Essays on Bargaining

Disstertation

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**Introduction**

Most of human interaction can be considered as negotiations of some form. As a consequence, literature on bargaining, be it theory or applications, is ubiquitous and, when it comes to applications, very diverse. Research topics range from marital bargaining, acknowledging the fact that married couples are almost constantly negotiating over a variety of matters to international relations concerned with negotiations among national governments on economic, environmental or territorial issues. Of course, also commodity prices are often the outcome of negotiations and analyzed by the means of bargaining theory. Quite generally, in this dissertation, I am interested in the variables that determine the outcome of negotiations and especially the factors that lead to their breakdown.

With respect to bargaining theory, basically all research can be traced back to the seminal works of Nash (1950) and Rubinstein (1982). I will give a brief overview of bargaining theory and anchor the theoretic concepts used in this dissertation in the section **Bargaining Theory**. I will then provide an outline of the specific problems and situations to which I apply bargaining theory in the section **Bargaining Applications**.

**Bargaining Theory**

In any bargaining situation, individuals see the possibility of reaching a mutually beneficial agreement but are not consent about how this agreement should look like. There is an inherent conflict of interest about how the gains from bargaining should be distributed among the involved parties. It is the aim of bargaining theory to identify solutions to these distributional problems.

Real world bargaining is usually tied to a bargaining process, and the outcome of bargain-
ing very much depends on the specific procedural features of this process, such as who can make offers and when. Nash (1950) abstracts away from such procedural features and considers only the set of agreements that satisfy “reasonable” properties, that are conditions that any outcome arrived at by rational decision makers should satisfy a priori. These conditions are treated as axioms, from which the outcome is deduced. The resulting Nash bargaining solution is pinned down only by the axioms of underlying expected utility, in addition to symmetry and independence of irrelevant alternatives. It has the remarkable property that the outcome is implemented cooperatively and uniquely by maximizing the players’ utilities.

Nash’s axiomatic approach to solve the two-person bargaining problem has become the foundation of modern bargaining theory. Still, even Nash felt the need to provide a non-cooperative foundation of his very abstract cooperative solution concept and came up with an explicitly modeled strategic game in Nash (1953). In the Nash demand game, two players make simultaneous demands and agreement is only reached if the combined demands are feasible. As is the problem with most non-cooperative models, the Nash demand game has multiple equilibria since any split constitutes a Nash equilibrium. Nash (1953) solves the problem by introducing uncertainty in the payoff function and obtains that the Nash bargaining solution is the unique limiting outcome of the demand game when uncertainty vanishes. Ever since, game theorists have set out to construct non-cooperative bargaining games with the purpose to validate the axiomatic solution concept and broaden the scope of its applicability.¹

The most famous non-cooperative approach in this vein, is the two-person bargaining game analyzed in Rubinstein (1982) in which players make alternating offers over the division of a pie that shrinks over time because of costs of delay. Rubinstein is able to show that the game has a unique solution, depending on who makes the first offer. Further, it is shown in Binmore (1987) that the non-cooperative equilibria of the Rubinstein game converge to the Nash bargaining solution when delay between offers goes to zero. Nevertheless, the Rubinstein outcome can only approximate Nash’s solution because the costs of delay can never become completely insignificant as they constitute the driving force behind the bargaining process.²

¹This endeavor is commonly referred to as the “Nash program”.
²Without costs of delay, the bargaining process is indeterminate. The bargaining could go on indefinitely because the players have no incentive to strike a deal today rather than tomorrow.
The axiomatic approach has the advantage that it determines a unique outcome by a fairly simple formula and is therefore very attractive to applied economics with a focus not especially on the bargaining process. It has the disadvantage that its application is limited to bargaining situation that fit the underlying axioms. Strategic models on the other hand can be tailored to the particular specifications of any bargaining situation. On the downside, this makes them very sensitive to procedural changes: even small changes in the rules of the bargaining process can have a decisive impact on the bargaining outcome.

In light of the different virtues and shortcomings of the cooperative and non-cooperative approach, it is now the prevalent view in bargaining literature that axiomatic and strategic models are complementary (see Sutton 1986). The reason for choosing an exclusively non-cooperative perspective in this thesis was to stay as close as possible to related literature.

Chapter 1 is based on a bargaining game with outside options and an infinite horizon in which a player is randomly chosen in every period to make a take-it-or-leave-it offer. In this game, reject and counteroffer is not a possible move but rejection of an offer immediately results in the breakdown of bargaining and the players taking up their respective outside options. In case of agreement, bargaining continues in the next period. The main ingredients in this set-up are the outside options and repeated interaction. Outside options are relevant because I focus on a crisis bargaining application in which outside options are usually modeled as the states payoffs from going to war.\(^3\) Repeated interaction is crucial because I study the effect of commitment problems on the bargaining outcome. Commitment problems only arise when a party involved cannot credibly commit to an agreement because it can demand revisions later in time. The inability to commit is only relevant in repeated interaction.

The game is an extension of the bargaining model used in Fearon (1995) and Powell (2006) in the context of crisis bargaining with commitment problems and has not been analyzed before. In their models only one player has the power to make a take-it-or-leave-it offer in every period which means that this player has the entire bargaining power. I relax this assumption by introducing random proposal power which gives both players some bargaining power.

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3\(^{\text{see for example Powell (2006)}}\)
As a variation of this bargaining set-up, in Chapter 2 I analyze a one-time bargaining game with the possibility of counteroffers and a possibly infinite horizon that is close to the Rubinstein game only that there are random proposers instead of alternating offers and two-sided outside options. I find that both bargaining models, the models in Chapter 1 and 2 have an equilibrium which generates identical subgame perfect equilibrium payoffs. The proof draws on Binmore, Shaked and Sutton (1989) who first studied the effect of outside options on the bargaining outcome and introduced the notion of ‘outside option principle’ by identifying the unique subgame-perfect equilibrium of a Rubinstein bargaining game with outside options.

In Chapter 3, I considerably reduce complexity with regard to the bargaining game in order to concentrate on the principal-agent relationship which is the driving force in this model. Since I am no longer interested in a temporal dimension, I skip repeated interaction so that the bargaining game boils down to a one-time take-it-or-leave-it offer game with outside options and random proposers. This has the advantage that the game is considerably simpler and easier to solve. Still, the important features of the crisis bargaining process are retained: both sides have bargaining power and the possibility to opt out and go to war. In addition, I introduce asymmetric information: the players outside options are now private information. The impact of such private information on the crisis bargaining outcome has first been analyzed in Powell (1996). Ever since, asymmetric information about the opponent’s outside option is considered a potential rational reason for the breakdown of international negotiations and the onset of war (see Fearon 1995). By showing that democracies can overcome such information asymmetries, I provide a rational reason for the ‘democratic peace’, that is, the empirical observation that democracies rarely fight wars with one another.

Chapter 4 retains information asymmetries as a potential source of breakdown but applies the bargaining set-up to a simple market structure. The market structure is a version of Akerlof’s (1970) market for lemons in which trade is decentralized and buyers and sellers are randomly matched. As an extension of Akerlof’s model, the buyer side is not completely uninformed about quality but a share of buyers is equally informed as the sellers. Once
matched, a seller makes a take-it-or-leave-it offer to the buyer who can either accept the offer or reject it. This is the most rudimentary version of a bargaining situation but it captures the common praxis in many markets, for example most retail markets, that prices are simply posted by sellers, without the buyer having much influence on the price. This set-up allows the analysis of the price formation process because, unlike Akerlof\’s centralized market version, equilibrium need not be exclusively defined by a single price equating supply and demand but may be characterized by different prices (see Wilson 1980).

**Bargaining Applications**

Chapters 1-3 of this dissertation apply bargaining theory to international relations and aim to shed light on rational reasons for war and peace. The idea to analyze war arguments on the basis of bargaining models has been initiated by Fearon (1995). In his seminal work on rationalist explanations for war, Fearon (1995) provides a coherent theory about the occurrence of war by introducing a formalization of the bargaining problem faced by two states in conflict and on these grounds establishes two main rational causes of war: *information asymmetries* and *commitment problems*. While commitment problems play the central role in Chapters 1 and 2, asymmetric information is explored in Chapter 3.

Commitment problems are present in many bargaining situations that are characterized by a temporal dimension. The parties involved in these interactions must be confident that agreements made in the present will be binding in future periods or else they might prefer to abstain from an agreement altogether. A commitment is not credible if one party has the incentive to renege on an earlier agreement. In the context of international relations, an increasingly powerful state may be unable to credibly commit to a current settlement because it can demand revisions later in time. Anticipating this, a declining state may have reason to fight in the present in order to guarantee itself a minimum of the stakes. It is Fearon (1995) who first connects commitment problems to the idea of preventive wars by

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4 Of course there are also markets in which prices are the outcome of actual bilateral negotiations with offers and counteroffers, for example bazaars, and there is a line of literature investigating the pros and cons of the two pricing institutions also in the context of asymmetric quality information. See for example Bester (1993) and Arnold and Lippman (1998).
showing that anticipated future shifts in power are a rational reason for a declining state to start war. Later Powell (2006) expands on Fearon’s analysis by showing that this mechanism also explains related phenomena like preemptive attacks and bargaining over issues that, by themselves, are sources of bargaining power.

While formalizing the argument that adverse shifts in power between states in conflict can lead to preventive war is an important step in understanding the reasons for war, there are still historic examples that contradict this prediction and one question remains: why do states not always respond to the anticipation of substantial negative power shifts with a strategy of preventive war and sometimes do fight preventively even though a shift in (military) power has not occurred? Chapters 1-2 aim to solve this puzzle by breaking down the concept of power and identifying the kind of power that can trigger preventive war.

Bargaining power can be defined as a measure of a player’s relative power to extract a share from the opponent during negotiation with respect to the opponent’s power to do the same. In this sense, a party’s bargaining power is captured by its share of the surplus. In war bargaining models, bargaining power is usually determined by the power to make proposals and the outside option payoff, which is the expected payoff, a country receives by going to war. This war payoff, in turn, is determined by a party’s probability to win the war and the pie it will receive in case of winning, decimated by the party’s costs of fighting. In the literature, so far, only a shift in military power has been analyzed. An increase in military power directly translates into increased bargaining power since it alters both parties’ outside option payoffs through the winning probability. But as pointed out above, there are other means to increase bargaining leverage than military power, namely changes in the parties’ respective costs of war and changes in the level of proposal power.

The models in Chapters 1-2 explicitly investigate these other means and find that war is only an equilibrium outcome if a state expects a reduction in its war payoff. On the other hand, war does not occur if the declining state’s outside option is unaffected by the rising state’s enhanced bargaining position.

Chapter 3 also draws on Fearon’s bargaining approach to war, in the attempt to provide a rational explanation of the “democratic peace”, that is, the empirical observation that
democracies tend not to fight wars with one another. I argue that democracies can overcome information asymmetries so that this rational reason for war dissolves when democracies are involved. The means by which democracies achieve this result, is successful signaling of their type due to general transparency within democracies. The model’s theoretic underpinning is the principal-agent nature of democratic political systems and the fact that the actual bargaining with a third party is delegated to an elected representative (agent), a practice which is common when one side involved in the bargaining process consists of a group of people (principal). There are other examples of delegated bargaining, like elected labor union leaders representing their union members when bargaining with management, politicians bargaining for their constituencies in domestic politics, and boards of directors bargaining on behalf of company shareholders. The model applies the idea of delegated bargaining to the literature on war initiation and analyzes how transparency within democracies and accountability of political representatives help overcome information asymmetries. The results of this analysis are then used to find out what degree of agency transparency is preferable in an international bargaining setting, comparing two scenarios, open-door bargaining in which the democratic public can observe the bargaining process between its representative and a third party and closed-door bargaining, in which the agent’s actions are partly hidden.

Chapter 4 is also concerned with bargaining in the presence of asymmetric information but shifts the focus to a competitive market situation in which the quality of the good is the sellers’ private information. Following Akerlof (1970), the prediction for such markets is that, when the average quality of the good held by sellers is low and buyers cannot distinguish quality, bad products drive out good products and only low-quality units trade in the competitive equilibrium. This dynamic is termed adverse selection and has been investigated in the context of various settings, such as health insurance and labor markets.

The model relaxes the general assumption that the buyer side is completely uninformed and integrates informed buyers into the lemons market. I investigate how the presence of informed buyers affects adverse selection and welfare by comparing two different market structures: one in which sellers cannot distinguish between informed and uninformed buyers and one in which sellers can learn whether a buyer is informed or not and price-discriminate...
on an individual level. I find that the presence of informed buyers reduces adverse selection and a sufficiently high share of informed buyers even induces an efficient fully-separating equilibrium. This is the reason why in most cases individual price discrimination leads to a welfare reduction.
Chapter 1

Commitment Problems in International Bargaining¹

Abstract

When contracts are not enforceable, bargaining can break down because of commitment problems. In the international context, standard models predict that a shift in military power can cause preventive war because it changes the relative bargaining position between states. We find that shifts in military power are not the only cause of war under commitment problems and that commitment problems per se are not necessarily a cause of war even if the relative bargaining position changes substantially.

¹see Amann, Erwin and Nadine Leonhardt (2013): Commitment Problems in International Bargaining, Ruhr Economic Papers 403
1.1 Introduction

Commitment problems arise if the relative bargaining position between states changes and an increasingly powerful state is unable to credibly commit to a current settlement because it can demand revisions later in time. Anticipating this, a declining state may have reason to fight now in order to still guarantee itself a minimum of the stakes.

Fearon (1995) and Powell (2006) formalize this argument in a bargaining model in which war constitutes the parties’ outside option. We describe the war payoff as the result of a costly lottery that is determined by a party’s military power and her costs of fighting. In the literature, so far, only shifts in military power have been analyzed and associated with commitment problems and the risk of war. The implications of changes in the parties’ respective costs of war have not been studied yet. Also, previous works have not explicitly modeled bargaining power so that the effects of changes in bargaining power are still unclear.

The present paper introduces variable proposal power which facilitates the analysis of situations in which both parties have some bargaining power. We show that a shift in bargaining power affects the distribution of the bargaining surplus, but does not lead to war. Also, an isolated decrease in one party’s costs of war can have an impact on the relative bargaining position but never causes war. On the other hand, war can occur in equilibrium if a party’s costs of war increase even though military power does not change.

1.2 The Basic Model

In every period, states $A$ (he) and $B$ (she) bargain about the distribution of an issue of size $\pi$. With probability $\alpha$ state $A$ can make a take-it-or-leave-it offer to $B$, with probability $1 - \alpha$ it is the other way around. A state’s proposal is denoted $x^t_i \in [0, \pi]$ where $x^t_i$ refers to the share of the pie that state $i$ receives in period $t$. A state can respond to a proposal in two ways: accept the offer or opt out. In case of agreement, the pie is shared according to the proposal and the game then proceeds to the next period. Otherwise, the negotiation is terminated.

\footnote{To our knowledge only Fearon (1995) and Powell (2006) have analyzed commitment problems theoretically.}

\footnote{see Fearon (1995) and Powell (2006)}
and the states fight. In case of war, state A wins with probability \( p \in [0,1] \) and state B with probability \( 1-p \), leading to future payoffs per period \( (\pi - c_A, 0) \) when A wins and \( (0, \pi - c_B) \) when B wins, where \( c_i \) represent the irreversible costs of war. Consequently, the expected values of the outside options are \( \frac{w_A}{1-\delta} = \frac{p(\pi - c_A)}{1-\delta} \) and \( \frac{w_B}{1-\delta} = \frac{(1-p)(\pi - c_B)}{1-\delta} \) with \( \delta \in [0,1] \) being the states’ common discount factor. Obviously, the two states have an incentive to reach an agreement if \( c_A + c_B \geq 0 \). Figure 1.1 below illustrates the game tree:

![Game tree of the repeated game](image)

**Figure 1.1:** Game tree of the repeated game
Lemma 1 In any subgame perfect equilibrium of the basic model, agreement is always reached and the equilibrium outcome is therefore Pareto efficient.

Proof. Let \((M_A, M_B)\) be the expected payoffs to A and B in a subgame perfect equilibrium of the game, or correspondingly any subgame starting with a move of nature. Let \(M_A^{\min} \leq M_A \leq M_A^{\max}\) and \(M_B^{\min} \leq M_B \leq M_B^{\max}\) be the corresponding interval for a specific set of SPE outcomes of this game. Then \(\frac{w_A + w_B}{1 - \delta} \leq M_A + M_B \leq \frac{\pi}{1 - \delta}\). Since the aggregate payoff in case of agreement in period \(t\) is always bigger than the war payoff, \(\frac{w_A + w_B}{1 - \delta} \leq \pi + \delta(M_A + M_B)\), war can occur only if the whole pie \(\pi (x_i = 0)\) is too small to meet the expectations of the opponent,

\[
\pi + \delta M_{-i} < \frac{w_{-i}}{1 - \delta}.
\]

However, \(M_{-i}^{\min} \geq \alpha \frac{w_{-i}}{1 - \delta} + (1 - \alpha) \frac{w_{-i}}{1 - \delta} = \frac{w_{-i}}{1 - \delta}\) since the opponent can always respond by choosing the outside option, and if he gets to make an offer, additionally extract potential efficiency gains in the current period and therefore expects to get at least his own outside option payoff. This, however, is in contradiction to Equation (1.1)

\[
\pi + \delta \frac{w_{-i}}{1 - \delta} \leq \pi + \delta M_{-i} < \frac{w_{-i}}{1 - \delta}
\]

as long as \(\pi \geq w_{-i}\).

Thus, in any subgame perfect equilibrium the equilibrium offer is always accepted and either makes the respondend indifferent between acceptance and war or provides the respondend with the minimal value \((x^*_i = \pi)\) in which case the outside option is not binding.

Figure 1.2 below depicts different regions of the player's proposals depending on whether or not the outside options are binding. Theorem 1 characterizes all subgame perfect equilibria.
Theorem 1 The equilibrium offers depend on the relative size of the outside options \( w_A \) and \( w_B \):

\[
\begin{align*}
    x^*_A &= \pi & \text{and} & & x^*_B &= \pi & \text{(case 1)} \\
    x^*_A &= \frac{\delta}{1-\delta} \left( \frac{(1-\alpha)}{(1-\delta)} \right) (\pi - w_B) & \text{and} & & x^*_B &= \frac{\delta}{1-\delta} \left( (1-\alpha)(\pi - w_A) - \alpha w_B \right) & \text{(case 2)} \\
    x^*_A &= \pi & \text{and} & & x^*_B &= \frac{\pi - w_A}{1-\delta} & \text{(case 3)} \\
    x^*_A &= \frac{\pi - w_A}{1-\delta} & \text{and} & & x^*_B &= \pi & \text{(case 4)}
\end{align*}
\]

Proof. In case 1 \((w_A \leq \delta \alpha \pi \text{ and } w_B \leq \delta (1-\alpha) \pi)\) both outside options are not binding since both players can claim the whole pie in all future periods and therefore have no incentive to end peaceful settlement. The respondend is better off accepting the minimal offer peacefully and hoping for future peaceful returns.

In case 3 \((\delta \alpha \pi < w_A \text{ and } w_B \leq \frac{\delta(1-\alpha)}{1-\delta \alpha} (\pi - w_A))\) player A’s outside option is binding even though he can claim the whole pie when he gets to make an offer in the future \(x^*_B < \pi\). Player B’s expected payoff, if it is her turn to accept the offer, must satisfy \(\frac{w_B}{1-\delta} \leq (\pi - x^*_A) + \delta M_B^{t+1}\).
In any stationary subgame perfect equilibrium if her outside option is not binding (A can extract the whole surplus, \( x_A = \pi \)) then

\[
\frac{w_B}{1 - \delta} \leq \delta M_B = \frac{(1 - \alpha)x_B}{1 - \delta} \quad \text{and} \quad \frac{w_A}{1 - \delta} = \pi - x_B + \delta M_A.
\]

\[
\pi - x_B + \delta M_A = \pi - x_B + \delta \alpha \pi + (1 - \alpha)(\pi - x_B) = \frac{1 - \delta \alpha}{1 - \delta} (\pi - x_B) + \delta \alpha \pi
\]

\[
x^*_B = \frac{\pi - w_A}{1 - \delta \alpha} \quad \text{and} \quad w_B \leq \delta (1 - \alpha) x^*_B
\]

**Case 4** (\( w_A \leq \frac{\delta \alpha}{1 - \delta (1 - \alpha)} (\pi - w_B) \) and \( \delta (1 - \alpha) \pi < w_B \)) and is analogous to case 3 with the roles of player A and B reversed.

**Case 2** (\( \frac{\delta \alpha}{1 - \delta (1 - \alpha)} (\pi - w_B) < w_A \) and \( \frac{\delta (1 - \alpha)}{1 - \delta \alpha} (\pi - w_A) < w_B \)), in which the size of net utility gained by peaceful settlement is not sufficient to cover the outside option payoff, describes all other conditions not covered in cases 1, 3 and 4, which implies that both outside options become binding.

\[
\frac{w_A}{1 - \delta} = (\pi - x_B) + \frac{\delta}{1 - \delta} (\alpha x_A + (1 - \alpha)(\pi - x_B))
\]

\[
\frac{w_B}{1 - \delta} = (\pi - x_A) + \frac{\delta}{1 - \delta} (\alpha(\pi - x_A) + (1 - \alpha)x_B)
\]

has the unique solution

\[
x^*_A = \frac{\delta}{1 - \delta} (\alpha(\pi - w_B) - (1 - \alpha)w_A) \quad \text{and} \quad x^*_B = \frac{\delta}{1 - \delta} ((1 - \alpha)(\pi - w_A) - \alpha w_B)
\]

Given the optimal proposals defined by Theorem 1, the player’s expected payoffs in any period are given by:
\[ M_A^* = \begin{cases} 
\alpha \pi & \text{in case 1} \\
\alpha(\pi - w_B) + (1 - \alpha)w_A & \text{in case 2} \\
\frac{(1-\delta)\pi\alpha + (1-\alpha)w_A}{1-\delta\alpha} & \text{in case 3} \\
\frac{\alpha(\pi - w_B)}{1-\delta(1-\alpha)} & \text{in case 4} 
\end{cases} \]

\[ M_B^* = \begin{cases} 
(1 - \alpha)\pi & \text{in case 1} \\
\alpha w_B + (1 - \alpha)(\pi - w_A) & \text{in case 2} \\
\frac{(1-\alpha)(\pi-w_A)}{1-\delta\alpha} & \text{in case 3} \\
\frac{(1-\delta)(1-\alpha)\pi + \alpha w_B}{1-\delta(1-\alpha)} & \text{in case 4} 
\end{cases} \]

### 1.3 Commitment Problems

Now we assume that the game tree is extended to include an additional stage \( t = 0 \) after which the players’ relative bargaining position changes. This change, beginning at \( t = 1 \), lasts for all periods to come and is fully expected by both players at the start of period \( t = 0 \) but not before. Note that every period of the extended game, from period \( t = 1 \) on, is strategically equivalent and equilibrium payoffs are determined by Theorem 1. In the following, we will only consider changes in the relative bargaining position in favor of player \( B \). This means that player \( A \)'s bargaining position deteriorates either because his military power decreases, his costs of war increase, \( B \)'s costs of war decrease or he loses proposal power. Suppose player \( B \) has no other means to buy off player \( A \) in the current period but to give him the entire pie, so that \( x_B^0 = 0 \).

Again, collective reasoning supports peaceful settlement. Player \( A \) has no incentive to trigger war because war would make him worse off than demanding the maximum acceptable share, since

\[
x_A^* + \delta M_A^1 < \frac{w_A^0}{1-\delta}, \quad \text{when } \pi - x_A^* + \delta M_B^1 = \frac{w_B^0}{1-\delta}
\]

\[
\Rightarrow \pi + \delta(M_A^1 + M_B^1) = \pi + \delta \frac{\pi}{1-\delta} < \frac{w_A^0 + w_B^0}{1-\delta} \quad \text{in contradiction to } w_A^0 + w_B^0 < \pi.
\]
The same argument applies to player B who also prefers bargaining over fighting.

A commitment problem can arise in this situation if player player B cannot credibly commit in \( t = 0 \) to not exploit her improved bargaining position in future periods.

### 1.3.1 Shift in military power

A shift in military power changes the states’ respective probabilities of winning war \( p \) and \( 1 - p \). When military power shifts in favor of B, A’s outside option decreases and B’s outside option increases, since \( w_A^{0} = \frac{p(\pi - c_A)}{1 - \delta} \) increases in \( p \) and \( w_B^{0} = \frac{(1-p)(\pi - c_B)}{1 - \delta} \) decreases in \( p \). This change in the players’ outside options can create a shift in the relative bargaining position if it alters the future distribution of the bargaining surplus. It can lead to preventive war if player A’s current outside option exceeds his expected future gains from bargaining. The war condition determines the critical value of \( w_A^{0} \) from which player A prefers going to war to bargaining. That is,

\[
\frac{w_A^{0}}{1 - \delta} > \pi + \frac{\delta}{1 - \delta} M_A^{1} \tag{1.3}
\]

Since the aggregate future bargaining payoff \( \frac{\delta}{1 - \delta} M_A^{1} \) depends on the 4 cases defined by Theorem 1, the war condition can be specified as follows:

- **case 1**: \( \frac{w_A^{0}}{1 - \delta} > \pi + \frac{\delta}{1 - \delta} \alpha \pi \)
- **case 2**: \( \frac{w_A^{0}}{1 - \delta} > \pi + \frac{\delta}{1 - \delta} \left( \alpha (\pi - w_B^{1}) + (1 - \alpha)w_A^{1} \right) \)
- **case 3**: \( \frac{w_A^{0}}{1 - \delta} > \pi + \frac{\delta}{1 - \delta} \left( \alpha (1 - \delta) \pi + (1 - \alpha)w_A^{1} \right) \)
- **case 4**: \( \frac{w_A^{0}}{1 - \delta} > \pi + \frac{\delta}{1 - \delta} \left( \alpha (\pi - w_B^{1}) \right) \) \( \left( \frac{1}{1 - \delta (1 - \alpha)} \right) \)

In all cases but Case 1, the war condition depends on at least one player’s outside option. This means that as long as Case 1 obtains in \( t = 0 \) and in \( t = 1 \), a shift in military power has no effect on the distribution of the bargaining surplus and can therefore not be the cause of preventive war. In all other cases, preventive war is possible and occurs if the war condition is fulfilled.

**Corollary 1** The fulfillment of the war condition decreases in \( \alpha \). It depends on the level of \( \alpha \), how substantial a change in military power has to be to cause preventive war.
Proof. The fulfillment of the war condition depends on $A$’s expected future bargaining payoff $M_A$. The bigger this payoff, the more he prefers bargaining over fighting. It can easily be verified that $M_A$ increases in $\alpha$ in all 4 cases:

$$
case ~ 1 : \quad \frac{\partial M_A}{\partial \alpha} = \frac{\delta}{1-\delta} \pi > 0 \\
\text{case} ~ 2 : \quad \frac{\partial M_A}{\partial \alpha} = \frac{\delta}{1-\delta} \left( \pi - (w_A^1 + w_B^1) \right) > 0 \\
\text{case} ~ 3 : \quad \frac{\partial M_A}{\partial \alpha} = \frac{\delta}{(1-\delta)^2} \left( \pi - w_A^1 \right) > 0 \\
\text{case} ~ 4 : \quad \frac{\partial M_A}{\partial \alpha} = \frac{\delta}{(1-\delta(1-\alpha))^2} \left( \pi - w_B^1 \right) > 0
$$

To show that the level of $\alpha$ affects the change in military power necessary to cause preventive war, we compare the limit cases $\alpha \in \{0, 1\}$ that are treated in Fearon (1995) and Powell (2006) respectively. For $\alpha \in \{0, 1\}$, the war condition looks as follows:\(^4\)

$$
\alpha = 1 : \quad \frac{w_A^0}{1-\delta} > \pi + \frac{\delta}{1-\delta} \left( \pi - w_B^1 \right) \\
\alpha = 0 : \quad \frac{w_A^0}{1-\delta} > \pi + \frac{\delta w_A^1}{1-\delta}
$$

The first case, $\alpha = 1$, coincides with Powell’s model in which a shift in military power takes place and player $A$ has all proposal power. This means that the players’ payoffs are determined only by player $B$’s outside option because player $A$ always offers player $B$ her war payoff and receives the residuum. Rearranging terms and subtracting $\frac{\delta w_A^1}{1-\delta}$ from both sides of the first inequality gives Powell’s general inefficiency condition:

$$
w_A^0 - \delta w_A^1 > \pi - \delta (w_A^1 + w_B^1) \quad (1.5)
$$

The second case, $\alpha = 0$, coincides with Fearon’s model in which a shift in military power takes place and player $B$ has all proposal power. In this war condition, $w_B^1$ is absent. When player $B$ always proposes, then the player’s payoffs are determined only by player $A$’s outside option because player $B$ always offers player $A$ his war payoff and receives the residuum. In

\(^4\)For $\alpha = 1$, Cases 1 and 3 disappear and the war conditions for Cases 2 and 4 are identical. For $\alpha = 0$ Cases 1 and 4 disappear and the war conditions for Cases 2 and 3 are identical.
this case, the war condition can be written as:

\[ w_A^0 - \delta w_A^1 > \pi - \delta \pi \] (1.6)

Comparing Condition 1.5 with Condition 1.6, it follows that the shift in military power necessary to trigger war, (left hand side of the conditions) is smaller for \( \alpha = 0 \) than for \( \alpha = 1 \) since \( \pi > w_B^1 \).

The findings confirm the standard argument that a shift in military power can change the relative bargaining position which can cause preventive war. But in contrast to Powell (2006) who concludes that the shift in military power necessary to cause preventive war needs to be “large and rapid”, we can show that the necessary shift depends on the parties’ respective bargaining power. When the declining party has little bargaining power and can only extract a small share of the bargaining surplus, then a smaller shift in military power is necessary to trigger war. When \( \alpha = 0 \), the necessary shift even goes to zero for \( \delta \rightarrow 1 \), as can be seen in Condition 1.6.

### 1.3.2 Increase in A’s costs of war

An increase in A’s costs of war reduces A’s outside option since \( \frac{w_A}{1-\delta} = \frac{\pi - c_A}{1-\delta} \) decreases in \( c_A \). It has no effect on player B’s outside option because \( \frac{w_B}{1-\delta} = \frac{(1-p)(\pi - c_B)}{1-\delta} \) does not depend on \( c_A \). A reduction in player A’s outside option can create a shift in the relative bargaining position if it alters the future distribution of the bargaining surplus and lead to preventive war if player A’s current outside option exceeds his expected future gains from bargaining.

As long as Case 1 obtains in \( t = 0 \) or Case 4 obtains in \( t = 0 \), an increase in A’s costs of war has no effect on surplus distribution because equilibrium offers do not depend on \( w_A \). In any other case, an increase in A’s costs of war changes the distribution of surplus and leads to preventive war if the war condition is fulfilled.

It is easy to verify that the war condition for the case of a shift in military power and the case of an increase in player A’s costs of war is identical when \( w_B^1 \) is not binding (Cases 1 and 3 in \( t = 1 \)), because the war condition in Cases 1 and 3 does not depend on B’s outside
option $w_B^1$.

In Cases 2 and 4, the war condition depends on $w_B^1$. In these cases, it makes a difference whether a change in military power causes $A$’s decline or an increase in his costs of war. A cost increase only affects $A$’s outside option while a shift in military power not only reduces $A$’s outside option but at the same time increases $B$’s outside option which further reduces $A$’s expected future gains from bargaining.

The finding that bargaining can break down not only because of a change in military power but also because of an isolated increase in one state’s costs of war is novel and has not yet been acknowledged in the formal literature on war initiation. Increased costs of war can result if, for example, one state intends to take measures to direct the blame of potential war to the adversary and secure diplomatic support. If this were the case, a shift in military power would not take place but still the adversary would expect to sustain a reduction in his outside option and possibly go to war in order to prevent this.

Next, we will present two cases of shifts in the relative bargaining position which, in contrast to military power shifts and costs increases, do not result in the breakdown of bargaining.

### 1.3.3 Decrease in $B$’s costs of war

A decrease in player $B$’s costs of war increases her outside option because

$$\frac{w_B}{1-\delta} = \frac{(1-p)(\pi-c_B)}{1-\delta}$$

decreases in $c_B$. This leads to a shift in the relative bargaining position in Cases 2 and 4 because in these cases, equilibrium offers depend on $w_B$. However, even though $A$’s future payoff deteriorates, he has no incentive to opt out because his outside option

$$\frac{w_A}{1-\delta} = \frac{p(\pi-c_A)}{1-\delta}$$

is independent of $c_B$.

**Corollary 2** If player $A$’s outside option remains constant, so that $w_A^0 = w_A^1$, then war is not an equilibrium outcome.

**Proof.** Player $A$ has no reason to opt for war in $t = 0$ because

$$\pi + \delta M_A^1 \geq \pi + \delta \frac{w_A^1}{1-\delta} > \frac{w_A^0}{1-\delta}$$

for $w_A^0 = w_A^1 < \pi$. ■
Notice that this result contradicts the standard argument that a shift in the player’s respective bargaining position can by itself be enough to make war a rational possibility. Here, the relative bargaining position of player $B$ can improve at the cost of diminished expected gains for player $A$, without involving inefficient outcomes.

The analysis concludes with the verification that changes in proposal power can also not be the cause of bargaining breakdowns.

### 1.3.4 Shift in proposal power

A shift in proposal power changes the players’ relative bargaining position in all cases because equilibrium offers always depend on $\alpha$. A negative shift in proposal power reduces player $A$’s expected future bargaining payoff $M_A$ and thus also positively affects the fulfillment of the war condition as shown in corollary 2. However, a negative shift in proposal power alone cannot cause preventive war because player $A$’s outside option does not decrease. This follows immediately from Corollary 2.

**Corollary 3** A reduction in player $A$’s proposal power does not lead to war.

### 1.4 Conclusion

Our model specifies the concept of commitment problems in international bargaining. We provide two main results. First, we show that a negative shift in the relative bargaining position problems does not necessarily lead to preventive war under commitment. Both, a decrease in one party’s costs of war and a loss of proposal power affect the parties’ relative bargaining position, and can diminish a party’s gains, but interestingly, cannot lead to preventive war. Second, we find that in addition to shifts in military power, increased costs of war can also result in preventive war under commitment problems.

This analysis builds the formal groundwork for preventive war arguments. It also allows conjectures about the role of third party intervention in international conflicts because it clarifies what kinds of power shifts between nations can actually induce preventive war. The model predicts that both economic (reduced costs of war) and military (higher probability
of winning war) support can improve a party’s bargaining position, while only military intervention can cause preventive war if it triggers a shift in the winning probabilities and/or increases the opponent’s costs of war.
Chapter 2

Note on the Equilibrium in a Rubinstein-type Bargaining Game with Random Proposers and Two-Sided Outside Options

Abstract

This note characterizes equilibrium behavior in a Rubinstein-type bargaining game with the possibility of counteroffers, random proposers and outside options and compares its unique SPE with the stationary SPE of the repeated bargaining game with take-it-or-leave-it offers presented in Chapter 1. It is shown that both games generate identical SPE payoffs. Since the war condition in Chapter 1 critically depends on the declining state’s future payoff, all results carry over to the one-time bargaining case discussed here.
2.1 Introduction

If a player’s bargaining power is captured by her share of the surplus, then beginning with the seminal work by Rubinstein (1982), bargaining literature has identified three independent key sources of bargaining power: the ability to propose an allocation, the ability to wait for agreement and the ability to quit the negotiation. The ability to wait, represented by the discount factor is an important element in the original model, indicating “shrinking cakes”. It also has the appealing feature of conveying bargaining power through the players’ respective valuation of time. The bargaining power resulting from the possibility of leaving the negotiation table permanently has first been analyzed by Shaked and Sutton (1984) and further explored in the works of Binmore, Shaked and Sutton (1989) and Ponsatí and Sákovics (1998). Proposal power on the other hand has long been considered the less attractive feature of the alternating offer protocol which conveys an undesired advantage to moving first.

The one-time bargaining model studied here is a variation of the Rubinstein alternating offer game with two-sided outside options. The alternating offer protocol is substituted with random determination of the proposer in each negotiation round. This constitutes a game in which all three sources of bargaining power - proposal power, discount factor and outside option - are variables. I find that this game has a unique equilibrium and that the players’ expected payoffs in this equilibrium coincide with the expected per period payoffs of the game in Chapter 1 which features infinitely repeated interaction but no counter offers. Because the expected payoffs are the same in both games and the analysis of changes in bargaining power depends on expected payoffs, all the results presented in Section 1.4 carry over to this game.

2.2 The Model

Negotiators A (he) and B (she) bargain about the distribution of an issue of size \( \pi > 0 \). Before the negotiation starts, each player’s proposal power is determined exogenously. Proposal power is measured by a fixed variable \( \alpha \in (0,1) \). More specifically, \( \alpha \) determines the probability that player A can make a proposal on the distribution in the current period. Players A and B discount future payoffs with a common discount factor \( 0 < \delta < 1 \). A player’s proposal
is denoted $x_i^t \forall t \in (0, \infty)$ where $x_i^t$ refers to the share of the pie that player $i$ receives in this period. A player can respond to a proposal in three different ways: accept the offer, reject the offer and make a counteroffer or opt out. In case of agreement, the pie is shared according to the proposal and the game ends. When the responder opts out, both players get their respective outside option payoffs $w_A$ and $w_B$ and the game ends. The game continues until one player accepts the other’s proposal or opts out. In case of perpetual disagreement each player’s payoff is zero. Consequently, player $A$’s and $B$’s expected utilities are defined as

$$u_A^t = \alpha x_A^t + (1 - \alpha)(\pi - x_B^t)$$

$$u_B^t = \alpha(\pi - x_A^t) + (1 - \alpha)x_B^t$$

in any subgame, in which both players accept.

*Figure 2.1* below illustrates the game tree:

---

**Proposition 1** In the unique subgame perfect equilibrium of the one-time bargaining model with random proposers and outside options, agreement is reached at $t = 0$ and the SPE is
Pareto efficient. Proposals are determined as follows:

- player A always proposes \( x_A^* \), always accepts an offer \( \pi - x_B \) if and only if \( x_B \leq x_A^* \) and always opts out if and only if \( w_A > \delta(\alpha x_A^* + (1 - \alpha)(\pi - x_B^*)) \)

- player B always proposes \( x_B^* \), always accepts an offer \( \pi - x_A \) if and only if \( x_A \leq x_B^* \) and always opts out if and only if \( w_B > \delta(\alpha(\pi - x_B^*) + (1 - \alpha)x_A^*) \), where

\[
x_A^* = \begin{cases} 
(1 - \delta(1 - \alpha))\pi & \text{if } w_A \leq \delta\alpha\pi \cap w_B \leq \delta(1 - \alpha)\pi \\
\pi - w_B & \text{if } w_A > \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
(1 - \delta)\pi + \delta(1 - \alpha)w_A & \text{if } w_A > \delta\alpha\pi \cap w_B \leq \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
\pi - w_B & \text{if } w_A \leq \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \delta(1 - \alpha)\pi
\end{cases}
\]

\[
x_B^* = \begin{cases} 
(1 - \delta)\pi & \text{if } w_A \leq \delta\alpha\pi \cap w_B \leq \delta(1 - \alpha)\pi \\
\pi - w_A & \text{if } w_A > \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
\pi - w_A & \text{if } w_A > \delta\alpha\pi \cap w_B \leq \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
\frac{(1 - \delta)\pi + \delta(1 - \alpha)w_B}{1 - \delta(1 - \alpha)} & \text{if } w_A \leq \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \delta(1 - \alpha)\pi
\end{cases}
\]

Given these optimal proposals, the player’s expected payoffs are exactly the same as the expected payoffs that result in any period in the repeated game with take-it-or-leave-it offers, given the optimal proposals defined by Theorem 1:1

\[
u_A^* = \begin{cases} 
\alpha\pi & \text{if } w_A \leq \delta\alpha\pi \cap w_B \leq \delta(1 - \alpha)\pi \\
\alpha(\pi - w_B) + (1 - \alpha)w_A & \text{if } w_A > \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
\frac{(1 - \delta)\pi + (1 - \alpha)w_A}{1 - \delta(1 - \alpha)} & \text{if } w_A > \delta\alpha\pi \cap w_B \leq \frac{\delta(1 - \alpha)(\pi - w_A)}{1 - \delta(1 - \alpha)} \\
\frac{\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} & \text{if } w_A \leq \frac{\delta\alpha(\pi - w_B)}{1 - \delta(1 - \alpha)} \cap w_B > \delta(1 - \alpha)\pi
\end{cases}
\]

1see p.13, Chapter 1
Proof. In any SPE that satisfies the properties of stationarity and no-delay player \( i \) is indifferent between accepting and not accepting player \( j \)'s equilibrium offer. That is

\[
\begin{align*}
\pi - x_A^* &= \max \{ \delta (\alpha (\pi - x_A^*) + (1 - \alpha)x_B^*) , w_B \} \\
\pi - x_B^* &= \max \{ \delta (\alpha x_A^* + (1 - \alpha)(\pi - x_B^*)) , w_A \}
\end{align*}
\]

The unique solution to these equations is stated in the proposition. Since the solution is unique there exists at most one SPE satisfying the properties of stationarity and no-delay. It is easy to verify that there does not exist another subgame perfect equilibrium by exploiting the stationary structure that underlies the random-proposer game: any two subgames have an identical strategic structure which means that the sets of SPE payoffs to each player in any two SPE are identical. Showing that the maximum and minimum values of the payoff sets are identical leads to the conclusion that the payoffs to each player in any two SPE are identical. Let \( M_i \) denote the supremum and \( m_i \) the infimum of equilibrium payoffs to player \( i \) in any subgame. The following must hold:

\[
\begin{align*}
m_A &\geq \pi - \max \{ \delta (\alpha (\pi - m_A) + (1 - \alpha)M_B) , w_B \} \quad (2.1) \\
M_A &\leq \pi - \max \{ \delta (\alpha (\pi - M_A) + (1 - \alpha)m_B) , w_B \} \quad (2.2) \\
m_B &\geq \pi - \max \{ \delta (\alpha M_A + (1 - \alpha)(\pi - m_B)) , w_A \} \quad (2.3) \\
M_B &\leq \pi - \max \{ \delta (\alpha m_A + (1 - \alpha)(\pi - M_B)) , w_A \} \quad (2.4)
\end{align*}
\]

Inequality (2.1) follows from the fact that, in equilibrium, player \( B \) must accept any offer.
$x_A$ with $\pi - x_A > \max\{\delta (\alpha(\pi - m_A) + (1 - \alpha)M_B), w_B\}$ because the right-hand side is the most that she can get from either refusing or opting out. Thus, in equilibrium player $A$ cannot get less than $x_A$, where $x_A < \pi - \max\{\delta (\alpha(\pi - m_A) + (1 - \alpha)M_B), w_B\}$, because he can always guarantee $x_A$ by making $x_A$ his offer.

Inequality (2.2) follows from the fact that, in equilibrium, player $B$ must get at least $x_B$, for each $x_B < \max\{\delta (\alpha(\pi - M_A) + (1 - \alpha)m_B), w_B\}$ because $x_B$ can be guaranteed either by refusing player $A$’s offer or by opting out. Hence player $A$ can get at most $\pi - \max\{\delta (\alpha(\pi - M_A) + (1 - \alpha)m_B), w_B\}$ in equilibrium. Inequalities (2.3) and (2.4) are just the same, but with the roles of players $A$ and $B$ reversed.

9 cases need to be distinguished:

1. $w_B \leq \delta (\alpha(\pi - M_A) + (1 - \alpha)m_B) \cap w_A \leq \delta (\alpha m_A + (1 - \alpha)(\pi - M_B))$
2. $w_B > \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B) \cap w_A > \delta (\alpha M_A + (1 - \alpha)(\pi - m_B))$
3. $w_B \leq \delta (\alpha(\pi - M_A) + (1 - \alpha)m_B) \cap w_A > \delta (\alpha m_A + (1 - \alpha)(\pi - m_B))$
4. $w_B > \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B) \cap w_A \leq \delta (\alpha m_A + (1 - \alpha)(\pi - M_B))$
5. $\delta (\alpha(\pi - M_A) + (1 - \alpha)m_B) < w_B \leq \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B) \cap w_A \leq \delta (\alpha m_A + (1 - \alpha)(\pi - M_B))$
6. $\delta (\alpha(\pi - M_A) + (1 - \alpha)m_B) < w_B \leq \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B) \cap w_A > \delta (\alpha M_A + (1 - \alpha)(\pi - m_B))$
7. $\delta (\alpha m_A + (1 - \alpha)(\pi - M_B)) < w_A \leq \delta (\alpha M_A + (1 - \alpha)(\pi - m_B)) \cap w_B \leq \delta (\alpha(\pi - M_A) + (1 - \alpha)m_B)$
8. $\delta (\alpha m_A + (1 - \alpha)(\pi - M_B)) < w_A \leq \delta (\alpha M_A + (1 - \alpha)(\pi - m_B)) \cap w_B > \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B)$
9. $\delta (\alpha(\pi - M_A) + (1 - \alpha)m_B) < w_B \leq \delta (\alpha(\pi - m_A) + (1 - \alpha)M_B) \cap \delta (\alpha m_A + (1 - \alpha)(\pi - M_B)) < w_A \leq \delta (\alpha M_A + (1 - \alpha)(\pi - m_B))$

For cases 1–4, equilibrium offers and payoffs can be uniquely defined as stated in Proposition 1. Cases 5–9 lead to contradictions. ■

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2.3 Conclusion

The note defines the unique equilibrium in a Rubinstein-type bargaining game with the possibility of counteroffers, random proposers and two-sided outside options. It shows that the equilibrium payoffs are the same as in the structurally different game in Chapter 1 in which the base game is infinitely repeated, counteroffers are excluded and the players only have the choice between acceptance and opting out.
Chapter 3

Crisis Bargaining, Democracy and the Transparency Argument

Abstract

Transparency, along with accountability of democratic representatives, has long been identified as a key component of democratic peace. The paper shows how these two arguments interact and facilitate democratic peace in a simple game-theoretic model. Based on this analysis, I investigate how the level of transparency within the principal-agent relationship of a democratic public and its elected leader affects the crisis bargaining outcome and whether more transparency and accountability always translates into better results for the democratic public.
3.1 Introduction

Democratic peace refers to the very stable empirical observation that democracies rarely go to war with one another, but are not immune from fighting wars with non-democracies.\(^1\)

The present paper is a contribution to the game-theoretic literature which aims to explain democratic peace with informational advantages of democracies.\(^2\) This strand of literature originated in Fearon’s (1994) famous paper on audience costs and highlights the importance of successful signaling in eliminating miscommunications between democracies that may otherwise lead states to miscalculate the willingness of the opponent to fight over an issue.

Audience costs theory claims that democracies are better able to signal their intentions because democratic leaders incur audience costs if they make threats that they later fail to follow through. In contrast, statements of politically unaccountable dictators are considered to lack that source of credibility because they are able to bluff without facing domestic costs.

Despite the prominence of audience costs theory (Snyder and Borghard (2011) count over 400 references in scholarly journals), the actual relevance of audience costs in real world crisis bargaining could not be verified in empirical studies.\(^3\) Also, Snyder and Borghard point out that, in historic cases, public threats are rarely unambiguous which prevents leaders to be held fully accountable for failed threats and the audience costs argument to unfold. Weeks (2008), on the other hand, argues that democracies need not be unique in their ability to raise audience costs. She identifies various sources of audience costs in autocracies and, on these grounds, concludes that a signaling advantage for democratic leaders based on audience costs does not exist.

Numerous scholars have since followed the signaling approach and added new arguments to the pool of reasons why democracies are able to signal successfully. Smith (1998) and Guisinger and Smith (2002) include democratic politics and endogenize the credibility of a threat to the leadership selection process, in the attempt to provide a rational underpinning as to why domestic audiences punish leaders who back down from a threat. Schultz (1998) takes the public behavior of an informed opposition into account, assuming that the opposition’s

\(^1\)see for example Oneal and Russett (1997) and Maoz and Abdolali (1989)
\(^2\)for a critical appreciation of democratic peace theory see Rosato (2003)
\(^3\)for an overview of empirical studies on audience costs see Gartzke and Lupu (2012)
rhetoric qualifies as a credible signaling device. Ramsay (2004) elaborates the domestic opposition angle showing that the opposition’s endorsement of the leader can work as a costly signal in crisis bargaining and eliminate information asymmetries.

In this study, I take one step back and show that democracies do not require a costly signaling device to credibly convey their intentions but that transparency (within democracies) and accountability (of democratic representatives) is sufficient. More specifically, when political leaders care sufficiently about reelection, they are ready to align their private interest with that of the public, and because public interests are in fact quite public in democracies, the opponent knows what to expect from a democracy, leading to the phenomenon of democratic peace in the dyadic case.

This approach simplifies the analysis because it is not build around a signaling device and thus creates room to address another important question that has usually been investigated in separate articles and separate models but is essentially related: The question of what level of transparency is optimal within the agency relationship between the political leader and the voters. Especially, why is crisis bargaining sometimes completely public while other times happening behind closed doors, and how does that affect the crisis bargaining outcome.

With that respect, I distinguish between two kinds of transparency: First, a general kind of transparency, including freedom of speech, free press and media, is the reason why all parties involved in crisis bargaining share the same information about the democratic majority’s preferences. Second, by agency transparency, I mean the ability of the principal to observe the agent’s behavior and its consequences.

The paper has two objectives: First, it is to provide a simple game-theoretic foundation of the most prevalent arguments for democratic peace: transparency and accountability. Second, it is to analyze what degree of agency transparency is preferable in an international bargaining setting by comparing two scenarios, open-door bargaining in which the democratic public can observe the bargaining process between its representative and a third party and closed-door bargaining, in which the agent’s actions are partly hidden.

I find that imperfect information can have two effects. Firstly and predictably, it can worsen the agent’s accountability and increase his incentive to follow his own preferences

\footnote{see Schultz (1999)}
instead of the principal’s which reduces the predictability of democracies and can increase the probability of a bargaining breakdown. But secondly and interestingly, less information and less accountability can also increase the opponent’s offer and lead to better outcomes for the democratic public.

The idea that revealing more information may not always be beneficial is fairly new and departs from earlier research proclaiming that agency relationships should be as transparent as possible because transparency improves accountability.\textsuperscript{5} Recent studies on career concern models suggest that in some circumstances, transparency may have detrimental effects if it leads to inefficient posturing. Prat (2005) employs a quite general model of career concerns for experts in which the agent’s type determines his ability to understand the state of the world and finds that more transparency has a negative effect if it induces the agent to disregard useful private signals and to act according to how an able agent is expected to act a priori.

The present paper is closer related to Stasavage (2004) who assumes that the public is uncertain about a representative’s preferences instead of his ability. He also finds that more transparency and less difficulty in inferring a representative’s type has the negative effect that unbiased representatives have more incentive to ignore their private signal (about the opponent’s minimum acceptable offer) and posture. Stasavage concludes that because posturing is the unique equilibrium under open-door bargaining as long as reputational concerns are sufficiently strong, “one should expect to see more uncompromising positions taken during open-door bargaining, greater polarization of debate, and more frequent breakdowns in bargaining than would otherwise be the case.” (Stasavage 2004)

In the present paper, however, a negative effect from posturing cannot arise because there is no private signal (about the opponent’s minimal offer) that could be neglected and posturing is never beneficial for unbiased agents. But even with the negative posturing effect under open-door bargaining missing, I find that closed-door bargaining can generate comparatively better results for the democratic public if it induces the opponent to increase his offer. This is why democracies may sometimes prefer closed-door bargaining even though it includes a higher risk of bargaining breakdown.

\textsuperscript{5}see Holmström (1999)
3.2 The Classical Crisis Bargaining Approach

Two parties, A and B, bargain about the distribution of an issue of size 1. Each player has an individual outside option $w_i$, which is private information. For simplification, I assume that one of the bargainers is selected at random to make a take-it-or-leave-it offer, which the opponent accepts if and only if this offer meets at least her outside option $w_i$. Otherwise, she opts out and both parties receive their respective outside option payoffs.

This modeling correlates to the literature on war bargaining that often treats war as a costly lottery which is won by party A with probability $\phi_A \in [0, 1]$ and party B with probability $\phi_B = 1 - \phi_A$. The expected gains from this lottery can be interpreted as the parties’ respective outside options $w_i = \phi_i - c_i$ of the bargaining game, where $c_i$ represent the costs of war. Note that, in this formulation, the term $c_i$ captures the relative net value that a party places on winning or losing the war. That is, $c_i$ reflects party i’s costs of war relative to any possible benefits. In practice, low costs of war translate into a high outside option or high resolve, which means that the issue at stake is highly valued and going to war a viable option at relatively small costs. On the other hand, if a party sees little to gain from winning war, then $c_i$ would be large even if the actual costs, incurred by war, were small.\(^6\)

War can occur when there is asymmetric information about the opponent’s resolve. When the opponent can either have a high outside option (high resolve) or a low outside option (low resolve), a state may have an incentive to screen the opponent’s type and make an offer which is only acceptable to the weak type, leading to war whenever the opponent is strong.

Asymmetric information about the opponent’s outside option either means that the parties have private information about the probability to win a war, which basically refers to military resources, or private information about the costs of war, which will be applied here. As stated above, high costs of war immediately result in a low outside option and low costs of war in a high outside option.

The asymmetry in information is modeled as follows: From A’s point of view, player B’s outside option is low ($w_B^l$) with probability $0 < \beta < 1$ and high ($w_B^h$) with probability $1 - \beta$, with $w_B^l < w_B^h$. Equally, player B believes that A’s outside option is low (weak type) with

\(^6\)see Fearon (1995)
probability $0 < \alpha < 1$ and high (strong type) with probability $1 - \alpha$. In contrast to the case of asymmetric information about military resources, in the case of asymmetric information about costs, a mutually beneficial bargaining outcome always exists, even when two strong types negotiate, $w^h_A + w^h_B \leq 1$.

The conditions for war are as follows:

**Conditions for War**

1. $A$ makes a screening offer $x = w^l_B$ if $\beta(1 - w^l_B) + (1 - \beta)w^i_A > 1 - w^h_B$

2. $B$ makes a screening offer $x = w^i_A$ if $\alpha(1 - w^i_A) + (1 - \alpha)w^j_B > 1 - w^h_A$

Condition 1 states that it is optimal for player $A$ given his type $w^i_A$ with $w^i_A \in \{w^l_A, w^h_A\}$ to make a screening offer. A screening offer (left hand side) gives him a profit of $1 - w^l_B$ with probability $\beta$ which is the probability that $B$ is weak and will accept a low offer $w^l_B$. With probability $1 - \beta$, however, $B$ is strong and refuses to accept such low offer. In this case, war occurs and $A$ obtains his outside option $w^i_A$. A pooling offer (right hand side) is accepted by weak and strong types and is thus a riskless profit for the proposer. Condition 2 essentially states the same with the roles of the players reversed. Generally, when a state makes a screening offer, the probability of war is positive and equals the probability that the opponent is strong. Making a screening offer is less attractive for weak types because they have a low outside option and therefore a smaller expected profit from making such offer compared to strong types. For example, when $B$ makes the offer, then the critical threshold (minimum $\alpha$) from which a screening offer is optimal for both types is given by

$$\alpha'' = \frac{1 - w^h_A - w^h_B}{1 - w^l_A - w^l_B}$$

The critical threshold from which screening is optimal for the strong but not the weak type is given by

$$\alpha' = \frac{1 - w^h_A - w^j_B}{1 - w^l_A - w^l_B}$$

with $\alpha' < \alpha''$. It follows that

(a) for $\alpha \leq \alpha'$, both types prefer a pooling offer.
(b) for $\alpha' < \alpha \leq \alpha''$, the weak type prefers a pooling and the strong type a screening offer.

(c) for $\alpha'' < \alpha$, both types prefer a screening offer.

A further illustration of the war condition is given in Figure 3.1 which depicts A’s optimal offer to B where $\tilde{\beta} = \frac{1 - w_B^{I} - w_A^{I}}{1 - w_B^{I} - w_A^{I}}$ is the critical $\beta$ from which A prefers to make a screening offer.

### 3.3 Extension of the Classical Approach

As an extension of the classical war bargaining approach, I take into account individual preferences regarding war within a state. I assume that not everyone in one state shares the same preferences regarding war but that these preferences differ. More specifically, there may be people who suffer high private costs in case of war, i.e. soldiers who do the actual fighting or pacifists who oppose the concept of war in general. On the other hand, there may be people who have much lower costs associated with war, either because they highly value the issue at stake or because their personal costs of war are much smaller, i.e. people who work in the arms industry.
Additionally, I follow in the steps of Fearon (1994), Jackson and Morelli (2007) and many others by assuming that states feature a principal-agent relationship with the agent being the political leader who engages in a crisis bargaining game with a third party on behalf of his principal, the people. While both regime types share this characteristic, the difference between democracy and autocracy is that the revelation of the principal’s preferences is private in autocracies but public in democracies. It is this general transparency within democracies that facilitates signaling to the opponent.

However, within the framework of this model, this transparency does not necessarily translate into efficient signaling if the agent’s preferences (regarding resolve for war) differ from majority’s preferences. Since, as opposed to classical principal-agent literature, in the context of political institutions, the principal cannot make a specific incentive compatible contract to induce the right behavior in the agent but can only elect and dismiss, the agent may still not be completely deterred from following his own and not majority’s preferences even under perfect information. The problem intensifies when information is imperfect and the agent’s action cannot be fully observed by the principal.

What is also interesting to note is that even though revealing weak resolve to the opponent through signaling cannot be in the principal’s best interest, otherwise there would be no reason to conceal the type and no grounds for miscommunications in the first place, the conclusion that successful signaling provides democracies with more peaceful yet disadvantageous bargaining outcomes turns out to be wrong. Although there are situations in which revealing a weak type through signaling has negative effects on the bargaining share, I will show that there are also situations in which democracies are offered comparably higher shares than autocracies in a similar position despite the public signal revealing a weak type.

The reason for this lies within the principal-agent relationship. Although the signal about majority’s preferences is public, the opponent does not know from the outset how a democratic leader will react in the bargaining game because the leader’s type is still private information. Since the public’s control over the agent is limited to removing him from office,

\footnote{In accordance with Bueno de Mesquita et al. (1999), I suppose that one decisive difference between democracy and autocracy is the size of the winning coalition, where the winning coalition subsumes all “people whose support is required to keep the incumbent in office” and is typically large in a democracy, while small in autocracies. Because the winning coalition is small in autocracies, I argue that the political leader can learn the principal’s preferences privately while in a democracy he cannot.}
and agents may be more interested in obtaining their preferable bargaining outcome than
being reelected, agents may still defect from majority’s preferences even if their actions can
be monitored. In order to satisfy such biased agents, the opponent may be ready to offer more
than he would to an autocracy. This will be discussed in Section 3.5.1 *Open-door bargaining*
and used as a reference case for Section 3.5.2 *Closed-door bargaining*. Under closed-door
bargaining, the public can monitor the agent’s choice of action imperfectly. But even though
imperfect information worsens the agent’s accountability and increases his incentive to follow
his own preferences, which counteracts successful signaling, there are situations in which less
information and less accountability leads to better outcomes for the democratic public.

3.4 The Model

Suppose there is a democratic state involved in crisis bargaining with another state, denoted
O (opponent, which can either be a democratic or an autocratic state). Let A denote the
leader (agent) of a democracy and P the democratic majority (principal). In the bargaining
game, A decides what offer to make to the opponent and what offer to accept, and majority
decides if the leader will remain in office afterwards.

The leader cares about the bargaining outcome as well as getting reelected. In the bar-
gaining game he decides whether to act in the principal’s interest (unbiased) or his own
interest (biased). The principal has two types, \( w \in \{w_l, w_h\} \) depending on the democratic
majority’s preferences, where majority is weak \( (w_l) \) with probability \( \beta \) and strong \( (w_h) \) with
probability \( 1 - \beta \). Majority’s type becomes known immediately to the agent and the oppo-
nent through a public signal. As any other individual in the democracy, the agent has two
types that differ in their preferences, he is either \( w_l \) with probability \( \beta \) or \( w_h \) with probability
\( 1 - \beta \), which is private information. I assume that the public is not interested in the agent’s
type but in his action. In the election that follows crisis bargaining, the voter’s concern is
to reelect an agent that *acted* unbiased and replace an agent that *acted* biased. The reason
for this assumption is that the bargaining issue is not a constant but subject to change so
that in the next bargaining case the leader’s preferences may be unaligned with majority’s
preferences. With this in mind, the voter is better off making his reelection decision based
on the leader’s action and not dismiss leaders who act in the public’s interest.

For simplicity, I assume that the public’s reelection decision is an increasing function of the public’s belief that the representative acted unbiased \( u \) according to the public’s preference, given the bargaining outcome \( \theta \), \( Pr(u | \theta) \).\(^8\)

The payoff to the agent is then given by

\[
U_A = \lambda \theta(w_A) + (1 - \lambda) Pr(u | \theta)
\]

with \( w_A \in \{w_l, w_h\} \), where \( \lambda \) measures the weight agents put on the bargaining outcome and \( 1 - \lambda \) the weight, they put on reputational concerns. The agent’s value of reelection is normalized to 1.

The payoff to the principal is a monotonic increasing function of the bargaining outcome and depends on his outside option \( w_P \), with \( w_P \in \{w_l, w_h\} \): \( U_P(\theta(w_P)) \).

The game proceeds as follows:

2. Public signal about principal’s type.
3. Agent chooses whether to act biased or unbiased.
4. Bargaining game: Upon random selection, either the opponent \( O \) or the agent \( A \) makes a take-it-or-leave-it offer which is either accepted or rejected.
5. Public either observes bargaining outcome and process (under open-door bargaining) or merely the bargaining outcome (under closed-door bargaining), and infers the agent’s choice of action.
6. \( A \) receives payoff based on the bargaining outcome and the public’s inferences (posterior of \( A \)’s choice).

\(^8\)See Ottaviani and Sørensen (2006) for a similar approach and review of the literature on agents with career concerns.
Because the signal about the principal’s type is private in autocracies, signaling is not possible for an autocracy within the framework of this model. The propensity of war in the Autocracy/Autocracy case therefore coincides with that predicted by classical crisis bargaining models as presented in Section 3.2. In the following equilibrium analysis I will distinguish between the Democracy/Autocracy case in which the opponent is an autocracy and the Democracy/Democracy case in which the opponent is also a democracy.

3.5 Equilibrium Analysis

Note that agents whose preferences are aligned with the principal’s preferences have every incentive to act in the principal’s interest because otherwise they would incur a payoff loss in terms of bargaining outcome and in terms of reputation. The following conditions ensure that agents whose preferences differ from the principal’s act in the principal’s interest when democracy responds to an offer:

Weak Agent

If \( w_p = w_h \) the weak agent has different preferences than the principal. In order to act unbiased he needs to be ready to reject an offer that is higher than his own outside option but below the principal’s outside option. He rejects an offer \( w_l \leq x < w_h \) as long as

\[
\lambda x + (1 - \lambda)Pr(u \mid x) \leq \lambda w_l + (1 - \lambda)Pr(u \mid war)
\]

\[\Leftrightarrow \lambda \leq \frac{Pr(u \mid war) - Pr(u \mid x)}{Pr(u \mid war) - Pr(u \mid x) + x - w_l}\]

The weak type is ready to reject an offer that is higher than his outside option if reputational concerns \((1 - \lambda)\) are high. In this case, he is ready to accept personal costs from the bargaining outcome, which is going to war despite high personal costs of war, because these costs are offset by higher reputation which also creates utility.

Note that \( Pr(u \mid war) \) and \( Pr(u \mid x) \in \{0, 1\} \) under open-door bargaining when the principal can observe the entire bargaining process but that \( Pr(u \mid war) \) and \( Pr(u \mid x) \) may well assume values between 0 and 1 under closed-door bargaining when the principal can
merely observe the bargaining outcome.

**Strong Agent**

If \( w_P = w_l \) the strong agent has different preferences than the principal. In order to act unbiased he needs to be ready to accept an offer that is below his own outside option but higher than the principal’s outside option. The strong agent accepts \( w_l \leq x < w_h \) if:

\[
\lambda w_h + (1 - \lambda) Pr(u \mid war) \leq \lambda x + (1 - \lambda) Pr(u \mid x)
\]

(3.2)

\[
\Leftrightarrow \lambda \leq \frac{Pr(u \mid x) - Pr(u \mid war)}{Pr(u \mid x) - Pr(u \mid war) + w_h - x}
\]

When the condition holds, the strong agent prefers accepting \( w_l \) over rejecting it because his reputational gains outweigh his personal costs from accepting an offer below his outside option.

### 3.5.1 Open-door bargaining

When the bargaining process is completely transparent, the game is one of perfect information. The only reason why signaling in the bargaining game might fail would be that the biased agent values the bargaining outcome so much that he is not restrained from acting on his own preferences despite negative reputation effects. Under open-door bargaining, the public can exactly discern whose turn it is to make an offer and what the offer looks like. It follows that the public is certain about whether the agent acted unbiased or biased. For the two subgames, the principal’s assessment is as follows:

1. When the opponent makes the offer and the agent responds:
   - If \( w_P = w_l \) then \( Pr(u \mid x \geq w_l) = Pr(u \mid war, x < w_l) = 1 \) and \( Pr(u \mid war, x \geq w_l) = 0 \).
   - If \( w_P = w_h \) then \( Pr(u \mid x \geq w_h) = Pr(u \mid war, x < w_h) = 1 \) and \( Pr(u \mid x < w_h) = 0 \).

2. When the agent makes the offer and the opponent responds:
• If the principal prefers a pooling offer \( x^p \) then \( \Pr(u \mid x = 1 - x^p) = 1 \) and \( \Pr(u \mid x \neq 1 - x^p) = 0. \)

• If the principal prefers a screening offer \( x^s \) then \( \Pr(u \mid x = 1 - x^s) = 1 \) and \( \Pr(u \mid x \neq 1 - x^s) = 0. \)

**Democracy/Autocracy**

I will first look at the case, that the opponent is an autocracy, meaning a single actor with an outside option \( w_O \in \{w_l, w_h\} \).

*Democracy makes the offer*

The democratic agent’s offer to the autocracy does not differ from the offer that an autocracy would make in this position as long as democratic agents are sufficiently interested in reelection. To see this, remember that the democratic principal is considered a unitary actor with preferences \( w_P \in \{w_l, w_h\} \). Just as an autocracy, he prefers to make a screening offer if the probability that the opponent is weak, \( \alpha \), is sufficiently high. The critical values for \( \alpha \) are the same as in the Autocracy/Autocracy case presented in Section 3.2. Note that when \( \alpha \leq \alpha' \), a pooling offer is optimal for both types and when \( \alpha'' < \alpha \) both types prefer to make a screening offer, so that there is no conflict of interest between the principal and the agent independent of their types. However, when \( \alpha' < \alpha \leq \alpha'' \) strong types prefer a screening offer while weak types prefer a pooling offer which may create a conflict of interest.

When \( \alpha' < \alpha \leq \alpha'' \), the following conditions guarantee that the agent acts unbiased:

When \( w_P = w_h \) and \( w_A = w_l \), the agent’s preferences differ from the principal’s. The weak agent acts unbiased if:

\[
\lambda(\alpha(1 - w_O^l) + (1 - \alpha)w_A^l) + (1 - \lambda)\Pr(u \mid x = 1 - x^s) \geq \lambda(1 - w_O^h) + (1 - \lambda)\Pr(u \mid x \neq 1 - x^s)
\]

For \( \Pr(u \mid x = 1 - x^s) = 1 \) and \( \Pr(u \mid x \neq 1 - x^s) = 0 \), the critical \( \tilde{\lambda}_s \) that guarantees that the weak agent acts unbiased is determined by

\[
\lambda \leq \frac{1}{1 - w_O^h + \alpha w_A^l + (1 - \alpha)(1 - w_A^l)} = \tilde{\lambda}_s
\]
When $w_P = w_l$ and $w_A = w_h$, the agent’s preferences differ from the principal’s. The strong agent acts unbiased if:

$$\lambda(1 - w_h^h) + (1 - \lambda)Pr(u \mid x = 1 - x^p) \geq \lambda(\alpha(1 - w_l^l) + (1 - \alpha)w_h^l) + (1 - \lambda)Pr(u \mid x \neq 1 - x^p)$$

For $Pr(u \mid x = 1 - x^p) = 1$ and $Pr(u \mid x \neq 1 - x^p) = 0$, the critical $\tilde{\lambda}_p$ that guarantees that the strong agent acts unbiased is determined by

$$\lambda \leq \frac{1}{\alpha(1 - w_l^l) + (1 - \alpha)w_h^l + w_h^O} = \tilde{\lambda}_p$$

**Corollary 4** When bargaining is open-door and reputation is sufficiently important to biased agents, so that $\lambda \leq \min\{\tilde{\lambda}_s, \tilde{\lambda}_p\}$, the optimal offer to an autocracy is independent of regime type.

Because screening offers are made as frequently as in the Autocracy/Autocracy case, given that reelection is important, the probability of war is also the same. That is why the following analysis concentrates on the offer that is accepted by the democracy because here the equilibrium offer changes with regime type when reputation is important.

**Democracy responds to an offer**

When $w_P = w_h$, the unbiased agent will accept no offer $x < w_h$. The opponent makes a pooling offer $x = w_h$, if he has no incentive to deviate to a screening offer acceptable only to the weak agent. The following condition must hold:

$$1 - w_h \geq \beta(1 - x) + (1 - \beta)w_O$$

(3.3)

The left hand side is the opponent’s payoff from making a high offer accepted by both types and the right hand side is his payoff from making an offer $x < w_h$ that is accepted with probability $\beta$ (by the weak type) and rejected with probability $1 - \beta$ (by the strong type) in which case war results leaving the opponent with his outside option $w_O$. An offer $x < w_h$ is acceptable to the weak agent if his incentive constraint, Condition 3.1, fails. For
\[ Pr(u \mid war, x < w_h) = 1 \] and \[ Pr(u \mid x < w_h) = 0, \] that is if
\[ x > \frac{1}{\lambda} + w_l - 1 = x_h \] (3.4)

So \( x_h \) defines the weak agent’s minimum acceptable offer. Let \( x'_h \) define the maximum screening offer the opponent is ready to make to the weak agent. It is determined by failure of Condition 3.3:
\[ x < 1 - w_O - \frac{1 - w_O - w_h}{\beta} = x'_h \] (3.5)

It follows from Conditions 3.4 and 3.5 that there is a screening equilibrium for \( x_h < x'_h \) in which the opponent makes a screening offer \( x = max\{w_l, x_h\} \) that is accepted by the weak type and rejected by the strong type, and there is a pooling equilibrium for \( x_h \geq x'_h \) in which the offer \( x = w_h \) is accepted by both types of agent.

Let \( \tilde{\lambda}_h \) define the maximum \( \lambda \) for which a pooling equilibrium exists. Then \( \tilde{\lambda}_h \) is determined by:
\[ x_h \geq x'_h \]
\[ \iff \frac{1}{\lambda} + w_l - 1 \geq 1 - w_O - \frac{1 - w_O - w_h}{\beta} \]
\[ \iff \lambda \leq \frac{\beta}{(1 - w_l)\beta - (1 - \beta)(1 - w_O) + w_h} = \tilde{\lambda}_h \] (3.6)

For \( \lambda > \tilde{\lambda}_h \) a screening equilibrium results.

If \( w_P = w_l \), the opponent makes a pooling offer \( x \geq w_l \) acceptable to both types if his payoff from making such an offer (right hand side of Condition 3.7 below) exceeds his payoff from making a screening offer \( x = w_l \) that is only accepted by the weak type (left hand side of Condition 3.7):
\[ \beta(1 - w_l) + (1 - \beta)w_O \leq 1 - x \] (3.7)

An offer \( w_l \leq x < w_h \) is acceptable to both types of agent if the strong type’s incentive constraint, Condition 3.2, holds. For \( Pr(u \mid war, x \geq w_l) = 0 \) and \( Pr(u \mid x \geq w_l) = 1 \), that is if
\[ x \geq 1 + w_h - \frac{1}{\lambda} = x_l \] (3.8)
So \( x_l \) defines the strong agent’s minimum acceptable offer. Let \( x'_l \) define the maximum pooling offer the opponent is ready to make, which results from Condition 3.7:

\[
x \leq 1 - \beta (1 - w_l) - (1 - \beta) w_O = x'_l
\]  
(3.9)

It follows from Conditions 3.8 and 3.9 that there is a screening equilibrium for \( x_l > x'_l \) in which the opponent makes a screening offer \( x = w_l \) that is accepted by the weak unbiased type and rejected by the strong biased type, and there is a pooling equilibrium for \( x_l \leq x'_l \) in which the offer \( x = \max\{x_l, w_l\} \) is accepted by both types of agent.

Let \( \tilde{\lambda}_l \) define the maximum \( \lambda \) for which a pooling equilibrium exists. Then \( \tilde{\lambda}_l \) is determined by:

\[
x'_l \geq x_l \\
\iff 1 - \beta (1 - w_l) - (1 - \beta) w_O \geq 1 + w_h - \frac{1}{\lambda} \\
\iff \lambda \leq \frac{1}{(1 - w_l)\beta + (1 - \beta)w_O + w_h} = \tilde{\lambda}_l
\]  
(3.10)

For \( \lambda > \tilde{\lambda}_l \) a screening equilibrium results.

**Corollary 5** When bargaining is open-door and reputation is sufficiently important to biased agents, so that \( \lambda \leq \min\{\tilde{\lambda}_l, \tilde{\lambda}_h\} \), the opponent makes a pooling offer \( x = w_h \) if \( w_P = w_h \) and \( x = \max\{w_l, x_l\} \) if \( w_P = w_l \) and democracy always accepts this offer in equilibrium. When \( \lambda > \tilde{\lambda}_h \) and \( w_P = w_h \), the opponent makes a screening offer \( x = \max\{w_l, x_h\} \) that is rejected with probability \( 1 - \beta \), by the strong unbiased type. When \( \lambda > \tilde{\lambda}_l \) and \( w_P = w_l \), the opponent makes a screening offer \( x = w_l \) that is rejected with probability \( 1 - \beta \), by the strong biased type.

Figures 3.2 and 3.3 below show the equilibrium offer in the two cases \( w_P = w_h \) and \( w_P = w_l \). We can see that as long as reelection concerns are sufficiently important \( (\lambda \leq \min\{\tilde{\lambda}_l, \tilde{\lambda}_h\}) \), the opponent makes a pooling offer that is peacefully accepted.

In the following, I will analyze whether democratic transparency, that is, revealing the democratic public’s type to the opponent in international bargaining leads to disadvantageous bargaining outcomes for democracies. To do so, I compare the equilibrium offer that the
opponent makes in the Democracy/Autocracy case with that in the Autocracy/Autocracy case. We know from Section 3.2 that in the Autocracy/Autocracy case, the opponent makes a pooling offer \( w_h \) if the probability that the autocrat is a weak type is sufficiently low, \( \beta \leq \tilde{\beta} \) and a screening offer \( w_l \) if \( \tilde{\beta} < \beta \).

It follows that a democracy is comparably worse off in terms of the bargaining share when \( \beta \leq \tilde{\beta} \) and \( w_P = w_l \) because \( x^* < w_h \), see Figure 3.3 below. When \( w_P = w_h \) and the agent cares sufficiently about reelection, so that \( \lambda \leq \tilde{\lambda}_h \) the outcome is the same for both regime types, a pooling offer \( w_h \), see Figure 3.2 below.

When \( \beta > \tilde{\beta} \), the opponent makes a screening offer \( w_l \) in the Autocracy/Autocracy case. In that case, the share that the opponent offers to a democracy is at least as high as what he would offer an autocracy. When \( w_P = w_h \) and the agent cares sufficiently about reelection, so that \( \lambda \leq \tilde{\lambda}_h \) the equilibrium offer is strictly higher \( w_h > w_l \) in the Democracy/Autocracy case, see Figure 3.2. When \( w_P = w_l \) and \( \lambda \leq \tilde{\lambda}_l \), the equilibrium offer is higher than in the Autocracy/Autocracy case if \( x^* = x_l > w_l \), otherwise it is the same.

---

This is shown in Figure 3.1.
Overall, the probability of war is smaller in the Democracy/Autocracy as long as democratic agents care sufficiently about reelection. Even though, democracies make screening offers as frequently as autocracies so that the probability of war is the same as in the Autocracy/Autocracy case whenever it is democracy’s turn to make an offer, when it is democracy’s turn to respond, the opponent’s (pooling) offer is accepted, so that the probability of war is zero in this case.

**Democracy/Democracy**

When the opponent is a democracy, there is the opposing state’s principal and agent to take into account. Let $O_A$ denote the agent from the opposing state and $O_P$ its principal, with $w_{O_P} \in \{w_l, w_h\}$ and $w_{O_A} \in \{w_l, w_h\}$ their respective outside options.

Democratic peace results if both states make pooling offers. First, I look at the case when the home state principal’s outside option is $w_P = w_h$. The opposing principal’s incentive condition for a pooling offer $x = w_h$ coincides with Condition 3.3. It follows that for $\lambda \leq \tilde{\lambda}_h$, $O_P$ prefers a pooling offer $x_h$ which is the minimum offer that a weak home agent is ready to accept and determined by Condition 3.4.
When \( w_{OP} = w_h \), then a pooling offer is always optimal independent of \( \lambda \) because if a pooling offer is optimal for the strong type, it is also optimal for the weak type. When \( w_{OP} = w_l \) and \( w_{OA} = w_h \), the opposing state’s agent \( O_A \) might have an incentive to make a screening offer. He is ready to make the pooling offer if

\[
\lambda(1 - w_h) + (1 - \lambda)Pr(u \mid x = 1 - x^P) \geq \lambda(\beta(1 - w_h) + (1 - \beta)w_{OA}) + (1 - \lambda)Pr(u \mid x \neq 1 - x^P)
\]

Let \( \tilde{\lambda}_{Oh} \) denote the maximum \( \lambda \) at which \( O_A \) is ready to make the pooling offer. For \( Pr(u \mid x = 1 - x^P) = 1 \) and \( Pr(u \mid x \neq 1 - x^P) = 0 \), \( \tilde{\lambda}_{Oh} \) is determined by

\[
\lambda \leq \frac{1}{\beta(1 - w_h) + (1 - \beta)w_{OA} + w_h} = \tilde{\lambda}_{Oh} \tag{3.11}
\]

When the home state principal’s outside option is \( w_P = w_l \), the opposing principal’s incentive condition for a pooling offer \( x \geq w_l \) coincides with Condition 3.7. It follows that for \( \lambda \leq \tilde{\lambda}_l \), \( O_P \) prefers the pooling offer \( x_l \) which is the minimum offer that is acceptable to both types of the home agent and determined by Condition 3.8. Again, when \( w_{OP} = w_h \), a pooling offer is always optimal independent of \( \lambda \). When \( w_{OP} = w_l \) and \( w_{OA} = w_h \), \( O_A \) might have an incentive to make a screening offer. He is ready to make the pooling offer if

\[
\lambda(1 - x) + (1 - \lambda)Pr(u \mid x = 1 - x^P) \geq \lambda(\beta(1 - w_l) + (1 - \beta)w_{OA}) + (1 - \lambda)Pr(u \mid x \neq 1 - x^P)
\]

Let \( \tilde{\lambda}_{Ol} \) denote the maximum \( \lambda \) at which \( O_A \) is ready to make the pooling offer. For \( Pr(u \mid x = 1 - x^P) = 1 \) and \( Pr(u \mid x \neq 1 - x^P) = 0 \), \( \tilde{\lambda}_{Ol} \) is determined by

\[
\lambda \leq \frac{1}{\beta(1 - w_l) + (1 - \beta)w_{OA} + x_l} = \tilde{\lambda}_{Ol} \tag{3.12}
\]

**Corollary 6** When reputation is sufficiently important to democratic agents, pooling offers are made and accepted both ways, which constitutes the democratic peace.
3.5.2 Closed-door bargaining

When bargaining takes place behind closed doors, there is informational asymmetry. The principal cannot observe who makes the offer, what the offer looks like and which offer has been rejected by whom.

Under closed-door bargaining, the agent’s payoff can change because \( Pr(u \mid war) \) is no longer certain. When an offer is accepted, the principal can still infer who made the offer and whether the agent acted in a biased manner.\(^{10} \) However, when it comes to the rejection of an offer and war occurs, the principal can no longer identify the reason, because war can result either if the agent rejects an offer or if the agent’s offer is rejected.

Democracy/Autocracy

Under closed-door bargaining, the principal cannot distinguish between responder and proposer which means that there are no longer two subgames that can be analyzed independently. The principal’s optimal offer to the opponent depends on the principal’s type and the probability that the opposing autocrat is weak, as explained in Section 3.2:

(a) \( \alpha \leq \alpha' \), both types prefer a pooling offer.

(b) \( \alpha' < \alpha \leq \alpha'' \), the weak type prefers a pooling and the strong type a screening offer.

(c) \( \alpha'' < \alpha \), both types prefer a screening offer.

When \( w_P = w_h \), the agent acts according to the principal’s preferences if he rejects \( x < w_h \) and, in Cases (b) and (c), makes a screening offer. It follows that in Cases (b) and (c) \( Pr(u \mid war) = 1 \), independent of the opponent’s offer because acting biased either means accepting an offer \( x < w_h \) or making a pooling offer to the opponent, both actions which do not lead to war. For \( Pr(u \mid x < w_h) = 1 \) (and \( Pr(u \mid x < w_h) = 0 \)) the weak type’s incentive constraint is the same as under open-door bargaining and equilibrium is determined as stated in Corollary 5.

In Case (a), it is optimal for both types to make a pooling offer to the opponent. In this case,

\(^{10}\) In case of agreement, the principal’s share is \( 1 - w^h_O \) when the agent made a pooling offer, \( 1 - w^l_O \) when the agent made a screening offer, and \( w_l \leq x \leq w_h < 1 - w^l_O \) when the opponent made a pooling offer.
\( Pr(u \mid war) \) is an off-equilibrium belief if the opponent’s best response is also a pooling offer. Otherwise, when the opponent makes a screening offer, \( Pr(u \mid war) = 0 \) which is the same as under open-door bargaining so that the results are the same. Only when the off-equilibrium belief \( Pr(u \mid war) > 0 \), the weak type’s incentive constraint, Condition 3.4 changes:

\[
x_{h,c} = \frac{1 - \lambda}{\lambda} Pr(u \mid war) + w_l \tag{3.13}
\]

where \( x_{h,c} < x_h \) for \( Pr(u \mid war) < 1 \). In terms of \( \lambda \), a screening equilibrium results if:

\[
x_{h,c} \geq x'_h \Rightarrow \frac{1 - \lambda}{\lambda} Pr(u \mid war) + w_l \geq 1 - w_O - \frac{1 - w_O - w_h}{\beta}
\]

so that the critical \( \lambda \) from which screening is optimal is determined by

\[
\lambda \leq \frac{\beta Pr(u \mid war)}{(Pr(u \mid war) - w_l)\beta - (1 - \beta)(1 - w_O) + w_h} = \hat{\lambda}_{h,c} \tag{3.14}
\]

Since \( \hat{\lambda}_{h,c} < \hat{\lambda}_h \) for \( Pr(u \mid war) < 1 \), the critical threshold for a screening equilibrium is lower compared to open-door bargaining for \( Pr(u \mid war) < 1 \).

However, \( Pr(u \mid war) < 1 \) is not a plausible off-equilibrium belief because it would mean that the principal believes that the agent did not act in his interest in case of war but somehow acted more aggressive. This is not possible since the principal is a strong type and the agent can either be equally strong or weaker. The only plausible off-equilibrium belief is \( Pr(u \mid war) = 1 \) for which the results do not differ from the results under open-door bargaining.

**Corollary 7** When \( w_P = w_h \), the results are the same as under open-door bargaining in Cases (b) and (c). In Case (a), the threshold for a pooling equilibrium may be below the threshold under open-door bargaining, however only for implausible off-equilibrium beliefs. For the only plausible off-equilibrium belief the results are the same as under open-door bargaining.

When \( w_P = w_l \) and Case (c), \( Pr(u \mid war) = 1 \) if the opponent makes a pooling offer \( x \leq w_h \) and the strong agent’s incentive constraint holds. But when \( Pr(u \mid war) = Pr(u \mid x \geq w_l) = \)

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1, the strong agent’s incentive constraint, Condition 3.2, only holds if \( x = w_h \), otherwise, for \( x < w_h \), it fails:
\[
\lambda x + (1 - \lambda) Pr(u | x) < \lambda w_h + (1 - \lambda) Pr(u | war)
\]

It follows that \( x = w_h \) is the only candidate for a pooling equilibrium.

The opponent is ready to offer \( x = w_h \) as long as
\[
\beta (1 - w_l) + (1 - \beta) w_O \leq 1 - w_h
\]
\[
\Leftrightarrow \beta \leq \frac{1 - w_O - w_h}{1 - w_O - w_l} = \tilde{\beta}
\]
(3.15)

For \( \beta > \tilde{\beta} \) a pooling equilibrium does not exist.

There is a screening equilibrium, in which the opponent offers \( x = w_l \) and the offer is accepted by the weak and rejected by the strong type. Since in Case (c) it is optimal to make a screening offer to the opponent, \( Pr(u | war) = \frac{\alpha \beta}{1 - \beta + \alpha} \) in the screening equilibrium. The opponent has no incentive to deviate to a pooling offer \( w_l < x \leq w_h \) acceptable to both types if such pooling offer generates less payoff than the screening offer \( x = w_l \). The critical offer \( x'_{l,c} \) for which the opponent still prefers pooling is determined by Condition 3.9, so that:
\[
x'_{l,c} = x_{l,c} = 1 - \beta (1 - w_l) - (1 - \beta) w_O
\]

As long as this offer is smaller than the minimum offer the strong agent is ready to accept, a screening equilibrium results. The strong agent’s minimum acceptable offer is determined by Condition 3.2. and given by
\[
x_{l,c} = w_h - \frac{1 - \lambda}{\lambda} (1 - Pr(u | war))
\]

It follows that a screening equilibrium results if,
\[
x_{l,c} > x'_{l,c}
\]
\[
\Leftrightarrow w_h - \frac{1 - \lambda}{\lambda} (1 - Pr(u | war)) > 1 - \beta (1 - w_l) - (1 - \beta) w_O
\]
Rearranging this condition gives the critical $\lambda$ from which a screening equilibrium results

$$\lambda > \frac{1 - Pr(u \mid war)}{(1 - w_l)\beta + (1 - \beta)w_O + w_h - Pr(u \mid war)} = \tilde{\lambda}_{l,c}$$

It is easy to check that $\tilde{\lambda}_l > \tilde{\lambda}_{l,c}$ for $Pr(u \mid war) > 0$. Since $Pr(u \mid war) = \frac{\alpha\beta}{1 - \beta + \alpha} > 0$ under closed-door bargaining, the threshold for a screening equilibrium is lower than under open-door bargaining.

When $\lambda \leq \tilde{\lambda}_{l,c}$, there are partial pooling equilibria in which the opponent offers $w_l < x < w_h$ which the weak type always accepts and the strong type accepts with probability $\theta$ and rejects with probability $1 - \theta$. In these equilibria, the opponent chooses $x$ so that the strong type is indifferent between accepting $x$ and rejecting it:

$$\lambda x + (1 - \lambda)Pr(u \mid x) = \lambda w_h + (1 - \lambda)Pr(u \mid war)$$

$$\iff x = w_h - \frac{1}{\lambda}Pr(u \mid x) - Pr(u \mid war))$$

For $Pr(u \mid x \geq w_l) = 1$ and $Pr(u \mid war) = \frac{\frac{1}{2}\alpha\beta}{(\alpha\beta + \frac{1}{2}(1 - \theta)(1 - \beta))(1 - \alpha)} = \frac{\alpha\beta}{(1 - \theta)(1 - \beta + \alpha) + \theta\alpha}$ we get

$$x = w_h - \frac{1 - \lambda}{\lambda}(1 - \theta)(1 - \beta)(1 + \alpha) = x_{l,c}$$

which defines the equilibrium offer.

In a partial pooling equilibrium, $\theta$ must be such that the opponent prefers making the pooling offer over making the screening offer $x = w_l$ acceptable only to the weak type:

$$\beta(1 - w_l) + (1 - \beta)w_O \leq (\beta + (1 - \beta)\theta)(1 - x) + (1 - \beta)(1 - \theta)w_O$$

Rearranging the condition above gives the maximum offer the opponent is ready to make in a partial pooling equilibrium:

$$x \leq 1 - \frac{\beta(1 - w_l) + (1 - \beta)\theta w_O}{\beta + (1 - \beta)\theta} = x'_{l,c}$$
In a partial pooling equilibrium, the opponent chooses $x = \max\{w, x_{l,c}\}$. Since $x_l < x_{l,c}$, the offer in the partial pooling equilibria is strictly higher than in the pooling equilibrium under open-door bargaining as long as $x_{l,c} > w_l$. Also, $x_{l,c}$ increases in $\theta$. The more often the strong type accepts the pooling offer, the higher the belief that war has been started in the principal’s interest, and the higher the strong type’s incentive to exploit this belief which induces the opponent to increase his offer. Since $x_l' > x_{l,c}'$ for all $\theta < 1$ and $x_l < x_{l,c}$, the threshold for (partial) pooling is lower than the threshold for pooling under open-door bargaining.

In Cases (a) and (b), the agent acts in the principal’s interest when he makes a pooling offer. Then $Pr(u | war)$ is an off-equilibrium belief if the opponent’s best response is also a pooling offer, and the strong agent is ready to accept and make a pooling offer. Otherwise, when either the opponent makes a screening offer or the strong agent makes a screening offer to the opponent, $Pr(u | war) = 0$ which is the same as under open-door bargaining so that the results are the same. Only when the off-equilibrium belief $Pr(u | war) > 0$, the strong agent’s incentive constraint differs from that under open-door bargaining and the minimum offer acceptable to both types changes:

$$x_{l,c} = w_h - \frac{1 - \lambda}{\lambda} (1 - Pr(u | war))$$  \hfill (3.18)

where $x_{l,c} > x_l$ for $Pr(u | war) > 0$. Since $x_{l,c} > x_l$ for $Pr(u | war) > 0$, a pooling equilibrium results for a lower range of $\lambda$. We can see this by comparing the minimum acceptable offer to both types $x_{l,c}$ with the maximum pooling offer the opponent is ready to make $x_l'$ which is defined by (3.9). A pooling equilibrium results if

$$x_{l,c} \leq x_l'$$

$$\Leftrightarrow w_h - \frac{1 - \lambda}{\lambda} (1 - Pr(u | war)) \leq 1 - w_O - \frac{1 - w_O - w_h}{\beta}$$

$$\Leftrightarrow \lambda \leq \frac{1 - Pr(u | war)}{w_h - Pr(u | war) + w_O - \frac{1 - w_O - w_h}{\beta}} = \tilde{\lambda}_{l,c}$$  \hfill (3.19)
which defines the critical $\lambda$ for which a pooling equilibrium results. Since $\tilde{\lambda}_{l,c} < \tilde{\lambda}_l$ for $Pr(u \mid war) > 0$, the critical threshold for a pooling equilibrium is lower compared to open-door bargaining for $Pr(u \mid war) > 0$.

However, $Pr(u \mid war) > 0$ is not a plausible off-equilibrium belief because it would imply that going to war was in the principal’s best interest when we know that the principal is weak and a weak type would always prefer to stick to his equilibrium action instead of going to war. The only plausible off-equilibrium belief is $Pr(u \mid war) = 0$ for which the results are the same as under open-door bargaining.

**Corollary 8** When $w_P = w_l$, and it is optimal for both types to make a screening offer to the opponent (Case (c)), there is a pooling equilibrium for $\beta \leq \tilde{\beta}$ in which the opponent offers $x = w_h$. In addition, there are partial pooling equilibria for $\lambda \leq \tilde{\lambda}_{l,c} < \tilde{\lambda}_l$, with a positive probability of war but potentially higher offers compared to open-door bargaining, since $x_{l,c} > x_l$, and a screening equilibrium for $\lambda > \tilde{\lambda}_{l,c}$ with the same offer as under open-door bargaining. In Cases (b) and (c), the results are the same as under open-door bargaining for the only plausible off-equilibrium belief.

When $w_P = w_l$, and it is optimal from the principal’s point of view to make a screening offer to the opponent, closed-door bargaining can generate a higher pooling offer from the opponent compared to open-door bargaining. The offer in the screening equilibrium is the same under open- and closed-door bargaining. However, the threshold $\lambda$ for which a screening equilibrium results is lower than under open-door bargaining. When $\lambda$ is below this threshold, $\lambda \leq \tilde{\lambda}_{l,c}$, partial pooling equilibria can be supported in which the offer may be higher than under open-door bargaining. The reason for this is that under open-door bargaining the principal can immediately observe whether an agent acts in the principal’s interest. More specifically, when the agent rejects an offer, the principal knows that the agent is biased. Here, observing war, does not necessarily mean that the agent acted biased because war can also occur when the agent acted in the principal’s interest by making a screening offer to the opponent. That is why, the posterior belief that the agent acted in the principal's interest given that war can be observed has to be positive, $Pr(u \mid war) > 0$. This higher $Pr(u \mid war)$ increases the strong agent’s incentive to reject the opponent’s offer so that the opponent
needs to make a higher offer in order to keep the strong agent satisfied under closed-door bargaining.

Besides these two equilibria types, there is a pooling equilibrium for $\beta \leq \tilde{\beta}$ that only exists under closed-door bargaining in which the opponent offers $x = w_h$ even though the public signal is low. This equilibrium coincides with the pooling equilibrium in the Autocracy/Autocracy case.

The results from Corollary 7 and 8 suggest that a democracy bargaining with an autocracy prefers closed-door bargaining whenever the democratic public has a weak resolve for war and is indifferent between open- and closed door bargaining when the democratic public is highly resolved.

**Democracy/Democracy**

Section 3.5.1 shows that when reelection is sufficiently important to democratic agents, pooling offers are made and accepted in equilibrium when the bargaining process is public. Under closed-door bargaining, this equilibrium also exists with $Pr(u \mid war)$ as an off-equilibrium belief.
3.6 Conclusion

The paper develops a simple game-theoretic model based on the bargaining approach to war. The analysis shows that general transparency within democracies and accountability of democratic leaders enables democracies to signal their resolve for war successfully, so that war can be avoided when two democracies interact, constituting the phenomenon of democratic peace. Further, I show that this revealing of preferences need not necessarily be to the democracy’s disadvantage, but that under some circumstances, democracies can capture higher shares in the bargaining game than an autocrat would in the same position.

Another contribution of the paper is to analyze how the level of agency transparency affects the outcome of crisis bargaining. In this regard, I analyze under which circumstances democracies prefer open- or closed-door bargaining.
Chapter 4

Dynamic Pricing in Lemons Markets

Abstract

In this paper, I study markets with two-sided asymmetric information: quality uncertainty on part of some buyers and uncertainty about the buyer’s knowledge of quality on part of the sellers. It will be shown that resolving one source of uncertainty, by giving sellers the opportunity to learn the buyer’s type and price discriminate, can lead to increased adverse selection and a decrease in overall welfare. The reason for this is that informed buyers can act as a deterrent for low quality sellers to mimic good quality when sellers can only quote one price to all buyers.
4.1 Introduction

Ever since Akerlof’s (1970) seminal paper, adverse selection has been widely investigated in different market models and under different assumptions. One assumption, that is surprisingly robust in the literature, is the idea that all buyers are equally uninformed about quality.

The present paper seeks to relax this assumption. Its starting point is motivated by previous research on customer-specific price discrimination which supports the idea that buyers are not homogenous in their knowledge of quality and that sellers use certain cues to identify informed buyers. There is evidence to this effect from traditional on-site transactions, and even more so from online transactions.

For example, Ayres and Siegelman (1995) study race and gender discrimination in on-site bargaining for a new car and find that black and female test buyers are quoted significantly higher prices than their male counterparts using identical, scripted bargaining strategies. As a possible explanation for this result, Ayres and Siegelman suggest that car dealers believe white males to be more knowledgeable of quality. They observe that “dealers were somewhat more likely to volunteer information about the cost of the car to white males than to the other testers, possibly because they believed that white males already had such information.” (Ayres and Siegelman 1995) The study suggests that there are important cues that salespeople can use in on-site transactions to gauge a customer’s information status and infer his willingness to pay.

Even though transactions over the internet obfuscate some of these consumer characteristics, the increasing development of information technologies and web-browser cookies allows firms to collect, keep, and process even more and possibly more precise information about consumers. Information about consumers’ previous buying or search behavior may be used to charge customer-specific prices, a pricing practice known in the literature as behavior-based price discrimination.¹

It is only a small step from there to conjecture that firms can use a consumer’s browser history to infer the consumer’s knowledge of quality and identify him as an informed or

uninformed buyer.

In this paper, I am interested in the question, if sellers can obtain information about the buyer’s knowledge of the product’s quality, how does that affect adverse selection, and efficiency in lemons markets in general. The purpose of the paper is twofold: First, it is to establish equilibria in markets susceptible to the lemons problem, in which some buyers also have knowledge of quality. Second, it is to clear what welfare effects customer-specific price discrimination has in such markets.

To that end, two settings are investigated, a uniform pricing regime in which sellers do not price discriminate and a dynamic pricing regime in which sellers can acquire consumer data, identify the buyer’s type and price discriminate on an individual level based on the buyer’s knowledge of quality.

In order to account for differences in buyer information, I employ a lemons market model in which sellers set individual prices and are matched randomly with buyers, a market model that has first been analyzed in Wilson (1980). In contrast to the centralized market approach in Akerlof (1970), buyers and sellers can be modeled heterogenously and there may be different prices in the market instead of only one price.

I find that, under uniform pricing, the existence of informed buyers can reduce adverse selection. Since informed buyers do not buy low quality at a high price, the low quality seller’s incentive to offer at a high price is diminished because he can only quote one price to all consumers and needs to take into account the informed buyers resistance to buy overpriced low quality goods. Anticipating the low quality seller’s reduced incentive to cheat, uninformed buyers are ready to buy at a high price more frequently which reduces adverse selection.

When dynamic pricing becomes possible (through browser histories or other methods), the market gets segmented based on consumer types. First, there is an informed buyers market in which trade can always be concluded. Second, there is an uninformed buyers market in which adverse selection may be more severe than under uniform pricing due to the absence of informed buyers. Only if average quality is sufficiently high and there is a pooling equilibrium in the uninformed buyer’s market, dynamic pricing leads to a welfare improvement. Whenever the sellers separate in their prices and quote different prices to uninformed buyers, adverse selection is more severe than in the unsegmented market because
low quality sellers no longer need to factor in informed buyers’ resistance to buy at a high price. In these cases, dynamic pricing leads to a welfare loss.

Related Literature

There is extensive literature on markets with quality uncertainty, which, following Akerlof (1970), are termed lemons markets. Only few papers have so far considered lemons markets with a partially informed buyer side. Bester and Ritzberger (2001) give consumers the possibility to buy a perfect quality signal and investigate how this affects equilibrium with a monopolistic seller. In Voorneveld and Weibull (2011) all buyers receive a costless but noisy private signal about quality in a competitive market with quality uncertainty. In both models, buyers are homogenous in the beginning because they face the same cost of acquiring information about quality.

In the present paper, buyers are heterogenous in their quality information from the beginning. Some buyers are informed about quality and some are not. With this respect, the paper is related to a range of signaling models showing that monopolistic sellers can signal high quality by charging high prices, in particular so when some informed consumers are present. Milgrom and Roberts (1986) and Bagwell and Riordan (1991) predict upward price distortion for signaling purposes. Most recently, Mahenc (2004) develops a signaling model, in which informed buyers are necessary for a monopolist to use prices as signals of product quality.

In contrast to this literature, the pricing side of the present model is not a monopoly but competitive. Most pertinent in this regard is Kessler (2001) who, like the present paper, stays within a competitive framework. Kessler (2001) can be considered as the theoretic counterpart to the present paper as she also extends Akerlof’s model by relaxing the strict assumptions to the information status of the players and compares market performance under two different informational structures. The crucial difference is that where I assume some buyers to be equally informed as the sellers, Kessler looks at the case in which some sellers are equally uninformed as the buyers.

The paper is also related to the literature on dynamic pricing, focussing mostly on its effect on profit and consumer surplus in different market structures. The diversity of topics
that is investigated in this context, includes: dynamic pricing in dynamic settings (Fudenberg and Tirole, 2000), the impact of dynamic pricing on consumer perceptions of price fairness (Haws and Bearden, 2006), the effect of dynamic pricing when consumers are strategic and avail themselves for the low price (Chen and Zhang, 2009) or avoid revealing information that will hurt them (Taylor, 2004).

There is no literature investigating the effect of price discrimination on adverse selection in markets with two-sided asymmetric information. In such markets, buyers have different knowledge of quality and sellers can learn the buyer’s type and tailor special prices on an individual level. The present article closes this gap, as it particularly addresses the impact of dynamic pricing on adverse selection.

The paper proceeds as follows: Section 2 lays down the market model with informed buyers, section 3 defines equilibria in this market with uniform prices based on the share of informed buyers. Section 4 defines equilibria with information acquisition and price discrimination. Section 5 concludes.

4.2 The Model

Consider the following market: There are potential sellers of an indivisible object and potential buyers. Each seller owns and each buyer wishes to purchase only one unit of the object. The market operates for only one period. All agents enter the market at the beginning of this period and each seller posts a fixed price. Agents are matched randomly. Sellers differ in the quality of the object they want to sell. A seller owns good quality $q_h$ with probability $\lambda$ and bad quality $q_l$ with probability $1 - \lambda$. Buyers can also be sorted into different, privately known types because their information on quality varies. A buyer is either perfectly informed about quality with probability $\theta$ or completely uninformed with probability $1 - \theta$. I consider the most simple model in which types are all binary. The seller’s and buyer’s expected payoffs $v_s$ and $v_b$ are given by

$$v_s = \sigma p + (1 - \sigma)\epsilon q_i$$

(4.1)

and

$$v_b = \sigma (q_i - p)$$

(4.2)
where \( \epsilon \in (0, 1) \) is a fixed valuation parameter, \( p \) is the price for quality \( q \) and \( \sigma \) is a binary variable with \( \sigma = 1 \) if the object is sold and \( \sigma = 0 \) otherwise.

If quality were publicly observable to all buyers, all objects would be sold at \( p_i = q_i \) \( \forall i \in \{l, h\} \), that is the buyer’s reservation value. The seller can capture the entire surplus from trade because he can make a take-it-or-leave-it offer (post a price) and the buyer’s outside option is zero.\(^2\) Throughout the paper, it will be assumed that \( \epsilon q_h > q_l \), so that \( \epsilon > \frac{q_l}{q_h} \) which implies that a high quality seller will not sell at a low price because his reservation price \( \epsilon q_h \) exceeds the maximum price that buyers are willing to pay for low quality \( q_l \).

When quality is observable only to informed buyers, buyers differ in the maximum amount they are willing to pay. Informed buyers are ready to buy as long as \( p \leq q \) and uninformed buyers form expectations about quality and upon these expectations they decide to buy or not. In the presence of informed buyers, the outcome depends on the seller’s ability to differentiate between the two buyer types. In the following, I will analyze the welfare effects of the two respective market environments of uniform pricing in which sellers are unaware of the buyers’ types and of dynamic pricing in which sellers learn the buyers’ types.

Independent of the market environment, in equilibrium, the price of a low quality object cannot be below the buyer’s maximum willingness to pay for low quality because a low quality seller offering for less than \( q_l \) could always increase his price to \( q_l \) and still sell for sure. It follows that, in equilibria with different prices, the price for low quality will be equal to the buyer’s maximum willingness to pay for low quality: \( p_l^* = q_l \). On the other hand, a buyer will not buy if the seller asks a price above the buyer’s maximum willingness to pay for high quality. Since a seller of high quality only sells above \( \epsilon q_h \), in equilibrium it must be that \( p_h^* \in [\epsilon q_h, q_h] \).

\(^2\)Here, the buyer’s outside option is zero because the market only operates for one period which leaves him without an opportunity to find an alternative seller. The fact that the seller captures the entire surplus from trade does not change even if we assume that the market operates for more than one period and the buyer can search for another seller. Following Diamond (1971), each seller can charge the monopoly price in this setting as long as search costs are positive.
4.3 Uniform Pricing

Under uniform pricing, each seller quotes one price to the buyer that he is matched with, independent of the buyer’s type. Quite generally, when quality is private information, low quality sellers have two options. They can either imitate high quality sellers and hope to be mistaken for high quality or they post a low price and make a sale for sure.

Due to the presence of informed buyers, low quality sellers face an additional restraint from offering at a high price because informed buyers do not buy low quality at a high price. When there are sufficiently many informed buyers, low quality sellers can even be deterred completely from mimicking high quality because the sale probability at the high price is too low for the low quality sellers who therefore prefer posting a low price which definitely results in trade. In this case, efficient fully separating equilibria can be obtained.

Otherwise, there are 3 different possible but inefficient types of equilibria: fully separating equilibria, semi-separating equilibria and, under some conditions, also a pooling equilibrium.

For a start, I investigate the existence of efficient fully separating equilibria. Such equilibria exist if all goods are traded, all high quality sellers post a high price \( p_h \) and all low quality sellers post a low price \( p_l \).

Efficient-Fully Separating Equilibria

When the share of informed buyers \( \theta \) is sufficiently high, then low quality sellers prefer posting a low price which definitely results in sale (left hand side of the condition below) over posting a high price which deters informed buyers (right hand side of the condition below):

\[
q_l - \epsilon q_l \geq (1 - \theta)(p_h - \epsilon q_l)
\]

\[\Leftrightarrow \theta \geq \frac{p_h - q_l}{p_h - \epsilon q_l}\]  \( (4.3) \)

**Proposition 2** There is an efficient separating equilibrium for \( \theta \geq \tilde{\theta} = \frac{q_h - q_l}{q_h - \epsilon q_l} \). Low quality sellers post a low price \( p_l^* = q_l \) and high quality sellers post a high price \( p_h^* = q_h \). All buyers buy at the posted price so that all goods are traded and the market is efficient.

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\( ^3 \)Since \( \epsilon q_h > q_l \), the minimum price acceptable to high quality sellers is higher than the buyer’s maximum willingness to pay for low quality.
Proof. It is obvious that the high quality seller has no incentive to deviate from the proposed strategy because he captures the entire surplus from trade and trades for sure. Given Condition 4.3, low quality sellers have no incentive to deviate from the proposed strategy. The buyers have no incentive to deviate because they are indifferent between buying and not.

The following results apply to situations in which $\theta < \tilde{\theta}$ and the share of informed buyers is below the critical threshold that guarantees the efficient separating equilibrium as stated in Proposition 3.

First of all, it is important to note that there are efficient separating equilibria even for $\theta < \tilde{\theta}$. Since a lower share $\theta < \tilde{\theta}$ can be substituted to some extent with a lower price, the high price needs to be smaller than in the efficient equilibrium shown above, $p_h < q_h$, in order to reduce the low quality seller’s incentive to mimic high quality in these equilibria. Rearranging Condition 4.3 gives an upper constraint on $p_h$ that guarantees that low quality sellers have no incentive to deviate to the high price:

$$p_h \leq \frac{q_l - \theta e q_l}{1 - \theta} < q_h$$  \hspace{1cm} (4.4)

We also need to make sure that the high price is not too small in order to prevent high quality sellers from posting the maximum high price $q_h$ and selling only to informed buyers. The following condition guarantees that:

$$p_h - \epsilon q_h \geq \theta(q_h - \epsilon q_h)$$

$$\Leftrightarrow p_h \geq \theta q_h + (1 - \theta)\epsilon q_h$$  \hspace{1cm} (4.5)

Proposition 3 For $\tilde{\theta} > \theta \geq \theta^*$, there are efficient fully-separating equilibria in which the low quality seller posts $p^*_l = q_l$ and the high quality seller posts $p^*_h = \frac{q_l - \theta e q_l}{1 - \theta} < q_h$. All buyers buy at the posted price so that all goods are traded and the outcome is efficient.

Proof. Buyers have no incentive to deviate because they are indifferent between buying and not buying when matched with low quality sellers and get positive utility when matched
with high quality sellers since $p_h^* < q_h$. In equilibrium, Condition 4.4, the low quality seller’s incentive constraint is binding because at any price $p \in \left[ \theta q_h + (1-\theta)\epsilon q_h, \frac{q_h - \theta \epsilon q_h}{1-\theta} \right]$, high quality sellers have an incentive to deviate to $p_h^* = \frac{2q_h - \theta \epsilon q_h}{1-\theta}$ in order to get a higher profit.

In equilibrium, the high quality seller’s incentive constraint (Condition 4.5) must hold at $p_h^*$:

$$\frac{q_l - \theta \epsilon q_l}{1-\theta} \geq \theta q_h + (1-\theta)\epsilon q_h$$

which gives the minimum $\theta$ for which these equilibria exist:

$$\theta = \left( q_h(1-2\epsilon) - q_l \epsilon \right) + \sqrt{\left( q_h(1-2\epsilon) - q_l \epsilon \right)^2 - 4(q_h(1-\epsilon)(q_l - \epsilon q_h)) \over 2(q_h - \epsilon q_h)} = \theta^*$$

\[ \square \]

**Inefficient Fully-Separating Equilibria**

For $\theta < \tilde{\theta}$ there are also inefficient fully-separating equilibria with prices $p_h^* = q_h$ and $p_l^* = q_l$. Given these prices, uninformed buyers are indifferent between buying and not buying (in both cases the expected payoff is zero). They can therefore also use any mixed strategy. With this in mind, equilibria can be constructed in which uninformed buyers put sufficient weight on not buying at a high price, so that the low quality seller is deterred from posting a high price. The low quality seller is deterred as long as his expected profit selling at a low price to all buyers weakly exceeds his expected profit selling at a high price only to those uninformed buyers who are ready to buy at a high price. Let $\beta$ be the probability with which uninformed buyers buy at a high price. The following must hold:

$$q_l - \epsilon q_l \geq (1-\theta)\beta(q_h - \epsilon q_l)$$

$$\Leftrightarrow \beta \leq \frac{q_l - \epsilon q_l}{(q_h - \epsilon q_l)(1-\theta)} = \tilde{\beta}$$

**Proposition 4** For $\tilde{\theta} > \theta \geq 0$, there is a range of inefficient fully-separating equilibria in which the high quality seller posts $p_h^* = q_h$ and the low quality seller posts $p_l^* = q_l$. Uninformed
buyers mix between “accept $p_l^*$ and $p_h^*$” and “accept only $p_l^*$”, choosing “accept $p_l^*$ and $p_h^*$” with probability $0 \leq \beta \leq \tilde{\beta}$.

**Proof.** Condition 4.6 together with the assumption that the uninformed buyer’s off-equilibrium belief that quality is low equals 1, assures that neither seller type has an incentive to post any other price. Buyers have no incentive to deviate because they are indifferent between buying at $p_l^*$ or $p_h^*$ and not buying.

It is easy to see that there are no other inefficient fully-separating equilibria because at any other high price $p_h < p_h^*$ (at $p_h > p_h^*$ no one will buy), uninformed buyers would strictly prefer to buy at $p_h$ and no longer be indifferent between buying and not buying. But if uninformed buyers always buy at $p_h$, then low quality sellers have an incentive to post $p_h$ instead of $p_l^*$ which would prevent full separation. ■

In the worst of these equilibria, $\beta = 0$ and uninformed buyers do not buy at a high price at all. Still, high quality goods are sold with probability $\theta$ to informed buyers. In the best of these equilibria, uninformed buyers buy at a high price with probability $\tilde{\beta}$ so that the sale probability of high quality goods equals $\theta + \tilde{\beta}(1 - \theta)$.

**Inefficient Semi-Separating Equilibria**

In addition to the efficient and inefficient fully-separating equilibria, there is also a range of semi-separating equilibria for $\theta < \tilde{\theta}$ in which low quality sellers mix between posting a high and a low price and uninformed buyers mix between buying at both prices and buying only at the low price.

For a mixed equilibrium to exist, players need to be indifferent between the pure strategies that are used in the mix. Suppose the low quality seller, that occurs with probability $1 - \lambda$, chooses $p_h$ with probability $\alpha$ and $p_l$ with probability $1 - \alpha$. The uninformed buyer chooses “accept $p_l$ and $p_h$” with probability $\beta$ and “accept only $p_l$” with probability $1 - \beta$. Consequently, the low quality seller is indifferent between $p_h$ and $p_l$ if:

$$(p_h - \epsilon q_l)(1 - \theta)\beta = p_l - \epsilon q_l$$
The uninformed buyer is indifferent between “accept $p_l$ and $p_h$” and “accept only $p_l$” if $E(q | p_h) - p_h = 0$, that is:

$$
(1 - \lambda)\alpha q_l + \lambda q_h - p_h = 0
$$

$$\Leftrightarrow \alpha = \frac{\lambda(q_h - p_h)}{(1 - \lambda)(p_h - q_l)}
$$

$p_h < q_h$ constitutes a lower bound on $\beta$: $\beta > \beta(q_h) = \tilde{\beta}$.

We also need to keep in mind that high quality sellers must be ready to offer their good, that is, their expected profit from offering their car at a price $p_h$ must be at least $\theta(q_h - \epsilon q_h)$, which is what high quality sellers can always get by selling at the maximum price only to informed buyers:

$$
(\beta(1 - \theta) + \theta)(p_h - \epsilon q_h) \geq \theta(q_h - \epsilon q_h)
$$

$$\Leftrightarrow p_h \geq \frac{\theta q_h + \beta(1 - \theta)\epsilon q_h}{\beta(1 - \theta) + \theta}
$$

**Proposition 5** For $\tilde{\theta} > \theta \geq 0$, there are semi-separating equilibria with prices $p_h^*$ and $p_l^*$, where $p_l^* = q_l$ and $p_h \leq p_h^* < q_h$. The high quality seller always posts $p_h^*$, while the low quality seller posts $p_h^*$ only with probability $\alpha^* = \frac{q_h - p_h^*}{p_h^* - q_l} \frac{\lambda}{1 - \lambda}$. The informed buyer buys at $p_h^*$ if the quality is high and the uninformed buyer buys at $p_h^*$ with probability $\beta^* = \frac{q_h - \epsilon q_h}{(p_h^* - \epsilon q_h)(1 - \theta)}$.

**Proof.** By construction, the low quality seller and uninformed buyer are indifferent between their pure strategies in the support of their mixed strategies. Given that the uninformed buyer’s off-equilibrium belief that quality is low is 1, Condition 4.8 prevents the high quality seller from deviating to selling only to informed buyers.

\[\blacksquare\]

**Inefficient Pooling Equilibrium**

Finally, there is an additional pooling equilibrium in which both types of sellers post the same price equal to expected quality, $p = \lambda q_h + (1 - \lambda)q_l$. Since $q_l < \lambda q_h + (1 - \lambda)q_l$, informed
buyers do not buy from low quality sellers at \( p \). Therefore, we need to make sure that low quality sellers are ready to post \( p \) instead of posting \( q_l \) and selling also to informed buyers. The following condition guarantees that the low quality seller’s expected profit from selling at \( p \) only to uninformed buyers weakly exceeds the profit from selling at \( q_l \) to all buyers:

\[
(p - \epsilon q_l) (1 - \theta) \geq q_l - \epsilon q_l
\]

\[
\Leftrightarrow (\lambda q_h + (1 - \lambda) q_l - \epsilon q_l) (1 - \theta) \geq q_l - \epsilon q_l
\]

\[
\Leftrightarrow \theta \leq \frac{\lambda(q_h - q_l)}{\lambda(q_h - q_l) + q_l - \epsilon q_l}
\]  \hspace{1cm} (4.9)

But also high quality sellers must be ready to offer their good at \( p \). Since high quality sellers always have the option to sell at the maximum price only to informed buyers, we need to make sure that their expected profit from offering at \( p \) must be at least \( \theta(q_h - \epsilon q_h) \), their profit from selling only to informed buyers:

\[
p - \epsilon q_h \geq \theta(q_h - \epsilon q_h)
\]

\[
\Leftrightarrow \lambda q_h + (1 - \lambda) q_l - \epsilon q_h \geq \theta(q_h - \epsilon q_h)
\]

\[
\Leftrightarrow \theta \leq \frac{\lambda(q_h - q_l) + q_l - \epsilon q_h}{q_h - \epsilon q_h}
\]  \hspace{1cm} (4.10)

Both high and low quality sellers only sell their good if the pooling price \( p \) is sufficiently high. \( p \) increases in the share of high quality goods \( \lambda \). The higher the share of informed buyers \( \theta \), the higher must be \( \lambda \) for the sellers’ incentive constraints to still be fulfilled. Rearranging conditions 4.8 and 4.9, we get the following constraints on \( \lambda \):

\[
\lambda \geq \frac{\theta(q_l - \epsilon q_l)}{(1 - \theta)(q_h - q_l)}
\]  \hspace{1cm} (4.11)

for the low quality seller and

\[
\lambda \geq \frac{\theta(q_h - \epsilon q_h) + \epsilon q_h - q_l}{q_h - q_l}
\]  \hspace{1cm} (4.12)
for the high quality seller.

**Proposition 6** If \( \theta \leq \min\left\{ \frac{\lambda(q_h-q_l)+q_l-\epsilon q_h}{q_h-q_l}, \frac{\lambda(q_h-q_l)}{\lambda(q_h-q_l)+q_l-\epsilon q_l} \right\} \), there is pooling equilibrium in which \( p^* = \lambda q_h + (1 - \lambda)q_l \). Sellers unanimously post \( p^* \) and uninformed buyers always buy at that price, whereas informed buyers only buy when matched with a high quality seller. *Therefore, the market is inefficient.*

Note that for \( \lambda = 1 \) Condition 4.9, approaches \( \tilde{\theta} = \frac{q_h-q_l}{q_h-\epsilon q_l} \) which implies that there is a unique efficient fully-separating equilibrium when the share of good quality approaches one. Also, for \( \lambda = 0 \) there is a unique efficient equilibrium with \( p = q_l \).

When \( 0 < \lambda < 1 \), the presence of informed buyers makes a pooling equilibrium less likely because an increasing share of informed buyers reduces the sale probability at \( p^* \) for low quality sellers and increases the incentive for high quality sellers to post \( q_h \) instead of \( p^* \) and sell only to informed buyers. In the pooling equilibrium, all high quality goods are traded, which means that there is no adverse selection. However, the situation is not efficient since informed buyers do buy low quality with probability \( \theta \). This inefficiency increases in the share of informed buyers \( \theta \).

Figure 4.1 below illustrates all equilibria of the model with informed buyers. With regard to efficiency, there is a unique efficient equilibrium when the share of informed buyers is above some critical threshold, \( \theta \geq \tilde{\theta} \). When the share of informed buyers is below that threshold, multiple equilibria may exist simultaneously. Efficient fully-separating equilibria only exist for \( \tilde{\theta} > \theta \geq \theta^* \).

Inefficient separating equilibria exist in the range \( \tilde{\theta} > \theta \geq 0 \). In the fully-separating equilibria, there is adverse selection since all low quality goods are traded but not all high quality goods. In the semi-separating equilibria, there are two sources of inefficiency: First, not all high quality goods are traded which constitutes some degree of adverse selection. Second, not all low quality goods are traded because informed buyers do not buy low quality selling at a high price.

The pooling equilibrium only exists for \( \theta \leq \min\left\{ \frac{\lambda(q_h-q_l)+q_l-\epsilon q_h}{q_h-q_l}, \frac{\lambda(q_h-q_l)}{\lambda(q_h-q_l)+q_l-\epsilon q_l} \right\} \). In this equilibrium, all high quality goods are traded which means that there is no adverse selection. In contrast to Akerlof’s model there is still inefficiency because not all low quality goods are
traded.

4.4 Dynamic Pricing

Now assume that a seller can identify the buyer’s type, possibly at some cost, and price discriminate between informed and uninformed buyers. This possibility leads to a market segmentation because each seller can quote a different price depending on the buyer’s type. In the informed buyers market, there is perfect information so that trade is always concluded at a price equal to quality. In the following, I will analyze how the absence of informed buyers affects equilibrium in the uninformed buyers market and, on that basis, explore the overall welfare implications.

Given that the sellers’ cost of acquiring customer information is not prohibitively high, efficient fully-separating equilibria no longer exist because the low quality seller offers prices that depend on the buyer type. He offers a low price to informed buyers which is accepted.

Figure 4.1: Equilibria with informed buyers

\[
\bar{\theta} = \frac{q_h - q_l}{q_h - \epsilon q_l}
\]

\[
\bar{\lambda} = \frac{\epsilon q_h - q_l}{q_h - q_l}
\]

\[
\frac{\lambda (q_h - q_l) + q_l - \epsilon q_h}{q_h - \epsilon q_h}
\]

\[
\frac{\lambda (q_h - q_l)}{\lambda (q_h - q_l) + q_l - \epsilon q_l}
\]
Therefore, the price that he offers to uninformed buyers is not affected by the informed buyers’ resistance to buy low quality at a high price. This means that the low quality seller has no restriction to cheat and mimic the high quality seller when offering to uninformed buyers. Anticipating this, uninformed buyers are not ready to buy at a high price unconditionally.

**Corollary 9** There are no longer efficient fully-separating equilibria when customer-specific price discrimination is possible.

There still exist the inefficient types of equilibria: fully separating equilibria, semi-separating equilibria and the pooling equilibrium in the uninformed buyers market, but now for the special case that the share of informed buyers is zero, $\theta = 0$.

**Inefficient Fully-Separating Equilibria**

In the inefficient fully-separating equilibria, uninformed buyers only buy at the high price with some probability $\beta$. But the possibility of price discrimination reduces the threshold $\beta$ in these equilibria because low quality sellers no longer need to factor in the loss of informed buyers when offering at a high price. As a consequence, uninformed buyers may buy at $p_h$ less often in order to deter low quality sellers from posting high prices. To see this, we need to look at the low quality seller’s incentive constraint (Condition 4.6) at $\theta = 0$:

$$\beta \leq \frac{q_l - eq_l}{q_h - eq_l} = \beta'$$

Comparing the threshold for $\beta$ in the fully-separating equilibria with and without dynamic pricing, we see that:

$$\beta' = \frac{q_l - eq_l}{q_h - eq_l} < \frac{q_l - eq_l}{(q_h - eq_l)(1 - \theta)} = \tilde{\beta}$$

(4.13)

All low quality goods are traded but there is adverse selection. Compared to the inefficient fully-separating equilibria without price discrimination, dynamic pricing negatively affects trade probability of highly priced goods in the uninformed buyers market if $\beta \in (\beta', \tilde{\beta})$ under uniform pricing. For $\beta < \beta'$, dynamic pricing has no effect on welfare.

From a welfare perspective, a lower probability of trade at the high price constitutes a negative welfare effect because high quality sellers trade less and adverse selection increases.
Since all other market participants are indifferent between the two regimes, we can say that overall welfare is negatively affected by price discrimination at least for $\beta \in (\tilde{\beta}', \tilde{\beta})$.\footnote{Both buyer types are indifferent between the two regimes because equilibrium prices are such that their expected payoff from trade is zero. Low quality sellers gain $q_l - \epsilon q_l$ under both regimes.}

**Corollary 10** Under dynamic pricing, in the inefficient fully separating equilibria, uninformed buyers accept a high price $p_h^* = q_h$ with probability $\beta$, where $0 \leq \beta < \tilde{\beta}' < \tilde{\beta}$. Compared to uniform pricing, welfare is worse in these equilibria for $\beta \in (\tilde{\beta}', \tilde{\beta})$ because of higher adverse selection.

Next, I analyze how the semi-separating equilibria are affected by the absence of informed buyers.

**Inefficient Semi-Separating Equilibria**

When the share of informed buyers is zero, $\theta = 0$, the low quality seller’s indifference condition (Condition 4.7) becomes:

$$\beta' = \frac{q_l - \epsilon q_l}{p_h - \epsilon q_l}$$

(4.14)

Comparing $\beta'$, the probability with which uninformed buyers buy at a high price in the semi-separating equilibria under dynamic pricing with $\beta^*$, the respective probability under uniform pricing (defined by Proposition 6), it follows that

$$\beta' = \frac{q_l - \epsilon q_l}{p_h - \epsilon q_l} < \frac{q_l - \epsilon q_l}{(1 - \theta)(p_h - \epsilon q_l)} = \beta^*$$

(4.15)

Since $\beta' < \beta^*$, uninformed buyers buy at the high price less often in the semi-separating equilibria when price discrimination is possible. Again, low quality sellers do not need to factor in the loss of informed buyers when offering a high price to uninformed buyers. The low quality sellers’ reduced incentive to post a low price is anticipated by uninformed buyers who buy at a high price less often. This, in turn, negatively affects the high quality sellers’ probability of trade.

Comparing the semi-separating equilibria with and without price discrimination, the following can be stated: The expected payoff from trading is $q_l - \epsilon q_l$ for low quality sellers and
0 for uninformed buyers with and without price discrimination.\footnote{When price discrimination is possible, low quality sellers trade with informed buyers at \( p_l = q_l \), so that the sellers’ payoff is \( q_l - \epsilon q_l \). Their expected payoff from trade with uninformed buyers is also \( q_l - \epsilon q_l \) because uninformed buyers buy less often at high prices \( (\beta' < \beta^*) \) in order to offset the low quality sellers’ increased incentive to offer at a high price.} Informed buyers gain under uniform pricing when buying from high quality sellers because \( p_h < q_h \) compared to \( p_h = q_h \) under dynamic pricing. But this payoff gain on part of informed buyers translates into a payoff loss on part of high quality sellers, so that welfare is unaffected. What negatively affects welfare though, is the increased reluctance to buy at a high price on part of uninformed buyers under dynamic pricing which results in a decrease in the trade of highly priced goods and thus an increase in adverse selection.

**Corollary 11** Under dynamic pricing, in the semi-separating equilibria, uninformed buyers accept a high price \( p_h^* < q_h \) with probability \( \beta' \), where \( \beta' < \beta^* \). Compared to uniform pricing, welfare is worse in these equilibria because of higher adverse selection.

**Efficient Pooling Equilibrium**

In the pooling equilibrium, sellers post a price equal to expected quality at that price, \( p = \lambda q_h + (1 - \lambda) q_l \), to uninformed buyers. High quality sellers are ready to offer the pooling price to uninformed buyers as long as this price exceeds their reservation value:

\[
\epsilon q_h \geq \lambda q_h + (1 - \lambda) q_l
\]

\[
\iff \lambda \geq \frac{\epsilon q_h - q_l}{q_h - q_l}
\]

When sellers quote the pooling price to uninformed buyers and \( p_i = q_i \) to informed buyers, then all goods are traded. The surplus from trade is captured entirely by the sellers, where low quality sellers benefit disproportionately high compared to high quality sellers in the uninformed buyers market because of the pooling price. Compared to the pooling equilibrium without price discrimination, this equilibrium generates a better result in terms of welfare because all low quality goods are traded while in the pooling equilibrium without price discrimination, low quality goods do not always trade.
Corollary 12 If \( \lambda \geq \frac{c_{h/q} - c_l}{q_{h/q} - q_l} \), there is an efficient pooling equilibrium in the uninformed buyers’ market in which sellers offer \( p^* = \lambda q_h + (1 - \lambda)q_l \) to uninformed buyers. Compared to uniform pricing, welfare is better in this equilibrium because all goods are traded.

With respect to consumer welfare, dynamic pricing and market segmentation is not beneficial for informed buyers because sellers can easily identify informed buyers and push them to their reservation value.\(^6\) In the case of uniform pricing, uninformed consumers can impart a benefit on informed buyers because prices may be below the informed buyer’s reservation value so to still address uninformed buyers with a higher reservation value due to their lack of information.

The overall welfare effects of price discrimination can be subsumed as follows:

Corollary 13 Price discrimination generates an efficient equilibrium only if the average quality is high and both types of sellers make a pooling offer to uninformed buyers. When the average quality is low or sellers separate in their offer to uninformed buyers, price discrimination can have a negative welfare effect because it increases adverse selection.

Figure 4.2 below depicts all equilibria in the uninformed buyers market when sellers can price discriminate. Because the market is segmented, equilibria in the uninformed buyers market are independent of the share of informed buyers.

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\(^6\)Uninformed buyers are indifferent between both pricing regimes because their expected payoff is zero in both cases.
4.5 Conclusion

In this paper, I investigate how market performance in a lemons market depends on the specific information structure of the buyer side by letting some buyers be equally informed as the sellers. I compare two different settings, one in which sellers cannot differentiate between buyer types, which constitutes a market with two-sided asymmetric information and one in which sellers can learn the buyer type and price discriminate on an individual level, which eliminates asymmetric information on the seller side.

Opposed to conventional wisdom, reducing asymmetric information by facilitating price discrimination can have a negative welfare effect in this setting because adverse selection can increase. The reason for this, is that without sellers being able to distinguish between informed and uninformed buyers, low quality sellers have less incentive to mimic high quality because their sale probability at a high price is lower when informed buyers do not buy high priced low quality. This is anticipated by uninformed buyers who are ready to buy at a high

\[ \tilde{\lambda} = \frac{\varepsilon q_h - q_l}{q_h - q_l} \]

Figure 4.2: Equilibria with price discrimination

\[ \tilde{\lambda} \leq 1 \]

\[ \text{share of informed buyers } \theta \]

\[ \text{share of good quality } \lambda \]

1

Inefficient fully-separating equilibria

Inefficient semi-separating equilibria

Efficient pooling equilibrium

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price more often when the market is unsegmented.

The results presented in this paper shed some light on the often neglected issue of intermediate degrees of informational asymmetry and reveal the connection between the level of market information and the level of adverse selection in a simple and intuitive manner which may be useful in applications.


Conclusion

The studies in this Dissertation show that bargaining theory is applicable to a wide range of problems because many, even seemingly unrelated ‘non-economic’ interactions have a bargaining problem at heart.

From a scientific point of view, the motivation for bargaining is not especially exciting because bargaining is mutually beneficial to all parties involved. The interesting part is to identify reasons why bargaining sometimes breaks down in spite of the potential gains.

Two different reasons for a breakdown of bargaining are explored in this Dissertation: commitment problems and asymmetric information. While Chapters 1 and 2 concentrate on commitment problems as a possible reason for the inability to reach an agreement in repeated interaction, Chapters 3 and 4 focus on asymmetric information in one-time bargaining.

Commitment Problems are a source of inefficiency when binding contracts are not enforceable because a powerful authority capable of guaranteeing agreements is absent. It follows that commitment problems are less prevalent in bargaining activities within the rule of law and more relevant with regard to illegal activities and negotiations on an international level.

The basic trouble with bargaining under commitment problems is the bargainers’ incentive to break up the negotiation before it even started when there is no guarantee that the other party sticks to the agreement. A good way to illustrate this dilemma is by thinking of the reason why crime witnesses are in danger of being ‘silenced forever’. Because a witness cannot credibly commit to not go to the police and report the crime, the criminal has no rational choice other than kill the witness, even though he would prefer the witness to live and not tell. Chapter 1 investigates this logic within the framework of crisis bargaining between autonomous states and preventive war as the result of bargaining breakdown.
Compared to commitment problems, the role of asymmetric information has been extensively analyzed in the bargaining literature and identified as a viable source of bargaining inefficiencies such as strikes, wars and costly delay. Despite the very different set-ups and fields of application, the bargaining problems in Chapters 3 and 4 boil down to a simple negotiation between two bargainers with privately known reservation prices. In this case, an agreement may not be struck because a bargainer with a low reservation price (low outside option in Chapter 3 or low quality good in Chapter 4) has the incentive to pretend to be a type with a high reservation price (high outside option in Chapter 3 or high quality good in Chapter 4).

The inability of one negotiator to distinguish between different types of his negotiating partner and the incentive to cheat on part of weak types can be the reason for a breakdown in bargaining. Of course, bargaining need not necessarily break down and in bargaining models with multiple rounds in which players have the option to reject offers and make counteroffers, agreement can usually be reached, albeit not without cost of delay. In such settings, inefficiencies are often the means by which bargainers become able to identify the other party’s type and finally conclude an agreement.

Since the models presented in Chapters 3 and 4 preclude the possibility of counteroffers, an agreement that can not be struck at once, immediately results in the worst possible outcome, the termination of bargaining. In Chapter 3, terminating negotiations means war, in Chapter 4, it means some buyer/seller matches do not result in trade even though trading generates surplus.
Bibliography


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