test is worse than the actual skill level with $X_{\text{real}}(\tau) = \xi(\tau) - 0.05$. The parameters used for this plot are $T - \tau = 2$, $e = 1$, $\delta = \frac{1}{2}\log 2$, $\gamma = 1/12$, $\xi(\tau) = \frac{1}{3}$, $\underline{\epsilon} = \overline{\epsilon} = 0.1$, $p = \frac{1}{2}$

to investigate the relative change in expected utility

$$\frac{\Delta \mathbb{E}u}{\mathbb{E}u} := \frac{\mathbb{E}u_{\text{new}} - \mathbb{E}u}{\mathbb{E}u}.$$

On the left-hand side of Figure 2.2 we see the relative utility change as a function of the predicted skill level $X_{pre}(\tau)$. Note that this figure assumes $\epsilon_\tau = 0$. Consequently, the mid-term test reveals the true skill level without uncertainty. The relative utility change decreases if the predicted skill level is near the real skill level. Therefore, utility decreases if the predicted skill level is far away from its real value.

The right-hand side of Figure 2.2 shows utility if learning time is based on the condition that it is optimal, that it is based on the self-assessment and that it is updated after the mid-term test. Utility increases for higher values of the predicted skill level. Here, utility is on its lowest level if learning time is based on a low predicted skill level. Note that learning time is optimal if the predicted skill level is equal to the real skill level. Assuming a flawed self-assessment, the student is able to increase his utility if he updates his learning activity according to the results of the mid-term test. Consequently, taking the exam, the student is able to increase his utility by updating his learning strategy. In contrast to this, utility stays on a low level if the student does not take the test and maintains his low learning time.

In contrast to Figure 2.2, Figure 2.3 assumes $\epsilon_\tau > 0$. Therefore, the mid-term test does not always reveal the real skill level. Not surprisingly, utility in the case of the updated learning time after the mid-term test only approaches the utility maximum, since the updated skill level is based on the uncertainty of the test situation. Still, a student who takes the mid-term test and updates his learning activity is mostly able to increase his utility. However, assuming the student's self-perception which reflects his actual skill level, i.e. $X_{\mathrm{pre}}(\tau) = \xi(\tau)$, the feedback of the mid-term test can lead to a less optimal learning behavior and decrease the student's utility if the test result is better or worse than the actual skill level. This is possible since the test result also depends on the concentration and luck-level. Nevertheless, this is only a minor case and the relative utility change is rather small see Figure 2.3.

In conclusion, the analysis of the utility with and without mid-term test emphasizes the positive impact of feedback. In most cases, students are able to increase their utility if they update their learning activity according to the gained information about their real skill level.

2.3.8 Outlook

Among all results we have derived six hypotheses which may be tested empirically. Since our research focuses on a theoretical analysis, further empirical research may follow.

Without knowing the student's predicted skill level, it is not possible to make statements about his under- or overconfidence. The introduction of a placement examination at the beginning of the semester may serve as a measure to receive information about the individual skill level. Based on the result of this placement examination, students are able to build their self-assessment. In addition, the result of the placement examination categorizes students into low-ability and average students. Since Hypothesis 1 and Hypothesis 2 deal with average students and Hypothesis 3, Hypothesis 4, Hypothesis 5 and Hypothesis 6 deal with a low-ability students, this distinction is necessary. Participating in the mid-term test, students have the opportunity to discover their under- or overconfidence. The learning activity before and after the mid-term test can be measured by considering the time spent on solving online exercises.

Hypothesis 1 says that an average student who discovers his under-confidence will increase his learning activity. It is possible to test this hypothesis by looking at a student whose mid-term test result is better than the result of his placement examination. In this case he should increase his learning activity because of the gained motivation.

Hypothesis 2 implies that an average student who discovers his overconfidence, will reduce his learning time. This hypothesis can be confirmed if the student performs badly in the mid-term test, although he expected a higher skill level according to the placement examination. Furthermore, after discovering his overconfidence he would feel frustrated and because of that reduce his learning activity .

Hypothesis 3 implies that a low-ability student will continue to learn if the result of his mid-term test is average. This hypothesis may be tested by looking at the learning activity before and after the mid-term test of a low-ability student with an average result.

Hypothesis 4 implies that a low-ability student will increase his learning activity if the result of his mid-term test is better. This hypothesis may be tested by looking at the learning activity before and after the mid-term test of a low-ability student with a better result.

Hypothesis 5 says that a low-ability student will learn more if the result of his mid-term test is bad. This hypothesis can be proved by observing the learning activity before and after the mid-term test of a low-ability student with a bad result.

Finally, Hypothesis 6 implies that a low-ability student who is highly overconfident and performs very badly in the mid-term test will give up and stop learning. It is possible to prove this hypothesis by showing that a low-ability student with a bad mid-term test result does not take the exam or stops solving online exercises.

2.4 Conclusion

This paper has extended the simple model of learning by doing in the context of university courses (see Chapter 1) by introducing the role of feedback. Exam outcomes are the result of accumulated skills through learning during the semester. Thereby, the offered blended learning opportunities enable a flexible and around-the-clock learning activity. Nevertheless, learning time is limited due to the trade-off between leisure and learning time. We introduce feedback with the aim to analyze its impact on students' flawed self-assessment. In this context, online exercises and mid-term tests serve as sources of feedback to check the individual skill level. Students who use blended learning are able to compare their expected skill level to the received feedback and may, based on this comparison, update their learning behavior.

Considering feedback on solving online exercises, we show that a student is able to determine his growth rate of skills if he constantly makes use of this source of feedback. Although feedback on online exercises rather offers hints about the learning improvement than on the actual skill level, it connects performance of online exercises to the learning progress. Thus, it has a huge influence on the utility function.

Assuming a mid-term test as source of feedback, students who take the mid-term test have the opportunity to compare their predicted to their real skill level. Nevertheless, the test does not exactly describe the current skill level because of unpredictable influences like luck and concentration. Furthermore, the student assumes that his predicted skill level falls within a certain range because he is not able to estimate it precisely. Therefore, the student is overconfident if his true skill level is smaller than the lower bound of this range and under-confident if his true skill level is higher than the upper bound. Otherwise, the student has a well-calibrated self-assessment. Making use of the gained knowledge about his true skill level, the student is able to update his learning strategy. However, depending on the range of his predicted skill level, it is also possible that an over- or under-confident student does not discover his flawed self-assessment. Furthermore, we show that high-ability students who enjoy learning, are motivated to learn more and low-ability students who should learn more learn less. This result may be explained by intrinsic motivation which influences the commitment to learn. While intrinsically motivated students are willing to learn more, students who are frustrated or perceive barriers reduce learning performance independent of the result of the mid-term test.

Furthermore, we analyze the decision-making of a low-ability student whose skill level does not guarantee to pass the exam. We are able to show that a mid-term test helps the low-ability student to discover his true skill level and, based on this information, to update his learning behavior by adjusting it upwards. This knowledge increases his probability to pass the exam. Nevertheless, we also show that students who perform badly in the mid-term test stop learning and do not take the exam. Hence, low-ability students face a high risk of perceiving barriers to learning and give up as a consequence.

In addition to improving the self-assessment of skills and reducing the failure rate, feedback also has a positive influence on utility. If students take the mid-term test and update their learning time according to the gained information about their real skill level, they mostly increase their utility.

Concluding, we are able to show that online exercises and mid-term tests as sources of feedback are able to improve flawed self-assessment, learning performance and to reduce the failure-rate if students make use of them on a regular basis.

Chapter 3

Blended learning as response to student heterogeneity

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Abstract

These days the student body is characterized by a considerable heterogeneity. Thus, higher education has to cope with new demands concerning heterogeneous students. This paper analyzes dimensions of student heterogeneity and corresponding measures in order to respond to these changes. Furthermore, the higher education system comprising bachelor's and master's degrees is analyzed by comparing the signaling power of different degrees by using Spence's (1973) signaling model. In contrast to prior one-signal-degrees, this more differentiating system enables students to choose different levels of higher education degrees and thus takes into consideration student heterogeneity. Nevertheless, entering higher education does not always ensure a successful completion. Dropout rates in higher education may be the result of wrong self-assessment, since signaling requires the knowledge of one's individual productivity and capabilities.

In order to implement interventions in the daily teaching routine, JACK as an example of blended learning is presented as a response to faulty self-assessment and the increasing heterogeneity of students. On the one hand, the implementation of JACK in university courses meets the growing demands of a heterogeneous student body through guiding self-study and through flexibility of time and space. On the other hand, JACK helps to correct flawed self-assessment with the aid of immediate feedback and opportunities to repeat course contents.

Consequently, this paper analyzes dimensions of student heterogeneity and, based on this analysis, proposes blended learning as a successful measure to adapt teaching routines.

3.1 Introduction

These days, higher education is characterized by an increasing heterogeneity of students (Hanft 2015). Besides traditional aspects of heterogeneity like skills, professional experiences, social and cultural background, diversity has further dimensions. New sources of heterogeneity are represented by a broader age span and by differences in organizing self-study among students (Schulmeister et al. 2012). The homogeneous university structures, teaching methods and courses with a huge number of students are confronted with these heterogeneous student characteristics. Thus, higher education has to cope with new demands caused by student heterogeneity. Until now, initiated actions like receiving inspections and preparatory courses concentrate mainly on the orientation phase of and access to higher education studies and do not take teaching routines into account (Hanft 2015). Instead of transforming heterogeneity into homogeneity, higher education might be forced to adapt course structures in order to accept and accommodate heterogeneous students (Biggs and Tang 2011; Wildt 2001).

Within the framework of the Bologna process, the higher education system has been restructured to bachelor's and master's degrees (EACEA 2012). In contrast to prior degrees like the Diplom in Germany, this highly differentiating system enables students to choose different levels of higher education degrees according to their cognitive abilities and commitment level and therefore has to cope with a higher heterogeneity within bachelor's degrees. In addition, the Bologna process aims to harmonize tertiary education among European countries. Under this common framework, bachelor's and master's degrees have been standardized to improve the equivalence between similar programs and to simplify students' international mobility.

In this connection, the completed degree serves as a signal for the employer, which reduces his uncertainty concerning the productivity of the graduate (Spence 1973). In order to compare the signaling power of different degrees, I compare a one-signal-degree and the bachelor's and master's degree as a two-signal-degree by using Spence's (1973) signaling model.

Spence's signaling model assumes students to be perfectly informed about their skills, but entering higher education does not always ensure a successful graduation in reality. Although the choice of bachelor's and master's degrees gives individuals differentiated possibilities of education, many students drop out. According to the results of the research of "OECD-Education at glance" (OECD 2013) and "Bologna Process Implementation Report" (EACEA 2012), only 70% of students who enter higher education graduate among OECD countries. This dropout rate in higher education may be the result of faulty self-assessment, since signaling requires the knowledge of one's individual productivity.

With the aim of accepting student heterogeneity and improving the flawed self-assessment, higher education should give more attention to the adaptation of course structures instead of purely concentrating on the selection process before the start of higher education studies. In order to implement actions in the daily teaching routine, I propose blended learning as a response to the frequently imperfect self-assessment and increasing heterogeneity of students.

Blended learning combines traditional face-to-face sessions with online learning opportunities which are available 24 hours per day and 7 days per week. E-learning exercises take individual preferences and living conditions into account, since they are characterized by a high flexibility of time and space. In addition, they help to organize the self-study and improve the retention of acquired course contents. The immediate feedback helps to validate acquired skills and to recognize one's progress in learning. Beside these advantages of blended learning, the current learning environment shows a trend towards online learning. Many surveys confirm this trend by emphasizing that online learning tools have significantly increased in students' learning experiences (Allen and Seaman 2006) and that students show a new attitude towards online learning (Sebastianelli and Tamimi 2011).

Thus, this paper analyzes dimensions of student heterogeneity, copes with the more heterogeneous educational offers and shows how differentiated and more flexible learning arrangements pay attention to these dimensions and improves a potentially flawed self-assessment.

3.2 Student heterogeneity

3.2.1 Dimensions of student heterogeneity

Nowadays, higher education is characterized by an increasing heterogeneity among students. This growing heterogeneity has multiple reasons. The first reason is caused by the increasing number of first-year students. Over the past 16 years, the number of graduates has increased by nearly 20 percent on average across OECD countries (OECD 2013). Furthermore, more than 50 percent of each age group start to study (OECD 2011). Looking at Germany, the number of first-year students doubled since the 80s and 90s, having arrived at a higher overall count than beginners of apprenticeships (Hanft 2015).

Secondly, universities are open to all social classes which had no opportunity to study in former times. Thus, in these days higher education studies are not reserved to the elites anymore (Middendorf 2015). Beyond this, our society is characterized by a social change with a multitude of background conditions to grow up with. Consequently, the student body shows an increasing age span and differences in regard to previous experiences, skills, motivation, goals and social background (Middendorf 2015). In this context, some policy changes in Germany lead to younger first-year students, while changing living conditions lead to older students. These are reasons for the described age span. The educational reform G8, for instance, shortened the overall duration of compulsory schooling by one year in Germany. In addition, conscription has been discontinued, i.e. young men are not obligated to compulsory military service anymore and thus begin to study earlier. Moreover, many students are gainfully employed during their studies or work before or between their bachelor's and master's degree (Hanft 2015). This additional workload may lead to a postponed graduation and an advanced age.

Alauddin and Ashman (2014) confirm these significant changes in the student body over the last two decades by adding an international dimension. Beside the increased number of students, the share of international students has expanded in particular. This trend is confirmed by the OECD Indicators (2015) which emphasize that international students constitute a significant share of higher education graduates among OECD countries. In more detail, it has been shown that 18% of students who attained a master's degree and 7% who attained a bachelor's degree are international students. The internationalization of higher education entails a highly diverse cultural background with individual differences in the philosophy of studying and learning (Johnson and Kumar 2010). This philosophy is shaped by previous experiences at home like expectations and pressure exerted by the family.

Beside traditional aspects like social background, age, skills and professional experience, diversity has further dimensions which go beyond the traditional ones. Schulmeister et al. (2012) emphasize a different learning behavior among students as new source of heterogeneity. By analyzing how students spend their time on self-study, attending courses and leisure time, they discover a wide range of time allocation leading to different results. Although students face the same homogeneous university structures, every student responds in a different manner of time management. The resulting diverse learning behavior leads to different learning outcomes and university success. Especially the time spent on self-study is characterized by a huge diversity among students. In contrast to this, the participation in courses is almost the same for all students. As a response to the diversity of learning behavior, Schulmeister et al. (2012) demand new teaching methods which support the organization and support of self-study.

As we have seen, student heterogeneity comprises many dimensions which provide higher education with new challenges.

3.2.2 Current measures to respond to student heterogeneity

Until now, initiated actions mainly concentrate on the introductory phase of higher education studies and do not take account of teaching routines (Hanft 2015). Counseling, tutorials and preparatory courses are offered in order to prepare students for their studies and in order to select the appropriate individuals. However, a change of university structures still has not yet been realized. Receiving inspections aim to create homogeneous student groups. All these actions aim to homogenize the student body without changing organization and structures of higher education. Thus, until now homogeneous university structures persist and only the orientation phase attempts to meet the requirements of the growing heterogeneity of students.

In this context, Wildt (2001) distinguishes two dimensions of measures. On the one hand, heterogeneity may be accepted and the universities can respond to the changing requirements. On the other hand, heterogeneity may be transformed into homogeneity. As described above, higher education so far mainly focuses on transforming heterogeneity into homogeneity. However, the first aspect, accepting heterogeneity, shall be given higher priority in the future, if the requirements of society are going to be met. This point of view is also supported by Bigg and Tang (2011), who argue that university teaching should be adapted to the needs of an increasingly heterogeneous student body. Section 3.4 focuses on this gap by adapting course structures and thus, accepting heterogeneity. Before we cope with this aspect, we start with a simple signaling model analysing the changing incentives with more differentiated educational programmes.

3.3 The signaling mechanism in higher education

The choice between differentiated higher education degrees is one possibility to cope with higher heterogeneity of students. Certificates in higher education like the bachelor's and the master's degree do not only represent an investment in education, they also serve as a signal for the employer in case of otherwise asymmetrical information. This section analyzes the signaling mechanism of degrees in higher education and compares their signaling power with respect to student heterogeneity.

3.3.1 Theoretical background

In 1973, Spence introduced the aspect of education as a signaling device on the job market for the first time. Since asymmetrical information prevails on the job market, the employer would like to learn about the productivity of the individual before hiring him. In order to counteract this uncertainty, according to Spence individuals have the opportunity to invest in education involving costs in terms of money and time which vary according to the individual skills. Obtaining a degree serves as an observable signal for the employer if skills and productivity are correlated. Consequently, in this context education can be a signal to reduce the inefficiencies associated with asymmetric information.

Spence (1973) distinguishes two groups of high and less productive individuals who are assumed to know their own skills. Every individual can choose to specifically invest in education. Signaling works, if costs are negatively correlated with individual skills and productivity, since highly productive individuals then have lower costs to reach a specific education level than low-ability individuals.

In equilibrium then a critical level of education y^* exists which distinguishes high and less productive individuals. According to this wage schedule, individuals choose their investment in education with the aim to maximize their utility, given by the difference between wage and signaling-costs. In a signaling equilibrium, high-ability individuals invest the critical level of education y^*, while low-ability individuals do not invest in education.

Thereby, the employer pays a wage according to education level, even if education has no impact on skills and thus on productivity. It is pure self selection that makes differences in wages and job offers. If employers are completely competitive, wages paid in equilibrium equal expected individual productivity.

In this equilibrium, there exists overinvestment in education, even if education has a positive impact on skills and productivity. There are also pooling equilibria without signalling. This is however in general instable if employers have credible beliefs about the equilibrium path. If signaling cost are high and the share of high skill workers is low, the pooling equilibrium dominates the separating equilibrium however.

Spence assumed a continuous spectrum of educational levels. In practice however there are only specific degrees. This might be unable to cope with larger or increasing heterogeneity. With only one specific degree, in Germany the Diplom, full separation would not be possible. Differentiating the degrees by splitting the Diplom into bachelor's and master's degrees allows to further differentiate the different skill level.

The following subsection analyses the different equilibria and asks, which group might be better off after the change and which group might be worse off.

3.3.2 Signals in higher education

Certificates in higher education like the bachelor's, the master's degree and the German Diplom represent an investment in education incurring costs, which allows students to distinguish themselves from less productive individuals by choosing a specific degree. If different talented students behave in a different manner according to their individual ability the chosen degree might serve as a signal for the employer and determine the individual wage.

An investment in higher education bears the possibility to indicate that students are able to learn sizeable quantities over a longer period of time, that they are able to pass exams, that they can work in a team, that they function in a social setting and that they are able to follow a predetermined schedule (Krassen 2014). Consequently, investing in higher education represents the acquisition of skills and the ability to achieve a goal. Obtaining a higher education degree serves as a signal if students reveal their ability depending on their graduation. Although signals give no information about the student's ability to do the job, they show to which extent the student invested effort and costs. Since the labor market is characterized by asymmetric information, costly signals offer a remedy to the resulting inefficiency by resolving asymmetries of information. Nevertheless, there is a risk of over-education which may result in the distortion of the signal and the bumping-down effect on less educated people. If more and more students complete higher education degrees, the pressure to further increase higher education arises. This situation is not desirable because the degree as a signal loses its power. Therefore, higher education should give better opportunities to differentiate.

In this connection, the switch to the system of bachelor's and master's programs offers more differentiating opportunities for students of higher education, in contrast to prior one-signal-degrees like the Diplom in Germany. The resulting offer of different degrees may reduce over-education.

In order to analyze different higher education degrees and its signaling power, I extend Spence's signaling model from 1973 by introducing a third level of productivity. Since nowadays higher education is characterized by a differentiated system of degrees comprising bachelor's and master's degrees, the model needs differentiated equilibrium cut off levels. In order to emphasize the signaling power of the bachelor's and master's degrees in contrast to the signaling power of the Diplom as a one-signal-example, I use a numerical example.

I assume three groups of individuals with different productivity levels θ. Individuals of Group 1 (low ability) have a productivity of $\theta_l = 1$ and signaling costs of $c_l = y$, individuals of Group 2 (mid ability) are characterized by a level of productivity of $\theta_m = 2$ and signaling costs of $c_m = \frac{y}{2}$ and finally, individuals of Group 3 (high ability) show a level of productivity of $\theta_h = 3$ and signaling costs of $c_h = \frac{y}{3}$. Population shares of the three groups are given by: q_l, q_m and $q_h = 1 - q_l - q_m$.

In the case of Bologna including bachelor's and master's degrees, the employer takes two critical levels of education y^B and y^M into consideration which distinguish high-, mid- and low-ability individuals. Consequently, if $y < y^B$ in equilibrium expected productivity equals 1, if $y^B \leq y < y^M$ expected productivity equals 2 and if $y \geq y^M$ expected productivity equals 3. In this case, there are three in equilibrium values of y: $y = 0$, $y = y^B$ and $y = y^M$.

In a signaling equilibrium, individuals of Group 1 set $y = 0$ if

$$1 \geq 2 - y^B.$$

Individuals of Group 2 set $y = y^B$ if

$$1 \leq 2 - \frac{y^B}{2},$$

and do not choose $y = y^M$ if

$$2 - \frac{y^B}{2} \geq 3 - \frac{y^M}{2}.$$

Finally, individuals of Group 3 set $y = y^M$ if

$$2 - \frac{y^B}{3} \leq 3 - \frac{y^M}{3}.$$

The levels of education have to satisfy the following inequality:

$$1 < y^B < 2 < y^M - y^B < 3.$$

For further calculation I focus on the efficient critical levels of education which are given by

$$y^B = 1 \qquad \text{and} \qquad y^M = 3.$$

These education levels do not reflect academic years but required effort and level of difficulty of the degree. In contrast to the bachelor's degree which implies three years of studies and the acquisition of basic knowledge, the master's degree includes two years of additional studies and the learning of more challenging and demanding knowledge. Therefore, completing a master's degree requires significantly more effort.

Consequently, equilibrium utility is given by

$$\begin{aligned} u_l &= 1, \\ u_m &= 2 - \frac{1}{2} = 1.5, \\ u_h &= 3 - \frac{3}{3} = 2. \end{aligned}$$

A signaling equilibrium is characterized by a high heterogeneity of graduates with bachelor's degree and a low heterogeneity among graduates with master's degree. While Group 2 and 3 complete the bachelor's degree, only Group 3 completes the master's degree.

In addition to this signaling equilibrium with two signals, different scenarios are possible in case of a one-signal-degree like the Diplom:

1. Every group signals.

2. Low-ability students do not signal, high- and mid-ability students signal.

3. Low- and mid-ability students do not signal, high-ability students signal.

4. No group signals.

In the first case, every group invests in education and an inefficient pooling equilibrium with $w = q_l + 2q_m + 3(1 - q_l - q_m) = 3 - 2q_l - q_m$ results. The last case gives the same expected wage, in which the low ability workers are best off, since they participate at the high-ability individuals' productivity.

In the second scenario low-ability individuals do not invest in higher education $(y = 0)$ if the productivity-wage $w = 1$ is greater than the mean wage of mid- and high-ability individuals minus education costs according to the following equation:

$$1 \geq \frac{3 - 3q_l - q_m}{1 - q_l} - y.$$

As a consequence, low-ability students earn $w_l = 1$ and mid- and high-ability graduates earn the mean wage $w_{2/3} = \frac{3-3q_l-q_m}{1-q_l}$ and the critical level of education equals $y^* = \frac{2-2q_l-q_m}{1-q_l}$.

In this case, utility is given by

$$u_l = 1,$$
$$u_m = \frac{4 - 4q_l - q_m}{2(1 - q_l)},$$
$$u_h = \frac{7 - 7q_l - 2q_m}{3(1 - q_l)}.$$

Highly productive individuals lose because they earn less, while mid-ability individuals win and make a profit of the mean productivity.

Finally, in the third case I assume that only high-ability students invest in education and complete the Diplom, while low- and mid-ability individuals set $y = 0$. Therefore, the following inequality holds

$$\frac{q_l + 2q_m}{q_l + q_m} \geq 3 - \frac{y}{2}.$$

Consequently, low- and mid-ability students earn the mean wage $w_{l/m} = \frac{q_l+2q_m}{q_l+q_m}$ and high-ability graduates earn $w_h = 3$ and the critical level of education equals $y^* = 6 - \frac{2(q_l+2q_m)}{q_l+q_m} = \frac{2(2q_l+q_m)}{q_l+q_m}$.

Therefore, utility is given by

$$
\begin{aligned}
u_l &= \frac{q_l + 2q_m}{q_l + q_m}, \\
u_m &= \frac{q_l + 2q_m}{q_l + q_m}, \\
u_h &= \frac{5q_l + 7q_m}{3(q_l + q_m)}.
\end{aligned}
$$

In this case high-ability individuals are payed according to their productivity. Low-ability individuals benefit from the mean wage, while mid-ability individuals lose.

Consequently, if I assume three productivity-level and only one signal, not every individual is payed according to his productivity. Every possible situation includes a mean wage which puts some individuals at a disadvantage and others at an advantage. Therefore, every equilibrium includes winners and losers.

Practical application

In order to illustrate the derived outcomes, I introduce examples for a particular case and calculate the critical level of education and individual utility for the three described scenarios.

For this purpose I apply the results of the survey "OECD-Education at glance" (2015). According to this survey, an average of 36% of populations in OECD countries received a bachelor's degree, while about 17% received a master's degree in 2013. Since I assume that high-ability individuals are able to complete the master's degree and mid-ability individuals the bachelor's degree, I assume for reasons of simplicity the following population-shares: $q_l = 0.5, q_m = 0.4, q_h = 0.1$.

Thus, in the first case assuming these population-shares a pooling equilibrium with $u_i = w_i = 1.6$ results. Since many students graduate with a degree, this equilibrium can be neglected.

In the second scenario, mid and high-ability students graduate with a Diplom and

$$y = 1.2; \quad u_l = 1;\ u_m = 1.6;\ u_h = 1.8.$$

Finally, if only high-ability students finish their studies with a Diplom, the following outcomes result

$$y = 3.12; \quad u_l = u_m = 1.44;\ u_h = 1.96.$$

In this context, descriptive statistics prove that graduates with a Diplom as degree earn on average 7200 Euro more per year than graduates with a bachelor's degree (Briedis and Minks 2005). This wage differential among graduates indicates a higher productivity among Diplom graduates in contrast to Bachelor's graduates. Thus, the third scenario in which only highly productive individuals signal appears to be an accurate depiction of reality. If no wage differential between bachelor's and Diplom graduates existed the second scenario would be more realistic.

After checking the relevance of the practical application for all scenarios, I am now able to compare the relevant two scenarios: the signaling equilibrium with bachelor's and master's degrees and the signaling equilibrium with high-ability individuals completing a Diplom.

In case of a signaling equilibrium with bachelor's and master's programs, mid- and high-ability individuals benefit in comparison to a signaling equilibrium with Diplom, in which only high-ability individuals signal. Since high-ability individuals have to put a distance between themselves and low- and mid-ability individuals, the critical level of education is higher in case of a Diplom. Consequently, their utility is less than their utility with a master's degree. In addition, mid-ability individuals do not have the opportunity to signal, meaning that their utility equals the mean wage of low- and mid-ability graduates which is less than their utility with a bachelor's degree. In contrast to this, low-ability individuals benefit from this mean wage which is greater than their wage in case of the signaling equilibrium with bachelor's and master's degrees. Consequently, the signaling equilibrium with bachelor's and master's degrees represents an improvement for high- and mid-ability individuals and a deterioration for low-ability individuals.

In order to confront these results with an example with differentiating levels of productivity, I take actual wage differentials into consideration. Among OECD countries, people with a higher education degree are characterized by an advantage in earnings. Relative earnings rise with educational attainment. Thereby, higher education graduates can expect to earn 60 % more than people with non-tertiary education (OECD 2015). In addition, according to the salary survey of the National Association of Colleges and Employers (NACE 2013) and according to the survey of Bispinck et al. (2012), in many career areas the starting salaries of master's graduates are about 20% higher than the starting salaries of bachelor's graduates.

Thus, in order to improve the practical orientation of the model, I adapt the values of the productivities of the three groups. The levels of productivity are changed according to the described and empirically proved wage differentials.

Since wage is equal to productivity, I assume that low-ability individuals are characterized by a productivity of $\theta_l = 1$, mid-ability individuals of $\theta_m = 1.6$ and high-ability individuals of $\theta_h = 1.9$, taking wage differentials into account. Signaling costs, population shares and previously derived assumptions remain unchanged.

In the case of a signaling equilibrium with two signals representing bachelor's and master's degrees, the critical levels of education are

$$y^B = 0.6 \qquad \text{and} \qquad y^M = 1.2$$

and utility is given by

$$u_l = 1; u_m = 1.3; u_h = 1.5.$$

If no signal exists, a pooling equilibrium with $u_i = w_i = 1.33$ results for all individuals.

Furthermore, in the second scenario there exists only one signal and mid- and high-ability individuals signal. Here, the critical level of education is $y = 0.66$, leading to the following values of utility

$$u_l = 1;\ u_m = 1.33;\ u_h = 1.44.$$

Finally, in the case of only one signal when high-ability individuals invest in education, the critical education level is $y = 1.26$ and utility is given by

$$u_l = u_m = 1.27;\ u_h = 1.48.$$

Now I compare the relevant two scenarios in the case of the adapted levels of productivity: the signaling equilibrium with bachelor's and master's degrees and the signaling equilibrium with high-ability individuals completing a Diplom.

In case of a signaling equilibrium with bachelor's and master's graduates, the high- and mid-ability individuals benefit while low-ability individuals lose in comparison to the one-signal equilibrium. Since low-ability individuals are payed according to their productivity instead of the mean-productivity, they lose. In contrast to this, mid-ability individuals have the possibility to distinguish themselves through a bachelor's degree and thus they are able to obtain a greater wage. High-ability individuals have to invest less in education in case of a master's degree. In case of a Diplom, high-ability individuals have to put a distance between themselves and low- and mid-ability individuals. Therefore, in this case the critical level of education is higher. Consequently, the bachelor's and master's degrees lead to an improvement for high- and mid-ability individuals and to a deterioration for low-ability individuals.

Although I have adapted levels of productivity according to empirically-proved wage differentials, the results remain the same. Utility varies according to the wage differentials but winners and losers remain the same.

On the one hand, I am able to emphasize that the bachelor's and master's system leads to an increase of utility for high- and mid-ability individuals because they are paid according to their productivity. On the other hand, I show that utility of low-ability individuals decreases, since they do not benefit anymore from the mean wage. In contrast to this, a Diplom leads to a very high investment in education because high-ability individuals have to distinguish themselves from the other two groups. In addition, low-ability individuals earn more and mid-ability individuals earn less than they contribute to the firm. Thus, a system with two signals is much more suitable to reduce asymmetrical information in comparison to a system with only one signal.

3.3.3 Self-assessment and dropout rate of students

Although the bachelor's and master's system enables individuals to choose an adequate degree, entering higher education does not always ensure successful graduation. According to the results of the research of "OECD-Education at glance" (OECD 2013) and "Bologna Process Implementation Report" (EACEA 2012), only 70% of the students who enter higher education graduate among OECD countries.

The students who do not complete their studies drop out. Thereby, dropout may have various reasons and motivations. On the one hand, students may drop out of university because the study program does not fit their interests, expectations or knowledge, or because they find an attractive employment opportunity. On the other hand, the higher education system may not meet their needs (OECD 2013).

The individual self-perception of ability might be a reason why university requirements and perceived knowledge do not fit and thus cause notable dropout. This paper focuses on flawed self-perception as a reason for dropout. Self-perception of ability plays a crucial role in the decision to invest in higher education, the choice of degree and the probability of graduation. On the one hand, underestimation may prevent enrollment and reduce success. On the other hand, overestimation may lead to dropout and disappointment. Especially overconfident individuals are more risky and therefore may fail more often than well-calibrated individuals (Barber and Odean 2001). Moreover, the "hard-easy effect" proves that individuals are overconfident concerning their abilities when they are confronted with a difficult task and underconfident being confronted with an easy task (Lichtenstein and Fischhoff 1977; Benoît and Dubra 2011).

In contrast to over- and underconfident students, March et al. (2005) make use of longitudinal data in order to show that students with a good self-concept show a more efficient allocation of study time and achieve better university outcomes. Thus, the authors emphasize the positive influence a good self-perception may exert.

A wide range of literature examines the self-perception of students and its impact on their studies. In this context, Chevalier et al. (2009) investigate the impact of students' self-perception on participation rates in higher education and find out that students misconceive their own performance and especially over-estimate their own ability. Thereby, a high self-perception supports the decision to attend higher education. The dimension of these effects varies according to class and gender gaps.

Furthermore, by examining self-assessment in real-world settings, Dunning et al. (2004) show that most students are not able to self-assess themselves accurately and even tend to be overconfident concerning acquired skills. Moreover, students are often not able to assess whether they have understood newly acquired skills because they focus on the quick acquisition of contents and neglect and underestimate the retention and the transfer of course contents. For this reason, authors propose feedback as an opportunity to improve pupil's self-assessment and check newly acquired contents, since feedback helps students to judge their own performance.

In addition, quantitative self-assessment studies which compare self-assessed grades and teacher grades in higher education show that the correlation between these grades is higher in advanced classes in contrast to introductory classes (Falchikov and Bound 1989). If I interpret the correlation of grades as a measure of correct self-perception, experience has a positive influence on self-assessment. Consequently, in contrast to students of introductory classes, the students of advanced classes are more experienced and therefore are able to self-assess themselves more precisely.

On the one hand, an extended offer of opportunities to repeat course contents might help students to acquire experiences in solving exercises and thus improve self-perception. On the other hand, feedback may be another solution to the flawed self-perception of students. Receiving feedback enables students to check and to update their self-assessment. In addition, repetition and feedback are means to pay attention to students' needs.

3.4 New measures as response

Section 3.2.2 has described current measures to respond to student heterogeneity. These actions mainly concentrate on the start of higher education studies and take no account of teaching routines. By taking reasons for dropout into account, Section 3.4 presents blended learning as a measure which is implemented in the daily teaching routine and may be a response to the flawed self-assessment and increasing heterogeneity of students.

3.4.1 Blended learning as response

As a response to the growing heterogeneity of students, Schulmeister et al. (2012) demand new teaching reforms which support students' self-study which is neglected in traditional teaching methods. In addition to face-to-face sessions, they need support, feedback and extra exercises to guide their self-study. Newly acquired knowledge has to be consolidated through application. If students have the possibility to apply and repeat course contents and if they receive feedback during this repetition, self-assessment and individual motivation may be increased.

In this context, online exercises may be a response to this heterogeneity since they offer different levels of difficulty and are available 24 hours per day during 7 days per week. Solving exercises represents a repetition and consolidation of acquired knowledge and entails feedback after entering a solution. Furthermore, students are free to choose how often and on which exercises they work.

Beside the advantages of online exercises, the current learning environment is characterized by a trend towards online learning. This is the reason why blended learning begins to take the place of traditional teaching methods. This trend is confirmed by various surveys that show that online learning tools in students' learning experience have increased significantly (Allen and Seaman 2006) and that students show a new attitude towards online learning (Sebastianelli and Tamimi 2011). In Germany, for example, nearly every university offers an e-learning platform (Henning 2015). However, the virtual learning environment places new demands on educators (Harasim et al. 1995). Beside the traditional requirements, guiding and helping students through formative feedback becomes more important. Attending to feedback, students are able to self-regulate their learning activity. Thereby, self-regulated students are able to estimate their own skills and to update and organize their further learning activities (Butler and Winne 1995). In the context of computer-supported learning, Zumbach and Reimann (2003) confirm the positive influence of external feedback on students' motivation and problem-solving.

As a response to the heterogeneity of students, blended learning offers extensive possibilities to repeat course contents and it offers guided learning and feedback. Consequently, the described advantages of blended learning might meet the requirements of heterogeneous students and serve to improve the flawed self-assessment.

3.4.2 JACK as example for blended learning

Description of JACK

JACK was developed and is provided by PALUNO (The Ruhr Institute for Software Technology) and represents a computer-aided system offering online exercises and their automatic assessment and grading (Goedicke et al. 2008). Teachers have the opportunity to generate multiple-choice and fill-in exercises. Thereby, JACK does not correct longer proofs and text sequences and thus is particularly suitable for sciences related to mathematics. Students are free to submit answers without time restriction and receive an immediate evaluation by the feedback-function of JACK. Randomized variables are used for every online exercise in order to offer a variety of opportunities to repeat course contents and in order to confront every student with different exercises. Access to JACK is provided to students which sign in with their matriculation number. Each user account contains information about solved exercises and their evaluation. Online tests are administrated on a different platform and their grading is published with a delay.

This section focuses on the courses in microeconomics of the Chair of Microeconomics at the University of Duisburg-Essen which use JACK as online tool. Beside offering weekly lectures and tutorial classes as usual, JACK is implemented as e-learning setting. While weekly lectures and tutorial classes aim to teach basic knowledge and demonstrate examples, the e-learning opportunities offered by JACK represent a possibility of around-the-clock self-training and assessment of course contents. Since the courses in microeconomics combine face-to-face sessions and e-learning opportunities, blended learning is applied.

The courses in microeconomics are characterized by a considerable number of students. Face-to-face sessions are visited by about 800 students while about 600 students take part in the final exam at the end of the semester. This huge student body places new demands on teachers. Here, JACK provides a measures of support. On the one hand, students have the opportunity to solve online exercises around-the-clock and on the other hand they can participate in a mid-term test with the aim to improve the final exam mark.

Every topic of the course is accompanied by various online exercises for the purpose of self-study. These exercises have the form of multiple-choice or fill-in tasks. Thereby, each student receives the same task but different numbers since variables are randomized. After solving the exercise by entering a numerical solution or marking with a cross, the student immediately gets a feedback whether his answer was partially or completely faulty or correct. In the case of a wrong answer the student has the possibility to repeat the exercise, to get a hint by pressing the hint-putting, or to obtain the solution. Exercises have several stages with increasing level of difficulty. Thus, in the case of the right answer or obtaining the solution, the student is guided to the next stage.

With the aim to avoid last-minute learning and to increase commitment, online mid-term tests with the possibility to gain bonus points for the final exam are also offered. The existence of a mid-term test enhances students' engagement in early learning and the seriousness of early learning. Thereby, students are free to choose whether they take part or not. A failed mid-term test has no negative effect. If the student passes the test, he gains bonus points which improve his final exam mark. Nevertheless, bonus points are only credited if the student passes the exam without bonus points. Thus, bonus points do not help to pass the exam but to improve the mark.

The key advantage of online tests lies in their fairness and the immediate electronic correction. Online tests have a substantially reduced workload of correction and are objective since correction is automatic, whereas correction by hand varies among different teachers and depends on the individual concentration and form on the day. Since the mid-term tests are offered online, students may solve them at home. Although they can work in groups or use documents and books, every student obtains randomized and therefore individualized numbers.

Consequently, the described courses in microeconomics are characterized by an extensive offer of blended learning. While weekly lectures and tutorial classes teach course contents, online exercises guide the self-study and mid-term tests entail the possibility to improve the final exam grade.

JACK's contribution

In consideration of the dimensions of student heterogeneity described in Section 3.2.1 and with regard to the flawed self-assessment described in Section 3.3.3, I now emphasize the contribution of JACK.

Since the number of first-year-students has increased significantly, courses are visited by a considerable number of students. Therefore, teachers are not able to guide students individually. JACK has the possibility to pay attention to every student by offering a huge number of exercises which repeat course contents. In addition, every student receives individual feedback after the submission of a solution.

Since universities are open to all social classes, financing of studies represents a burden for students of low-income families. In this connection, e-learning offers a variety of learning opportunities which are free of charge.

Furthermore, many students are gainfully employed during their studies and show differences in the organization of their daily life. Since online exercises are available around-the-clock and since they can be solved everywhere, they offer a high flexibility of time and place. This flexibility enables students to learn according to their individual preferences of time. Flexibility and different levels of difficulty are also means to pay attention to international students with different cultural background.

In addition, the diversity of individual learning behavior and especially the organization of self-study is described as a new source of student heterogeneity. Applying JACK, students start solving exercises at the beginning of the semester and maintain this effort on self-study during the whole semester, instead of learning mainly at the end of the semester before the final exam. Thus, the possibility to learn around-the-clock represents an incentive to learn on a regular basis during the semester. Thereby, overall learning time is higher than usual before the mid-term test. Consequently, the test serves as an incentive to early learning and boosts the motivation to invest time in self-study.

Section 3.2.2 demands new measures to respond to student heterogeneity by accepting heterogeneity instead of transforming it into homogeneity. These measures include the adaptation of courses structures. The described gains of JACK prove that the adaptation of traditional courses to blended learning courses pays attention to student heterogeneity and thus meets this demand.

Finally, JACK may be a solution to correct flawed self-assessment since it offers a wide range of opportunities for repetition and feedback on a regular basis. In order to improve the retention of taught contents, many online exercises are offered for each topic of a course. Therefore, students have the possibility to repeat course contents by solving as many exercises as they want and need to. Moreover, randomized variables enable several repetitions of the same exercise. These possibilities of repetition support the acquisition of experience in solving exercises. Furthermore, every time students hand in a solution, it is evaluated and a response is given immediately. In addition, mid-term tests assess the level of skills and give information about the learning progress. Therefore, students who obtain feedback through JACK are able to check their self-assessment and update their learning activities according to the received feedback.

Consequently, on the one hand, the implementation of JACK in courses meets the growing demands of a heterogeneous student body through guiding self-study and through flexibility of time and space. On the other hand, JACK helps to correct flawed self-assessment with the aid of immediate feedback and possibilities to repeat course contents.

3.5 Conclusion

This paper has analyzed dimensions of student heterogeneity and how higher education system copes with the resulting new challenges.

Nowadays, the student body is characterized by considerable heterogeneity because the number of students has increased significantly and students differ in age, living conditions, learning behavior as well as social and cultural background. Since initiated measures until now mainly concentrate on the opening phase of higher education, they only aim to make heterogeneous students fit to homogeneous university structures.

Furthermore, the higher education system offers differentiated degrees like the bachelor's and master's degree which enable heterogeneous students to choose a degree according to their skills and future plans.

Contrasting the predominant higher education system comprising bachelor's and master's degrees with prior one-signal degrees, I show that mid- and high-ability individuals benefit and low-ability individuals are disadvantaged. While mid- and high-ability individuals are able to graduate with an advanced degree and earn higher wages, low-ability individuals do not gain an advantage from a mean wage. Compared to a one-signal degree, high-ability individuals have to invest more in education since they do have to put a distance between themselves and educated mid-ability individuals. Moreover, mid-ability individuals have the chance to complete an intermediate degree and therefore they are able to distinguish themselves from low-ability individuals. Consequently, the higher education system meets the needs of heterogeneous students but on the cost of higher effort. The signaling model assumes students to be perfectly informed about their skills but, in reality, dropout rates prove that students who enter higher education do not always complete their studies. One main reason for dropout might be a flawed self-assessment. Individual self-perception of skills plays a crucial role in investing in higher education, the choice of degree and the probability of graduation.

In order to take heterogeneity into account and to improve the potentially flawed self-assessment, I propose automatic self-assessment tools.

As an example of blended learning the online learning tool JACK is introduced in order to support traditional face-to-face sessions. The around-the-clock offer of online exercises provides individual feedback, flexibility of time and space and pays attention to the individual learning progress of every student. Thus, online exercises consider different living conditions, learning preferences and social and cultural background and guide students' self-study. By introducing online mid-term tests, the students' motivation to repeat course contents from the beginning of the semester is enhanced and they receive a feedback which evaluates the acquired skills. These effects improve flawed self-assessment, retention of newly acquired skills and regularity of learning behavior during the semester.

Consequently, the implementation of JACK in courses takes the growing demands of a heterogeneous student body, especially within the bachelor's degree, into account and improves flawed self-assessment. Students are able to learn according to their individual preferences and backgrounds and their self-study is evaluated and guided during the semester, decreasing dropout rates. In order to discourage low-ability individuals, better tools within the orientation phase are needed.

Conclusion

The studies of this dissertation have analyzed the influence of blended learning on students' learning behavior and on the heterogeneity of students.

Nowadays, higher education faces many challenges due to the growing heterogeneity of students. Beside traditional aspects like skills, professional experience, as well as social and cultural background, heterogeneity displays new dimensions. These new dimensions are the number of first year students, the broader age span and the organization of self-study. Furthermore, today's learning environment is characterized by a new trend towards online learning. Since traditional teaching methods are taken up by this new trend, blended learning becomes more important. This dissertation investigates from a theoretical point of view how blended learning influences dynamic utility maximization and heterogeneity in a higher education setting.

Chapter 1 and Chapter 2 develop a simple model of learning by doing in the context of university courses. Final exam marks are the result of skills accumulated through learning during the semester. Thereby, students are able to make use of the online learning opportunities which are available around-the-clock. In addition to the offered online exercises, they have the opportunity to take an online mid-term test with the aim to improve the final exam mark or to receive feedback. However, learning time is limited due to the trade-off between learning and leisure. Therefore, students have to decide upon the optimal time allocation between learning and leisure depending on their individual preferences for time management.

Chapter 1 assumes that students are perfectly informed about their individual skill level. For this case, dynamic utility maximization emphasizes that around-the-clock online learning opportunities lead to a nearly constant learning activity in time. In the case of preferences for long-term learning, students learn totally constant during the semester. Students with preferences for short-term learning are characterized by a little increase in learning time, nevertheless last-minute learning is avoided. In this connection, we also show that constant learning increases the learning progress. Furthermore, the introduction of an online mid-term test leads to a higher level of learning time before the test and consequently shifts the learning activity towards the beginning of the semester. This shift of learning time promotes early learning instead of last-minute learning. The importance of these results is supported by cognitive psychology (Ebbinhaus 1885; Webb 2007). Many cognitive psychologists have shown that learning is most effective if students repeat course contents at regular intervals in a longer time frame.

Since studies on flawed self-perception indicate that students are not able to self-assess themselves accurately, Chapter 2 assumes imperfect knowledge about the individual ability and focuses on feedback as a solution. In this context, online exercises and a mid-term test serve as sources of feedback to evaluate the individual skill level. Thereby, students who make use of these online learning opportunities have the possibility to compare their predicted skill level to the received feedback and may update their learning time according to this comparison. We are able to show that average students who really are or perceive to be high-ability are motivated to learn more, while students who are frustrated because of their overconfidence reduce learning. These results support the concept of intrinsic motivation which influences the commitment to learn.

In contrast to these findings, low-ability students benefit from the received feedback and increase their learning activity. This update of learning time increases their possibility to pass the exam. Nevertheless, since they face a high risk of perceiving barriers to learn, they may also drop out of the course. Finally, we show that online feedback has a positive influence on the utility. If students take the mid-term test and update their learning strategy according to the gained information, they are able to increase their utility even if the mid-term test result does not improve the final exam mark.

While Chapter 1 and 2 focus on the influence of blended learning on students' learning behavior, Chapter 3 investigates the impact of blended learning on student heterogeneity. These days, students are characterized by a considerable heterogeneity which comprises many dimensions. The comparison of the current higher education system comprising bachelor's and master's degrees to prior one-signal degrees shows that today's system meets the needs of heterogeneous students. High-ability individuals have to invest less in education, since they do not have to put a distance between themselves and two groups of less productive individuals. Moreover, mid-ability individuals are able to complete an intermediate degree and to distinguish themselves from low-ability individuals. Finally, low-ability individuals do not benefit from a mean-wage. Consequently, students have the possibility to choose a degree which fits their future plans and individual abilities.

In the connection of teaching routines, universities mainly focus on the opening phase by preparing and selecting heterogeneous students for homogeneous university structures. In order to implement new teaching structures which accept student heterogeneity, Chapter 3 proposes blended learning. Since online exercises are available around-the-clock, they show a high flexibility of time and space. This flexibility considers different preferences, living conditions and backgrounds. In addition, online exercises and mid-term tests offer feedback and incentives to early learning which guide and evaluate students' self-study. Thus, these incentives improve the flawed self-assessment, the retention of acquired skills and increase the continuity of learning during the semester.

On the one hand, Chapter 3 shows that blended learning guides the self-study of students and improves flawed self-assessment and on the other hand, blended learning is able to transform teaching routines in higher education to adapt to the heterogeneous student body.

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