Prediction of driving behavior in cooperative guidance and control: a first game-theoretic approach

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Abstract: In this paper, we introduce a novel approach for analyzing the behavior of a vehicle that is controlled by both a human driver and a cognitive and cooperative automation at the same time. In a situation where a dynamic and a static obstacle force the driver-vehicle system to engage in a lane changing maneuver at a sooner or later point in time, the objective is to maximize the driving comfort, minimize risks, and, ultimately, avoid a crash with either obstacle. Hereby, the dynamic obstacle is another driver-vehicle system with similar properties as the first one. Therefore, their interaction can be modeled as a sequential game with imperfect information, since neither can perfectly predict the behavior of the other. As a proxy for comfort and safety, we use the time to collision (TTC) and compare it to the preferred TTC of each system. A further component of the utility function is the velocity, which serves to evaluate the time consumed to perform a maneuver as well as the costs related to it. Each system puts different weights on the individual components and, therefore, chooses different actions at different points in time. It can influence the velocity as well as the lane on which to drive. Using a simulation, we evaluate the behavior of both systems at several points in time until the static obstacle is passed.

1. INTRODUCTION

Nowadays, state-of-the-art assistance systems in vehicles already offer several aspects of automation for supporting the driver. Systems, such as the Lane Departure Assistant or the Night Vision system, alert the driver upon detecting possible risks in the close environment, while using various channels of communication to reach the human driver. Furthermore, some assistance systems even introduce automation in parts of the driving task, in the lateral as well as in the longitudinal direction. Covering the longitudinal aspect, Adaptive Cruise Control (ACC) is already designed to control a vehicle in such a way that it automatically follows a vehicle ahead at an appropriate (timewise) distance by adjusting the speed accordingly. An example for lateral control can be found in the Active Lane Keeping Assistance system (LKAS), which ensures that the vehicle does not deviate from the current lane by optically tracking the markings for lane delimitations. When combining these two systems, the driving task can already be executed in a partially automated way, at least while staying on highways such as the German Autobahn. Highly-automated driving has been explored in several research and development projects over the last decades (Hoeger et al., 2011; Flemisch et al., 2014). Important issues of controlling such a system and keeping the human driver in the loop in critical traffic situations are addressed in the approach for cooperative driving (Flemisch et al., 2012; Winner and Hakuli, 2006). A crucial element of cooperative guidance and control in the driving task is

the use of an automation, which should, in principle, be able to steer the vehicle autonomously. While previous game-theoretic approaches for behavior of drivers mainly consider human drivers without the aspects of automation (Kita, 1999; Lubashevsky et al., 2003, f.ex.), we attempt to find optimal maneuvers or strategies for human-machine systems. The first step in doing so is to define the utility of the integrated system. Relevant components are (required) time, costs, and (perceived) safety. It should be noted that preferences vary with the type of human driver. To illustrate this, one can think of drivers who are mainly interested in completing the driving task as quickly as possible, while costs are rated as less important (and enter the utility function with a lower weighting). The highly-automated vehicle controlled by such a driver should then execute a maneuver more swiftly than a system concerned about cost factors. As aggressive driving is often associated with smaller (time) gaps and therefore leads, in comparison to defensive driving, to a higher criticality of the traffic situation, one also needs to consider safety aspects. Using parameter estimation methods, information about current states and probabilities of future states can be extracted. For discrete instants of time in the considered period, equilibrium choices of driving maneuvers can be found. Analyzing these equilibria, expected behavior can be predicted. The introduced game-theoretic method for analyzing the cooperation of human-machine systems can be used for online estimations of expected driving behavior. This leads to an enhanced comprehension of the functioning and interaction of several human-machine systems

that conform to the principles of cooperative guidance and control. Moreover, such an estimation method can be used as part of an automation for assisted, partially and highly automated vehicles, because it provides an assessment of the behavior of other road users without the need of (more costly) solutions like Car-2-X-communication.

2. THEORETICAL BACKGROUND: COOPERATIVE GUIDANCE AND CONTROL

2.1 Assistance and automation

Cooperative guidance and control of vehicles occurs when at least one human and one machine work together to determine the behavior of at least one vehicle (Flemisch et al., 2014). However, Flemisch et al. (2014) point out that this concept can also be extended to cooperative technical systems, such as vehicles and infrastructure that communicate and work together towards a common goal (as in Car2Car or Car2Infrastructure). An important aspect hereby is that a human and a machine work on the control task at the same time, which can be called "shared control" (Griffiths and Gillespie, 2004; Mulder et al., 2012) or "shared authority" (Flemisch et al., 2012). If this is not optimal or not desired for any other reason, tasks or subtasks can also be delegated to the different agents in a hierarchical manner (cf. Rasmussen, 1983; Hollnagel and Woods, 1983). The automation can thereby be adaptive as well as adaptable (e.g. Sheridan and Parasuraman, 2006). In this sense, cooperative guidance and control can be understood as the implementation of an automation that can rather autonomously steer a vehicle, but where the human driver remains in the loop. She can partly or fully take over control with a seamless transition whenever she likes.

2.2 Cooperative guidance and control in the H-Mode

A first definition of semi- and high automation is given by Gasser et al. (2012), a slightly different one is given in Hoeger et al. (2011). Overcoming this gap, we differentiate between partially-/highly automated (Loose Rein), and highly/temporarily fully automated driving (Secured Rein). Thus, we denominate the area where most of the control task is carried out by the automation as partially-/highly automated. When the human driver is only assisted in selected tasks, such as in lateral control by a Lane Keeping Assistance system, we enter the semi-autonomous area. Figure 1 illustrates this using the assistance and automation chart, which is a simplified model of one-dimensional division of control between human and automation. Challenges and opportunities for highly-
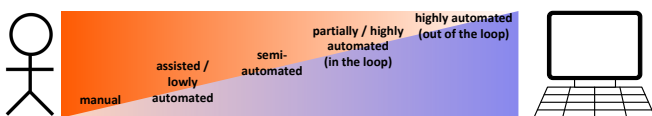

Fig. 1. Assistance and automation scale (e.g. Flemisch et al., 2014)

automated driving are increasing with growing technical achievements for vehicle assistance. The H-Mode is based

on the H-Metaphor (e.g. **?**), which describes the interaction of a cooperative or highly-automated vehicle and a human driver. The metaphor uses the natural example of the relationship between a rider and a horse, or a driver and a horse cart, as a blueprint for the interaction between the human and the technical subsystem. This also means that the most important element in cooperative guidance and control is the capability of the automation of working together with the human driver. In order to enable a human to take part in vehicle guidance and control, the technical system needs to be comprehensive and act predictably (Löper et al., 2008). H-Mode as an approach to cooperative guidance and control takes into account the possibilities of dynamically dividing the tasks between human and machine as well as the issue of implementing appropriate takeover and reaction times. In the course of several research projects, three specific automation modes with different degrees of control distribution between the human and the machine emerged (e.g. Flemisch et al., 2014). In particular, these are an assisted mode, a partially/highly- automated mode and a temporarily fully-automated mode. Figure 2 illustrates these modes graphically. Another important requirement is that the
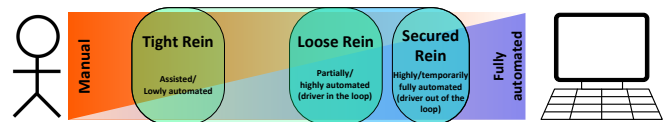

Fig. 2. Automation spectrum with specific modes (cf. Flemisch et al., 2014)

automation should be compatible with the driver in terms of inner and outer compatibility (**?**Bubb, 1993).

2.3 Automation cooperative vehicle guidance and control

In addition to the requirement that the automation should in principle be capable of driving autonomously, the concept of inner compatibility (Bubb, 1993) is a decisive component. Hereby inner compatibility denotes, among others, the fitting of the mental model of the human to the behavior of her technical counterpart. It can also be understood as cognitive compatibility (**?**), which includes common goals and norms of the human and the automation ensuring consistent evaluations of upcoming situations. In order to increase inner compatibility, the automation needs to be modeled according to the mental model of the human driver. In doing so, one obtains a multi-layered model, which considers several planning and execution horizons of the human driver. Cognitive models for vehicle guidance and control are discussed in Donges (1982), while more general models on the interaction with automated technical systems can be found in Rasmussen (1983) and Parasuraman et al. (2000). Löper et al. (2008) apply models in the context of vehicle guidance and control in the H-Mode. The three-layered model of Donges (1982) introduces the levels of stabilization, guidance, and navigation. From a technical perspective, guidance is implemented as anticipatory control (Donges, 2012). Löper et al. (2008) further distinguish two layers within the level of guidance, namely the maneuver and the trajectory level. These result in a four-layered model of vehicle guidance

and control, including state control, trajectory planning, maneuver planning, and navigation. Maneuvers are hereby defined as spatially and temporally related processes. We consider this four-layered model as the basis of the cognitive design of the automation, which is required to exhibit similar levels to achieve the aforementioned inner compatibility.

2.4 Maneuver-based vehicle guidance and control in partially automated mode (Loose Rein)

In this paper, we focus on the partially/highly- automated mode of cooperative guidance and control in the H-Mode. This driving mode describes vehicle guidance as maneuver-based assignments by the human driver. In this mainly sequential model, the automation takes over the lower levels of trajectory planning and control, while the human remains with the higher levels of maneuver planning and navigation. As decisions and interaction between human and machine can only be taken on these levels, we limit ourselves in this study to the investigation of lane change and speed adjustments.

3. THEORETICAL BACKGROUND: GAME-THEORETIC MODELING OF DRIVING BEHAVIOR

Most applications of game-theoretic approaches consider use cases which involve lane changing maneuvers or the gap between two cars on the same lane. Bell and Cassir (2002) model an optimal routing choice problem under consideration of the estimated duration of the route and the possibility of an unexpected increase in costs for a specific branch. A similar problem is considered in Garcia et al. (2000), but in form of a cooperative game. Hereby total utility, which is equivalent to the social welfare discussed in the end of this paper, is to be maximized by an optimal routing choice, i.e. the average travel time of all players is to be minimized. Fotakis et al. (2002) models this as a non-cooperative game and compares the social costs of all possible Nash equilibria. Wie (1993) expands the situation of a traffic jam with flexible departure times and thereby broadens the applicability of the model to also include aviation. Although Vetta (2002) and Fotakis et al. (2002) both use traffic planning problems as examples, they concentrate on the development of efficient algorithms that are capable of solving for the Nash equilibria. The emphasis is on the relatively high complexity in computation, which is why so far no algorithms in polynomial time have been found.

Hollander and Prashker (2006) classify game-theoretic modeling in the traffic domain as games against a demon, who tries to do the largest possible damage (i.e. minimize the objective function), games between institutions or companies, which play a minor role, games between institutions and travelers and games between travelers. The model at hand can be categorized as the latter, because it includes two players that are users with a similar perspective and on an equal level in the traffic system. Most other games of this category consider strategic planning of departure or traveling times to achieve maximum utility. Usually, this is modeled as a conflict of shortest branch and route with least traffic. Preventing traffic jams
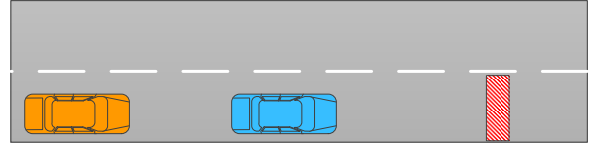

Fig. 3. Initial driving situation at time t_0

by implementing appropriate measures for traffic planning and management falls into the category of games between institutions or games between institutions and travelers. This can include the planning of public transport facilities, traffic legislation, tolls, or extension of routes and networks.

Closest to our study is Kita (1999). He develops a game-theoretic model for two vehicles that are about to change lanes on a street with two lanes. The starting point of this is the mutual influence of both vehicles. The strategy space encompasses, similar to our model, the options of merging or waiting on the one hand, and giving way or not on the other hand. The utility function depends on the distance between the two vehicles or rather the TTC (time to collision). The model is solved with a mixed strategy equilibrium in a game of perfect information. In a case study, the model parameters are determined to validate the model.

4. INVESTIGATED USE CASE

The scene is composed of a multi-lane road, i.e. one that is composed of more than one lane, and a fixed obstacle on the rightmost lane. All lanes run in the same direction, which means we do not consider oncoming traffic. A real world example of such a scenario would be a highway, such as the German Autobahn. Dynamic elements in this context are two human-machine systems, which are composed of a vehicle that is capable of highly-automated, cooperative driving and a human driver. At initial time t_0, both of the vehicles are on the right lane in front of the obstacle. Although they are driving on the same lane, the second vehicle is on an earlier section of the route than the first vehicle. This is how a typical overtaking situation emerges. The rear vehicle is driving at a higher speed, at least in the initial state (3).

4.1 Scenario

The driving situation on a multi-lane road without oncoming traffic, as described in the previous section, is used for modeling the decision problem. Hereby, we will limit ourselves again to the two human-machine systems. Actions concerning the motion guidance of both vehicles are mutually interdependent, because an obstacle on one lane forces changing lanes for at least one of the vehicles. Hereby, the obstacle can either be static (f. ex. a car that broke down, an object or similar) or dynamic (a vehicle driving at a lower speed level). For better illustration, let us now assume that the obstacle is on the right lane and that both human-machine systems, hereafter vehicle 1 and 2, are driving on that same lane.

5. MODELING THE DRIVING BEHAVIOR

The game at hand can be regarded as a classical Stackelberg leader-follower game (von Stackelberg, 1934). The central component hereby is that the second player can observe the actions of the first player and can, therefore, react with his Best Response. In a game of perfect information, player 1 anticipates this and chooses his strategy in such a way that his own utility is maximized. However, this assumes complete and perfect information from the start. In the above mentioned game, however, we only have incomplete information, since each player only knows his own type with certainty. In order to be able to solve such a game, we need to perform a Harsanyi transformation. Thereby an additional player, Nature, is introduced. Nature takes all random actions which are not performed by the actual players. By doing so, we obtain a game with complete, but imperfect information (Harsanyi, 2004). As the probability distribution of all possible moves of Nature is known, the game can be solved using Bayes' theorem (cf. Rasmussen, 1989, for the role of information in games). Since the driving situation described above cannot only be described at a discrete point in time, but continuously, the chosen strategy of the driver-vehicle system is also influenced by the course of time. Games with consideration of the time component are discussed in Fudenberg and Tirole (1991) or Rasmussen (1989), for example.

For every driver-vehicle system, there is a set of possible actions that can be defined to cope with the driving task. This set is a subset of the entire action space A, which includes all theoretically possible actions. An action hereby describes the execution of one aspect of the driving task on the maneuver level. In the proposed scenario, both human-machine systems have, in principal, a similar strategy space, which can be defined by the momentarily possible maneuvers. In detail, these are accelerate, brake, and change lane (left or right):

$$A \in \mathcal{A}$$
$$A_j = \{Accelerate, Brake, Lane\ Change\ left, \quad (1)$$
$$Lane\ Change\ right\}$$

As we assume there to be two lanes with an obstacle on the right lane, we can reduce the possible/feasible actions for the second vehicle, which also decreases the complexity of the described driving, and thereby also decision, situation.

$$A_1 = \{Accelerate, Brake, Lane\ Change\ left\}$$
$$A_2 = \{Accelerate, Brake, Lane\ Change\ left\} \quad (2)$$

For modeling the behavior of a driver-vehicle system we assume that the executed or preferred maneuvers can be described as intentional actions. We further assume that the choice of a maneuver happens iff that specific maneuver leads to the largest possible utility. Such a description of the considered system offers the possibility of a game-theoretic approach, especially since the attainable utility is co-determined by the chosen maneuver of the other party. A player in the described scenario is a driver-vehicle system, where the fixed system limit specifically entails both subsystems, i.e. the human and the machine. The driving maneuvers describe the strategies which can be followed for reaching the goals, i.e. maximizing utility. By choosing an optimal strategy, a player tries to maximize her expected utility. For doing so, traditional game theory defines costs and payoffs for each set of actions. Accordingly, we can describe the payoffs as the acquired utility, composed of several factors. In analogy, the payoffs can be modeled as the obtained utility, however, the transfer to the illustrated traffic scenario requires considering several factors.

5.1 Identification of input factors

The currently available strategies are constrained by the (relative) geographical position. The underlying logic becomes obvious when considering the consequences of a lane change while the other vehicle is driving in parallel. A change of lane is, thus, only possible when the two vehicles are not directly next to each other. These circumstances are, however, not modeled in a discrete way, but as a continuum of all possible coordinates. As further constraints, we also need to determine the points of origin as coordinates as well as initial velocities of both cars.

As the utility function is specific to each driver, i.e. for every user of the system, the final values for the variables can only be determined empirically. However, we still want to introduce the underlying formulas in preparation in the next section.

5.2 Model synthesis

In order to be able to analyze the behavior of the human-machine system under consideration, we develop a utility function on the basis of the input factors described above. In our model, the utility function is influenced by the position of the own vehicle, the velocity as well as the position of the obstacle and the other vehicle. Hereby, we assume that the TTC highly influences the decision about the action to be taken. To take into account the user-specific preferences, coefficients are introduced that specify the weights of the elements of the utility function. For simplicity, costs are assumed to be proportional to the velocity:

$$c = \hat{\beta}\ v_i(t) \quad (3)$$

While the velocity is a negative factor in the cost function, and therefore also becomes a negative factor in the expected utility function, it exhibits an otherwise positive impact in the utility function. This positive interdependence results from the time needed to conduct a driving maneuver. For numerical reasons, we hereby introduce a reference time $T_{ref,i}$, which can be surpassed or undercut. The time difference between reference time and time actually consumed is weighted with the coefficient γ and enters the utility function as follows:

$$\gamma\ (T_{ref,i} - T_{consumed}) \quad (4)$$

Hereby the time consumed is composed of a reference distance that is based on the reference time and the velocity.

$$\gamma \frac{s_{ref,i}}{v_i} \text{ with } s_{ref,i} = v_{ref,i} t_{ref,i} \quad (5)$$

With such a formulation, it is assured that the positive influence of the velocity has a sensible upper limit. At the same time, we achieve that very small velocities have a

clear negative impact on the utility function, which also means that stopping the vehicle is hardly a preferable option.

The TTC has a positive impact on the utility, because the perceived and probably also the actual safety positively correlate with the TTC. However, this part of the utility is limited, since one can assume that from a certain TTC onwards the utility does not increase any further. In the utility function described here, this is considered by limiting the TTC by a user-specific reference time $TTC_{ref,i}$ from above

$$TTC = max(TTC_{ref,i}, \frac{d}{v_i}) \qquad (6)$$

Every human-machine system, i.e. every player, tries to maximize its own utility. As mentioned in the preceding paragraph, we assume three possible driving maneuvers.

$$A = \{Accelerate, Brake, Lane\ change\} \qquad (7)$$

The utility can, therefore, be influenced by the choice of the lane and the velocity. When considering another vehicle or an obstacle in the driving situation, the distance d can be regarded as the gap between the two vehicles in the driving direction. For simplicity, it can be assumed that the two vehicles are moving along their lanes, which means that the velocity vectors overlap in case they choose the same lane. From here, the TTC can be determined as the ratio of the distance in driving direction and the relative velocity. When introducing a local coordinate system, which defines the orientation of y in the driving direction, the TTC can be represented as follows:

$$TTC = \frac{y_2 - y_1}{v_1 - v_2} = \frac{d}{v_{diff}} \qquad (8)$$

The utility function of a driver-vehicle system i therefore becomes:

$$U_i = \alpha\ \frac{d}{v_{diff}} + \hat{\beta}\ v_i + \gamma\ (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i}) \qquad (9)$$

5.3 Weighting user-specific factors

Central aspect of a game theoretic modeling is the utility for each driver-car system. Obviously, a perceived utility is most likely to be user specific. Therefore we explicitly allowed the Time to collision constraint to vary for each simulated human machine system. Moreover, we introduced user-specific factors, describing the weight of each term of the utility function. To this end, we introduce suitable coefficients: ling is the utility for each driver-car system. Obviously, a perceived utility is most likely to be user specific. Therefore we explicitly allowed the Time to collision constraint to vary for each simulated human machine system. Moreover, we introduced user-specific factors, describing the weight of each term of the utility function. To this end, we introduce suitable coefficients:

$$\alpha,\ \gamma > 0$$
$$\hat{\beta} < 0 \qquad (10)$$

As explained in an earlier paragraph, we expect that the element costs has a negative impact on total utility. This aspect is accounted for with the negative coefficients $\hat{\beta}$ in equation 9 and 10. As this fact is already considered when

formulating the corresponding element, the coefficient can be substituted with positive coefficients as in equation (9). From here we obtain an alternative formulation for the utility function:

$$U_i = \alpha\ \frac{d}{v_{diff}} + \beta\ v_i + \gamma\ (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i}) \qquad (11)$$
$$\text{with } \beta = -\hat{\beta}$$

5.4 Parametrization

Sensible values are attributed to the user-specific parameters of the model. Please note that we subsequently plan to evaluate the model with human drivers in a driving simulator. Therefore, it is currently sufficient to use some plausible dimensioning for the values, also because the focus of this study is on the resulting driving situation.

$$TTC_{ref} = 12.0$$
$$\alpha = 1.5 \qquad (12)$$
$$\beta,\ \gamma = 1.0$$

6. SIMULATION AND ANALYSIS

6.1 Implementation

The model described above is implemented using MATLAB. The driver-car systems are modeled in an object-orientated way and is based on a model for simulating the longitudinal and lateral vehicle dynamics. For the optimization of the utility function the MATLAB Optimization Toolbox is used, applying a NLP Solver in the fmincon environment. For better illustration, the schematics of this process can be found in Figure 4. As mentioned before, the TTC is considered to have a certain maximum, which results in an optimal perceived safety and therefore a further increase would have no effect on the utility. Thus, two objective functions are implemented. One function uses the maximum impact of the TTC, while the other objective function considers a dynamic influence of the TTC.

$$max(\alpha\ \frac{d}{v_{diff}} - \beta\ v_i + \gamma\ (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i}))$$
$$\text{or}$$
$$max(\alpha\ TTC_{ref,i} - \beta\ v_i + \gamma\ (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i})) \qquad (13)$$
$$\text{s.t}$$
$$TTC \geq 1$$
$$0 \leq v_i \leq v_{max,i}$$

The switching point between both functions is relative to the constraints of the velocity.

$$v_{bound} = \frac{d}{TTC_{ref}} \qquad (14)$$

Obviously, a crash is to be strictly avoided, because in this case the utility would, of course, reach a minimum. To ensure that no incident in form of a crash occurs, the utility function has a constraint that requires the TTC to be above a certain threshold, which is set at 1.0. The fmincon function of the MATLAB Optimization Toolbox provides a solver only for minimization problems. Therefore, an equivalent objective function of the corresponding
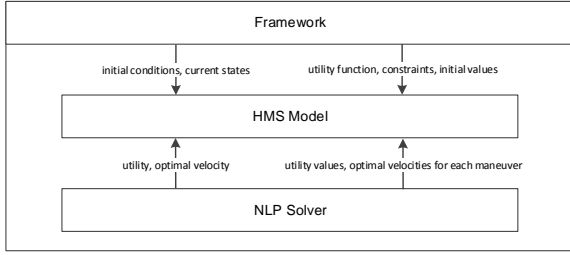
Fig. 4. Schematic description of the simulation environment

minimization problem was formulated by inverting the result.

$$\tilde{U} = -U \tag{15}$$

For both cases, the static TTC and the dynamic TTC case, the associated objective function is minimized individually to avoid a switching structure of the optimization problem. Subsequently, the minimal term is used while the other term is neglected. The overall optimization problem can be described as:

$$min(min(-(\alpha \; \frac{d}{v_{diff}} - \beta \; v_i + \gamma \; (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i})),$$
$$min(-(\alpha \; TTC_{ref,i} - \beta \; v_i + \gamma \; (\frac{s_{ref,i}}{v_{ref}} - \frac{s_{ref,i}}{v_i}))))) \tag{16}$$
$$\text{s.t}$$
$$TTC \geq 1$$
$$0 \leq v_i \leq v_{max,i}$$

This optimization problem is solved for both vehicles at every time step. A driver-car system can observe the behavior of the opponent system, but its intentions remain unknown. Therefore, every system decides upon its own strategy with the information given by a simple observation without taking actual intentions of the opponent into account. In each round, both human-machine systems choose their optimal strategy by comparing the attainable utilities of every possible action and choosing the one with the highest expected score. This leads to the selection of a driving lane as well as to an adjustment regarding the speed guide.

All user-specific parameters as input have been randomly allocated according to the normal distribution:

$$\begin{aligned} TTC_{ref} : \quad & \mu = 12.0, \; \sigma = 1.0 \\ \alpha : \quad & \mu = 1.5, \; \sigma = 0.10 \\ \beta : \quad & \mu = 1.0, \; \sigma = 0.05 \\ \gamma : \quad & \mu = 1.0, \; \sigma = 0.05 \end{aligned} \tag{17}$$

The resulting distributions as used during the simulation can be seen in Figure 5. The starting position in the driving direction for the rear car (vehicle 2) is $y = 200$, for vehicle 1 $y = 300$. The obstacle is placed on the right lane at $y = 1000$. At initial conditions, the velocity of vehicle 2 is 25.0, while the velocity of vehicle 1 is 20.0. These values result from the use case under consideration, i.e. two vehicles on the right lane of a highway in a typical overtaking situation as described above. Therefore, vehicle 1 is driving at 90% of the relevant speed limit for trucks, while the other vehicle is approaching at a higher speed with a non-critical TTC of 20 seconds.
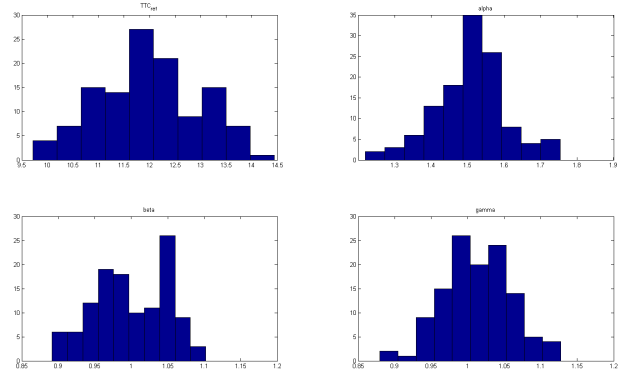

Fig. 5. Distribution of input parameters

6.2 Analysis of the model

The model has been simulated in 60 simulation runs. The corresponding parameters have been drawn randomly in accordance to equation 17. All simulation runs were successful, i.e. they ended without interruption or errors and the given constraints were never violated. Hardware-wise, we used a desktop PC with Windows 7 as operating system, 8GB RAM and four processor cores with a clock rate of 3.1 GHz. The model has been implemented and simulated in MATLAB 2013b.

Having described the traffic situation as a dynamic game, the next step is to analyze possible equilibria of the model. In general, the characteristics of an equilibrium are always that no player has an incentive to change her behavior as long as the other player does not change her behavior. In the situation at hand, we need to extend this to include the environment, i.e. the distance to the obstacle. As long as each system is satisfied with the distances to the other two components as well as with its traveling speed, it will not change its behavior. If, however, the TTCs drop below the level perceived as comfortable, some action needs to be taken. The same applies upon the arousal of dissatisfaction with the traveling speed. As mentioned above, the actions that can be taken are limited by the position of the car and influenced by possible actions of the other player. Three possible constellations can occur:

- Both vehicles are on the right lane with the obstacle.
- Both vehicles are on the left lane.
- One vehicle is on the left lane, one vehicle is on the right lane.

From the perspective of the later vehicle following the earlier vehicle a need for action always arises when the vehicle in front is driving at a (much) lower speed. This is irrespective of the lane they are using, but can be influenced by the distance to the obstacle. This situation can never constitute an equilibrium.

When the difference in speed is not relevant, a temporal equilibrium can emerge when the obstacle is far enough, such that the TTC does not require an immediate reaction. This means in case 1, if the velocity of the first vehicle is at least as high as that of the second vehicle a temporal equilibrium is sustained as long as the distance to the obstacle is large enough. Case 3 is similar, however, only the car on the right lane is concerned about the obstacle.
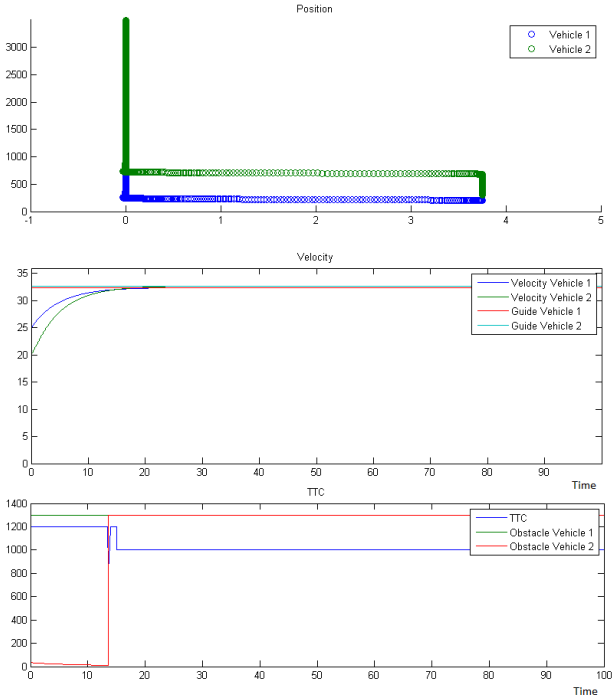
Fig. 6. Position, speed and TTC

An example of a situation which can arise during the simulation, can be seen in Figure 6. In the middle part of the figure, one can observe that vehicle 1 drives at a higher speed than vehicle 2 and, therefore, has an incentive to immediately switch lanes. In the lower part of that same figure, one can also see that the TTC of vehicle 2 (which is still on the right lane) and the obstacle decreases constantly, while the vehicle is approaching the obstacle. At time $t = 13$, it undercuts the reference TTC of vehicle 2, which then, of course, also moves to the right lane. In Figure 7 one can see how the utility develops. It becomes obvious that vehicle 1 is initially indifferent between remaining on the right lane or changing to the left lane. This, however, changes very quickly when she approaches vehicle 2 and the obstacle, which forces her to switch to the left lane to avoid a crash. This is mirrored in the accelerating utility (remember that we converted the maximization problem into a minimization problem, which means that strongly negative values can be translated into very high levels of utility). In the middle of Figure 7 the distance between the two vehicles is depicted as a result of the chosen maneuvers. It can be seen that when it is minimal, the utility for changing the lane starts striving.

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When speed is not an issue, case 2 can constitute a general equilibrium. This also means that both players are best off if they can coordinate in such a way that they are both driving on the left lane, with the faster car in front and the slower car following behind. This situation emerges when both cars have switched to the left lane (cf. Figure 6).

7. LIMITATIONS AND FURTHER RESEARCH

While the utility function introduced in this paper already offers a good approximation and results from the simu-
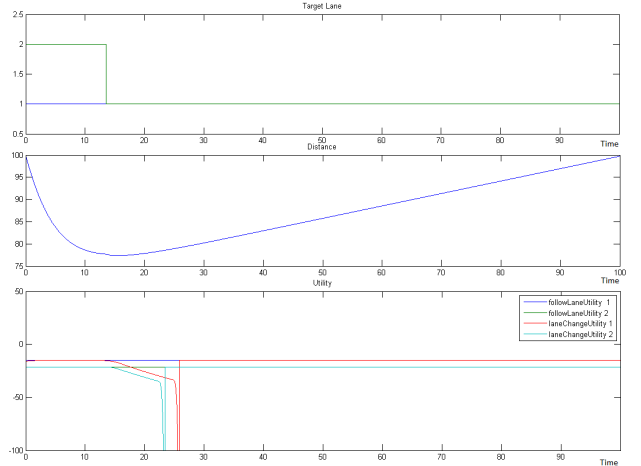

Fig. 7. Target lane, distance between the two cars and value of the utility function

Lane Changes	1	2	3	4	5	11	12
Vehicle 1	44	8	5	0	2	1	0
Vehicle 2	46	12	0	1	0	0	1

Tab. 1. Amount of initiated lane change maneuvers for each vehicle in 60 simulations
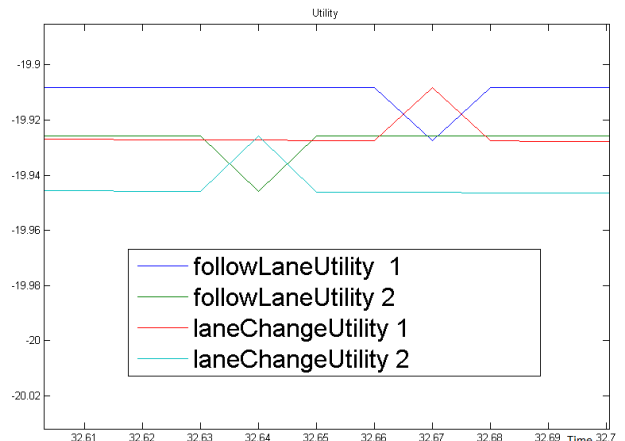

Fig. 8. Utility functions during the most critical run with multiple lane change maneuvers

lation seem to be robust, the real-world utility function might look differently to include further factors that are important to a driver-vehicle system.

7.1 Extending the utility function

An alternative model of the utility function might look as follows:

$$U = \alpha S + \tilde{\beta}C + \tilde{\gamma}T + \delta J + \epsilon W \qquad (18)$$

While some common elements remain, such as the intuition of term T (introduced in equations (4) and (5)) which is necessary for performing the maneuver with some weighting factor $\tilde{\gamma}$, some new elements now additionally enter the function. Some of these already find consideration in the original model in other terms. For example, we introduced TTC as a proxy for perceived safety. However,

other factors like vehicle properties or the condition of the road might also play a role. Therefore, here S denotes the perceived safety of the individual driver-vehicle system, which is composed of not only the distance to the other vehicle and the obstacle, but also other internal and external factors. Furthermore, experienced joy might be crucial for the overall evaluation of the situation and the resulting payoffs. Term J is supposed to be constructed from the joy of driving, which again depends on the perceived dynamics and the perceived success in performing the driving task. As at least most human beings exhibit some altruistic traits and since other goals like reducing travel times on certain roads might want to be accomplished with the introduction of a cooperative system, social effects need to be considered. Term W describes the social welfare, which, unlike all the other elements, can be assumed to be identical for all systems under consideration. Coefficient ϵ considers the individual weights that each human-machine systems puts on social welfare.

Perceived risk is highly influenced by the risk attitude of the individual agent. In principal, it can be assumed that time and risk are negatively correlated, since maneuvers that are perceived as risky, such as swift acceleration, close tailgating, and slipping into short gaps in the traffic flow, are usually performed to decrease travel time and often really do so. Nevertheless, as this interdependence cannot be regarded as generally applicable, both elements still need to be modeled in detail. To maximize utility, all elements except for the time consumed and the emerging costs need to be maximized. The time term $\tilde{\gamma}T$ and the cost term $\tilde{\beta}C$ tend to decrease when approaching the global optimum, since performing the driving task in a faster way or lower costs result in a higher utility. These circumstances are mirrored in the negative weighting factors $\tilde{\gamma}$ and $\tilde{\beta}$.

Perceived safety   In order to be able to describe perceived safety in quantitative terms, one can resort to the approach of collision avoidance and warning, known in aviation. Hereby, in analogy to aviation, we define speed dependent areas around the considered vehicle. To define these areas, the delimitation of the lanes at the lateral axis of the vehicle as well as in the middle of the road are used. For better illustration, one can describe the emerging areas as a sort of tube around the middle of the lane. For evaluating the perceived safety, the lowest resulting distance between the vehicle in question and a third-party vehicle is additionally determined. For a driver-vehicle system, the trajectory belonging to a specified maneuver is known for the own vehicle, but those of the other road users are not predictable. The perceived safety can be modeled under consideration of the following elements: TTC, lowest resulting distance and collision avoidance under consideration of the braking distance depending on speed and environment .

Perceived joy of driving   For quantifying the joy of driving, several joy supporting and hindering activities are linked to the driving maneuvers. Furthermore, some meta-elements need to be considered, such as the contribution of a maneuver to fulfilling the driving task and possibly resulting feedback. For estimating the joy of driving, several components are further broken down and their valuation is ranked on an ordinal scale. The theoretically

achievable maximal value of the perceived joy of driving is used for normalization.

$$J_i = \quad \frac{\sum_k j_{i,k}}{J_{max}} \qquad (19)$$

Economic costs   The economic costs of a driving maneuver are composed of energy and wear-out costs. The energy part of costs consists of the consumed energy. In particular, this can be fuel consumption or the amount of energy taken from a battery. Fuel consumption corresponds to the time integral of the molar flux entering the combustion engine over the period of the driving maneuver. This depends on the course of the position of the throttle valve.

$$C_i = \quad (1 - \frac{c - c_{min}}{c_{max} - c_{min}}) \qquad (20)$$

7.2 Introducing expectations and risk preferences

Keeping in mind that a player does not know a priori which type his opponent has been attributed, one can expect that she forms some sort of expectations about the behavior of her opponents. As this can significantly influence the choice probabilities of a strategy, it should be incorporated into the model. One usually assumes that the distribution function of risk types is commonly known and that a risk type is uniformly or normally distributed. An approach for solving such a dynamic situation with incomplete information is the perfect Bayesian Nash equilibrium. Hereby one assumes that a player can improve and update her expectations starting with previous probabilities and by observing the behavior of other players. Hereby, she can adjust her own behavior to reach a better solution. Expectations are formed under consideration of the Bayesian rules. This can be wrapped-up as follows:

- In a sequential game, nature chooses first and draws the individual attributes from a normal distribution.
- Player 1 gets to know his characteristic, player 2 knows the probability distribution.
- Player 1 can only choose between the available actions. By choosing the action which does not correspond to his own type, a player therefore accepts a lower payoff, but might be able to elicit a certain behavior from the other player.
- Player 2 has similar options as player 1 and can choose with the same consequences.

The limitations mentioned here can also be understood as challenges that need to be addressed by future research. It should be evaluated whether the model performs better with the inclusion of these factors. Using experiments with human subjects, it can be investigated which model is closer to actual human behavior.

8. CONCLUSION AND OUTLOOK

In this paper, we modeled the behavior of two driver-vehicle systems on a road with a static obstacle as a game of imperfect information. Using TTC and velocity as main components of the utility function, each system strives for its maximization. We find that this leads to lane changes at different points in time, where one system can also change

lanes more than once, i.e. more than might be necessary for just avoiding the static obstacle. A crash – which has, of course, the lowest utility – does not occur in any of the cases we investigated. At least in the model, user-specific preferences lead to highly differentiated behavior and fluctuating dynamics in the investigated driving situation.

In order to complement and validate our findings from the simulation, we plan to conduct experiments with human test subjects in a driving simulator which disposes of a cooperative automation such as the one described in this paper. We will then evaluate how well the existing model predicts the system's behavior and adjust it, if necessary, accordingly. In a second step, we plan to extend the model to encompass more than just two players to be applicable to traffic jams with many vehicles or to investigate swarm-like behavior.

On a broader level, findings from this and following studies can also be adopted in the field of vehicle automation. Applying a suitable parameter estimation method, a tendency in the probability of observing a certain driving behavior of other road users can be identified. Among other factors, this can be used as input for the decision making process of a cognitive automation in automated guidance and control.

REFERENCES

Bell, M. G. and Cassir, C. (2002). Risk-averse user equilibrium traffic assignment: an application of game theory. *Transportation Research Part B*, 36:671–681.

Bubb, H. (1993). Systemergonomische Gestaltung. In Schmidtke, H., editor, *Ergonomie*, volume 3. Munchen: Carl Hanser.

Donges, E. (1982). Aspekte der aktiven Sicherheit bei der Führung von Personenkraftwagen. *Automobil-Industrie*, 27(2):183–190.

Donges, E. (2012). Fahrerverhaltensmodelle. In Winner, H., Hakuli, S., and Wolf, G., editors, *Handbuch Fahrerassistenzsysteme - Grundlagen, Komponenten und Systeme fur aktive Sicherheit und Komfort*, chapter 2, pages 15–23. Vieweg+Teubner, Wiesbaden, 2. edition.

Flemisch, F., Heesen, M., Hesse, T., Kelsch, J., Schieben, A., and Beller, J. (2012). Towards a dynamic balance between humans and automation: authority, ability, responsibility and control in shared and cooperative control situations. *Cognition, Technology & Work*, 14(1):3–18. 10.1007/s10111-011-0191-6.

Flemisch, F. O., Bengler, K., Bubb, H., Winner, H., and Bruder, R. (2014). Towards a cooperative guidance and control of highly automated vehicles: H-mode and conduct-by-wire. *Ergonomics*, 57(3):343–360.

Fotakis, D., Kontogiannis, S., Koutsoupias, E., Mavronicolas, M., and Spirakis, P. (2002). The structure and complexity of nash equilibria for a selfish routing game. Springer Berlin Heidelberg.

Fudenberg, D. and Tirole, J. (1991). *Game Theory*. The MIT Press.

Garcia, A., Reaume, D., and Smith, R. L. (2000). Fictitious play for finding system optimal routings in dynamic traffic networks. *Transportation Research Part B*, 34:147–156.

Gasser, T. M., Arzt, C., Ayoubi, M., Bartels, A., Bürkle, L., Eier, J., Flemisch, F., Häcker, D., Hesse, T., Huber, W., Lotz, C., Maurer, M., Ruth-Schumacher, S., Schwarz, J., and Vogt, W. (2012). Rechtsfolgen zunehmender Fahrzeugautomatisierung - Gemeinsamer Schlussbericht der Projektgruppe. Fahrzeugtechnik F 83, Bundesanstalt fur Straßenwesen (bast).

Griffiths, P. and Gillespie, R. B. (2004). Shared control between human and machine: haptic display of automation during manual control of vehicle heading. In *Haptic Interfaces for Virtual Environment and Teleoperator Systems, 2004. HAPTICS '04. Proceedings. 12th International Symposium on*. IEEE.

Harsanyi, J. C. (2004). Games with incomplete information played by "bayesian" players, i-iii. *Management Science*, 50(12):1804–1817.

Hoeger, R., Zeng, H., Hoess, A., Kranz, T., Boverie, S., Strauss, M., Jakobsson, E., Beutner, A., Bartels, A., To, T.-B., Stratil, H., Furstenberg, K., Ahlers, F., Frey, E., Schieben, A., Mosebach, H., Flemisch, F. O., Dufaux, A., Manetti, D., Amditis, A., Mantzouranis, I., Lepke, H., Szalay, Z., Szabo, B., Luithardt, P., Gutknecht, M., Schomig, N., Kaussner, A., Nashashibi, F., Resende, P., Vanholme, B., Glaser, S., Allemann, P., Seglo, F., and Nilsson, A. (2011). Deliverable d61.1final report. Technical report, DLR.

Hollander, Y. and Prashker, J. N. (2006). The applicability of non-cooperative game theory in transport analysis. *Tranportation*, 33(5):481–496.

Hollnagel, E. and Woods, D. D. (1983). Cognitive systems engineering: New wine in new bottles. *International Journal of Man- Machine Studies*, 18:583–600.

Kita, H. (1999). A merging giveway interaction model of cars in a merging section: a game theoretic analysis. *Transportation Research Part A*, 33:305–312.

Löper, C., Kelsch, J., and Flemisch, F. O. (2008). Kooperative, manöverbasierte Automation und Arbitrierung als Bausteine fur hochautomatisiertes Fahren. In *Gesamtzentrum fur Verkehr Braunschweig (Hrsg.): Automatisierungs-, Assistenzsysteme und eingebettete Systeme fur Transportmittel. GZVB, Braunschweig*, pages 215–237.

Lubashevsky, I., Wagner, P., and Mahnke3, R. (2003). Rational-driver approximation in car-following theory. *Physical Review E*, 68(5):056109.

Mulder, M., Abbink, D. A., and Boer, E. R. (2012). Sharing control with haptics: Seamless driver support from manual to automatic control. *Human Factors*, The Journal of the Human Factors and Ergonomics Society October 2012 54(5):786–798.

Parasuraman, R., Sheridan, T. B., and Wickens, C. D. (2000). A model for types and levels of human interaction with automation. *IEEE Transactions on Systems, Man, and Cybernetics - Part A: Systems and Humans*, 30(3):286–297.

Rasmussen, E. (1989). *Games and Information: An Introduction to Game Theory*. Blackwell Publishers Inc.

Rasmussen, J. (1983). Skills, rules and knowledge; signals, signs, and symbols, and other distinctions in human performance models. *IEEE Transactions on Systems, Man, and Cybernetics*, 3(3):257–266.

Sheridan, T. B. and Parasuraman, R. (2006). Human-automation interaction. volume 18, pages 89–129. Human Factors and Ergonomics Society.

Vetta, A. (2002). Nash equilibria in competitive societies, with applications to facility location, traffic routing and auctions. Foundations of Computer Science.

von Stackelberg, H. F. (1934). *Marktform und Gleichgewicht (Market Structure and Equilibrium).* Habilitation.

Wie, B.-W. (1993). A differential game model of nash equilibrium on a congested traffic network. *Networks*, 23(6):557–565.

Winner, H. and Hakuli, S. (2006). Conduct-by-wire - following a new paradigm for driving into the future. In *FISITA 2006 World Automotive Congress : 22 - 27 October, Yokohama, Japan.* FISITA; SAE of Japan.