Economics of Long-term Portfolio Management in Electricity Markets

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Abstract

Electricity systems around the world are facing massive investments to replace aged and add new generation capacity. Thereby, investments in power generation assets bear considerable financial risks in view of the typically high capital investments and long-lasting asset lifetimes between 20 and 50 years. The regulatory frame set by policy makers and the investment decisions of companies of today influence the socioeconomic costs of tomorrow in liberalized electricity markets. Thus, both policy makers and investors should reflect costs (respectively return) and monetary risks of their investment decisions carefully to build economical and long-term sustainable electricity systems.

This thesis aims to contribute to the theory of decision making under uncertainty in the field of electricity generation investments and to analyze efficient generation portfolios both from a societal and from a company perspective. For that, the research proposed in this thesis combines and extends concepts of capacity planning and peak load pricing on the one hand and Mean-Variance Portfolio theory based on Markowitz on the other hand—which are well-discussed in economic literature individually—in an integrated modelling approach. Thereby, the research in this thesis is focused on financial risks induced by technology-specific fuel price uncertainties which are inherent to all non-renewable generation technologies.

Having briefly recaped the fundamentals of decision theory under uncertainty, we propose a model that captures the investment decision as a formal optimization problem. From the latter, quantitative diversification criteria are derived and analytical solutions for cost-risk efficient generation portfolios are determined from a welfare perspective. The results show that diversification of generation portfolios is—even under high societal risk aversion—not beneficial per-se. The technology mix in efficient portfolios depends rather on the specific risk of each technology. Consequently, generation technologies with traditionally low fuel price fluctuations (e.g. nuclear or lignite plants) are preferred compared with technologies with higher price fluctuations (e.g. gas) with increasing societal risk aversion.

While commonly neglected in literature, the effect of reversal risks in the short-term order of dispatch ("merit order") is analytically studied and quantified in this thesis. It is shown that this risk factor can impact the efficient technology mix substantially especially given long-term investment horizons. While existing literature in the field of capacity planning and Mean-Variance Theory relies predominantly on the key assumptions of perfect markets, we show how risk-averse investor behavior may shift the technology structure in the market equilibrium significantly away from the welfare optimum.

Finally, we resume the focus on the investor perspective and empirically study the impact of the fuel mix structure in power generation portfolios on expected stock returns for major European power companies. It is shown that the generation fuel mix has a significant impact on the historical stock returns of the investigated companies. Thus, these results provide theoretical and practical benefit to determine adequate riskadjusted capital costs for typical generation technologies from an investor perspective.

Zusammenfassung

Weltweit erfordern Erneuerungen und Erweiterungen der Stromversorgungssysteme hohe Investitionen in neue Stromerzeugungsanlagen. Die hierfür erforderlichen Kraftwerksinvestitionen bergen aufgrund der hohen Baukosten und der langen Lebensdauern zwischen 20 und 50 Jahren erhebliche finanzielle Risiken sowohl für einzelne Investoren als auch für die Wettbewerbsfähigkeit ganzer Volkswirtschaften; denn der von Energiepolitikern gesetzte regulatorische Marktrahmen und im Markt getroffenen Investitionsentscheidungen von heute beeinflussen die volkswirtschaftlichen Kosten für die Energieversorgung von morgen. Daher sollten Energiepolitiker bei der Förderung bestimmter Kraftwerkstechnologien und Investoren bei ihren Entscheidungen zu Kraftwerksneubauten sowohl die Kosten (bzw. Renditen) als auch die finanziellen Risiken im Hinblick auf das Erzeugungsportfolio im Markt berücksichtigen.

Diese Arbeit zielt darauf ab, die Entscheidungstheorie unter Unsicherheit im Hinblick auf Kraftwerksinvestitionen weiterzuentwickeln und effiziente Erzeugungsportfolios sowohl von einer volkswirtschaftlichen als auch von einer Investoren-Perspektive zu erforschen. Dazu setzt diese Arbeit auf den in der Literatur bereits intensiv diskutierten Konzepten zur Kapazitätsplanung sowie des "Peak-load Pricing" einerseits sowie der Markowitz'schen Portfoliotheorie andererseits auf und entwickelt diese in einem integrierten Modellansatz weiter. Dabei stehen finanzielle Risiken aus den spezifischen Brennstoffpreis-Unsicherheiten unterschiedlicher Erzeugungs-Technologien im Fokus dieser Arbeit.

Nach einer kurzen Zusammenfassung elementarer Grundlagen zur Entscheidungstheorie wird die Investitionsentscheidung als ein formales Optimierungsproblem modelliert. Hiervon werden quantitative Kriterien zur Diversifikation des Erzeugungsportfolios in Abhängigkeit von der Risikoaversion abgeleitet und analytische Lösungen für bezogen auf Kosten und Risiken effiziente Erzeugungsportfolios aus einer Wohlfahrtperspektive bestimmt. Die Ergebnisse belegen, dass Diversifikation per se selbst bei hoher gesellschaftlicher Risikoaversion nicht zwingend vorteilhaft ist. Der effiziente Technologiemix im Erzeugungsportfolio ist vielmehr durch die spezifischen Risiken der einzelnen Technologien selbst bestimmt. Folglich werden mit steigender gesellschaftlicher Risikoaversion jene Erzeugungstechnologien mit geringen Brennstoffpreisrisiken (beispielsweise Kernkraft oder Braunkohle) gegenüber Technologien mit hohen Preisschwankungen (beispielsweise Gas) bevorzugt.

Im Gegensatz zu bestehenden Forschungsarbeiten, in denen Veränderungen der Grenzpreis basierten Angebotskurve ("Merit Order") aufgrund von Schwankungen in den Brennstoffkosten nicht betrachtet werden, wird dieses Risiko in der vorliegenden Arbeit explizit analysiert und quantifiziert. Im Ergebnis wird gezeigt, dass Merit Order Risiken insbesondere bei langen Investitionszeiträumen den Technologiemix in effizienten Erzeugungsportfolios erheblich beeinflussen. Während bestehende Literatur zum Thema Kapazitätsplanung und Portfoliotheorie gemeinhin die Annahme vollkommener Märkte voraussetzt, wird in dieser Arbeit analytisch gezeigt, wie Risikoaversion von Investoren die sich im Marktgleichgewicht einstellende Struktur des Erzeugungsportfolios beeinflussen und zu erheblichen Abweichungen vom wohlfahrtsoptimalen Technologiemix führen kann.

Schließlich wird wieder aus der Investorenperspekive empirisch der Einfluss des Technologiemixes im Erzeugungsportfolio auf die erwartete Rendite von großen europäischen Energieversorgern untersucht. Es kann gezeigt werden, dass die Erzeugungsstruktur einen signifikanten Einfluss auf die historischen Aktienrenditen der untersuchten Unternehmen hat. Die Ergebnisse liefern einen theoretischen und praktischen Beitrag zur Ermittlung risikoadjustierter Kapitalkosten für typische Erzeugungstechnologien aus einer Investorenperspektive.

List of Publications

This dissertation includes in Chapters 3-6 the following articles and working papers which are in the process of being published:

[A] Sunderkötter, M., Weber, C., 2012. Valuing fuel diversification in power generation capacity planning. Energy Economics, 2012, Vol. 34, Issue 5, Pages 1664–1674.
http://dx.doi.org/10.1016/j.eneco.2012.02.003

[B] Sunderkötter, M., Weber, C., 2011. Mean-variance optimization of power generation portfolios under uncertainty in the merit order. University of Duisburg-Essen, EWL working paper 05/2011. Submitted to Annals of Operations Research.
http://ideas.repec.org/p/dui/wpaper/1105.html

[C] Sunderkötter, M., Weber, C., Ziegler, D., 2013. Perfect competition versus riskaverse agents: Technology portfolio choice in electricity markets. University of Duisburg-Essen, EWL working paper 02/2013. Submitted to the Journal of Industrial Economics.
http://ideas.repec.org/p/dui/wpaper/1303.html

[D] Sunderkötter, M., 2011. Fuel mix characteristics and expected stock returns of European power companies. University of Duisburg-Essen, EWL working paper 06/2011. Submitted to Managerial Finance.
http://ideas.repec.org/p/dui/wpaper/1106.html

Contents

Chapter 1

Introduction

1.1 Objectives and academic relevance

The global electricity generation sector faces an immense need for investments in the next 25 years. The IEA (2010) estimates that "total capacity additions, to replace obsolete capacity and to meet demand growth, amount to more than 5900 GW globally in the period 2009–2035; over 40% of this is installed by 2020." The cumulative global investment required solely for new generation plants is about US$ 9.6 trillion (in year-2009 dollars) over 2010–2035. Determining and ensuring an optimal capital and capacity allocation with respect to the generation technology mix thus represents a crucial problem faced both by investors on liberalized markets but also by governments and policy makers.

Thereby, energy policy has to reflect three main criteria with respect to an efficient generation technology mix: Security of supply, sustainability, and competitiveness of supply. Many European electricity markets have undergone a transition with respect to the relevance of these three criteria in the last decades. Taking Germany as an example, security of supply was implicitly the primary objective before market liberalization until the late 1990s. The following decade of market liberalization changed the focus in favor of competitiveness. Starting with the Renewable Energy Act from 2000, the German energy policy turned increasingly towards a sustainable development of the electricity sector. Nevertheless, cost competitiveness remains in the long-run a necessary condition for affordable power prices and therewith for public acceptance of the generation fuel mix.

Since generation investments bear considerable risks in particular in view of the high capital requirements and the long-lasting commitment periods with life cycles of power plants of 30–50 years, competitiveness of supply is also driven by the specific financial risks incurred. Both investors and society as a whole should reflect the technological and financial risks of different generation technologies. Thereby, a risk-averse behavior will typically by applied and influence the investment decision. As in financial markets, financial risk management has become an important prerequisite to succeed in liberalized electricity markets. Hence, optimal generation capacity allocation under cost and risk targets represents a key problem from an energy policy view. Companies face the analogue problem of efficient fuel mix selection under risk and return objectives with investment decisions in electricity generation assets.

This thesis aims to analyze the economics of investments in power plant portfolios under uncertainty. Thereby, the primary objective is to improve the understanding of how specific generation technologies affect the trade-off between long-term risks[1] and expected benefits (respectively expected costs) in power generation portfolios. To serve this goal, this thesis includes four research papers on long-term risk management and related decision problems with respect to investments in power generation portfolios in electricity markets.

Although the topic itself is not new in academia and a substantial number of studies has been published in recent years on optimal capacity allocation in risk-return frameworks, the majority of this literature applies numerical methods. In contrast, this dissertation aims to contribute to the topic with a special emphasis on analytically derived insights which enable a better understanding of interdependencies of parameters and results in efficient portfolios.

As a second goal of this work, the proposed models aim to be applicable for analytical decision support for both electricity market investors and political decision makers to determine the efficient fuel mix in power generation portfolios with respect to financial risk and return. Thereby, the exact numbers in the applications presented within this thesis have primarily illustrative character and are to be considered as exemplary case studies. However, the models are sufficiently generic to be easily transferred to other electricity markets and investment decisions.

This thesis is based on and integrates the work of four separate research papers which analyze capacity allocation and investment problems under uncertainty focusing on power generation assets in electricity markets. Thereby, mean-variance efficient investment strategies in electricity markets are analyzed from the perspective of a representative societal decision maker and individual investor as well as a market equilibrium problem.

By reviewing existing work and developing novel theory and applications based on a broad methodological spectrum with respect to specific aspects of investment and portfolio optimization problems, the papers in this thesis contribute to different areas of research within the field of energy economics. Thereby, the connecting element between all papers is the central question of the efficient power generation fuel mix from a societal and investors' perspective.

1.2 Summary and structure of this thesis

The overall introduction to the research topic of this thesis is provided in Chapter 1. Chapter 2 reviews elementary cornerstones and concepts of decision theory under uncertainty and by that aims to serve as a useful foundation of the following research papers. Chapters 3-6 provide the research

[1] Denton et al. (2003) distinguish risks of asset operators in electricity markets by three different time horizons: Firstly operational/earnings risks over the short term (less than one month), secondly trading and operational risks over the intermediate term (one month to one year), and asset valuation/equity risks over a long (more than one year) time-frame.

papers which deal with different aspects and methodologies of long-term portfolio management in electricity markets.

Chapter 3 and 4 focus on mean-variance optimization of power generation portfolios and contribute methodologically to applied operations research. Both papers consider the impact of technology-specific fuel price risks on welfare-optimal investments and derive quantitative diversification criteria for efficient generation portfolios. Chapter 3 has been published as Sunderkötter and Weber (2012) and presents a novel analytical approach combining conceptual elements of peak-load pricing and MVP theory to derive optimal portfolios consisting of an arbitrary number of plant technologies given uncertain fuel prices.

Building on the modeling principles of Chapter 3, Chapter 4 (corresponding to Sunderkötter and Weber, 2011) relaxes one of the core assumptions and no longer excludes uncertainty in the short-term order of variable operating costs (merit order). This relaxation is of particular importance if competing technologies face operating costs with only a small difference in mean, but high variances and imperfect correlation. The extended model framework results in a non-convex optimization model. Both article conclude with an examination of the proposed frameworks and the results for the German electricity market.

Chapter 5 (corresponding to Sunderkötter et al., 2013) compares market imperfections in form of risk-averse company behavior in investment decisions with the market outcome under perfect competition. Implications on the market investment equilibrium are analyzed in a partial equilibrium model. It is shown that risk-averse company behavior applying mean-variance portfolio optimization does typically not coincide with market equilibrium in perfectly competitive markets.

It can be shown that the mean variance based decision models discussed in Chapters 3-5 are under weak conditions consistent with the well-known postulations of the capital asset pricing model (CAPM). Hence it arises the question from an investor perspective whether there are significant differences in the systematic risk of different power generation technologies. To answer this question, Chapter 6 empirically investigates interdependencies between the power generation fuel mix and the systematic risk of power companies. The analysis is based on historical evaluations of expected stock returns for 22 European power generation companies. Differences in the debt-adjusted market betas can be found between the sample companies that own and operate generation portfolios consisting of different technologies. Based on these observations, technology-specific beta factors are identified and tested for significance in differences for coal, natural gas, nuclear, and renewable generation technologies.

1.2.1 Paper I: Mean-Variance optimization of power generation portfolios

Deterministic capacity planning problems in electricity systems can be solved by comparing technology specific long-term and short-term marginal costs. In an uncertain market environment, Mean-Variance Portfolio (MVP) theory provides a consistent framework to balance risk and return in power generation portfolios. Focusing on fuel price risks, MVP theory can be adopted to determine the welfare efficient system generation technology mix.

Existing literature on MVP applications in electricity generation markets uses predominantly numerical methods to characterize portfolio risks. In contrast, this article presents a novel analytical approach combining conceptual elements of classical capacity planning models and MVP theory to derive the efficient portfolio structure consisting of an arbitrary number of plant technologies given uncertain fuel prices. For this purpose, we provide a static optimization model which allows to fully capture fuel price risks in a mean variance portfolio framework. The analytically derived optimality conditions contribute to a better understanding of the optimal investment policy and its risk characteristics compared to existing numerical methods. Furthermore, we demonstrate an application of the proposed framework and provide results for the German electricity market which has been hardly treated in MVP literature on electricity markets.

This article provides easily interpretable analytical optimality conditions for efficient generation portfolios from a societal point of view and therewith contributes to a better understanding of MVP in electricity applications.

In the following paper, the discussion of risk-cost efficient capacity allocation is deepened by relaxing one of the core assumption, i.e. a stable merit order:

1.2.2 Paper II: Optimization of power portfolios under uncertainty in the merit order

In this article we discuss welfare-optimal capacity allocation of different electricity generation technologies available for serving system demand. While the classical peak load pricing theory derives the efficient portfolio structure from a deterministic marginal production cost curve ("merit order"), we investigate in particular the implications of possible reversals in the merit order—so-called merit order risks or fuel switch risks—induced by uncertain operating costs.

We propose a static, non-convex optimization model combining the classic peak load pricing model with elements of mean-variance portfolio (MVP) theory and analytically discuss possible solution cases and important optimality properties. We examine the approach in a case study on the efficient structure of generation portfolios consisting of CCGT and hard coal technologies in Germany. With special emphasis, we study the emergence of overcapacities (exceeding maximal demand) in efficient portfolios and show that diversification is not beneficial per-se. The results show that the efficient technology mix may be significantly impacted by the merit order risk, especially given a difference time series of operating costs without mean-revering behavior. Therefore, our findings support the importance of considering this risk factor especially with long-term investment horizons.

The model is applicable to various investment problems related to production of non-storable goods under price uncertainty of input factors. Similar problems can e.g. be found in transportation systems or in the process industry.

1.2.3 Paper III: Perfect competition versus riskaverse agents: Technology portfolio choice in electricity markets

Investments in power generation assets are risky due to high construction costs and long asset lifetimes. Technology diversification in generation portfolios represents one option to reduce long-term investment risks for risk-averse decision makers. In this article, we analyze the impact of market imperfections induced by risk-aversion on the long-term investment portfolio structure in the market. We show that risk-averse electricity market agents who receive a managerial profit share may shift the technology structure in the market significantly away from the welfare optimum. A numerical example provides estimates on the potential scale of this effect and discusses sensitivities of key parameters.

In contrast to the previous papers, the following article focuses on the investor perspective:

1.2.4 Paper IV: Fuel mix characteristics and expected stock returns of European power companies

This article investigates the impact of the fuel mix structure in power generation portfolios on expected stock returns for major European power companies. The 22 largest publicly listed European power producers are examined between January 2005 and December 2010. Based on the capital asset pricing model (CAPM) and multi-factor market models, the systematic risk of the power companies relative to the overall market performance and other typical energy and macroeconomic risk factors is analyzed. The full-information approach is used to determine technology-specific betas and risk factor sensitivities from the sample. Although most companies are not exclusively in the power producing business, it is shown that the generation fuel mix has a significant impact on the historical stock returns of the investigated companies. In particular, the sample companies exhibit significant differences in the systematic risk of gas and nuclear generation technologies compared with renewable technologies measured by technology-specific, delevered beta factors.

This study extends existing literature and contributes new insights in two ways: Firstly, this is to our knowledge the first empirical analysis comparing the financial risk of different electricity generation technologies. Secondly, the results provide practical benefit to determine adequate risk-adjusted capital costs for typical generation technologies. Therewith, this study is relevant for evaluating all kinds of power plant investments.

Chapter 2

Decisions under uncertainty: A brief review of theory

Before addressing any specific decision and portfolio problems, it seems helpful to approach the topic with some more general reflections on how risk may influence the satisfaction of a decision maker who bears the risk. Hence, this chapter aims to provide a brief introduction to decision theory under uncertainty and therewith sets the basis for the following chapters in which we discuss specific decision problems.

2.1 Risk and utility

The economic literature provides many different definitions of risk, uncertainty, and ambiguity. On the basis of Knight (1921), modern economic textbooks commonly distinguish the following three different categories of unknowns (e.g. Stirling, 1994, Domschke and Scholl, 2003, Trautmann, 2006): *Risk* describes situations in which the decision maker has full information on all decision alternatives, on all possible outcomes, and the probability for the realization of each outcome. In contrast, there is no information about the distribution of probabilities under *uncertainty* (in a narrower sense). This definition typically corresponds also to *ambiguity*. Last, *ignorance* describes situations in which individuals are not able to form beliefs of probabilities, for instance if there is no knowledge about the possible outcomes at all.

In the following, we will focus our considerations to decision situations under risk. These are based on the assumption that the decision maker has full information on all decision alternatives, on all possible outcomes, and the probability for the realization of each outcome. Classic economic theory characterizes decisions in risk situations by four attributes (cf. e.g. Gollier, 1999):

- The space of possible *lotteries* or actions $\mathcal{L} = \{L_1, \ldots, L_I\}$ describes the set of mutually exclusive decision alternatives, the decision maker has at a certain point in time.
- The *state space* $\Omega = \{\omega_1, \ldots, \omega_S\}$ which characterizes a set of potential states of the environment that may exogenously influence the decision or action. For simplicity reasons, we assume a finite set of possible states.

- The *information structure* over the state space $\Omega : P = (p_1, \ldots p_S)$ which contains the probabilities for the realization of each state of the environment.

- The space of possible *outcomes* $X = \{x_{11}, \ldots, x_{IS}\}$, where each outcome x_{ij} is defined by a function $f(L_i, \omega_j)$ with $(L_i, \omega_j) \mapsto x_{ij} = f(L_i, \omega_j)$. The element x_{ij} represents the consequence of the lottery L_i given the environment state ω_j. The latter is realized at a likelihood of $p_j = p(\omega_j)$.

We can characterize a simple lottery L_i by the vector $(x_{i1}, p_{i1}; x_{i2}, p_{i2}; \ldots; x_{iS}, p_{iS})$. A compound or multi-stage lottery is a lottery whose outcomes are again lotteries. Consider a compound lottery L which yields lottery $L_a = (x_{a1}, p_{a1}; \ldots; x_{aS}, p_{aS})$ with probability α and lottery $L_b = (x_{b1}, p_{b1}; \ldots; x_{bS}, p_{bS})$ with probability $(1-\alpha)$, i.e. $L = \alpha L_a \oplus (1-\alpha) L_b$. The probability that the outcome of lottery L is x_1 equals $p_1 = \alpha p_{a1} + (1-\alpha) p_{b1}$. For all decision situations considered in this thesis, we assume that $\mathcal{L}, \Omega, P, X$ are known.

To analyze which lottery yields in a decision situation an optimal outcome, it is necessary to define a decision principle that allows to evaluate the outcomes with respect to a specified set of characteristics. This is performed by a preference relation "$\preceq$" which allows to rank the outcomes of different decision alternatives or the decision alternatives itself based on the decision makers' preferences. For instance, $L_i \succ L_j$ means that decision alternative L_i is strictly preferred to L_j. While a preference relation allows to compare and rank alternatives only on an ordinal scale, it is desirable in stochastic settings to quantify the utility of different alternatives on a cardinal scale to derive utility maximizing decision strategies (see e.g. Trautmann, 2006).

The utility from decision alternative L_j can be expressed by a real number u, the utility index. A function U assigning a utility index to every combination of outcomes, $U : X(\mathcal{L}, \Omega) \to \mathbb{R}$ is called *utility function*. With the natural order of the real numbers, the utility function enables to rank and—even more—to quantitatively value the investor's utility from all combinations of goods. Utility functions can be based on different underlying criteria which reflect the individual benefits of the decision maker. Most common in microeconomic theory are utility functions which reflect the expected value and the risk incurred with a decision alternative. These utility functions are also called *risk utility functions*.

Utility functions characterize and represent different risk attitudes. Risk-neutral individuals are characterized by linear utility functions (cf. $U_0(x)$ in Fig. 2.1): These persons would accept a certain payment equal to the expected value of the outcome—regardless of the risk incurred. A risk-averse person is represented by a concave utility function ($U_1(x)$ in Fig. 2.1): The decision maker would accept a certain payment (certainty equivalent) of less than the expected value, rather than taking the gamble and possibly receiving nothing. Conversely, a convex utility function applies for risk-affine individuals ($U_2(x)$ in Fig. 2.1).

In economic theory, individuals are typically assumed to behave risk-averse which is equivalent to a concave utility function. Thereby, the degree of risk aversion is characterized by the second derivative of the utility function. Since the risk attitude characterized by a utility function is invariant with respect to affine-linear transformations of the utility function, however not the

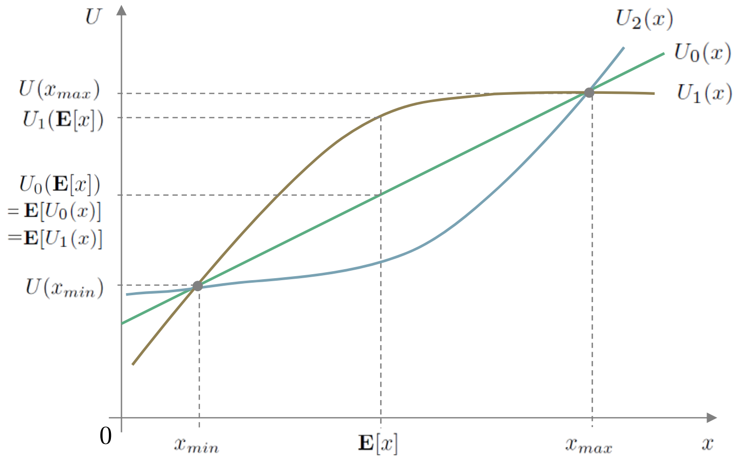


Fig. 2.1: Utility functions corresponding to different risk attitudes: $U_0(x)$ represents a utility function of a risk-neutral decision maker, $U_1(x)$ the utility function of a risk-averse (risk-avoiding) individual, and $U_2(x)$ the utility function of a risk-affine (risk-seeking) person.

second derivative of the utility function, a suitable normalization is necessary. The normalization can be done with the first derivative, yielding to the *Arrow-Pratt coefficient of absolute risk aversion* (ARA) as described by Arrow (1971), Pratt (1964):

$$ARA(x) := -\frac{U''(x)}{U'(x)} \tag{2.1}$$

The *coefficient of relative risk aversion* (RRA) is defined as

$$RRA(x) := -\frac{x \cdot U''(x)}{U'(x)} = x \cdot ARA(x) \tag{2.2}$$

The interpretation of (2.1) and (2.2) is straightforward: An investor with decreasing (increasing) absolute risk aversion would agree a larger (smaller) absolute potential loss for increasing capital invested. Hence a decreasing ARA seems plausible. An investor with decreasing (increasing) relative risk aversion would agree a larger (smaller) potential loss as a percentage of his capital invested for increasing capital invested. Here, the assumption of a constant relative risk aversion seems most intuitive.

The most common examples for utility functions representing risk aversion in microeconomic theory are characterized by different absolute and relative risk aversions:

- **Quadratic utility function**

  $$U(x) = ax - \frac{1}{2}bx^2 \tag{2.3}$$

  with $a \geq 0$, $b \geq 0$. For all quadratic utilities, $ARA(x)$ and $RRA(x)$ are increasing in x. The main problem with quadratic utility is that it has the odd behavior that for a sufficiently large return the utility function leads to a situation where a smaller return is preferred, i.e. $U'(x) < 0$ for all $x > \frac{a}{b}$. Thus it is advisable to restrict the domain to $x < \frac{a}{b}$ or extend it with a constant $U(x) = \frac{a^2}{2b^2}$ for $x > \frac{a}{b}$.

- **Exponential utility function**

$$U(x) = -\exp(-ax) \tag{2.4}$$

with $a > 0$. Here, $ARA(x)$ is constant whereas $RRA(x)$ is increasing in x. Thereby, the utility function exhibits the upper limit $\lim_{x\to\infty} U(x) = 0$.

- **Power utility function**

$$U(x) = \frac{x^{1-\gamma} - 1}{1-\gamma} \tag{2.5}$$

with $0 \leq \gamma \leq 1$. $ARA(x)$ decreases in x while $RRA(x)$ is constant with $RRA(x) = \gamma$.

2.2 Fundamental decision principles

2.2.1 Expected utility

The probably most fundamental principle for rational decisions under uncertainty going back to Cramer (1728) and Bernoulli (1938) comprises maximization of expected utility. The work of von Neumann and Morgenstern (1944) was the first important application of this principle in modern economic theory. Today's axiomatic foundation of utility theory is furthermore built on the work of Savage (1954).

The theory of expected utility is based on four axioms about agents' preferences for rational decision making in uncertain environments. We consider again an agent in the following decision situation: $L_j \in \mathcal{L}$ denotes the set of possible strategies or decision alternatives with risky outcomes (lotteries). Each lottery has a finite number i of uncertain outcomes x_{ij}, where for each outcome the agent knows its probability p_{ij}. Let $\preceq$ be a complete and transitive binary relation defined on $\mathcal{L} \times \mathcal{L}$, representing the agent's preference ordering over the lotteries. As usual, (strict) preference of L_1 over L_2 is denoted as $L_1 \preceq L_2$ ($L_1 \prec L_2$). Indifference between lotteries L_1 and L_2 is denoted by $L_1 \sim L_2$.

Based on the original axioms developed by von Neumann and Morgenstern (1944) which describe the properties of the preference structure over all alternatives, different refined formulations have been discussed in literature (see e.g. Gollier, 1999):

(A1) **Completeness:** There is a complete weak order over all alternatives, i.e. for any two alternatives L_1, L_2, either $L_1 \preceq L_2$ or $L_1 \succeq L_2$ holds.

(A2) **Transitivity:** If $L_1 \preceq L_2$ and $L_2 \preceq L_3$ than also $L_1 \preceq L_3$ is true. Together, (A1) and (A2) are often referred to as the "weak order" axioms.

(A3) **Convexity/continuity:**[1] For all alternatives with $L_1 \preceq L_2 \preceq L_3$, there is a $p, q \in (0, 1)$ such that $pL_1 \oplus (1-p)L_3 \preceq L_2 \preceq qL_1 \oplus (1-q)L_3$.
This axiom can be interpreted as the indifference between a pair of lotteries and a simple one-stage lottery. In this case, $pL_1 \oplus (1-p)L_2$ is a two stage lottery which yields either alternative

[1] This axiom is in literature sometimes also called *Archimedean property*.

L_1 with probability p and alternative L_3 with probability $(1-p)$ in the first stage. (A3) states that given any three alternatives preferred to each other, then there exists a two-stage lottery with $p \in (0,1)$ combining the most and least preferred alternative in a way such that the compound of L_1 and L_3 is preferred to the middling alternative L_2. Furthermore, there exists another two-stage lottery with $q \in (0,1)$ so that the middling alternative L_2 is strictly preferred to the compound of L_1 and L_3.

(A4) **Independence:** For all alternatives L_1, L_2, L_3 and $p \in [0,1]$, $L_1 \preceq L_3$ if and only if $pL_1 \oplus (1-p)L_2 \preceq pL_3 \oplus (1-p)L_2$.
The Independence axiom (A4) claims that the preference between two alternatives L_1, L_3 is unaffected if they are both combined in the same way with a third alternative L_2. Again, this can be envisaged as a choice between a pair of two-stage lotteries. $pL_1 \oplus (1-p)L_2$ is a two stage lottery which yields either lottery L_1 with probability p or lottery L_2 with probability $(1-p)$ in the first stage. Using the same interpretation for $pL_3 \oplus (1-p)L_2$, then preferences between the two-stage lotteries ought to depend entirely on the agent's preferences between the alternative lotteries in the second-stage, L_1 and L_3, since both combinations lead to L_2 with the same probability $(1-p)$ in the first stage so that the agent is indifferent if this case occurs.

So far we have always assumed a finite state space Ω and therewith a finite set of possible outcomes X. For the following part of this chapter, we extend this assumption to a continuous notation: The random variable $\tilde{x}_i$ defined on the probability space $(\Omega_i, \mathcal{A}(\mathbb{R}), \mathbf{P}_i)$ with $\tilde{x}_i : \Omega_i \mapsto \mathbb{R}$ defines the set of continuous outcomes of lottery $L_i \in \mathcal{L}$. $F_i(\tilde{x}_i) : \mathbb{R} \mapsto [0,1]$ is the cumulative density function of lottery L_i with the associated probability density $\varphi_i(\tilde{x}_i)$.

The above-mentioned axioms represent the basis for the characterization of the expected utility function by von Neumann and Morgenstern (1944):

Theorem 2.2.1. *(von Neumann and Morgenstern, 1944) Let "$\preceq$" be a binary preference relation on $\mathcal{L}$. Then "$\preceq$" satisfies axioms (A1)-(A4) if and only if there is a real-valued function $U : L \mapsto \mathbb{R}$ such that:*

a) U represents "$\preceq$", i.e. $\forall\, L_1, L_2 \in L$, it holds $L_1 \preceq L_2 \Leftrightarrow U(L_1) \leq U(L_2)$.

b) U is affine, i.e. $\forall\, L_1, L_2 \in L$, and $\forall\, p \in (0,1)$ it is
$U(pL_1 \oplus (1-p)L_2) = pU(L_1) + (1-p)U(L_2)$.

Moreover, if there is another preference representing utility function $V : L \to \mathbb{R}$, then $\exists\, a, b \in \mathbb{R}$ with $a > 0$ such that $V = aU + b$, i.e. U is unique up to a positive linear transformation. This implies a cardinal

In literature, U is frequently called *von Neumann-Morgenstern* utility function. Going beyond an ordinal preference order, the utility function implies a cardinal preference which is especially desirable for applications in stochastic settings.

von Neumann and Morgenstern (1944) have proven that the obtained utility function U has a expected utility representation of the form

$$U(L_i) = \mathbf{E}\big[U(\tilde{x}_i)\big] = \sum_j p_{ij} U_{ij} = \sum_j p_{ij} U(x_{ij}). \tag{2.6}$$

where $U : x \to \mathbb{R}$ is an (elementary) utility function on the underlying outcomes L_i. The expected utility of lottery L_i equals the probability-weighted sum of the utilities of all possible outcomes of L_i. The expected utility can thus be considered as an ex-ante utility function while the utility of the possible outcomes is an ex-post utility function. We conclude this section with

Corollary 2.2.1 (Expected utility principle)**.** *Let "$\preceq$" be a binary preference relation on $\mathcal{L}$ satisfying axioms (A1)-(A4). A rational agent will prefer lottery 2 compared to lottery 1, if the expected utility from the uncertain outcomes $\tilde{x}_2$ of lottery 2 is greater then the expected utility from outcomes $\tilde{x}_1$ of lottery 1, i.e.*

$$L_1 \preceq L_2 \quad \Leftrightarrow \quad \mathbf{E}[U(\tilde{x}_1)] \leq \mathbf{E}[U(\tilde{x}_2)] \tag{2.7}$$

Notably, this decision principle may be very different from the maximization of the expected outcome itself. Since its axiomatic foundation, the expected utility principle[2] is regarded as one of the most fundamental principle for rational decisions under uncertainty. There are no limiting requirements on the utility function, i.e. a rational preference structure induced by the expected utility principle is rational regardless of the assumed utility function provided that it exists any utility function with expectation on this preference structure.

2.2.2 Mean-variance utilities

The classic mean variance portfolio theory (Markowitz, 1952, 1959) as well as the CAPM (Sharpe, 1964, Lintner, 1965, Mossin, 1966) are based on the assumption that investors are comparing investments solely based on the expected value and its variance:

Definition 2.2.2 (Mean variance principle)**.** *Let "$\preceq$" be a binary preference relation on $\mathcal{L}$. An investor prefers lottery 2 compared to 1 if its uncertain future payoffs $\tilde{x}_2$ are greater than or equal in expectation at a smaller variance, or if they are greater in expectation at a smaller than or equal variance compared to the payoffs $\tilde{x}_1$ of alternative 1, i.e.*

$$L_1 \preceq L_2 \quad \Leftrightarrow \quad \Big(\mathbf{E}[\tilde{x}_1] \leq \mathbf{E}[\tilde{x}_2] \,\wedge\, Var[\tilde{x}_1] > Var[\tilde{x}_2]\Big) \vee \Big(\mathbf{E}[\tilde{x}_1] < \mathbf{E}[\tilde{x}_2] \,\wedge\, Var[\tilde{x}_1] \geq Var[\tilde{x}_2]\Big) \tag{2.8}$$

For more than two decision alternatives, the mean variance decision principle (2.8) will in general not result in a complete ordering of alternatives, i.e. there may be alternatives L_1 and L_2 for which neither $L_1 \preceq L_2$ nor $L_2 \preceq L_1$ holds. An unambiguous order among these can only be obtained with additional information about the risk attitude of the investors and/or about the distribution of payoffs. Furthermore, decisions according to the mean variance principle are not necessarily

[2] The expected utility principle is alternatively frequently denoted as Bernoulli's principle.

consistent with the expected utility principle (Corollary 2.2.1). The interrelation of both decision principles will be discussed in the following.

Note that the mean variance principle implies the investors' utilities being of the form $U(\tilde{x}) = f(\mu, \sigma^2)$, with $\mu = \mathbf{E}[\tilde{x}]$ denoting the expected value and $\sigma^2 = \mathrm{Var}[\tilde{x}]$ the variance of $\tilde{x}$. Levy and Markowitz (1979) studied several approximations to the expected utility where the approximation depends only on the mean and the variance of the distribution. The authors have shown that for the most common utility functions the expected utility can be reasonably well approximated by a function of only mean and standard deviation using a truncated Taylor series approximation around the mean:

$$\mathbf{E}[U(\tilde{x})] \approx f(\mu, \sigma) = U(\mu) + \frac{1}{2}U''(\mu)\sigma^2 \tag{2.9}$$

For example, with a quadratic utility function of the form $U(\tilde{x}) = x - \frac{\alpha}{2}(x - x_0)^2$ it is

$$f(\mu, \sigma) = \mu - \frac{\alpha}{2}\left(\sigma^2 + (\mu - x_0)^2\right) \tag{2.10}$$

As it can be seen, a quadratic utility function corresponds to the first three terms of the Taylor series expansion of any utility function and represents in many cases a reasonable well approximation. Tsiang (1972, p. 356) states in this context: "If the convergence of the series is sufficiently fast, so that, for fairly close approximation, the terms beyond the second moments can be neglected, then indeed the expected utility can be approximately determined by the first two moments, mean and variance, even if the utility function is not quadratic, and the uncertain outcomes not normally distributed." However, the negation of third and higher moments is only acceptable for sufficiently small standard deviations (Tsiang, 1972, p. 356): "Since risk (variance) is assumed to be infinitesimally small, higher order central moments are assumed to be of even smaller orders and thus all omitted."

Theorem 2.2.3 (Consistency with expected utility principle)**.** *Rule 2.2.2 is consistent with the expected utility principle (Corrolary 2.2.1) and the von Neumann-Morgenstern axioms of rational preferences under uncertainty either*

a) for investors with quadratic utility functions, or

b) (multivariate) normal distributed payoffs and any concave utility function

The proof is given in Tobin (1958).

Note that Tobin (1958) has shown sufficient criteria for the mean variance principle to be consistent with the expected utility principle. However, he could not show (or allege) these conditions as being also necessary.

2.3 Mean-variance preference functionals

The difficulty of the risk-utility functions discussed above is that they allow to determine the utility for single outcomes (or even a utility distribution with respect to the distribution of the outcomes), but they do not capture aggregate metrics or risk measures. *Preference functionals* allow to directly evaluate the level of satisfaction of the decision maker by incorporating certain (quantitative) risk measures. An alternative decision principle related to definition 2.2.2 is based on the maximization of a *mean variance preference functional.* In contrast to the mean variance decision principle, it ensures a complete ordering of alternatives.

Definition 2.3.1 (Mean variance preference functional). *Let "$\preceq$" be a binary preference relation on $\mathcal{L}$. An investor prefers lottery 2 compared to 1 if its preference value is greater, i.e.*

$$L_1 \preceq L_2 \quad \Leftrightarrow \Psi(L_1) \leq \Psi(L_2) \tag{2.11}$$

with $\Psi(L_i) := \mathbf{E}[\tilde{x}_i] - \frac{A}{2}\,Var[\tilde{x}_i]$.

The parameter A denotes the investor's risk attitude and reflects for $A = 0$ risk neutrality, $A > 0$ risk aversion and $A < 0$ risk proclivity. This preference functional allows to combine mean and variance in one objective which is technically very convenient in optimization problems.

Theorem 2.3.2 (Consistency with expected utility principle). *Rule 2.3.1 is consistent with the expected utility principle (Corrolary 2.2.1) and the von Neumann-Morgenstern axioms of rational preferences under uncertainty for investors with exponential utility functions of the form* $U(x) = -\exp(-Ax), A > 0$, *and normal distributed payoffs.*

Proof. Theorem 2.3.2 can bee seen as follows:

$$\begin{aligned}
\mathbf{E}[U(\tilde{x})] &= \int_{-\infty}^{\infty} U(x) f(x) \mathrm{d}x = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} -\exp(Ax)\exp\left(-\frac{1}{2}\frac{(\mu - x)^2}{\sigma^2}\right) \mathrm{d}x \\
&= -\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\frac{x^2 - 2x\mu + \mu^2 + 2Ax\sigma^2}{\sigma^2}\right) \mathrm{d}x \\
&= -\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\frac{(x - (\mu - A\sigma^2))^2}{\sigma^2}\right) \exp\left(-\frac{1}{2}\frac{-(\mu - A\sigma^2)^2 + \mu^2}{\sigma^2}\right) \mathrm{d}x
\end{aligned}$$

Applying the transformation $y := \frac{x - (\mu - A\sigma^2)}{\sigma}$, we obtain

$$\begin{aligned}
\mathbf{E}[U(\tilde{x})] &= -\frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left(-\frac{1}{2}\frac{(x - (\mu - A\sigma^2))^2}{\sigma^2}\right) \exp\left(-\frac{1}{2}y^2\right) \sigma \mathrm{d}x \\
&= -\exp\left(-\frac{1}{2}\frac{(x - (\mu - A\sigma^2))^2}{\sigma^2}\right) = -\exp\left(-A\left(\mu - \frac{A}{2}\sigma^2\right)\right)
\end{aligned}$$

Thus, the expected utility increases monotone with the preference functional $\Psi(L_i) := \mathbf{E}[\tilde{x}_i] - \frac{A}{2}\mathrm{Var}[\tilde{x}_i]$. □

Note, that the condition stated in Theorem 2.3.2 is *sufficient* for consistency of maximization of the mean variance preference functional and expected utility maximization. Furthermore, Schneeweiss (1965) has shown that for normal distributed payoffs, exponential utilities are necessarily required for consistency of preference functions of the form $\Psi(L_i) := \mathbf{E}[\tilde{x}_i] - \frac{A}{2}\mathrm{Var}[\tilde{x}_i]$ with expected utility maximization.

Inadvertently, theorems 2.2.3 and 2.3.2 can be found frequently mixed up in literature in a way that quadratic utilities are treated as sufficient condition for consistency of maximization of a preference functional Ψ with the expected utility principle. The falsity of the general conclusion can, however, is important to see: Under the assumption of a quadratic utility function, the expected utility in an uncertain investment environment can be stated as

$$\begin{aligned}\mathbf{E}[U(\tilde{x})] &= \mathbf{E}[a\tilde{x} - \frac{1}{2}b\tilde{x}^2] \\ &= a\mathbf{E}[\tilde{x}] - \frac{1}{2}b\mathbf{E}[\tilde{x}^2] \\ &= a\mathbf{E}[\tilde{x}] - \frac{1}{2}b\left(\mathrm{Var}[\tilde{x}] + \mathbf{E}[\tilde{x}]^2\right) \end{aligned} \tag{2.12}$$

It can be easily seen that only for zero expected payoffs $f = \mathbf{E}[\tilde{x}] - \frac{A}{2}\mathrm{Var}[\tilde{x}]$ represents a reasonable approximation of the expected utility. This, however, limits the application of the considered mean variance preference functionals with quadratic utilities to very few cases.

Due to its simplicity, mean variance preference functionals represent one commonly used approach to model risk return trade-offs, e.g. in decision support models. Overall, it can be seen that this approach is reasonable if the precondition of normally distributed payoffs and exponential utilities are satisfied. The following chapters will hence build on this decision principle to derive efficient portfolios properties.

Chapter 3

Mean-variance optimization of power generation portfolios

3.1 Introduction

In the next two decades, the European power industry will face an increasing need for investments to renew and extend its aging power plant fleet (cf. e.g. IEA, 2008). In parallel, EU and several national governments have implemented different development schemes which politically influence investment decisions and thereby the fuel mix of the respective country. For an economic evaluation of power plant investments from a welfare perspective, it is crucial to take into account both the expected total life-cycle costs and the economic risks conveyed with investment and operation of the plant fleet. With fuel prices fluctuating considerably, cost volatility becomes a severe risk over a plant's lifetime that influences the return of an investment and therewith the system optimal fuel mix.

Mean-Variance Portfolio (MVP) theory has been established as a clear framework to capture the two aspects of risk and return in a single decision support model since the work of Markowitz (1952, 1959) has set the stage for MVP theory in financial markets. Markowitz' theory builds on the premise that a compound portfolio of assets shows reduced variance characteristics in case each pair of assets shows only imperfect correlation. Similarly, portfolio cost risks can be reduced in a portfolio of well-chosen generation technology options as a result of less than perfect correlations between their cost characteristics. This approach can also be applied to derive efficient electricity generation portfolios from a risk-cost perspective.

Standard MVP models on electricity portfolios use numerical simulation or quadratic optimization techniques to derive efficient power generation portfolios. This methodology makes it possible to solve even very complex optimization problems with numerous plants and technologies, but it naturally complicates the understanding for the exact interplay of the different input parameters. This article provides easily interpretable analytical optimality conditions for efficient generation portfolios from a societal point of view and therewith contributes to a better understanding of

MVP in electricity applications. Moreover, we analyze the sensitivities of the optimal solution on different parameters. Our findings include the at a first sight counterintuitive result that higher risk aversion can yield to less diversified generation portfolios. The proposed model is finally applied to the German electricity market. The model is generic enough to be easily transfered to other electricity systems in order to support policy makers in their decisions and strategy adoptions with respect to the generation mix.

This paper is structured as follows: We begin by reviewing relevant literature and briefly discussing the selection of adequate risk and return measures in Section 3.2. Section 3.3 contains the formulation and proposes a general solution approach of the portfolio optimization problem. For a simplified case with two technologies, optimality conditions are analytically derived and discussed. The insights from the theoretical model are demonstrated and interpreted in a case study on the German generation portfolio in Section 3.4. Section 3.4.3 provides an in-depth discussion of the two-technology case with CCGT and coal technologies. The article concludes in Section 3.5 with a summary of key results and an outlook of related interesting areas for future research.

3.2 Relevant literature

Capacity planning problems in energy systems face the particularity of (nearly) non-storability of electricity for serving demand. Classical generation capacity planning models go back to the work of Steiner (1957), Hirshleifer (1958), Boiteux (1960) on peak load pricing and capacity planning. Various expansions as linear, non-linear and dynamic programming models have been proposed later (see e.g. Anderson, 1972).

To incorporate risk in the planning problems, portfolio optimization techniques have emerged by adopting Mean-Variance Portfolio (MVP) theory based on the work of Markowitz (1952, 1959). As one of the first who adopted a MVP approach to long-term portfolio optimization in electricity markets, Bar-Lev and Katz (1976) discuss the problem of fuel-cost optimization of fossil plants in the U.S. utility industry. The topic re-appeared widely on the academic agenda after the millennium with the emergence of numerical simulation techniques in economic research:

Intending to provide decision support to energy policy makers, one major group of studies analyses total system costs and risks to derive the efficient power generation mix with varying focus on the considered regional/national markets and the specific risk factors. Thereby, early studies (such as e.g. Awerbuch and Berger, 2003, Awerbuch, 2004, 2006, Jansen et al., 2008) share however the major drawback of using unit costs (total generation cost per MWh, including average fixed costs) as input parameters. Yet this would only be valid if full load hours of all considered technologies were not influenced by the portfolio composition. In energy policy considerations, however, effective operating hours and unit cost cannot be assumed as being independent from the optimal technology mix. Instead, fixed and variable costs should be treated separately to reflect varying operating times due to changes in the fuel mix structure.

A second group of studies focuses on the efficient generation mix from an investor's perspective (Roques et al., 2006a,b, 2008). As the major drawback here, these studies assume a stable electricity price distribution derived from historical data. This consequently implies a net present value (NPV) distribution that neglects the fact that portfolio choices will also influence electricity prices in the long run. To avoid the problem of modeling technology-specific adjustments of full load hours and implications on the electricity price distribution, Roques et al. (2008) explicitly restrict their model to base-load portfolios in which all technologies are assumed to operate at the same full load hours. Although this assumption avoids inconsistencies in the modeling results, it however prevents to derive conclusions on the system optimum.

To avoid the inaccuracy from exogenous operating times, a correct long-term model framework aiming to allow conclusions on the optimal generation technology mix for an electricity market as a whole should therefore reflect the actual operating ours. Correspondingly, unit costs need to be separated into operating and investment costs. Based on this modeling principle, Gotham et al. (2009) suggest a single-period cost-based model for optimal capacity allocation in a mean-variance framework with different load segments to be served. Delarue et al. (2011) have proposed more recently a numerically solved optimization model capturing endogenous dispatch hours as well as uncertain availability of renewable technologies such as wind.

This article builds on the work of the latter-mentioned authors and differentiates between (cost of) installed capacity and (costs of) produced electricity to discuss the value of technology diversification in efficient portfolios. Going beyond, we extend the model to allow not only for different load segments but for a continuous load duration curve being served. In contrast to all other studies on MVP optimization of electricity portfolios we know of being published, this article proposes an analytical study of optimality conditions instead of using simulation techniques or numerical methods to characterize the efficient generation fuel mix. We believe that the analytical approach is not only more exact but allows also a better understanding of the functional impact of the different model parameters on the efficient generation portfolio mix.

3.3 Model formulation

3.3.1 Deterministic capacity planning problem

The classical lowest-cost capacity planning can be formulated as a two-stage optimization problem building as shown in Eqs. (3.1) to (3.4): We consider an electricity system with $u \in \{1, \ldots, n\}$ generation technologies available. From a societal perspective, the objective is to minimize the sum of total operating costs, $C_{op,u}$, plus annualized capacity investment costs, $C_{inv,u}$, summed over the available technologies u and over the total planning period $[0;T]$ (e.g. a year). Let the latter be broken down into time steps of equal length $t \in [0;T]$ (e.g. hours). Eqn. (3.4) represents the demand constraint. Without loss of generality, total system demand is assumed to be given in a decreasing order (i.e. rearranged in form of the load duration curve) by the function $D : [0;T] \to \mathbb{R}_+, t \mapsto D(t)$ which we assume to be strictly monotone with $D(0) = D_{max}$. Furthermore, demand is assumed to be price inelastic which can be considered as a simplifying but within a wide range of operating costs fairly realistic assumption. The capacity constraint which assures that the output of each plant, $y_{u,t}$, is less than or equal its capacity, K_u, is given in Eqn. (3.3):

$$C^* = \min_{y_{u,t},K_u} C(y_{u,t}, K_u) \tag{3.1}$$

$$\text{s.t. } C = \int_0^T \sum_u C_{op,u} \mathrm{d}t + \sum_u C_{inv,u} = \int_0^T \sum_u y_{u,t} \cdot c_{op,u,t} \mathrm{d}t + \sum_u K_u \cdot c_{inv,u} \tag{3.2}$$

$$y_{u,t} - K_u \leq 0 \qquad \forall \quad t, u \tag{3.3}$$

$$\sum_u y_{u,t} \geq D(t) \qquad \forall \quad t \tag{3.4}$$

Operating costs at time t are a function $C_{op,u}(c_{op,u,t}, y_{u,t})$ of specific operating costs $c_{op,u,t}$ (€/MWh) and the instantaneous output level $y_{u,t}$ (MW). In addition, we will write the investment costs in the following sections as $C_{inv,u}(K_u, c_{inv,u})$, indicating the dependency on the installed capacity K_u and the specific investment costs $c_{inv,u}$ (€/MW$_{el}$). Therefore, the plant capacities K_u and the corresponding output levels $y_{u,t}$ are the decision variables to be optimized.

To allow a better understanding of the results, we assume in this model formulation full capital flexibility which can realistically only be assumed over a very long planning horizon.[1] Furthermore, we neglect plant indivisibilities and other technology-specific constraints not reflected in the average operating costs such as ramp-up costs and times. However, these assumptions can easily be implemented in any numerical large-scale model setup and are not in focus of this analysis.

3.3.2 Risk-adjusted investment optimum with uncertain fuel prices

To derive efficient frontiers of asset combinations, classic MVP theory assumes that investors' portfolio preferences depend solely on mean and variance of the expected return. The portfolio with the smaller variance of return at the same level of expected return or the portfolio with the higher expected return at the same level of return variance will be preferred. Thereby, consistency of the (μ, σ^2) decision principle with maximization of expected utility[2] requires either investors to act based on quadratic utility functions or returns to be normally distributed and investors to behave risk aversely.[3]

Frequently used in optimization literature are preference functions of the form $\Psi(a) := \mathbb{E}[X(a)] - \frac{A}{2}\mathrm{Var}[X(a)]$, where a denotes a decision alternative and X the corresponding random payoff. Schneeweiss (1965) has shown that for normally distributed payoffs, exponential utilities with constant absolute risk aversion are necessary and sufficient for consistency of the preference $\Psi(\mu, \sigma^2, a)$ with the rational principle of expected utility maximization. Furthermore, Meyer (1987) proposes with the location and scale condition a more general condition to test two-moment decision rules

[1] In a differentiated modeling of the capacity planning problem regarding new and historic investments as e.g. done by Gotham et al. (2009), historic (sunk) investment costs are neglected. This enables the model to capture the optimal transition from an existing plan fleet the future long-term optimum, which is however not the main purpose of our analysis.

[2] In economic textbooks frequently referred to as "Bernoulli's principle".

[3] See e.g. Tobin (1958).

for consistency with expected utility maximization which is widely fulfilled by well-diversified portfolios regardless of a specific utility function.

The proposed approach can be straightforwardly transferred to model generation portfolio risks induced by fuel price uncertainty by using the preference function as the objective function of the capacity planning model. We assume societal preferences being described by the preferences of a representative consumer with an exponential utility function of the form $U(x) = -\frac{1}{A}\exp(-Ax)$. Then the expected dis-utility can then be approximated by the following (μ, σ^2) preference as a function of the expected generation costs and the corresponding variance:

$$L = \mathbb{E}[C] + \frac{1}{2}A \cdot \mathrm{Var}[C], \tag{3.5}$$

The parameter A denotes the society's risk attitude and reflects for $A = 0$ risk neutrality, $A > 0$ risk aversion and $A < 0$ risk proclivity.

The proposed model focuses on input price risks of electricity generation. Most important, fuel price fluctuations can financially affect generation costs in principle both in the long-term and in the short-term. Unlike short-term risks, which can be hedged on energy forward markets, long-term fuel price uncertainties remain as a major risk factor. Therefore, we conceive the optimal generation portfolio selection problem as a two-stage problem: At the first stage, investment is carried out, i.e. capacities are selected based on known investment cost and uncertain fuel cost. The second stage covers the power plant operation over a representative period, i.e. typically a year. At this stage, the actual fuel prices are revealed. Fuel price fluctuations within the operating period are disregarded in this article, assuming that those may be eliminated through hedging. Non-market risks, e.g. operational or technical risk factors such as availability or construction cost risks, are not considered in the model either.

To capture the long-term fuel price fluctuations, specific operating costs of each technology $\tilde{c}_{op,u}$ are modeled as random variables[4] with obtained realizations being taken as constant throughout the operating period $[0; T]$.[5] In line with other MVP studies (see e.g. Roques et al. (2008)) we assume that levels of fuel prices—and therewith operating costs—are normally distributed. This assumption can be justified by the fact that independently and identically distributed price increments—even if not normally distributed—result in price levels that follow a normal limiting distribution. Expected operating costs are denoted by $\bar{c}_{op,u} := \mathbb{E}[\tilde{c}_{op,u}]$. The covariance in specific operation costs of plants u and v is denoted by σ_{uv}, i.e. $\tilde{c}_{op,u}$ are n-variate jointly distributed. For a shorter notation, we denote Q_u the energy produced by technology u in the period $[0; T]$, i.e. $Q_u := \sum_t y_{u,t}$. Thereby, Q_u is determined as a result of the fixed investment and the deterministic merit order. Then, as shown in A.2.1, the expected dis-utility capturing expected total generation costs and (fuel) cost risk can be specified as

$$L = \sum_u Q_u \bar{c}_{op,u} + \sum_u C_{inv,u} + \frac{A}{2}\left(\sum_u \sum_v \sigma_{uv} Q_u Q_v\right). \tag{3.6}$$

[4] Throughout this article, random variables are indicated by a "~", whereas their realizations are written as plain letters.

[5] Because operating costs are constant within the planning period, we write $c_{op,u}$ instead of $c_{op,u,t}$.

Without loss of generality, the n technologies are ordered by increasing operating costs, i.e. $\forall\, u, v \in \{1, \ldots, n\}$, $(\bar{c}_{op,u} < \bar{c}_{op,v})$. We exclude the possibility of reversals in the merit order such that no realization of operating costs with $c_{op,u} \geq c_{op,v}$ can occur.[6] To solve the second-stage of the optimization problem, the optimal technology dispatch can now be determined based on the merit order of expected generation costs. For the technology with the lowest operating costs, i.e. technology 1, the upper bound of operating duration is always $t_0 = T$. The lower bound is given through $D(t_1) = K_1$, since technology one will run at full capacity as soon as demand exceeds capacity K_1. Similarly for technology two, the upper bound for operation hours is given by $D(t_1) = K_1$, and the lower by $D(t_2) = K_1 + K_2$ and so forth (see Figure 3.1). Finally, it can be seen that the lower bound of the operating time of the n-th technology is zero, i.e. $t_n = 0$. By introducing the cumulative capacity $K_u^c = \sum_{j=1}^{u} K_j$, and defining $R(K)$ as the inverse of the monotonously decreasing function $D(t)$, we may write $t_u = R(K_u^c)$. Now, solving the first-stage portfolio selection problem is equivalent to determining the cumulative capacities K_u^c. We additionally define the integral to the inverse demand function

$$Q^I(K_u^c) = \int_0^{K_u^c} R(\kappa) \mathrm{d}\kappa, \tag{3.7}$$

hence $Q_u(K_u^c, K_{u-1}^c) = Q^I(K_u^c) - Q^I(K_{u-1}^c)$. The optimization problem may now be reformulated, using only K_u^c as decision variables. In time-continuous notation, this yields

$$L^* = \min_{K_u^c} L \tag{3.8}$$

$$\text{s.t.} \quad L = \sum_{u=1}^{n} c_{inv,u} \left(K_u^c - K_{u-1}^c \right) + \bar{c}_{op,u} Q_u + \frac{A}{2} \left(\sum_{u=1}^{n} \sum_{v=1}^{n} \sigma_{uv} Q_u Q_v \right) \tag{3.9}$$

$$K_u^c - K_{u-1}^c \geq 0 \qquad (1 \leq u \leq n,\, K_0^c = 0) \tag{3.10}$$

$$Q_u = Q^I(K_u^c) - Q^I(K_{u-1}^c) \qquad (1 \leq u \leq n) \tag{3.11}$$

$$Q_E \leq Q^I(K_n^c) \tag{3.12}$$

where Q_E denotes total energy demand over the considered period, i.e. $Q_E = \int_0^T D(t) \mathrm{d}t$. To properly reflect a limited willingness-to-pay of power customers, the last technology n may also be interpreted as an accepted load shedding with the corresponding generation costs representing the value of lost load. The corresponding Lagrangian writes:

$$\begin{aligned} \mathcal{L}_n = \sum_{u=1}^{n} c_{inv,u} \left(K_u^c - K_{u-1}^c \right) + \bar{c}_{op,u} Q_u + \frac{A}{2} \left(\sum_{u=1}^{n} \sum_{v=1}^{n} \sigma_{uv} Q_u Q_v \right) + \\ + \sum_{u=1}^{n} \mu_u \left(K_{u-1}^c - K_u^c \right) + \lambda \left(Q_E - Q^I(K_n^c) \right) \end{aligned} \tag{3.13}$$

[6] This simplification can be justified because the empirically estimated year-to-year risk for reversals in the merit order is less than 1% for all considered technologies and hence extremely low (cf. Section 3.4).

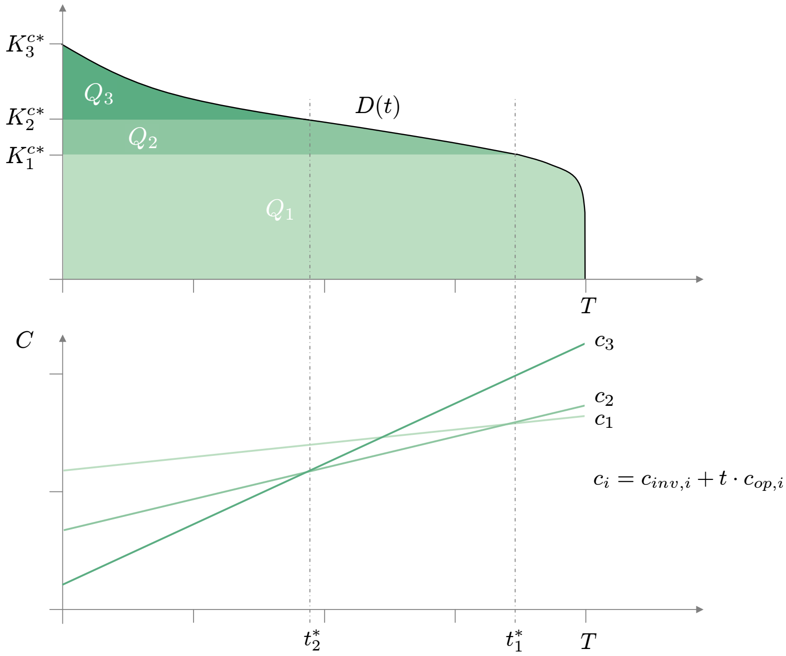


Fig. 3.1: Graphical solution of the deterministic capacity planning problem from load duration curve and full-cost curves

Before exploring this general optimization problem in more detail, two specific configurations are investigated, contained as extremes in the general portfolio problem: One limiting case obviously embedded in the general formulation is the purely cost-minimal capacity planning problem, corresponding to $A = 0$. The other case to be looked at is the purely variance-minimizing problem, to which the general problem converges as $A \to +\infty$.

3.3.3 Standard solution for purely cost efficient portfolios with n technologies

As discussed in Crew and Kleindorfer (1986) for the deterministic capacity planning problem, the purely cost-minimal problem with $A = 0$ and n technologies may also be solved graphically using the load duration curve and the full-cost curves of the respective technologies (see Figure 3.1). Formally, the Karush-Kuhn-Tucker (KKT) conditions corresponding to the Lagrangian (3.6) are here:

$$\frac{\partial \mathcal{L}_n}{\partial K_u^c} = c_{inv,u} - c_{inv,u+1} + (\bar{c}_{op,u} - \bar{c}_{op,u+1})\, t_u - \mu_u + \mu_{u+1} \geq 0, \quad \perp \quad K_u^c \geq 0,\ (1 \leq u \leq n-1) \tag{3.14}$$

$$\frac{\partial \mathcal{L}_n}{\partial K_n^c} = c_{inv,n} + \bar{c}_{op,u} t_n - \mu_n - \lambda t_n \geq 0, \qquad \perp \quad K_n^c \geq 0, \tag{3.15}$$

$$\frac{\partial \mathcal{L}_n}{\partial \mu_u} = K_{u-1}^c - K_u^c \leq 0, \qquad \perp \quad \mu_u \geq 0,\ (1 \leq u \leq n) \tag{3.16}$$

$$\frac{\partial \mathcal{L}_n}{\partial \lambda} = Q_E - Q^I(K_n^c) \leq 0, \qquad \perp \quad \lambda \geq 0. \tag{3.17}$$

Obviously, the last set of conditions pushes K_n^c to be at least equal to $D(0)$, so that all energy is provided by the total capacity installed and consequently even the peak-load demand is covered. Given that $t_n = R(K_n^c) = 0$ follows from $K_n^c = D(0)$, (3.15) simultaneously then implies that $\mu_n = c_{inv,n}$. This result reflects the finding from classical peak-load-pricing theory stating that the shadow price in the (here infinitesimal) peak-load moment covers the full investment costs of the peak technology. From (3.14) optimal lower bounds of operating hours may be written

$$t_u = \frac{c_{inv,u} - c_{inv,u+1} - \mu_u + \mu_{u+1}}{c_{op,u+1} - c_{op,u}},\ (1 \leq u < n), \tag{3.18}$$

If $K_u > 0$ is assumed throughout, (3.16) implies that all μ_u are equal to zero. Consequently (3.18) may be directly used to compute capacities based on the following propositions.

Proposition 3.3.1. *Let be*

$$t_u^o := \frac{c_{inv,u} - c_{inv,u+1}}{c_{op,u+1} - c_{op,u}},\ (1 \leq u < n) \tag{3.19}$$

If $t_u^o < t_{u-1}^o$ for all $1 \leq u < n$, then the cost-minimal portfolio consists of all technologies, i.e. $K_u^* > 0$ for all $1 \leq u < n$.

Optimal capacities can then be obtained by $K_u^* = K_u^{c*} - K_{u-1}^{c*}$, $(1 \leq u \leq n)$ with $K_u^{c*} = D(t_u^o)$. In fact the t_u^o may be used generally to check the validity of the assumption $K_u > 0$:

Proposition 3.3.2. *Technology u is part of the cost-minimal portfolio, i.e. $K_u^* > 0$, only if $t_u^o < t_{u-1}^o$.*

Stated in other words, $t_u^o \geq t_{u-1}^o$ implies that technology u is not included in the cost-efficient portfolio, i.e. $K_u^* = 0$. The proofs and further interpretation of Propositions 3.3.1 and 3.3.2 are provided in A.2.2.

3.3.4 Standard solution for purely variance efficient portfolios with n technologies

For the second limiting case with $A \to +\infty$, the Lagrangian (3.6) may be rewritten as:

$$\mathcal{L}_n = \sum_{u=1}^{n}\sum_{v=1}^{n}\sigma_{uv}Q_uQ_v - \sum_{u=1}^{n}\mu_u^Q Q_u + \lambda^Q\left(Q_E - \sum_{u=1}^{n} Q_u\right) \tag{3.20}$$

Thereby the Q_u are directly used as decision variables, since this allows a more convenient treatment. Yet through $Q_u = Q^I(K_u^c) - Q^I(K_{u-1}^c)$ a unique mapping between the Q_u 's and K_u^c 's is established, which may be later used to transform results. The KKT-conditions are here:

$$\frac{\partial \mathcal{L}_n}{\partial Q_u} = \sum_{v=1}^{n}\sigma_{uv}Q_v - \mu_u^Q - \lambda^Q \geq 0, \qquad \perp \qquad Q_u \geq 0, \quad (1 \leq u \leq n) \tag{3.21}$$

$$\frac{\partial \mathcal{L}_n}{\partial \mu_u^Q} = -Q_u \leq 0, \qquad \perp \qquad \mu_u^Q \geq 0, \quad (1 \leq u \leq n) \tag{3.22}$$

$$\frac{\partial \mathcal{L}_n}{\partial \lambda^Q} = Q_E - \sum_{u=1}^{n} Q_u \leq 0 \qquad \perp \qquad \lambda^Q \geq 0 \tag{3.23}$$

A matrix notation is advantageous for the further treatment, hence the key conditions are rewritten with $\mathbf{i} = (1, \ldots, 1)^T$ denoting the n-dimensional one vector and $\mathbf{\Sigma}$ the covariance matrix of operating costs:

$$\mathbf{\Sigma Q} - \boldsymbol{\mu}^Q - \lambda^Q \mathbf{i} = \mathbf{0} \tag{3.24}$$

$$Q_E - \mathbf{i}^T\mathbf{Q} = 0 \tag{3.25}$$

In fact the case $Q_u = 0$ does not immediately lead to a determinate value for μ_u^Q since each μ_u^Q only appears in one inequality (3.21). Therefore we assume without loss of generality the left part of (3.21) to be fulfilled with equality. The left part of condition (3.23) holds also with equality, since the opposite would imply that in a cost minimization framework excess quantities were available for free.[7]

Obviously two cases have to be distinguished for the determination of the variance-minimal portfolio: (i) The assets in the portfolio are linearly independent. (ii) The assets are linearly dependent. In the latter case, at least the stochastic variation of one asset may be replicated by a combination of the others. Then obviously the variance minimizing portfolio may also be not unique and without further restrictions also a non-trivial risk free portfolio may be constructed. This case is therefore not further considered here. For the first case, we continue with the following propositions:

Proposition 3.3.3. *Let be a portfolio with n generation technologies with linearly independent operating costs $c_{op,u}, 1 \leq u \leq n$. Then the covariance matrix $\mathbf{\Sigma}$ is positive definite and hence*

[7] Alternatively, equality could be required in the left part of condition (3.21) from the beginning. The chosen formulation has the advantage that λQ_i is known to be positive.

invertible and the central optimality condition of the variance minimal portfolio is given by

$$\mathbf{Q} = \mathbf{\Sigma^{-1}} \left(\lambda^Q \mathbf{i} + \boldsymbol{\mu}^Q \right) \quad \textit{with} \tag{3.26}$$

$$\lambda^Q = \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1} i}} \left(Q_E - \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right) \tag{3.27}$$

See A.2.3 for the proof. Given strict convexity of the optimization problem, a straight-forward procedure may be used to check whether all technologies u are included in the risk-minimal portfolio.

Proposition 3.3.4. *The pure variance-minimal portfolio problem is convex in* $\mathbf{Q}$*. If and only if* $\mathbf{\Sigma}$ *is positive definite, then the optimization problem is strictly convex in* $\mathbf{Q}$*.*

Proposition 3.3.5. *Let be*

$$\mathbf{Q}^o = \frac{Q_E}{\mathbf{i}^T \mathbf{\Sigma^{-1} i}} \mathbf{\Sigma^{-1} i} \tag{3.28}$$

with $\mathbf{i} = (1, \ldots, 1)^T$*. The variance-minimal portfolio consists of all available technologies, i.e.* $Q_u > 0$ *for all* $1 \leq u \leq n$*, if and only if* $Q_u^o > 0$ *for all* $1 \leq u \leq n$*. Then,* $\mathbf{Q}^* = \mathbf{Q}^o$ *is a solution to the variance minimal optimization problem. The solution is unique if* $\mathbf{\Sigma}$ *is positive definite.*

Note that the element Q_u^o from Eqn. (3.28) corresponds to:

$$Q_u^o = \frac{Q_E}{\mathbf{i}^T \mathbf{\Sigma^{-1} i}} \sum_{v=1}^{n} \left\{ \mathbf{\Sigma^{-1}} \right\}_{uv}$$

Only if the row-sums of the inverse covariance matrix $\mathbf{\Sigma}^{-1}$ are all positive, it is Q_u^o and thus all available technologies are part of the variance-minimal portfolio. Then, the optimal amount of energy produced by technology u is obtained as weighted share of the total energy produced, where the u-th row sum of the inverted covariance matrix $\mathbf{\Sigma}^{-1}$ is used as weighting factor. Remark that for instance $\mathbf{\Sigma^{-1}}$ being strictly diagonally dominant with positive diagonal entries can thus guarantee that all technologies are part of the risk-minimal portfolio.[8] Furthermore it can be shown that the variance minimization problem has a unique solution if $\mathbf{\Sigma}$ is positive definite (cf. A.2.3).

3.3.5 Standard solutions to the combined portfolio problem with n technologies

The optimal portfolio in the general case of combined cost-risk optimization may in principle be derived using a combination of the two previously shown approaches. This is however complicated by differences in notation and differences in solution logics between the two cases. The risk-minimization case requires matrix-inversion, therefore introducing matrix-notation is also necessary

[8] A Hermitian, strictly diagonally dominant matrix with positive diagonal entries is also positive definite (cf. Horn and Johnson, 1985, Corollary 7.2.3). However, the converse cannot be concluded in general.

for the cost-minimization part. This can be achieved by introducing the lag-operator in matrix form $\mathbf{L}$ through:

$$\mathbf{L} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix}$$

With $\mathbf{I}$ denoting the n-dimensional identity matrix and $\mathbf{K^c} = (K_1^c, \ldots, K_n^c)^T$ the vector of cumulated capacities, the Lagrangian may be written

$$\begin{aligned} \mathcal{L}_n = & \mathbf{c_{inv}}^T (\mathbf{I} - \mathbf{L})\, \mathbf{K^c} + \bar{\mathbf{c}}_{\mathbf{op}}^T (\mathbf{I} - \mathbf{L}) Q^I(\mathbf{K^c}) + \frac{A}{2} \left((\mathbf{I} - \mathbf{L}) Q^I(\mathbf{K^c})\right)^T \mathbf{\Sigma} (\mathbf{I} - \mathbf{L}) Q^I(\mathbf{K^c}) \\ & + \boldsymbol{\mu}^T (\mathbf{I} - \mathbf{L})\, \mathbf{K^c} + \lambda \left(Q_E - Q^I(K_n^c)\right) \end{aligned} \tag{3.29}$$

Denoting $\frac{\partial \mathcal{L}_n}{\partial \mathbf{K^c}} = \left(\frac{\partial \mathcal{L}_n}{\partial K_1^c}, \ldots, \frac{\partial \mathcal{L}_n}{\partial K_n^c}\right)^T$ and $\frac{\partial \mathcal{L}_n}{\partial \boldsymbol{\mu}} = \left(\frac{\partial \mathcal{L}_n}{\partial \mu_1}, \ldots, \frac{\partial \mathcal{L}_n}{\partial \mu_n}\right)^T$, the corresponding KKT conditions can be derived using matrix calculus:

$$\begin{aligned} \frac{\partial \mathcal{L}_n}{\partial \mathbf{K^c}} = & (\mathbf{I} - \mathbf{L})^T \mathbf{c_{inv}} + \operatorname{diag}\big(t(\mathbf{K^c})\big) (\mathbf{I} - \mathbf{L})^T \mathbf{c_{op}} + (\mathbf{I} - \mathbf{L})^T \boldsymbol{\mu} \\ & + \lambda (\mathbf{i_n})^T t(\mathbf{K^c}) + A \operatorname{diag}\big((\mathbf{I} - \mathbf{L}) t(\mathbf{K^c})\big) \mathbf{\Sigma} (\mathbf{I} - \mathbf{L}) Q^I(\mathbf{K^c}) \geq \mathbf{0}, \quad \perp \quad \mathbf{K^c} \geq \mathbf{0}, \end{aligned} \tag{3.30}$$

$$\frac{\partial \mathcal{L}_n}{\partial \boldsymbol{\mu}} = (\mathbf{I} - \mathbf{L})\, \mathbf{K^c} \leq \mathbf{0}, \quad \perp \quad \boldsymbol{\mu} \geq \mathbf{0}, \tag{3.31}$$

$$\frac{\partial \mathcal{L}_n}{\partial \lambda} = Q_E - Q^I(K_n^c) \leq 0, \quad \perp \quad \lambda \geq 0. \tag{3.32}$$

Similar to the previous problem, we can again assume $Q_E = Q^I(K_n^c)$ and condition (3.30) to be fulfilled with equality. Hence, $\frac{\partial \mathcal{L}_n}{\partial \mathbf{K^c}} = \mathbf{0}$ is the remaining optimality condition to be solved. From $K_n^c = D(0)$, it is however clear that $t_n = 0$ and consequently λ is eliminated from the optimality condition and we end up with the following:

Proposition 3.3.6. *The central optimality condition for the combined portfolio problem is given by*

$$-A\, \mathbf{\Sigma} Q(\mathbf{K^c}, \mathbf{L}\mathbf{K^c}) = \operatorname{diag}\big((\mathbf{I} - \mathbf{L}) t(\mathbf{K^c})\big)^{-1} \Big((\mathbf{I} - \mathbf{L})^T (\mathbf{c_{inv}} + \boldsymbol{\mu}) + diag\big(t(\mathbf{K^c})\big) (\mathbf{I} - \mathbf{L})^T \mathbf{c_{op}}.\Big) \tag{3.33}$$

The proof is proved in A.2.4. Focusing on solutions which include all technologies, we assume again $\boldsymbol{\mu} = \mathbf{0}$. At a closer look, optimality condition (3.33) represents an n-dimensional equation system consisting of two functional terms $\mathbf{l}, \mathbf{r}$ of the form

$$\begin{pmatrix} l_1(Q_1(K_1^c), \ldots, Q_n(K_n^c, K_{n-1}^c)) \\ l_2(Q_1(K_1^c), \ldots, Q_n(K_n^c, K_{n-1}^c)) \\ \vdots \\ l_n(Q_1(K_1^c), \ldots, Q_n(K_n^c, K_{n-1}^c)) \end{pmatrix} = \begin{pmatrix} r_1(t_1(K_1^c)) \\ r_2(t_2(K_2^c), t_1(K_1^c)) \\ \vdots \\ r_n(t_n(K_n^c), t_{n-1}(K_{n-1}^c)) \end{pmatrix} \tag{3.34}$$

Assuming $c_{inv,u} > c_{inv,u+1}$ for all $u = 1, \ldots, n-1$, the existence of a $\mathbf{K^c}$ satisfying optimality condition (3.34) is clearly given, because row u of $\mathbf{r}$ is monotone decreasing in t_u, while row u of $\mathbf{l}$ is monotone increasing in t_u (respectively in K_u^c). Furthermore it holds the following proposition as shown in A.2.4:

Proposition 3.3.7. *Let be $A > 0$ and $c_{inv,u} > c_{inv,u+1}$ for all $(u = 1, \ldots, n-1)$. Then the combined portfolio problem (3.8)-(3.12) is convex in $\mathbf{Q}$. If and only if $\mathbf{\Sigma}$ is positive definite, then the optimization problem is strictly convex in $\mathbf{Q}$ and hence has a unique solution.*

Therewith, we have shown that it exists a unique solution to the risk-adjusted capacity planning problem under a deterministic and strictly monotone load duration function with uncertain fuel prices.[9] Deeper insights may be gathered from the solution in the two-technology case. Therefore the following paragraph is devoted to this special case.

3.3.6 Results in the two-technology case

In this example we consider two competitive generation technologies (i.e. $u = \{1, 2\}$) with $(c_{inv,1} > c_{inv,2}) \wedge (c_{op,1} < c_{op,2})$ being available to meet demand. For this case with $n = 2$, the central condition to be satisfied for an interior solution with $K_1, K_2 > 0$ can be obtained from KKT condition $\frac{\mathrm{d}\mathcal{L}_2}{\mathrm{d}K_1} = 0$ by applying the equality $Q_1 = Q_E - Q_2$ as

$$A\big((\sigma_1^2 - \sigma_{12})Q_E - (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\big) = \frac{c_{inv,2} - c_{inv,1}}{t_1} - \bar{c}_{op,1} + \bar{c}_{op,2}, \tag{3.35}$$

with $Q_2 = \int_0^{t_1} \big(D(t) - D(t_1)\big)\mathrm{d}t$. As shown in A.2.5, it holds:

Proposition 3.3.8. *The combined portfolio problem (3.8)-(3.12) with two technologies $u = \{1, 2\}$ has a unique solution if*

$$AQ_E(\sigma_2^2 - \sigma_{12}) \geq \frac{1}{T}(c_{inv,1} - c_{inv,2}) + \bar{c}_{op,1} - \bar{c}_{op,2}. \tag{3.36}$$

As a first optimality property, two limiting parameter configurations can be observed from Eqn. (3.35) for the case of purely variance-efficient portfolios (i.e. $A \to +\infty$):

Proposition 3.3.9. *Let $\rho := \frac{\sigma_{12}}{\sigma_1 \sigma_2}$ denote the coefficient of correlation of operating costs. The variance-efficient portfolio does not include technology 2 for $\frac{\sigma_1}{\sigma_2} < \rho$, while it includes only technology 2 for $\frac{\sigma_2}{\sigma_1} < \rho$, i.e.*

$$Q_2 = \begin{cases} 0 & \textit{for } \frac{\sigma_1}{\sigma_2} \leq \rho, \\ Q_E & \textit{for } \frac{\sigma_2}{\sigma_1} \leq \rho \end{cases}$$

[9] Note that the solutions obtained for Q_u may be unambiguously transformed into t_u given the assumed load duration function $D(t_u)$"

Note that the parameter ranges derived in Proposition 3.3.9 are consistent with those derived in Section 3.3.4 for the variance-minimal portfolio.[10] Since per definition $0 \leq |\rho| \leq 1$, we can further conclude the following

Corollary 3.3.1. *Technology 2 will always be included in variance-efficient portfolios for $\sigma_1 \geq \sigma_2$ while technology 1 will be included in any variance-efficient portfolio for $\sigma_2 > \sigma_1$ and arbitrary levels of correlation.*

For a better understanding of efficient portfolio characteristics, we next discuss sensitivity properties of the efficient fuel mix with respect to variations of the covariance matrix and the risk aversion parameter A.

Proposition 3.3.10. *For a risk-cost-efficient portfolio, the sensitivity of optimal operating hours (and respectively capacities) of the considered technologies with respect to the risk-aversion parameter A is only dependent on the covariance of operating costs with*

$$\frac{dt_1^*}{dA} \lesseqgtr 0, \quad \textit{for } \frac{\sigma_1}{\sigma_2} \lesseqgtr \rho. \tag{3.38}$$

The proof is provided in A.2.6. Note that risk-cost-efficient operating hours (respectively capacities) of technology 1 and consequently also of technology 2 are independent from the risk-aversion parameter A if (and only if) $\frac{\sigma_1}{\sigma_2} = \rho$. This parameter configuration at the same time implies that the purely cost-efficient portfolio equals the purely variance-efficient portfolio. Only if $\frac{\sigma_1}{\sigma_2} > \rho$, the efficient run time of the technology 2 increases with increasing risk aversion and vice versa. For $\frac{\sigma_1}{\sigma_2} < \rho$, which is in general satisfied as the operating costs of the peak technology are much higher than those of the base technology (see Section 3.4), an increasing risk aversion leads ceteris paribus to a shorter optimal run time and therewith to smaller optimal capacities of technology 2. Thus, increasing risk aversion leads in the latter case to a decline of fuel mix diversification in the considered portfolio. This is due to the fact that the total risk contribution of technology 2 is higher than of technology 1.

Proposition 3.3.11. *Alternatively, a comparison of optimal operating times for the purely cost-efficient portfolio, t_1^{c*}, and for the purely risk-efficient portfolio, t_1^{r*}, can provide evidence on the sensitivity. As shown in A.2.6, it equivalently holds*

$$\frac{dt_1^*}{dA} \lesseqgtr 0, \quad \textit{for } t_1^{c*} \gtreqless t_1^{r*}. \tag{3.39}$$

[10] To show that, we first compute the inverted covariance matrix $\mathbf{\Sigma^{-1}}$ for the case with two technologies from its adjoint (cf. Horn and Johnson, 1985, p. 21) as:

$$\mathbf{\Sigma^{-1}} = \frac{1}{\det(\mathbf{\Sigma})}\operatorname{adj}(\mathbf{\Sigma}) = \frac{1}{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}\begin{pmatrix} \sigma_2^2 & -\sigma_{12} \\ -\sigma_{12} & \sigma_1^2 \end{pmatrix} \tag{3.37}$$

Then, $\mathbf{Q_u^o}$ can be computed from Eqn. (3.28) as $Q_1^o = Q_E \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2+\sigma_2^2-2\sigma_{12}}$, $Q_2^o = Q_E \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2+\sigma_2^2-2\sigma_{12}}$. It can be seen that Q_1^o, Q_2^o to be greater than zero requires $\frac{\sigma_2}{\sigma_1} > \rho$ and $\frac{\sigma_1}{\sigma_2} > \rho$, respectively. If satisfied, both technologies are part of the variance-minimal portfolio.

This is an interesting finding, since it implies by taking into account Properties 3.3.9 and 3.3.10, that the case $t_1^{c*} < t_1^{r*}$ cannot occur if both technologies are part of the variance-efficient portfolio.

For the following sensitivity properties of the solution to the two-technology problem corresponding to Eqs. (3.8) to (3.12) with $n = 2$, we will hence concentrate on the more interesting case where both technologies are part of the variance-efficient portfolio, i.e. $\frac{\sigma_1}{\sigma_2} > \rho$ and $\frac{\sigma_2}{\sigma_1} > \rho$ as shown in A.2.6.

Proposition 3.3.12. *Given technologies 1 and 2 being part of the cost-efficient and the variance-efficient portfolio, the following parameter conditions are sufficient for the stated sensitivity properties of optimal operating hours (respectively capacities) of technology 2:*

$$\frac{dt_1^*}{d\sigma_1} \geq 0 \qquad \text{for all } \sigma_1, \sigma_2 \geq 0,\ 0 \leq \rho \leq 1, \tag{3.40}$$

$$\frac{dt_1^*}{d\sigma_2} \leq 0 \qquad \text{for all } \sigma_1, \sigma_2 \geq 0,\ 0 \leq \rho \leq 1, \tag{3.41}$$

$$\frac{dt_1^*}{d\rho} \leq 0 \qquad \text{for all } \sigma_1, \sigma_2, \geq 0,\ -1 \leq \rho \leq 1. \tag{3.42}$$

Reciprocal sensitivity properties are obtained for optimal operating hours (respectively capacities) of technology 1.

3.4 Case study: Exemplary model application to the German electricity market

In the following section, we will deepen the insights gained from the analyses on risk-efficient generation portfolios in a case study. We use the German electricity market as an exemplary application for three reasons: For the first, there will be a considerable need for new investments in generation assets in Germany in the next decades due to the age of the existing plant fleet and to fulfill the ambitious climate protection targets. Secondly, the obtained results can be transferred easily to other countries with comparable cost parameters and demand patterns. In particular, the key structural findings apply to Continental Europe as a whole as we use market prices for the key cost input parameters. Thirdly, a comprehensive MVP analysis of the German electricity market has not yet been published before to our best knowledge.[11]

3.4.1 Parameter estimates

Estimation of plant costs and fuel prices

The technical and economic key parameters for five typical generation technologies are based on 2007 values derived from Konstantin (2009) as summarized in Table 3.1. Annualized specific investment costs per kW are quoted with respect to the gross installed capacity including plant consumption of auxiliaries. For all technologies, capital costs are calculated based on the annuity method with a uniform interest rate of 10% after tax and the quoted economic lifetimes.

[11] Westner and Madlener (2009) consider the German context, yet they focus on CHP plants.

Tab. 3.1: Key parameters for typical coal, gas and nuclear plants (source: Konstantin, 2009, own analysis).

Parameter	Unit	Coal	Lignite	CCGT	OCGT	Nuclear
Technology index u		3	2	4	5	1
Thermal efficiency	MWh_e/MWh_t	0.46	0.43	0.56	0.34	0.37
Carbon emission rate	tCO_2/MWh_t	0.34	0.41	0.20	0.20	0.0
Total net investment costs	€/KW	1419	1934	608	456	3225
Technical lifetime	a	45	45	30	25	50
Fixed O&M, overhead	€/KW a	36.06	43.26	13.97	9.69	74.06
Variable O&M, transport	€/KWh_e	2.9	1.7	5.5	20.0	0.0

Tab. 3.2: Distribution parameters for fuel costs 1986–2008 (Germany), EUA costs included from 2005 on (source: BAFA, 2009, UxC, 2009, ECX, 2009, StaBu, 2009, own analysis).

	Coefficient of correlation				Std.-dev.	Mean 2006-08
	Gas	Coal	Lignite	Nuclear	€/MWh_t	€/MWh_t
Gas	1.00	0.92	0.88	0.77	7.48	28.73
Hard coal	0.92	1.00	0.94	0.67	4.32	15.83
Lignite	0.88	0.94	1.00	0.62	3.33	11.41
Nuclear EPR	0.77	0.67	0.62	1.00	0.30	2.66

To account for fuel price risks, total operating costs $c_{op,u}$ are modeled as normal random variables calculated as the sum of the respective fuel prices plus the emission factor weighted price of CO_2 emission rights divided by the technology specific efficiency rate, i.e. $c_{op,u} = \frac{p_{f,u}+e_u p_{co_2}}{\eta_u}$. Since valuing the influence of fuel price fluctuations on the long-term investment optimum in a MVP approach requires a reliable long-term estimate of the covariance matrix which captures all underlying price risks, we estimate variance and covariance for (pairs of) total fuel prices including CO_2 for the considered generation technologies over the sample period 1986–2008. Relevant sources and results are provided in Table 3.2. As fuel prices are expected to possess the martingale property[12], we use 2006-2008 average fuel prices instead of long-term means to compute expected generation costs as depicted in Table 3.2.

Under the assumption of normal distributed fuel price levels, it can be seen that the probability for reversals in the merit order, $\mathbf{P}(\tilde{c}_{op,1} > \tilde{c}_{op,2})$, is negligibly small: Applying the transformation $\tilde{z} = \tilde{c}_{op,1} - \tilde{c}_{op,2}$, where $\tilde{z} \sim \mathcal{N}(\bar{c}_{op,1} - \bar{c}_{op,2}, \sigma_1^2 + \sigma_2^2 - 2\sigma_{12})$[13], the year-to-year likelihood for a

[12] Weber (2005) points out that "as for any storable equity, arbitrage opportunities would arise if the (discounted and risk-adjusted) product price would not follow a martingale process, i.e. a stochastic process where the observed value today corresponds to the risk adjusted expected value for tomorrow."

[13] It is well-known that the sum of n jointly normal distributed random variables X_i, with $X_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ is also

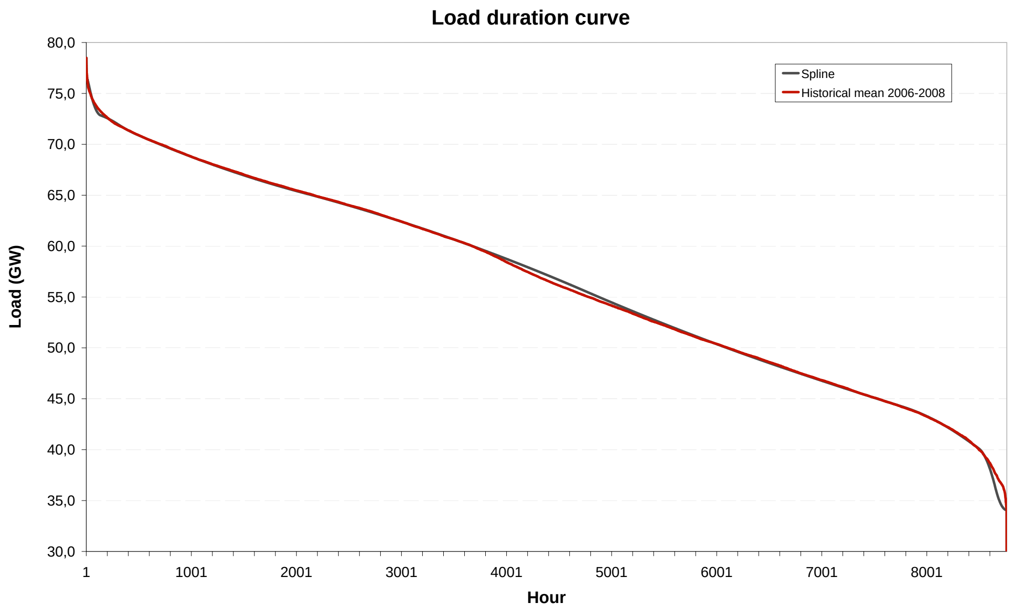


Fig. 3.2: Historical load duration curve (red) and fitted spline function (source: ENTSO-E (2009); own analysis).

reversal in the merit order is given by

$$\mathbf{P}(\tilde{c}_{op,1} > \tilde{c}_{op,2}) = \mathbf{P}_z(\tilde{z} > 0) = 1 - \int_{-\infty}^{0} \phi_z(z)\mathrm{d}z = 1 - \Phi_z(0). \tag{3.43}$$

where Φ_z denotes the cumulated distribution function of $\tilde{z}$. With the empirical data from Table 3.2 we obtain for the pairwise reversal likelihoods of coal and gas $\mathbf{P}(\tilde{c}_{op,3} \geq \tilde{c}_{op,4}) = 0.06\%$, lignite and coal $\mathbf{P}(\tilde{c}_{op,2} \geq \tilde{c}_{op,3}) = 0.47\%$, and nuclear and lignite technologies $\mathbf{P}(\tilde{c}_{op,1} \geq \tilde{c}_{op,2}) = 0.13\%$. In case of very high carbon or coal prices, however, the gas technology would displace coal and could even lead to a reversal in the merit order. A detailed discussion of the risk of reversal in the merit order and its impact on the efficient technology mix is provided in Sunderkötter and Weber (2011). For a better tracability of the solution, we thus exclude this case in the following.

We use historical load data for Germany provided in an hourly resolution by ENTSO-E (2009) for the years 2006–2008. A historical reference load duration curve can then be generated from the hourly means of the historic data. To accomplish the further analysis with a continuous function $\tilde{D}(t)$, a spline function is fitted to the historical data as shown in Fig. 3.2

Determination of the risk aversion level

The optimal selection of a specific portfolio combination from the efficient frontier depends on the risk aversion parameter, A. This parameter determines the willingness in an economy to bear risk or

normal distributed with mean and variance

$$\mu = \sum_{i=1}^{n} \mu_i, \quad \sigma^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_{ij} = \sum_{i=1}^{n}\sum_{j=1}^{n} \sigma_i\sigma_j\rho, \quad \text{with } \rho = \frac{\sigma_{ij}}{\sigma_i\sigma_j}.$$

For a proof see e.g. Elishakoff (1999).

equivalently the willingness to accept extra costs to reduce the risk of the plant portfolio. Although A is scale-variant and therefore in general unknown, the price of risk can easily be observed on capital markets and can be used to determine the corresponding risk aversion parameter A as shown in Sunderkötter et al. (2013).

3.4.2 Results: Efficient fuel mix characteristics

We characterize the cost-risk efficient generation fuel mix in a "green-field" analysis, i.e. regardless to the existing fuel mix, limited availabilities and other possible constraints. This intentionally over-idealizing study allows to first identify long-term technology equilibria before looking in more detail to a narrowed-down path of more realistic investment options in the mid-term.

With respect to fuel price risks, we investigate open and combined cycle gas turbine (OCGT, CCGT), lignite, coal and nuclear (EPR) plants as risky technologies. In addition, an enforced share of base-load serving technologies are incorporated as indicator for existing renewable technologies (mainly wind, water) which can be considered as nearly risk-free in terms of operating costs.[14] OCGTs are in all cases the technology with the highest, nuclear with the lowest operating costs.

The efficient fuel mix for varying risk aversion in the first scenario with nuclear technologies is shown in Figure 3.3: Based on the historically estimated covariance characteristics of total fuels prices including EUAs, higher risk aversion leads always to an increase of nuclear generation in efficient portfolios. In contrast, lignite dominates the efficient portfolio in the second scenario calculation without nuclear technologies with increasing risk aversion (Figure 3.4). This result is mainly driven by the fact that lignite generation costs have been very stable compared to other commodities even when taking into account volatile EUA markets.[15] Vice versa, gas-fired generation decreases in all cases with increasing risk aversion. Surprisingly, hard coal does not represent a substantial share of generation in any considered case as it cannot compete with the low operating costs and low variance characteristics of nuclear and lignite generation for base load generation nor with the low investment costs for gas technologies tailored to mid and peak load generation.

A standard representation of efficient portfolios for varying risk aversion A is shown in Figure 3.5. For all portfolio combinations on the efficient frontier, the expected generation costs can be reduced only by increasing the portfolio risk. The risk-return profile of the generation mix including nuclear technologies is clearly more favorable compared to the one without nuclear generation.

These last analyses show that nuclear and lignite technologies play an important role in cost-risk efficient generation portfolios. Although geologically feasible, the mid-term potential for lignite

[14] We are aware that a solid valuation of renewable technologies in electricity generation systems would require a much deeper treatment of specific technology characteristics as for instance reflected in the work of Vogel and Weber (2009) on MVP optimization and wind power. Since this empirical application aims to demonstrate in a traceable manner the analytical results discussed before, we kindly refer readers with special interest in renewable technologies to this and other dedicated existing publications.

[15] The low volatility of lignite prices is caused by predominantly long-term contract based sourcing of lignite from local mines. In contrast to other fuel commodities, very high transportation costs prevent the development of a liquid world market for lignite.

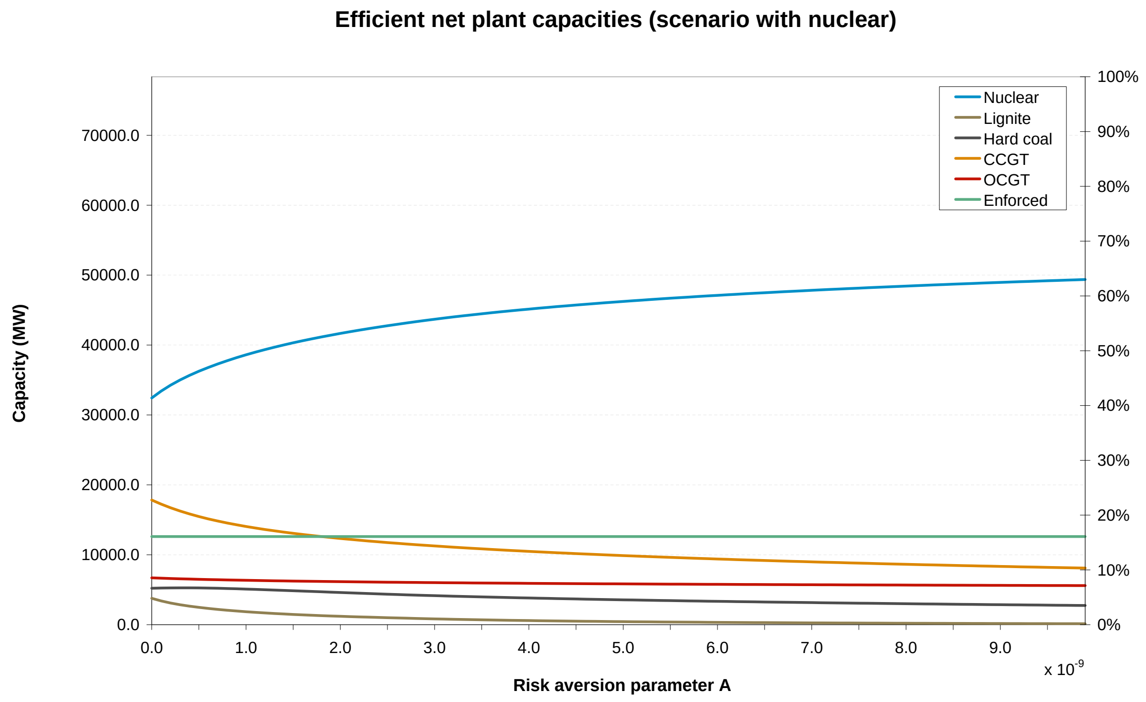


Fig. 3.3: Efficient fuel mix of OCGT, CCGT, lignite, coal, nuclear technologies (in GW) for varying risk aversion A.

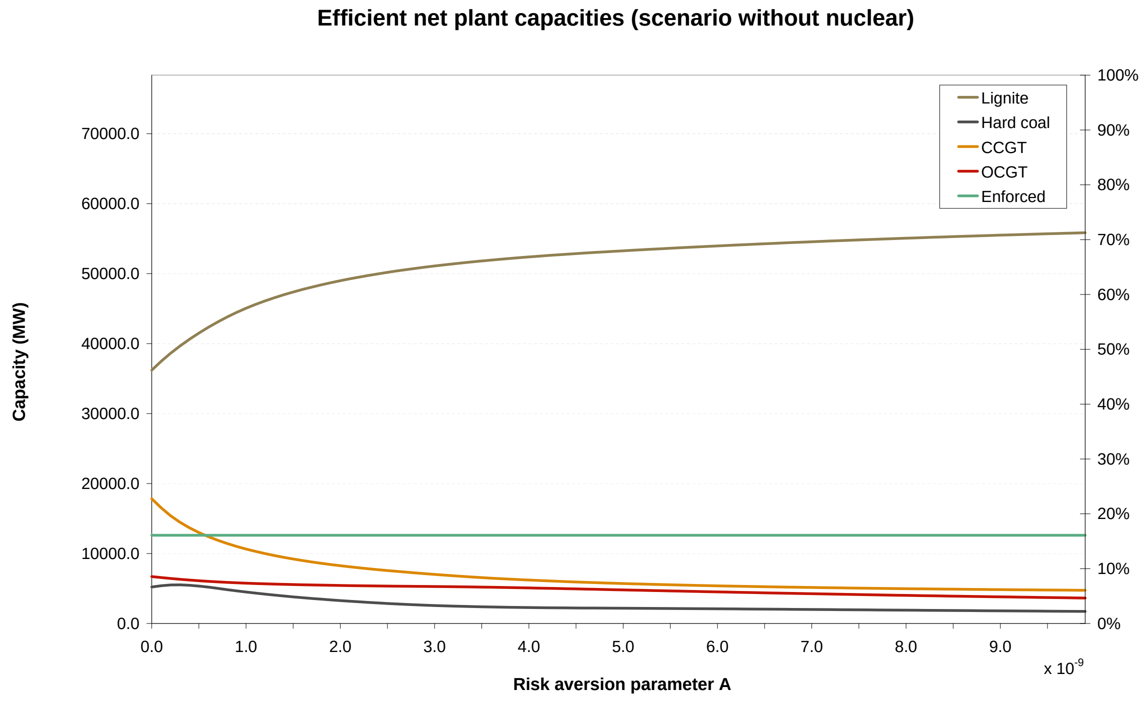


Fig. 3.4: Efficient fuel mix of OCGT, CCGT, lignite, and coal technologies (in GW) for varying risk aversion coefficient A.

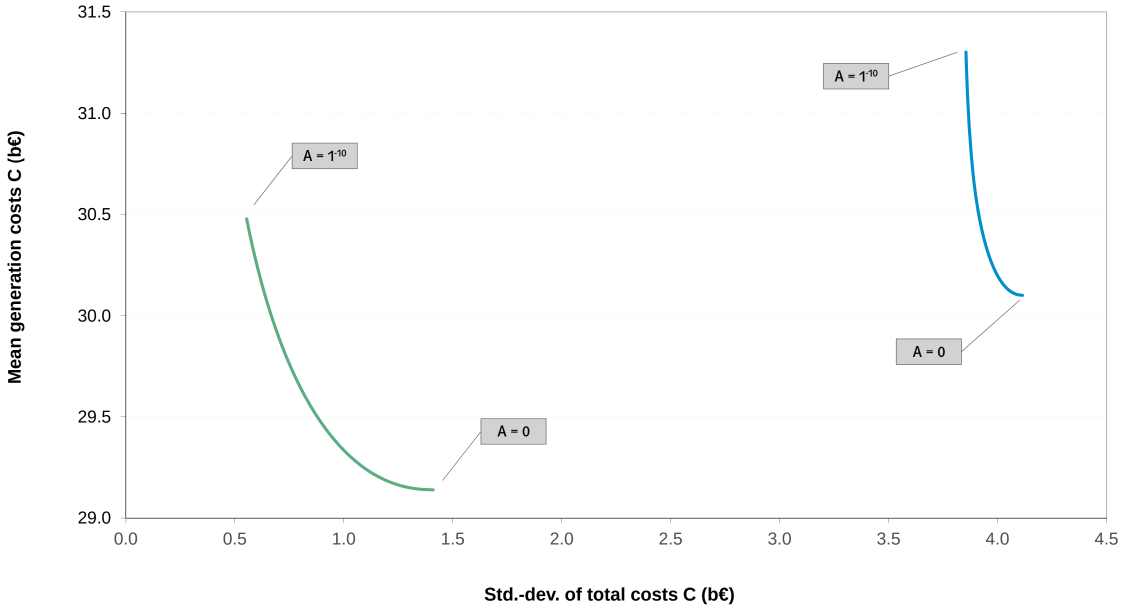


Fig. 3.5: Efficient frontier of portfolios with (left curve) and without (right curve) nuclear generation technologies (in billion Euro).

generation capacity extensions in Germany is realistically very limited as the exploitation of new open-cast mines seems unrealistic due to little public and political acceptance. With the German parliament's decision on the nuclear phase-out from July 2011 (Deutscher Bundestag, 2011), new nuclear plants are however an option in Germany any longer. Therefore, we will focus in the following paragraphs on the comparison of hard coal and CCGT technologies in optimal portfolios in more detail.

3.4.3 Trade-off between coal and gas fired technologies

To analyze the trade-off between the two technologies, we now consider that only CCGT and hard coal technologies are available to serve the load function. Optimal CCGT plant capacities and corresponding generation costs for varying risk aversion coefficient A and fuel price correlation ρ are shown in Figure 3.6. In addition to the technology cost characteristics and the form of the load duration curve, the optimal portfolio selection is directly determined by the society's risk attitude, A. Since risk proclivity can be considered as abnormal for power plant investments, we will concentrate on the case $A \geq 0$.

Consistent with the results of section 3.3.6, we observe that with increasing risk aversion the optimal combination of capacities in the portfolio moves in general away from the risk-free optimum with $A = 0$. As already discussed in Property 3.3.10, the portfolio selection is equal to the risk-free case if $\frac{\sigma_1}{\sigma_2} = \rho$. Hence at a correlation coefficient of 0.6 the variance-minimal portfolio corresponds to the cost-minimal portfolio. Consequently at this particular level of correlation, the portfolio

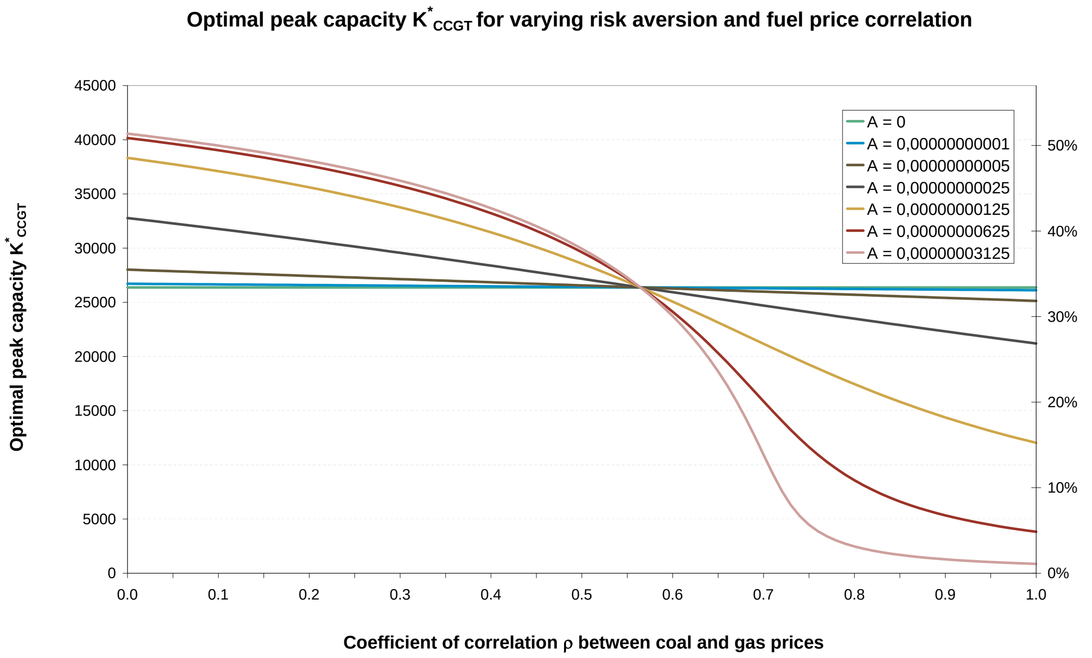


Fig. 3.6: Optimal CCGT peak plant capacities, K_4^* (in MW) in the two technology-case for varying risk aversion A and fuel price correlation ρ.

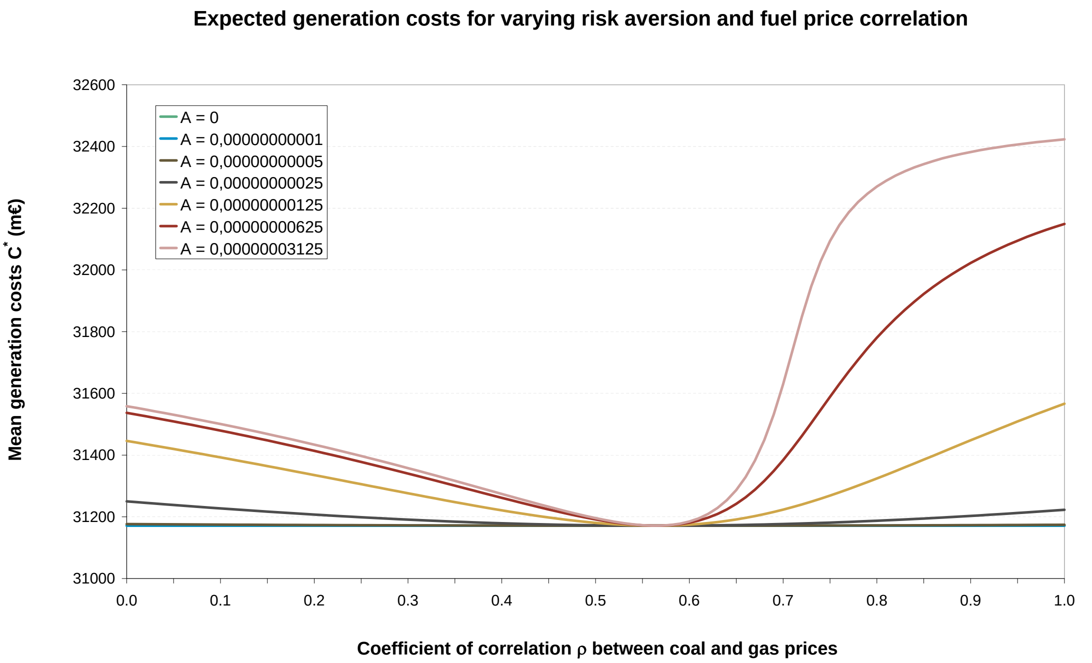


Fig. 3.7: Expected total generation costs C^* (in billion EUR) in the two technology-case for varying risk aversion A and fuel price correlation ρ.

composition is independent of the risk aversion. At higher correlations, as indicated in section 3.3.6, higher risk aversion reduces the share of the gas peak technology, since the diversification effect is lower than the addition in variance due to the higher price volatility of gas. If correlations were however below 0.6, risk aversion would induce an increase in the proportion of the gas technology. These results clearly emphasize the need for appropriate correlation estimates. Given that the portfolio components have lifetimes of 30 years and more, long term correlations as those used here, estimated based on price levels, are certainly more adequate than (typically lower) short term correlations of price changes. With increasing correlation between total fuel prices of the two technologies, the optimal selection becomes additionally more and more a binary decision. Total expected generation costs increase as expected with increasing risk aversion as shown in Figure 3.7. However, the effect of increasing risk aversion is diminishing. When an increase in A leads to a complete elimination of the more risky asset in the portfolio, a further increase of A does not lead to a different portfolio selection.

3.5 Concluding remarks

This paper has analyzed the impact of fuel mix diversification on the long-term optimum of electricity generation portfolios. By combining Mean-Variance Portfolio theory and peak-load pricing theory, we have formulated a convex optimization problem to derive cost-risk efficient generation portfolios. Sufficient conditions have been formulated for the existence of a unique solution of the optimization problem. Optimality conditions for cost-minimal, risk-minimal and cost-risk efficient portfolios have been derived and analyzed based on a continuous load duration curve. Thereby, we have shown that the degree of diversification in the efficient portfolio depends on the covariance matrix of operating costs of the technologies and on the societal risk-aversion. A comparison of the cost-minimal and of the risk-minimal portfolios may be used to determine the sensitivity of the cost-risk efficient portfolio structure on the risk aversion.

The proposed model has been used to demonstrate quantitatively the derivation of efficient generation fuel mixes for Germany. The calculations show that fuel mix diversification can considerably influence the total standard deviation of generation costs.

With respect to the current debate on security of supply, the results indicate that increasing risk aversion implies a higher share of lignite and nuclear generation in efficient portfolios and conversely to a decrease of gas-fired generation. Consistent with the results of Fan et al. (2010), the optimal fuel mix shows independently from the risk attitude a high sensitivity to the price and/or the allocation method of CO_2 emission rights. The results indicate that with full auctioning of CO_2, efficient portfolios at historically observed CO_2 price levels consist of more nuclear and lignite and less coal-fired generation compared to the current fuel mix. If nuclear and lignite capacities are reduced or fixed at the current level, hard coal is the most economical technology instead.

The case study demonstrates in consistency with previous studies (see e.g. Roques et al., 2008, Delarue et al., 2011, Gotham et al., 2009) that fuel-mix diversification does not provide reduced risk characteristics per se. Blind diversification without consideration of technology costs and price risks as well as the correlation risks may even be counterproductive.

Some simplifying assumptions within the MVP based model framework proposed in this article open the stage for potential future research directions: Firstly, this article relies on a central planner perspective, but the derived efficient portfolio structure is not necessarily congruent with the market equilibrium on liberalized markets. Since risk aversion represents one kind of market imperfection, a comparison of welfare optimum, investor optimum, and market equilibrium would be desirable (see Ziegler et al., 2010, for further discussions). Secondly, this article uses variance as a risk measure for the benefit of the clarity of the MVP framework. A comparison of variance with more sophisticated and coherent risk measures (e.g. lower partial moments, conditional value-at-risk) in electricity portfolio applications could be an interesting and yet missing building block in this area of research. Thirdly, the capacity investment problem is reduced in this article for the sake of simplicity to a static decision problem and neglects dynamic constraints (e.g. existing plant structure, retirements over time, optionality to postpone projects). Within the proposed MVP framework, the model could be expanded straightforwardly to capture existing plant structures (see e.g. Gotham et al., 2009, for a similar approach). Furthermore, the portfolio analysis can be carried out for multiple discrete points in time to allow for conclusions on the efficient trajectory of the efficient technology mix. Including recursive decisions coherently in the MVP framework is however a challenging further research task.

Chapter 4

Optimization of power generation portfolios under uncertainty in the merit order

4.1 Introduction

In the next decades, the European power industry will face an immense need for investments to renew and extend its power plant fleet. The challenge is massively increased by the required transformation to reach emission reduction targets and reduce the carbon-intensity of the power system: For a 25% reduction in greenhouse-gas emissions by 2020 compared with 1990, the IEA (2010) estimates required generation capacity additions in European OECD countries of 337GW between 2010 and 2020 and another 498GW between 2021 and 2035. The more ambitious reduction target of 80% by 2050 (respectively 40% by 2020) as agreed by the European representants during the G8 meeting in L'Aquila in July 2009 would require consequently a much more drastic change of the generation system. To reach the reduction targets, the EU and national states have implemented several measures and development schemes which aim to politically influence investment decisions in new generation capacities directly or through monetary incentives.

For quantifying the costs and effects of these measures and subsidies on the long-term optimal system fuel mix from a welfare perspective, it is not only crucial to valuate expected total life-cycle costs, but also the economic risks conveyed with investment and operation of the plant fleet. With fuel prices fluctuating considerably, cost volatility becomes a severe risk that influences the investment economics and therewith the optimal fuel mix within the system. Thereby, short-term fuel price shocks as well as longer-term structural changes in the commodity markets can lead to changes in the merit order represented by the marginal costs of production of the generation technologies. Under these circumstances, it is not possible to derive an unambiguous merit order as assumed in the classic peak-load capacity planning literature. In the analytical literature on generation investments, the risk of reversals in the merit order (in the following denoted as merit order risk) through fuel price fluctuations is commonly neglected. Although simulation-based investment optimization approaches (as e.g. discussed in Weber, 2005, Fleten et al., 2007) may capture this type of uncertainty, analytical solutions of peak-load pricing based frameworks limit fuel price risk—if captured at all—to the extent that unambiguity of the merit order is still satisfied (cf. Sunderkötter and Weber, 2012). However, long-term investment decisions such as for power plants with lifetimes of several decades may be heavily impacted by possible reversals in the short-term order of dispatch.

The optimization problem and the solutions proposed in this paper can be easily transferred to investment decisions in several other industries besides power generation where similar problem characteristics can be found. Thereby, three main properties characterize the considered type of investment problems: (a) Availability of alternative production technologies, (b) non-storability of produced good, and (c) price uncertainty of input factors. Potential areas of application in other industries are

- **Transportation systems:** Vehicle fleet operators as e.g. taxi companies or logistics providers face a trade-off between investment costs and operating costs of different engine types when deciding on new investments. Thereby, the investment decision is typically subject to an expected or pre-scheduled annual transportation performance. While retail prices for diesel fuels were traditionally lower compared to gasoline in past decades, the picture changed in many European countries in 2008 due to changes in demand and refinery capacities (Eurostat, 2009). In the U.S., the order of retail diesel and gasoline fuel prices changed several times between 2007 and 2009 (EIA, 2009). Moreover, the expected further emergence of hybrid and electric technologies may lead to significant changes in the order of operating costs among all vehicle technologies.

- **Process industry:** Many industrial production processes, e.g. in chemical industry, show similar trade-offs between investment costs and uncertain costs of required input factors. Replacement of a certain chemical in case of price shocks is usually only a mid-term option, since process changes usually cause investment costs.[1]

This paper is structured as follows: After a brief recall of the deterministic peak load pricing concept in Section 4.2.1, the risk-extended optimization problem is proposed in Section 4.2.2. The analytical solution and its properties are discussed starting in Section 4.2.3 for the simplified case without merit order risk, before the general portfolio problem is treated. Thereby, the two special cases of purely cost-efficient and purely risk-efficient portfolios are elucidated in Sections 4.2.4 and 4.2.5, before the combined problem is discussed in Section 4.2.6. Section 4.3 treats determining the likelihood for reversals in the merit order, at first for one period and then in a multi-period quantification over a plant's lifetime. In Section 4.4, we examine the model in a case study on the

[1] Recently, severe delivery shortages of a chemical mass product caught particular attention (cf. e.g. Bonilla, 2010): Acetonitrile, a by-product of acrylnitrile which is widely used in the plastics and textile industry, is a frequently used solvent in laboratories and industrial processes. Due to the world-wide demand drop of plastic products combined with temporarily decommissioned production capacities, the price of acetonitrile exploded in 2008/2009 from few euro per liter up to a hundred. The "Great Acetonitrile Shortage", as it has come to be known in the industry, forces chemists to reduce the required solvent consumption or switch to other solvents. However, these alternatives are usually conveyed with additional investment costs.

cost-risk efficient structure of generation portfolios consisting of CCGT and hard coal technologies in Germany. The paper concludes in Section 4.5 with a summary and critical acclaim of our key findings.

4.2 Modelling optimal investment policies in electricity markets given uncertainty in the merit order

4.2.1 Deterministic peak-load pricing problem

Our investment model for the electricity system is based on a two-stage decision problem following the classic peak load pricing literature (see e.g. Crew et al., 1995): On the first stage, the decision to invest in a portfolio of different available plant technologies $u \in \{1, \ldots, n\}$ with capacity K_u is made. At this point in time, investors are assumed to have full information about investment costs $C_{inv,u}$ of each technology and about the distribution of the uncertain fuel prices $\tilde{c}_u$. However, actual fuel price realizations are not revealed until the second stage. Then, the optimal deployment decision of each plant within the portfolio selected on the first stage is made for the total planning period $[0;T]$ (e.g. a year), which we assume to be broken down into time steps of equal length $t \in [0;T]$ (e.g. hours). Price inelastic system demand is deterministically given by the demand function $D : [0;T] \to \mathbb{R}_+, t \mapsto D(t)$, which is assumed to be continuous and (at least) two-times differentiable.

The objective is to minimize the total system cost of electricity production, C, consisting of total operating costs, $C_{op,u}$, plus annualized capacity investment costs, $C_{inv,u}$, summed over the available technologies u. In fact, operating costs at time t are a function $C_{op,u}(y_{u,t}) = y_{u,t} \cdot c_{u,t}$ of the instantaneous output level $y_{u,t}$ (MW) times the specific operating costs $c_{u,t}$ (€/MWh). Furthermore, total investment costs are determined by the installed capacity K_u and the specific investment costs $c_{inv,u}$ (€/MW$_{el}$) and can be expressed by $C_{inv,u}(K_u) = K_u \cdot c_{inv,u}$. Therefore, the plant capacities K_u and the corresponding output levels $y_{u,t}$ are the decision variables to be optimized. Taking into account that demand must never exceed available capacities, the deterministic optimization problem may be written as

$$C^* = \min_{y_{u,t}, K_u} C(y_{u,t}, K_u) \tag{4.1}$$

$$\text{s.t. } C = \int_0^T \sum_u y_{u,t} \cdot c_{u,t} \mathrm{d}t + \sum_u K_u \cdot c_{inv,u} \tag{4.2}$$

$$y_{u,t} - K_u \leq 0 \qquad \forall \quad t, u \tag{4.3}$$

$$\sum_u y_{u,t} \geq D(t) \qquad \forall \quad t \tag{4.4}$$

In the following paragraph, we well extend this problem to reflect not only the expected costs but also cost risks in the optimization.

4.2.2 The risk-adjusted portfolio problem

In classic portfolio theory, investors are assumed to select efficient portfolios solely based on the expected return and risk (in the form of variance of return) of the available assets. We adopt this decision principle also for the cost based investment decision of generation assets in the system portfolio. Following Jansen et al. (2008) and Gotham et al. (2009), we thus use total system costs instead of "return" and variance of total costs as the relevant risk measure.

As frequently used in investment literature, we assume that society's preferences are represented by a function of the form

$$L = \mathbf{E}[C] + \frac{1}{2}A \cdot \mathrm{Var}[C], \tag{4.5}$$

where A denotes the investors' risk attitude (see e.g. Trautmann, 2006, Sunderkötter and Weber, 2012). For normally distributed payoffs, this preference function is induced by exponential utilities with constant absolute risk aversion and shows in maximization problems consistency with the rationale of expected utility maximization.

To allow for a better traceability of the solution, we confine in the following to the case with only two technologies, i.e. $n = 2$.[2] Given uncertain fuel prices which lead to fluctuating operating costs $\tilde{c}_1$ and $\tilde{c}_2$, two possible scenarios are to be distinguished on the second stage for the merit order. These states of the merit order are indicated by the state variable $\tilde{s}$ with realizations $s_i, i \in \{1; 2\}$ defined as

$$\tilde{s} := \begin{cases} s_0, & \text{for } c_1 \leq c_2 \text{ ("default order")} \\ s_1, & \text{for } c_1 > c_2 \text{ ("reverse order")} \end{cases} \tag{4.6}$$

Let periodic operating costs $\tilde{c}_u, u \in \{1; 2\}$ be represented by bivariate jointly distributed random variables with joint probability density function $\varphi_{1,2}(c_1, c_2, \rho)$ and mean $\bar{c}_u$.[3] For each technology u, $\tilde{c}_u$ is a (univariately) distributed random variable on the probability space $(\Omega, \mathcal{A}(\mathbb{R}), \mathbf{P})$ with $\tilde{c}_u : \Omega \to \mathbb{R}$ with marginal probability density function $\varphi_u(c_u)$ and marginal cumulative distribution $\Phi_u(c_u)$ and corresponding mean $\bar{c}_u$, variance σ_u^2, and correlation ρ. Then, $\tilde{s}(\tilde{c}_1, \tilde{c}_2)$ is itself a (discrete) random variable generated by $\tilde{c}_1, \tilde{c}_2$ on the probability space $(\Omega', \mathcal{B}(\mathbb{R}^2), \mathbf{P}')$, with $\tilde{s} : \Omega' \to \{s_0; s_1\}, \Omega' = \Omega_1 \times \Omega_2$.

The likelihood for a reversal in the merit order is denoted by $\mathbf{P}(\tilde{s} = s_1) := \mathbf{P}(c_1 > c_2)$. By definition, $\mathbf{P}(\tilde{s} = s_0) := \mathbf{P}(c_1 \leq c_2) = 1 - \mathbf{P}(\tilde{s} = s_1)$ is thus the probability that no reversals of annual operating costs in the merit order occur. Intuitively, the merit order risk depends on the assumed time series model for the operating costs c_1 and c_2. We will come back to the computation of the merit order risk in Section 4.3. Taking into account unambiguity of the plant merit order, the classic peak load pricing framework to model total system costs has to be extended. Considering two generation technologies with the resulting possible fuel price orders according to definition (4.6), the following states of the plant output variable y_t have to be distinguished:

[2] The solution to the extended portfolio problem with n technologies given the case of a deterministic merit order is discussed in Sunderkötter and Weber (2012).

[3] Throughout this article, random variables are indicated by a "~", whereas their realizations are written as plain letters.

$$\tilde{y}_{1,t} = \begin{cases} D(t), & \text{for } \tilde{s} = s_0 \wedge D(t) < K_1 \\ K_1, & \text{for } \tilde{s} = s_0 \wedge D(t) \geq K_1 \\ 0, & \text{for } \tilde{s} = s_1 \wedge D(t) < K_2 \\ D(t) - K_2, & \text{for } \tilde{s} = s_1 \wedge D(t) \geq K_2 \end{cases}$$
$$\tilde{y}_{2,t} = \begin{cases} 0, & \text{for } \tilde{s} = s_0 \wedge D(t) < K_1 \\ D(t) - K_1, & \text{for } \tilde{s} = s_0 \wedge D(t) \geq K_1 \\ D(t), & \text{for } \tilde{s} = s_1 \wedge D(t) < K_2 \\ K_2, & \text{for } \tilde{s} = s_1 \wedge D(t) \geq K_2 \end{cases} \tag{4.7}$$

We assume without limitation of generality $D(t)$ to represent demand in a decreasing order and thus to be strictly monotone decreasing in t with $D(0) = D_{max}$. This allows to simplify problem (4.1)-(4.4) by using the minimum[4] operating duration O_u as decision variables instead of $y_{u,t}$.

Given the strict monotony of the load duration curve, there is a unique mapping between capacities and operating times for each merit order state. Obviously, the upper bound of the optimal operating time of technology 1 equals T in case of the default order where technology 1 represents the base load technology. By defining $R(K)$ as the inverse of the demand function $D(t)$, the minimum operating duration of the base technology 1 in the default merit order is determined by $O_1 = t_1 = R(K_1)$. Equivalently, t_1 determines the upper bound of the operating time of the respective peak technology. For the latter, the minimum operating duration equals zero. The minimum operating duration of technology one is reduced to a value $O_1 = 0$ in the reverse case in favor of technology 2, which then becomes the new base load technology running at least during $O_2 = t_u$. The lower bound of the ex-ante optimal operation time of technology 1 reduces with increasing merit order risk. Therefore, both t_1^* (lower bound of the optimal operating time of technology 1 given the default order) and t_2^* (lower bound of the optimal operating time of technology 2 given the reverse order) have to be determined endogenously in the optimization problem. Recapitulating the optimal plant dispatch, the minimum operating times O_1, O_2 of technology 1 and 2 over the two considered merit order states can be formulated as:

$$O_1 = \begin{cases} t_1 & \text{for } \tilde{s} = s_0 \\ 0 & \text{for } \tilde{s} = s_1 \end{cases} \qquad O_2 = \begin{cases} 0 & \text{for } \tilde{s} = s_0 \\ t_2 & \text{for } \tilde{s} = s_1 \end{cases} \tag{4.8}$$

Note that through the invertible function $D(t)$, we can use t_u and K_u interchangeably as decision variables. Finally, we introduce for the operating costs of technology u the conditional expectation $\mathbf{E}[\tilde{c}_u|\tilde{s}] \equiv \bar{c}_{u|\tilde{s}}$ and the conditional variance $Var[\tilde{c}_u|\tilde{s}] \equiv \sigma^2_{u|\tilde{s}}$. The computation of these conditional parameters from a bivariate density $\varphi_{1,2}(c_1, c_2)$ is derived in B.2.1.

Based on these pre-considerations, the extended portfolio optimization problem with uncertain merit order can now be rewritten taking into account that the total variance is the sum of inter-

[4] Analogously, the problem could be formulated using maximum operating durations as decision variables.

scenario and intra-scenario variance as shown in B.2.2:[5]

$$L^* = \min_{K_1,K_2} L \tag{4.9}$$

$$\text{s.t. } L = \sum_{u=1}^{2}\left(K_u c_{inv,u} + \mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\left[\tilde{c}_u|\tilde{s}\right] + \frac{A}{2}Q^2_{u|\tilde{s}}\left(\mathrm{Var}\left[\tilde{c}_u|\tilde{s}\right] + \mathbf{E}\left[\tilde{c}_u|\tilde{s}\right]^2\right)\Big] - \frac{A}{2}\mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\left[\tilde{c}_u|\tilde{s}\right]\Big]^2\right)$$
$$+ A\mathbf{E}\Big[Q_{1|\tilde{s}}Q_{2|\tilde{s}}\big(\mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|\tilde{s}] + \mathbf{E}\left[\tilde{c}_1|\tilde{s}\right]\cdot\mathbf{E}\left[\tilde{c}_2|\tilde{s}\right]\big)\Big] - A\mathbf{E}\Big[Q_{1|\tilde{s}}\mathbf{E}\left[\tilde{c}_1|\tilde{s}\right]\Big]\cdot\mathbf{E}\Big[Q_{2|\tilde{s}}\mathbf{E}\left[\tilde{c}_2|\tilde{s}\right]\Big] \tag{4.10}$$

$$D(0) - K_1 - K_2 \leq 0 \tag{4.11}$$

$$K_1, K_2 \geq 0. \tag{4.12}$$

As visualized in Figure 4.1, $Q_{u|s_i}$ denotes the energy produced by plant technology u over the planning period $[0,T]$ given the fuel price state s_i with

$$Q_{1|s_0} = \int_0^{K_1} R(\kappa)\mathrm{d}\kappa \qquad Q_{2|s_0} = \int_{K_1}^{D(0)} R(\kappa)\mathrm{d}\kappa = Q_E - Q_{1|s_0} \tag{4.13}$$

$$Q_{2|s_1} = \int_0^{K_2} R(\kappa)\mathrm{d}\kappa \qquad Q_{1|s_1} = \int_{K_2}^{D(0)} R(\kappa)\mathrm{d}\kappa = Q_E - Q_{2|s_1}. \tag{4.14}$$

Here, $Q_E := \int_0^T D(t)\mathrm{d}t = \int_0^{D(0)} R(\kappa)\mathrm{d}\kappa$ denotes the total energy demand in $[0,T]$.

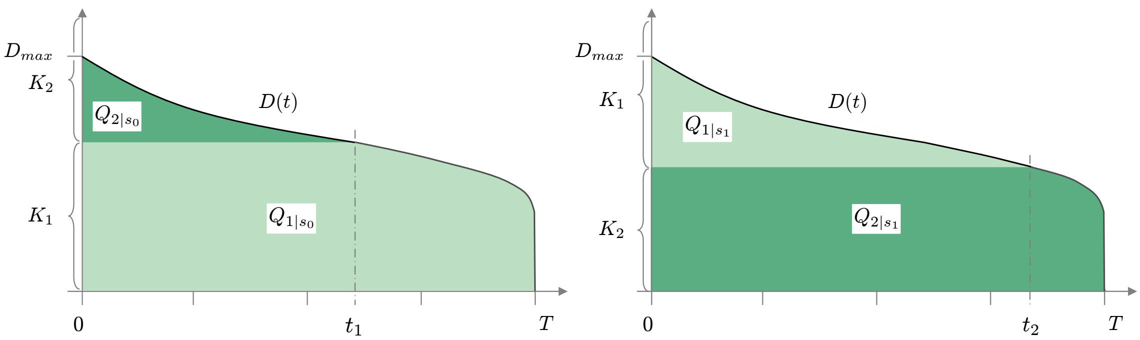


Fig. 4.1: Reversals in the merit order influence the produced energy of technologies 1 and 2 in the default (left) and reverse case (right). Note that total installed capacity may exceed the maximal demand for $\lambda = 0$.

4.2.3 Standard solutions to the portfolio problem in the two-technology case with a deterministic merit order

Before approaching the unrestricted problem, we will first discuss possible solution cases assuming a deterministic merit order without merit order risk, i.e. $\mathbf{P}(\tilde{s} = s_1) = 0$. Then, problem (4.9)-(4.12)

[5] Notably, the objective function is different from the plain expected value of the preference function over the two fuel price order scenarios which would be $\mathbf{E}\big[\mathbf{E}[C|\tilde{s}] + \frac{1}{2}A \cdot \mathrm{Var}[C|\tilde{s}]\big] \neq L$.

reduces to

$$L_0^* = \min_{K_1,K_2} L_0 \tag{4.15}$$

$$\text{s.t.} \quad L_0 = \sum_{u=1}^{2} c_{inv,u}K_u + c_uQ_u + \frac{A}{2}\left(\sum_{u=1}^{2}\sum_{v=1}^{2}\sigma_{uv}Q_uQ_v\right) \tag{4.16}$$

$$D(0) - K_1 - K_2 \leq 0 \tag{4.17}$$

$$K_1, K_2 \geq 0 \tag{4.18}$$

As shown in Sunderkötter and Weber (2012), the central optimality condition can be obtained from the KKT-conditions as

$$A\big((\sigma_1^2 - \sigma_{12})Q_E - (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\big) = \frac{c_{inv,2} - c_{inv,1}}{t_1} - \bar{c}_1 + \bar{c}_2, \tag{4.19}$$

with

$$Q_2 = \int_0^{t_1} D(t) - D(t_1)\mathrm{d}t, \qquad Q_E = \int_0^{T} D(t)\mathrm{d}t = Q_1 + Q_2.$$

In Eqn. (4.19), the risk term and the cost term are separated, each to one side of the equation. Although an explicit formulation of the optimal operating time (and therewith capacities) is not possible for a generic load duration function $D(t)$, the optimality condition allows to draw conclusions on the structure of efficient portfolios. For that, the risk term on the left side and the cost term on the right side of Eqn. (4.19), respectively, are denoted by

$$l_0(t_1) := A\big((\sigma_1^2 - \sigma_{12})Q_E - (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\big), \tag{4.20}$$

$$r_0(t_1) := \frac{c_{inv,2} - c_{inv,1}}{t_1} - \bar{c}_1 + \bar{c}_2. \tag{4.21}$$

Thereby, the intersection of $l_0(t_1)$ and $r_0(t_1)$ characterizes a stationary point t_1^o which is necessary for the solution of the cost-variance efficient portfolio. Analysis of $l_0(t_1)$ and $r_0(t_1)$ shows that three characteristic portfolio structures can be distinguished for the purely cost-efficient and the variance efficient portfolio with either technology 1, technology 2, or both technologies being part of the efficient portfolio. For the cost efficient portfolio with $A = 0$, three different solution cases can be derived from the condition $r_0(t_1) = 0$:

Property 4.2.1. *The purely cost efficient portfolio with $A = 0$ consists of*

Only technology 1	*for*	$c_{inv,1} - c_{inv,2} \leq 0,$	(4.22)
Technology 1 and 2	*for*	$0 < \frac{c_{inv,1} - c_{inv,2}}{\bar{c}_2 - \bar{c}_1} < T,$	(4.23)
Only technology 2	*for*	$\frac{c_{inv,1} - c_{inv,2}}{\bar{c}_2 - \bar{c}_1} \geq T.$	(4.24)

Similarly, three solution cases can be stated for the variance minimal portfolio with $A \to \infty$:

Property 4.2.2. *The purely variance efficient portfolio with $A \to \infty$ consists of*

Only technology 1	*for*	$\sigma_1^2 \leq \sigma_{12},$	(4.25)
Technology 1 and 2	*for*	$(\sigma_1^2 > \sigma_{12}) \wedge (\sigma_2^2 > \sigma_{12}),$	(4.26)
Only technology 2	*for*	$\sigma_2^2 \leq \sigma_{12}.$	(4.27)

As a consequence of Properties 4.2.1 and 4.2.2, stationary points in nine different cases can be distinguished to determine the general set of solutions for combined cost-variance efficient portfolios as shown in Figure 4.2. Since the optimization problem is only convex in the case with both technologies being part of the cost-minimal and the variance-minimal portfolios (case (V), Figure 4.2) as shown in Sunderkötter and Weber (2012), first derivatives can be used as sufficient test for a local minimum in the other cases: Thereby, according to the mean value theorem, a stationary point t_1^o is a minimum to $\mathcal{L}_0$ only if there exists a $r \in \mathbb{R}_+$ such that for all $t_1 \in (t_1^o - r, t_1^o]$ it is $\frac{\partial \mathcal{L}_0}{\partial K_1} = l_0(t_1) - r_0(t_1) \leq 0$, and for every $t_1 \in [t_1^o, t_1^o + r)$ it is $\frac{\partial \mathcal{L}_0}{\partial K_1} = l_0(t_1) - r_0(t_1) \geq 0$.

The identified stationary points allow to formulate the following properties for the structure of cost-variance efficient portfolios given a deterministic merit order ($\mathbf{P}(s_1) = 0$):

Property 4.2.3. *If both technologies 1 and 2 are included in the purely cost-efficient portfolio (i.e. $A = 0$) and in the purely variance-efficient portfolio (i.e. $A \to +\infty$), then they are also included in all cost-variance efficient portfolios with $A > 0$.*

Property 4.2.4. *If technology u is neither included in the purely cost-efficient portfolio nor in the purely variance-efficient portfolio, then u is not included in any cost-variance efficient portfolio with $A > 0$.*

Property 4.2.5. *If the purely cost-efficient (purely variance-efficient) portfolio consists only of technology $u \in \{1, 2\}$ and the purely variance-efficient (purely cost-efficient) portfolio consists of both technologies, then there exists an A_0 such that technology u is included in all efficient portfolios for $A > A_0$ ($A < A_0$) and excluded in all efficient portfolios for $A < A_0$ ($A > A_0$).*

4.2.4 Standard solutions to the purely cost efficient portfolio with uncertainty of the merit order

As an extreme case of the general optimization problem, we will at first consider the purely cost-efficient portfolio with an unstable merit order, i.e. the case $A = 0$. Under this premise, the Lagrangian simplifies to

$$\mathcal{L}_c = \sum_{u=1}^{2} \Big(K_u c_{inv,u} + \mathbf{E}\left[Q_{u|\tilde{s}} \mathbf{E}\left[\tilde{c}_u|\tilde{s}\right]\right] \Big) + \lambda \cdot (D(0) - K_1 - K_2) \tag{4.28}$$

With z denoting the difference in operating costs of technology 1 and 2, i.e. $z := c_2 - c_1$, the corresponding KKT-conditions can be derived as shown in B.2.3 as

$$\frac{\partial \mathcal{L}_c}{\partial K_1} = c_{inv,1} - \lambda - t_1 \mathbf{P}(s_0) \cdot \mathbf{E}[\tilde{z}|s_0] \geq 0 \quad \perp \quad K_1 \geq 0 \tag{4.29}$$

$$\frac{\partial \mathcal{L}_c}{\partial K_2} = c_{inv,2} - \lambda + t_2 \mathbf{P}(s_1) \cdot \mathbf{E}[\tilde{z}|s_1] \geq 0 \quad \perp \quad K_2 \geq 0 \tag{4.30}$$

$$\frac{\partial \mathcal{L}_c}{\partial \lambda} = D(0) - K_1 - K_2 \leq 0 \quad \perp \quad \lambda \geq 0 \tag{4.31}$$

Two cases can be distinguished: For $\lambda > 0$, KKT condition (4.31) is binding. Hence, total capacity will meet but not exceed the demand maximum in the optimum, i.e. $K_1 + K_2 = D(0)$.

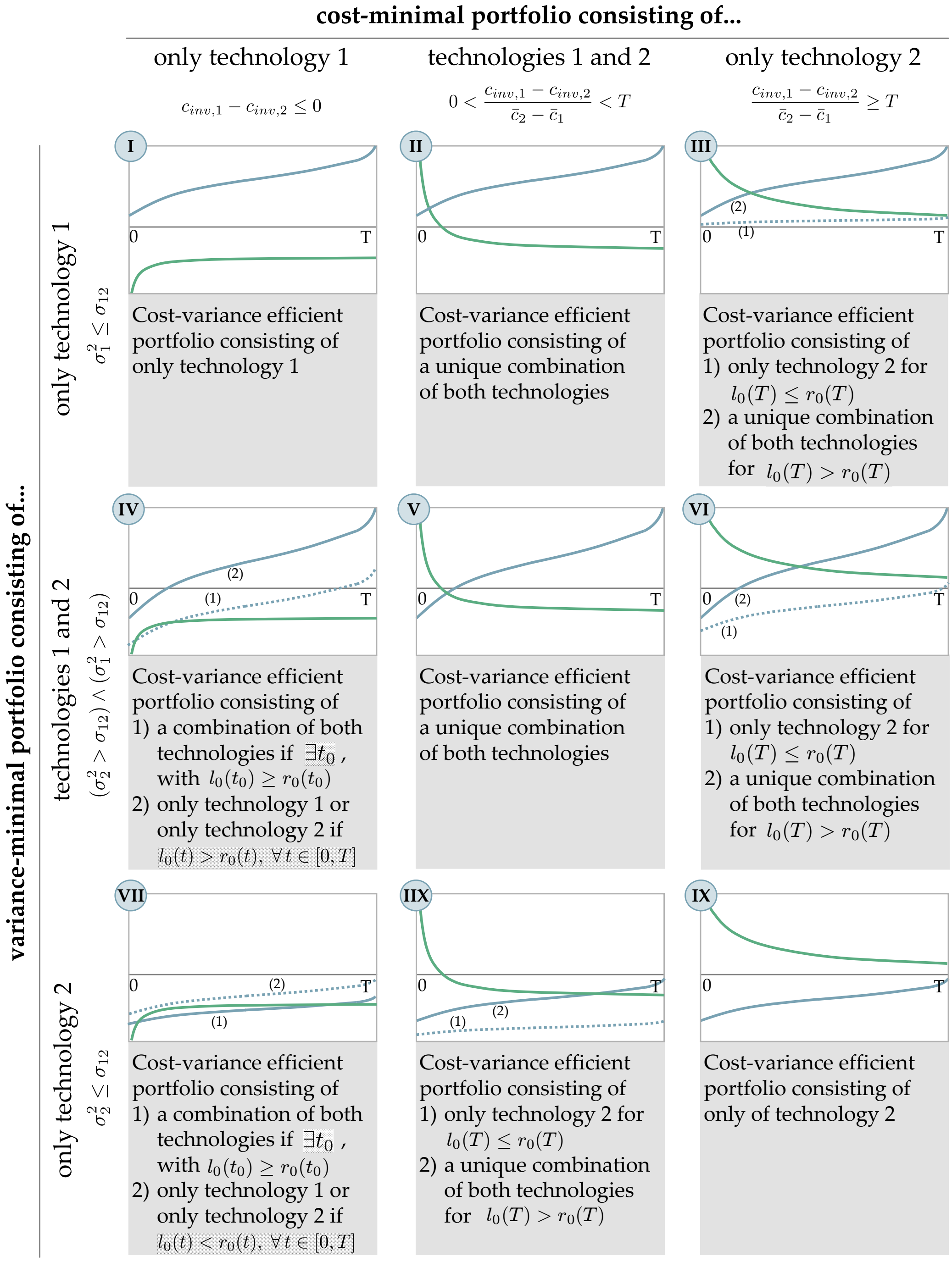


Fig. 4.2: Solution cases for the portfolio problem without merit order risk, i.e. $\mathbf{P}(\tilde{s} = s_1) = 0$. Stationary points for cost-variance efficient portfolio combinations are characterized by the intersection of $r_0(t)$ (green) and $l_0(t)$ (blue).

For $\lambda = 0$, total installed capacity may exceed the maximal demand, i.e. $K_1 + K_2 \geq D(0)$. The potential "overcapacities" in this case will be economically favorable, if the probability weighted savings from operating costs in the reversed merit order case exceed the additional investment costs for the capacity surplus. The excess capacity thus increases the fleet's operating flexibility. Nevertheless, the optimal energy produced in any scenario cannot exceed total energy demand, thus it still holds

$$Q_E = \int_0^T D(t)\mathrm{d}t = Q_{1|s_0} + Q_{2|s_0} = Q_{1|s_1} + Q_{2|s_1}.$$

It is intuitively clear that in the considered model framework with ideal and deterministic plant availabilities excess capacities may only be economical if the technologies' investment costs are relatively small compared to the expected difference in operating costs. This becomes evident in view of KKT conditions (4.29) and (4.30): The shadow price λ is linearly decreasing in t_1 and t_2, respectively, with

$$\lambda = c_{inv,1} - t_1\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0] = c_{inv,2} + t_2\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_1] \tag{4.32}$$

Clearly, λ is positive for $t_1 = 0, t_2 = 0$ and reaches its minimum at $t_1 = T$ or $t_2 = T$. Since the solution for the cost efficient portfolio with excess capacities requires $\lambda = 0$, it can be concluded that this case may only exist if expected operating costs and investment costs satisfy the necessary condition

$$(c_{inv,1} - T\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0] < 0) \quad \wedge \quad (c_{inv,2} + T\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_1] < 0) \tag{4.33}$$

Notably, condition (4.33) will rarely be satisfied for applications related to power generation portfolios with the cost characteristics of conventional plant technologies as shown in Section 4.4.

Similar to the special case with a deterministic merit order (cf. Section 4.2.3), three solution cases can be distinguished for purely cost efficient portfolios with uncertainty of the merit order B.2.3:

Proposition 4.2.1. *With $z := c_2 - c_1$ denoting the difference in operating costs of technologies 1 and 2, the purely cost-minimal portfolio with merit order risk $\mathbf{P}(s_1)$ consists of technology 2 if and only if*

$$c_{inv,1} - c_{inv,2} \geq T\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0]. \tag{4.34}$$

In contrast, the portfolio consists of technology 1 if and only if

$$c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_1]. \tag{4.35}$$

With other words, the purely cost efficient portfolio consists of both technologies if and only if

$$T\mathbf{P}(s_1) \cdot \mathbf{E}[\tilde{z}|s_1] < c_{inv,1} - c_{inv,2} < T\mathbf{P}(s_0) \cdot \mathbf{E}[\tilde{z}|s_0]. \tag{4.36}$$

4.2.5 Standard solutions to the purely variance efficient portfolio with uncertainty of the merit order

The variance efficient portfolio represents another extreme case of the general portfolio problem obtained as $A \to \infty$. The Lagrangian may be written as

$$\mathcal{L}_v = \lambda \cdot \big(D(0) - K_1 - K_2\big) + \frac{1}{2}\sum_{u=1}^{2}\left(\mathbf{E}\left[Q_{u|\tilde{s}}^2 \cdot \left(\mathrm{Var}\left[\tilde{c}_u\middle|\tilde{s}\right] + \mathbf{E}\left[\tilde{c}_u\middle|\tilde{s}\right]^2\right)\right] - \mathbf{E}\left[Q_{u|\tilde{s}}\mathbf{E}\left[\tilde{c}_u\middle|\tilde{s}\right]\right]^2\right)$$
$$+\mathbf{E}\left[Q_{1|\tilde{s}}Q_{2|\tilde{s}} \cdot \left(\mathrm{Cov}[\tilde{c}_1, \tilde{c}_2|\tilde{s}] + \mathbf{E}\left[\tilde{c}_1\middle|\tilde{s}\right] \cdot \mathbf{E}\left[\tilde{c}_2\middle|\tilde{s}\right]\right)\right] - \mathbf{E}\left[Q_{1|\tilde{s}}\mathbf{E}\left[\tilde{c}_1\middle|\tilde{s}\right]\right] \cdot \mathbf{E}\left[Q_{2|\tilde{s}}\mathbf{E}\left[\tilde{c}_2\middle|\tilde{s}\right]\right] \tag{4.37}$$

The corresponding KKT-conditions can be derived as:

$$\frac{\partial \mathcal{L}_v}{\partial K_1} = -\lambda - \mathbf{P}(s_0)t_1 \cdot \sum_{u=1}^{2}(-1)^u Q_{u|s_0}\Bigg(\sigma_{u|s_0}^2 - \sigma_{12|s_0} + $$
$$+\,\mathbf{P}(s_1) \cdot \left(\bar{c}_{u|s_0}^2 - \bar{c}_{2|s_0}\bar{c}_{1|s_0} - \bar{c}_{u|s_0}\sum_{v=1}^{2}\bar{c}_{v|s_1}\frac{Q_{v|s_1}}{Q_{u|s_0}}\right)\Bigg) \geq 0, \qquad \perp \qquad K_1 \geq 0 \tag{4.38}$$

$$\frac{\partial \mathcal{L}_v}{\partial K_2} = -\lambda + \mathbf{P}(s_1)t_2 \cdot \sum_{u=1}^{2}(-1)^u Q_{u|s_1}\Bigg(\sigma_{u|s_1}^2 - \sigma_{12|s_1} + $$
$$+\,\mathbf{P}(s_0) \cdot \left(\bar{c}_{u|s_1}^2 - \bar{c}_{2|s_1}\bar{c}_{1|s_1} - \bar{c}_{u|s_1}\sum_{v=1}^{2}\bar{c}_{v|s_0}\frac{Q_{v|s_0}}{Q_{u|s_1}}\right)\Bigg) \geq 0, \qquad \perp \qquad K_2 \geq 0 \tag{4.39}$$

$$\frac{\partial \mathcal{L}_v}{\partial \lambda} = D(0) - K_1 - K_2 \leq 0, \qquad \perp \qquad \lambda \geq 0 \tag{4.40}$$

Since investment costs are neglected in the purely variance optimal investment decision, it can be concluded from the existence of a variance-efficient portfolio without overcapacities that an increase in the capacity of any technology is also efficient. With other words: If additional capacity has zero cost, increasing the capacity of an efficient portfolio cannot negatively influence its optimality as long as the expected technology deployment on the second stage of the optimization problem is not changed compared to the situation without overcapacities.[6] Therefore, we will restrict in the following the variance minimization problem to the more interesting case in which total installed generation capacity matches system demand. Then, the hitherto constraint inequality (4.11) is replaced by the equality

$$D(0) - K_1 - K_2 = 0 \tag{4.41}$$

Under this assumption it is not clear any longer whether the efficient portfolio consists of a single technology or a mix of both technologies. With (4.41), t_2 can be expressed as a function of the t_1 with $t_2(t_1) = R(D(0) - D(t_1))$. The latter expression may be used to reformulate the production volumes as defined in Eqs. (4.13), (4.14) in order to express the problem solely dependent on t_1.

[6] Notably, for a purely variance efficient solution the actual plant deployment on the second stage is irrelevant and could theoretically be realized arbitrarily. Only the *expected* plant deployment is relevant for the variance minimal investment decision on the first stage of the problem.

For an interior solution with $K_1^*, K_2^* > 0$, the KKT conditions (4.38) and (4.39) have to be satisfied with equality as necessary optimality condition. Eliminating λ through subtraction of these conditions yields the central optimality condition $\frac{\partial \mathcal{L}_R}{\partial K_1} - \frac{\partial \mathcal{L}_R}{\partial K_2} = 0$ which allows us to derive the following proposition:

Proposition 4.2.2. *Under the restriction that total installed generation capacity must match total demand, i.e.* $\lambda \neq 0$*, the purely variance minimal portfolio with a merit order risk* $\mathbf{P}(s_1) > 0$ *corresponding to optimization problem (4.9)-(4.12) consists of both technologies 1 and 2 if*

$$\left(-\frac{\sigma_{12|s_0} - \sigma^2_{1|s_0}}{\mathbf{E}[\tilde{z}|s_0]\mathbf{P}(s_1)} > \bar{c}_{1|s_1} - \bar{c}_{1|s_0}\right) \wedge \left(\bar{c}_{1|s_1} - \bar{c}_{1|s_0} < \frac{\sigma^2_{1|s_1} - \sigma_{12|s_1}}{\mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)}\right) \tag{4.42}$$

The proof and supplementary discussions on other solution cases for the variance minimal portfolio are provided in B.2.4

4.2.6 Standard solutions to the combined portfolio problem with uncertainty of the merit order

We start solving the general non-convex problem (4.9)-(4.12) by using a standard Lagrange approach for the relaxed assumption of $\mathbf{P}(\tilde{s} = s_1) \geq 0$ to identify stationary points as necessary conditions in the optimum. For ease of computation, again t_1, t_2 are used as equivalent decision variables. Having obtained optimal values for t_1^*, t_2^*, we can subsequently derive K_1^*, K_2^* from Eqn. (4.8) to complete the solution. Then, the Lagrangian $\mathcal{L}$ writes

$$\mathcal{L} = L + \lambda \cdot \big(D(0) - K_1 - K_2\big), \tag{4.43}$$

and the corresponding KKT-conditions may be written as:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_1} &= c_{inv,1} - \lambda - \mathbf{P}(s_0) t_1 \cdot \sum_{u=1}^{2} (-1)^u \cdot \Bigg(\bar{c}_{u|s_0} + A Q_{u|s_0} \cdot \Bigg(\sigma^2_{u|s_0} - \sigma_{12|s_0} \\
&\quad + \mathbf{P}(s_1) \cdot \Bigg(\bar{c}^2_{u|s_0} - \bar{c}_{2|s_0}\bar{c}_{1|s_0} - \bar{c}_{u|s_0} \sum_{v=1}^{2} \bar{c}_{v|s_1} \frac{Q_{v|s_1}}{Q_{u|s_0}}\Bigg)\Bigg)\Bigg) \geq 0, \qquad \perp \quad K_1 \geq 0 \qquad (4.44) \\
\frac{\partial \mathcal{L}}{\partial K_2} &= c_{inv,2} - \lambda + \mathbf{P}(s_1) t_2 \cdot \sum_{u=1}^{2} (-1)^u \cdot \Bigg(\bar{c}_{u|s_1} + A Q_{u|s_1} \cdot \Bigg(\sigma^2_{u|s_1} - \sigma_{12|s_1} \\
&\quad + \mathbf{P}(s_0) \cdot \Bigg(\bar{c}^2_{u|s_1} - \bar{c}_{2|s_1}\bar{c}_{1|s_1} - \bar{c}_{u|s_1} \sum_{v=1}^{2} \bar{c}_{v|s_0} \frac{Q_{v|s_0}}{Q_{u|s_1}}\Bigg)\Bigg)\Bigg) \geq 0, \qquad \perp \quad K_2 \geq 0 \qquad (4.45) \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= D(0) - K_1 - K_2 \leq 0, \qquad \perp \quad \lambda \geq 0 \qquad (4.46)
\end{aligned}$$

The optimal portfolio mix (K_1^*, K_2^*) to problem (4.9)-(4.12) is either obtained as a boundary solution with $K_1^* = 0$ or $K_2^* = 0$ or as an interior solution with K_1^*, $K_2^* > 0$. The necessary condition for an interior solution is determined by the non-convex equation system with Eqs. (4.44), $\frac{\partial \mathcal{L}}{\partial K_1} = 0$, and (4.45), $\frac{\partial \mathcal{L}}{\partial K_2} = 0$. Both equations represent functions of K_1 and K_2, respectively t_1 and t_2.

As for the purely cost-minimal portfolio, two cases for λ have to be distinguished: For $\lambda = 0$, total installed capacity can exceed the maximal demand, i.e. $K_1 + K_2 \geq D(0)$, while for $\lambda > 0$ total capacity will meet but not exceed the demand maximum, i.e. $K_1 + K_2 = D(0)$.

First, we consider the case $\lambda > 0$. Here, λ can be eliminated in Eqs. (4.44) and (4.45) by subtracting $\frac{\partial \mathcal{L}}{\partial K_1} - \frac{\partial \mathcal{L}}{\partial K_2} =: \upsilon$. The resulting optimality condition can then be written as $\upsilon = 0$ with

$$\begin{aligned} \upsilon(t_1) =& A\mathbf{P}(s_0)\mathbf{P}(s_1)\sum_{i=0}^{1} t_{i+1}\Big(Q_{i+1|s_i}\mathbf{E}[\tilde{z}|s_i]^2 - Q_E(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}[\tilde{z}|s_i](-1)^i - \mathbf{E}[\tilde{z}|s_1]\mathbf{E}[\tilde{z}|s_0]Q_{i+1|s_i}\Big) \\ &+ c_{inv,1} - c_{inv,2} - \mathbf{E}\big[(t|\tilde{s}) \cdot \mathbf{E}[\tilde{z}|\tilde{s}]\big] + A\mathbf{E}\Big[(t|\tilde{s}) \cdot Q_{1|\tilde{s}}\mathrm{Var}[\tilde{z}|\tilde{s}] - Q_E(\sigma^2_{2|\tilde{s}} - \sigma_{12|\tilde{s}})\Big] \end{aligned} \quad (4.47)$$

Remark that $\upsilon(t_1)$ is solely dependent on the decision variable t_1, respectively K_1, if we use the relation $t_2(t_1) = R(D(0) - D(t_1))$. The combined cost-risk efficient portfolio structure given the merit order risk $\mathbf{P}(s_1) \geq 0$ can be characterized by the following property as shown in B.2.5:

Proposition 4.2.3 (Existence and uniqueness of an interior solution for the case $\lambda \neq 0$)**.** *Let be $c_{inv,1} > c_{inv,2}$ and total installed capacity matching maximum demand, i.e. $\lambda \neq 0$ in constraint (4.12). If both technologies 1 and 2 are included in the purely cost-efficient portfolio (i.e. $A = 0$) and in the purely variance-efficient portfolio (i.e. $A \to +\infty$) satisfying condition (4.42), then all cost-variance efficient portfolios with $A > 0$ corresponding to problem (4.9)-(4.12) consist of a unique combination of both technologies.*

Based on the discussion in Section 4.2.4, the economic benefit of excess capacities in purely cost-efficient portfolios is limited to investment settings with relatively small investment cost compared to a large difference in operating costs and a severe merit order risk (see condition (4.33)). In a risk-cost investment setting, excess capacities are beneficial from an economic point of view if the difference in *risk-adjusted* expected operating costs is relatively large compared to the investment costs of the generation technologies. Thus, the economic benefit of overcapacities increases with the level of social risk aversion A.

Although thus rare in electricity investment applications, we continue to characterize this solution case with $\lambda = 0$ in the following propositions (proofs are provided in B.2.5):

Proposition 4.2.4 (Implicit functions)**.** *For the implicit function $\frac{\partial \mathcal{L}}{\partial K_1}(\check{t}_1, \check{t}_2) = 0$ with $\frac{\partial \mathcal{L}}{\partial K_1}(t_1, t_2) : [0,T] \times [0,T] \to Z_1 \subseteq \mathbb{R}_+$, there exists a unique function $\zeta_1(t_2) : (0,T] \to Z_1 \subseteq \mathbb{R}_+$. Similarly, for the implicit function $\frac{\partial \mathcal{L}}{\partial K_2}(\check{t}_1, \check{t}_2) = 0$ with $\frac{\partial \mathcal{L}}{\partial K_2}(t_1, t_2) : [0,T] \times [0,T] \to Z_2 \subseteq \mathbb{R}_+$, there exists a unique function $\zeta_2(t_1) : (0,T] \to Z_2 \subseteq \mathbb{R}_+$.*

Proposition 4.2.5 (Monotony)**.** *For given expected fuel prices $\bar{c}_{1|s_0} \leq \bar{c}_{2|s_0}$ and $\bar{c}_{2|s_1} \leq \bar{c}_{1|s_1}$ and $\zeta_1(t_2)$ and $\zeta_2(t_1)$ being functions represented by the implicit functions $\frac{\partial \mathcal{L}}{\partial K_1}(t_1, t_2) = 0$ from Eqn. (4.44), and $\frac{\partial \mathcal{L}}{\partial K_2}(t_1, t_2) = 0$ from Eqn. (4.45), respectively, $\zeta_1(t_2)$ and $\zeta_2(t_1)$ are both monotone increasing in t_2 and t_1, respectively. For $\bar{c}_{1|s_0} < \bar{c}_{2|s_0}$ and $\bar{c}_{2|s_1} < \bar{c}_{1|s_1}$, it follows strict monotony of $\zeta_1(t_2)$ and $\zeta_2(t_1)$.*

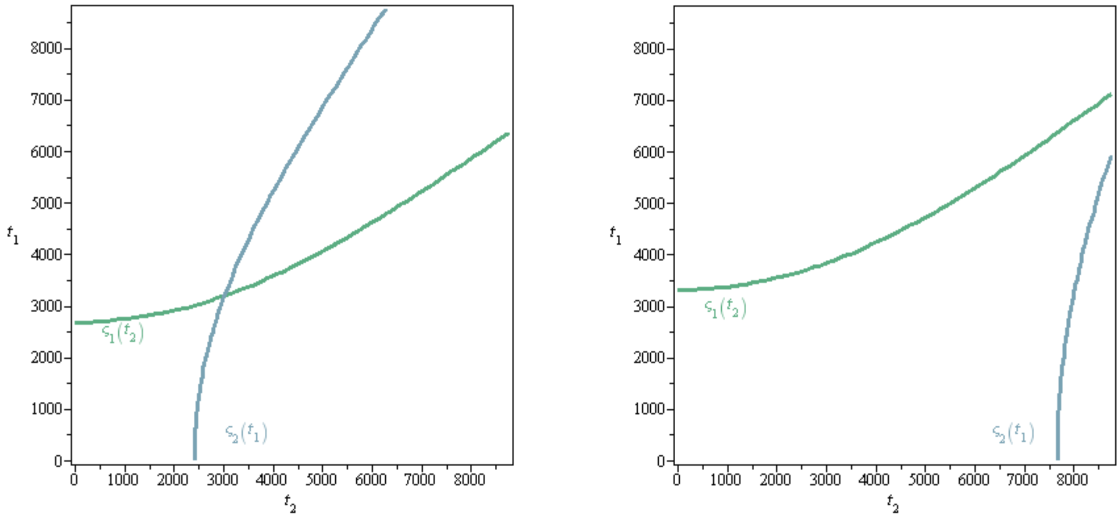


Fig. 4.3: Optimality condition for the portfolio problem as implicit functions of t_1, t_2. In the left diagram, the intersection of $\zeta_1(t_2)$ and $\zeta_2(t_1)$ represents the solution to the optimization problem for a symmetric, exemplary parameter configuration. For the asymmetric parameter configuration in the right diagram, the problem exhibits a corner solution.

With $t_1 = \zeta_1(t_2), t_2 = \zeta_2(t_1)$ being the functions represented by the implicit function $\frac{\partial \mathcal{L}}{\partial K_1}(t_1, t_2) = 0$, and $\frac{\partial \mathcal{L}}{\partial K_2}(t_1, t_2) = 0$, respectively, the existence of at least one stationary tuple $(t_1^o, t_2^o) \in (0;T) \times (0;T)$ with $\zeta_1(t_2^o) = t_1^o$ and $\zeta_2(t_1^o) = t_2^o$ is necessary prerequisite for an interior solution. This tuple represents graphically the intersection point of $\zeta_2(t_2)$ and $\zeta_1(t_2)$ as shown in cf. Figure 4.3 for a typical parameter set. An explicit, analytical solution formulation is however infeasible for the case of a general demand function as both equations contain the objective variables as integration limits in $Q_{u|\tilde{s}}$. Thereby, the following solution cases may occur in the case $\lambda = 0$:

I) Corner solution with the efficient portfolio consisting of
   i) only one technology, i.e. $K_1^* = D(0), K_2^* = 0$, or $K_2^* = D(0), K_1^* = 0$, respectively,[7]
   ii) both technologies, i.e. $K_1^* = K_2^* = D(0)$,

II) Interior solution with the efficient portfolio consisting of both technologies, i.e. $0 < K_1^*, K_2^* \leq D(0)$.

Proposition 4.2.6 (Existence and uniqueness of an interior solution for the case $\lambda = 0$)**.** *If the technology parameters satisfy*

$$\frac{c_{inv,1}}{T\mathbf{P}(s_0)} - \mathbf{E}[\tilde{z}|s_0] - AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0 \tag{4.48}$$

and

$$\frac{c_{inv,2}}{T\mathbf{P}(s_1)} + \mathbf{E}[\tilde{z}|s_1] - AQ_E\left(\sigma^2_{1|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0, \tag{4.49}$$

[7] Overcapacities may only be economical if they increase the operating flexibility of the generation portfolio. However, with a boundary solution with only one technology in the portfolio, there is no increase in flexibility and therefore a boundary solution excludes excess capacities

then the cost-variance efficient portfolios with $A > 0$ corresponding to problem (4.9)-(4.12) consist of a unique combination of both technologies. The total installed generation capacity of the cost-risk efficient portfolio may exceed total demand, implying $\lambda = 0$ in constraint (4.12).

Notably, if there exists a local minimum of $\mathcal{L}$ in $(K_1^*, K_2^*, \lambda^*)$ with $\lambda^* = 0$ according to Proposition 4.2.6, the installed capacity in the cost-variance efficient portfolio does not necessarily exceed maximum demand. In addition, there may also exist a local minimum of $\mathcal{L}$ in another point $(K_1^{**}, K_2^{**}, \lambda^{**})$ with $\lambda^{**} > 0$. Finally, the corner points as discussed above have to be checked for optimality due to the non-convexity of the problem.

4.3 Quantifying the merit order risk

The probability of a reversal in the merit order depends on the joint distribution of operating costs $\tilde{c}_1, \tilde{c}_2$, or more precisely on the distribution of the difference in operating costs $\tilde{z} = \tilde{c}_2 - \tilde{c}_1$. Before we propose a general computation method for the merit order risk over the plant's lifetime, we will briefly discuss the basic calculation technique for the periodical risk in the next section.

4.3.1 Front year merit order risk

Given the distribution of fuel prices in the following period, the risk $\mathbf{P}(c_1 > c_2)$ for a reversal in the merit order in this period can be computed from the two-dimensional density $\varphi_{1,2}$ as

$$\mathbf{P}(c_1 > c_2) = 1 - \int_{-\infty}^{\infty} \int_{-\infty}^{c_2} \varphi_{1,2}(c_1, c_2) \mathrm{d}c_1 \mathrm{d}c_2 = 1 - \int_{-\infty}^{\infty} \int_{c_1}^{\infty} \varphi_{1,2}(c_1, c_2) \mathrm{d}c_2 \mathrm{d}c_1 \tag{4.50}$$

Instead of computing the reversal likelihood directly from Eqn. (4.50), we can use the more convenient transformation[8] $\tilde{z} = \tilde{c}_2 - \tilde{c}_1$ with $\mathbf{E}[\tilde{z}] = \bar{c}_2 - \bar{c}_1$ and $\mathrm{Var}[\tilde{z}] = \sigma_1^2 + \sigma_2^2 - 2\sigma_{12}$. In knowledge of the cumulative distribution function of $\Phi(z)$ the likelihood for reversals can be calculated as

$$\mathbf{P}(c_1 > c_2) = \mathbf{P}(z < 0) = \Phi(0), \tag{4.51}$$

4.3.2 merit order risk over the plant's lifetime

Up to now, we have calculated the probability $\mathbf{P}(c_2 < c_1)$ for a single realization of fuel prices such that operating costs of technology 2 exceed those of technology 1. In fact, this calculation captures only the merit order risk for the period $\tau + 1$ given all information at τ, or more precisely $\mathbf{P}_\tau(c_{2,\tau+1} < c_{1,\tau+1}) := \mathbf{P}(c_{2,\tau+1} < c_{1,\tau+1} | \mathcal{F}_\tau)$. Beyond that, the investment decision requires to take into account the merit order risk in all subsequent periods of the plants' lifetime. Valuing

[8] It is well-known that the sum of n jointly normal distributed random variables X_i, with $X_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$ is also normal distributed with mean and variance

$$\mu = \sum_{i=1}^{n} \mu_i, \quad \sigma^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} \sigma_i \sigma_j \rho, \quad \text{with } \rho = \frac{\sigma_{ij}}{\sigma_i \sigma_j}.$$

For a proof see e.g. Elishakoff (1999).

the optimal fuel mix over all periods of the considered plant portfolio would in general require a relatively complex multi-period model, but since the investment is limited to one point in time, the problem can be reduced to a single-period model by calculating the average merit order risk over all periods of the plants' lifetime.

It is intuitively clear that the merit order risks for multiple periods ahead depends on the type of the assumed underlying stochastic fuel price process and as a consequence on the type of the resulting process of differences in operating costs. To study the impact on the merit order risk over the plant's lifetime, we will discuss in the following two fundamental stochastic processes which are typically applied in financial energy-related applications:

First, let differences in periodic operating costs $\tilde{z}_\tau = \tilde{c}_{\tau,2} - \tilde{c}_{\tau,1}$ be represented by a *random walk* defined on the probability space $(\Omega, \mathcal{F}_\tau, \mathbf{P})$ of the form

$$\Delta z_\tau = \sigma \varepsilon_\tau \sqrt{\Delta \tau}, \tag{4.52}$$

where ε denotes the standardized white noise with $\varepsilon_\tau \overset{i.i.d.}{\sim} \mathcal{N}(0,1), \tau \in \mathbb{N}$. Since the distribution of $z_{\tau+k}$ given all information at time t is non-stationary with constant mean but linearly increasing variance $Var_t[z_{\tau+k}] = k \cdot \sigma^2$, the likelihood for a reversal in the merit order k periods ahead will also increase with k as shown in Figure 4.4. Typical solution approaches for these kinds of problems apply multi-period optimization frameworks. To keep the simplicity of the annualized valuation framework proposed in Section 4.2, we use instead the compound periodical likelihood of reversals, calculated as the weighted average of the single-period merit order risk during the lifetime $\tau \in [1, \ldots, \hat{\tau}]$ of the considered plants. Thereby, the discount factor is used as weighting factor, i.e.

$$\mathbf{P}(\tilde{s} = s_1) \equiv \mathbf{P}(z_{[1,\hat{\tau}]} < 0) = \frac{q^{\hat{\tau}} \cdot i}{q^{\hat{\tau}} - 1} \cdot \sum_{k=1}^{\hat{\tau}} \mathbf{P}(z_k < 0) \cdot q^{-k}, \tag{4.53}$$

with $q = 1 + i$ and i denoting the discount rate.

While the random walk model excludes any predictability of the difference in operating costs of the two technologies, application of a *mean-reversion* model follows the idea that there is a long-term equilibrium for both technologies. This rationale can be motivated with the long-term substituting effects of commodities in many industries and is supported by various studies on cointegration of commodity prices (cf. e.g. Schwartz, 1997, Alexander, 1999, Pindyck, 1999, Schwartz and Smith, 2000, Pindyck, 2001, Geman, 2007, Mohammadi, 2009). In fact, mean-reverting behavior of the differences of two stochastic processes does even imply cointegration, i.e. stationarity of a linear combination of two stochastic processes. In addition to short-term deviations of operating costs caused by fluctuations in supply and demand of the underlying fuel types, variations in the long-run equilibrium may occur caused e.g. by technological progress impacting investment costs of generation technologies. Hence, the distribution of price differences in operating costs of two technologies is usually time-dependent and not constant over time.

We consider a mean-reversion process (corresponding to an AR(1) time series model) on the probability space $(\Omega, \mathcal{F}_t, \mathbf{P})$ of the form

$$\Delta z_\tau = \theta(\mu - z_\tau)\Delta\tau + \sigma\varepsilon_\tau\sqrt{\Delta\tau}, \tag{4.54}$$

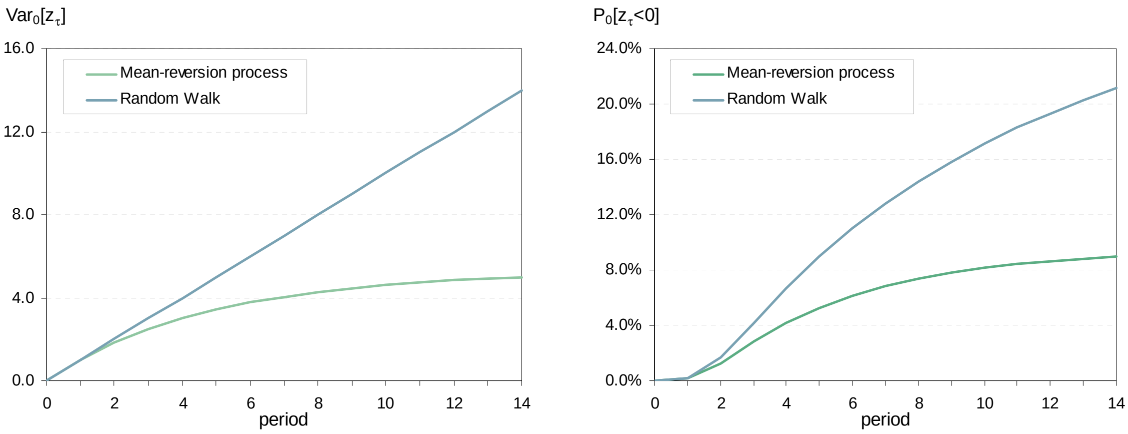


Fig. 4.4: Comparison of variance $\text{Var}_0[z_t]$ (left) and corresponding periodical merit order risk for $\mathbf{P}_0(\tilde{z}_t < 0)$ (right) for a random walk and a mean-reversion process ($\theta = 0.8$). The expected mean difference is $\mathbf{E}[z] = 3$ and the standard deviation $\sigma = 1$ for both processes.

where ε_τ is again assumed to be the standardized white noise. For $|\theta| < 1$ and $\Delta\tau \to \infty$, the process is (weakly) stationary with bounded variance $\text{Var}[z_\tau] = \frac{\sigma_\epsilon^2}{2\theta}$ and constant expectation $\mathbf{E}[z_\tau] = \mu$. As a consequence, the periodical merit order risk given mean-reverting difference in operating costs is also limited to the merit order risk implied by the variance supremum (see Figure 4.4). If the mean reversion parameter θ is sufficiently small, this upper bound may also be used as a fair approximation for the periodical merit order risk, i.e.

$$\mathbf{P}(\tilde{s} = s_1) \equiv \mathbf{P}(z_{[1,\hat{\tau}]} < 0) \approx \Phi_{z_{\hat{\tau}}}(0), \tag{4.55}$$

where $\Phi_{z_{\hat{\tau}}}$ is the unconditional cumulated normal probability distribution of $z_{\hat{\tau}}$ with $\tilde{z}_{\hat{\tau}} \sim \mathcal{N}(\mu, \frac{\sigma_\epsilon^2}{2\theta})$.

4.4 Application: Optimal generation portfolios of coal and CCGT technologies for the German electricity market

To illustrate the results, the proposed model is calibrated on the German electricity market using historical market data. For that, typical new CCGT and hard coal technologies are considered for serving demand. Note that the purpose of the calibration is to allow us to derive practically relevant results. The numbers thereby serve primarily as an illustration whereas this paper does not claim to derive a complete picture on the efficient power generation fuel mix.

4.4.1 Estimation of model parameters

Economic and technical key parameters of the coal and CCGT plant technologies based on Konstantin (2009) are depicted in Table 4.1. Total operating costs are calculated based on fuel, CO_2 emission, and variable O&M costs. To account for fuel price risks, total operating costs $\tilde{c}_u$ are modeled as normally distributed random variables calculated as the sum of the respective fuel prices plus the emission factor weighted price of CO_2 emission rights divided by the technology specific efficiency rate, i.e. $c_u = (p_{f,u} + e_u p_{co_2})/\eta_u$. A two-step approach is used to determine the merit order risk and the conditional distribution parameters as described in further detail in the following paragraphs:

- In the first step, the compound periodical merit order risk is determined based on the historical difference time series in operating costs of coal and CCGT technologies.
- In the second step, conditional distribution parameters for the individual time series of operating costs of coal and CCGT are computed based on the corresponding unconditional distribution parameters and the likelihood for reversals in the merit order.

Tab. 4.1: 2007 based key parameters for new conventional coal and CCGT technologies (source: Konstantin, 2009, own analysis).

Parameter	**Unit**	**Hard coal**	**CCGT**
Total net investment costs	€/KW	1419	608
Technical lifetime	a	45	30
Fixed O&M, overhead	€/KW a	36.06	13.97
Annualized investment costs c_{inv}	€/KW	179.905	78.442
Variable O&M, transport	€/KWh$_e$	2.9	5.5
Thermal efficiency	MWh$_e$/MWh$_t$	0.46	0.56
Carbon emission rate	tCO_2/MWh$_t$	0.34	0.20

Estimation of the merit order risk

Time series of monthly coal and natural gas import prices 1970–2010 are used based on the price indices provided by the German Federal Statistical Office (StaBu, 2010) and absolute data of the German Federal Office of Economics and Export Control (BAFA, 2010). The price data reflects the average cross-border price converted to €/MWh$_t$ for all contracted deliveries in the respective month. Starting with the beginning of the European Union Emission Trading System in 2005, total fuel prices are computed including the costs of CO_2 emission allowances (EUA) based on front year price data from EEX (2011). EUAs are modeled to be purchased at market conditions (full auctioning) as it has been announced by the EU for ETS Phase III starting in 2013. Levels of differences in variable generation costs of new CCGT and hard coal technologies are computed from the nominal time series, i.e. $z_\tau = c_{\tau,ccgt} - c_{\tau,coal}$, as shown in Figure 4.5.[9]

[9]Instead of using nominal data, we also considered deflating the data into real terms. This methodology, however, may yield biasing results since selection of an appropriate deflator is an ambiguous process. Having tested wholesale price indices for deflation, the time series properties of the difference s in operating costs did not much change.

Fig. 4.5: Differences in total operating costs of new CCGT and hard coal plants in Germany 1970–2010 (source: BAFA (2010); StaBu (2010); EEX (2011); own analysis).

The difference time series is then analyzed with respect to random walk and mean reversion properties: Table 4.2 provides "regular" and augmented Dickey-Fuller (ADF) test statistics on unit roots of the difference time series z_τ. While the DF test statistic does not allow to reject the null hypothesis of unit roots, the ADF test allows a rejection at a weak 10% level indicating trend-stationary time series characteristics. Following from both tests results, non-stationarity for the differences time series in operating costs cannot be excluded. However, this hypothesis is conflicting with the principles of a long-term market equilibrium: Since the gap in operating costs will in the long-run influence new built decisions of power plant investors as well as substituting effects in other industries, a mean-reverting behavior in the differences in operating costs would to be expected in the long-term market equilibrium.

Tab. 4.2: Regular and augmented Dickey-Fuller tests on unit roots and estimated parameters for the difference time series of variable generation costs $z_\tau = c_{\tau,ccgt} - c_{\tau,coal}$, 1970–2010. The merit order risk is based on the long-term mean difference in operating costs 1970–2010 for $\mathbf{P}_l(\tilde{s} = s_1)$ and on the short-term period 2007–2009 for $\mathbf{P}_s(\tilde{s} = s_1)$, respectively.

Time series z_τ	**Test statistic**		**Parameter estimates** (t-statistics)			**merit order risk**	
	DF	ADF	θ	μ	σ_ε	$\mathbf{P}_l(s_1)$	$\mathbf{P}_s(s_1)$
Mean reversion	-1.772	-3.171*	0.013* (1.772)	5.667 (1.268)	1.088	0.202	0.038
Random walk			-	3.988 (0.583)	6.840	0.401	0.240

Note: * significant at the 10% level, ** significant at the 5% level

To compare the impact of a mean-reverting process versus a random walk assumption for the difference time series in operating costs with regard to the efficient capacity allocation, the subsequent analysis is carried out applying both types of time series.[10] Parameter estimates and t-statistics are also depicted in Table 4.2 for annual levels of gas and coal import prices 1970–2009. Correspondingly, the table provides the computed lifetime annuities for a merit order risk for coal and CCGT technologies calculated according to Eqs. (4.53) and (4.55). Thereby, $\mathbf{P}_l(\tilde{s} = s_1)$ denotes the likelihood for reversals under the long-term mean difference in operating costs, $\bar{z}$, over the full estimation period 1970–2009. In contrast, $\mathbf{P}_s(\tilde{s} = s_1)$ is based on the mean difference in operating costs $\bar{z}$ estimated from the short-term period 2007–2009. Using the short-term period for estimating mean operating costs is most suitable in our view since it ensures appropriate long-term estimates for variance and covariance while it takes into account recent shifts in the means of operating costs. The long-term estimate for expected operating costs would yield in contrast severe inconsistencies with respect to the other key technology parameters which refer to new built power plants based on recent data. Thus, we use $\mathbf{P}_s(\tilde{s} = s_1)$ to accomplish the further analysis. As expected, the limiting periodical merit order risk given mean-reverting differences in operating costs is with 3.8% much lower compared to compound periodic merit order risk of 24.0% under the random walk hypothesis.

Unconditional and conditional distribution of operating costs

Having determined the compound merit order risk during the plant's lifetime from the difference time series of operating costs, the corresponding unconditional mean and variance are determined for each time series of operating cost both under the mean reversion and random walk hypothesis. Next, conditional means and variances are computed for each technology as shown in B.2.1 (cf. Table 4.3).

Specification of the load duration curve

The estimation of a load duration function is performed as described in Sunderkötter and Weber (2012): Historical load data for Germany provided in an hourly resolution by ENTSO-E (2009) for the years 2006–2008 provide the basis for the fitting procedure. For comparability reasons, we adjust the data sets for the general increase in energy consumption by 1.02% in 2007 and 0.4% in 2008, respectively. A historical reference load duration curve is then generated from the hourly means of the historic data. To accomplish the further analysis in Matlab with a continuous inverted load duration function $\hat{R}(K) = \hat{D}^{-1}(t)$, we use OLS regression to fit a polynomial function of the form

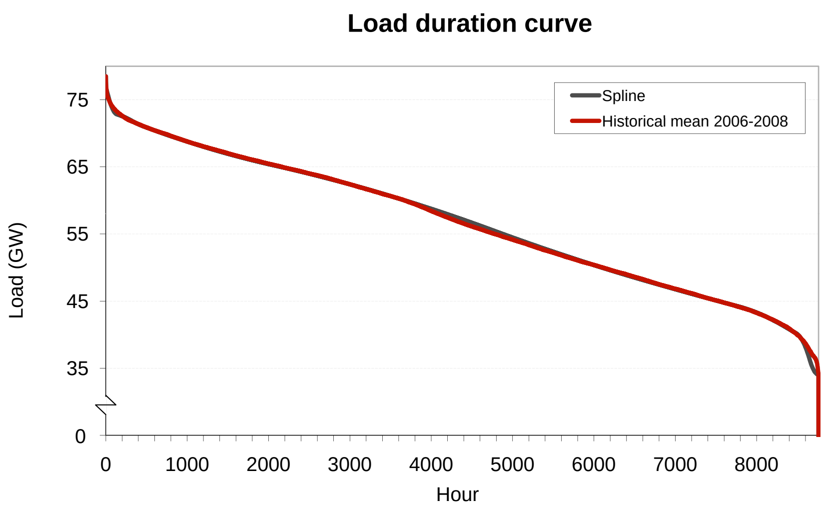
$$\tilde{R}(K) \approx \begin{cases} \sum_{j=0}^{q} A_j \cdot K^j, & \text{for } K \geq D(T) \\ T, & \text{for } K < D(T). \end{cases} \tag{4.56}$$

with $T = 8760$ hours and for a load ranging from $D(T) = 35031$ MW to $D(0) = 78332$ MW. Parameter estimates for a polynomial function of degree $q = 7$ are provided in Fig. 4.6.

[10] Further research remains necessary to provide empirical evidence on the question of random walk versus mean-reverting time series behavior of difference of operating costs. Since the main objective of this section is to provide an illustrative application of the analytical discussion, we kindly refer the reader to existing literature

Tab. 4.3: Distribution parameters for operating costs of coal and CCGT technologies. Unconditionally expected operating costs represent historical mean costs 2007–2009. Conditional distribution parameters were calculated based on the merit order risk given a random walk and a mean reversion process.

	Empirical estimate	Random walk	Mean reversion
$\mathbf{E}[\tilde{c}_{coal}]$	45.912	45.912	45.912
$\mathbf{E}[\tilde{c}_{ccgt}]$	57.975	57.975	57.957
$\mathbf{P}_s(\tilde{s} = s_1)$	-	0.238	0.0383
$\mathrm{Var}[\tilde{c}_{coal}]$	84.447	513.422	83.305
$\mathrm{Var}[\tilde{c}_{ccgt}]$	195.574	1189.223	192.957
$\mathrm{Cov}[\tilde{c}_{coal}, \tilde{c}_{ccgt}]$	116.561	188.858	115.779
$\mathbf{E}[\tilde{c}_{coal}\vert s_0]$	-	50.606	46.315
$\mathbf{E}[\tilde{c}_{coal}\vert s_1]$	-	30.871	35.795
$\mathbf{E}[\tilde{c}_{ccgt}\vert s_0]$	-	69.505	58.958
$\mathbf{E}[\tilde{c}_{ccgt}\vert s_1]$	-	20.952	33.067
$\mathrm{Var}[\tilde{c}_{coal}\vert s_0]$	-	452.673	79.818
$\mathrm{Var}[\tilde{c}_{coal}\vert s_1]$	-	411.257	64.404
$\mathrm{Var}[\tilde{c}_{ccgt}\vert s_0]$	-	821.499	171.853
$\mathrm{Var}[\tilde{c}_{ccgt}\vert s_1]$	-	570.799	78.548
$\mathrm{Cov}[\tilde{c}_{coal}, \tilde{c}_{ccgt}\vert s_0]$	-	559.261	106.415
$\mathrm{Cov}[\tilde{c}_{coal}, \tilde{c}_{ccgt}\vert s_1]$	-	457.363	68.49



Spline parameter	Estimate
A_0	$4.590 \cdot 10^{6}$
A_1	$-6.213 \cdot 10^{2}$
A_2	$3.552 \cdot 10^{-2}$
A_3	$-1.110 \cdot 10^{-6}$
A_4	$2.047 \cdot 10^{-11}$
A_5	$-2.330 \cdot 10^{-16}$
A_6	$1.330 \cdot 10^{-21}$
A_7	$-3.350 \cdot 10^{-27}$

Fig. 4.6: Historical and fitted load duration curve and parameter specification of the polynomially fitted inverse load duration function $R(K) = D^{-1}(t)$ (ENTSO-E, 2009, own analysis).

on this topic as mentioned above.

4.4.2 Results I: The cost and variance efficient portfolio fuel mix

We start interpreting the results by first investigating the efficient technology mix under the two extreme cases with $A = 0$ and $A \to \infty$.

In a first approximation, the merit order risk may be neglected to assess the solution case for the efficient portfolio structure as discussed in Section 4.2.3. Assuming risk-neutrality (i.e. $A = 0$) given the estimated technology parameterization (cf. Tab. 4.3), the cost efficient portfolio includes both generation technologies according to Property 4.2.1, since it holds $(c_{inv,1} - c_{inv,2})(\bar{c}_2 - \bar{c}_1) = 8411 \in (0; 8760)$. The purely cost efficient portfolio (i.e. $A = 0$) consists of a balanced mix of both technologies with about 48% CCGT and 52% hard coal capacity (Fig. 4.7). It can be seen that the cost efficient technology mix both under the random walk and the mean-reversion assumption is fairly well approximated by the calculation with neglected merit order risk. The absolute discrepancy between the calculations with and without merit order risk is marginal.

From a pure risk perspective, a comparison of the unconditional variance of operating costs shows the superiority of coal compared to the CCGT technology. Neglecting the merit order risk and applying Property 4.2.2, it can bee seen that the variance efficient portfolio will only contain coal technologies since $\sigma_1^2 - \sigma_{12} = 84.447 - 116.561 < 0$. The picture does not change by taking into account the merit order risk: Under the random walk assumption, the conditional variance of the CCGT technology exceeds the conditional variance of the coal technology in both merit order states. Put differently, the coal technology dominates the CCGT technology with respect to the scenario variance. Hence, diversification is not efficient. As expected, the sufficient condition (4.42) for variance efficient portfolios including both technologies as formulated in Property 4.2.2 is not satisfied.

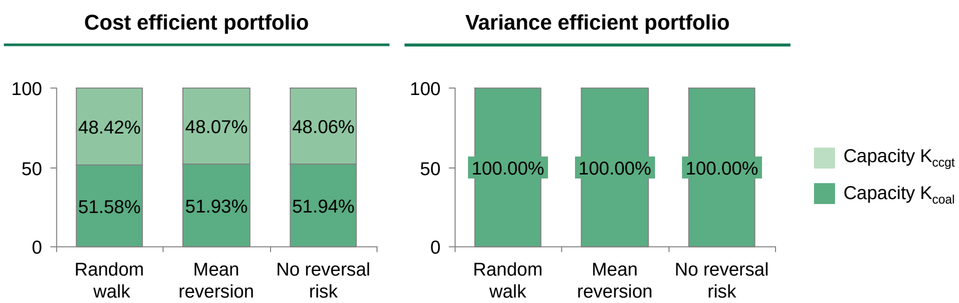


Fig. 4.7: Capacity structure of purely cost and variance efficient generation portfolios consisting of new CCGT and hard coal technologies under the random walk and mean reversion hypothesis and for the approximating calculation with neglected merit order risk.

4.4.3 Results II: Cost-risk efficient fuel mix under the random walk and the mean reversion hypothesis

To assess the technology structure of cost-risk efficient portfolios, we compute efficient capacities and total system costs of hard coal and CCGT technologies for levels of risk aversion in a range of

$A = [0, \ldots, 10^{-9}]$ under the two alternative assumption that differences in operating costs follow a random walk or a mean reversion process.

Figure 4.8 presents the efficient capacity mix of CCGT and hard coal technologies under the assumption that differences in operating costs follow a random walk. Correspondingly, Figure 4.9 depicts the efficient portfolio structure under the mean-reversion assumption. For reference purposes, the hypothetical efficient portfolio structure without merit order risk is indicated by a dashed line.

Despite the relatively high merit order risk under the random walk assumption, the share of CCGT generation in the efficient portfolio is decreasing with increasing levels of risk aversion. This is due to the specific risks of coal and CCGT technologies: The higher the risk aversion factor A, the stronger is the impact of the variance in the objective function. With the risk contribution of CCGT technologies being higher than that of coal technologies, the higher overall fiancial risk in the case with merit order risk implies that the efficient portfolios contain higher shares of coal generation compared to the calculations without merit order risk.

Under the random walk hypothesis, the share of coal generation in efficient generation portfolios is much higher than under a mean-reverting difference time series of operating costs. This is due to the fact that the higher probability of merit orders in the merit order results also in a higher absolute portfolio risk (measured both by conditional and unconditional variances) than under the mean-reversion assumption. Due to the relatively small merit order risk in the latter case, the efficient portfolio structure under the mean-reversion assumption is fairly well approximated by a model formulation with neglected merit order risks.

Remarkably, neither the efficient portfolios under the random walk nor under the mean reversion hypothesis exhibit any overcapacities in the numerical example. This phenomenon will be analyzed in detail in the following paragraph.

4.4.4 Results III: Overcapacities in efficient portfolios

In a world with deterministic peak demand and full information about plant availabilities, there is no need to install more generation capacity than maximum demand as long as uncertainty of generation costs does not lead to changes in the merit order.

The picture may change drastically given uncertainty in the merit order as already discussed earlier in this paper: If there is a substantial risk that the CCGT plant may run as the base-load plant in certain periods, then it may be economical to install a higher share of CCGT generation capacity compared to the situation where CCGT is only expected to run as peak plant. Thereby, the question whether to build more generation capacity than maximum demand depends highly on the relation of capacity investment costs compared to the difference of operating costs: The lower plant-specific investment costs, the more economical it becomes to build overcapacities. Considering the extreme case with zero capacity investment costs, the cost-minimal generation portfolio would include generation capacities of each technology at maximum demand, i.e. the total installed capacity would be twice the maximum demand.

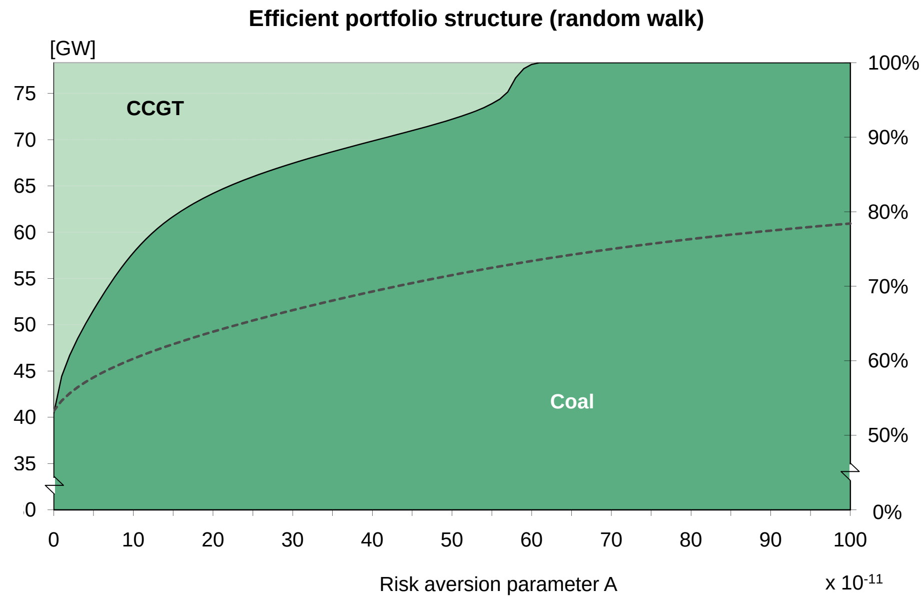


Fig. 4.8: Capacity structure of efficient generation portfolios consisting of new CCGT and hard coal technologies for varying risk aversion parameter, A, given that differences in operating costs follow a random walk. The hypothetical efficient portfolio structure without merit order risk is shown by the dashed line.

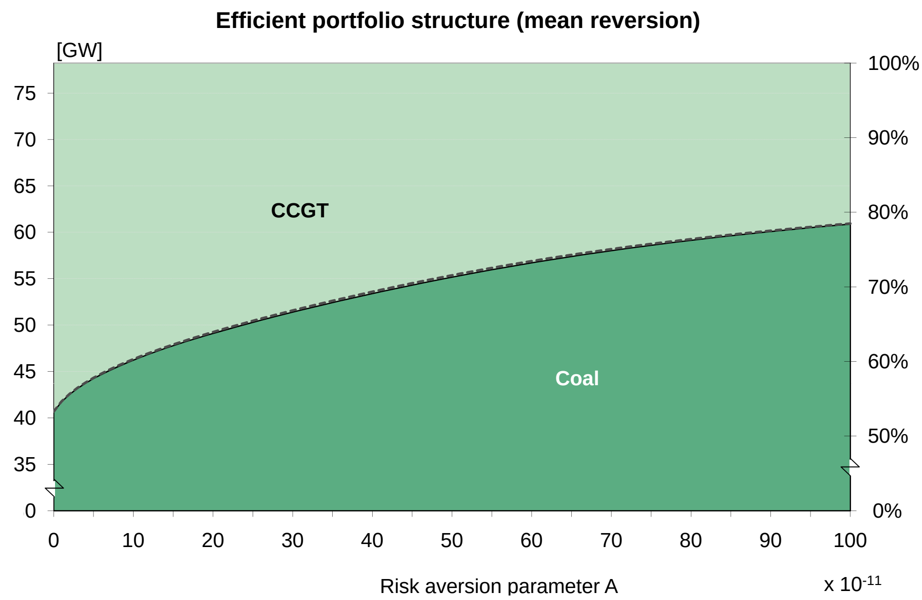


Fig. 4.9: Capacity structure of efficient generation portfolios consisting of new CCGT and hard coal technologies for varying risk aversion parameter, A, given that differences in operating costs follow a mean-reversion process. The hypothetical efficient portfolio structure without merit order risk is shown by the dashed line.

For the case with risk-neutrality (i.e. $A = 0$), a necessary parameter condition for plant overcapacities has been formulated in Eqn. (4.33). For this case, the interdependency between total excess capacity (measured by the system capacity ratio which equals total installed capacity divided by maximal demand) and specific investment costs is shown in Figure 4.10 (left): While the efficient portfolio does not include any overcapacity for the empirically estimated plant investment costs (indicated by the red stack), the system capacity ratio increases up to 200% for decreasing investment costs. Figure 4.10 (right) indicates at which parameter combination the necessary condition for overcapacities, Eqn. (4.33), is satisfied.

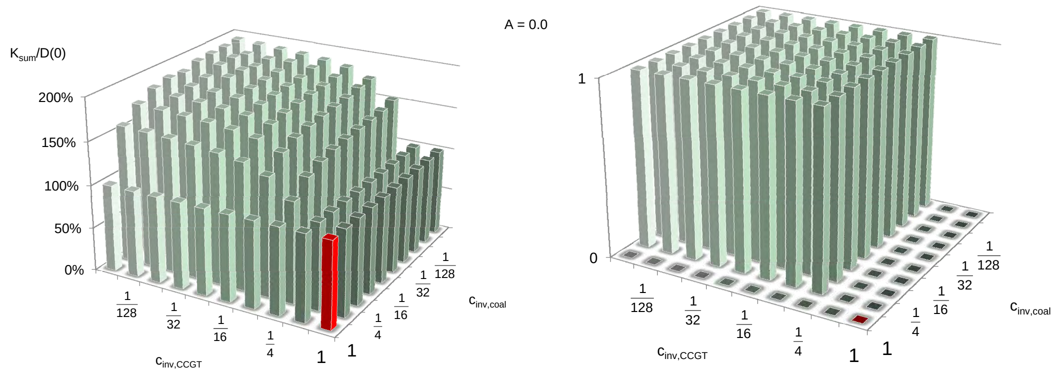


Fig. 4.10: System capacity ratio $\left(K^*_{coal} + K^*_{CCGT}\right)/D(0)$ for varying specific investment costs of coal and CCGT technologies for the purely cost efficient portfolio, i.e. $A = 0$ (left). The red stack represents the efficient capacity for the empirically derived annual investment costs. The graph on the right indicates, for which parameter combinations the necessary condition for overcapacities discussed in Eqn. (4.33) is satisfied.

For the risk-extended portfolio problem, one could expect that the system capacity ratio would increase the higher the levels of risk aversion. However, the opposite can be observed as shown in Figure 4.11 assuming that operating costs follow a random walk: With increasing societal risk aversion A, the installed overcapacity in efficient portfolios decreases. This is at first sight contra-intuitive as one would expect that with increasing risk aversion, the limiting impact of investment costs on the portfolio diversification decreases in favor of a more flexible plant portfolio. However, the reason is again the specific risk of the coal and the CCGT technologies: The risk contribution of the CCGT technology is higher compared to the coal technology. Thus, the higher the risk aversion factor A, the greater is the impact of the variance in the objective function (4.5) and hence the less attractive is diversification into the CCGT technology.

4.4.5 Results IV: Impact of increased risk of the coal technology

Based on the historically estimated operating costs parameters (cf. Tab. 4.3), the coal technology is superior compared to the CCGT technology from a pure risk perspective (measured by variance of operating costs). Yet, the variance of operating costs of the considered technologies is driven by changes of the fuel and CO_2 price levels and an increase of CO_2 and coal price volatility seems possible in the near to mid future. One reason are the ambitious EU emission reduction targets

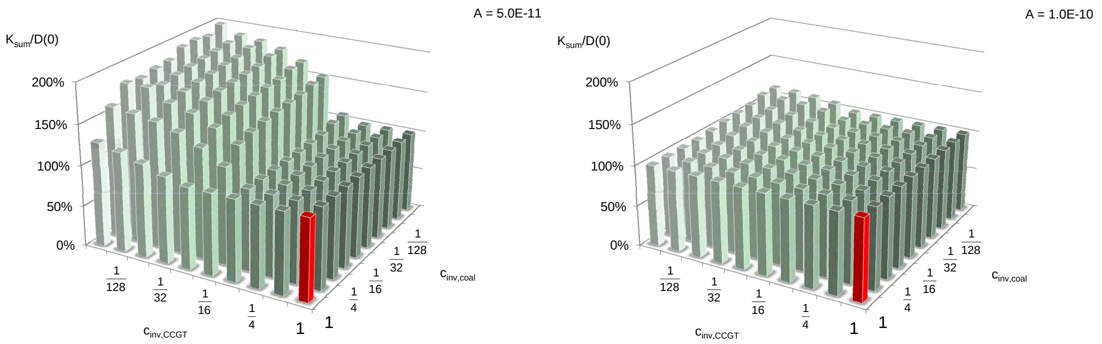


Fig. 4.11: System capacity ratio $\left(K^*_{coal} + K^*_{CCGT}\right)/D(0)$ for varying specific investment costs of coal and CCGT technologies at risk aversion levels $A = 5.0 \cdot 10^{-11}$ (left), and $A = 1.0 \cdot 10^{-10}$ (right). Differences of operating costs are assumed to follow a random walk. The red stack represents the efficient capacity for the empirically derived annual investment costs.

and another is the increasing correlation of world market prices for coal with highly volatile oil prices. This could improve the relative riskiness of the CCGT technology compared to coal. To assess the impact of such a scenario, we assume the standard deviation of the coal operating costs to be increased by 50%. As the mean operating costs are assumed to remain constant, the variance increase implies also changed conditional distribution parameters for both technologies. Furthermore, the (unconditional) variance of the difference time series decreases, resulting in a slightly reduced likelihood for reversals in the merit order of 21.4%.

The resulting impact on the structure of the purely cost and the purely variance efficient portfolio is depicted in Fig. 4.12: While the cost efficient portfolio structure remains widely unchanged, the variance efficient portfolio includes now both technologies with a capacity share of about one third coal and two third gas technology. Notably, there is again a significant discrepancy in the efficient fuel mix based on the calculation with neglected merit order risk and the calculation with a merit order risk under the random walk hypothesis. The calculation with the mean reversion hypothesis again yields results close the case with neglected merit order risk.

We start by assuming again risk-neutrality (i.e. $A = 0$) to assess the benefit of overcapacities (cf. Fig. 4.13, upper left): As in the previous section, building overcapacities becomes only efficient for decreased levels of investment costs. Thereby, the required reduction of CCGT investment costs is even higher compared to the previous section with the lower coal variance. This is plausible keeping in mind the necessary condition for overcapacities discussed in Eqn. (4.33): Since the higher coal variance implies a reduced merit order risk, $\mathbf{P}(s_1)$, the CCGT investment costs must even be smaller to satisfy the left part of condition (4.33).

With both technologies being included in the cost and in the variance efficient portfolio, it could intuitively be expected that with increasing risk aversion building overcapacities becomes more efficient. However, it can still be observed that with increasing societal risk aversion overcapacities become less attractive (cf. Fig. 4.13). This at first sight surprising result is due to the fact that

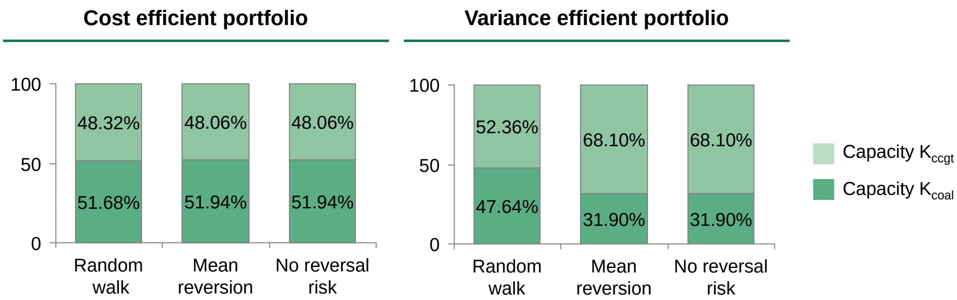


Fig. 4.12: Capacity structure (as a percentage of maximum demand $D(0)$) of purely cost and variance efficient generation portfolios consisting of new CCGT and hard coal technologies under the random walk and mean reversion hypothesis and for the approximating calculation with neglected merit order risk. The standard deviation of the coal technology is increased by 50% compared to the empirically estimated values from Tab. 4.3.

the variance term in the objective function is not only driven by the specific variance but also by the expected energy produced by each technology which is subject to the second stage of the optimization problem. To determine the expected value of the produced energy, we consider—as in the classic peak load pricing theory—at the second stage of the optimization a technology dispatch based on the merit order of generation costs: The technology with the lowest operating costs is used as base load technology, the other to serve peak load demand. This cost-based—and not variance-based—dispatch order influences the expected energy production for each technology. Installing overcapacities in a plant portfolio can hence increase operating flexibility, but may lead to the situation in which the expected mix of energy produced deviates from the variance optimum. Put differently: The most flexible technology mix which would include overcapacities, does typically not minimize the variance of operating costs.

4.5 Concluding remarks

This article analyzes efficient capacity allocation in electricity systems under uncertainty. Special emphasis is put on the impact of the merit order risk due to long-term shifts in fuel prices. In particular, technologies with operating costs characterized by little difference in mean, high variance and imperfect correlation are affected by these changes in the merit order. The model approach and the obtained insights are also relevant for investment decisions in other industries where different technologies are to be selected to supply an (expected) demand pattern, such as e.g. in transportation applications.

Our results show that risk in (variable) operating cost—measured by its variance—can heavily affect efficient capacity allocation among different technologies. Thereby, two levers of impact can be distinguished:

- Firstly, the cost risk affects the optimal capacity mix given a stable merit order of variable production costs and a firm order of dispatch. However, the efficient technology mix deviates

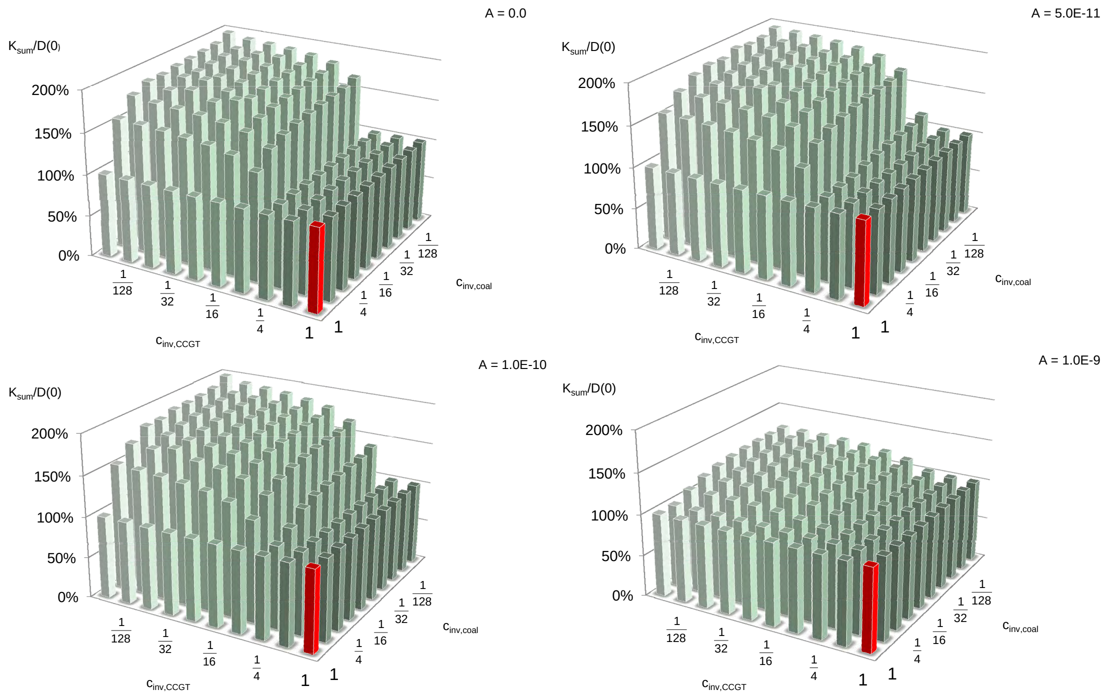


Fig. 4.13: System capacity ratio $\left(K^*_{coal} + K^*_{CCGT}\right)/D(0)$ for varying specific investment costs of coal and CCGT technologies at risk aversion levels $A = 0$ (upper left), $A = 5.0 \cdot 10^{-11}$ (upper right), $A = 1.0 \cdot 10^{-10}$ (lower left), and $A = 1.0 \cdot 10^{-9}$ (lower right). Differences of operating costs are assumed to follow a random walk. The red stack represents the efficient capacity for the empirically derived annual investment costs. The standard deviation of the coal technology is increased by 50% compared to the empirically estimated values from Tab. 4.3.

from the purely mean-based optimum only under risk-averse social preferences.

- Secondly, fuel price fluctuations may result in reversals in the merit order which may significantly influence the efficient technology mix. This risk is of particular importance if the considered technologies are characterized by only small differences in their mean costs and exhibit high, uncorrelated cost variances.

In a model application with CCGT and hard coal technologies in the German market environment, we find that a cost efficient portfolio includes a balanced mix of both technologies. In contrast, only coal is the risk efficient technology given the variance of operating costs based on historical long-term estimates.

Thereby, the characteristic of the underlying difference time series in operating costs in the considered technologies remains a crucial assumption of the model. Assuming random walk versus mean reverting properties of the difference time series may result in significantly different likelihoods for reversals in the merit order and therewith impacts massively the efficient technology mix. Given a mean reverting difference time series, the optimal portfolio structure deviates only marginally from the efficient portfolio structure with neglected merit order risk.

With an increasing merit order risk, overcapacities exceeding maximum demand may become economically favorable if investment costs are sufficiently low compared to the expected difference in operating costs. Hence, under risk-neutral preferences with the sole objective to minimize expected generation costs, an increasing merit order risk increases the degree of diversification in efficient technology portfolios. However, our example shows that at current investment costs and historically estimated cost variances, overcapacities are inefficient for CCGT and coal technology portfolios.

The benefits of diversification may change in a risk-averse investment environment where the total variance of operating costs is sought to be reduced: They are reduced if one technology dominates the other(s) with respect to the specific cost risk induced by this technology. Then increasing levels of risk aversion can even lead to efficient technology portfolios which are fully non-diversified and consist of only one technology. In this case, there is also no economic benefit from installing overcapacities—even if these are without additional costs. Hence, diversification of the generation portfolio is—even under risk aversion—not beneficial per-se.

Chapter 5

Perfect competition vs. riskaverse agents: Technology portfolio choice in electricity markets

5.1 Introduction

More than 10 years after the liberalization of electricity markets in Europe, there is a continuing discussion among energy researchers and policy makers whether fully liberalized electricity markets provide an adequate framework in which competition leads also to welfare optimal results (e.g. Roques et al., 2008, Müsgens, 2006, Bunn and Day, 2009). Following neoclassical economic theory, the set of decisions by individual investors should also lead to a socially optimal capacity allocation in efficient markets.[1]

In fact, there are several indications for insufficiencies observable on real electricity markets that could lead to a suboptimal degree of fuel mix diversification from a welfare perspective:[2] Thereby, one potential conflict of interests between individual investors and social welfare is caused by different valuation of market risks inherent to a generation portfolio with a particular fuel mix: Given that all technology portfolios realize the same level of expected costs and profits, risk averse plant investors on electricity markets will favor the plant portfolio with least variability in the net cash flow (income minus operating costs). In contrast, the welfare optimal technology mix would minimize the variability of total operating costs. This indicates that pricing mechanisms in particular market settings do not allow an adequate risk transfer from investors to consumers so

[1] However, there are several important preconditions for this to hold: In addition to perfect competition (Pigou, 1932), complete financial spot and forward markets or perfect foresight, risk neutrality (or risk-sharing opportunities), and convex production possibilities, which imply non-increasing returns to scale, are required to obtain a match of investor and wealth optimum (e.g. Arrow and Debreu, 1954, Debreu, 1959).

[2] Imperfect competition on electricity markets has drawn great attention in academic literature. Market imperfections are usually analyzed using game theory and price equilibrium models (Bolle, 1992, Green and Newbery, 1992, Hobbs et al., 2000, Hobbs, 2001, Kleindorfer et al., 2001, Moitre, 2002). Several models have been discussed in literature, including Cournot and Bertrand models (Joskow and Tirole, 2007, Ellersdorfer, 2005) and supply function models (Day et al., 2002).

that investor incentives lead to a market equilibrium which is also optimal from a social wealth point of view.

This article aims to investigate impact of risk aversion on the choice of technology portfolios on liberalized electricity markets. To our knowledge, a related study has only been published by Meunier (2012): The author proposes a simple equilibrium model taking into account correlation between technology costs and their implication on the firms' revenues. Yet specific operating times of the different technologies are neglected, implying an unrealistic derivation of the electricity market price. Instead, we use an equilibrium model based on the peak-load pricing concept to analyze the influence of risk aversion of the electricity market agents on the market outcome.

5.1.1 Market imperfections and risk-averse agents

Risk averse investor behavior represents one market imperfection which could lead to deviations from a welfare optimal investment policy. The fact that many companies commit significant resources to corporate risk management and portfolio management indicates the existence of risk-averse behavior at the company level although this practice raises doubts in view of neoclassical microeconomic theory. Following the validity of the CAPM (Sharpe, 1964, Lintner, 1965, Mossin, 1966) and the APT (Ross, 1976), investors on efficient capital markets value their investment decisions solely based on the ratio of expected return and systematic risk of an investment, whereas the unsystematic (i.e. firm-specific) risk is eliminated by diversification in other financial investments. In such a world, corporate risk management has to be questioned as a whole because its impact on the firm's risk position would be irrelevant for the investors.

While the applicability of CAPM and APT with their idealistic assumptions on market perfection became increasingly questioned in view of more and more empirically observed market anomalies in the last decades, valuation of corporate risk management in view of the firm's value has attracted substantial interest in economic literature. The rationale behind different corporate risk management strategies such as corporate portfolio management including R&D project or technology portfolios optimization hedging, has been intensively investigated and is summarized e.g. in Bartram (2000) and Gossy (2008). Three main lines of argumentation can be identified that may justify corporate risk management and risk averse behavior at the company level:

Firstly, agency-theory explains risk-averse management behavior through personal interests of the management. The management's wealth including future compensation is often little diversified, so that in-company diversification is in the management's interest (Stulz, 1984, Smith and Stulz, 1985). Furthermore, volatility reducing risk management activities allow an exacter corporate planning and protect from negative outliers which makes it easier for the management to deliver the promised performance and avoid situations which could be interpreted by the equity holders as managerial incompetence.

Secondly, the costs of financial distress have been intensively discussed as a reason why firms hedge their risk exposure (see e.g. Stulz, 1996, Bartram, 2000): If firms cannot meet their payment obligations and enter the stage of insolvency, direct costs arise, e.g. for legal expenses, as well as indirect costs as e.g. the loss of tax shields. But even before this point, financial distress can induce high costs due to e.g. higher financing costs as a result of a lower credit rating. The thread of bankruptcy can furthermore yield a loss of reputation in view of employees and (potential) customers, resulting in higher costs for human resources and customer discounts. If shareholders see bankruptcy and financial distress as a real risk, corporate risk management can therefore increase the firm's value by reducing this risk.

Thirdly, effective capital market imperfections, such as agency costs, transaction costs (especially with not publicly listed companies such as many utilities) and taxes hinder the equity holders' from sufficient diversification in their financial portfolios. Instead, some investors may prefer adequate risk management on the company level to reduce their risk exposure. Especially equity holders in the electricity industry (to a high degree public entities) often have a strategic and long-term interest in their investments which reduces the possibilities for diversification due to limited funds.

5.1.2 Mean-Variance optimization of corporate portfolios

Although mean-variance optimization of corporate portfolios has been variously discussed in corporate finance literature, there is only one work transferring this approach to electricity generation portfolios of a utility company: Roques et al. (2008) propose an optimization framework for generation portfolios from an investor perspective.

However, the applicability of the proposed model is limited to base load generation portfolios, because the authors base their model on the assumption of a stable electricity price distribution derived from historical data. The consequence is a net present value (NPV) distribution which neglects the fact that the portfolio composition will also affect electricity prices and therewith technology-specific full load hours in the long run. If however rational investors would apply Mean-Variance Portfolio (MVP) theory market-wide, the resulting optimal technology mix will clearly influence the shape of the price duration curve and therewith specific NPVs of the considered generation technologies. By limiting the model to base load generation portfolios, Roques et al. (2008) circumvent the problem of modeling technology-specific adjustments of full load hours and implications on the electricity price distribution. While this limitation avoids inconsistencies in the modeling results, it however prevents to derive conclusions about the optimal generation portfolio for an electricity market as a whole and about the long-term market equilibrium which can have – even for base load portfolios – a very different electricity price distribution due to changes in the generation portfolio.[3] Hence, a solid long-term modeling framework should therefore be based on the integrated modeling of the long-term market optimum taking into account operating and investment costs instead of unit costs.

[3] One central question remaining open concerns the market implications if all investors apply the proposed form of portfolio optimization.

5.1.3 Structure of this article

This article is structured as follows: Section 5.2 describes the general economic assumptions for the capital and the electricity markets as the basics for the following considerations. In section 5.3, we analyze the generation portfolio structure in the long-run equilibrium under perfect competition. Thereafter, we reformulate in section 5.4 the problem as a decentralized market model with risk-averse agents. The resulting market equilibria are compared in a numerical example in section 5.5.

5.2 General market assumptions

To study optimal investment equilibria in electricity generation portfolios, we consider a stylized economy with perfect competitive[4] electricity and capital markets. All other parts of the economy may fairly be represented through the capital market. Investors decide on the amount of money they want to invest in each market and on the allocation of capital to the different available assets within each market. The capital market consists of a risk-free security with interest rate r_0 and a complete set of risky assets represented through the market portfolio with rate of return r_m. The yield of all assets and therewith the return of the market portfolio are random variables, and all investors have full information and the same perception of its distribution.

In addition to the security market, the considered economy provides opportunity to invest in generation assets on the electricity market. We assume that investment and production follows a two stage process:

At the first stage, the suppliers choose their generation capacities from a set of different technologies U without knowledge of the real production costs but in full awareness of their distribution parameters. Each technology $u \in U$ with capacity K_u is assumed to be fully flexible and completely described through its deterministic specific investment costs $c_{inv,u}$ and its normally distributed operating costs $\tilde{c}_{op,u}$ with mean $\bar{c}_{op,u}$ and standard deviation σ_u.[5] All technologies are numbered in an increasing order of expected operating costs with $u = 1$ indicating the base load technology with the least operating costs. We assume a deterministic order of operating costs and exclude the risk of reversals in the merit order due to fuel price fluctuations, i.e. $c_{op,u} < c_{op,u+1}$ for all realizations.[6]

Different ways to model consumers' willingness-to-pay have been discussed in literature (see e.g. Weber, 2005). One simple concept to cope with the idea is the introduction of an additional back-stop technology of infinite capacity which can also be interpreted as a repurchase of demand, e.g. by large industrial consumers or as a price-cap as it can be found in some electricity market designs.

[4] Perfect competition includes in particular atomistic and profit maximizing behavior of all market participants, perfect information and precludes personal or corporate taxes, bankruptcy penalties, fees and other types of transaction costs.

[5] This idealization is justifiable for most fossil thermal plants, which represent by far the biggest share of the European generation mix.

[6] See Sunderkötter and Weber (2011) for a discussion of mean-variance efficient generation portfolios given uncertainty in the merit order.

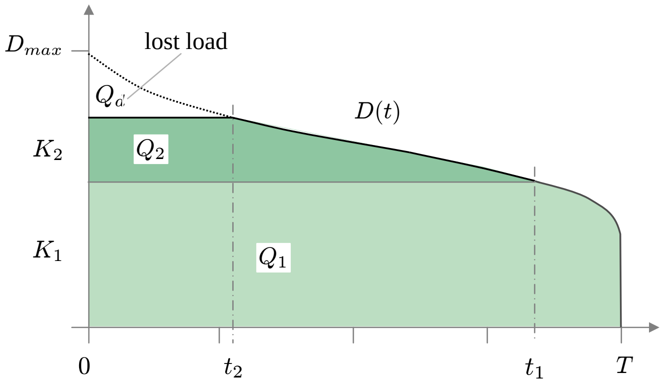


Fig. 5.1: As in the standard peak-load pricing model, the welfare optimal generation schedule can be equivalently characterized by lower bounds of optimal operating times (t_u), optimal capacities (K_u), and produced energies (Q_u) of the generation technologies.

In the following, we assume that operating costs of this technology c_d are fix with investment cost equal to zero.[7]

At the second stage, electricity is produced and traded on the wholesale market given the realized cost levels within the period $[0;T]$ (e.g. a year). The system's energy demand is assumed to be deterministic and inelastic, given in form of the load duration curve $D : [0;T] \to \mathbb{R}_+, t \mapsto D(t)$. Then, the efficient production schedule can be determined as in the standard peak-load pricing problem: Obviously from Figure 5.1, the upper bound of the optimal operating time of technology 1 equals $t_0 = T$ while the lower bound of the optimal operating time of the backstop technology equals $t_d = 0$. In fact, the lower bound of the efficient operating time of technology u is given through $D(t_u) = K_u^c$, where $K_u^c = \sum_{i=1}^{u} K_i$ denotes the cumulative capacity (with $K_0^c := 0$).

The cumulated energy in period $[0,T]$ by technologies $1,\ldots,u$ is denoted by Q_u^c, whereas the energy Q_u generated by each technology u is defined as visualized in Figure 5.1 by

$$Q_u^c(K_u^c) = \int_0^{K_u^c} R(\kappa)\mathrm{d}\kappa; \qquad Q_u(K_{u-1}^c, K_u^c) = Q_u^c(K_u) - Q_u - 1^c(K_{u-1}) = \int_{K_{u-1}^c}^{K_u^c} R(\kappa)\mathrm{d}\kappa, \quad (5.1)$$

By setting $K_d^c = D(0)$ it is assured that the total energy produced (including demand reduction from the backstop technology) does exactly match maximal demand as a residual, i.e. $Q_d = Q_E - \sum_u Q_u$, with Q_E denoting the total energy demand in period $[0,T]$.

Since K_u, K_u^c, and Q_u, Q_u^c are invertible functions of t_u, there is a unique mapping between capacities and operating times. By defining $R(K)$ as the inverse of the monotone decreasing function $D(t)$, we may write $t_u = R(K_u)$. Consequently, t_u, K_u, and K_u^c can be used interchangeably as decision variables.

For a better traceability of the optimality conditions, we will limit our considerations in the following to the case with two generation technologies ($U \in \{1,2\}$) and a backstop technology available for serving demand.

[7] The costs of the backstop technology, c_d, can also be interpreted as the *value of lost load*.

5.3 Market equilibrium under perfect competition

Among economists it is without controversy that optimal investment decisions of individual companies can as well be considered as the result of an optimal central planning if the characteristics of perfect competition are fulfilled.[8] Thus, we consider in this section the investment problem from the perspective of a representative, central planning agent with limited budget $B > 0$. Two decisions have to be made: (a) The optimal capital allocation between the electricity market and other industries, and (b) the technology structure of the electricity generation portfolio.

5.3.1 Formulation of the optimization problem

Since the assumed inelastic electricity demand implies an infinite consumer surplus, welfare maximization in the electricity market is equivalent to minimizing total electricity generation costs including the value of lost load. Additionally, the alternative use of capital in the financial market has to be considered in the welfare function. Thus, the total welfare function W takes the form

$$W = B + r_0 X_0 + \tilde{r}_m X_m - \sum_{u \in U} (c_{inv,u} \cdot K_u + \tilde{c}_{op,u} Q_u) - c_d Q_d, \tag{5.2}$$

with X_0 denoting the capital invested at risk-free return r_0 in the risk-free security and X_m the capital invested at risky return $\tilde{r}_m$ in the financial market portfolio. As a restriction, total investment in the capital market and in the electricity market must not exceed the budgeting limit B, i.e.

$$B \geq X_0 + X_m + \sum_{u \in U} c_{inv,u} K_u \tag{5.3}$$

From an economic perspective, it is clear that (5.3) will always be fulfilled with equality, since maximal profit requires that all capital is invested either in the capital market or in the electricity market.

To determine the optimal investment in the market equilibrium given the uncertainty of market return and operating costs, expected utility maximization can be applied as one of the most generic decision principles under uncertainty.[9] Let societal utility be represented by an exponential utility function of the form $U(W) = -\frac{1}{A}\exp(-AW)$ with risk aversion parameter A with normally distributed profits W. It has been shown by Schneeweiss (1965) that in this case the exponential utility function induces a unique preference function of the form

$$\Psi = \mathbf{E}[W] - \frac{1}{2} A \mathrm{Var}[W] \tag{5.4}$$

Maximization of this preference function is consistent with the decision principle of expected utility maximization. Based on these pre-considerations, the complete welfare optimization problem can

[8] This proposition, also known as the first theorem of welfare economics, is described in many economic textbooks and goes back to the Adam Smith's postulations of the "'invisible hand"'. Among others, Pigou (1932), chapter II, contributed fundamentally to a precise formulation of this theorem and its prerequisites. One of the first mathematical proofs was published by Lange (1942).

[9] This decision criterion is sometimes also referred to as the *Bernoulli* principle.

be formulated as:

$$\max_{X_0, X_m, K_u} \mathbf{E}[W] - \frac{1}{2} A \mathrm{Var}[W] \tag{5.5}$$

$$\text{with} \quad W = B + r_0 X_0 + \tilde{r}_m X_m - \sum_{u \in U} (c_{inv,u} K_u + \tilde{c}_{op,u} Q_u) - c_d Q_d \tag{5.6}$$

$$\text{s.t.} \;\; B \geq X_0 + X_m + \sum_{u \in U} c_{inv,u} K_u, \tag{5.7}$$

$$X_m \geq 0, \quad X_0 \geq 0, \quad K_u \geq 0 \; \forall u \in U. \tag{5.8}$$

5.3.2 Solving the optimization problem

Problem (5.5)-(5.8) can be solved under weak assumptions as shown in Sunderkötter and Weber (2012) for the n-technology case: Given $c_{inv,u} > c_{inv,u+1}$ the problem is strictly concave and thus has a unique solution, if and only if the covariance matrix of the market return and the technologies' operating costs is positive definite. However, an explicit formulation of the solution will in general not be possible.

In a first general solution approach, we will focus on inner solutions which include investments in both technologies and the market portfolio, i.e. $K_1, K_2, X_m > 0$.[10]

Assuming that the total investment in the economy is sufficiently large compared to the electricity market, i.e. $X_m \gg \sum_u c_{inv,u} K_u$ and $X_m \gg \sum_u Q_u$, we can state the following approximation for the optimality conditions:

Proposition 5.3.1. *[Market equilibrium under perfect competition] Let be a stylized economy and as defined in Section 5.2 and an electricity market with two generation technologies. Furthermore, a strictly positive societal risk aversion, i.e. $A > 0$ is assumed. If an interior solution to problem (5.5)-(5.8) exists with $K_1, K_2, X_m > 0$, and under the assumption that $X_m \gg \sum_u c_{inv,u} K_u$ and $X_m \gg \sum_u Q_u$, the optimal investments into the market portfolio and the risk-free security are given by*

$$X_m = \frac{1}{A} \frac{\bar{r}_m - r_0}{\sigma_m^2}, \tag{5.9}$$

$$X_0 = B - X_m - \sum_{u \in U} c_{inv,u} K_u. \tag{5.10}$$

The optimal capacity structure within the generation portfolio is characterized by the following optimality conditions, which are only dependent on the decision variables K_1, K_2 (or equivalently on $t_1(K_1), t_2(K_1, K_2)$, and $Q_1(K_1), Q_2(K_1, K_2)$):

$$\frac{(1 + r_0)\,(c_{inv,1} - c_{inv,2})}{t_1} = \bar{c}_{op,2} - \bar{c}_{op,1} + \frac{\bar{r}_m - r_0}{\sigma_m^2}(\sigma_{1m} - \sigma_{2m}), \tag{5.11}$$

$$\frac{(1 + r_0) c_{inv,2}}{t_2} = c_d - \bar{c}_{op,2} + \frac{\bar{r}_m - r_0}{\sigma_m^2} \sigma_{2m}. \tag{5.12}$$

[10] For the existence of corner solutions with only one technology in the efficient portfolio c.f. Sunderkötter and Weber (2012).

For the proof and the detailed optimality conditions without the assumptions $X_m \gg \sum_u c_{inv,u} K_u$ cf. C.2.1. Thus risk aversion affects the welfare-optimal solution only through the correlations $\sigma_{1,m}$ and $\sigma_{2,m}$ of the fuel prices with the market returns. Positive correlations decrease the corresponding expected costs, since then the electricity generation costs in the welfare term act as a hedge to the financial market returns.

5.4 Market equilibrium with risk-averse agents

In the last section, we discussed the structure of the welfare optimal generation portfolio from a central planning perspective which equals the market equilibrium under perfect competition. However, risk-aversion of the electricity market agents may impact the market equilibrium substantially. Therefore, we now consider a stylized economy consisting of households, an imperfect electricity market, and a perfect financial market.

Definition 5.4.1 (Electricity market agents)**.** *The electricity market is represented by profit maximizing agents with the following key properties:*

1. ***Profit share:*** *Each agent $j \in J$ (one could simplifyingly say, the managers) receives a certain fraction $\alpha_j \in (0, 1)$ of the profit of his firm.*
2. ***Risk aversion:*** *The agents are risk-averse. The preferences of agent $j \in J$ are represented by a mean-variance preference functional with an absolute risk aversion A_j.*
3. ***Diversification:*** *The agents diversify their investments into a set of different electricity generation technologies from the index set $U = 1, ..., u$.*
4. ***Homogeneity:*** *All agents $j \in J$ have homogeneous risk aversion, receive identical profit fractions α_j, and thus invest in the same technologies.*[11]

The optimization rationales of the market participants are characterized in the following.

5.4.1 Formulation of the individual optimization problems

Households can invest their capital up to a budgeting limit B in a risk-free security with interest rate r_0 or in the economy's market portfolio with uncertain return $\tilde{r}_m$ which together may represent a complete set of assets. The amount of capital invested by the households in the risk-free and risky asset are denoted with X_0, and X_m, respectively. In addition, households may invest an amount of capital $x_{el,j}$ in a security dedicated to electricity generation companies $j \in J$ at an uncertain return $r_{el,j}$. Thereby, the gross security return of company j is defined as the sum of profits $\Pi_{u,j}$ from all generation technologies in the portfolio per capital invested, i.e.

$$r_{el,j}(\widetilde{\zeta}) := \frac{\sum_u \Pi_{u,j}}{x_{el,j}}. \tag{5.13}$$

[11] We will first formulating the market equilibrium without the assumption of homogeneous agents and later come back to this assumption.

Thus, companies have to fully pay out total profits in each period without any internal accumulation of funds.

Objective of the households is to optimally allocate funds into the capital and into the electricity market so that risk-adjusted expected returns minus expected electricity costs are maximized. Being $\alpha_j \in (0,1)$ the share of profit paid to the managers (e.g. executive bonuses), then the profit share paid to the shareholders is given by $(1-\alpha_j) \cdot r_{el,j} \cdot x_{el,j}$. The electricity costs C_{el} for the households consist of the electricity market price $p_{el}(t,\tilde{\zeta})$ for the amount of consumed electricity plus the incurred utility losses at costs c_d through undelivered load which can be measured by the value of lost load, i.e.

$$C_{el} = \int_0^T p_{el}(t,\tilde{\zeta})D(t)\mathrm{d}t - \int_0^T (p_{el}(t,\tilde{\zeta}) - c_d)y_d(t)\mathrm{d}t \tag{5.14}$$

Assuming again exponential utilities with constant absolute risk aversion A_h, the households' optimization problem can be written as:

$$\max_{X_0, X_m, x_{el,j,u}, y_d} \mathbf{E}[V_h] - \frac{A_h}{2}\mathrm{Var}[V_h] \tag{5.15}$$

$$\text{with} \quad V_h := r_0 X_0 + \tilde{r}_m X_m + \sum_{j \in J}(1-\alpha_j)r_{el,j}(\tilde{\zeta})x_{el,j} - C_{el}(\tilde{\zeta}) \tag{5.16}$$

$$\text{s.t.} \quad B \geq X_m + X_0 + \sum_{j \in J} x_{el,j}, \tag{5.17}$$

$$X_m \geq 0, \quad X_0 \geq 0, \quad x_{el,j} \geq 0 \; \forall j \in J. \tag{5.18}$$

Here, the random vector $\tilde{\zeta} := (\tilde{c}_{op,1}, \tilde{c}_{op,2}, \tilde{r}_m)$ denotes the vector of exogenous risk factors defined on the probability space $(\Omega, \mathcal{A}(\mathbb{R}), \mathbf{P})$ with $\zeta : \Omega \mapsto \mathbb{R}^3$.

Each electricity market agent seeks to maximize the expected profit $\Pi_{el,j}$ adjusted by its variance. Thereby the profit $\Pi_{el,j}$ is given by the contribution margin minus investment costs of each generation asset reduced by the interest payable to the households $(1-\alpha_j) \cdot r_{el,j} \cdot x_{el,j}$. Therewith, the companies' optimization problem can be stated as

$$\max_{k_{u,j}, y_{u,j}(t,\tilde{\zeta})} \mathbf{E}[\Pi_{el,j}(\tilde{\zeta})] - \frac{A_j}{2}\mathrm{Var}[\Pi_{el,j}(\zeta)] \tag{5.19}$$

$$\text{with } \Pi_{el,j}(\widetilde{\zeta}) := \sum_u \Pi_{u,j}(\widetilde{\zeta}) - (1-\alpha_j)r_{el,j}(\widetilde{\zeta})x_{el,j}; \tag{5.20}$$

$$\Pi_{u,j}(\widetilde{\zeta}) := \int_0^T (p_{el}(t,\widetilde{\zeta}) - \widetilde{c}_{op,u})y_{u,j}(t,\widetilde{\zeta})dt - c_{inv,u}k_{u,j} \tag{5.21}$$

$$\text{s.t. } y_{u,j}(t,\tilde{\zeta}) \leq k_{u,j} \; \forall j \in J, \; \forall t, \; \forall \zeta, \tag{5.22}$$

$$\sum_u c_{inv,u}k_{u,j} \leq x_{el,j} \; \forall j \in J \tag{5.23}$$

Thereby, the capacity constraint (5.22) ensures that at every point in time the electricity produced by plant (u,j) does not exceed its capacity. The budget constraint (5.23) ensures that the total investment costs for the capacity installed by company j does not exceed its available funds $x_{el,j}$. Ex post, the remaining profit of each electricity generation company equals the managerial profit share from all payoffs of the generation units due to Eqn. (5.13), i.e.

$$\Pi_{el,j}(\widetilde{\zeta}) = \alpha_j \sum_{u} \Pi_{u,j}. \tag{5.24}$$

In addition, the supply constraint (5.25) has to be satisfied as a market clearing condition: Total production must meet or exceed system demand less the load of the backstop technology at any point in time. It will exactly meet demand minus the load of the backstop technology unless the electricity price is zero.

$$\sum_{u \in U} \sum_{j \in J_u} y_{u,j}(t,\tilde{\zeta}) \geq D(t) - y_d(t,\tilde{\zeta}) \qquad \perp \qquad p_{el}(t,\tilde{\zeta}) \geq 0 \; \forall \, t \in [0,T], \; \forall \, \tilde{\zeta} \tag{5.25}$$

To determine the market equilibrium, the intertwined optimization problems of the households and the generation companies can be split up into two stages as discussed in Section 5.2. Thereby, we will first determine the technology dispatching and electricity price formation at the second stage (with given generation capacities) before we turn back to the investment decision of generation companies and households at the first stage of the model. At the first stage all investment decisions are made without knowledge of the values of $\tilde{\zeta}$, i.e. the decision variables $X_0, X_m, x_{el,j,u}, u \in U_j, j \in J$ for the households and $k_{u,j}, u \in U_j, j \in J$ for the electricity market agents, respectively, are set. At the second stage these values are fix and no longer decision variables. The realization of $\tilde{\zeta}$ is now revealed and decisions are made with respect to $y_{u,j}(t,\tilde{\zeta})$, and $y_d(t,\tilde{\zeta})$.

5.4.2 Second stage of the market equilibrium: Technology dispatch and electricity price formation

In a perfectly competitive power market the spot price will always reflect short term marginal costs of the last producing unit as long as there is sufficient power generation capacity to meet demand. In situations when demand comes close to available capacity the end-users' willingness to pay for electricity (value of lost load) determines the price. During these periods of peak demand the resulting scarcity rent would pay off the investment cost of peak load units, and also contribute to cover the fix costs for all other plants. This intuitive result can be easily derived from the equilibrium model as formulated above (see C.2.2 for a formal proof).

Proposition 5.4.1. *Let be a stylized economy as defined in Section 5.2. Then, the wholesale electricity price is given by the function:*

$$p_{el}(t,\tilde{\zeta}) = \begin{cases} \tilde{c}_{op,1}, & \text{if } t > D^{-1}(K_1) \\ \tilde{c}_{op,2}, & \text{if } D^{-1}(K_1) \geq t > D^{-1}(K_2 + K_1) \\ c_d, & \text{if } D^{-1}(K_2 + K_1) \geq t \end{cases} \tag{5.26}$$

Note that this proposition holds both for the case of risk-neutral and risk-averse electricity market agents, as it is solely derived at the second stage of the investment problem, where uncertainty has been resolved. As a consequence, the electricity price formation can be characterized as shown in

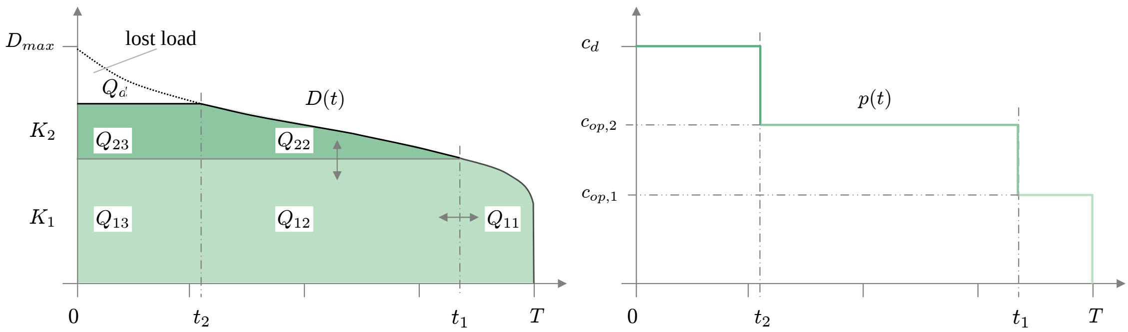


Fig. 5.2: Load duration curve $D(t)$ (left) and the price duration curve $p(t)$ (right) in the analysis period $[0, T]$.

Figure 5.2 for the two-technology case: Given a firm order of variable costs with $c_{op,1} < c_{op,2} < c_d$ it can be concluded that the load $D(t)$ is solely served by the base load technology 1 at a market price of $p_{el}(t, \tilde{\zeta}) = c_{op,1}$ at any time when demand is smaller than its installed capacity $K_1 = \sum_j k_{1,j}$. With $D(t)$ given in a decreasing order as load duration curve, this phase can be characterized by the time segment between t_1 and T. When demand exceeds capacity K_1 but is still below $K_1 + K_2$, technology 2 is dispatched to serve all additional demand at price $p_{el}(t) = c_{op,2}$. In this time segment during t_2 and t_1, the base technology earns an operational margin of $c_{op,2} - c_{op,1}$ per produced unit. Its production in this segment equals to the square $Q_{12} = \sum_j q_{12,j}$ between t_2 and t_1 and with height K_1.[12] Finally, when both technologies are operating at their capacity limits, demand response comes in implying lost load and resulting in an electricity price equal to the value of lost of load, i.e. $p_{el}(t) = c_d$. Both technologies, base and peak, earn contribution margins of $c_d - c_{op,1}$ and $c_d - c_{op,2}$ per produced unit respectively with the produced amounts $Q_{13} = \sum_j q_{13,j}$ and $Q_{23} = \sum_j q_{23,j}$ which are represented by the squares between 0 and t_2 with heights $K_1 = \sum_j k_{1,j}$ and $K_2 = \sum_j k_{2,j}$. As a result, it holds for the cumulated production quantities

$$q_{12,j} = (t_1 - t_2)k_{1,j}, \qquad q_{13,j} = t_2 k_{1,j}, \qquad q_{23,j} = t_2 k_{2,j}. \tag{5.27}$$

We thereby have also shown (again, under the given assumptions, in particular for a stable merit order) the simple intuitive result that production of each technology is not dependent at the realization of the stochastic parameters $\tilde{\zeta}$, i.e. the levels of t_2 and t_1 and consequently of Q_{12}, Q_{13} and Q_{23} are deterministic functions of the choice of K_1 and K_2 at the first stage while the specific contribution margins $\tilde{c}_{op,2} - \tilde{c}_{op,1}$, $c_d - \tilde{c}_{op,1}$ and $c_d - \tilde{c}_{op,2}$ are stochastic at that point in time.

Based on these considerations at the second stage of the model, we can summarize the following

Proposition 5.4.2. *The profits of the individual technologies $\Pi_{u,j}(\tilde{\zeta})$ as defined in (5.21) can be*

[12] The subscription is to be read as follows: the first number is the producing technology, the second is the price-setting technology.

rewritten as:

$$\Pi_{1,j}(\tilde{\zeta}) = (t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1}) + t_2(c_d - \tilde{c}_{op,2}) - c_{inv,1})k_{1,j} \tag{5.28}$$

$$\Pi_{2,j}(\tilde{\zeta}) = (t_2(c_d - \tilde{c}_{op,2}) - c_{inv,2})k_{2,j} \tag{5.29}$$

Proposition 5.4.3. *The return $r_{el,j}(\tilde{\zeta})$ as defined in (5.13) can be specified as:*

$$r_{el,j}(\tilde{\zeta}) = \frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})k_{1,j} + t_2(c_d - \tilde{c}_{op,2})(k_{1,j} + k_{2,j})}{c_{inv,1}k_{1,j} + c_{inv,2}k_{2,j}} - 1 \tag{5.30}$$

5.4.3 First stage of the market equilibrium: Investment optimum

Under consideration of the market price formation and the efficient dispatch structure at the second stage of the optimization problem, we can derive the equilibrium conditions for the optimization problems of the households and of the electricity market agents. At first, the optimality condition of the agents can be derived from the KKT conditions as shown in C.2.3:

Proposition 5.4.4. *[Electricity market agents' optimality condition] Let be a stylized economy as defined in Section 5.2 and Definition 5.4.1.*

Under the assumption of homogeneous market agents with identical risk aversion A_j $\forall j \in J$ the necessary optimality condition for an interior solution with $k_1, k_2 > 0$ for the optimization problem of the electricity agents as stated in Eqn. (5.19)-(5.23) is given by:

$$\begin{aligned} &t_2\left(\frac{c_d - \tilde{c}_{op,2}}{c_{inv,2}} - \frac{A_j\alpha_j}{Nc_{inv,2}}\left(t_1K_1(\sigma_{12} - \sigma_2^2) + t_2(K_1 + K_2)\sigma_2^2\right)\right) \\ =&t_1\left(\frac{\tilde{c}_{op,2} - \tilde{c}_{op,1}}{c_{inv,1} - c_{inv,2}} - \frac{A_j\alpha_j}{N(c_{inv,1} - c_{inv,2})}\left(t_1K_1(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + t_2(K_1 + K_2)(\sigma_{12} - \sigma_2^2)\right)\right) \end{aligned} \tag{5.31}$$

Under the assumption of homogeneity of the market agents, we can substitute the households' decision variables $x_{el,j}$ by $X_{el} = Nx_{el,j}$.

The optimality condition for the households' optimization problem can then straightforwardly be derived as shown in C.2.4. For a better traceability of the solution, we thereby assume that the relative share of the returns which is paid to the managers is very small, i.e. we consider the limiting case with $\alpha_j \to 0$, yielding:

Proposition 5.4.5. *[Households' optimality condition] Let be a stylized economy as defined in Section 5.2 and Definition 5.4.1. We assume a neglectable managerial profit share, i.e. $1-\alpha_j \approx 1$, and total investments in the economy being sufficiently large compared to the electricity market, i.e. $X_m \gg X_{el}$. Then, the necessary optimality condition for an interior solution with $x_{el,j} >, \forall j \in J$ to the optimization problem of the households as stated in Eqn. (5.15)-(5.18), is given by:*

$$\begin{aligned} &\frac{t_2(K_1 + K_2)(c_d - \tilde{c}_{op,2}) + t_1K_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})}{c_{inv,1}K_1 + c_{inv,2}K_2} \\ &\qquad = 1 + r_0 + X_m\frac{\bar{r}_m - r_0}{\sigma_m^2}\cdot\frac{t_1K_1\sigma_{m,1} - ((t_1 - t_2)K_1 - t_2K_2)\sigma_{m,2}}{c_{inv,1}K_1 + c_{inv,2}K_2} \end{aligned} \tag{5.32}$$

Remark that the left side of Eqn. (5.32) equals the electricity portfolio return $r_{el,j}$ as derived in Proposition 5.4.3. Hence, the portfolio return equals the risk-free rate in case of $A_h = 0$ or $\sigma_{m,1} = \sigma_{m,2} = 0$. In all other cases the term $A_h X_m(\cdot)$ on the right side of Eqn. (5.32) describes the risk premium in the market.

We have now received two necessary conditions for an optimal solution of our equation system. In total, this system originally contained the decision variables K_1, K_2 (equaling $k_{1,j} \cdot N, k_{2,j} \cdot N$ under common homogeneity assumptions) and $y_{u,j}(t, \tilde{\zeta})$ for the agents and X_m, X_{el}, X_0, and y_d for the households. We have outlined that the optimal values for the $y_{u,j}(t, \tilde{\zeta})$ and y_d are uniquely defined by the optimal values for K_1^* and K_2^* at the second stage of the model.

The optimal investment into the non-electricity market, X_m^* can be straigtforwardly derived from the first order condition $\frac{\partial \mathcal{L}_h}{\partial X_m}$ of the household's Lagrangian (cf. C.2.2). The obtained investment X_m^* equals the optimal investment in the welfare optimum as stated in Eqn. (5.9). The relation to the value of X_0 is given by the budget restriction (5.17) so that the optimal investment into the risk-free security is given as in the welfare optimum in Eqn (5.10). Therewith, we have solved the combined optimization problem of electricity market companies and households:

Corollary 5.4.1. *[Market equlibrium] The long-term investment equilibrium with electricity market agents $j \in J$ with homogeneous risk aversion A_j and households with risk-aversion A_h and decision variables X_0, X_m, t_1, t_2 is given by the equation system (5.9), (5.10), (5.31) and (5.32) if an inner solution exists.*

Although an explicit solution of the equation system cannot be provided in general, it can be seen that the first order optimality conditions deviate from those of the welfare optimum discussed previously (cf. Eqs. (5.11)-(5.12)). We will further assess the deviations between welfare optimum and long-term market equilibrium in the following numerical example.

5.5 A numerical example

To illustrate the results, the proposed model is calibrated to the German electricity market using historical market data. The numbers thereby serve primarily as an illustration whereas it is not intended to derive a complete picture on the efficient power generation fuel mix in Germany. For the example, typical CCGT (peak) and hard coal (base) technologies are considered being available for serving demand.

5.5.1 Model calibration and parameter estimation

Market parameters

The proposed model requires asumptions on mean and standard deviation of the market portfolio and on the risk-free rate of return. Dimson et al. (2006) have analyzed historical equity returns and equity premiums for different countries over the period 1900–2005. The authors report a global average equity risk premium of 5.15% p.a. (relative to bonds) at a standard deviation of 14.96%.

Given recent developments of German bond interest rates, we assume a risk-free rate of $r_0 = 2\%$, yielding an expected market portfolio return of $\bar{r}_m = 7.2\%$.

Generation technologies and value of lost load

Economic and technical key parameters of the coal and CCGT plant technologies are based on Konstantin (2009) as depicted in Table 5.1. We exogeneously assume costs of capital of the elctricity firms being 7.2%. This implies an investment cost annuity of 179.9 €/kW for the coal technology, and 78.442€/kW for the CCGT technology, respectively.

Total operating costs are based on fuel, CO_2 emission, and variable operating and maintenance costs. Thereby, long-term time series of monthly coal and natural gas import prices 1970–2010 are used based on the price indices provided by the German Federal Statistical Office (StaBu, 2010) and absolute data of the German Federal Office of Economics and Export Control (BAFA, 2010) to estimate variance and covariance parameters. The price data reflect the average cross-border price converted to €/MWh$_t$ for all contracted deliveries in the respective month. Starting with the beginning of the European Union Emission Trading System in 2005, total fuel prices are computed including the costs of CO_2 emission allowances (EUA) based on front year price data from ECX (2010). EUAs are modeled to be purchased at market conditions (full auctioning) as it has been put in place by the EU for ETS Phase III starting in 2013. The mean operating costs are estimated from the same data set over the short-term period 2006–2008. This combination of estimation periods is most suitable in our view since it allows appropriate long-term estimates for variance and covariance while it takes into account recent shifts in the means of operating costs. The estimated covariance of the operating cost levels of each technology with the market return are very small. Since there is also no theoretical evidence for a linear dependency between the returns and the cost levels, these parameters are set to zero.

Empirical studies on the value of lost load vary by country, by customer segment, and according the applied research methodology. Following Gilmore et al. (2010) for the U.S., typical values are ranging between 2$/kWh and 16$/kWh for the U.S. market. Reflecting these findings, we assume a value of lost load of 5000€/MWh$_e$.

Load duration curve

The estimation of a load duration function is based on 2006–2008 load data for Germany provided in an hourly resolution by ENTSO-E (2009). For comparability reasons, we adjust the data sets for the general increase in energy consumption by 1.02% in 2007 and 0.4% in 2008, respectively. A reference load duration curve is then fitted as a polynomial function to the hourly means of the historical data using OLS regression. The resulting maximum system load is $D(0) = 78377$ MW.

Tab. 5.1: Key parameters new conventional coal and CCGT technologies (source: Konstantin, 2009, Sunderkötter, 2011, BAFA, 2010, StaBu, 2010, ECX, 2010, own analysis).

Parameter	Unit	Base	Peak
Total net investment costs	€/KW	1419	608
Technical lifetime	a	45	30
Fixed O&M, overhead	€/KW a	36.1	14.0
Annualized investment costs $c_{inv,u}$	€/KW a	179.9	78.4
Variable O&M, transport	€/KWh$_e$	2.9	5.5
Thermal efficiency	MWh$_e$/MWh$_t$	0.46	0.56
Carbon emission rate	tCO_2/MWh$_t$	0.34	0.20
Mean operating costs $\bar{c}_u$	€/KWh$_e$	37.3	56.8
Variance of operating costs σ_u	€/KWh$_e$	84.5	195.6
Covariance of operating costs σ_{12}		116.6	116.6
Covariance of operating costs σ_{mi}		0.0	0.0
Value of lost load c_d	€/KWh$_e$	5000.0	

Risk aversion parameters

We first determine the societal risk aversion coefficient A_h. For given X_m, it can be seen from the Lagrangian of the households' optimization problem and the corresponding first order conditions[13]

$$A_h X_m = \frac{r_m - r_0}{\sigma_m^2}. \tag{5.33}$$

Thereby X_m can be estimated from total gross asset investments in Germany which amounted to 469B€ in 2010 (Statistisches Bundesamt, 2012). This yields a societal risk aversion in the order of magnitude of $A_h \approx 5 \cdot 10^{-12}$. Claiming a similar level of relative risk aversion for the market agents on their ideosyncratic risk factor $r_{el}(\tilde{\zeta})$ requires

$$\alpha_j A_j \frac{X_{el}}{N} = \frac{r_m - r_0}{\sigma_m^2}. \tag{5.34}$$

Power generation asset investments account for approximately 1% of total investments in the German economy[14], i.e. approximately 5B€. Furthermore, we assume $N = 50$ power producers and an agents' profit share of $\alpha_j = 0.001$. Consequently, we obtain $A_j \approx A_h \cdot 5 \cdot 10^6$.

[13] Eqn. (5.33) can be derived after some simple transformations from the first order conditions

$$\frac{\partial \mathcal{L}_h}{\partial X_m} = r_m - A_h X_m \sigma_m^2 - \mu_h = 0, \qquad \frac{\partial \mathcal{L}_h}{\partial X_0} = r_0 - \mu_h = 0.$$

[14] Statistisches Bundesamt (2012) reports gross asset investments in the German elctricity industry of 13B€in 2010, whereof estimated one third is attributable to generation assets.

5.5.2 Determining the optimal technology mix

First, we determine the welfare optimal technology mix straightforwardly as discussed in Proposition 5.3.1. The optimal values for t_1^*, t_2^* (and subsequently K_1^*, K_2^*) can be determined directly from Eqs. (5.11) and (5.12) for the case with two generation technologies.

The mix of base and peak load technology in the decentralized market equilibrium case can be determined from the optimality conditions (5.31) and (5.32) for varying firms' risk aversion A_j. Plotting these implicit functions of t_1 and t_2 allows to determine graphically the stationary points for the households' and the agents' optimization problem as shown in Fig. 5.3 for different values of A_j: For risk-neutral companies, the electricity market agents' optimality condition is represented by a linear function $t_1(t_2)$ (Fig. 5.3, left). For increasing values of A_j, the intersection point of both functions represents the market equilibrium (Fig. 5.3, middle). With A_j exceeding a certain threshold, there is no stationary point for an inner solution within the domain of t_1, t_2 (Fig. 5.3, right). In this case the market equilibrium is characterized by a corner solution.

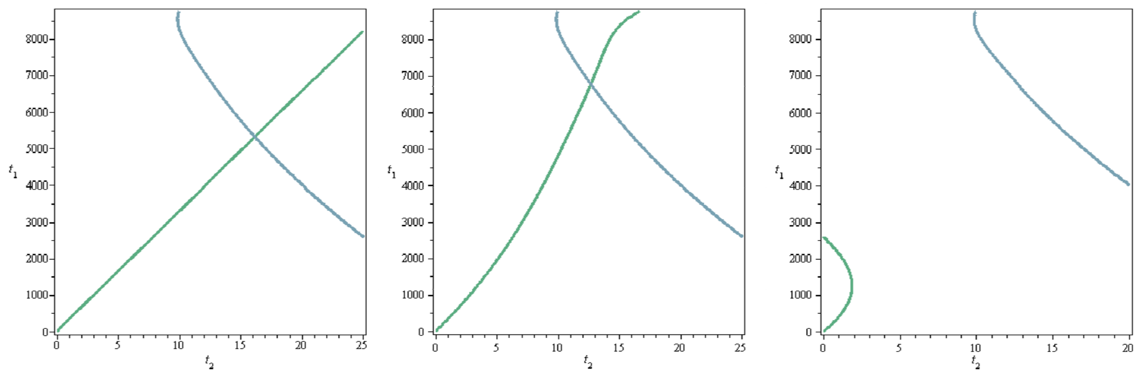


Fig. 5.3: Optimality conditions (5.14) and (5.14) plotted as implicit functions of t_1, t_2. The parametrization is $\alpha_j = 0.001$ for the managerial profit share, $A_h = 5 \cdot 10^{-12}$ for the societal risk aversion and firm's risk aversion $A_j = 0$ (left), $A_j = A_h \cdot 5 \cdot 10^7$ (middle), and $A_j = A_h \cdot 5 \cdot 10^8$ (right).

5.5.3 Results I: Impact of agents' risk aversion

Based on the analytical considerations in the previous sections, we compare the efficient portfolio structure under perfect competition with the market equilibrium under imperfect competition given risk averse electricity market agents. The efficient portfolio structures for varying agent risk aversion A_j and resulting technology returns are summarized in Fig. 5.4. Thereby, we first assume that the operating costs of technologies 1 and 2 and the return of the market portfolio are uncorrelated, i.e. $\sigma_{m1} = \sigma_{m2} = 0$.

In the market equilibrium under perfect competition and societal risk aversion, the equilibrium investment portfolio consists 67.7% base-load technology, 31.9% peak-load technology and 0.5% loss of load. Thereby, the results derived according to Proposition 5.3.1 match the portfolio structure derived from Corrolary 5.4.1 for the case $A_j = 0$.

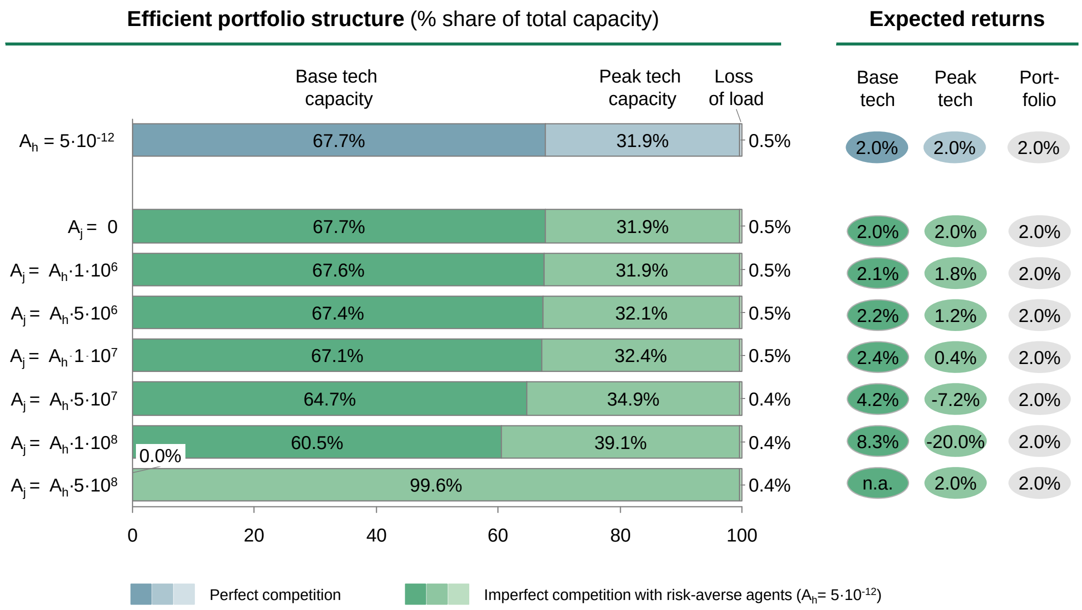


Fig. 5.4: Efficient portfolio technology mix structures (as shares of installed capacity) in the long-term market equilibrium under perfect competition (blue) and under imperfect competition with risk-averse firms (green) for varying risk aversion of the electricity market agents A_j at a managerial profit share of $\alpha_j = 0.001$ and societal risk aversion $A_h = 5 \cdot 10^{-12}$.

A risk averse investment behavior at the company level can induce structural changes in the long-term equilibrium portfolio: Given a company risk aversion of $A_j = A_h \cdot 5 \cdot 10^6$, the portfolio structure in the market equilibrium consists of 67.4% peak load and 32.1% base load technology. The share of peak load technology steadily increases with higher levels of risk aversion. At risk aversion levels of $A_j \geq A_h \cdot 5 \cdot 10^8$, the market equilibrium is characterized by a corner solution with solely peak load technology in the portfolio. The increasing investment in peak load can be explained by the fact that it sets the price most of the time and is therefore less risky from the investor's point of view.

These results are consistent with the observations of Roques et al. (2008): "High degrees of correlation between gas and electricity prices—as observed in most European markets—reduce gas plant risks and make portfolios dominated by gas plant more attractive." Our model shows that in the long-term market equilibrium risk-averse firms would clearly invest into higher shares of peak technologies—even more if the companies receive a substantial profit share.

The change in the technology structure in the decentralized market equilibrium leads also to remarkable changes in the expected return $\mathbf{E}[r_{i,j}]$ for each generation technology.[15] The investment returns for each technology in the welfare optimum equal the risk-free return given that we have assumed so far no correlation between the fuel price risk and the market returns.

[15] Remember that the (non risk-adjusted) return on investment is obtained by the cumulated cash flow devided by the investment costs, i.e. $r_{el,j} = \frac{\Pi_{el,j}}{\sum_{i \in j} c_{inv,i} K_i}$; $r_{1,j} = \frac{(t_1(c_{op,2} - c_{op,1}) + t_2(c_d - c_{op,2}) - c_{inv,1}) k_{1,j}}{c_{inv,1} k_{1,j}}$; $r_{2,j} = \frac{(t_2(c_d - c_{op,2}) - c_{inv,2}) k_{2,j}}{c_{inv,2} k_{2,j}}$.

Independently from A_j, the total expected returns $\mathbf{E}[r_{el,j}]$ of the electricity generation portfolio remains constant and equal to the risk-free rate for the case $\sigma_{m1} = \sigma_{m2} = 0$. This becomes obvious from the right side of optimality condition (5.32). The returns deviate with increasing company risk aversion substantially between the two technologies. Since the loss of load remains almost constant in all considered portfolios, the increasing company risk aversion and therewith the increasing share of the less risky peak load technology lead to lower returns of investment for the peak load and higher returns for the base load technology. Interestingly, the return of the peak technology turns negative at risk aversion levels of $A_j = A_h \cdot 5 \cdot 10^7$. Nevertheless an increasing share of the peak technology is beneficial from a company perspective as it helps decreasing the variability of cash flow.

5.5.4 Results II: Impact of correlation between risk factors

In this section, we will relax the assumption $\sigma_{m1} = \sigma_{m2} = 0$ and investigate the impact of different levels of correlation between operating costs and the return of the market portfolio. Market agents' and societal risk aversion are kept constant with $A_j = A_h \cdot 5 \cdot 10^6$ and $A_h = 5 \cdot 10^{-12}$ at a managerial profit share of $\alpha_j = 0.001$. For the purpose we use definition of the correlation coefficient

$$\rho_{m,u} := \frac{\sigma_{m,u}}{\sigma_u \sigma_m}, \quad u \in \{1; 2\} \tag{5.35}$$

As shown in Fig. 5.5, the capacity share of a generation technology in the long-term equilibrium portfolio increases with increasing levels of correlation between operating costs of the respective technology and the market portfolio return. This holds both for the market equilibrium under perfect competition and under imperfect competition given risk averse market agents. However, risk-averse behavior of electricity market agents diminishes the degree of diversification compared to the case with perfect competition. The assumed degree of correlation has significant impact at the expected portfolio returns—even on the portfolio level: While the overall expected portfolio return turns negative in the equilibrium portfolios for $\rho_{m,1} = 0.7$ and $\rho_{m,2} = 0$, we obtain a clearly higher expected portfolio return for the inverse case $\rho_{m,1} = 0$ and $\rho_{m,2} = 0.7$.

5.6 Conclusion

This article compares optimal technology portfolio choices under market imperfections. Taking investment decisions on electricity markets as an example, we first propose a partial equilibrium model to determine the optimal portfolio consisting of two generation technologies with different cost and risk characteristics under the assumption of perfect competition. The resulting portfolio matches the welfare optimal technology mix, i.e. a generation portfolio minimizing the total risk-adjusted costs of households over consumption and investments. Efficient generation portfolios are derived from exogenous factors such as demand, risk aversions of the market participants, costs and available budget on the basis of classic mean-variance-preference calculus and peak load pricing theory. This immediately implies a distribution of prices with respective consumer and investor

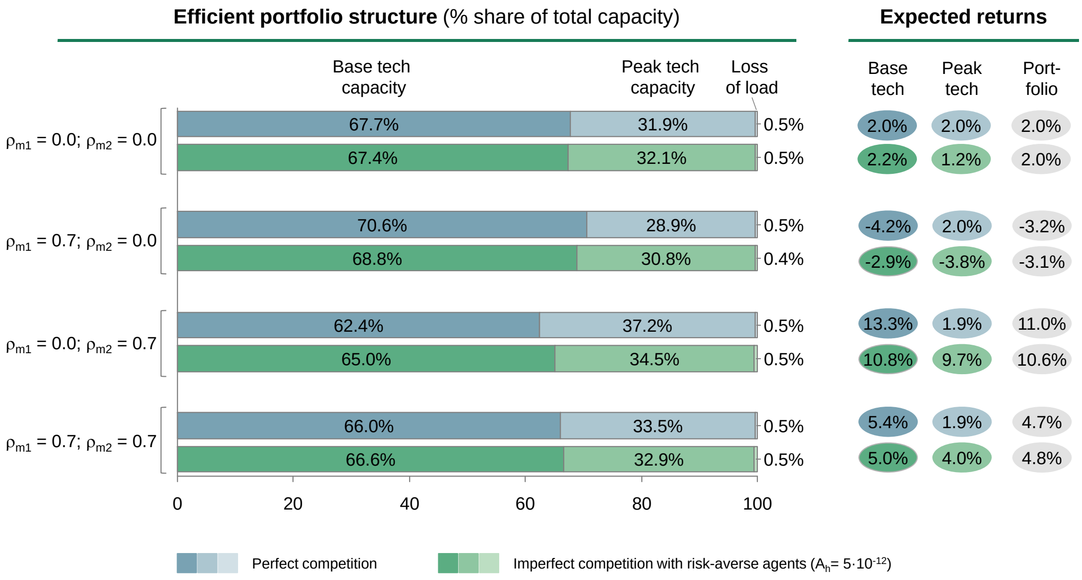


Fig. 5.5: Efficient portfolio technology mix (as shares of installed capacity) in the long-term market equilibrium under perfect competition (blue) and under imperfect competition with risk-averse firms (green) for varying levels of correlation between generation costs and market return, $\rho_{m,1}, \rho_{m,2}$. The risk aversion of the market agents is $A_j = A_h \cdot 5 \cdot 10^6$ at a managerial profit share of $\alpha_j = 0.001$ and societal risk aversion $A_h = 5 \cdot 10^{-12}$.

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surpluses. In a second step, the model is extended by introducing electricity market agents (i.e. companies) and market imperfections based on agency-theoretic considerations. By considering that market agents receive a certain profit share and are risk-averse, we focus on two—in our view crucial—imperfections which may be caused by lacking transparency in investment decisions.

In case of risk averse agents receiving a profit share, the long-term investment equilibrium can substantially deviate from the welfare efficient portfolio mix. This implies that the risk-preferred technology from a societal perspective (i.e. minimizing total cost risks) deviates from the risk-preferred technology from a company perspective (i.e. minimizing total cost and revenue risks).

For a better traceability of the results, we have conceded a couple of strong simplifications in the model. While these may open the need for further research, we are convinced that they do not bias our principal findings in a substantial way. Many of these simplifications refer to the design of the underlying peak-load pricing model and have been considered before by other authors: We consider only two technologies but most arguments can easily be transferred to the n-technology case (cf. Sunderkötter and Weber, 2012). In addition, we assume a deterministic merit order and exclude the possibility of a fuel switch in our calculations. Sunderkötter and Weber (2011) discuss this topic in detail and show that a fuel switch risk requires generation technologies with little difference in the mean operating costs. Furthermore, one may criticize that the assumption of a constant non-stochastic backstop technology is a too simple representation. Including stochastic shocks or a description of the backstop price as an increasing function of load may be suitable for making the model more realistic at this point. However, we expect that implementing these extensions would improve the quality of the numerical estimates but not lead to structural changes of key results of this article.

Chapter

6

Fuel mix characteristics and expected stock returns of European power companies

6.1 Introduction

Valuation of power generation assets under uncertainty represents one of the core issues for individuals and companies investing in power plants on liberalized electricity markets. Thereby, a thorough analysis of risk and return is particularly important due to three reasons: Firstly, new-build plants are capital intensive and require—depending on size and selected technology—up to billions of euros. Secondly, plants typically have long life cycles of 30 to 50 years resulting in long periods of tied-up capital. Thirdly, investors face cost-, price-, and volume-risks which directly impact the return on investment on liberalized markets.

Following Leahy and Whited (1996), one fundamental dimension to classify investment theories under uncertainty is the scope of considered assets. Thereby, it can be distinguished between theories that look at a firm or investor in isolation and capture the risk of some aspect of the firm's environment in total and theories that look at the firm or investor in relation to other market participants and emphasize the covariance in the returns of different investments. While in the first case the absolute value of a risk measure matters, uncertainty is only relevant in the second case as far it affects covariances with respect to some market measures. As the most prominent representative of the first class of models, mean-variance portfolio optimization based on the work of Markowitz (1952) applies variance of return as the relevant risk measure to derive an efficient frontier of asset combinations. Thereby, it is not distinguished between systematic and unsystematic asset fluctuations. In contrast, the CAPM (Sharpe, 1964, Lintner, 1965, Mossin, 1966) and other market models rely on the assumption that capital market investors will only value the systematic risk component of assets since firm-specific (i.e. unsystematic) risks can be eliminated through diversification and are thus irrelevant. Despite some ongoing controversy on the empirical validity of the CAPM, both approaches are widely used in academia and practice for asset pricing application and managerial decision support, although the models differ fundamentally in the treatment of unsystematic risk.

Investment decisions in the electricity industry bear the complexity that electricity can hardly be stored on a large-scale and thus needs to be instantly generated to serve demand. Taking into account this particularity, different authors have proposed optimization models for decision support tailored to long-term investment and portfolio management decisions on competitive electricity markets.

Different concepts to adapt mean-variance portfolio optimization to power plant investments under uncertainty from an investor perspective have been proposed in recent literature: Applying Monte-Carlo simulation, Roques et al. (2008) come to the result that portfolios with a high share of gas plants are most attractive in view of risk and return due to a high correlation of gas and electricity prices observable on many liberalized markets. Another set of publications uses partial equilibrium models to value the trade-off between risk and return in investment decisions on liberalized electricity markets: Chuang et al. (2001) present a model for generation expansion planning based on an equilibrium formulation in a Cournot oligopoly. In a setting with separate energy and capacity markets, the authors find greater reserve capacities and thus system reliability in Cournot competition than in centralized planning. Zöttl (2008) theoretically compares equilibrium fuel mixes and electricity prices in markets with centralized planning, perfect and imperfect competition. Botterud et al. (2003) use stochastic dynamic programming to identify an optimal generation investment strategy from a profit-oriented investor perspective. Different from equilibrium models, the spot price is empirically estimated as a function of load level and installed generation capacity. More complex market interactions with several market constraints can be simulated with agent-based models (e.g. Gnansounou et al., 2004).

Although these studies indicate that power generation technologies differ fundamentally in terms of risk as measured e.g. by the absolute variance of generation costs, there is so far no empirical evidence to support the same hypothesis for the systematic risk with respect to the overall market and/or other risk factors. In other words, the proposed models may be adequate to manage all kinds of risks inherent in generation asset portfolios, but it is questionable whether these risks are relevant at all for decisions of capital market investors.

Empirical studies on asset pricing and costs of equity in the utility industry and for power producers are very rare: Bower et al. (1984) investigate U.S. utility stocks over the period 1971–1979. The authors come to the conclusion that multi-factor models can better approximate expected returns of utility companies and should therefore be preferred to model risk compared to the CAPM. Extending this study, Bubnys (2005) cannot confirm the superiority of multi-factor models compared to the CAPM based on an analysis of 128 public utility companies over a longer period of time. Sadorsky (2001), Boyer and Filion (2007) present a multi-factor market model to estimate the expected returns of Canadian oil and gas industry stock prices. In recent years, alternative energy companies have become another focus of research in the field of empirical works related to asset pricing in the energy industry (see e.g. Henriques and Sadorsky, 2007). However, all those articles are neither focusing on power generation companies nor on liberalized markets. Furthermore, the systematic risk characteristics of different power generation technologies and their implications on the cost of equity of power generation companies have not been investigated before to our best knowledge. Due to the lack of empirical evidence on technology-specific risk factors of power plants, utilities and power producers typically still rely on valuation approaches based on weighted average costs of capital (WACC). These, however, may massively bias an investment decision as the average company risk is assumed also for a specific investment project.

In this paper, the systematic risk characteristics of different power generation technologies (i.e. hard coal, lignite, nuclear, natural gas, and renewables) and their impact on (individual) stock returns of the power generation companies are investigated using an approach based on the capital asset pricing model (CAPM) and multi-factor market models. The analysis involves 22 major power generation companies that are publicly listed at European stock exchanges, representing together the biggest European listed power generation firms. Thereby, one core question is whether different power generation technologies face significant differences in the systematic risk. Furthermore, this study aims to analyze the overall explanatory power of a technology-beta oriented market model.

The remainder of this paper is structured as follows: Section 6.2 provides an overview of the considered models and data. Empirical results for the estimated models focusing on the explanatory power of the models are discussed in section 6.3. The article concludes with section 6.4.

6.2 Models and data

6.2.1 Considered models

Following the well-known CAPM (Sharpe, 1964, Lintner, 1965, Mossin, 1966), the expected return of any asset i can be explained by the company-specific (market-)beta factor β_i and the expected excess return of the overall market performance by

$$\mathbf{E}[r_i] = r_f + \beta_i(\mathbf{E}[r_m] - r_f), \tag{6.1}$$

where r_i denotes the return of stock i and r_m the return of the market portfolio, and r_f the risk-free rate of return. OLS regression can be used to estimate from each asset return time series the average abnormal return over the expected return $\beta_i(r_{m,t} - r_{f,t})$, $\hat{\alpha}_i$, and $\hat{\beta}_i$ for each asset i from the equation

$$r^*_{i,t} = \hat{\alpha}_i + \hat{\beta}_i r^*_{m,t} + \epsilon_{i,t}. \tag{6.2}$$

Here, $r^*_{i,t} := r_{i,t} - r_{f,t}$ denotes the excess return over the risk-free rate for stock i and $\epsilon_{i,t}$ the error term. This yields the beta estimator $\hat{\beta}_i = \text{Cov}(r_m, r_i)/\text{Var}(r_i)$. The form of the model is identical with a standard one-factor model

$$r^*_{i,t} = \hat{\lambda}_{0,i} + \hat{\lambda}_{1,i} F_t + \epsilon_{i,t} \tag{6.3}$$

with the excess return of the market portfolio, $F_t \equiv r^*_m := r_{m,t} - r_{f,t}$ assumed as the only risk factor.

If the fuel mix of a power generation company has an impact on its systematic risk, then there will exist technology-specific beta factors β_u representing the systematic risk sensitivity of technology u in the market. Assuming the same capital structure for all companies, the technology betas are constant across the industry. Typically, the generation asset portfolio of a power producing company i consists of a mix of different generation technologies $u \in U$. Since in an arbitrage-free market the portfolio's beta must equal the weighted average of the constituent asset betas, the company-specific beta-factor β_i from Eqn. (6.1) can be decomposed into a weighted sum of technology-specific betas β_u yielding

$$\mathbf{E}[r_i] = \sum_{u \in U} w_{u,i} \cdot \mathbf{E}[r_{u,i}] = r_f + \sum_{u \in U} w_{u,i} \beta_u (\mathbf{E}[r_m] - r_f), \tag{6.4}$$

where $w_{u,i}$ denotes the value-based weighting factor of technology u in portfolio i with $\sum_{u \in U} w_{u,i} = 1$. Note that the technology beta β_u equals the company-specific beta in case of a "pure-play" power generator who operates only generation assets of technology u.

In fact, Eqn. (6.4) ignores differences in the companies' capital structure. However, "borrowing from whatever source, while maintaining a fixed amount of equity, increases the risk of the investor" (Hamada, 1972) and companies with higher debt-to-equity ratio (leverage) face a higher systematic risk in the equity since debt is not subject to market risk. To realistically compare the systematic risk across the sample and identify the technology-specific impact, the impact on estimated betas induced by differences in the capital structure of the analyzed firms needs thus to be removed. To correct for differences in the sample companies' leverage, delevered betas β_i^d are calculated from equity betas following Hamada (1972). Specifically, it is

$$\beta_i^d = \frac{\beta_i}{1 + D_i/E_i(1 - \tau_i)} \tag{6.5}$$

where τ_i is the corporate tax rate, and D_i and E_i denote the market value of debt and equity, respectively.

Assuming that portfolio weights $w_{u,i}$ are constant over time, unlevered technology betas can be obtained from a cross-sectional multiple least square regression of the form

$$\beta_i^d = \sum_{u \in U} w_{u,i} \hat{\beta}_u^d + \varepsilon_i \tag{6.6}$$

The selection of an adequate measure to determine weighting factors is crucial. Financial theory suggests to use weighting factors based on the actual market value of the respective asset in the portfolio with respect to the total portfolio market value. Since market values of power plants cannot be observed directly, one possibility would be to derive technology weights from actual installed capacities (in GW) times the average specific investment costs of the respective technologies (€/GW). Despite the difficulty of estimating time-constant specific investment costs for power plants of varying age, this methodology would ignore the technology characteristics of specific operational costs such as fuel costs and the resulting dispatch.

Following the peak-load pricing concept (see e.g. Oren et al., 1985), the electricity markets will reflect both fixed and variable costs in the (long-term) equilibrium. Operating hours of each technology can thus be determined from the full-cost characteristics of the different technologies. Therefore, actual electricity generation data (in TWh) should be a fairly good proxy for the relative market value of different technology classes. Actual production data from annual company reports is used to calculate the portfolio weights by dividing the electricity produced from one of five fuel type classes (hard coal/lignite, natural gas, nuclear, renewables and miscellaneous[1] technologies) by the total production of the respective year. Thereby, electricity purchases are not included. Since fuel type specific energy production data is not publicly reported by all companies for each year, time-constant average weighting factors are used for the periods 2005–2007 and 2008–2010 derived from the reported data.[2]

Estimation of technology-specific beta factors from the actual fuel mix of the considered companies proceeds along the lines proposed by Boquist and Moore (1983), Ehrhardt and Bhagwat (1991), Kaplan and Peterson (1998) for deriving full-information industry betas. Thereby, estimation of technology betas is performed in two steps: First, firm-specific beta factors β_i are estimated from an OLS time-series regression on historical returns 2005–2010 using Eqn. (6.2). In a second step, technology-specific betas β_u can be estimated from a multiple, cross-sectional regression based on Eqn. (6.6).

In addition to the one-factor models, the explanatory power of different multi-factor models (see e.g. Ross, 1976) of the form

$$r^*_{i,t} = \hat{\lambda}_{0,i} + \sum_{j=1}^{k} \hat{\lambda}_{i,j} F_{j,t} + \epsilon_{i,t}. \tag{6.7}$$

is investigated. Thereby, different energy-related risk factors are discussed and tested (cf. section 6.3.3). The full-information approach can be applied straightforwardly to derive technology-specific sensitivities for each risk factor.

6.2.2 Model tests

One classical approach for CAPM tests is based on cross-sectional analysis. The principle of these tests relies on the fact that given validity of the CAPM, average abnormal returns α_i must jointly equal zero. Adopting the well-known test of Fama and MacBeth (1973) with the regression equation

$$r^*_{i,t} = \gamma_{0,t} + \gamma_{1,t}\beta_i + \xi_{i,t}, \tag{6.8}$$

the hypothesis $\bar{\gamma}_0 = 0, \bar{\gamma}_1 > 0$ can be tested using the t-statistic $\gamma_j/\sigma_{\gamma_j}$. Thereby, the time-series averages are used as estimates of expected coefficient values, i.e. $\bar{\gamma}_j = \sum_{t=1}^{T} \hat{\gamma}_{j,t}, j = 1, 2$. However, this test requires the β_i to be known, whereas these coefficients need to be estimated in practice. Since the limited sample size in our application prevents from building sufficiently diversified asset

[1] The "miscellaneous" technology category includes reported generation from oil-fired plants, waste, combined heat and power, as well as generation from unreported sources and rounding differences.

[2] Since investments and divestments affect the fuel mix of the sample companies over time, just taking the average fuel mix over the total period could bias the results. Since, however, the annual changes remain marginal and are subject to reporting inaccuracies, distinguishing two sub-periods seems most suitable.

portfolios to circumvent this problem, time-series tests are used instead. Thereby, the finite sample GRS test proposed by Gibbons et al. (1989) is applied to test the hypothesis whether the estimated α_i are jointly zero. Under the assumption of normal, homoskedastic, and independent disturbances over time, the test statistic is given by

$$W = \hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \cdot \frac{T-N-1}{N} \cdot \left(1 + \left(\frac{\hat{\mu}_m^*}{\hat{\sigma}_m}\right)^2\right)^{-1} \sim F_{N,T-N-1}, \quad (6.9)$$

where $\hat{\alpha}$ denotes the N-dimensional vector of estimated intercepts, $\hat{\Sigma}$ the residual covariance matrix computed from the vector of residuals ϵ_t by $\hat{\Sigma} := \mathbf{E}[\epsilon_t \epsilon_t']$, and $\hat{\mu}_m^*$ and $\hat{\sigma}_m$ sample mean and standard deviation of the excess return $r_{m,t}^* := r_{m,t} - r_{f,t}$. An overview of this test is e.g. provided in Cochrane (2001).

To ensure time-consistent results and exclude potential biasing effects from a specific selection of the analysis period, the Chow (1960) test is used to test for equality of coefficients over time versus structural breaks within the time series. For that, the time series is split up in sub-periods a and b. The CAPM regressions are then performed both for the combined period and for each sub-period separately. Let S be the sum of squared residuals from the combined data, S^a be the sum of squared residuals from the first sub-period, and S^b be the sum of squared residuals from the second sub-period. Furthermore, T^a and T^b denote the number of observations in each group and k the total number of parameters. Then the Chow test statistic is

$$\frac{(S - S^a - S^b)/(k+1)}{(S^a + S^b)/(T^a + T^b - 2(k+1))} \sim F_{k+1,T-2(k+1)}, \quad (6.10)$$

Having determined technology-specific beta-factors from the multiple regression according to Eqn. (6.6), the question arises whether the obtained $\beta_u,\ u \in U$ are significantly different from each other. To provide evidence on this question, we can pairwise test for the following hypothesis:

$$H_0 : \beta_u = \beta_{u'},\ u, u' \in U, u \neq u' \quad \text{versus} \quad H_1 : \beta_u \neq \beta_{u'},\ u, u' \in U, u \neq u'$$

If the null hypothesis is correct, a reduced regression model with five (or less) independent variables should explain as much variance in the delevered company beta factor β^d as a the initial regression model with six independent variables. Testing for instance the null hypothesis that coal and gas technologies have equal beta coefficients, the initial regression model from Eqn. (6.6) would be restricted to

$$\beta_i^d = (w_{coal,i} + w_{gas,i})\hat{\beta}_{coal,gas}^d + \ldots + w_{renew,i}\hat{\beta}_{renew}^d + \varepsilon_i$$

The pairwise hypotheses can be tested by comparing the unrestricted model with six independent variables with a restricted model with a reduced number of variables in an F-test with the following test statistic (cf. Greene and Zhang, 2003):

$$\frac{(R_u^2 - R_r^2)}{(1 - R_u^2)/(N-1-1)} \sim F_{1,N-2}, \quad (6.11)$$

where R_u^2 and R_r^2 denote the coefficient of determination of the unrestricted and the restricted model, respectively.

6.2.3 The data

The following analyses are based on monthly returns of the 22 biggest power generation companies listed at different European stock exchanges in the period 2005–2010.[3] Although most empirical stock return studies are based on longer analysis periods, we abstained from a longer time frame for two reasons: Firstly, electricity market liberalization started in most European countries in the late 1990s, triggered by the EU Directive 96/92/EC. While continental European countries had opened their electricity markets on average to less than 25% in 1999, the value increased to more than 75% in 2005 (Haas et al., 2006). Although the intensity of electricity market competition across European countries still varies, European electricity wholesale markets have reached sufficient comparability in the fundamental competitive structures since the mid-decade. Secondly, it has to be ensured that markets had fully absorbed all consequences of the Enron bankruptcy from 2001/2002 which were likely to disturb asset pricing in the whole energy sector for years.

In total, the considered time period yields $T = 72$ observations of monthly returns for each of the considered firms.[4] The considered firms exhibit an annual average production from owned assets ranging from 18 to 621 TWh at an installed capacity between 4 and 131 GW. Annual electricity production data of the sample companies was then systematically categorized into five technology classes (gas, coal/lignite, nuclear, renewable, and miscellaneous) as illustrated in Figure 6.1. The companies' generation portfolios differ widely in the technology mix: Only two companies (Drax Group and Iberdrola Renowables) show "pure-play" generation portfolios consisting solely of hard coal respectively renewable generation technologies. A detailed description of the sample companies and corresponding operational and financial key data can be found in D.1.

For the classical CAPM specification, historical one-week Euribor rates provided by DB (2011) are used for the risk-free rate r_f of return and the Dow Jones Euro Stoxx Utility index (ECB, 2011) to represent the return of the relevant market portfolio r_m. Following and extending the work of Sadorsky (2001), the impact of potential risk factors such as commodity prices and economic sentiment indicators are investigated in one-factor and multi-factor models in addition to the classical CAPM specification. Where available, futures prices are used rather than spot prices since spot prices are more affected by short-run price fluctuations due to temporary market imbalances. All considered risk factors in this study are measured by excess returns r^* in monthly granularity as shown in Table 6.1. The relative development of all risk factor returns is depicted in Figure 6.2.

[3] Note that only companies with stock price data available in at least four years within the period 2005–2010 are included in the analysis. Companies that were de-listed during the period due to takeovers or mergers are also excluded.

[4] Shortened time series were accepted for Edf (listed since November 2005), Drax Power (listed since December 2005), and Iberdrola Renowables (listed since January 2008).

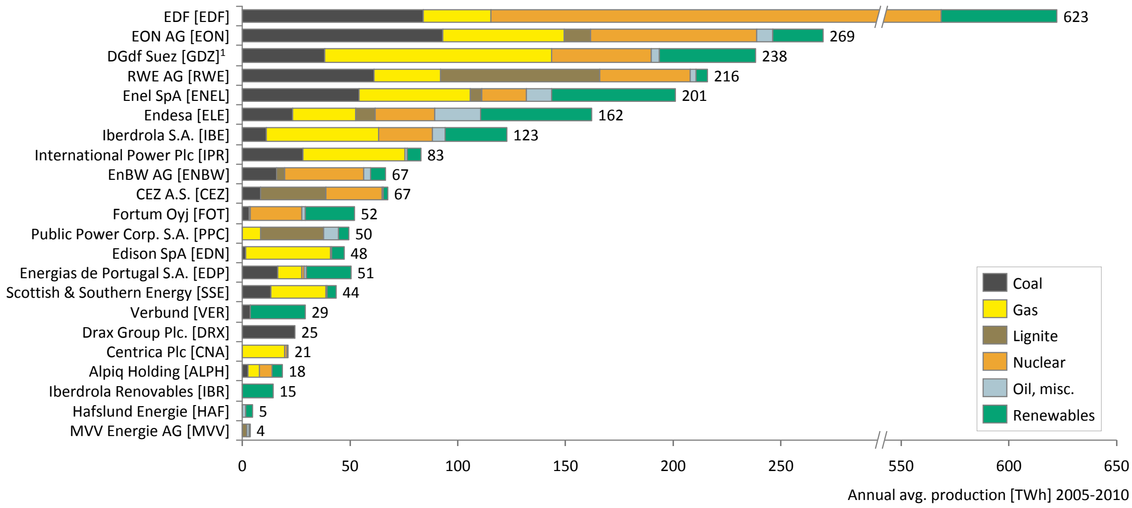

Fig. 6.1: Annual average power production and generation fuel mix of the sample companies 2005–2010

Tab. 6.1: Captured risk factors and corresponding data sources in this study.

Symbol	Description	Source
r^*_m	Market portfolio return	Dow Jones Euro Stoxx Utility index (ECB, 2011)
r^*_{eua}	Carbon price return	EUA front year futures (EEX, 2011)
r^*_{el}	Electricity price return	EEX Phelix year-ahead base electricity futures (EEX, 2011)
r^*_{es}	Economic sentiment index	ifo German Business Climate index (IFO, 2011)
r^*_{oil}	Oil price return	WTI crude oil futures, four months to delivery (EIA, 2011)
r^*_{gas}	Gas price return	German cross-border gas import prices (BAFA, 2010)

6.3 Empirical results

This section provides the estimation results and corresponding tests on the explanatory power of the described models. Since the estimation of technology betas is performed in two steps, results for the standard CAPM with estimation of firm-specific beta factors β_i are presented first in section 6.3.1. Subsequently, technology-specific betas estimates β_u in the CAPM framework are provided in section 6.3.2. Similarly, firm-specific results of different multi-factor market model specifications are analyzed in section 6.3.3, before the implied technology characteristics are discussed in section 6.3.4.

6.3.1 Firm characteristics in the one-factor models

Table 6.2 provides sample means of the estimated regression coefficients and coefficients of determination R^2 for the CAPM and other one-factor models.[5]The standard CAPM provides the best model fit measured by the coefficient of determination at an average R^2 of 0.22, indicating

[5]Detailed regression results and coefficient estimates are provided in D.2.

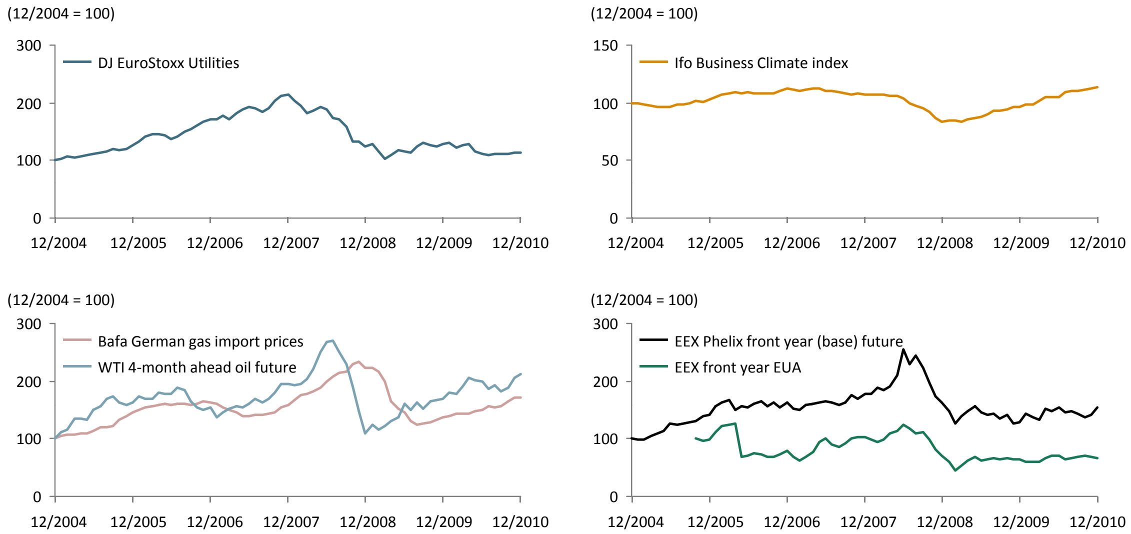

Fig. 6.2: Relative development of DJ Euro Stoxx Utility index (ECB, 2011), IFO Business climate index for Germany (IFO, 2011), WTI oil futures four months to delivery (EIA, 2011), German gas import prices BAFA (2010), EEX Phelix year-ahead base and EUA year futures (EEX, 2011).

that about 22% of the variation in returns of the considered energy companies may be explained through variations of the market portfolio represented through the DJ Euro Stoxx Utility index. The characteristics of the estimated coefficients from single time series support this hypothesis: For the standard CAPM, all estimated coefficients β_i are significantly greater than zero even beyond a 5% level of significance. In contrast, in all other one-factor models a non-zero number of stocks have β_i estimates which are not significantly different from zero. In particular, applying gas import price returns and the economic sentiment indicator as regressors yield 6 respectively 7 insignificant coefficients (cf. Tab. D.8 and Tab. D.6).

Tab. 6.2: Summary results for 2005–2010 univariate OLS regressions with varying risk factors. For each considered risk factor, the table provides sample averages of coefficient estimates α_i, β_i, standard errors are provided in parenthesis.

	$\bar{\alpha}_i$		$\bar{\beta}_i$		$\bar{R}_i^2$
r_m^*	0.002	(0.008)	0.780	(0.183)	0.22
r_{eua}^*	0.001	(0.010)	0.205	(0.090)	0.11
r_{el}^*	0.002	(0.009)	0.238	(0.134)	0.07
r_{es}^*	0.003	(0.009)	0.910	(0.501)	0.05
r_{oil}^*	0.002	(0.009)	0.132	(0.116)	0.04
r_{gas}^*	0.005	(0.009)	-0.335	(0.227)	0.04

The standard CAPM specification is tested for structural breaks within the analysis period. For that, two additional OLS regressions are performed covering the sub-periods 2005–2007 and 2008–2010. The coefficient estimates are tested for significant cross-periodic differences applying the test procedure proposed by Chow (1960).[6] The test results indicate only for 2 of 22 companies a structural break at a significance level of 10% as shown in D.2, Table D.3. Thus, we continue the majority of the following analyses for the combined period 2005–2010.

As described in the previous section, the specified CAPM is tested on abnormal returns using the GRS methodology. In consistency with the CAPM assumptions, the null hypothesis of jointly zero abnormal returns, i.e. $\alpha_i = 0$ $(i = 1, \ldots, N)$, cannot be rejected given a test statistic of $W = 1.06$ (p-value 0.418) for the full analysis period 2005–2010. For the sub-periods, the GRS tests confirm this result with $W = 0.525$ (p-value 0.911) for the sub-period 2005–2007 and $W = 0.281$ (p-value 0.996) for the sub-period 2008–2010.

6.3.2 Technology characteristics in the CAPM

Before the technology-specific beta factors are derived from the multiple regression, the data set is assessed for multicollinearity by computing the correlation matrix of the technology weighting factors $w_{u,i}$. This is important as neglecting multicollinearity in the multiple regression could yield misleading and erratic results. As shown in Table 6.3, the absolute coefficients of correlation of the pairwise analysis of weighting factors are consistently less than 0.5 and thus do not indicate multicollinearity.

Tab. 6.3: Pairwise coefficients of correlation of the technology weighting factors $w_{u,i}$ during the period 2005–2010.

R	w_{gas}	w_{coal}	w_{nuc}	w_{misc}	w_{renew}
w_{gas}	1.00	-0.45	-0.48	-0.17	-0.28
w_{coal}		1.00	-0.19	0.07	-0.41
w_{nuc}			1.00	-0.10	-0.13
w_{misc}				1.00	0.04
w_{renew}					1.00

Having estimated the firm-specific beta factors β_i from the historical returns, corporate tax rates 2008 from KPMG (2008) and 2008 debt and equity data from Bloomberg are used to calculate delevered company betas β_i^d. Next, a second pass cross-sectional OLS regression without constant according to Eqn. (6.6) is applied to estimate delevered technology-specific betas β_u^d. Coefficient estimates and standard errors are provided in Table 6.4. For the total analysis period 2005–2010 all coefficient estimates except for the miscellaneous technology class are greater than zero at a 5% level of significance at minimum. This is generally confirmed when considering the two sub-periods separately.

Note that the firm-specific β_i^d which are used as the left hand-side of the second pass regression

[6] Application of the Chow test requires knowledge about the timing of a structural break. Although there is no evidence on a structural break end of 2007, we abstain from testing for other potential timing of structural breaks due to the limited length of the considered analysis period.

Tab. 6.4: Delevered technology-specific coefficient estimates $\hat{\beta}^d_u$, standard errors, and adjusted coefficient of determination R^2_{adj} from multiple OLS regression. Dependent variable is β_i. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

	Period (2005–2010)		Period I (2005–2007)		Period II (2008-2010)	
$\hat{\beta}^d_{gas}$	0.262**	(0.204)	0.094	(0.129)	0.301**	(0.109)
$\hat{\beta}^d_{coal}$	0.367***	(0.230)	0.337**	(0.136)	0.337**	(0.123)
$\hat{\beta}^d_{nuc}$	0.431**	(0.299)	0.555***	(0.186)	0.28**	(0.158)
$\hat{\beta}^d_{misc}$	0.112	(1.049)	0.248	(0.538)	-0.27**	(0.436)
$\hat{\beta}^d_{renew}$	0.645***	(0.227)	0.897***	(0.194)	0.587***	(0.112)
R^2_{adj}	0.860	(0.166)	0.825	(0.204)	0.810	(0.171)

(see Eqn. (6.6)) are subject to estimation errors. Given that beta factors are constant over time, this measurement error in beta declines as the time-series sample size, T, increases. Since the measurement errors occur only in the dependent variable while weighting factors $w_{u,i}$ as independent variables are without errors, the standard OLS regression model accounts sufficiently for errors in β_i.

The coefficient of determination of $R^2_{adj} = 0.86$ indicates a very high explanatory power for the second pass regression. This is particularly remarkable taking into account that most of the considered companies are not pure power generators but companies with other utility-related business activities such as sales, trading, transmission, and distribution. Over the total period 2005–2010, all coefficients except β_{misc} are significantly greater than zero, with the beta factors of coal and nuclear technologies showing significance even at the 1% level. Renewables (i.e. predominantly hydro) exhibit absolutely the highest beta factors while gas technologies show the lowest β-sensitivity to the market portfolio. Hypothetically, this might be due to the fact that hydro technologies face only little volatility in generation costs. Consequently, the operational margin of these technologies highly depends on the electricity price which is expected to be cointegrated or even correlated with the overall market performance. The lower gas technology betas might be explained following the argumentation of Roques et al. (2008): Caused by a high correlation of earnings from electricity sales and costs from gas purchases, the resulting operational margin of gas technologies may be expected to be less volatile compared to nuclear or hydro generation, justifying a lower beta factor. These first hypotheses and potential explanations will be further investigated and tested in the following sections using different multi-factor market models.

As discussed before, we test for equality of technology-specific beta coefficients by applying the pairwise F-test as discussed in Section 6.2.2. As shown in Table 6.5, the null hypothesis of equal beta coefficient can be rejected at a confidence level of at least 5% for gas and renewables as well as for coal and renewable technologies based on the analysis of the combined period and period I. In period II, the null hypothesis can only be rejected at a confidence level of 10% for gas and renewables technologies. This supports the hypothesis that the unlevered market beta factor of renewable technologies is significantly higher compared to the betas of gas and coal technology classes.

Tab. 6.5: Pairwise test on equality of technology-specific beta coefficients. The table provides the F-distributed test statistics applied on unlevered betas according to Eqn. (6.11). Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

		$\hat{\beta}^d_{coal}$	$\hat{\beta}^d_{nuc}$	$\hat{\beta}^d_{misc}$	$\hat{\beta}^d_{renew}$
Combined period (2005–2010)	$\hat{\beta}^d_{gas}$	0.414	0.976	0.168	4.812**
	$\hat{\beta}^d_{coal}$		0.140	0.439	3.216*
	$\hat{\beta}^d_{nuc}$			0.611	1.531
	$\hat{\beta}^d_{misc}$				4.959**
Period I (2005–2007)	$\hat{\beta}^d_{gas}$	1.529	4.057*	0.111	7.718**
	$\hat{\beta}^d_{coal}$		0.751	0.000	5.194**
	$\hat{\beta}^d_{nuc}$			0.387	1.574
	$\hat{\beta}^d_{misc}$				1.173
Period II (2008–2010)	$\hat{\beta}^d_{gas}$	0.099	0.026	1.747	3.046*
	$\hat{\beta}^d_{coal}$		0.101	1.637	2.621
	$\hat{\beta}^d_{nuc}$			1.636	2.321
	$\hat{\beta}^d_{misc}$				3.061*

6.3.3 Firm characteristics in the multi-factor models

Based on the insights gained from the analysis of the one-factor models, the explanatory power of a combination of risk factors with respect to stock returns and technology characteristics of power generation companies is investigated in multi-factor models. Thereby, we consider the same risk factors as in the univariate analysis. Multi-factor models, however, bear the risk that risk factors are not significantly different from zero or exhibit linear interdependencies among each other. To assess this risk of multicollinearity, the coefficient of correlation for each pair of risk factors is analyzed as shown in Table 6.6. With a maximum coefficient of correlation of $R = 0.45$ for the cross-correlation in returns between returns of EEX electricity futures and EEX EUA future prices, a risk for multicollinearity among the risk factors can be neglected.

The selection of risk factors for an optimal multi-factor specification is performed consistently for all sample companies $i = 1, \ldots, N$ by backward selection starting with the complete 6-factor market model. The risk factor with the lowest F-stat over the sample average is removed for the next regression unless it is significantly different from zero at the 10%-level at minimum. Following this rule, all multi-factor models have to be rejected since only the market return shows significant difference from zero over the sample average. Even in the 2-factor model, only the sensitivity

Tab. 6.6: Pairwise coefficients of correlation in returns of the market portfolio r_m^*, electricity price r_{el}, oil price, r_{oil}, economic sentiment r_{es}, EUA price r_{eua}, and clean spark spread r_{sp}.

R	r_m^*	r_{eua}^*	r_{el}^*	r_{es}^*	r_{oil}^*	r_{gas}^*
r_m^*	1.00	0.29	0.29	0.42	0.31	-0.11
r_{eua}^*		1.00	0.45	0.21	0.35	-0.21
r_{el}^*			1.00	0.18	0.43	-0.02
r_{es}^*				1.00	0.41	-0.12
r_{oil}^*					1.00	-0.02
r_{gas}^*						1.00

factors for 10 out of 22 companies are significantly different from zero with respect to the emission certificate price return (see Table D.9).

All multi-factor model specifications show an improved explanatory power compared to the CAPM measured by the adjusted coefficient of determination[7] $\bar{R}^2_{adj}$ as shown in Table 6.7.

Tab. 6.7: Comparison of sample averages of adjusted coefficients of determinations for the considered one-factor and multi-factor model specifications.

$\bar{R}^2_{adj,i}$	r_m^*	r_{eua}^*	r_{el}^*	r_{es}^*	r_{oil}^*	r_{gas}^*
1-factor models	0.21	0.10	0.19	0.04	0.03	0.03
2-factor model	- - - 0.27 - - -					
3-factor model	- - - - - - - 0.3 - - - - - - -					
4-factor model	- - - - - - - - - - 0.31 - - - - - - - - - -					
5-factor model	- - - - - - - - - - - - - - 0.25 - - - - - - - - - - - - - -					
6-factor model	- - - - - - - - - - - - - - - - - - 0.36 - - - - - - - - - - - - - - - - - -					

6.3.4 Technology characteristics in the multi-factor models

Again, multiple, cross-sectional OLS regressions according to Eqn. (6.6) are performed to determine technology-specific sensitivity factors. Thereby, the analysis is limited to the 2-factor model specification with market return and emission certificate price return representing the regressors, since the other risk factors yield non-significant coefficients for even more companies.

In this two-factor analysis, renewable (i.e. mostly hydro), gas, and coal technology classes exhibit in the second pass regression sensitivities significantly greater than zero with respect to the market portfolio (cf. Table 6.8). As in the one-factor model specification, we can again observe significant

[7] The adjusted coefficient of determination is computed by $R^2_{adj,i} = 1 - (1 - R_i^2)\frac{T-1}{T-K-1}$, with K denoting the number of applied risk factors (without constant).

Tab. 6.8: Unlevered technology-specific coefficient estimates $\hat{\lambda}^d_m$, $\hat{\lambda}^d_{eua}$ and standard errors from multiple OLS regressions corresponding to the two-factor model using market return and EUA future price return as relevant regressors. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

	$\hat{\lambda}^d_{m,i}$		$\hat{\lambda}^d_{eua,i}$	
$\hat{\lambda}^d_{\cdot,gas}$	0.274***	(0.094)	0.004	(0.046)
$\hat{\lambda}^d_{\cdot,coal}$	0.267**	(0.104)	0.075	(0.051)
$\hat{\lambda}^d_{\cdot,nuc}$	0.216	(0.136)	0.14*	(0.067)
$\hat{\lambda}^d_{\cdot,misc}$	0.033	(0.375)	-0.06	(0.184)
$\hat{\lambda}^d_{\cdot,renew}$	0.516***	(0.100)	0.068	(0.049)

differences between the market risk factor coefficients for gas-renewable and nuclear-renewable technology combinations. In contrast, most technology classes do not show significant sensitivities with respect to the EUA price returns: Only the nuclear technology shows a sensitivity coefficient which satisfies a weak confidence level of 10%. This could be the fact that high certificate prices lead to high electricity prices and therewith to higher profits of the nuclear technology while the EUA price represents for all other thermal technologies also a cost factor.

6.4 Concluding remarks

This article investigates the systematic risk of European power generation companies relative to the overall market using an approach based on the CAPM and multi-factor market models. The analysis of historical stock returns of 22 European power companies over the period 2005–2010 supports the validity of the CAPM with respect to the sample. However, the explanatory power of the standard CAPM can be improved by including other energy-related and macroeconomic measures such as EUA prices, power prices, oil prices, gas prices, or an economic sentiment indicator as additional risk factors in multivariate model specifications.

A simple but powerful estimation approach is examined to derive technology-specific beta factors for the standard CAPM. The results indicate that gas and coal technologies face significantly lower beta factors with respect to the market portfolio as renewable (dominated by hydro) technologies, which show the highest market portfolio sensitivity across the sample. While the proposed model specification can be transferred to various markets, one should note that the estimated technology betas refer only to the European liberalized electricity markets. For companies with operational focus in non-liberalized electricity systems or in markets that differ in basic regulatory setting, technology betas may vary substantially. Therewith, this paper contributes an easy-applicable valuation approach which may be used in practice both for single power plant valuations as well as for portfolio considerations.

Chapter 7

Concluding evaluation

7.1 Review of results and conclusions

The four papers presented in this thesis analyze the economics of capacity allocation and investment problems under uncertainty focusing on power generation assets in electricity markets. The papers serve the primary objective of this thesis to improve the understanding on how the portfolio selection of different generation technologies affects the trade-off between financial risk and return in power generation portfolios. As a second objective, the proposed models aim to provide analytical decision support for both electricity market investors and political decision makers to determine the efficient fuel mix in power generation portfolios with respect to financial risk and return. Thereby, the exact numbers in the presented applications have primarily illustrative character and are to be considered as exemplary case studies. However, the models are sufficiently generic to be easily transferred to the context of other electricity markets and investment decisions.

By reviewing existing work and developing novel theory and applications based on a broad methodological spectrum with respect to specific aspects of investment and portfolio optimization problems, the papers in this thesis contribute to different areas of research within the field of energy economics. Thereby, the connecting element between all papers is the central question of the efficient power generation fuel mix from a societal and investors' perspective.

The study from Chapter 3 has analyzed the impact of fuel mix diversification on the long-term optimum of electricity generation portfolios. By integrating Mean-Variance Portfolio theory into a classic peak-load pricing framework, conditions for efficient capacity allocation in power generation portfolios have been derived and analyzed.

Applied to the German electricity market, the model provides evidence on the following insights: First, the calculations show that fuel mix diversification can considerably influence the total standard deviation of generation costs by more than 10%. However, the exemplary results for the German generation mix have also demonstrated that fuel-mix diversification does not provide reduced risk characteristics per se. Blind diversification without consideration of technology costs and price risks as well as the correlation of risks may even be counterproductive. Second, with respect to the current debate on security of supply, the results indicate that increasing risk aversion implies a higher share of lignite and nuclear generation in efficient portfolios and conversely a decrease of gas-fired generation. Consistent with the results of Fan et al. (2010), the optimal fuel mix shows independently from the risk attitude a high sensitivity to the price and/or the allocation method of CO_2 emission rights. Third, the results indicate that with full auctioning of CO_2, efficient portfolios at historically observed CO_2 price levels consist of more nuclear and lignite and less coal-fired generation compared to the current fuel mix. If nuclear and lignite capacities are reduced or fixed at the current level, hard coal is the most economical technology instead.

Building on the insights gained from Chapter 3, Chapter 4 has investigated with special emphasis the impact of merit order risks due to fluctuations and long-term shifts of fuel prices. It has been shown that an increasing degree of diversification in technology portfolios is efficient for increasing merit order risks—even under risk-neutral preferences. The merit order risk is of particular importance if the considered technologies are characterized by only small differences in their mean costs and high, uncorrelated cost variances and for difference time series of operating costs of two technologies without mean reverting behavior. Given a substantial likelihood for reversals in the merit order, even overcapacities exceeding maximum demand can become economically favorable if investment costs are sufficiently low compared to the expected difference in operating costs. However, these excess capacities do rarely occur for typical parametrizations of thermal technologies and the efficient amount of reserve capacity in electricity systems is hence much more determined by other factors not considered in this article, such as demand uncertainty and risk of operating defaults.

The results show that the main driver for the merit order risk is the underlying time series of differences in operating costs of the considered technologies. Thereby, the efficient portfolio structure with merit order risks given a mean reverting difference time series of operating costs is very close to the efficient portfolio structure with neglected merit order risk. In contrast, the efficient technology mix is strongly affected by a random walk proporty of the difference time series of operating costs.

Although the provided model applications both of Chapters 3 and 4 focus on the German electricity market, the general model frameworks and several of the obtained insights are also relevant for investment decisions in other energy markets and even in other industries where different technologies may be selected to serve an (expected) demand pattern, such as e.g. in transportation applications.

The optimality conditions from Chapters 3 and 4 for welfare efficient power generation portfolios rely on a central planning perspective. In Chapter 5, these results are compared to the competitive market equilibrium based on decentralized investment decisions of electricity generation companies which are considered as representative market agents. Thereby, the market equilibrium under perfect competition matches welfare optimal results. The picture changes with market imperfections induced by agency theoretic causes: Both the risk attitude and the profit share of the electricity market companies influence the technology structure in the competitive market equilibrium: While the competitive market equilibrium is congruent with the welfare optimal solution given risk-neutral market agents and vanishing managerial profit shares, the market equilibrium given risk-averse agents yields considerably higher shares of the peakload technology. With managerial profit shares substantially greater than zero, the preference for the peakload technology increases strongly. The efficient share of the baseload technology is by far the highest in the decentralized market equilibria with risk-averse companies. Given the possibility of financial diversification at the capital markets, investment in the peak load technology equals zero. In consistency with previous discussions in literature (cf. e.g. Roques et al., 2008), the model shows that in the long-term market equilibrium risk-averse firms would clearly invest into higher shares of peak technologies—even more if the companies receive a substantial profit share. With empirical studies supporting the hypothesis that agents' behavior is risk-averse in management decisions, it is likely that in real-world settings market equilibrium and welfare optimum do not coincide.

It can be shown that the mean variance based decision models are consistent with the well-known postulations of the CAPM given the existence of a perfect capital market and investors with homogeneous expectations on the available assets (cf. Section 7.2.2). Hence, the question arises from an investor perspective whether there are significant differences in the systematic risk of different power generation technologies. To answer this question, Chapter 6 leaves the mean-variance based framework and applies the CAPM and other multi-factor models for the analysis of technology-specific differences in power generation portfolios.

For that, the study empirically investigates the systematic risk of European power generation companies relative to the overall market using an approach based on the CAPM and multi-factor market models. The analysis of historical stock returns of 22 European power companies over the period 2005–2010 supports the validity of the CAPM with respect to the sample. However, the explanatory power of the standard CAPM can be improved by including other energy-related and macroeconomic measures such as EUA prices, power prices, oil prices, gas prices, or an economic sentiment indicator as additional risk factors in multivariate model specifications. A simple but powerful estimation approach is examined to derive technology-specific beta factors for the standard CAPM. The results indicate that gas and coal technologies face significantly lower beta factors with respect to the market portfolio renewable (dominated by hydro) technologies, which show the highest market portfolio sensitivity across the sample. It is thus shown that a valuation of power plant projects based on non-differentiated weighted average cost of capital (WACC)—in practice frequently applied due to its simplicity—leads to biased investment decisions.

These insights are relevant both for financial investors with regard to their risk-adjusted valuation of power generation companies which may vary in the structure of their generation portfolios and for power generation companies itself with regard to the valuation of new power generation projects.

7.2 Practical applicability of mean-variance decision models

Compared to other (scenario based) stochastic models, the mean-variance based decision models presented in this thesis exhibit a relatively high modeling simplicity. This is due to two main assumptions: Firstly, the assumption of normally distributed payoffs which allow to fully describe the distribution of random variables by their mean and variance. Secondly, the reduction of the problem to a one-period model reduces the complexity massively. This allows to study analytically mean-variance efficient portfolios and serve the main objective of this thesis, i.e. contributing to a better understanding of risk-return trade-offs associated with power plant investments.

Besides simplicity, the applicability of the economic models presented in Chapters 3-4 in practice depends on three crucial criteria which will be discussed in the following sections:

- Straightforward and unambiguous model parametrization—most critically the risk aversion parameter A
- Consistency with other well-accepted decision models such as the CAPM
- Acceptable model limitations

7.2.1 Model parametrization and estimation of the absolute level of risk aversion

To apply the mean-variance models proposed in Chapters 3-5 in real-world portfolio planning problems, a correct parametrization of the models is crucial. In most cases, the parametrization will be done based on historical data. Although the discussed estimation procedures are straightforward for all of the proposed models, using historical data implies two inherent dilemmas: Firstly, the general validity of using historical data as input for future-oriented decisions remains questionable but is a necessary evil as there is no better data available. However, risk managers should not solely rely on historical data but also reflect sensitivities to switches in the market regime which possibly have not been observed before. Secondly, practitioners will still face a general lack of sufficiently long and consistent energy market data, especially with regard to data from younger energy markets as e.g. in Europe. However the usage of shorter time series data implies typically ambiguity of results and reduces the robustness of the model.

One of the most crucial model parameters to be estimated throughout Chapters 3-5 is the level of risk aversion, A. Recap the preference functional Ψ with $\tilde{r}_i$ denoting the return on investment on the capital invested x_i

$$\max \Psi = \mathbf{E}[\tilde{r}_i \cdot x_i] - \frac{A}{2} \cdot \mathrm{Var}[\tilde{r}_i \cdot x_i], \tag{7.1}$$

If there is a unique *market price of risk*[1] observable in the market, this implies the societal risk aversion in the economy. For instance, a zero market price of risk implies risk neutrality. However,

[1] In financial literature, the *market price of risk* is mostly defined within the CAPM framework as the expected excess return that is required in the market for each unit of risk held in the equilibrium. It represents the slope of the *security market line* (cf. Sharpe (1964)).

the risk aversion parameter in the mean-variance preference functional, A, is scale-variant and depends on the absolute amount of capital invested, x_i.

Given a capital market with a risk-free asset and a market portfolio, the market price of risk and the implied absolute market risk aversion can be determined straightforwardly: For the market to be arbitrage free, it can be concluded that the utility from capital x invested into the risky market asset with expected return $\bar{r}_m$ and variance σ_m equals the utility of an investment into a risk-free asset with return r_0. With other words, the risk-adjusted return of the risky market asset equals the risk-free return. Under mean-variance preferences, the following equation must thus be satisfied

$$\Psi(r_m \cdot x_i) = \mathbf{E}[\tilde{r}_m] \cdot x_i - \frac{A \cdot x_i^2}{2} \mathrm{Var}[\tilde{r}_m] = r_0 \cdot x_i \tag{7.2}$$

Solving (7.2) for A yields then absolute risk aversion (ARA) in the mean-variance preference functional with respect to the capital invested for the capital one unit of the market portfolio:

$$A = \frac{2(\mathbf{E}[\tilde{r}_m] - r_0)}{x_i \cdot \mathrm{Var}[\tilde{r}_m]} \tag{7.3}$$

In fact, it becomes obvious from the last equation that $A(x_i)$ is a function of the capital invested. Conversely, a constant absolute risk aversion A would imply an linearly increasing relative risk aversion (RRA), since the variance of return increases by the second order while the expected value increases linearly in x_i.

The application of the scale-variant risk aversion $A(x_i)$ is, however, is a severe weakness of the mean-variance preference approach, since the capital invested is in general not known ex-ante for portfolio applications as discussed in this thesis. Hence, an iterative approach is advisable to determine $A(x_i)$ these cases:

1. First, the capital investment x_0^r, x_0^v is determined for the purely return maximal portfolio (i.e. $A = 0$), respectively for the purely variance minimal portfolio (i.e. $A \to \infty$).

2. The corresponding absolute market price of risk $A(x_0^r)$, and $A(x_0^v)$ is computed according to Eqn. (7.3)

3. The optimal investment x_1^r, x_1^v is determined for the efficient portfolio for $A(x_0^r)$, respectively for $A(x_0^v)$.

4. Steps 2 and 3 are repeated iteratively, until the stopping criterion $|x_u^r - x_u^v| \leq \alpha, \alpha \in \mathbb{R}$ is satisfied after the uth iteration.

An numerical estimation of the societal risk aversion for the German market is provided in Section 5.5.1.

7.2.2 Comparison of mean-variance based decision models and the CAPM

The mean-variance based approach is not a stand-alone concept but is closely related to other well-known concepts from financial risk theory. Thereby, the capital asset pricing model (CAPM

Sharpe, 1964, Lintner, 1965, Mossin, 1966) represents one of the most widely used concepts for valuation of risky assets in practice. In this section we will discuss the similarities and differences of the mean-variance optimization discussed in the previous section and risk-return optimization based on the classic CAPM framework and show how the two modeling approaches are interrelated.

Same as mean-variance models which are based on the preference functional (7.1), the CAPM builds on the fundamental assumption of risk-averse investors who apply the mean variance principle for their decisions (cf. Definition 2.2.2). Furthermore, the CAPM imposes a set of additional assumptions on the capital market:

- It exists an efficient (i.e. complete and frictionless) capital market with a risk-free rate of return r_0 at which investors may borrow or lend unlimited amounts.
- All investors have homogeneous expectations on the distribution of returns of all traded asset.

Following the CAPM, investors on efficient capital markets value their investment decisions solely based on the ratio of expected return and *systematic* (i.e. market-specific) risk of an investment. The *unsystematic* (i.e. firm-specific) risk component can be eliminated through diversification and is thus irrelevant for the investment decision. Given arbitrage-free markets, the *security market line* (SML) says that the expected rate of return of an individual asset i is a linear function its systematic, non-diversifiable risk (i.e. its beta):

$$\mathbf{E}[r_i] = r_0 + \beta_i(\mathbf{E}[\tilde{r}_m] - r_0), \tag{7.4}$$

where $\beta_i := \mathrm{Cov}(r_m, r_i)/\mathrm{Var}(r_i)$ denotes the beta factor of asset i, r_m the return of the market portfolio, and r_0 the risk-free rate of return.

Different approaches have been proposed in literature to derive the SML: Sharpe (1964) considers an arbitrary portfolio p as a combination of a risky asset i and the market portfolio m and concludes that in the market equilibrium there is a unique ratio, γ, of the marginal return contribution to the marginal risk contribution for any asset i in the market portfolio with $\gamma = (\mathbf{E}[\tilde{r}_m] - r_0)/\sigma_m$. In contrast, Lintner (1965) derives the SML from the optimality conditions of the efficient portfolio selection problem.

Similarly to the latter approach, the SML can also be obtained from a mean-variance preference functional given the existence of an efficient capital market: In an efficient and arbitrage-free market, the value of the preference functional for a diversified portfolio p must equal the return of the risk-free rate, i.e. $\Psi(r_p) = \Psi(r_0)$ which is equivalent

$$\mathbf{E}[\tilde{r}_p] - \frac{A(r_m)}{2}\mathrm{Var}[\tilde{r}_p] = r_0 \tag{7.5}$$

Inserting the risk aversion $A(r_m)$ that is implied by the market price of risk as derived in Eqn. (7.3) followed by simple transformation yields

$$\mathbf{E}\left[\sum_i \omega_i \tilde{r}_i\right] = r_0 + \frac{\mathbf{E}[\tilde{r}_m] - r_0}{\sigma_m^2} \cdot \left(\omega_i^2\sigma_i^2 + \sum_{j=1, i\neq j} \omega_i\omega_j \mathrm{Cov}(r_i, r_j)\right). \tag{7.6}$$

with $r_m = \sum_{i=1}^{n} \omega_i r_i$.

To determine that portfolio combination which maximizes the expected profit, the partial derivatives with respect to the share of the asset i, $\frac{\partial \Psi(r_p)}{\partial \omega_i}$, $i = [1, \ldots, n]$ are required to equal zero. This yields for the asset i

$$\mathbf{E}[\tilde{r}_i] = r_0 + \frac{\mathbf{E}[\tilde{r}_m] - r_0}{\sigma_m^2} \cdot \underbrace{\left(2\omega_i \sigma_i^2 + \sum_{j=1, i \neq j} \omega_j \mathrm{Cov}(r_i, r_j) \right)}_{=\mathrm{Cov}(r_i, r_m)} \tag{7.7}$$

With perfect diversification for $n \to \infty$, the risk contribution of asset i is reduced to its systematic risk, i.e. the covariance with the market portfolio. The remainder, i.e. the cost changes that are uncorrelated with the market portfolio return, is the uncorrelated risk component.

This shows that the optimization approach based on the mean-variance preference functional (7.1) is consistent with the CAPM given the existence of a perfect capital market and investors with homogeneous expectations on the available assets.

7.2.3 (Ir-)relevancy of reflecting unsystematic risk

Based on the previous section the question arises whether energy politicians as well as investors should only reflect the systematic risk incurred with a generation portfolio. If so, a CAPM based valuation of generation assets would be advisable. Otherwise, if unsystematic risk is to be valued as well, a modeling approach based on the mean-variance preference functional from Definition 2.2.2 seems appropriate.

From an energy policy view, neglecting the unsystematic risk from generation portfolios requires the capital market being sufficiently large compared to the electricity market. Under this premise the society has the opportunity to eliminate almost entirely the impact of the unsystematic risk induced by the generation assets by applying and overall diversification of investments. The same rationale applies from an investor's perspective on efficient capital markets: If investors can optimally diversify their investment portfolio, then companies and their managers do not need not act risk aversely and apply diversification of the investment portfolio on the corporate level.

Nevertheless, many companies commit significant resources to corporate portfolio management due to several reasons (cf. also Section 5.1.1): Firstly, agency theory suggests it may be in the manager's interest to diversify his personal risk position by diversifying the company portfolio. Secondly, costs of financial distress and illiquidity might be significant and could therefore motivate corporate diversification. Thirdly, capital market imperfections such as agency costs, transaction costs and taxes might hinder the equity holders from sufficient diversification in their financial portfolios. In this context it also has to be considered that public investors in many European countries may have an eligible interest to hold shares of energy companies (from the viewpoint of easier implementation of the energy politic targets), while their possibilities for diversification are limited due to public budget constraints and limited public interest in other industries. This could additionally motivate utilities to diversify their business—also in the interest of their shareholders.

Since the question whether unsystematic risk in power generation investments should be reflected or not cannot be answered in general, this thesis discusses both approaches in parallel: The models proposed in Chapters 3 and 4 do not incorporate the wider capital market and rely on the societal perspective that both the unsystematic and the systematic risk of generation investments are relevant. In Chapter 5, the wider capital market is added to the set of assumptions and opens the the possibility for diversification also from a welfare perspective. Finally, Chapter 6 extends the traditional CAPM framework which is mostly used in corporate finance applications to the valuation of generation technologies. Thereby, the model is based on the premise that the electricity market investments are small compared to the overall market and thus only systematic risk is relevant for the corporate investment decision.

7.2.4 Model limitations and prospects for future research

Although the relative simplicity of mean-variance decision models provides a wide applicability in academia and practice, the inherent model limitations are to be kept in mind to avoid biasing results under certain circumstances.

Constant absolute risk aversion

As previously discussed, mean-variance preference functionals as defined in Definition 2.3.1 require knowledge of the absolute risk aversion parameter A. The latter depends—as discussed in Section 7.2.1—on the absolute amount of capital invested. In typical investment decisions, however, the invested capital itself depends the portfolio structure and therewith again on the risk aversion. This interdependency require either an iterative solution procedure or yield to approximate solutions in case A has been estimated from historic investments decisions as applied in the numerical example presented in Section 5.5.

Assumptions for consistency with expected utility maximization

Decision models based on the mean-variance preference functional as defined in Definition 2.3.1 impose particular requirements on the utility function of the decision maker and on the distribution of returns to be consistent with the expected utility principle: Firstly, it requires exponential utilities of the decision maker and, secondly, normally distributed payoffs (cf. Theorem 2.3.2). While the exponential utility function could be avoided in case of zero expected payoffs (which itself is an unrealistic assumption), normally distributed payoffs remain a necessary requirement.

However, normally distributed payoffs cannot be assumed to be fulfilled in all practical applications although this is frequently done. The findings from Chapter 4 illustrate this limitation: Given a substantial risk for reversals of the merit order, risk-averse society, and power plant investment costs close to or equal zero, a clearly dominant strategy with respect to both states of the merit order would be to install as much capacity from each technology that maximal demand is covered, i.e. the total installed capacity is twice the maximal demand. However, the empirical results from Section 4.4 show that even for minimal investment costs the case for overcapacities becomes less beneficial the higher the society's risk aversion. This result seems paradox and is due to the fact that total costs given the risk for reversals in the merit order are not normally distributed.

Selection of risk measures

A general point concerning all models presented in this thesis refers to the selection of adequate risk measures: Although allowing a relatively high simplicity in analytical approaches, application of variance as a measure of risk is generally problematic. For further practical applications of the proposed model frameworks aiming to provide direct decision support in risk management issues, the adaptation to other, more powerful risk measures is advisable. In financial economic literature, different properties have been discussed as requirements for a risk measure for being able to aggregate financial risks as good as possible. One of the most widely applied property sets for risk measures has been formulated by Artzner et al. (1999). The authors define a *coherent* risk measure R as a function $R : \mathcal{X} \mapsto \mathbb{R}$, with $\mathcal{X}$ denoting the set of real random variables defined on an appropriate probability space, based on the following four properties:

1. **Translation invariance:** $R(X + c) = R(X) + c,$ for all $c \in \mathbb{R}$
2. **Sub-additivity:** $R(X_1 + X_2) \leq R(X_1) + R(X_2)$
3. **Positive homogeneity:** $R(cX) = cR(X),$ for all $c \geq 0$
4. **Monotonicity:** $X \leq Y \Rightarrow R(Y) \leq R(X).$

Clearly, the variance operator is neither homogeneous nor monotone, nor sub-additive and thus violates the properties of coherent risk measures in multiple dimensions. Although widely used in academia and risk management applications at the firm level, *value-at-risk* is in general not a coherent risk measure either, since it does not fulfill the sub-additivity property.[2] Due to its relative simplicity compared to other risk measures, *conditional value-at-risk*[3] (Acerbi and Tasche, 2002) is one of the practically most accepted coherent risk measures.

Extending the mean-variance approach to the inclusion of coherent measures of risk seems promising—in particular in numerical applications since this will probably reduce the analytical traceability of solutions.

Technical power plant restrictions

Throughout this thesis, generation technologies are only characterized by their fixed and variable costs. For simplicity reasons, other technical parameters have been ignored. Since these restrictions are relevant for the dispatch decision, they should also be reflected for the investment decision. In particular, the following parameters and restrictions are relevant to be reflected:

- Ramp-up times and corresponding costs

[2] Value at risk is only coherent under certain distributional assumptions with respect to the underlying risk factor, such as e.g. for normally distributed risks.

[3] Conditional value-at-risk is sometimes also referred to as expected shortfall.

- Plant indivisibilities and minimum capacities per plant
- Minimum and maximum run-times per plant/technology
- In-availabilities of plants/technologies, e.g. due to maintenance

Consideration of other risk factors

The mean-variance models proposed in Chapters 3-5 exclusively focus on long-term risks in power plant investments and more precisely on risks in operating cost and electricity price risks due to volatile fuel prices. This narrow focus implies of course a high degree of simplification from real world complexity—as with most economic models. The simplifications are necessary to allow an easy traceability of solutions and therewith an in-depth understanding of results. However, the simplifications limit the direct applicability of the models for energy politics and investment decisions as other important risk factors are neglected. Thus it makes sense to keep the time-frame focus on long-term risks and not try to model all other kinds of mid- and short-term risks (cf. Denton et al. (2003)) within the same model. Nevertheless, the following relaxations of simplifying assumptions and extensions to the risk factors captured in the discussed models should be considered to increase the model's fit to reality and the relevance for decision support:

- Capturing demand uncertainty instead of a deterministic load duration curve as one important part of reserve capacity planning. Thereby, different modeling approaches might be considered: For instance, a stochastic load function $D_s(t)$ could be created as a superposition of a deterministic and a stochastic component as $D_s(t) = D(t) + \tilde{d}$.
- On the supply side, an extension of the peak-load pricing model to capture fluctuating renewable energy supply and stochastic plant availabilities would enable the model to study comprehensive portfolio applications—in particular for renewables.
- Furthermore, the peak-load pricing model could be extended by storage technologies—similarly to the model proposed by Steffen and Weber (2011), however under the assumption of risk-averse investors.

Notably, the suggested extensions will rise complexity and might risk the analytical solvability of the models. Ultimately, by including all above-mentioned potential extensions, the model might converge to a fundamental market model which is typically used for scenario-based market studies and relies on Monte Carlo simulation to calculate development paths for a considered market.

Time series properties and the risk for reversals in the merit order

The results of Chapter 4 have shown that considering potential reversals in the merit order of thermal plants may largely affect the technology structure in efficient power generation portfolios. Thereby, the assumptions on the underlying time series of differences in operating costs (e.g. mean reversion versus random walk) of the considered technologies massively drive the likelihood for reversals in the merit order. In this context, further research may be necessary to provide empirical evidence on random walk versus mean reverting behavior for the difference time series in operating costs for typical (thermal) generation technologies.

Managerial view on differences in the systematic risks of different generation technologies

The empirical study presented in Chapter 6 indicates considerable differences in the systematic risk of gas and coal technologies compared to renewable (dominated by hydro) technologies. A larger sample—both with respect to the length of the considered asset return time series but most importantly with respect to the number of analyzed companies—would be desirable but will realistically be hard to obtain in the near future. While the total number of listed electricity companies in Europe exceeds the selected sample size, all other companies known to the author are disqualified by high shares of non-generation related business activities. Moreover, the results open the need for further empirical investigations on the management side: Based on the observations of significant differences in the systematic risk of different power generation technologies observable in the market, an empirical survey among plant investors and market analysts could confirm the application of technology-specific internal rates of return at the decision maker level.

Appendix

A

Appendix to Chapter 3

A.1 Symbols and model notation

Indices

u		Generation technology
t	hours	Time step during analysis period $[0; T]$

Vectors and matrices

$\mathbf{i}$		One vector
$\mathbf{i}_n$		Unit vector of dimension n
$\mathbf{I}$		Identity matrix
$\mathbf{\Sigma}$		Covariance matrix
$\mathbf{L}$		Lag operator

Other

$\mathrm{Var}[\,\cdot\,]$		Variance operator
$\mathbf{E}[\,\cdot\,]$		Expected value operator
$\mathbf{P}(\,\cdot\,)$		Probability measure
$\mathcal{L}$		Lagrangian

Parameters and variables

A	1/€	Risk aversion parameter
D_t	MW	Total system demand at time t
t_u	hours	Lower bound of operating hours of technology u during analysis period $[0; T]$
$p_{u,t}$	€/MWh$_{th}$	Fuel price of technology u in period t
η_u	MWh$_e$/MWh$_{th}$	thermal efficiency of technology u
h_u	MWh$_{th}$/MWh$_e$	heat rate of technology u
e_u	tCO$_2$/MWh$_{th}$	emission rate of technology u
K_u	MW	Installed capacity of technology u
Q_u	MWh	Energy produced of technology u in period $[0; T]$
Q_E	MWh	Total energy produced in the system in period $[0; T]$
$y_{u,t}$	MW	Output level of plant u at time t
C	€	Total generation costs
$C_{inv,u}$	€	Annuity of "overnight" investment costs of technology u
$c_{inv,u}$	€/MW$_e$	Annuity of specific overnight costs of plant u per capacity K_u
$C_{op,u,t}$	€	Operating costs of plant u in period t
$c_{op,u,t}$	€/MWh$_e$	Specific operating costs of plant u in period t per output $y_{u,t}$
$\bar{c}_{op,u}$	€/MWh$_e$	Mean operation costs of plant u
σ_u	€/MWh$_e$	Standard deviation of total operation costs of technology u
σ_{uv}	€^{2}/MWh$_e^2$	Covariance of total operation costs of technologies u and v
ρ	-	Coefficient of correlation

A.2 Proofs and mathematical appendix

A.2.1 Portfolio variance and variance of operating costs

Proof. As in Section 3.3.2, we assume specific operating costs per MWh to be uncertain but constant within the planning period. More specific, let specific operating costs for technology u be represented by multivariate distributed random variables, $\tilde{c}_{op,u}$, with mean $\bar{c}_{op,u}$, variance $\text{Var}(c_{op,u}) = \sigma_u^2$ and corresponding covariance σ_{uv}. Then, the relation between variance of total operating costs for technology u, $\text{Var}(C_{op,u})$, and the variance of specific operating costs, $\text{Var}(c_{op,u}) = \sigma_u^2$, can be calculated as follows:

$$\begin{aligned}
\text{Var}[C_{op,u}] &= \text{Var}\left[Q_u c_{op,u}\right] = \text{Var}\left[\int_t y_{u,t} c_{op,u} \mathrm{d}t\right] = \mathbb{E}\left[\left(\int_t y_{u,t} c_{op,u} - y_{u,t}\bar{c}_{op,u} \mathrm{d}t\right)^2\right] \\
&= \mathbb{E}\left[\left(\int_t y_{u,t}(c_{op,u} - \bar{c}_{op,u})\mathrm{d}t\right)^2\right] = \mathbb{E}\left[\left(\int_t y_{u,t}\mathrm{d}t\right)^2 (c_{op,u} - \bar{c}_{op,u})^2\right] \\
&= \left(\int_t y_{u,t}\mathrm{d}t\right)^2 \mathbb{E}\big[(c_{op,u} - \bar{c}_{op,u})^2\big] = \sigma_u^2 \left(\int_t y_{u,t}\mathrm{d}t\right)^2 = Q_u^2 \text{Var}\left[c_{op,u}\right].
\end{aligned}$$

The variance of specific operating costs, $\text{Var}(c_{op,u}) = \sigma_u^2$, can be calculated from the technology-specific heat rate, h_u, and from the variance of the underlying fuel price, $\text{Var}(p_u)$, as

$$\text{Var}\left[c_{op,t}\right] = h_u^2 \text{Var}\left[p_{u,t}\right]$$

Thus, the total variance of operating costs for the set of all plants $u = \{1, \dots, n\}$ can be calculated as:

$$\begin{aligned}
\text{Var}[C_{op}] &= \text{Var}\left[\sum_u \left(\int_t y_{u,t} c_{op,u} \mathrm{d}t\right)\right] = \sum_u \sum_v \left(\int_t y_{u,t}\mathrm{d}t\right)\left(\int_t y_{v,t}\mathrm{d}t\right)\sigma_{uv} = \\
&= \sum_u \sigma_u^2 \left(\int_t y_{u,t}\mathrm{d}t\right)^2 + \sum_u \sum_{v, v\neq u} \sigma_{uv} \left(\int_t y_{u,t}\mathrm{d}t\right)\left(\int_t y_{v,t}\mathrm{d}t\right) \\
&= \sum_u \sigma_u^2 Q_u^2 + \sum_u \sum_{v,v\neq u} \sigma_{uv} Q_u Q_v.
\end{aligned}$$

□

A.2.2 Optimal technology selection for purely cost minimal portfolios with n technologies

Proposition 3.3.1. *Let be*

$$t_u^o := \frac{c_{inv,u} - c_{inv,u+1}}{c_{op,u+1} - c_{op,u}}, \ (1 \leq u < n) \tag{3.19}$$

If $t_u^o < t_{u-1}^o$ for all $1 \leq u < n$, then the cost-minimal portfolio consists of all technologies, i.e. $K_u^* > 0$ for all $1 \leq u < n$.

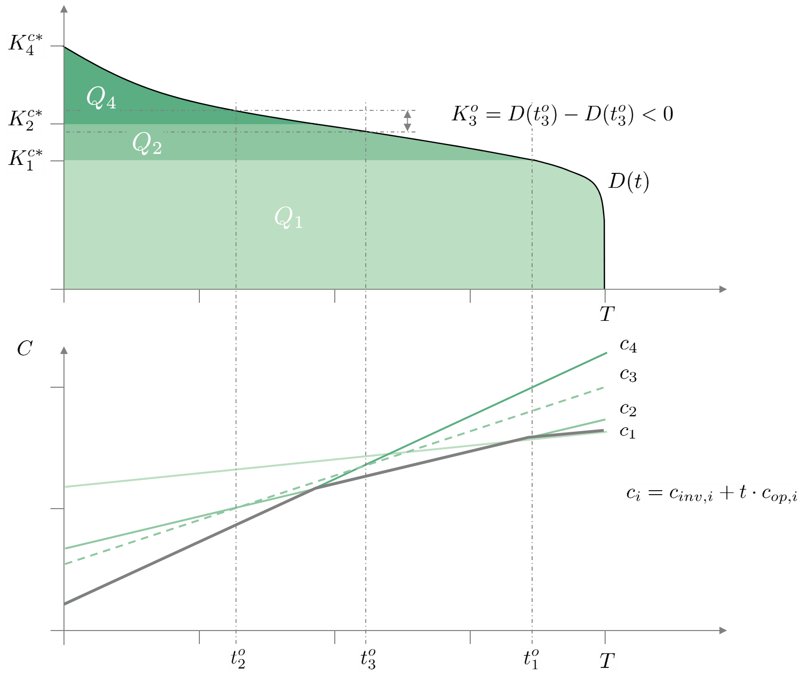

Fig. A.1: Graphical solution of the deterministic capacity planning problem from the load duration function and full-cost production functions. With $t_3^o > t_2^o$, technology 3 will not be included in the cost-efficient portfolio.

Proposition 3.3.2. *Technology u is part of the cost-minimal portfolio, i.e. $K_u^* > 0$, only if $t_u^o < t_{u-1}^o$.*

A graphical interpretation of these propositions is depicted in Figure A.1: Comparison of the full cost curves shows that although technology 3 (dashed line) is not "dominated" by any other technology v such that $c_{op,2'}t + c_{inv,2'} > c_{op,v}t + c_{inv,v}$ for all admissible t, it is not part of the cost-minimal portfolio because the condition formulated in Proposition 3.3.1 is violated. The cost-efficient technology mix is characterized by the lowest envelope of the different cost functions which yield piece-wise linear efficient cost curve per capacity unit as function of operating time (gray line in Figure A.1). Only if all intersections of the full cost curves are obtained in a decreasing order the cost-minimal portfolio will consist of all technologies.

Proof of Proposition 3.3.2. The proposition is proved by contradiction: From $K_u = D(t_u^*) - D(t_{u-1}^*) > 0$, it can be concluded from KKT condition (3.10) that $\mu_u = 0$. Assuming contrarily to the proposition that $t_u^o \geq t_{u-1}^o$. We can compute straightforwardly $t_u^* = t_u^o + \frac{\mu_{u+1}}{c_{op,u+1} - c_{op,u}}$ and $t_{u-1}^* = t_{u-1}^o - \frac{\mu_{u-1}}{c_{op,u} - c_{op,u-1}}$ from Eqn. (3.18). Given that $\mu_{u-1}, \mu_{u+1} \geq 0$, this implies $t_u^* > t_{u-1}^*$. The strict monotony of $D(t)$ then yields $D(t_u^*) - D(t_{u-1}^*) < 0$, which is in contradiction with the initial hypothesis $K_u > 0$. □

Proof of Proposition 3.3.1. To show the implication

$$t_u^o < t_{u-1}^o \text{ for all } u \Rightarrow K_u^* > 0 \text{ for all } u,$$

we proceed again by contradiction. Taking $t_u^o < t_{u-1}^o$ for all u as given, we assume for one single u that $K_u = D(t_u^*) - D(t_{u-1}^*) = 0$. Without much limitation of the generality for the succeeding technology $u+1$ and the preceding technology $u-1$, we assume $K_{u+1} > 0, K_{u-1} > 0$, implying $\mu_{u+1} = 0$ and $\mu_{u-1} = 0$.[1] Straightforwardly, $t_u^* = t_u^o - \frac{\mu_u}{c_{op,u+1} - c_{op,u}}$ and $t_{u-1}^* = t_{u-1}^o + \frac{\mu_u}{c_{op,u} - c_{op,u-1}}$ may be computed which yields $t_u^* < t_{u-1}^*$ for all $\mu_u \geq 0$. This, however, implies $K_u = D(t_u^*) - D(t_{u-1}^*) > 0$, in contradiction to the starting assumption. □

A.2.3 Solution to the pure variance minimization problem with n technologies

Proposition 3.3.3. *Let be a portfolio with n generation technologies with linearly independent operating costs $c_{op,u}, 1 \leq u \leq n$. Then the covariance matrix $\mathbf{\Sigma}$ is positive definite and hence invertible and the central optimality condition of the variance minimal portfolio is given by*

$$\mathbf{Q} = \mathbf{\Sigma^{-1}} \left(\lambda^Q \mathbf{i} + \boldsymbol{\mu}^Q \right) \quad \textit{with} \tag{3.26}$$

$$\lambda^Q = \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(Q_E - \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right) \tag{3.27}$$

Proof of Proposition 3.3.3. Given linearly independent generation technologies, the covariance matrix of operating costs $\mathbf{\Sigma}$ will be not only positive semi-definite as satisfied per definition (cf. e.g. Horn and Johnson, 1985, p. 392), but even positive definite. Consequently it is also invertible and the two equations (3.21) and (3.23) may be combined to yield unique solutions for λ^Q and $\mathbf{Q}$ as a function of $\mu^{\mathbf{Q}}$ (cf. A.2.3). Now, the optimality conditions (3.27) and (3.26) can be derived as follows: Starting with $\lambda^Q \mathbf{i} = \mathbf{\Sigma Q} - \boldsymbol{\mu}^Q$ from Eqn. (3.24), the positive definiteness of matrix $\mathbf{\Sigma}$ allows multiplication with $\mathbf{i}^T \mathbf{\Sigma}^{-1}$ from the left followed by division through the scalar $\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}$ yielding

$$\begin{aligned}
& \left(\mathbf{i}^T \mathbf{\Sigma^{-1}}\right) \lambda^Q \mathbf{i} = \left(\mathbf{i}^T \mathbf{\Sigma^{-1}}\right) \mathbf{\Sigma Q} - \left(\mathbf{i}^T \mathbf{\Sigma^{-1}}\right) \boldsymbol{\mu}^Q \\
\Leftrightarrow \qquad & \lambda^Q = \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(\left(\mathbf{i}^T \mathbf{\Sigma^{-1}}\right) \mathbf{\Sigma Q} - \left(\mathbf{i}^T \mathbf{\Sigma^{-1}}\right) \boldsymbol{\mu}^Q \right) = \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(\mathbf{i}^T \mathbf{Q} - \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right)
\end{aligned}$$

Finally, we apply $Q_E = \mathbf{i}^T \mathbf{Q}$ from Eqn. (3.25) to obtain optimality condition (3.27):

$$\lambda^Q = \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(A Q_E - \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right).$$

By inserting λ^Q in Eqn. (3.24), $\mathbf{Q}$ can be computed as

$$\begin{aligned}
\mathbf{Q} &= \mathbf{\Sigma^{-1}} \left(\lambda^Q \mathbf{i} + \boldsymbol{\mu}^Q \right) = \mathbf{\Sigma^{-1}} \left(\frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(Q_E - \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right) \mathbf{i} + \boldsymbol{\mu}^Q \right) \\
&= \mathbf{\Sigma^{-1}} \left(\frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \left(Q_E \mathbf{i} - \mathbf{i} \mathbf{i}^T \mathbf{\Sigma^{-1}} \boldsymbol{\mu}^Q \right) + \boldsymbol{\mu}^Q \right) \\
&= \frac{Q_E}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \mathbf{\Sigma^{-1}} \mathbf{i} + \left(\mathbf{\Sigma^{-1}} - \frac{1}{\mathbf{i}^T \mathbf{\Sigma^{-1}} \mathbf{i}} \mathbf{\Sigma^{-1}} \mathbf{i} \mathbf{i}^T \mathbf{\Sigma^{-1}} \right) \boldsymbol{\mu}^Q.
\end{aligned}$$

□

[1] In the more general case with possibly several subsequent technologies with zero capacities, a recursive procedure of elimination of inefficient technologies has to be started.

Proposition 3.3.4. *The pure variance-minimal portfolio problem is convex in* $\mathbf{Q}$*. If and only if* $\mathbf{\Sigma}$ *is positive definite, then the optimization problem is strictly convex in* $\mathbf{Q}$*.*

Proof of Proposition 3.3.4. For the purely variance minimal portfolio, the objective function from problem (3.8) can be rewritten as

$$L^r(\mathbf{Q}) = \mathbf{Q}^T \mathbf{\Sigma} \mathbf{Q}$$

The Hessian of the objective function can be derived straightforwardly with matrix calculus as $\mathbf{H}^r = \mathbf{\Sigma}$. Taking into account that an arbitrary covariance matrix $\mathbf{\Sigma}$ is positive semi-definite (cf. Horn and Johnson, 1985, p. 392), convexity of L^r can be concluded. Furthermore, the Hessian is positive definite and consequently L^r strictly convex if and only if $\mathbf{\Sigma}$ is positive definite. Using $\mathbf{Q}$ as decision variable, constraints (3.10) and (3.12) can be rewritten as $-\mathbf{Q} \leq \mathbf{0}$ and $Q_E - \mathbf{Qi} \leq \mathbf{0}$, so that linearity and hence also convexity of both constraints become obvious. □

Proposition 3.3.5. *Let be*

$$\mathbf{Q}^o = \frac{Q_E}{\mathbf{i}^T \mathbf{\Sigma}^{-1} \mathbf{i}} \mathbf{\Sigma}^{-1} \mathbf{i} \tag{3.28}$$

with $\mathbf{i} = (1, \ldots, 1)^T$*. The variance-minimal portfolio consists of all available technologies, i.e.* $Q_u > 0$ *for all* $1 \leq u \leq n$*, if and only if* $Q_u^o > 0$ *for all* $1 \leq u \leq n$*. Then,* $\mathbf{Q}^* = \mathbf{Q}^o$ *is a solution to the variance minimal optimization problem. The solution is unique if* $\mathbf{\Sigma}$ *is positive definite.*

Proof of Proposition 3.3.5. For notational brevity, we define $\mathbf{R} := \omega \mathbf{\Sigma}^{-1} \left(\left(\mathbf{i}^T \mathbf{\Sigma}^{-1} \mathbf{i} \right) \mathbf{I} - \mathbf{i}\mathbf{i}^T \mathbf{\Sigma}^{-1} \right)$ with $\omega := \left(\mathbf{i}^T \mathbf{\Sigma}^{-1} \mathbf{i} \right)^{-1}$. Then, we can rewrite Eqn. (3.26)

$$\mathbf{Q} = \mathbf{Q^o} + \mathbf{R} \boldsymbol{\mu}^Q.$$

Remark that the symmetric matrix $\mathbf{R}$ is in general indefinite, even for $\mathbf{\Sigma}^{-1}$ being positive definite.[2] Suppose $Q_u > 0$, which implies $\mu_u^Q = 0$ according to KKT condition (3.10). Consequently, $Q_u > 0$ for all $1 \leq u < n$ implies $Q_u^o > 0$ for all $1 \leq u < n$. Therewith, $Q_u^o > 0$ for all $1 \leq u < n$ represents the necessary condition for the variance-minimal portfolio to consist of all available technologies.

The condition is even sufficient for the variance-minimal solution, since $Q_u = Q_u^o$, $\mu_u = 0$ for all $1 \leq u < n$ represents a solution to the equation system (3.27)-(3.26) and therewith a local variance minimum to the considered portfolio problem. Taking into account convexity of the optimization problem as shown in Proposition 3.3.4, it is clear that any local variance minimum is also global. For $\mathbf{\Sigma}$ being positive definite, the optimization problem is strictly convex and hence the obtained solution is unique. □

[2] Only nonnegative linear combinations of positive semi-definite matrices are again positive definite (cf. Horn and Johnson, 1985, Observation 7.1.3), however, $\mathbf{R}$ represents a negative linear combination of definite matrices.

A.2.4 Solution to the general portfolio problem with n technologies

Proposition 3.3.6. *The central optimality condition for the combined portfolio problem is given by*

$$-A\,\mathbf{\Sigma}Q(\mathbf{K^c},\mathbf{LK^c}) = \mathrm{diag}\big((\mathbf{I}-\mathbf{L})t(\mathbf{K^c})\big)^{-1}\Big((\mathbf{I}-\mathbf{L})^T(\mathbf{c_{inv}}+\boldsymbol{\mu}) + diag\big(t(\mathbf{K^c})\big)(\mathbf{I}-\mathbf{L})^T\mathbf{c_{op}}.\Big) \tag{3.33}$$

Proof of Proposition 3.3.6. The central solution condition (cf. Eqn. (3.33)) for the general risk-adjusted portfolio problem can be derived from $\frac{\partial \mathcal{L}_n}{\partial \mathbf{K^c}}$ as follows:

$$\begin{aligned}
& A\,\mathrm{diag}\big((\mathbf{I}-\mathbf{L})t(\mathbf{K^c}))\big)\mathbf{\Sigma}(\mathbf{L}-\mathbf{I})Q^I(\mathbf{K^c}) = (\mathbf{I}-\mathbf{L})^T(\mathbf{c_{inv}}+\boldsymbol{\mu}) + \mathrm{diag}\big(t(\mathbf{K^c})\big)(\mathbf{I}-\mathbf{L})^T\mathbf{c_{op}}\\
\Leftrightarrow\quad & A\,\mathbf{\Sigma}(\mathbf{L}-\mathbf{I})Q^I(\mathbf{K^c}) = \mathrm{diag}\big((\mathbf{I}-\mathbf{L})t(\mathbf{K^c})\big)^{-1}\Big((\mathbf{I}-\mathbf{L})^T(\mathbf{c_{inv}}+\boldsymbol{\mu}) + \mathrm{diag}\big(t(\mathbf{K^c})\big)(\mathbf{I}-\mathbf{L})^T\mathbf{c_{op}}\Big)\\
\Leftrightarrow\quad & -A\,\mathbf{\Sigma}Q(\mathbf{K^c},\mathbf{LK^c}) = \mathrm{diag}\big((\mathbf{I}-\mathbf{L})t(\mathbf{K^c})\big)^{-1}\Big((\mathbf{I}-\mathbf{L})^T(\mathbf{c_{inv}}+\boldsymbol{\mu}) + \mathrm{diag}\big(t(\mathbf{K^c})\big)(\mathbf{I}-\mathbf{L})^T\mathbf{c_{op}}\Big)\\
\Leftrightarrow\quad & \mathbf{Q} = -\frac{1}{A}\mathbf{\Sigma^{-1}}\mathrm{diag}\big((\mathbf{I}-\mathbf{L})t(\mathbf{K^c})\big)^{-1}\Big((\mathbf{I}-\mathbf{L})^T(\mathbf{c_{inv}}+\boldsymbol{\mu}) + \mathrm{diag}\big(t(\mathbf{K^c})\big)(\mathbf{I}-\mathbf{L})^T\mathbf{c_{op}}\Big)
\end{aligned}$$

□

Proposition 3.3.7. *Let be $A > 0$ and $c_{inv,u} > c_{inv,u+1}$ for all $(u = 1, \dots, n-1)$. Then the combined portfolio problem (3.8)-(3.12) is convex in $\mathbf{Q}$. If and only if $\mathbf{\Sigma}$ is positive definite, then the optimization problem is strictly convex in $\mathbf{Q}$ and hence has a unique solution.*

Proof of Proposition 3.3.7. As shown in Proposition 3.3.4, the pure variance-minimization problem is convex (strictly convex) in $\mathbf{Q}$ if and only if $\mathbf{\Sigma}$ is positive semi-definite (positive definite). For the second part of the proof, we consider the objective function of the pure cost-minimization problem

$$L^c(\mathbf{Q}) = \mathbf{c_{inv}}^T(\mathbf{I}-\mathbf{L})\,\mathbf{K^c}(\mathbf{Q}) + \mathbf{\bar{c}}_{\mathbf{op}}^T\mathbf{Q}$$

Note that the first summand is linear in $\mathbf{K^c}(\mathbf{Q})$, which itself is a nonlinear function of $\mathbf{Q}$. From the definition of Q_u^I in Eqn. (3.7) it is known that $Q_u^I(K_u^c) = f(K_u^c)$ is an increasing and concave function of K_u^c since $\frac{\mathrm{d}\left(Q_u^I(K_u^c)\right)^2}{\mathrm{d}^2K_u^c} = \frac{\mathrm{d}R(K_u^c)}{\mathrm{d}K_u^c} \le 0$ since $D(t)$ is monotone decreasing. Hence, it can be concluded that the inverse function $K_u^c(Q_u^I) = f^{-1}(Q_u^I)$ is convex in Q_u^I. In fact, Q_u^I can be expressed as the nonnegative linear combination $Q_u^I = \sum_{i=1}^{u} Q_i$, hence $K_u^c(Q_u^I) = K_u^c(Q_1, \dots, Q_u)$ is also convex in each Q_i, $(i = 1, \dots, u)$. Finally, $\mathbf{c_{inv}}^T(\mathbf{I}-\mathbf{L})\,\mathbf{K^c}(\mathbf{Q})$ is convex as a nonnegative linear combination of convex functions if $c_{inv,u} > c_{inv,u+1}$ for all $(u = 1, \dots, n-1)$.

The second summand of L^c is linear in $\mathbf{Q}$ and therefore also convex in $\mathbf{Q}$. Thus, also the objective function $L = L^c + L^r$ of the general cost variance optimization problem is convex as a nonnegative linear combination of convex functions if $c_{inv,u} > c_{inv,u+1}$ for all $(u = 1, \dots, n-1)$. □

A.2.5 Proof of uniqueness and existence of the portfolio optimum with two technologies

Proposition 3.3.8. *The combined portfolio problem (3.8)-(3.12) with two technologies $u = \{1, 2\}$ has a unique solution if*

$$AQ_E(\sigma_2^2 - \sigma_{12}) \geq \frac{1}{T}(c_{inv,1} - c_{inv,2}) + \bar{c}_{op,1} - \bar{c}_{op,2}. \tag{3.36}$$

Proof of Proposition 3.3.8. Rewriting the optimality condition as given by Eqn. (3.35) leads to

$$A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2 - A(\sigma_1^2 - \sigma_{12})Q_E = \frac{c_{inv,1} - c_{inv,2}}{t_1} + \bar{c}_{op,1} - \bar{c}_{op,2}$$

Here, the risk-free term is separated from the risk-term, each to one side of the optimality condition. For brevity, we denote the left hand-side of the latter equation with $l(t_1)$ and the right hand-side with $r(t_1)$, i.e.

$$l(t_1) := A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2 - A(\sigma_1^2 - \sigma_{12})Q_E, \tag{A.1}$$

$$r(t_1) := \frac{c_{inv,1} - c_{inv,2}}{t_1} + \bar{c}_{op,1} - \bar{c}_{op,2}. \tag{A.2}$$

Because it holds $(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) \geq 0$ for all $\sigma_1, \sigma_2, \sigma_{12} \geq 0, |\rho| \leq 1$ and since $Q_2 = \int_0^{t_1} D(t) - D(t_1)\mathrm{d}t$ is monotone increasing in t_1, it can be concluded that also $l(t_1)$ is monotone increasing in t_1, i.e. $\frac{\partial l(t_1)}{\partial t_1} \geq 0$. In contrast, it can be seen that $r(t_1)$ is hyperbolically decreasing in t_1 thus $\frac{\partial r(t_1)}{\partial t_1} \leq 0$, given $(c_{op,1} < c_{op,2}) \wedge (c_{inv,1} > c_{inv,2})$.

The optimal operating time t_1^* satisfying condition (3.35) is given by the intersection of $l(t_1)$ and $r(t_1)$ (see Figure A.2). This value represents the optimal operating time of the peak-load technology and captures the trade-off of the variance-minimal and the cost-minimal run-time of the peak load technology. A unique intersection point is obtained if $l(T) \geq r(T)$, i.e.

$$AQ_E(\sigma_2^2 - \sigma_{12}) \geq \frac{1}{T}(c_{inv,1} - c_{inv,2}) + \bar{c}_{op,1} - \bar{c}_{op,2}$$

and the two functions will cross exactly once in the interval $[0, T]$, resulting in a unique solution from the optimality condition (3.35). The latter assumption will generally be fulfilled as empirically shown in Section 3.4. In the rare case of $AQ_E(\sigma_2^2 - \sigma_{12}) < \frac{1}{T}(c_{inv,1} - c_{inv,2}) + \bar{c}_{op,1} - \bar{c}_{op,2}$, $l(t_1)$, and $r(t_1)$ have no intersection in the interval $[0, T]$. Hence, in this case there is no interior solution to problem (3.8) to (3.12). □

A.2.6 Proof of sensitivity properties of the cost-variance efficient portfolio

Proposition 3.3.10. *For a risk-cost-efficient portfolio, the sensitivity of optimal operating hours (and respectively capacities) of the considered technologies with respect to the risk-aversion parameter A is only dependent on the covariance of operating costs with*

$$\frac{dt_1^*}{dA} \lesseqgtr 0, \quad \textit{for } \frac{\sigma_1}{\sigma_2} \lesseqgtr \rho. \tag{3.38}$$

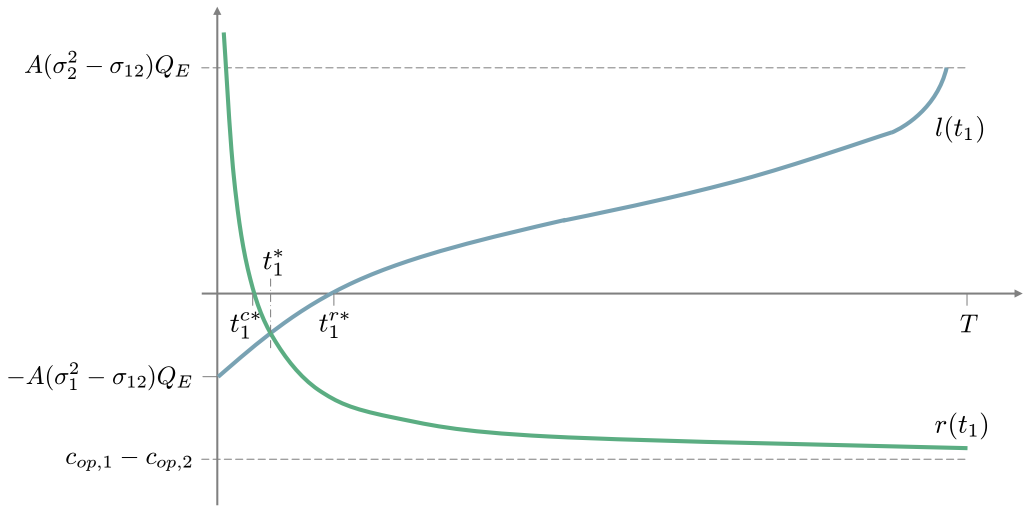

Fig. A.2: Graphical proof of the uniqueness of results from the optimality condition. The intersection of $r(t_1)$ and $l(t_1)$ represents the optimal operating time.

Proof of Proposition 3.3.10. To derive sensitivity properties of the optimal portfolio fuel mix on the risk attitude factor A, we use in the two-technology case the first order derivative of the optimality condition itself. Total differentiation of Eqn. (3.35) with respect to A and following reallocation leads to

$$\frac{\mathrm{d}t_1^*}{\mathrm{d}A}\left(\bar{c}_{op,1} - \bar{c}_{op,2} + AQ_E(\sigma_1^2 - \sigma_{12}) - A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\right) +$$
$$+\, t_1^*\left((\sigma_1^2 - \sigma_{12})Q_E - Q_2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + A\left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right)t_2 D'(t_1^*)\frac{\mathrm{d}t_1^*}{\mathrm{d}A}\right) = 0$$
$$\Leftrightarrow \quad \frac{\mathrm{d}t_1^*}{\mathrm{d}A} = \frac{(t_1^*)^2\left(Q_2(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) - Q_E(\sigma_1^2 - \sigma_{12})\right)}{c_{inv,2} - c_{inv,1} + (t_1^*)^3 A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})D'(t_1^*)}. \tag{A.3}$$

At first, suppose $\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \leq 0$. Taking into account the negativity of the denominator in Eqn. (A.3), this requires consequently the enumerator in the latter term to be non-negative, i.e. $\frac{Q_2}{Q_E} \geq \frac{\sigma_1^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}$. By inserting 0 as the lower bound of $\frac{Q_2}{Q_E}$, it follows $\forall\, \sigma_1, \sigma_2, \rho$ with $\left(\frac{\sigma_1}{\sigma_2} \leq \rho\right) \Rightarrow \left(\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \leq 0\right)$.

From the upper bound $\frac{Q_2}{Q_E} \leq 1$ it can be concluded in this case $\left(\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \leq 0\right) \Rightarrow \left(\frac{\sigma_2}{\sigma_1} \geq \rho\right)$. Remark that condition $\frac{\sigma_1}{\sigma_2} \leq \rho$ can be considered as *sufficient* and $\frac{\sigma_2}{\sigma_1} \geq \rho$ as *necessary* for the case $\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \leq 0$. Per definition of the coefficient of correlation it is $|\rho| \leq 1$. Therefore, $\frac{\sigma_1}{\sigma_2} \leq \rho$ implies $\sigma_2 \geq \sigma_1$. The latter again implies $\frac{\sigma_2}{\sigma_1} \geq \rho$. Hence, with $\left(\frac{\sigma_1}{\sigma_2} \leq \rho\right) \Rightarrow \left(\frac{\sigma_2}{\sigma_1} \geq \rho\right)$, the *necessary* condition implies the *sufficient* condition and we can simply state

$$\left(\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \leq 0\right) \Leftrightarrow \left(\frac{\sigma_1}{\sigma_2} \leq \rho\right).$$

Now, the case $\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \geq 0$ follows directly from the negation of this equivalence:

$$\left(\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \geq 0\right) \Leftrightarrow \left(\frac{\sigma_1}{\sigma_2} \geq \rho\right)$$

□

Proposition 3.3.11. *Alternatively, a comparison of optimal operating times for the purely cost-efficient portfolio, t_1^{c*}, and for the purely risk-efficient portfolio, t_1^{r*}, can provide evidence on the sensitivity. As shown in A.2.6, it equivalently holds*

$$\frac{dt_1^*}{dA} \lesseqgtr 0, \quad \text{for } t_1^{c*} \gtreqless t_1^{r*}. \tag{3.39}$$

Proof of Proposition 3.3.11. Alternatively, the sensitivity of t_1^* on the parameter A can be checked by comparison of the optimal operating times of the purely cost-efficient portfolio, t_2^{r*}, and the purely risk-efficient portfolio, t_2^{r*}:

$$\left(\frac{\mathrm{d}t_1^*}{\mathrm{d}A} \lesseqgtr 0\right) \Leftrightarrow \left(t_1^{c*} \lesseqgtr t_2^{r*}\right)$$

This can be seen from the risk term and the cost term of optimality condition (cf. Eqn. (3.35)) as previously defined in Eqs. (A.1) and (A.2). With $l(t_1)$ monotone increasing and $r(t_1)$ monotone decreasing in t_1, we can indirectly derive sensitivity properties of t_1^* from the sensitivities of $l(0), l(T)$, and the variance minimal operating time t_1^{r*} with $l(t_1^{r*}) = 0$ (cf. Figure A.2). Knowing that $Q_2(t_1^{r*})$ is monotone increasing in t_1^{r*}, it can be seen from that the variance minimal operating time is independent from the parameter A:

$$Q_2(t_1^{r*}) = Q_E \frac{\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}}{\sigma_1^2 - \sigma_{12}} > 0 \qquad \text{for } \sigma_1 \gtrless \sigma_2.$$

Therefore, it holds

$$\left(\frac{\partial l(0)}{\partial A} \gtreqless 0\right) \wedge (t_1^{c*} < t_1^{r*}) \Rightarrow \left(\frac{\partial t_1^*}{\partial A} \gtreqless 0\right), \qquad \left(\frac{\partial l(T)}{\partial A} \lesseqgtr 0\right) \wedge (t_1^{c*} > t_1^{r*}) \Rightarrow \left(\frac{\partial t_1^*}{\partial A} \gtreqless 0\right).$$

For l and its partial differentials we can state

$$\begin{aligned} l(0) &= -A(\sigma_1^2 - \sigma_{12})Q_E \lesseqgtr 0 \quad \text{for } \rho \lesseqgtr \frac{\sigma_1}{\sigma_2}, & \frac{\partial l(0)}{\partial A} &= -(\sigma_1^2 - \sigma_{12})Q_E \lesseqgtr 0 \quad \text{for } \rho \lesseqgtr \frac{\sigma_1}{\sigma_2}, \\ l(T) &= A(\sigma_2^2 - \sigma_{12})Q_E \gtreqless 0 \quad \text{for } \rho \lesseqgtr \frac{\sigma_2}{\sigma_1}, & \frac{\partial l(T)}{\partial A} &= (\sigma_2^2 - \sigma_{12}) \gtreqless 0 \quad \text{for } \rho \lesseqgtr \frac{\sigma_2}{\sigma_1}. \end{aligned}$$

Note that from $l(0) < 0$ follows $\frac{\partial l(0)}{\partial A} < 0$, similarly $l(T) > 0$ implies $\frac{\partial l(T)}{\partial A} > 0$. Thus, within the boundaries where both technologies are part of the purely risk-efficient portfolio, i.e. $\rho \leq \frac{\sigma_2}{\sigma_1}$ and $\rho \leq \frac{\sigma_1}{\sigma_2}$ (cf. Property 3.3.9), we can conclude

$$\left(t_1^{c*} \lesseqgtr t_1^{r*}\right) \Leftrightarrow \left(\frac{\partial t_1^*}{\partial A} \gtreqless 0\right).$$

□

Proposition 3.3.12. *Given technologies 1 and 2 being part of the cost-efficient and the variance-efficient portfolio, the following parameter conditions are sufficient for the stated sensitivity prop-*

erties of optimal operating hours (respectively capacities) of technology 2:

$$\frac{dt_1^*}{d\sigma_1} \geq 0 \qquad \textit{for all } \sigma_1, \sigma_2 \geq 0,\ 0 \leq \rho \leq 1, \tag{3.40}$$

$$\frac{dt_1^*}{d\sigma_2} \leq 0 \qquad \textit{for all } \sigma_1, \sigma_2 \geq 0,\ 0 \leq \rho \leq 1, \tag{3.41}$$

$$\frac{dt_1^*}{d\rho} \leq 0 \qquad \textit{for all } \sigma_1, \sigma_2, \geq 0,\ -1 \leq \rho \leq 1. \tag{3.42}$$

Proof 1 for Proposition 3.3.12. Total differentiation of optimality condition Eqn. (3.35) with respect to σ_1 yields

$$\begin{aligned}
&\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1}\left(c_{op,1} - c_{op,2} + AQ_E(\sigma_1^2 - \sigma_{12}) - A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\right) + \\
&\qquad + t_1^* A\left((2\sigma_1 - \sigma_2\rho)Q_E - 2Q_2(\sigma_1 - \sigma_2\rho) + \left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right) t_1^* D'(t_1^*)\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2}\right) = 0 \\
\Leftrightarrow\ &\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} = \frac{(t_1^*)^2 A\big((\sigma_2\rho - 2\sigma_1)Q_E - 2Q_2(\sigma_2\rho - \sigma_1)\big)}{c_{inv,2} - c_{inv,1} + (t_1^*)^3 A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})D'(t_1^*)}.
\end{aligned} \tag{A.4}$$

Consider $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \geq 0$ which requires the enumerator in Eqn. (A.4) to be negative. Consequently, two cases have to be differentiated:

I) For non-negativity of the term $Q_2(\cdot)$ in Eqn. (A.4), let be $\rho > \frac{\sigma_1}{\sigma_2}$: Consequently, $\frac{Q_2}{Q_E} \geq \frac{\sigma_2\rho - 2\sigma_1}{2\sigma_2\rho - 2\sigma_1}$ has to hold. Using $\frac{Q_2}{Q_E} \geq 0$ as the lower bound, it follows $\rho \leq \frac{2\sigma_1}{\sigma_2}$.

II) For negativity of the term $Q_2(\cdot)$ in Eqn. (A.4), let be $\rho < \frac{\sigma_1}{\sigma_2}$: Then $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \geq 0$ requires $\frac{Q_2}{Q_E} \leq \frac{\sigma_2\rho - 2\sigma_1}{2\sigma_2\rho - 2\sigma_1}$. Using $\frac{Q_2}{Q_E} \leq 1$ as the upper bound, it follows $\rho \geq 0$.

Hence, within the boundaries $\frac{\sigma_1}{\sigma_2} > \rho$ and $\frac{\sigma_2}{\sigma_1} > \rho$, it can be concluded $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \geq 0$ for all $\sigma_1, \sigma_2, \rho \geq 0$.[3] □

Proof 2 for Proposition 3.3.12. Total differentiation of Eqn. (3.35) with respect to σ_2 yields

$$\begin{aligned}
&\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2}\left(c_{op,1} - c_{op,2} + AQ_E(\sigma_1^2 - \sigma_{12}) - A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\right) - \\
&\qquad - t_2 A\left(\sigma_1\rho Q_E + 2Q_2(\sigma_2 - \sigma_1\rho) - \left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right) t_1^* D'(t_1^*)\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2}\right) = 0 \\
\Leftrightarrow\ &\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2} = \frac{(t_1^*)^2 A\big(\sigma_1\rho Q_E + 2Q_2(\sigma_2 - \sigma_1\rho)\big)}{c_{inv,2} - c_{inv,1} + (t_1^*)^3 A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})D'(t_1^*)}.
\end{aligned} \tag{A.5}$$

The case $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2} \leq 0$ requires the enumerator in Eqn. (A.5) to be non-negative. It needs to be distinguished between the following cases:

I) For non-negativity of the term $Q_2(\cdot)$ in Eqn. (A.5), let be $\rho < \frac{\sigma_2}{\sigma_1}$, i.e. $\frac{Q_2}{Q_E} \geq \frac{\sigma_1\rho - 2}{2(\sigma_1\rho - \sigma_2)}$. Using $\frac{Q_2}{Q_E} \geq 0$ as the lower bound, it follows $\rho \geq 0$.

[3] For $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \leq 0$, we can proceed vice versa to obtain a sufficient condition for σ_1, σ_2, ρ fulfilling the assumption. Since we do only obtain the null set, the existence of a parameter set with $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \leq 0$ remains unproven.

II) For negativity of the term $Q_2(\cdot)$ in Eqn. (A.5), let be $\rho > \frac{\sigma_2}{\sigma_1}$: Then $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2} \leq 0$ requires $\frac{Q_2}{Q_E} \leq \frac{\sigma_2\rho - 2\sigma_1}{2\sigma_2\rho - 2\sigma_1}$. Using $\frac{Q_2}{Q_E} \leq 1$ as the upper bound, it follows $\rho \leq \frac{2\sigma_2}{\sigma_1}$.

Taken both cases together and considering the boundaries $\frac{\sigma_1}{\sigma_2} > \rho$ and $\frac{\sigma_2}{\sigma_1} > \rho$, we obtain for all $\sigma_1, \sigma_2, \rho \geq 0$, $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2} \leq 0$.[4] □

Proof 3 for Proposition 3.3.12. As shown, total differentiation of the optimality condition Eqn. (3.35) with respect to ρ leads to

$$\frac{\mathrm{d}t_1^*}{\mathrm{d}\rho}\left(c_{op,1} - c_{op,2} + AQ_E(\sigma_1^2 - \sigma_{12}) - A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})Q_2\right) -$$

$$- t_1^* A\left(\sigma_1\sigma_2 Q_E - 2\sigma_1\sigma_2 Q_2 - \left(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}\right) t_2 D'(t_2)\frac{\mathrm{d}t_1^*}{\mathrm{d}\rho}\right) = 0$$

$$\Leftrightarrow \quad \frac{\mathrm{d}t_1^*}{\mathrm{d}\rho} = \frac{(t_1^*)^2 A\sigma_1\sigma_2\left(Q_1 - Q_2\right)}{c_{inv,2} - c_{inv,1} + (t_1^*)^3 A(\sigma_1^2 + \sigma_2^2 - 2\sigma_{12})D'(t_1^*)} \tag{A.6}$$

For $\frac{\mathrm{d}t_1^*}{\mathrm{d}\rho} \leq 0$, it can be reasoned from Eqn. (A.6) that $(Q_1 - Q_2 \geq 0) \Leftrightarrow \left(\frac{Q_2}{Q_E} \leq \frac{1}{2}\right)$ has to be fulfilled. As seen in proof 2, if $\frac{\sigma_1}{\sigma_2} \geq \rho$ the relation $\frac{Q_2}{Q_E} \leq \frac{\sigma_1^2 - 2\sigma_{12}}{\sigma_1^2 + \sigma_2^2 - \sigma_{12}}$ holds and represents an upper bound for the quotient $\frac{Q_2}{Q_E}$. Thus, we can conclude

$$\left(\frac{\sigma_1^2 - \sigma_1\sigma_2\rho}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho} \leq \frac{1}{2}\right) \quad \Leftrightarrow \quad (\sigma_1 \leq \sigma_2)\,.$$

Hence, within the boundaries $\frac{\sigma_1}{\sigma_2} > \rho$ and $\frac{\sigma_2}{\sigma_1} > \rho$, we can conclude $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_1} \leq 0$ for all $\sigma_1, \sigma_2 \geq 0$ with $\sigma_1 \leq \sigma_2$. □

[4] Applying the analogue estimation for $\frac{\mathrm{d}t_1^*}{\mathrm{d}\sigma_2} \geq 0$, however, cannot prove the existence of a feasible set of σ_1, σ_2, ρ as we only obtain the null set.

Appendix

B

Appendix to Chapter 4

B.1 Symbols and model notation

Indices		
u		Plant technology
s_i		Merit order state
t	hours	Intra-period time step during analysis period $[0; T]$
τ	years	Period time step during considered plant lifetime $[0; \hat{\tau}]$
Operators		
$\mathbf{Var}[\,\cdot\,]$		Variance operator
$\mathbf{Var}[\,\cdot\,\vert s_i]$		Conditional variance operator given scenario s_i
$\mathbf{E}[\,\cdot\,]$		Expectation operator
$\mathbf{E}[\,\cdot\,\vert s_i]$		Conditional expectation operator given scenario s_i
$\mathbf{P}(\,\cdot\,)$		Probability measure

Parameters and variables

A	1/€	Social risk attitude
D_t	MW	Total system demand at time t
t_u	hours	Minimal operating duration of u when representing the base technology
O_u	hours	Minimal operating duration of technology u
$p_{u,t}$	€/MWh$_{th}$	Fuel price of technology u in period t
η_u	MWh$_e$/MWh$_{th}$	thermal efficiency of plant technology u
h_u	MWh$_{th}$/MWh$_e$	heat rate of plant technology u
e_u	tCO_2/MWh$_{th}$	emission rate of plant technology u
K_u	MW	Installed capacity of plant technology u
Q_u	MWh	Energy produced of plant technology u in period $[0; T]$
Q_E	MWh	Total energy produced in the system in period $[0; T]$
$y_{u,t}$	MW	Output level of plant u at time t
$C_{inv,u}$	€	Annuity of overnight costs (total investment costs) of plant u
$c_{inv,u}$	€/MW$_e$	Annuity of specific overnight costs of plant u per capacity K_u
$C_{u,t}$	€	Operating costs of plant u in period t
$c_{u,t}$	€/MWh$_e$	Specific operating costs of plant u in period t per output $y_{u,t}$
$\bar{c}_u$	€/MWh$_e$	Mean operating costs of plant u
σ_u	€/MWh$_e$	Standard deviation of operating costs of plant u
σ_{uv}	€^{2}/MWh$_e^2$	Covariance of operating costs of plant u and v
$\bar{c}_{u\|s_i}$	€/MWh$_e$	Conditional mean operating costs o given scenario s_i
$\sigma_{u\|s_i}$	€/MWh$_e$	Conditional standard deviation of op. costs given scenario s_i
$\sigma_{uv\|s_i}$	€^{2}/MWh$_e^2$	Conditional covariance of op. costs given scenario s_i
ρ	-	Coefficient of correlation

B.2 Mathematical Appendix

B.2.1 Calculation of conditional expectations and variances

To calculate the conditional expectations and variances used in the optimization problem from Eqs. (4.9)-(4.12), we start with the conditional joint distribution of $\tilde{c}_1, \tilde{c}_2$ given $\tilde{c}_1 \leq \tilde{c}_2$ which can be obtained as the truncated distribution (see Figure B.1) with density

$$f_{1,2}(c_1, c_2|\tilde{s} = s_0) = f_{1,2}(c_1, c_2|c_1 < c_2) = \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\mathbf{P}(\tilde{s} = s_0)}, \text{ for } -\infty < c_1 \leq c_2 < \infty, \tag{B.1}$$

where $\mathbf{P}(\tilde{s} = s_0)$ denotes the fuel-switch likelihood which can be computed from the distribution of differences in operating costs $\tilde{z} = \tilde{c}_2 - \tilde{c}_1$ as discussed in Eqn. (4.51). Next, the conditional densities of $\tilde{c}_1, \tilde{c}_2$ under the condition $\tilde{c}_1 \leq \tilde{c}_2$ (see Figure B.2) and given a fixed value of c_2, c_1, respectively, are determined as:

$$f_1(c_1|c_1 \leq c_2) = \frac{f_{1,2}(c_1, c_2|c_1 \leq c_2)}{\varphi_2(c_2)} = \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_2(c_2)} \tag{B.2}$$

$$f_2(c_2|c_1 \leq c_2) = \frac{f_{1,2}(c_1, c_2|c_1 \leq c_2)}{\varphi_1(c_1)} = \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_1(c_1)} \tag{B.3}$$

From these, we can straightforwardly derive the (single) conditional expectations as

$$\mathbf{E}[\tilde{c}_1|c_2 \wedge (c_1 \leq c_2)] = \int_{-\infty}^{c_2} c_1 f_1(c_1|c_1 \leq c_2)\mathrm{d}c_1 = \int_{-\infty}^{c_2} c_1 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_2(c_2)}\mathrm{d}c_1 \tag{B.4}$$

$$\mathbf{E}[\tilde{c}_1|c_2 \wedge (c_1 > c_2)] = \int_{c_2}^{\infty} c_1 f_1(c_1|c_1 > c_2)\mathrm{d}c_1 = \int_{c_2}^{\infty} c_1 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{(1-\Phi(0))\varphi_2(c_2)}\mathrm{d}c_1 \tag{B.5}$$

$$\mathbf{E}[\tilde{c}_2|c_1 \wedge (c_1 \leq c_2)] = \int_{c_1}^{\infty} c_2 f_2(c_2|c_1 \leq c_2)\mathrm{d}c_2 = \int_{c_1}^{\infty} c_2 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_1(c_1)}\mathrm{d}c_2 \tag{B.6}$$

$$\mathbf{E}[\tilde{c}_2|c_1 \wedge (c_1 > c_2)] = \int_{-\infty}^{c_1} c_2 f_2(c_2|c_1 > c_2)\mathrm{d}c_2 = \int_{-\infty}^{c_1} c_2 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{(1-\Phi(0))\varphi_1(c_1)}\mathrm{d}c_2 \tag{B.7}$$

Here, conditional expectations $\mathbf{E}\left[\tilde{c}_1|c_2 \wedge \left(c_1 \lesseqgtr c_2\right)\right] = g(c_2)$ and $\mathbf{E}\left[\tilde{c}_2|c_1 \wedge \left(c_1 \lesseqgtr c_2\right)\right] = g(c_1)$ represent functions which are solely dependent on c_1 and c_2, respectively. Hence it makes sense to define the conditional expectation $\bar{c}_{u|s_0} \equiv \mathbf{E}[\tilde{c}_u|\tilde{s} = s_0]$ given the default fuel cost order scenario s_0 as the double expectation $\mathbf{E}\big[\mathbf{E}[\tilde{c}_1|c_2 \wedge (c_1 \leq c_2)]\big]$. Straightforwardly, we obtain

$$\mathbf{E}[\tilde{c}_1|\tilde{s} = s_0] := \mathbf{E}\big[\mathbf{E}[\tilde{c}_1|c_2 \wedge (c_1 \leq c_2)]\big] = \int_{-\infty}^{\infty}\int_{-\infty}^{c_2} c_1 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 \tag{B.8}$$

$$\mathbf{E}[\tilde{c}_1|\tilde{s} = s_1] := \mathbf{E}\big[\mathbf{E}[\tilde{c}_1|c_2 \wedge (c_1 > c_2)]\big] = \int_{-\infty}^{\infty}\int_{c_2}^{\infty} c_1 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{(1-\Phi(0))\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 \tag{B.9}$$

$$\mathbf{E}[\tilde{c}_2|\tilde{s} = s_0] := \mathbf{E}\big[\mathbf{E}[\tilde{c}_2|c_1 \wedge (c_1 \leq c_2)]\big] = \int_{-\infty}^{\infty}\int_{c_1}^{\infty} c_2 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{\Phi(0)\varphi_1(c_1)}\mathrm{d}c_2\mathrm{d}c_1 \tag{B.10}$$

$$\mathbf{E}[\tilde{c}_2|\tilde{s} = s_1] := \mathbf{E}\big[\mathbf{E}[\tilde{c}_2|c_1 \wedge (c_1 > c_2)]\big] = \int_{-\infty}^{\infty}\int_{-\infty}^{c_1} c_2 \frac{\varphi_{1,2}(c_1, c_2, \rho)}{(1-\Phi(0))\varphi_1(c_1)}\mathrm{d}c_2\mathrm{d}c_1 \tag{B.11}$$

Finally, it is worthwhile to note that $\mathbf{E}\big[\mathbf{E}[\tilde{c}_u|\tilde{s}]\big] = \mathbf{E}[c_u]$. Thus, the probability-weighted sum of the conditional expectations of operating costs $\mathbf{E}[\tilde{c}_u|\tilde{s} = s_i]$ given both fuel cost scenarios, equals the unconditional expectation of $\tilde{c}_u$, i.e.

$$\mathbf{E}\big[\mathbf{E}[\tilde{c}_u|\tilde{s}=s_i]\big] = \sum_{i=1}^{2}\left(\mathbf{P}(\tilde{s}=s_i)\mathbf{E}[\tilde{c}_u|\tilde{s}=s_i]\right) = \mathbf{E}[\tilde{c}_u]. \tag{B.12}$$

Recall that the conditional variance of a random variable $\tilde{x}$ given $\tilde{y}$ is defined as

$$\mathrm{Var}[\tilde{x}|\tilde{y}] := \mathbf{E}\big[\big(\tilde{x}-\mathbf{E}[\tilde{x}|\tilde{y}]\big)^2\big|y\big]. \tag{B.13}$$

In analogy to the conditional expectation, we denote the conditional variance given the fuel price scenario s_0

$$\sigma^2_{u|s_0} \equiv \mathrm{Var}[\tilde{c}_u|\tilde{s}=s_0] := \mathbf{E}\big[\mathrm{Var}[\tilde{c}_u|c_2\wedge(c_1\leq c_2)]\big] = \mathbf{E}\big[\mathbf{E}[\tilde{c}_u^2|c_2\wedge(c_1\leq c_2)]\big] - \mathbf{E}[\tilde{c}_u|s_0]^2 \tag{B.14}$$

$$\sigma^2_{u|s_1} \equiv \mathrm{Var}[\tilde{c}_u|\tilde{s}=s_1] := \mathbf{E}\big[\mathrm{Var}[\tilde{c}_u|c_2\wedge(c_1> c_2)]\big] = \mathbf{E}\big[\mathbf{E}[\tilde{c}_u^2|c_2\wedge(c_1> c_2)]\big] - \mathbf{E}[\tilde{c}_u|s_1]^2 \tag{B.15}$$

It can be obtained by incremental computation from the square expectations[1] with

$$\mathrm{Var}[\tilde{c}_1|\tilde{s}=s_0] = \int_{-\infty}^{\infty}\int_{-\infty}^{c_2} c_1^2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{\Phi(0)\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 - \big(\mathbf{E}[c_1|\tilde{s}=s_0]\big)^2 \tag{B.16}$$

$$\mathrm{Var}[\tilde{c}_1|\tilde{s}=s_1] = \int_{-\infty}^{\infty}\int_{c_2}^{\infty} c_1^2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{(1-\Phi(0))\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 - \big(\mathbf{E}[c_1|\tilde{s}=s_1]\big)^2 \tag{B.17}$$

$$\mathrm{Var}[\tilde{c}_2|\tilde{s}=s_0] = \int_{-\infty}^{\infty}\int_{c_1}^{\infty} c_2^2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{\Phi(0)\varphi_1(c_1)}\mathrm{d}c_2\mathrm{d}c_1 - \big(\mathbf{E}[c_2|\tilde{s}=s_0]\big)^2 \tag{B.18}$$

$$\mathrm{Var}[\tilde{c}_2|\tilde{s}=s_1] = \int_{-\infty}^{\infty}\int_{-\infty}^{c_1} c_2^2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{(1-\Phi(0))\varphi_1(c_1)}\mathrm{d}c_2\mathrm{d}c_1 - \big(\mathbf{E}[c_2|\tilde{s}=s_1]\big)^2 \tag{B.19}$$

In a straightforward manner we obtain for the conditional covariance $\sigma_{12|s_i} \equiv \mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|\tilde{s}=s_0]$

$$\begin{aligned}\mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|\tilde{s}=s_0] &= \mathbf{E}[\tilde{c}_1\tilde{c}_2|\tilde{s}=s_0] - \mathbf{E}[\tilde{c}_1|\tilde{s}=s_0]\cdot\mathbf{E}[\tilde{c}_2|\tilde{s}=s_0] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{c_2} c_1c_2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{\Phi(0)\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 - \mathbf{E}[\tilde{c}_1|\tilde{s}=s_0]\mathbf{E}[\tilde{c}_2|\tilde{s}=s_0]\end{aligned} \tag{B.20}$$

$$\begin{aligned}\mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|\tilde{s}=s_1] &= \mathbf{E}[\tilde{c}_1\tilde{c}_2|\tilde{s}=s_1] - \mathbf{E}[\tilde{c}_1|\tilde{s}=s_1]\cdot\mathbf{E}[\tilde{c}_2|\tilde{s}=s_1] \\ &= \int_{-\infty}^{\infty}\int_{c_2}^{\infty} c_1c_2\frac{\varphi_{1,2}(c_1,c_2,\rho)}{(1-\Phi(0))\varphi_2(c_2)}\mathrm{d}c_1\mathrm{d}c_2 - \mathbf{E}[\tilde{c}_1|\tilde{s}=s_1]\mathbf{E}[\tilde{c}_2|\tilde{s}=s_1]\end{aligned} \tag{B.21}$$

[1] Alternatively, the expected conditional variance could be derived from the law of total variance, i.e. $\mathrm{Var}[\tilde{x}] = \mathrm{Var}[\tilde{x}|\tilde{y}] + \mathrm{Var}\big[\mathbf{E}[\tilde{x}|\tilde{y}]\big]$. Based on the latter, the expected conditional variance can be written as

$$\mathbf{E}\big[\mathrm{Var}[\tilde{x}|\tilde{y}]\big] = \mathrm{Var}[\tilde{x}] - \mathbf{E}\big[\mathrm{Var}\big[\mathbf{E}[\tilde{x}|\tilde{y}]\big]\big]$$

.

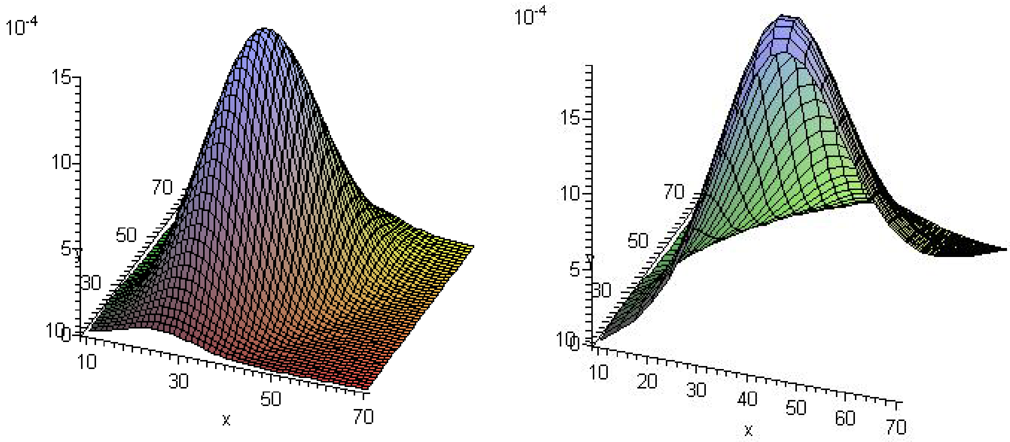

Fig. B.1: Unconditional bivariate density function $\varphi_{1,2}(c_1, c_2, \rho)$ (left) and conditional (truncated) bivariate density function $f_{1,2}(c_1, c_2|c_1 < c_2)$ (right).

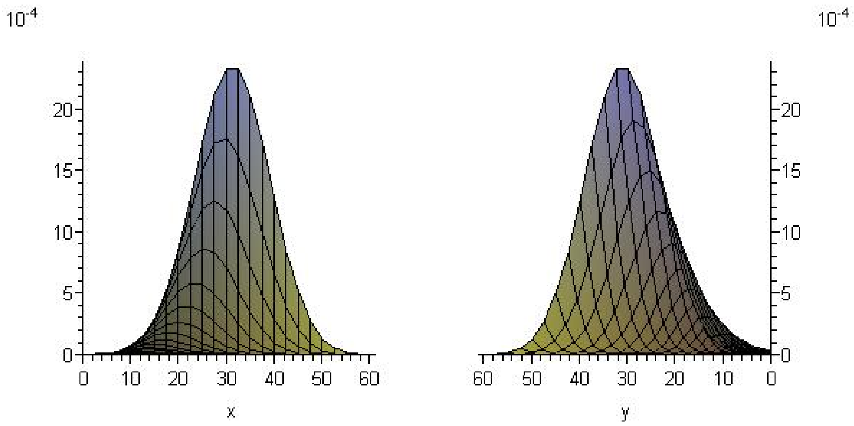

Fig. B.2: Marginal densities $f_1(c_1, c_2|c_1 < c_2)$ (left) and $f_2(c_1, c_2|c_1 < c_2)$ (right).

B.2.2 The risk-adjusted portfolio problem with merit order risk

Proof of Eqn. (4.10). Total expected generation costs can be calculated as

$$\begin{aligned}\mathbf{E}[C_{op}] &= \mathbf{E}\big[\mathbf{E}[C_{op}|\tilde{s}]\big] = \mathbf{E}\left[\mathbf{E}\Big[\sum_u \left(Q_{u|\tilde{s}}\tilde{c}_u\right)\Big|\tilde{s}\Big]\right] = \mathbf{E}\left[\sum_u \mathbf{E}\Big[\left(Q_{u|\tilde{s}}\tilde{c}_u\right)\Big|\tilde{s}\Big]\right] \\ &= \mathbf{E}\left[\sum_u Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\right] = \sum_u \mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big] = \sum_u\sum_i \left(\mathbf{P}(\tilde{s}=s_i)Q_{u|s_i}\mathbf{E}[\tilde{c}_u|\tilde{s}=s_i]\right)\end{aligned} \tag{B.22}$$

Similarly, conditional variance is used to calculate total variance of generation costs. Thereby, the total variance consists of intra-scenario variance and inter-scenario variance. For the variance of generation costs of technology u we obtain by inserting the conditional expectation as calculated above followed by rewriting

$$\begin{aligned}\mathrm{Var}[C_u] &= \mathbf{E}\Big[\mathbf{E}\big[(Q_{u|\tilde{s}})^2\tilde{c}_u^2|\tilde{s}\big]\Big] - \Big(\mathbf{E}\Big[\mathbf{E}\big[Q_{u|\tilde{s}}\tilde{c}_u\big|\tilde{s}\big]\Big]\Big)^2 \\ &= \mathbf{E}\Big[\mathbf{E}\big[(Q_{u|\tilde{s}})^2\tilde{c}_u^2|\tilde{s}\big]\Big] - \mathbf{E}\Big[\big(\mathbf{E}\big[Q_{u|\tilde{s}}\tilde{c}_u|\tilde{s}\big]\big)^2\Big] + \mathbf{E}\Big[\big(Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\big)^2\Big] - \Big(\mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\Big)^2 \\ &= \mathbf{E}\Big[\mathbf{E}\big[(Q_{u|\tilde{s}})^2\tilde{c}_u^2|\tilde{s}\big] - \big(\mathbf{E}\big[Q_{u|\tilde{s}}\tilde{c}_u|\tilde{s}\big]\big)^2\Big] + \mathbf{E}\Big[\big(Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\big)^2\Big] - \Big(\mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\Big)^2 \\ &= \mathbf{E}\Big[\mathrm{Var}\big[Q_{u|\tilde{s}}\tilde{c}_u\big|\tilde{s}\big]\Big] + \mathrm{Var}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big] \\ &= \mathbf{E}\Big[\big(Q_{u|\tilde{s}}\big)^2\cdot\mathrm{Var}\big[\tilde{c}_u\big|\tilde{s}\big]\Big] + \mathbf{E}\Big[\big(Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\big)^2\Big] - \Big(\mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\Big)^2 \\ &= \mathbf{E}\Big[\big(Q_{u|\tilde{s}}\big)^2\cdot\Big(\mathrm{Var}\big[\tilde{c}_u\big|\tilde{s}\big] + \big(\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\big)^2\Big)\Big] - \Big(\mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\Big)^2\end{aligned} \tag{B.23}$$

Similarly, the covariance of operating costs of technologies u, v can be derived as

$$\begin{aligned}\mathrm{Cov}[C_u, C_v] &= \mathbf{E}\Big[\mathbf{E}\big[(Q_{u|\tilde{s}})\tilde{c}_u\big|\tilde{s}\big]\cdot\mathbf{E}\big[(Q_{v|\tilde{s}})\tilde{c}_v\big|\tilde{s}\big]\Big] - \mathbf{E}\Big[\mathbf{E}\big[Q_{u|\tilde{s}}\tilde{c}_u\big|\tilde{s}\big]\Big]\cdot\mathbf{E}\Big[\mathbf{E}\big[Q_{v|\tilde{s}}\tilde{c}_v\big|\tilde{s}\big]\Big] \\ &= \mathbf{E}\Big[Q_{u|\tilde{s}}Q_{v|\tilde{s}}\mathrm{Cov}\big[\tilde{c}_u,\tilde{c}_v\big|\tilde{s}\big]\Big] + \mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]Q_{v|\tilde{s}}\mathbf{E}\big[\tilde{c}_v\big|\tilde{s}\big]\Big] \\ &\quad - \mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\mathbf{E}\Big[Q_{v|\tilde{s}}\mathbf{E}\big[\tilde{c}_v\big|\tilde{s}\big]\Big] \\ &= \mathbf{E}\Big[Q_{u|\tilde{s}}Q_{v|\tilde{s}}\Big(\mathrm{Cov}\big[\tilde{c}_u,\tilde{c}_v\big|\tilde{s}\big] + \mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\mathbf{E}\big[\tilde{c}_v\big|\tilde{s}\big]\Big)\Big] \\ &\quad - \mathbf{E}\Big[Q_{u|\tilde{s}}\mathbf{E}\big[\tilde{c}_u\big|\tilde{s}\big]\Big]\mathbf{E}\Big[Q_{v|\tilde{s}}\mathbf{E}\big[\tilde{c}_v\big|\tilde{s}\big]\Big]\end{aligned} \tag{B.24}$$

Hence, the total variance of operating costs for the set of all technologies $u = \{1, \ldots, n\}$ can be calculated as:

$$\mathrm{Var}[C_{op}] = \mathrm{Var}\left[\sum_u C_u\right] = \sum_u \mathrm{Var}[C_u] + \sum_u\sum_{v,v\neq u}\mathrm{Cov}[C_u, C_v]. \tag{B.25}$$

By inserting $\mathbf{E}[C_{op}]$ and $\mathrm{Var}[C_{op}]$ as derived above into Eqn. (4.5), we obtain as objective function of the optimization problem as formulated in Eqs. (4.9)-(4.12):

$$
\begin{aligned}
L =& \sum_u \left(K_u c_{inv,u} + \mathbf{E}\left[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\right] \right) \\
&+ \frac{A}{2} \sum_u \sum_v \left(\mathbf{E}\Big[Q_{u|\tilde{s}} Q_{v|\tilde{s}} (\mathrm{Cov}[\tilde{c}_u, \tilde{c}_v|\tilde{s}] + \mathbf{E}[\tilde{c}_u|\tilde{s}] \cdot \mathbf{E}[\tilde{c}_v|\tilde{s}])\Big] - \mathbf{E}\Big[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\Big] \cdot \mathbf{E}\Big[Q_{v|\tilde{s}} \mathbf{E}[\tilde{c}_v|\tilde{s}]\Big] \right) \\
=& \sum_u \left(K_u c_{inv,u} + \mathbf{E}\left[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\right] + \frac{1}{2} A \left(\mathbf{E}\Big[(Q_{u|\tilde{s}})^2 (\mathrm{Var}[\tilde{c}_u|\tilde{s}] + (\mathbf{E}[\tilde{c}_u|\tilde{s}])^2)\Big] - \left(\mathbf{E}\Big[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\Big]\right)^2 \right.\right. \\
&\left.\left. + \sum_{v, v \neq u} \left(\mathbf{E}\Big[Q_{u|\tilde{s}} Q_{v|\tilde{s}} (\mathrm{Cov}[\tilde{c}_u, \tilde{c}_v|\tilde{s}] + \mathbf{E}[\tilde{c}_u|\tilde{s}] \cdot \mathbf{E}[\tilde{c}_v|\tilde{s}])\Big] - \mathbf{E}\Big[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\Big] \cdot \mathbf{E}\Big[Q_{v|\tilde{s}} \mathbf{E}[\tilde{c}_v|\tilde{s}]\Big] \right) \right) \right) \\
=& \sum_u \left(K_u c_{inv,u} + \mathbf{E}\left[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\right] + \frac{1}{2} A \left(\mathbf{E}\Big[(Q_{u|\tilde{s}})^2 (\mathrm{Var}[\tilde{c}_u|\tilde{s}] + (\mathbf{E}[\tilde{c}_u|\tilde{s}])^2)\Big] - \left(\mathbf{E}\Big[Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}]\Big]\right)^2 \right) \right) \\
&+ A \left(\mathbf{E}\Big[Q_{1|\tilde{s}} Q_{2|\tilde{s}} (\mathrm{Cov}[\tilde{c}_1, \tilde{c}_2|\tilde{s}] + \mathbf{E}[\tilde{c}_1|\tilde{s}] \cdot \mathbf{E}[\tilde{c}_2|\tilde{s}])\Big] - \mathbf{E}\Big[Q_{1|\tilde{s}} \mathbf{E}[\tilde{c}_1|\tilde{s}]\Big] \cdot \mathbf{E}\Big[Q_{2|\tilde{s}} \mathbf{E}[\tilde{c}_2|\tilde{s}]\Big] \right) \\
=& \sum_u \left(K_u c_{inv,u} + \sum_{i=0}^{1} \mathbf{P}(\tilde{s} = s_i) \left(Q_{u|s_i} \mathbf{E}[\tilde{c}_u|s_i] + \frac{1}{2} A \sum_v Q_{u|s_i} Q_{v|s_i} (\mathrm{Cov}[\tilde{c}_u, \tilde{c}_v|s_i] + \mathbf{E}[\tilde{c}_u|s_i] \mathbf{E}[\tilde{c}_v|s_i]) \right) \right. \\
&\left. + A Q_{1|\tilde{s}} Q_{2|\tilde{s}} (\mathrm{Cov}[\tilde{c}_1, \tilde{c}_2|\tilde{s}] + \mathbf{E}[\tilde{c}_1|\tilde{s}] \cdot \mathbf{E}[\tilde{c}_2|\tilde{s}]) \right) \\
&- \frac{A}{2} \sum_{i=0}^{1} \mathbf{P}(\tilde{s} = s_i) \left(\sum_u Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}] \right)^2 + \frac{A}{2} \left(\sum_{i=0}^{1} \mathbf{P}(\tilde{s} = s_i) Q_{u|\tilde{s}} \mathbf{E}[\tilde{c}_u|\tilde{s}] \right)^2
\end{aligned}
\tag{B.26}
$$

□

B.2.3 Standard solutions to the purely cost efficient portfolio with uncertainty in the merit order

Proof of Eqs. (4.29) and (4.30). In an extensive form, the Lagrangian (4.28) can be written as

$$
\begin{aligned}
\mathcal{L}_c = & \sum_{u=1}^{2} K_u c_{inv,u} + Q_{1|s_0} \mathbf{P}(s_0) \bar{c}_{1|s_0} + Q_{1|s_1} \mathbf{P}(s_1) \bar{c}_{1|s_1} \\
& + Q_{2|s_0} \mathbf{P}(s_0) \bar{c}_{2|s_0} + Q_{2|s_1} \mathbf{P}(s_1) \bar{c}_{2|s_1} + \lambda(D(0) - K_1 - K_2)
\end{aligned}
\tag{B.27}
$$

Denoting $z := c_2 - c_1$, the KKT-conditions (4.29) and (4.30) can be derived from $\mathcal{L}_c$ as follows:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_c}{\partial K_1} &= c_{inv,1} - \lambda + \frac{\partial Q_{1|s_0}}{\partial K_1} \cdot \mathbf{P}(s_0) \bar{c}_{1|s_0} + \frac{\partial Q_{2|s_0}}{\partial K_1} \cdot \mathbf{P}(s_0) \bar{c}_{2|s_0} \\
&= c_{inv,1} - \lambda + t_1 \mathbf{P}(s_0) \bar{c}_{1|s_0} - t_1 \mathbf{P}(s_0) \bar{c}_{2|s_0} = t_1 \mathbf{P}(s_0) \cdot \mathbf{E}[\tilde{z}|s_0]
\end{aligned}
\tag{B.28}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_c}{\partial K_2} &= c_{inv,2} - \lambda + \frac{\partial Q_{1|s_1}}{\partial K_2} \cdot \mathbf{P}(s_1) \bar{c}_{1|s_1} + \frac{\partial Q_{2|s_1}}{\partial K_2} \cdot \mathbf{P}(s_1) \bar{c}_{2|s_1} \\
&= c_{inv,1} - \lambda - t_2 \mathbf{P}(s_1) \bar{c}_{1|s_1} + t_2 \mathbf{P}(s_1) \bar{c}_{2|s_1} = t_2 \mathbf{P}(s_1) \cdot \mathbf{E}[\tilde{z}|s_1]
\end{aligned}
\tag{B.29}
$$

□

Proposition 4.2.1. *With $z := c_2 - c_1$ denoting the difference in operating costs of technologies 1 and 2, the purely cost-minimal portfolio with merit order risk $\mathbf{P}(s_1)$ consists of technology 2 if and only if*

$$
c_{inv,1} - c_{inv,2} \geq T\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0]. \tag{4.34}
$$

In contrast, the portfolio consists of technology 1 if and only if

$$c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_1]. \tag{4.35}$$

Proof of Proposition 4.2.1. We will first prove the equivalence $K_2^* = 0 \Leftrightarrow c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\left(\bar{c}_{2|s_1} - \bar{c}_{1|s_1}\right)$ by showing the validity of the two implications:

Given $K_2^* = 0$, it can be concluded in the case $\lambda > 0$ (i.e. no overcapacities in the optimum) $t_1^* = 0$ since $K_1 = D(0) = D(t_1^*)$ according to Eqn. (4.31). Since for the lower bound of the operating time of the respective base load technology it holds $D(t_u^*) = K_u^*$, it must furthermore be $t_2^* = T$. Since Eqn. (4.29) holds with equality, λ can be be eliminated by subtraction of Eqs. ((4.29) and ((4.30) yielding

$$c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\left(\bar{c}_{2|s_1} - \bar{c}_{1|s_1}\right)$$

The case $\lambda = 0$ (i.e. there may be overcapacities in the optimum) can be rejected with the initial assumption: Since $K_2^* = 0$ implies $t_1^* = 0$, KKT condition (4.30) yields the contradiction $c_{inv,1} = 0$.[2]

To prove the converse implication, we assume for $c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\left(\bar{c}_{2|s_1} - \bar{c}_{1|s_1}\right)$ without limitation of the generality $K_2^* > 0$. This implies $0 \leq t_2^* < T$, $0 < t_1^* \leq T$. Hence KKT condition (4.30) has to be fulfilled with equality. Eliminating λ through subtraction of KKT conditions (4.29) and (4.30) yields

$$c_{inv,1} - c_{inv,2} \geq t_1^*\mathbf{P}(s_0)(\bar{c}_{2|s_0} - \bar{c}_{1|s_0}) + t_2^*\mathbf{P}(s_1)(\bar{c}_{2|s_1} - \bar{c}_{1|s_1})$$

However, the latter inequality is contradictory to the initial assumption for all feasible $0 \leq t_2^* < T$, $0 < t_1^* \leq T$ since $(\bar{c}_{2|s_0} - \bar{c}_{1|s_0}) > 0$ and $(\bar{c}_{2|s_1} - \bar{c}_{1|s_1}) < 0$. Thus it follows $K_2^* = 0$ from $c_{inv,1} - c_{inv,2} \leq T\mathbf{P}(s_1)\left(\bar{c}_{2|s_1} - \bar{c}_{1|s_1}\right)$.

The proof of the analogue equivalence $K_1^* = 0 \Leftrightarrow c_{inv,1} - c_{inv,2} \geq -T\mathbf{P}(s_0)\left(\bar{c}_{2|s_0} - \bar{c}_{1|s_0}\right)$ can be obtained in a straightforward manner analogue to this proof. □

B.2.4 Standard solutions to the purely variance efficient portfolio with uncertainty in the merit order

Proposition 4.2.2. *Under the restriction that total installed generation capacity must match total demand, i.e.* $\lambda \neq 0$*, the purely variance minimal portfolio with a merit order risk* $\mathbf{P}(s_1) > 0$ *corresponding to optimization problem (4.9)-(4.12) consists of both technologies 1 and 2 if*

$$\left(-\frac{\sigma_{12|s_0} - \sigma^2_{1|s_0}}{\mathbf{E}[\tilde{z}|s_0]\mathbf{P}(s_1)} > \bar{c}_{1|s_1} - \bar{c}_{1|s_0}\right) \wedge \left(\bar{c}_{1|s_1} - \bar{c}_{1|s_0} < \frac{\sigma^2_{1|s_1} - \sigma_{12|s_1}}{\mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)}\right) \tag{4.42}$$

Proof of Proposition 4.2.2. Given the initial assumption $\lambda \neq 0$, t_2 can be expressed as a function of t_1 throughout this proof with $t_2(t_1) = R(D(0) - D(t_1))$. Hence, t_1 can be used as the only decision

[2] The rejection of the case $\lambda = 0$ is intuitively plausible from an economical perspective, since it does not make sense to build overcapacities in the optimum if only one technology is selected.

variable in the problem since K_1, K_2, t_2 are all functions of t_1. Theoretically, locating the root $\frac{\mathrm{d}\mathcal{L}_v(t_1,t_2(t_1))}{\mathrm{d}t_1} = 0$ would allow to further discuss the considered solution case.[3] Since an analytical discussion of the latter derivative seems practically impossible, we use an alternative approach in the proof:

For an interior solution with $0 < t_1^* < T$ (and hence $K_1^*, K_2^* > 0$), KKT conditions (4.38) and (4.39) have to be satisfied with equality as necessary optimality condition. Eliminating λ through subtraction of these conditions yields $\frac{\partial \mathcal{L}_v}{\partial K_1} - \frac{\partial \mathcal{L}_v}{\partial K_2} = 0$. For $t_1 > 0$, the latter condition may be equivalently transformed by division through t_1. Substitution of $Q_{2|s_i}$ by utilizing the relation $Q_{2|s_i} = Q_E - Q_{1|s_i}$, $i \in \{1,2\}$, finally yields the equivalent optimality condition $l(t_1) = r(t_1)$ with

$$\begin{aligned} l(t_1) := \mathbf{P}(s_0)\Big(\Big(Q_{1|s_0}\left(\mathrm{Var}[\tilde{z}|s_0] + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0]^2\right) - \mathbf{P}(s_1)Q_{1|s_1}\mathbf{E}[\tilde{z}|s_1]\mathbf{E}[\tilde{z}|s_0]\Big) - \\ - Q_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}[\tilde{z}|s_0]\right)\Big) \end{aligned} \tag{B.30}$$

$$\begin{aligned} r(t_1) := \mathbf{P}(s_1)\frac{t_2}{t_1}\Big(\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0]Q_{1|s_0}\mathbf{E}[\tilde{z}|s_1] - \left(\mathrm{Var}[\tilde{z}|s_1] + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1]^2\right)Q_{1|s_1} + \\ + Q_E\left(\sigma^2_{2|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)(\bar{c}_{2|s_1} - \bar{c}_{2|s_0})\mathbf{E}[\tilde{z}|s_1]\right)\Big) \end{aligned} \tag{B.31}$$

Thereby, the corresponding boundaries for $l(t_1)$ can straightforwardly be obtained as

$$l(0) = \mathbf{P}(s_0)Q_E\left(\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0](\bar{c}_{1|s_1} - \bar{c}_{1|s_0}) + \sigma^2_{1|s_0} - \sigma_{12|s_0}\right), \tag{B.32}$$

$$l(T) = -\mathbf{P}(s_0)Q_E\left(\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0](\bar{c}_{2|s_0} - \bar{c}_{2|s_1}) + \sigma^2_{2|s_0} - \sigma_{12|s_0}\right). \tag{B.33}$$

By applying the derivatives

$$\frac{\mathrm{d}t_2(t_1)}{\mathrm{d}t_1} = \frac{\mathrm{d}}{\mathrm{d}t_1}\big(R(D(0) - D(t_1))\big) = -R'(D(0) - D(t_1)) \cdot D'(t_1) < 0 \quad \forall\, t_1 \in [0,T], \tag{B.34}$$

$$\frac{\mathrm{d}Q_{1|s_0}(t_1)}{\mathrm{d}t_1} = \frac{\mathrm{d}}{\mathrm{d}t_1}\left(\int_0^{D(t_1)} R(\kappa)\mathrm{d}\kappa\right) = D'(t_1) \cdot t_1 < 0 \quad \forall\, t_1 \in [0,T], \tag{B.35}$$

$$\frac{\mathrm{d}Q_{1|s_1}(t_1)}{\mathrm{d}t_1} = \frac{\mathrm{d}}{\mathrm{d}t_1}\left(\int_{D(0)-D(t_1)}^{D(0)} R(\kappa)\mathrm{d}\kappa\right) = -R'(D(0) - D(t_1)) \cdot D'(t_1) < 0 \quad \forall\, t_1 \in [0,T], \tag{B.36}$$

it becomes obvious that $l(t_1)$ is monotone decreasing on its domain, i.e. $\frac{\mathrm{d}l(t_1)}{\mathrm{d}t_1} < 0\ \forall t_1 \in [0;T]$. Function $r(t_1)$ can be considered as being of the form $r(t_1) = f(t_1)/t_1$, where $f(t_1)$ is defined as the function $f(t_1) := r(t_1) \cdot t_1$. We obtain for $f(0)$ and the boundaries of $r(t_1)$

$$f(0) = \mathbf{P}(s_1)TQ_E\left(\mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)(\bar{c}_{1|s_1} - \bar{c}_{1|s_0}) - \sigma^2_{1|s_1} + \sigma_{12|s_1}\right), \tag{B.37}$$

$$r(T) = 0, \qquad \lim_{t_1 \searrow 0} r(t_1) = \begin{cases} +\infty & \text{for } f(0) > 0 \\ 0 & \text{for } f(0) = 0 \\ -\infty & \text{for } f(0) < 0 \end{cases} \tag{B.38}$$

[3] Note that the derivative $\frac{\mathrm{d}\mathcal{L}_v(t_1,t_2(t_1))}{\mathrm{d}t_1}$ equals the directional derivative $\nabla_{\vec{t}}\mathcal{L}_v(K_1,K_2) = \frac{\partial \mathcal{L}_v(K_1,K_2)}{\partial K_1} \cdot t_1 + \frac{\partial \mathcal{L}_v(K_1,K_2)}{\partial K_2} \cdot t_2(t_1)$ with the directional vector $\vec{t} = (t_1, t_2(t_1))^T$ of unit length, i.e. $|\vec{t}| = 1$.

Furthermore, it can be concluded that $r(t_1)$ is monotone increasing (and decreasing, respectively) in an interval $(0;\xi)$ for arbitrary means and variances, before it may start to decrease if $f(0) < 0$ (and increase if $f(0) > 0$, respectively). Thereby, non-monotony of $r(t_1)$ can only occur if

$$g := \sigma^2_{2|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)(\bar{c}_{2|s_1} - \bar{c}_{2|s_0})\mathbf{E}[\tilde{z}|s_1] \begin{cases} > 0 \text{ for } f(0) < 0 \\ < 0 \text{ for } f(0) > 0 \end{cases}$$

where g refers to the last summand of $f(t_1)$.

Since an explicit formulation of the stationary points is not possible and the problem is non-convex in variable t_1, we use the first derivatives to test whether one of the possibly multiple stationary points is a local minimum or a local maximum: If there exists a $r \in \mathbb{R}_+$ such that for every $t_1 \in (t_1^o - r, t_1^o]$ it holds $\frac{\partial \mathcal{L}_v}{\partial K_2}(t_1) - \frac{\partial \mathcal{L}_v}{\partial K_1}(t_1) \leq 0$, and for every $t_1 \in [t_1^o, t_1^o + r)$ it is $\frac{\partial \mathcal{L}_v}{\partial K_2}(t_1) - \frac{\partial \mathcal{L}_v}{\partial K_1}(t_1) \geq 0$, then $\mathcal{L}_v$ has a local minimum at t_1^o according to the mean value theorem. Consequently, a local minimum in t_1^o implies that $r(t_1) - l(t_1) \leq 0$, $\forall t_1 \in (t_1^o - r, t_1^o]$ and $r(t_1) - l(t_1) \geq 0$, $\forall t_1 \in [t_1^o, t_1^o + r)$. The obtained stationary points $t_1^o \in (0;T)$ with $l(t_1^o) = r(t_1^o)$ and therewith $\frac{\partial \mathcal{L}_v}{\partial K_1}(t_1^o) = \frac{\partial \mathcal{L}_v}{\partial K_2}(t_1^o) = 0$ can be characterized as follows corresponding to Figure B.3:

I) For $l(0) \geq 0 \wedge f(0) \leq 0$, there may exist up to two stationary points t_1^o:

- For two stationary points to exist, it is required $g > 0$. Then $\mathcal{L}_v(t_1^o)$ takes a minimum if $r(t_1) - l(t_1) \leq 0$, $\forall t_1 \in (t_1^o - r, t_1^o]$ and $r(t_1) - l(t_1) \geq 0$, $\forall t_1 \in [t_1^o, t_1^o + r)$. Hence, the efficient portfolio consists of both technologies.
- For $g \leq 0 \wedge l(T) \leq 0$ there exists a unique local minimum at t_1^o and the variance efficient portfolio consists of a combination of both technologies.
- Otherwise, if a feasible stationary point t_1^o does not exist, the variance efficient portfolio consists only of technology 2.[4]

II) For $l(0) < 0 \wedge f(0) \leq 0$ (equivalent to condition (4.42)), there exists a unique local minimum at t_1^o. Hence, the variance efficient portfolio consists of both technologies.

III) For $l(T) \geq 0 \wedge f(0) > 0$, there exists a unique local maximum at t_1^o, since it is $r(t_1) - l(t_1) \geq 0$, $\forall t_1 \in (t_1^o - r, t_1^o]$ and $r(t_1) - l(t_1) \leq 0$, $\forall t_1 \in [t_1^o, t_1^o + r)$. Hence, it can be concluded that the variance efficient portfolio consists only of technology 2.

IV) For $l(T) < 0 \wedge l(0) > 0 \wedge (f(0) > 0)$, there may exist up two feasible stationary points t_1^o:

- In case of two stationary points , $\mathcal{L}_v(t_1^o)$ takes the minimum if $r(t_1) - l(t_1) \leq 0$, $\forall t_1 \in (t_1^o - r, t_1^o]$ and $r(t_1) - l(t_1) \geq 0$, $\forall t_1 \in [t_1^o, t_1^o + r)$. Hence, the efficient portfolio consists of both technologies.
- Otherwise, if a feasible stationary point t_1^o does not exist, the variance efficient portfolio consists only of technology 1.

[4]Since $l(T) \geq 0$ requires the variance of technology 1 to be much greater than the variance of technology 2, it can be concluded that the variance efficient portfolio will only consist of technology 2, not of technology 1.

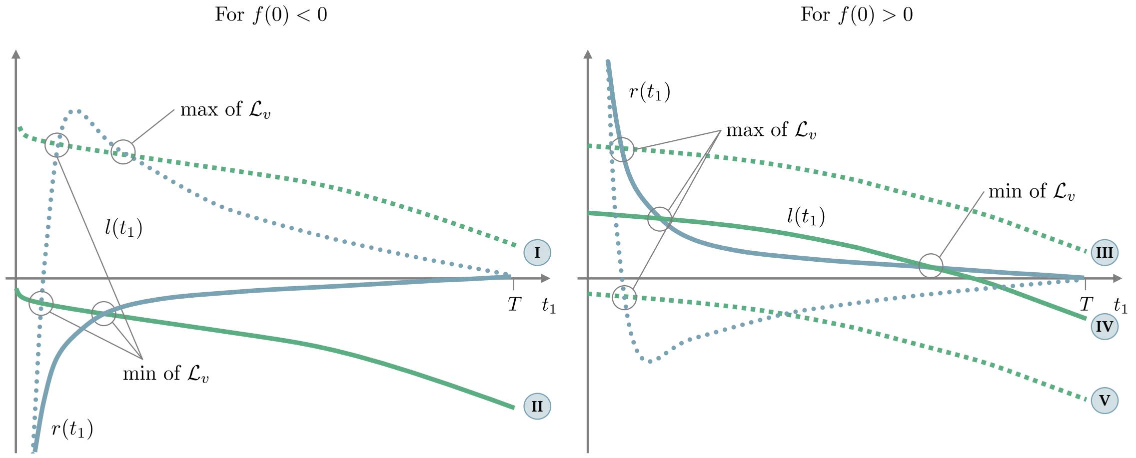

Fig. B.3: Stationarity conditions for the purely variance-minimal portfolio in the case $\lambda > 0$. The intersection of $l(t_1)$ (green line) and $r(t_1)$ (blue line) represents the optimal operating time.

V) For $l(0) \leq 0 \wedge f(0) > 0$, there may exist up two feasible stationary points t_1^o:

– For two stationary points to exist, it is required $g < 0$. Then, $\mathcal{L}_v(t_1^o)$ takes the minimum if $r(t_1) - l(t_1) \leq 0$, $\forall t_1 \in (t_1^o - r, t_1^o]$ and $r(t_1) - l(t_1) \geq 0$, $\forall t_1 \in [t_1^o, t_1^o + r)$. In this case, the efficient portfolio consists of both technologies.

– Otherwise, if a feasible stationary point t_1^o does not exist (for which $g > 0$ is sufficient), the variance efficient portfolio consists only of technology 1.[5]

□

[5]Since this case requires the variance of technology 2 to be much greater than the variance of technology 1, the efficient portfolio consists of technology 1 and not of technology 2.

B.2.5 Standard solutions to the combined portfolio problem with uncertainty in the merit order

Proof of Eqs. (4.44) and (4.45). For the case with two technologies, the KKT conditions (4.44) and (4.45) can be derived from the Lagrangian (4.43) as follows:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_1} &= c_{inv,1} - \lambda - \mathbf{P}(s_0)t_1\Bigg(\sum_{u=1}^{2}(-1)^u\Bigg(\mathbf{E}[\tilde{c}_u|s_0] + AQ_{u|s_0}\Bigg(\mathrm{Var}[\tilde{c}_u|s_0] - \mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|s_0] \\
&\quad + \mathbf{P}(s_1)\Bigg(\mathbf{E}[\tilde{c}_u|s_0]^2 - \mathbf{E}[\tilde{c}_2|s_0]\mathbf{E}[\tilde{c}_1|s_0] - \mathbf{E}[\tilde{c}_u|s_0]\sum_{v=1}^{2}\mathbf{E}[\tilde{c}_v|s_1]\frac{Q_{v|s_1}}{Q_{u|s_0}}\Bigg)\Bigg)\Bigg)\Bigg) \\
&= t_1\mathbf{P}(s_0)A\Big(Q_{1|s_0}\left(\mathrm{Var}[\tilde{z}|s_0] + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0]^2\right) \\
&\quad - Q_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}[\tilde{z}|s_0]\right)\Big) \\
&\quad - t_1\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0]\left(\mathbf{P}(s_1)Q_{1|s_1}\mathbf{E}[\tilde{z}|s_1] + 1\right) + c_{inv,1} - \lambda
\end{aligned} \tag{B.39}$$

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial K_2} &= c_{inv,2} - \lambda + \mathbf{P}(s_1)t_2\Bigg(\sum_{u=1}^{2}(-1)^u\Bigg(\mathbf{E}[\tilde{c}_u|s_1] + AQ_{u|s_1}\Bigg(\mathrm{Var}[\tilde{c}_u|s_1] - \mathrm{Cov}[\tilde{c}_1,\tilde{c}_2|s_1] \\
&\quad + \mathbf{E}[\tilde{c}_u|s_1]\sum_{v=1}^{2}\mathbf{E}[\tilde{c}_v|s_0]\frac{Q_{v|s_0}}{Q_{u|s_1}}\Bigg)\Bigg)\Bigg) \\
&= t_2\mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_1]\left(A\mathbf{P}(s_0)Q_{1|s_0}\mathbf{E}[\tilde{z}|s_0] + 1\right) - t_2\mathbf{P}(s_1)A\Big(Q_{1|s_1}\left(\mathrm{Var}[\tilde{z}|s_1] + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1]^2\right) \\
&\quad - Q_E\left(\sigma^2_{2|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)(\bar{c}_{2|s_1} - \bar{c}_{2|s_0})\mathbf{E}[\tilde{z}|s_1]\right)\Big) + c_{inv,2} - \lambda
\end{aligned} \tag{B.40}$$

□

Proposition 4.2.3 (Existence and uniqueness of an interior solution for the case $\lambda \neq 0$)**.** *Let be $c_{inv,1} > c_{inv,2}$ and total installed capacity matching maximum demand, i.e. $\lambda \neq 0$ in constraint (4.12). If both technologies 1 and 2 are included in the purely cost-efficient portfolio (i.e. $A = 0$) and in the purely variance-efficient portfolio (i.e. $A \to +\infty$) satisfying condition (4.42), then all cost-variance efficient portfolios with $A > 0$ corresponding to problem (4.9)-(4.12) consist of a unique combination of both technologies.*

Proof of Proposition 4.2.3. According to the assumption $\lambda \neq 0$, t_2 can be expressed as a function of the t_1 throughout this proof with $t_2(t_1) = R(D(0) - D(t_1))$. Herewith, the solution condition $v(t_1) = 0$ in Eqn. (4.47) can be derived as follows: For an interior solution with $K_1^*, K_2^* > 0$, KKT conditions (4.44) and (4.45) have to be satisfied with equality as necessary optimality condition. Eliminating λ through subtraction of these conditions yields $\frac{\partial \mathcal{L}}{\partial K_1} - \frac{\partial \mathcal{L}}{\partial K_2} = 0$. For $t_1 > 0$, the latter condition may equivalently be written as $l_3(t_1) = r_3(t_1)$ defined by

$$\begin{aligned}
l_3(t_1) &:= \mathbf{P}(s_0)A\Big(Q_{1|s_0}\left(\mathrm{Var}[\tilde{z}|s_0] + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0]^2\right) - \left(\mathbf{P}(s_1)Q_{1|s_1}\mathbf{E}[\tilde{z}|s_1] + 1/A\right)\mathbf{E}[\tilde{z}|s_0]\Big) \\
&\quad + \frac{c_{inv,1} - c_{inv,2}}{t_1} \\
r_3(t_1) &:= \mathbf{P}(s_0)AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}[\tilde{z}|s_0]\right) \\
&\quad + \mathbf{P}(s_1)A\frac{t_2}{t_1}\Big(Q_E\left(\sigma^2_{2|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)(\bar{c}_{2|s_1} - \bar{c}_{2|s_0})\mathbf{E}[\tilde{z}|s_1]\right) && \text{(B.41)} \\
&\quad - \left(\mathrm{Var}[\tilde{z}|s_1] + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1]^2\right)Q_{1|s_1} + \left(\mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0]Q_{1|s_0} + 1/A\right)\mathbf{E}[\tilde{z}|s_1]\Big) && \text{(B.42)}
\end{aligned}$$

By taking into account the sign of the derivatives as shown in Eqn. (B.34)-(B.36) in the previous proof, the behavior of $l_3(t_1), r_3(t_1)$ can be characterized at the boundaries as follows:

$$\lim_{t_1 \searrow 0} l_3(t_1) = \begin{cases} +\infty & \text{for } c_{inv,1} > c_{inv,2} \\ 0 & \text{for } c_{inv,1} = c_{inv,2} \\ -\infty & \text{for } c_{inv,1} < c_{inv,2} \end{cases}, \tag{B.43}$$

$$l_3(T) = \frac{c_{inv,1} - c_{inv,2}}{T} - \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_0], \tag{B.44}$$

$$\lim_{t_1 \searrow 0} r_3(t_1) = \begin{cases} +\infty & \text{for } f_1(0) > 0 \\ 0 & \text{for } f_1(0) = 0 \\ -\infty & \text{for } f_1(0) < 0 \end{cases}, \tag{B.45}$$

$$r_3(T) = \mathbf{P}(s_0)AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}\left[\tilde{z}|s_0\right]\right) \tag{B.46}$$

with the definition $f_1(0) := \mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)(\bar{c}_{1|s_1} - \bar{c}_{1|s_0}) + \frac{\mathbf{E}[\tilde{z}|s_1]}{Q_E A} - \sigma^2_{1|s_1} + \sigma_{12|s_1}$.

In case $c_{inv,1} > c_{inv,2}$, it can be concluded that $\frac{\mathrm{d}l_3(t_1)}{\mathrm{d}t_1} < 0$ and hence $l_3(t_1)$ is monotone decreasing as shown in Figure B.4. From our initial assumption that the purely cost-minimal portfolio consists of both technologies, we can further conclude $l_3(T) < 0$ according to Proposition 4.2.1. Thus, a unique intersection point of $l_3(t_1)$ and $r_3(t_1)$ and therewith a unique interior solution with $0 < t_1^*, t_2^* < T$ is obtained if and only if $r_3(T) > l_3(T)$. Given that the variance-minimal portfolio consists of both technologies satisfying condition (4.42), it can be concluded $f(0) \leq 0$. This implies also $f_1(0) \leq$ and therewith $\lim_{t_1 \searrow 0} r_3(t_1) \leq 0$. Furthermore, it follows $r_3(T) > 0$ because the purely variance efficient portfolio consists of technology 2 if and only if $r_3(T) \leq 0$ according to Proposition 4.2.2.

Assuming $c_{inv,1} < c_{inv,2}$ yields again $l_3(T) < 0$ and with the purely variance-minimal portfolio consisting of both technologies $r_3(T) > 0$. Hence a unique intersection point of $l_3(t_1)$ and $r_3(t_1)$ (cf. Figure B.4) and therewith a unique interior solution with $0 < t_1^*, t_2^* < T$ is obtained if and

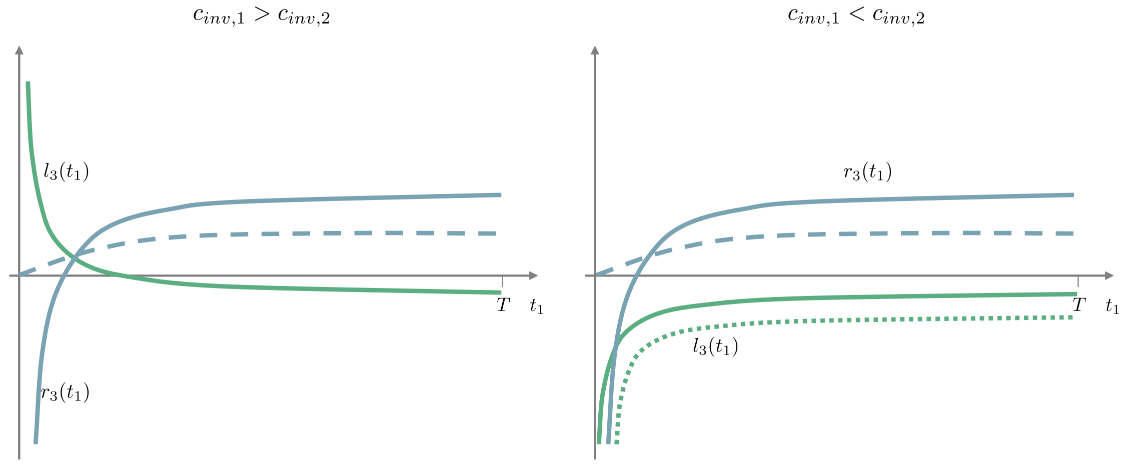

Fig. B.4: Stationarity conditions for the cost-variance efficient portfolio in the cases $c_{inv,1} > c_{inv,2}$ (left) and $c_{inv,1} < c_{inv,2}$ (right). The intersection of $l_3(t_1)$ (green line) and $r_3(t_1)$ (blue line) represents the optimal operating time.

only if

$$\lim_{t_1 \searrow 0} r_3(t_1) - l_3(t_1) \leq 0$$
$$\Leftrightarrow \qquad f_1(0) - c_{inv,1} + c_{inv,2} \leq 0$$
$$\Leftrightarrow \qquad f(0) + \frac{\mathbf{E}[\tilde{z}|s_1]}{Q_E A} - c_{inv,1} + c_{inv,2} \leq 0$$

Notably, the latter inequality is however only satisfied if A is greater than a defined threshold A_0. This is why for $c_{inv,1} > c_{inv,2}$, the cost-variance efficient portfolio does not necessarily consist of both technologies. □

Proposition 4.2.4 (Implicit functions)**.** *For the implicit function $\frac{\partial \mathcal{L}}{\partial K_1}(\check{t}_1, \check{t}_2) = 0$ with $\frac{\partial \mathcal{L}}{\partial K_1}(t_1, t_2)$: $[0,T] \times [0,T] \to Z_1 \subseteq \mathbb{R}_+$, there exists a unique function $\zeta_1(t_2) : (0,T] \to Z_1 \subseteq \mathbb{R}_+$. Similarly, for the implicit function $\frac{\partial \mathcal{L}}{\partial K_2}(\check{t}_1, \check{t}_2) = 0$ with $\frac{\partial \mathcal{L}}{\partial K_2}(t_1, t_2) : [0,T] \times [0,T] \to Z_2 \subseteq \mathbb{R}_+$, there exists a unique function $\zeta_2(t_1) : (0,T] \to Z_2 \subseteq \mathbb{R}_+$.*

Proof of Proposition 4.2.4. According to our initial assumptions $D(t)$ is strictly monotone decreasing, the likelihood for a fuel switch is non-negative (i.e. $\mathbf{P}(s_0) \geq 0$) and all individuals act strictly risk averse (i.e. $A > 0$). By partial differentiation of Eqs. (4.44), (4.45) we obtain

$$\frac{\partial}{\partial t_i}\frac{\partial \mathcal{L}}{\partial K_i}(t_1, t_2) = At_1t_2\mathbf{P}(s_1)\mathbf{P}(s_0) \cdot (c_{1|s_0} - c_{2|s_0})(c_{1|s_1} - c_{2|s_1})\frac{\mathrm{d}D(t_i)}{\mathrm{d}t_i} \neq 0 \quad \forall t_i > 0, \ (i = 1, 2)$$

According to the well-known implicit function theorem, the existence of the functions $t_1 = \zeta_1(t_2)$ and $t_2 = \zeta_2(t_1)$ is hence given for $t_2 > 0$ and $t_1 > 0$, respectively. □

Proposition 4.2.5 (Monotony)**.** *For given expected fuel prices $\bar{c}_{1|s_0} \leq \bar{c}_{2|s_0}$ and $\bar{c}_{2|s_1} \leq \bar{c}_{1|s_1}$ and $\zeta_1(t_2)$ and $\zeta_2(t_1)$ being functions represented by the implicit functions $\frac{\partial \mathcal{L}}{\partial K_1}(t_1, t_2) = 0$ from Eqn.*

(4.44), and $\frac{\partial\mathcal{L}}{\partial K_2}(t_1,t_2) = 0$ *from Eqn. (4.45), respectively,* $\zeta_1(t_2)$ *and* $\zeta_2(t_1)$ *are both monotone increasing in* t_2 *and* t_1*, respectively. For* $\bar{c}_{1|s_0} < \bar{c}_{2|s_0}$ *and* $\bar{c}_{2|s_1} < \bar{c}_{1|s_1}$*, it follows strict monotony of* $\zeta_1(t_2)$ *and* $\zeta_2(t_1)$*.*

Proof of Proposition 4.2.5. The necessary condition for an interior solution is given by the non-linear equation system with Eqs. (4.44), $\frac{\partial\mathcal{L}}{\partial K_1} = 0$, and (4.45), $\frac{\partial\mathcal{L}}{\partial K_2} = 0$ with both equations representing functions of K_1, K_2, and t_1, t_2, respectively. By rewriting and applying $Q_{1|s_i} = Q_E - Q_{2|s_i}, (i = 0, 1)$, we can bring optimality condition from Eqs. (4.44), (4.45) in a form where t_1, t_2 are separated to the two sides of the equation, i.e.

$$\frac{\partial\mathcal{L}}{\partial K_1}(t_1,t_2) = 0 \Leftrightarrow l_1(t_1) = r_1(t_2) \quad \text{and} \quad \frac{\partial\mathcal{L}}{\partial K_2}(t_1,t_2) = 0 \Leftrightarrow l_2(t_2) = r_2(t_1)$$

Then, we obtain

$$\begin{aligned} l_1(t_1) &:= \frac{c_{inv,1}}{t_1\mathbf{P}(s_0)} + AQ_{1|s_0}\left(\mathrm{Var}[\tilde{z}|s_0] + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0]^2\right) \\ &\quad - AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\mathbf{E}[\tilde{z}|s_0]\right) && \text{(B.47)} \\ r_1(t_2) &:= \mathbf{E}[\tilde{z}|s_0]\left(A\mathbf{P}(s_1)Q_{1|s_1}\mathbf{E}[\tilde{z}|s_1] + 1\right) && \text{(B.48)} \\ l_2(t_2) &:= \frac{c_{inv,2}}{t_2\mathbf{P}(s_1)} - AQ_{1|s_1}\left(\mathrm{Var}[\tilde{z}|s_1] + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1]^2\right) \\ &\quad + AQ_E\left(\sigma^2_{2|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)(\bar{c}_{2|s_1} - \bar{c}_{2|s_0})\mathbf{E}[\tilde{z}|s_1]\right) && \text{(B.49)} \\ r_2(t_1) &:= -\mathbf{E}[\tilde{z}|s_1]\left(A\mathbf{P}(s_0)Q_{1|s_0}\mathbf{E}[\tilde{z}|s_0] + 1\right) && \text{(B.50)} \end{aligned}$$

The first derivatives of $l_1(t_1)$, $r_1(t_2)$, $l_2(t_2)$, $r_2(t_1)$ can be derived as follows:

$$\begin{aligned} \frac{\mathrm{d}l_1(t_1)}{\mathrm{d}t_1} &= \frac{-c_{inv,1}}{t_1^2\mathbf{P}(s_0)} + At_1\cdot\left(\mathrm{Var}[\tilde{z}|s_0] + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0]^2\right)\left(\frac{\mathrm{d}D(t_1)}{\mathrm{d}t_1}\right) \le 0 && \forall\, t_1 > 0 && \text{(B.51)} \\ \frac{\mathrm{d}r_1(t_2)}{\mathrm{d}t_2} &= -At_2\mathbf{P}(s_1)\cdot\mathbf{E}[\tilde{z}|s_1]\mathbf{E}[\tilde{z}|s_0]\left(\frac{\mathrm{d}D(t_2)}{\mathrm{d}t_2}\right) \le 0 && \forall\, t_2 > 0 && \text{(B.52)} \\ \frac{\mathrm{d}l_2(t_2)}{\mathrm{d}t_2} &= \frac{-c_{inv,2}}{t_2^2\mathbf{P}(s_1)} + At_2\cdot\left(\mathrm{Var}[\tilde{z}|s_1] + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1]^2\right)\left(\frac{\mathrm{d}D(t_2)}{\mathrm{d}t_2}\right) \le 0 && \forall\, t_2 > 0 && \text{(B.53)} \\ \frac{\mathrm{d}r_2(t_1)}{\mathrm{d}t_1} &= -At_1\mathbf{P}(s_0)\cdot\mathbf{E}[\tilde{z}|s_0]\mathbf{E}[\tilde{z}|s_1]\left(\frac{\mathrm{d}D(t_1)}{\mathrm{d}t_1}\right) \le 0 && \forall\, t_1 > 0 && \text{(B.54)} \end{aligned}$$

Since it holds for all feasible parameter sets $\frac{dl_1}{dt_1} \le 0$ and $\frac{dr_1}{dt_2} \le 0$, it follows for the equation $l_1(t_1) - r_1(t_2) = 0$ that t_1 must (strictly) monotone increase (decrease) for (strictly) monotone increasing (decreasing) variable t_2. Consequently, the corresponding (explicit) function $\zeta_1(t_2)$ is (strictly) monotone increasing in t_2. In the same way, it can be concluded from $\frac{\mathrm{d}l_2}{\mathrm{d}t_1} \le 0$ and $\frac{\mathrm{d}r_2}{\mathrm{d}t_2} \le 0$ that $\zeta_2(t_1)$ is strictly monotone increasing in t_1. ☐

Proposition 4.2.6 (Existence and uniqueness of an interior solution for the case $\lambda = 0$)**.** *If the technology parameters satisfy*

$$\frac{c_{inv,1}}{T\mathbf{P}(s_0)} - \mathbf{E}[\tilde{z}|s_0] - AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0 \qquad (4.48)$$

$$\text{and} \qquad \frac{c_{inv,2}}{T\mathbf{P}(s_1)} + \mathbf{E}[\tilde{z}|s_1] - AQ_E\left(\sigma^2_{1|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0, \qquad (4.49)$$

then the cost-variance efficient portfolios with $A > 0$ corresponding to problem (4.9)-(4.12) consist of a unique combination of both technologies. The total installed generation capacity of the cost-risk efficient portfolio may exceed total demand, implying $\lambda = 0$ in constraint (4.12).

Proof of Proposition 4.2.6. Necessary prerequisite for an interior solution is the existence of at least one stationary tuple $(t_1^o, t_2^o) \in (0;T) \times (0;T)$ with $\zeta_1(t_2^o) = t_1^o$ and $\zeta_2(t_1^o) = t_2^o$. This tuple represents graphically the intersection point of $\zeta_2(t_2)$ and $\zeta_1(t_2)$. The existence of a unique stationary point (t_1^o, t_2^o) can be shown in two steps: (a) At first, it can be proved that it holds $\zeta_1(t_2 = 0) > 0$ and $\zeta_2(t_1 = 0) > 0$. (b) Secondly, it can be shown that it is $\zeta_1(t_2 = T) < T$ and $\zeta_2(t_1 = T) < T$ for a defined set of parameters. Taking into account the monotony of $\zeta_1(t_2), \zeta_2(t_1)$ (cf. Proposition 4.2.5), this consequently implies the existence of a unique intersection point. Therefore, we next consider the limits of the local functions $\zeta_1(t_2), \zeta_2(t_1)$:

a) First, $\zeta_2(t_1 = 0) > 0$ can be concluded from

$$\lim_{t_2 \to 0} l_2(t_2) = \infty \text{ and } \lim_{t_1 \to \xi} r_2(t_1) < \infty \;\forall\, \xi \in [0, T]$$

Similarly, $\zeta_1(t_2 = 0) > 0$ can be concluded since it holds

$$\lim_{t_1 \to 0} l_1(t_1) = \infty \text{ and } \lim_{t_2 \to \xi} r_1(t_2) < \infty \;\forall\, \xi \in [0, T].$$

b) Next, we will derive a condition which is sufficient for $\zeta_1(t_2 = T) < T$ and $\zeta_2(t_1 = T) < T$: Note that $t_1 = T$ implies $Q_{1|s_0} = 0, Q_{2|s_0} = Q_E$, and similar $t_2 = T$ implies $Q_{2|s_1} = 0, Q_{1|s_1} = Q_E$. Since $l_1(t_1)$ is monotone decreasing, it can then be concluded that $\zeta_1(t_2 = T) < T$ if and only if

$$\begin{aligned} & l_1(t_1 = T) - r_1(t_2 = T) < 0 \\ \Leftrightarrow \quad & \frac{c_{inv,1}}{T\mathbf{P}(s_0)} - \mathbf{E}[\tilde{z}|s_0] - AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{P}(s_1)\mathbf{E}[\tilde{z}|s_0](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0. \end{aligned} \tag{B.55}$$

Similarly, it follows due to the monotony of $r_2(t_1)$ that $\zeta_2(t_1 = T) < T$ if and only if

$$\begin{aligned} & l_2(t_2 = T) - r_2(t_1 = T) < 0 \\ \Leftrightarrow \quad & \frac{c_{inv,2}}{T\mathbf{P}(s_1)} + \mathbf{E}[\tilde{z}|s_1] - AQ_E\left(\sigma^2_{1|s_1} - \sigma_{12|s_1} + \mathbf{P}(s_0)\mathbf{E}[\tilde{z}|s_1](\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right) < 0. \end{aligned} \tag{B.56}$$

Therewith, it must exist a stationary point $(t_1^o, t_2^o) \in (0;T) \times (0;T)$ with $\zeta_1(t_2^o) = t_1^o$ and $\zeta_2(t_1^o) = t_2^o$. Finally, it can be verified that the identified stationary point $(t_1^o, t_2^o) \in (0;T) \times (0;T)$ satisfying (B.55) and (B.56) represents a local minimum of the optimization problem. Utilizing the mean value theorem, $\mathcal{L}_v$ has a local minimum at t_1^o it there exists a $r \in \mathbb{R}_+$ such that for every tuple $(t_1^o, t_2^o) \in (t_1^o - r, t_1^o] \times (t_2^o - r, t_2^o]$ it is $\frac{\partial \mathcal{L}}{\partial K_1} \le 0$, $\frac{\partial \mathcal{L}}{\partial K_2} \le 0$, and for every $(t_1^o, t_2^o) \in [t_1^o, t_1^o + r) \times [t_2^o, t_2^o + r)$ it is $\frac{\partial \mathcal{L}}{\partial K_1} \ge 0$, $\frac{\partial \mathcal{L}}{\partial K_2} \ge 0$. By inserting, it can be verified

$$
\begin{aligned}
&\frac{\partial \mathcal{L}}{\partial K_1}(D(0),K_2) = c_{inv,1} > 0\\
&\frac{\partial \mathcal{L}}{\partial K_1}(0,K_2) = c_{inv,1} - \mathbf{P}(s_0)T\mathbf{E}[\tilde{z}|s_0] - \\
&\qquad - \mathbf{P}(s_0)TA\left(\mathbf{E}[\tilde{z}|s_0]\mathbf{P}(s_1)\left(Q_{1|s_1}\mathbf{E}[\tilde{z}|s_1] + Q_E(\bar{c}_{2|s_0} - \bar{c}_{2|s_1})\right) + Q_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0}\right)\right)\\
&\qquad < c_{inv,1} - \mathbf{P}(s_0)T\left(\mathbf{E}[\tilde{z}|s_0] + AQ_E\left(\sigma^2_{2|s_0} - \sigma_{12|s_0} + \mathbf{E}[\tilde{z}|s_0]\mathbf{P}(s_1)(\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right)\right) < 0\\
&\frac{\partial \mathcal{L}}{\partial K_2}(K_1,D(0)) = c_{inv,2} > 0\\
&\frac{\partial \mathcal{L}}{\partial K_2}(K_1,0) = c_{inv,2} + \mathbf{P}(s_1)T\mathbf{E}[\tilde{z}|s_1] - \\
&\qquad - \mathbf{P}(s_1)TA\left(\mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)\left(Q_E(\bar{c}_{2|s_0} - \bar{c}_{1|s_1}) - Q_{1|s_0}\mathbf{E}[\tilde{z}|s_0]\right) + Q_E\left(\sigma^2_{1|s_1} - \sigma_{12|s_1}\right)\right)\\
&\qquad < c_{inv,2} + \mathbf{P}(s_1)T\left(\mathbf{E}[\tilde{z}|s_1] - AQ_E\left(\sigma^2_{1|s_1} - \sigma_{12|s_1} + \mathbf{E}[\tilde{z}|s_1]\mathbf{P}(s_0)(\bar{c}_{2|s_0} - \bar{c}_{1|s_1})\right)\right) < 0
\end{aligned}
$$

Consequently, condition (B.55)-(B.56) is sufficient for an interior local optimum with $\lambda = 0$. Due to the convexity of the Lagrangian $\mathcal{L}$ in λ, the obtained local minimum is also a global minimum. □

Appendix C

Appendix to Chapter 5

C.1 Symbols and model notation

Indices

u	Plant technology
j	Electricity market company
t	Time step during analysis period $[0; T]$

Operators

$\mathrm{Var}[\,\cdot\,]$	Variance operator
$\mathbf{E}[\,\cdot\,]$	Expected value operator

Parameters and variables

A_h	1/€	Households' coefficient of risk aversion
A_j	1/€	Market agents' coefficient of risk aversion
α_j		market agents' profit share
$r_{el,j}$		Rate of return of electricity company j
r_m		Rate of return of the market portfolio j
r_0		Risk-free rate of return
$D(t)$	MW	Total system demand at time t
t_u	hours	Minimal operating duration of u
$p_{el}(t)$		Electricity market price at time t
B	€	Investment budget
X_0	€	Total investment into the risk-free asset
X_m	€	Total investment into the market portfolio
X_{el}	€	Total investment into the electricity market
$x_{el,j}$	€	Investment into the electricity market company j
K_u	MW	Total installed capacity of plant technology u
$k_{u,j}$	MW	Installed capacity of plant technology u of company j
Q_E	MWh	Total energy produced (incl loss of load) in period $[0;T]$
Q_u	MWh	Total energy produced of plant technology u in period $[0;T]$
Q_d	MWh	Total loss of demand (c.f. backstop technology) in period $[0;T]$
$q_{u,j}$	MWh	Energy produced by plant technology u of company j in $[0;T]$
$y_{u,j}(t)$	MW	Output level by plant u of company j at time t
$y_d(t)$	MW	Loss of demand (load of the backstop technology) at time t
$c_{inv,u}$	€/MW$_e$	Annuity of specific investment costs of plant u per capacity K_u
$c_{op,u}(t)$	€/MWh$_e$	Specific operating costs of plant u in period t per output $y_{u,t}$
c_d	€/MWh$_e$	Specific value of lost load (operating costs backstop technology)
$\bar{c}_u$	€/MWh$_e$	Mean operation costs of plant u
σ_u	€/MWh$_e$	Standard deviation of total operation costs of plant u
σ_m	€/MWh$_e$	Standard deviation of the market portfolio return
σ_{uv}	€^{2}/MWh$_e^2$	Covariance of total operation costs of plant u and v

C.2 Mathematical Appendix

C.2.1 Optimality conditions to the market equilibrium under perfect competition

Proposition 5.3.1. *[Market equilibrium under perfect competition] Let be a stylized economy and as defined in Section 5.2 and an electricity market with two generation technologies. Furthermore, a strictly positive societal risk aversion, i.e. $A > 0$ is assumed. If an interior solution to problem (5.5)-(5.8) exists with $K_1, K_2, X_m > 0$, and under the assumption that $X_m \gg \sum_u c_{inv,u} K_u$ and $X_m \gg \sum_u Q_u$, the optimal investments into the market portfolio and the risk-free security are given by*

$$X_m = \frac{1}{A} \frac{\bar{r}_m - r_0}{\sigma_m^2}, \tag{5.9}$$

$$X_0 = B - X_m - \sum_{u \in U} c_{inv,u} K_u. \tag{5.10}$$

The optimal capacity structure within the generation portfolio is characterized by the following optimality conditions, which are only dependent on the decision variables K_1, K_2 (or equivalently on $t_1(K_1), t_2(K_1, K_2)$, and $Q_1(K_1), Q_2(K_1, K_2)$):

$$\frac{(1 + r_0)\,(c_{inv,1} - c_{inv,2})}{t_1} = \bar{c}_{op,2} - \bar{c}_{op,1} + \frac{\bar{r}_m - r_0}{\sigma_m^2}(\sigma_{1m} - \sigma_{2m}), \tag{5.11}$$

$$\frac{(1 + r_0) c_{inv,2}}{t_2} = c_d - \bar{c}_{op,2} + \frac{\bar{r}_m - r_0}{\sigma_m^2}\sigma_{2m}. \tag{5.12}$$

Proof of Proposition 5.3.1. For the welfare optimal solution discussed in Section 5.3, the Lagrangian can be obtained as follows:

$$\begin{aligned} \mathcal{L}_W := &\bar{r}_m X_m + r_0 X_0 - \sum_{u=1}^{2} (c_{inv,u} K_u + \bar{c}_{op,u} Q_u) - c_d Q_d - \mu \left(X_m + X_0 + \sum_{u=1}^{2} c_{inv,u} K_u - B \right) \\ &- \frac{1}{2} A \left(\sigma_m^2 X_m^2 + \sum_{u=1}^{2} \sigma_u^2 Q_u^2 - 2\sigma_{1m} X_m Q_1 - 2\sigma_{2,m} X_m Q_2 + 2\sigma_{12} Q_1 Q_2 \right) \end{aligned} \tag{C.1}$$

Taking into account the following (partial) derivatives for the energy produced by the respective technologies

$$\begin{aligned} \frac{\mathrm{d}Q_1(K_1)}{\mathrm{d}K_1} &= t_1 & \frac{\partial Q_2(K_1, K_2)}{\partial K_1} &= t_2 - t_1 & \frac{\mathrm{d}Q_d(K_1, K_2)}{\mathrm{d}K_1} &= -t_2 \\ \frac{\mathrm{d}Q_1(K_1)}{\mathrm{d}K_2} &= 0 & \frac{\partial Q_2(K_1, K_2)}{\partial K_2} &= t_2 & \frac{\mathrm{d}Q_d(K_1, K_2)}{\mathrm{d}K_2} &= -t_2, \end{aligned}$$

we obtain for the derivatives of the Lagrangian:

$$
\begin{aligned}
\frac{\partial \mathcal{L}_W}{\partial K_1} =& -(1+\mu)c_{inv,1} - \left(\frac{\partial Q_2}{\partial K_1}\right)\cdot\left(\bar{c}_{op,2} + \frac{1}{2}A\left(2\sigma_2^2 Q_2 + 2\sigma_{12}Q_1 - 2\sigma_{2,m}X_m\right)\right) \\
& - \left(\frac{\mathrm{d}Q_1}{\mathrm{d}K_1}\right)\cdot\left(\bar{c}_{op,1} + \frac{1}{2}A\left(2\sigma_1^2 Q_1 + 2\sigma_{12}Q_2 - 2\sigma_{1m}X_m\right)\right) - c_d\left(\frac{\mathrm{d}Q_d}{\mathrm{d}K_2}\right), \\
=& (t_1 - t_2)\left(\bar{c}_{op,2} - \bar{c}_{op,1} - A\left((\sigma_{2m} - \sigma_{1m})X_m - (\sigma_2^2 - \sigma_{12})Q_2 + (\sigma_1^2 - \sigma_{12})Q_1\right)\right) + c_d t_2 - (1+\mu)c_{inv,1} \\
=& (t_1 - t_2)\left(\sum_{u=1}^{2}(-1)^u\left(\bar{c}_{op,u} - A\left(\sigma_{u,m}X_m - (\sigma_u^2 - \sigma_{12})Q_u\right)\right)\right) + c_d t_2 - (1+\mu)c_{inv,1},
\end{aligned}
\tag{C.2}
$$

$$
\begin{aligned}
\frac{\partial \mathcal{L}_W}{\partial K_2} =& -(1+\mu)c_{inv,2} - \left(\frac{\partial Q_2}{\partial K_2}\right)\cdot\left(\bar{c}_{op,2} + \frac{1}{2}A\left(-2\sigma_{2,m}X_m + 2\sigma_2^2 Q_2 + 2\sigma_{12}Q_1\right)\right) - c_d\left(\frac{\mathrm{d}Q_d}{\mathrm{d}K_2}\right) \\
=& t_2\left(c_d - \bar{c}_{op,2} - A\left(-\sigma_{2,m}X_m + \sigma_2^2 Q_2 + \sigma_{12}Q_1\right)\right) - (1+\mu)c_{inv,2},
\end{aligned}
\tag{C.3}
$$

$$
\frac{\partial \mathcal{L}_W}{\partial X_m} = \bar{r}_m - \mu - A\left(\sigma_m^2 X_m - \sigma_{2,m}Q_2 - \sigma_{1m}Q_1\right) \tag{C.4}
$$

$$
\frac{\partial \mathcal{L}_W}{\partial X_0} = r_0 - \mu \tag{C.5}
$$

$$
\frac{\partial \mathcal{L}_W}{\partial \mu} = X_0 + X_m + \sum_{u=1}^{2} c_{inv,u}K_u - B \tag{C.6}
$$

For $K_1, K_2, X_m > 0$, it follows that $\frac{\partial \mathcal{L}_W}{\partial K_1^c}, \frac{\partial \mathcal{L}_W}{\partial K_2^c}, \frac{\partial \mathcal{L}_W}{\partial X_0}, \frac{\partial \mathcal{L}_W}{\partial X_m}, \nu_1, \nu_2$ must equal zero. Furthermore, it is $\mu = r_0$. X_m can be eliminated by solving $\frac{\partial \mathcal{L}_W}{\partial X_m} = 0$ for X_m, yielding:

$$
X_m = \frac{\bar{r}_m - r_0}{A\sigma_m^2} + \frac{Q_2\sigma_{m2} + Q_1\sigma_{m1}}{\sigma_m^2}. \tag{C.7}
$$

By inserting X_m into Eqs. (C.2) and (C.4), we obtain the following first order conditions:

$$
(1+r_0)c_{inv,2} = t_2\left(c_d - \bar{c}_2 + A\left(\sigma_{m2}\left(\frac{\bar{r}_m - r_0}{A\sigma_m^2} + \frac{Q_2\sigma_{m2} + Q_1\sigma_{m1}}{\sigma_m^2}\right) + \sigma_2^2 Q_2 + \sigma_{12}Q_1\right)\right) \tag{C.8}
$$

$$
\begin{aligned}
&(1+r_0)(c_{inv,2} - c_{inv,1}) = \\
&-t_1\left(\bar{c}_2 - \bar{c}_1 - A\left((\sigma_{m2} - \sigma_{m1})\left(\frac{\bar{r}_m - r_0}{A\sigma_m^2} + \frac{Q_2\sigma_{m2} + Q_1\sigma_{m1}}{\sigma_m^2}\right) - (\sigma_2^2 - \sigma_{12})Q_2 + (\sigma_1^2 - \sigma_{12})Q_1\right)\right)
\end{aligned}
\tag{C.9}
$$

Assuming that the total investment in the economy is sufficiently large compared to the electricity market, i.e. $X_m \gg \sum_u c_{inv,u}K_u$ and $X_m \gg \sum_u Q_u$, we can neglect all terms with Q_1 and Q_2. Consequently, we obtain optimality conditions (5.11) and (5.12).

□

C.2.2 Proof of price formation at the second stage of the model

Proposition 5.4.1. *Let be a stylized economy as defined in Section 5.2. Then, the wholesale electricity price is given by the function:*

$$p_{el}(t,\tilde{\zeta}) = \begin{cases} \tilde{c}_{op,1}, & \text{if } t > D^{-1}(K_1) \\ \tilde{c}_{op,2}, & \text{if } D^{-1}(K_1) \geq t > D^{-1}(K_2 + K_1) \\ c_d, & \text{if } D^{-1}(K_2 + K_1) \geq t \end{cases} \tag{5.26}$$

Proof of Proposition 5.4.1. For the proof, we derive the KKT conditions of the household's and the electricity company's optimization problem at the second stage. The Lagrangian of the household's optimization problem (5.15)-(5.18) can be stated as

$$\mathcal{L}_h(t,\tilde{\zeta}) = \mathbf{E}\left[V_h(\tilde{\zeta})\right] - \frac{A_h}{2}\text{Var}\left[V_h(\tilde{\zeta})\right] - \mu_h(X_0 + X_m + \sum_j x_{el,j} - B) \tag{C.10}$$

At the second stage, the realization of all risk factors is ζ known and the Lagrangian simplifies to

$$\begin{gathered}\hat{\mathcal{L}}_h(t,\zeta) = r_0 X_0 + \tilde{r}_m(\zeta) X_m + \sum_{j\in J}(1-\alpha_j)\cdot r_{el,j}(\zeta)x_{el,j} - \\ \int_0^T p_{el}(t,\zeta)D(t)\mathrm{d}t + \int_0^T (p_{el}(t,\zeta) - c_d)\cdot y_d(t,\zeta)\mathrm{d}t - \mu_h \cdot \left(X_m + X_0 + \sum_{j\in J} x_{el,j} - B\right)\end{gathered} \tag{C.11}$$

For the electricity market agents, the Lagrangian referring to optimization problem (5.19)-(5.23) writes:

$$\begin{gathered}\mathcal{L}_{el,j}(t,\tilde{\zeta}) = \mathbf{E}\left[\Pi_{el,j}(\tilde{\zeta})\right] - \frac{A_j}{2}\text{Var}\left[\Pi_{el,j}(\tilde{\zeta})\right] - \iint_0^T \lambda_{u,j}(t,\tilde{\zeta})\left(y_{u,j}(t,\tilde{\zeta}) - k_{u,j}\right)\mathrm{d}t\mathrm{d}\tilde{\zeta} - \\ \mu_j\left(\sum_u c_{inv,u}k_{u,j-x_{el,j}}\right)\end{gathered} \tag{C.12}$$

Given the deterministic realization of ζ at the second stage of the model, the Lagrangian simplifies to

$$\hat{\mathcal{L}}_{el,j(t,\zeta)} = \Pi_{el,j} - \int_0^T \lambda_{u,j}(t,\zeta)\cdot(y_{u,j}(t,\zeta) - k_{u,j})\,\mathrm{d}t - \mu_j\left(\sum_u c_{inv,u}k_{u,j-x_{el,j}}\right) \tag{C.13}$$

$$\text{with } \Pi_{el,j} := \alpha_j \sum_u \left(\int_0^T (p_{el}(t,\zeta) - c_{op,u})y_{u,j}(t,\zeta)\mathrm{d}t - c_{inv,u}k_{u,j}\right) \tag{C.14}$$

We can now straightforwardly derive the KKT conditions for the (decision) variables at the second stage of the model. The KKT conditions derived from the Lagrangian (C.13) of the suppliers' problem at the second stage have to be satisfied for each company j and each generation technology u are:

$$\frac{\partial\hat{\mathcal{L}}_{el,j}}{\partial y_{u,j}}(t,\zeta) = \alpha_j(p_{el}(t,\zeta) - c_{op,u}) - \lambda_{u,j}(t,\zeta) \leq 0 \quad \perp \quad y_{u,j}(t,\zeta) \geq 0\;\forall\, t\in[0,T],\;\forall\zeta \tag{C.15}$$

$$\frac{\partial\hat{\mathcal{L}}_{el,j}}{\partial \lambda_{u,j}}(t,\zeta) = y_{u,j}(t,\zeta) - k_{u,j} \leq 0 \quad \perp \quad \lambda_{u,j}(t,\zeta) \geq 0\;\forall\, t\in[0,T],\;\forall\zeta \tag{C.16}$$

In addition, the KKT condition for the households' optimization problem for the only decision variable at the second stage, y_d, is:

$$\frac{\partial \hat{\mathcal{L}}_h}{\partial y_d}(t,\zeta) = p_{el}(t,\zeta) - c_d \leq 0 \qquad \perp \qquad y_d(t,\tilde{\zeta}) \geq 0 \tag{C.17}$$

Therefore it holds $p_{el}(t,\tilde{\zeta}) = c_d$ at all points in time where $y_d(t,\tilde{\zeta}) > 0$. Hence, the upper bound of the electricity price $p_{el}(t,\tilde{\zeta})$ is the value of lost load c_d and it equals this value if and only if $y_d(t,\tilde{\zeta}) > 0$.

It becomes visible in condition (C.16) that the shadow price of capacity is zero whenever production is beneath the corresponding capacity, i.e. $\lambda_{u,j}(t,\zeta) = 0\ \forall\{t \in [0,T] | y_{u,j}(t,\zeta) < k_{u,j}\}$. Equation (C.15) implies that the shadow price of capacity $\lambda_{u,j}(t,\zeta)$ must equal company's share of the operational margin $\alpha_j(p_{el}(t,\zeta) - c_{op,u})$. Hence, the electricity price equals the marginal production costs whenever production of the respective technology is beneath its installed capacity, i.e. $p_{el}(t,\zeta) = c_{op,u}\ \forall\{t \in [0,T] \mid 0 < y_{u,j}(t,\zeta) < k_{u,j}\}, \forall\zeta$. It can also be concluded that at time of operation of technology u with costs $c_{op,u} > 0$, the electricity price must always be positive. If $y_d(t,\zeta) > 0$, it is known from (C.17) that $p_{el}(t,\zeta) = c_d$. When $y_d(t,\zeta) = 0$, the market clearing condition (5.25) requires that at least one technology is operating since $D(t) > 0\ \forall\ t \in [0,T]$. For this technology, equation (C.15) requires that $p_{el}(t,\tilde{\zeta}) = c_{op,u} + 1/\alpha_j \lambda_{u,j}(t,\tilde{\zeta})\ \forall\ t \in [0,T]$ implying $p_{el}(t,\tilde{\zeta}) > 0$ due to the non-negativity of the Lagrange multiplier $\lambda_{u,j}(t,\tilde{\zeta}) \geq 0$. Furthermore, the market clearing condition (5.25) holds with equality, i.e. $\sum_{u\in U}\sum_{j\in J_u} y_{u,j}(t,\tilde{\zeta}) = D(t) - y_d(t,\tilde{\zeta})\ \forall\ t \in [0,T]$ (but only if $c_{op,u} > 0, u \in U$).

□

Proposition 5.4.2. *The profits of the individual technologies $\Pi_{u,j}(\tilde{\zeta})$ as defined in (5.21) can be rewritten as:*

$$\Pi_{1,j}(\tilde{\zeta}) = (t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1}) + t_2(c_d - \tilde{c}_{op,2}) - c_{inv,1})k_{1,j} \tag{5.28}$$

$$\Pi_{2,j}(\tilde{\zeta}) = (t_2(c_d - \tilde{c}_{op,2}) - c_{inv,2})k_{2,j} \tag{5.29}$$

Proof of Proposition (5.4.2). Starting with the definition of $\Pi_{u,j}$,

$$\Pi_{u,j}(\widetilde{\zeta}) = \int_0^T (p_{el}(t,\widetilde{\zeta}) - \widetilde{c}_{op,u}) \cdot y_{u,j}(t,\widetilde{\zeta})dt - c_{inv,u}k_{u,j} \tag{C.18}$$

, and with prices as derived in Equation (5.26) we can argue:

For $t > t_1$, it holds $p_{el}(t,\widetilde{\zeta}) = c_1$ and consequently

$$(p_{el}(t,\widetilde{\zeta}) - \widetilde{c}_{op,1})y_{1,j}(t,\widetilde{\zeta}) = (\widetilde{c}_{op,1} - \widetilde{c}_{op,1})y_{u,j}(t,\widetilde{\zeta}) = 0 \tag{C.19}$$

for technology 1 and

$$(p_{el}(t,\widetilde{\zeta}) - \widetilde{c}_{op,2})y_{2,j}(t,\widetilde{\zeta}) = (p_{el}(t,\widetilde{\zeta}) - \widetilde{c}_{op,1}) \cdot 0 = 0 \tag{C.20}$$

for technology 2, i.e. neither technology earns any contribution margin for all $t \in (t_1, T]$. Analogously, one can conclude that technology 2 does not earn any margin for all $t \in (t_2, t_1]$.

Hence, it follows that one can write t_u as the upper bound of the integral describing the total operational margin, $\int_0^T (p_{el}(t, \widetilde{\zeta}) - \widetilde{c}_{op,u}) y_{u,j}(t, \widetilde{\zeta}) dt = \int_0^{t_u} (p_{el}(t, \widetilde{\zeta}) - \widetilde{c}_{op,u}) y_{u,j}(t, \widetilde{\zeta}) dt$. For the points in time t with $t < t_u$, we have shown that the prices are constant within the intervals $[0, t_1]$ and $(t_2, T]$ as given in Eqn.(5.26). With the given definitions of $q_{12,j}$, $q_{13,j}$ and $q_{23,j}$ we can then rewrite the operational margins $\Pi_{1,j}$ and $\Pi_{2,j}$ as:

$$\Pi_{1,j}(\widetilde{\zeta}) = q_{12,j} \cdot (\widetilde{c}_{op,2} - \widetilde{c}_{op,1}) + q_{13,j} \cdot (\widetilde{c}_d - \widetilde{c}_{op,1}) \tag{C.21}$$

$$\Pi_{2,j}(\widetilde{\zeta}) = q_{23,j} \cdot (\widetilde{c}_d - \widetilde{c}_{op,2}) \tag{C.22}$$

Replacing $q_{12,j}$, $q_{13,j}$ and $q_{23,j}$ as in Eqn.(5.27) immediately delivers the proof of Proposition (5.4.2). □

Proposition 5.4.3. *The return $r_{el,j}(\tilde{\zeta})$ as defined in (5.13) can be specified as:*

$$r_{el,j}(\tilde{\zeta}) = \frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})k_{1,j} + t_2(c_d - \tilde{c}_{op,2})(k_{1,j} + k_{2,j})}{c_{inv,1}k_{1,j} + c_{inv,2}k_{2,j}} - 1 \tag{5.30}$$

Proof of Proposition (5.4.3). $r_{el,j}(\widetilde{\zeta})$ is defined by: $r_{el,j}(\widetilde{\zeta}) := \frac{\sum_u \Pi_{u,j}}{x_{el,j}}$ (cp. Eqn.5.13). Putting the expressions for $\Pi_{1,j}$ and $\Pi_{2,j}$ from Proposition (5.4.2) into this definition yields the new term for $r_{el,j}$ as given in Eqn.(5.30). □

C.2.3 Proof of Market Agent Optimality Condition

Proposition 5.4.4. *[Electricity market agents' optimality condition] Let be a stylized economy as defined in Section 5.2 and Definition 5.4.1.*

Under the assumption of homogeneous market agents with identical risk aversion A_j $\forall j \in J$ the necessary optimality condition for an interior solution with $k_1, k_2 > 0$ for the optimization problem of the electricity agents as stated in Eqn. (5.19)-(5.23) is given by:

$$
\begin{aligned}
&t_2 \left(\frac{c_d - \tilde{c}_{op,2}}{c_{inv,2}} - \frac{A_j \alpha_j}{N c_{inv,2}} \left(t_1 K_1 (\sigma_{12} - \sigma_2^2) + t_2 (K_1 + K_2) \sigma_2^2 \right) \right) \\
=&t_1 \left(\frac{\tilde{c}_{op,2} - \tilde{c}_{op,1}}{c_{inv,1} - c_{inv,2}} - \frac{A_j \alpha_j}{N (c_{inv,1} - c_{inv,2})} \left(t_1 K_1 (\sigma_1^2 + \sigma_2^2 - 2\sigma_{12}) + t_2 (K_1 + K_2)(\sigma_{12} - \sigma_2^2) \right) \right)
\end{aligned} \tag{5.31}
$$

Proof of Proposition 5.4.4. Starting from the Lagrangian of the market agents' optimization problem,

$$
\begin{aligned}
\mathcal{L}_{el,j}(t, \tilde{\zeta}) = \mathbf{E}\left[\Pi_{el,j}(\tilde{\zeta})\right] - \frac{A_j}{2} \mathrm{Var}\left[\Pi_{el,j}(\tilde{\zeta})\right] - \iint_0^T \lambda_{u,j}(t, \tilde{\zeta}) \left(y_{u,j}(t, \tilde{\zeta}) - k_{u,j} \right) \mathrm{d}t\mathrm{d}\tilde{\zeta} - \\
\mu_j \left(\sum_u c_{inv,u} k_{u,j - x_{el,j}} \right)
\end{aligned} \tag{C.23}
$$

The first order condition with respect to $x_{el,j}$ yields

$$
\frac{\partial \mathcal{L}_{el,j}}{\partial x_{el,j}} = -(1 - \alpha_j) \frac{\mathbf{E}\left[\sum_u \Pi_{u,j}\right]}{x_{el,j}} + A_j (1 - \alpha_j) \alpha_j \frac{1}{x_{el,j}} \mathrm{Var}\left[\sum_u \Pi_{u,j} \right] + \mu_j = 0. \tag{C.24}
$$

After substituting $\sum_u \Pi_{u,j} / x_{el,j}$ with $r_{el,j}$, a defining equation for the shadow price of the investment capital from the market agent's perspective is given by

$$
\Leftrightarrow \mu_j = (1 - \alpha_j)(\mathbf{E}\left[r_{el,j}(\tilde{\zeta})\right] - A_j \alpha_j x_{el,j}(\tilde{\zeta}) \mathrm{Var}\left[r_{el,j}(\tilde{\zeta})\right]) \tag{C.25}
$$

From Proposition 5.4.2, we know that $r_{el,j}(\tilde{\zeta})$ can be written directly in terms of t_i and $k_{i,j}$, i.e. instead of Eqn. (C.25), we can write:

$$
\begin{aligned}
(1 - \alpha_j) \Bigg(&\frac{t_1 k_{1,j} \mathbf{E}[\tilde{c}_{op,2} - \tilde{c}_{op,1}] + t_2 (k_{1,j} + k_{2,j}) \mathbf{E}[c_d - \tilde{c}_{op,2}]}{c_{inv,1} k_{1,j} + c_{inv,2} k_{2,j}} - 1 - A_j \alpha_j \Bigg(\frac{t_1^2 k_{1,j}^2 \mathrm{Var}[\tilde{c}_{op,2} - \tilde{c}_{op,1}]}{c_{inv,1} k_{1,j}} \\
&+ \frac{t_2^2 (k_{1,j} + k_{2,j})^2 \mathrm{Var}[c_d - \tilde{c}_{op,2}]}{c_{inv,1} k_{1,j}} + \frac{2 t_1 t_2 k_{1,j} (k_{1,j} + k_{2,j}) \mathrm{Cov}[\tilde{c}_{op,2} - \tilde{c}_{op,1}, c_d - \tilde{c}_{op,2}]}{c_{inv,1} k_{1,j} + c_{inv,2} k_{2,j}} \Bigg) \Bigg) = \mu_j
\end{aligned} \tag{C.26}
$$

Additionally, equations (5.28) and (5.29) enable us to express the condition for the shadow price of investment also as a function of t_i and $k_{i,j}$:

$$\begin{aligned}(1+\mu_j)c_{inv,2} =& t_2\mathbf{E}[c_d - \tilde{c}_{op,2}] - A_j\alpha_j(t_2t_1k_{1,j}\mathrm{Cov}[(c_d-\tilde{c}_{op,2}),(\tilde{c}_{op,2}-\tilde{c}_{op,1})] \\ &+ t_2^2(k_{1,j}+k_{2,j})\mathrm{Var}[c_d-\tilde{c}_{op,2}]) \qquad (C.27)\\ (1+\mu_j)c_{inv,1} =& t_2\mathbf{E}[c_d - \tilde{c}_{op,2}] + t_1\mathbf{E}[\tilde{c}_{op,2}-\tilde{c}_{op,1}] - A_j\alpha_j\Big(t_2t_1k_{1,j}\mathrm{Cov}[(c_d-\tilde{c}_{op,2}),(\tilde{c}_{op,2}-\tilde{c}_{op,1})] \\ &+ t_2^2(k_{1,j}+k_{2,j})\mathrm{Var}[c_d-\tilde{c}_{op,2}] + t_1^2k_{1,j}\mathrm{Var}[\tilde{c}_{op,2}-\tilde{c}_{op,1}] \\ &+ t_2t_1(k_{1,j}+k_{2,j})\mathrm{Cov}[(\tilde{c}_{op,2}-\tilde{c}_{op,1}),(c_d-\tilde{c}_{op,2})]\Big) \qquad (C.28)\end{aligned}$$

We can equate these two by dividing them by $c_{inv,1}$ and $c_{inv,2}$, respectively. At the same time, we can replace the invidivual capacities $k_{i,j}$ by K_i/N and use simplified expressions to write the variances and covariances, $\sigma_i^2 := \mathrm{Var}[\tilde{c}_{op,i}]$ und $\sigma_{i,j} := \mathrm{Cov}[\tilde{c}_{op,i},\tilde{c}_{op,j}]$ and under consideration of $\mathrm{Cov}[(c_d-\tilde{c}_{op,2}),\tilde{c}_{op,2}-\tilde{c}_{op,1})] = (\sigma_{12}-\sigma_2^2)$, $\mathrm{Var}[\tilde{c}_{op,2}-\tilde{c}_{op,1}] = (\sigma_1^2+\sigma_2^2-2\sigma_{12})$, we obtain:

$$\begin{aligned}&\frac{1}{c_{inv,1}}\Big(t_2\mathbf{E}[c_d-\tilde{c}_{op,2}] + t_1\mathbf{E}[\tilde{c}_{op,2}-\tilde{c}_{op,1}] \\ &-\Big(A_j\alpha_j\Big(t_2t_1\frac{K_1}{N}(\sigma_{12}-\sigma_2^2) + t_2^2\frac{K_1+K_2}{N}\sigma_2^2 + t_1^2\frac{K_1}{N}(\sigma_1^2+\sigma_2^2-2\sigma_{12}) + t_2t_1\frac{K_1+K_2}{N}(\sigma_1^2+\sigma_2^2-2\sigma_{12})\Big)\Big)\Big) \\ &= \frac{1}{c_{inv,2}}\Big(t_2\mathbf{E}[c_d-\tilde{c}_{op,2}] - \Big(A_j\alpha_j\Big(t_2t_1\frac{K_1}{N}(\sigma_1^2+\sigma_2^2-2\sigma_{12}) + t_2^2\frac{K_1+K_2}{N}\sigma_2^2\Big)\Big)\Big)\end{aligned} \qquad (C.29)$$

Finally, some simple rearrangements (in particular, separating the parts related to the investment costs of the peak technology $c_{inv,2}$ and to the extra investment costs $c_{inv,1}-c_{inv,2}$ for the baseload technology) lead us to the condition of Proposition 5.4.4:

$$\begin{aligned}&t_2\left(\frac{c_d-\tilde{c}_{op,2}}{c_{inv,2}} - \frac{A_j\alpha_j}{Nc_{inv,2}}\left(t_1K_1(\sigma_{12}-\sigma_2^2) + t_2(K_1+K_2)\sigma_2^2\right)\right) = \\ &t_1\left(\frac{\tilde{c}_{op,2}-\tilde{c}_{op,1}}{c_{inv,1}-c_{inv,2}} - \frac{A_j\alpha_j}{N(c_{inv,1}-c_{inv,2})}\left(t_1K_1(\sigma_1^2+\sigma_2^2-2\sigma_{12}) + t_2(K_1+K_2)(\sigma_{12}-\sigma_2^2)\right)\right)\end{aligned} \qquad (C.30)$$

□

C.2.4 Proof of Households' Optimality Condition

Proposition 5.4.5. *[Households' optimality condition] Let be a stylized economy as defined in Section 5.2 and Definition 5.4.1. We assume a neglectable managerial profit share, i.e.* $1-\alpha_j \approx 1$*, and total investments in the economy being sufficiently large compared to the electricity market, i.e.* $X_m \gg X_{el}$*. Then, the necessary optimality condition for an interior solution with* $x_{el,j} >, \forall j \in J$ *to the optimization problem of the households as stated in Eqn. (5.15)-(5.18), is given by:*

$$\begin{aligned}\frac{t_2(K_1+K_2)(c_d-\tilde{c}_{op,2})+t_1K_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})}{c_{inv,1}K_1+c_{inv,2}K_2}\\ = 1+r_0+X_m\frac{\overline{r}_m-r_0}{\sigma_m^2}\cdot\frac{t_1K_1\sigma_{m,1}-((t_1-t_2)K_1-t_2K_2)\sigma_{m,2}}{c_{inv,1}K_1+c_{inv,2}K_2}\end{aligned} \tag{5.32}$$

Proof of Proposition 5.4.5. Starting point is the Lagrangian $\mathcal{L}_h$ of the households' optimization problem:

$$\mathcal{L}_h(t,\tilde{\zeta}) = \mathbf{E}\left[V_h(\tilde{\zeta})\right] - \frac{A_h}{2}\mathrm{Var}\left[V_h(\tilde{\zeta})\right] - \mu_h(X_0+X_m+\sum_j x_{el,j}-B) \tag{C.31}$$

Before deriving the first order conditions, we first compute the derivatives of $\mathrm{Var}[V_h]$ with respect to $x_{el,j}$. Thereby we use

$$\begin{aligned}\mathrm{Var}[V_h(\tilde{\zeta})] &= \mathrm{Var}\left[r_0X_0+r_m(\tilde{\zeta})X_m+\sum_j(1-\alpha_j)r_{el,j}-C_{el}(\tilde{\zeta})\right]\\
&= \int(r_0X_0+r_m(\tilde{\zeta})X_m+\sum_j(1-\alpha_j)r_{el,j}-C_{el}(\tilde{\zeta})-\mathbf{E}[V_h(\tilde{\zeta})])^2d\tilde{\zeta}\\
&= \int((r_m(\tilde{\zeta})-\mathbf{E}[r_m])X_m+\sum_j(1-\alpha_j)(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])x_{el,j}-(C_{el}(\tilde{\zeta})-\mathbf{E}[C_{el}]))^2d\tilde{\zeta}\\
&= X_m^2\int(r_m(\tilde{\zeta})-\mathbf{E}[r_m])^2d\tilde{\zeta}+\int(\sum_j(1-\alpha_j)(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])x_{el,j})^2d\tilde{\zeta}\\
&+\int(C_{el}(\tilde{\zeta})-\mathbf{E}[C_{el}])^2d\tilde{\zeta}+2X_m\int(r_m(\tilde{\zeta})-\mathbf{E}[r_m])(\sum_j(1-\alpha_j)(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])x_{el,j})d\tilde{\zeta}\\
&-2X_m\int(r_m(\tilde{\zeta})-\mathbf{E}[r_m])(C_{el}(\tilde{\zeta})-\mathbf{E}[C_{el}])d\tilde{\zeta}\\
&-2\int(\sum_j(1-\alpha_j)(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])x_{el,j})(C_{el}(\tilde{\zeta})-\mathbf{E}[C_{el}])d\tilde{\zeta}\end{aligned} \tag{C.32}$$

Thus, we get for the derivative

$$\begin{aligned}\frac{\partial\mathrm{Var}[V_h(\tilde{\zeta})]}{\partial x_{el,j}} &= 2(1-\alpha_j)^2\int(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])(\sum_{j'}(r_{el,j'}(\tilde{\zeta})-\mathbf{E}[r_{el,j'}])x_{el,j'})d\tilde{\zeta}\\
&+2X_m(1-\alpha_j)\int(r_m(\tilde{\zeta})-\mathbf{E}[r_m])(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])d\tilde{\zeta}\\
&-2(1-\alpha_j)\int(r_{el,j}(\tilde{\zeta})-\mathbf{E}[r_{el,j}])(C_{el}(\tilde{\zeta})-\mathbf{E}[C_{el}])d\tilde{\zeta}\end{aligned} \tag{C.33}$$

Under the premise of symmetric market agents it follows

$$\frac{\partial \mathrm{Var}[V_h(\tilde{\zeta})]}{\partial x_{el,j}} = 2(1-\alpha_j)((1-\alpha_j)X_{el}\mathrm{Var}[r_{el,j}(\tilde{\zeta})] + X_m Cov[r_{el,j}(\tilde{\zeta}), r_m(\tilde{\zeta})] - Cov[r_{el,j}(\tilde{\zeta}), C_{el}(\tilde{\zeta})]) \tag{C.34}$$

Based on these pre-considerations, the first order conditions with repect to $x_{el,j}, X_0, X_m$ can be derived as:

$$\frac{\partial \mathcal{L}_h}{\partial x_{el,j}} = \mathbf{E}[r_{el,j}] - A_h((1-\alpha_j)X_{el}\mathrm{Var}[r_{el,j}] + X_m\mathrm{Cov}[r_{el,j}r_m] - \mathrm{Cov}[r_{el,j}C_{el}]) - \frac{1}{(1-\alpha_j)}\mu_h = 0 \tag{C.35}$$

$$\frac{\partial \mathcal{L}_h}{\partial X_0} = r_0 - \mu_h = 0 \tag{C.36}$$

$$\frac{\partial \mathcal{L}_h}{\partial X_m} = r_0 - A_h X_m \mathrm{Var}[\tilde{r}_m] - \mu_h \tag{C.37}$$

From (C.35), the expected portfolio return of company j is given by:

$$\mathbf{E}[r_{el,j}] = \frac{1}{(1-\alpha_j)}r_0 + A_h((1-\alpha_j)X_{el}\mathrm{Var}[r_{el,j}] + X_m\mathrm{Cov}[r_{el,j}r_m] - \mathrm{Cov}[r_{el,j}C_{el}]). \tag{C.38}$$

Likewise, the expected portfolio return of company j as defined in (5.13) can be specified with the price formation at the second stage according to Proposition 5.4.1

$$r_{el,j}(\tilde{\zeta}) = \frac{(t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1}) + t_2(c_d - \tilde{c}_{op,2}) - c_{inv,1})k_{1,j} + (t_2(c_d - \tilde{c}_{op,2}) - c_{inv,2})k_{2,j}}{c_{inv,1}k_{1,j} + c_{inv,2}k_{2,j}} \tag{C.39}$$

$$= \frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})k_{1,j} + t_2(c_d - \tilde{c}_{op,2})(k_{1,j} + k_{2,j})}{c_{inv,1}k_{1,j} + c_{inv,2}k_{2,j}} - 1 \tag{C.40}$$

Equating (C.38) and (C.40) and with $C_{el} = c_d \int_0^{t_2} y_d(t)dt + \tilde{c}_{op,2}\int_{t_2}^{t_1} y_d(t)dt + \tilde{c}_{op,1}\int_{t_1}^{T} y_d(t)dt = c_d Q_d(t_2) + \tilde{c}_{op,2}Q_2 + \tilde{c}_{op,1}Q_1(t_1)$ we obtain:

$$\begin{aligned} \mathbf{E}\left[\frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})K_1 + t_2(c_d - \tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1 + c_{inv,2}K_2} - 1\right] = \frac{1}{1-\alpha_j}r_0 + \\ A_h\Bigg((1-\alpha_j)(c_{inv,1}K_1 + c_{inv,2}K_2)\mathrm{Var}\left[\frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})K_1 + t_2(c_d - \tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1 + c_{inv,2}K_2} - 1\right] + \\ X_m\mathrm{Cov}\left[\frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})K_1 + t_2(c_d - \tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1 + c_{inv,2}K_2} - 1, r_m\right] - \\ \mathrm{Cov}\left[\frac{t_1(\tilde{c}_{op,2} - \tilde{c}_{op,1})K_1 + t_2(c_d - \tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1 + c_{inv,2}K_2} - 1, c_dQ_d + \tilde{c}_{op,2}Q_2 + \tilde{c}_{op,1}Q_1(t_1)\right]\Bigg) \end{aligned} \tag{C.41}$$

With $1-\alpha_j \approx 1$ and some transformations we obtain:

$$\begin{aligned}\mathbf{E}\left[\frac{t_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})K_1+t_2(c_d-\tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1+c_{inv,2}K_2}-1-r_0\right] = \\ A_h\Bigg((c_{inv,1}K_1+c_{inv,2}K_2)\mathrm{Var}\left[\frac{t_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})K_1+t_2(c_d-\tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1+c_{inv,2}K_2}\right]+ \\ X_m\mathrm{Cov}\left[\frac{t_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})K_1+t_2(c_d-\tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1+c_{inv,2}K_2}, r_m\right]- \\ \mathrm{Cov}\left[\frac{t_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})K_1+t_2(c_d-\tilde{c}_{op,2})(K_1+K_2)}{c_{inv,1}K_1+c_{inv,2}K_2}, c_dQ_d+\tilde{c}_{op,2}Q_2+\tilde{c}_{op,1}Q_1(t_1)\right]\Bigg)\end{aligned} \tag{C.42}$$

In knowledge of the propoerties for variance of sums this equation can be written as

$$\begin{aligned}\frac{t_2(K_1+K_2)c_d+((t_1-t_2)K_1-t_2K_2)\mathbf{E}[\tilde{c}_{op,2}]-t_1K_1\mathbf{E}[\tilde{c}_{op,1}]}{c_{inv,1}K_1+c_{inv,2}K_2}-1-r_0 = \\ \frac{A_h}{c_{inv,1}K_{1,j}+c_{inv,2}K_{2,j}}\Bigg(\big(((t_1-t_2)K_1-t_2K_2)^2\mathrm{Var}[\tilde{c}_{op,2}]-2((t_1-t_2)K_1-t_2K_2)t_1K_1\mathrm{Cov}[\tilde{c}_{op,1},\tilde{c}_{op,2}] \\ +(t_1K_1)^2\mathrm{Var}[\tilde{c}_{op,1}]\big)+X_m\big(((t_1-t_2)K_1-t_2K_2)\mathrm{Cov}[\tilde{c}_{op,2},r_m]-t_1K_1\mathrm{Cov}[\tilde{c}_{op,1},r_m]\big)- \\ \mathrm{Cov}\big[t_1K_1(\tilde{c}_{op,2}-\tilde{c}_{op,1})+t_2(c_d-\tilde{c}_{op,2})(K_1+K_2), c_dQ_d(t_2)+\tilde{c}_{op,2}Q_2(t_2,t_1)+\tilde{c}_{op,1}Q_1(t_1)\big]\Bigg)\end{aligned} \tag{C.43}$$

For $X_m \gg X_{el}$, all summands without X_m on the right side of the equation can be neglected and the optimality condition can be simplified to:

$$\frac{t_2(K_1+K_2)c_d+((t_1-t_2)K_1-t_2K_2)(\mathbf{E}[\tilde{c}_{op,2}]-A_hX_m\sigma_{m,2}-t_1K_1(\mathbf{E}[\tilde{c}_{op,1}]-A_hX_m\sigma_{m,1})}{c_{inv,1}K_1+c_{inv,2}K_2} = 1+r_0 \tag{C.44}$$

After some transformations we obtain Equation (5.32). □

Appendix D

Appendix to Chapter 6

D.1 Sample company overview

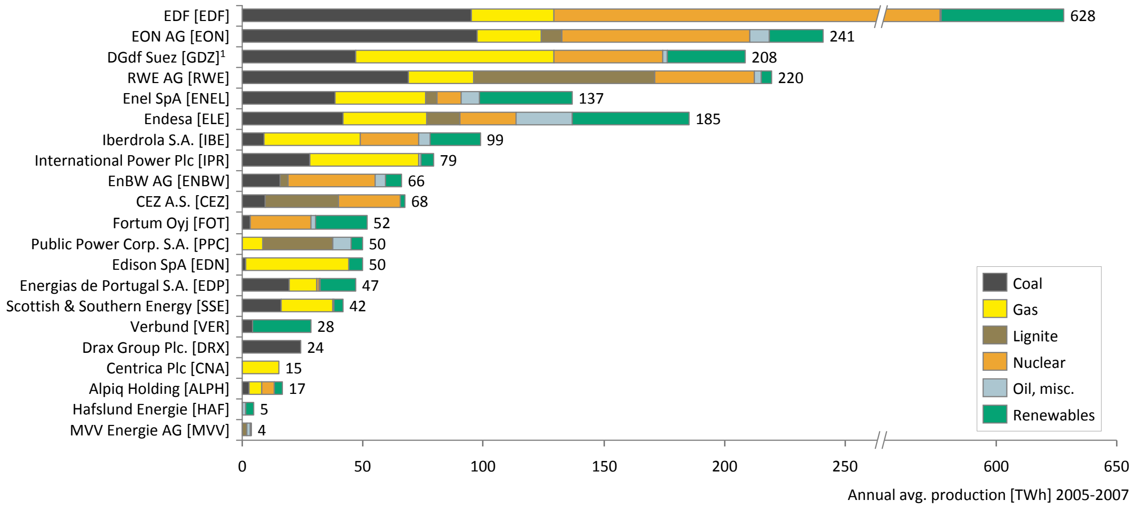

Fig. D.1: Annual average power production and generation fuel mix of the sample companies 2005–2007

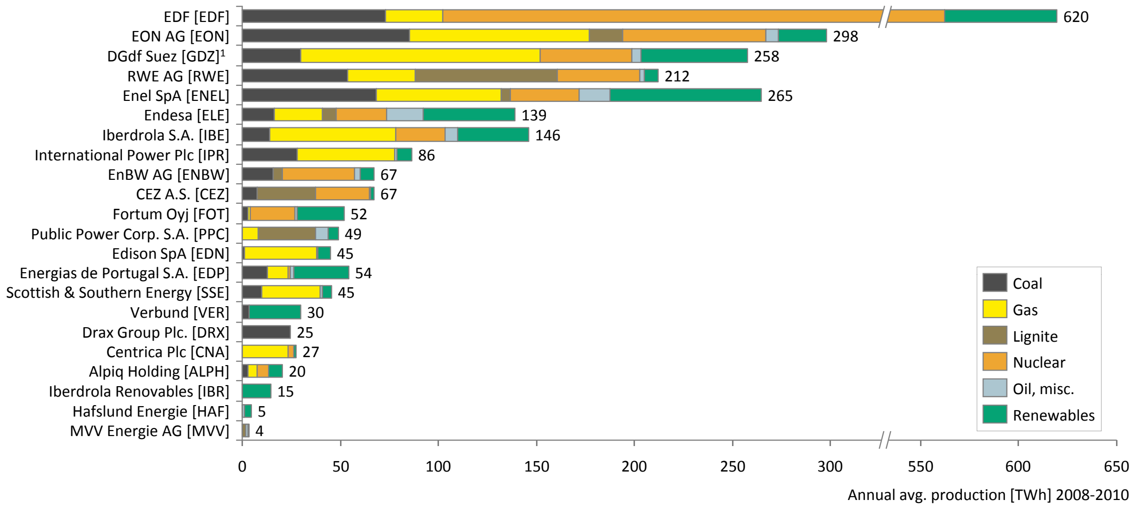

Fig. D.2: Annual average power production and generation fuel mix of the sample companies 2008–2010

Tab. D.1: This table provides company names, respective stock exchange, summary statistics for the share price return r_i, and debt to equity rations (D/E) for the sample companies in the relevant analysis periods.

			Combined period (2005-2010)			Period I (2005-2007)			Period II (2008-2010)		
Name	Symbol	Market	$\bar{r}_i$	σ_i	D/E	$\bar{r}_i$	σ_i	D/E	$\bar{r}_i$	σ_i	D/E
EDF	EDF	Paris	0.004	0.091	2.30	0.042	0.076	1.73	-0.023	0.092	2.86
EON AG	EON	Xetra	0.003	0.071	1.66	0.023	0.044	1.12	-0.017	0.086	2.20
Gdf Suez	GDZ	Paris	0.003	0.067	1.23	0.019	0.052	0.83	-0.008	0.075	1.62
RWE AG	RWE	Xetra	0.005	0.061	2.17	0.025	0.048	1.91	-0.016	0.066	2.43
Enel SpA	ENEL	Milan	-0.006	0.053	2.68	0.004	0.028	1.57	-0.015	0.069	3.78
Endesa	ELE	Madrid	0.006	0.096	1.39	0.023	0.071	1.23	-0.011	0.114	1.54
Iberdrola S.A.	IBE	Madrid	0.007	0.076	1.44	0.024	0.062	0.93	-0.011	0.086	1.95
International Power Plc	IPR	London	0.019	0.090	1.74	0.032	0.056	1.35	0.006	0.114	2.13
EnBW AG	ENBW	Xetra	0.007	0.062	2.34	0.022	0.066	1.81	-0.009	0.054	2.87
CEZ A.S.	CEZ	Prague	0.015	0.083	0.50	0.042	0.080	0.28	-0.012	0.076	0.72
Fortum Oyj	FOT	Helsinki	0.010	0.079	0.59	0.025	0.070	0.45	-0.005	0.086	0.74
DEI (Public Power Corporation)	DEI	Athens	-0.004	0.095	2.45	0.018	0.071	1.49	-0.027	0.110	3.41
Edison SpA	EDN	Milan	-0.005	0.080	1.27	0.010	0.045	0.95	-0.020	0.102	1.59
Energias de Portugal S.A.	EDP	Lisbon	0.003	0.059	2.25	0.020	0.043	1.58	-0.014	0.067	2.91
Scottish & Southern Energy	SSE	London	0.006	0.048	1.06	0.018	0.040	0.68	-0.007	0.052	1.43
Verbund	VER	Vienna	0.011	0.089	0.51	0.033	0.075	0.36	-0.010	0.097	0.66
Drax Group Plc	DRX	London	-0.002	0.093	0.63	0.009	0.100	0.49	-0.010	0.088	0.77
Centrica Plc	CNA	London	0.008	0.057	1.01	0.013	0.044	1.07	0.003	0.068	0.96
Alpiq Holding	ALPH	Zurich	0.012	0.084	1.09	0.035	0.083	1.26	-0.011	0.080	0.91
MVV Energie AG	MVV	Xetra	0.010	0.051	1.62	0.022	0.063	1.96	-0.003	0.030	1.28
Hafslund Energie	HNA	Oslo	0.013	0.101	1.04	0.044	0.095	0.72	-0.017	0.098	1.36
Iberdrola Renovables S.A.	IBR	Madrid	-0.014	0.104	0.89	NA	NA	NA	-0.014	0.104	0.89

D.2 Univariate regression results

Tab. D.2: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{m,i}$ for the standard CAPM model with DJ Euro Stoxx Utilities index as the relevant market portfolio. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{m,i}$		R^2
EDF	0.002	(0.009)	1.195***	(0.199)	0.38
EON	-0.001	(0.007)	0.8***	(0.162)	0.26
GDZ	0.002	(0.008)	0.522***	(0.174)	0.13
RWE	0.001	(0.006)	0.633***	(0.142)	0.22
ENEL	-0.01*	(0.005)	0.696***	(0.114)	0.35
ELE	0.002	(0.011)	0.752***	(0.238)	0.12
IBE	0.002	(0.008)	0.762***	(0.180)	0.20
IPR	0.013	(0.008)	1.378***	(0.174)	0.47
ENBW	0.003	(0.007)	0.415**	(0.158)	0.09
CEZ	0.01	(0.009)	0.909***	(0.190)	0.25
FOT	0.006	(0.008)	0.929***	(0.179)	0.28
DEI	-0.009	(0.010)	0.795***	(0.232)	0.14
EDN	-0.009	(0.009)	0.784***	(0.190)	0.20
EDP	0	(0.006)	0.619***	(0.137)	0.23
SSE	0.003	(0.005)	0.457***	(0.114)	0.19
VER	0.006	(0.009)	0.962***	(0.206)	0.24
DRX	-0.004	(0.011)	0.733***	(0.236)	0.14
CNA	0.005	(0.007)	0.36**	(0.146)	0.08
ALPH	0.007	(0.009)	0.92***	(0.194)	0.24
MVV	0.016**	(0.007)	0.444***	(0.115)	0.21
HNA	0.008	(0.010)	1.267***	(0.220)	0.32
IBR	-0.006	(0.017)	0.827**	(0.319)	0.17
Mean	0.002	(0.008)	0.780	(0.183)	0.22

Tab. D.3: 2005–2007 and 2008–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{m,i}$ for the standard CAPM model with DJ Euro Stoxx Utilities index as the relevant market portfolio. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

	Period I (2005-2007)					Period II (2008-2010)					Chow test	
Company	$\hat{\alpha}_i$		$\hat{\beta}_{m,i}$		R^2	$\hat{\alpha}_i$		$\hat{\beta}_{m,i}$		R^2	t-stat	(p-value)
EDF	0.015	(0.017)	1.242**	(0.443)	0.26	-0.01	(0.013)	0.369***	(0.247)	0.33	0.14	(0.868)
EON	0.012	(0.009)	0.435*	(0.243)	0.09	-0.007	(0.013)	0.331***	(0.248)	0.23	0.97	(0.385)
GDZ	0.004	(0.011)	0.738**	(0.303)	0.21	-0.004	(0.013)	0.199*	(0.239)	0.05	0.23	(0.799)
RWE	0.011	(0.009)	0.601**	(0.256)	0.14	-0.01	(0.011)	0.193**	(0.199)	0.14	1.44	(0.245)
ENEL	-0.006	(0.005)	0.363**	(0.148)	0.15	-0.005	(0.009)	0.234***	(0.179)	0.38	1.70	(0.190)
ELE	0.015	(0.014)	0.311	(0.408)	0.02	-0.001	(0.018)	0.408**	(0.349)	0.12	0.51	(0.603)
IBE	0.009	(0.012)	0.644*	(0.338)	0.10	-0.002	(0.013)	0.315***	(0.255)	0.18	0.20	(0.817)
IPR	0.011	(0.010)	0.934***	(0.285)	0.24	0.027**	(0.013)	0.653***	(0.250)	0.55	2.66	(0.077)
ENBW	0.006	(0.013)	0.716*	(0.363)	0.10	-0.008	(0.009)	0.069	(0.179)	0.01	1.79	(0.174)
CEZ	0.018	(0.015)	1.129**	(0.422)	0.17	-0.004	(0.012)	0.439***	(0.226)	0.19	1.65	(0.199)
FOT	0.006	(0.013)	0.848**	(0.376)	0.13	0.007	(0.012)	0.624***	(0.235)	0.31	0.04	(0.961)
DEI	0.007	(0.014)	0.446	(0.404)	0.03	-0.017	(0.018)	0.225**	(0.339)	0.12	0.62	(0.543)
EDN	0	(0.009)	0.356	(0.253)	0.06	-0.009	(0.016)	0.433***	(0.301)	0.19	0.73	(0.487)
EDP	0.005	(0.008)	0.658***	(0.225)	0.20	-0.008	(0.011)	0.167**	(0.204)	0.14	0.73	(0.484)
SSE	0.01	(0.008)	0.308	(0.228)	0.05	-0.002	(0.008)	0.221***	(0.154)	0.18	0.60	(0.552)
VER	0.009	(0.014)	1.082***	(0.395)	0.18	0	(0.015)	0.572***	(0.285)	0.19	0.29	(0.746)
DRX	-0.006	(0.024)	0.589	(0.652)	0.04	0	(0.014)	0.52***	(0.257)	0.20	0.32	(0.729)
CNA	0.005	(0.009)	0.246	(0.254)	0.03	0.008	(0.011)	0.249*	(0.215)	0.07	0.16	(0.848)
ALPH	0.014	(0.016)	0.98**	(0.451)	0.12	-0.002	(0.012)	0.465***	(0.226)	0.25	0.51	(0.601)
MVV	0	(0.011)	1.031***	(0.320)	0.23	-0.003	(0.005)	0.04	(0.097)	-0.01	6.32	(0.003)
HNA	0.013	(0.017)	1.446***	(0.493)	0.20	-0.004	(0.014)	0.553***	(0.266)	0.31	0.73	(0.484)
IBR	NA	NA	NA	NA	NA	-0.006	(0.017)	0.523**	(0.319)	0.14	NA	NA
Mean	0.007	(0.012)	0.719	(0.346)	0.13	-0.003	(0.013)	0.355	(0.240)	0.19		

Tab. D.4: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{el,i}$ with returns of EEX Phelix base front year futures (EEX, 2011) as considered risk factor. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{el,i}$		R^2
EDF	0.001***	(0.012)	0.309*	(0.165)	0.06
EON	-0.001***	(0.008)	0.299**	(0.121)	0.08
GDZ	0.001***	(0.009)	0.173	(0.123)	0.03
RWE	0.001***	(0.007)	0.242**	(0.104)	0.07
ENEL	-0.008***	(0.006)	0.036	(0.095)	0.00
ELE	0.004***	(0.011)	0.07	(0.171)	0.00
IBE	0.004***	(0.009)	0.069	(0.136)	0.00
IPR	0.015***	(0.011)	0.287*	(0.157)	0.05
ENBW	0.004***	(0.007)	0.101	(0.111)	0.01
CEZ	0.009***	(0.009)	0.575***	(0.130)	0.22
FOT	0.004***	(0.008)	0.593***	(0.123)	0.25
DEI	-0.006***	(0.011)	-0.021	(0.169)	0.00
EDN	-0.008***	(0.009)	0.204	(0.140)	0.03
EDP	0.002***	(0.007)	-0.089	(0.104)	0.01
SSE	0.003***	(0.006)	0.167**	(0.083)	0.05
VER	0.005***	(0.009)	0.698***	(0.135)	0.28
DRX	-0.005***	(0.011)	0.535***	(0.156)	0.17
CNA	0.006***	(0.007)	0.01	(0.102)	0.00
ALPH	0.008***	(0.010)	0.303**	(0.146)	0.06
MVV	0.007***	(0.006)	0.081	(0.089)	0.01
HNA	0.009***	(0.012)	0.419**	(0.173)	0.08
IBR	-0.016***	(0.018)	0.167	(0.213)	0.02
Mean	0.002	(0.009)	0.238	(0.134)	0.07

Tab. D.5: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{oil,i}$ with returns of WTI crude oil futures with four months to delivery (EIA, 2011) as considered risk factor. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{oil,i}$		R^2
EDF	0	(0.012)	0.225	(0.148)	0.04
EON	-0.002	(0.008)	0.218**	(0.104)	0.06
GDZ	0.002	(0.009)	-0.049	(0.111)	0.00
RWE	0.001	(0.007)	0.122	(0.090)	0.03
ENEL	-0.008	(0.006)	0.066	(0.080)	0.01
ELE	0.007	(0.011)	-0.247*	(0.142)	0.04
IBE	0.005	(0.009)	-0.045	(0.115)	0.00
IPR	0.014	(0.011)	0.265**	(0.133)	0.05
ENBW	0.003	(0.007)	0.106	(0.094)	0.02
CEZ	0.009	(0.009)	0.304**	(0.119)	0.08
FOT	0.003	(0.009)	0.43***	(0.109)	0.18
DEI	-0.007	(0.011)	0.064	(0.143)	0.00
EDN	-0.01	(0.009)	0.257**	(0.117)	0.06
EDP	0.001	(0.007)	0.059	(0.089)	0.01
SSE	0.003	(0.006)	0.045	(0.072)	0.01
VER	0.005	(0.010)	0.349***	(0.128)	0.10
DRX	-0.006	(0.012)	0.353**	(0.144)	0.09
CNA	0.007	(0.007)	-0.072	(0.086)	0.01
ALPH	0.008	(0.010)	0.138	(0.126)	0.02
MVV	0.007	(0.006)	0.043	(0.076)	0.00
HNA	0.008	(0.012)	0.307**	(0.148)	0.06
IBR	-0.016	(0.018)	-0.033	(0.189)	0.00
Mean	0.002	(0.009)	0.132	(0.116)	0.04

Tab. D.6: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{es,i}$ with monthly returns of the IFO Business Climate Index for Germany (IFO, 2011) as considered risk factor. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{es,i}$		R^2
EDF	0.002	(0.012)	1.205*	(0.615)	0.06
EON	0.001	(0.008)	1.124**	(0.449)	0.08
GDZ	0.002	(0.009)	0.4	(0.464)	0.01
RWE	0.003	(0.007)	0.611	(0.393)	0.03
ENEL	-0.008	(0.006)	1.028***	(0.329)	0.12
ELE	0.004	(0.011)	0.96	(0.622)	0.03
IBE	0.005	(0.009)	0.862*	(0.492)	0.04
IPR	0.017*	(0.010)	1.644***	(0.563)	0.11
ENBW	0.005	(0.007)	0.796*	(0.401)	0.05
CEZ	0.013	(0.009)	1.26**	(0.524)	0.08
FOT	0.008	(0.009)	1.812***	(0.479)	0.17
DEI	-0.006	(0.011)	0.753	(0.619)	0.02
EDN	-0.007	(0.009)	1.194**	(0.507)	0.07
EDP	0.001	(0.007)	0.788**	(0.376)	0.06
SSE	0.004	(0.006)	0.517*	(0.309)	0.04
VER	0.009	(0.010)	1.13*	(0.571)	0.05
DRX	-0.004	(0.012)	0.555	(0.635)	0.01
CNA	0.006	(0.007)	0.649*	(0.372)	0.04
ALPH	0.01	(0.010)	0.558	(0.552)	0.01
MVV	0.008	(0.006)	0.489	(0.328)	0.03
HNA	0.012	(0.012)	1.043	(0.654)	0.04
IBR	-0.016	(0.018)	0.648	(0.769)	0.02
Mean	0.003	(0.009)	0.910	(0.501)	0.05

Tab. D.7: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{eua,i}$ with returns of EUA front year futures (EEX, 2011) as considered risk factor. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{eua,i}$		R^2
EDF	0.002	(0.011)	0.392***	(0.098)	0.22
EON	0	(0.008)	0.349***	(0.078)	0.25
GDZ	0.002	(0.008)	0.257***	(0.074)	0.17
RWE	-0.001	(0.007)	0.223***	(0.069)	0.15
ENEL	-0.007	(0.007)	0.105	(0.067)	0.04
ELE	0.002	(0.013)	0.011	(0.118)	0.00
IBE	0.003	(0.010)	0.181*	(0.093)	0.06
IPR	0.014	(0.012)	0.234**	(0.110)	0.07
ENBW	-0.002	(0.006)	0.161***	(0.060)	0.11
CEZ	0.005	(0.008)	0.375***	(0.074)	0.30
FOT	0.008	(0.008)	0.351***	(0.079)	0.25
DEI	-0.004	(0.013)	0.122	(0.119)	0.02
EDN	-0.009	(0.010)	0.216**	(0.096)	0.08
EDP	0.001	(0.008)	0.136*	(0.073)	0.06
SSE	0.003	(0.006)	0.092	(0.058)	0.04
VER	0.004	(0.010)	0.446***	(0.093)	0.28
DRX	-0.004	(0.012)	0.185*	(0.108)	0.05
CNA	0.007	(0.008)	0.055	(0.072)	0.01
ALPH	0.009	(0.010)	0.245***	(0.090)	0.11
MVV	0.007	(0.006)	0.169***	(0.053)	0.15
HNA	0.007	(0.012)	0.247**	(0.113)	0.07
IBR	-0.016	(0.018)	-0.034	(0.187)	0.00
Mean	0.001	(0.010)	0.205	(0.090)	0.11

Tab. D.8: 2005–2010 regression results with estimates for firm-specific equity coefficients α_i and $\beta_{gas,i}$ with German gas import prices (BAFA, 2010) as considered risk factor. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\alpha}_i$		$\hat{\beta}_{gas,i}$		R^2
EDF	0.002	(0.012)	-0.476	(0.288)	0.04
EON	0.004	(0.008)	-0.522^{**}	(0.200)	0.09
GDZ	0.002	(0.009)	-0.229	(0.215)	0.02
RWE	0.003	(0.007)	-0.091	(0.178)	0.00
ENEL	-0.005	(0.006)	-0.438^{***}	(0.148)	0.11
ELE	0.007	(0.011)	-0.421	(0.278)	0.03
IBE	0.009	(0.009)	-0.591^{***}	(0.213)	0.10
IPR	0.02^{*}	(0.011)	-0.478^{*}	(0.260)	0.05
ENBW	0.007	(0.007)	-0.369^{**}	(0.179)	0.06
CEZ	0.017^{*}	(0.010)	-0.544^{**}	(0.234)	0.07
FOT	0.01	(0.009)	-0.307	(0.232)	0.02
DEI	-0.003	(0.011)	-0.481^{*}	(0.273)	0.04
EDN	-0.003	(0.009)	-0.611^{***}	(0.223)	0.10
EDP	0.003	(0.007)	-0.236	(0.171)	0.03
SSE	0.005	(0.006)	-0.123	(0.140)	0.01
VER	0.011	(0.011)	-0.263	(0.260)	0.01
DRX	-0.005	(0.012)	0.152	(0.296)	0.00
CNA	0.007	(0.007)	-0.127	(0.169)	0.01
ALPH	0.011	(0.010)	-0.174	(0.247)	0.01
MVV	0.008	(0.006)	0.015	(0.149)	0.00
HNA	0.015	(0.012)	-0.519^{*}	(0.290)	0.04
IBR	-0.016	(0.017)	-0.527	(0.357)	0.06
Mean	0.005	(0.009)	-0.335	(0.227)	0.04

D.3 Multivariate regression results

Tab. D.9: Two-factor model: 2005–2010 regression results with estimates for firm-specific equity coefficients $\lambda_{0,i}$, $\lambda_{m,i}$ and $\lambda_{eua,i}$ with excess returns of the DJ Euro Stoxx Utilities index (ECB, 2011) and EUA front year futures (EEX, 2011) as considered risk factors. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\lambda}_{0,i}$		$\hat{\lambda}^*_{m,i}$		$\hat{\lambda}^*_{eua,i}$		R^2_{adj}
EDF	0.002	(0.009)	1.007***	(0.191)	0.263***	(0.084)	0.47
EON	-0.001	(0.008)	0.642***	(0.168)	0.267***	(0.074)	0.40
GDZ	0.001	(0.008)	0.357**	(0.168)	0.21***	(0.074)	0.23
RWE	-0.001	(0.007)	0.508***	(0.152)	0.158**	(0.067)	0.28
ENEL	-0.008	(0.006)	0.712***	(0.130)	0.014	(0.057)	0.36
ELE	0.001	(0.012)	0.775***	(0.263)	-0.087	(0.116)	0.13
IBE	0.002	(0.009)	0.684***	(0.205)	0.094	(0.090)	0.21
IPR	0.012	(0.009)	1.32***	(0.199)	0.067	(0.088)	0.47
ENBW	-0.003	(0.006)	0.212	(0.142)	0.134**	(0.062)	0.14
CEZ	0.004	(0.007)	0.541***	(0.162)	0.306***	(0.071)	0.41
FOT	0.008	(0.007)	0.751***	(0.163)	0.256***	(0.072)	0.45
DEI	-0.005	(0.012)	0.801***	(0.265)	0.02	(0.117)	0.15
EDN	-0.01	(0.010)	0.699***	(0.211)	0.127	(0.093)	0.22
EDP	0	(0.007)	0.607***	(0.155)	0.059	(0.068)	0.25
SSE	0.002	(0.006)	0.427***	(0.128)	0.038	(0.056)	0.19
VER	0.003	(0.009)	0.654***	(0.205)	0.362***	(0.090)	0.39
DRX	-0.004	(0.011)	0.665***	(0.248)	0.1	(0.108)	0.15
CNA	0.007	(0.008)	0.361**	(0.167)	0.009	(0.073)	0.08
ALPH	0.008	(0.009)	0.774***	(0.191)	0.146*	(0.084)	0.30
MVV	0.006	(0.006)	0.204	(0.124)	0.143**	(0.054)	0.18
HNA	0.005	(0.010)	1.12***	(0.227)	0.104	(0.100)	0.34
IBR	-0.006	(0.017)	0.892**	(0.329)	-0.147	(0.176)	0.19
Mean	0.001	(0.009)	0.669	(0.191)	0.120	(0.085)	0.27

Tab. D.10: Three-factor model: 2005–2010 regression results with estimates for firm-specific equity coefficients $\lambda_{0,i}$, $\lambda_{m,i}$, $\lambda_{eua,i}$, and $\lambda_{el,i}$ with excess returns of the DJ Euro Stoxx Utilities index (ECB, 2011), EUA front year futures (EEX, 2011), and Phelix base front year futures (EEX, 2011) as considered risk factors. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\lambda}_{0,i}$		$\hat{\lambda}^*_{m,i}$		$\hat{\lambda}^*_{eua,i}$		$\hat{\lambda}^*_{el,i}$		R^2_{adj}
EDF	0.002	(0.009)	1.033***	(0.195)	0.289***	(0.092)	-0.099	(0.142)	0.48
EON	-0.001	(0.008)	0.644***	(0.173)	0.269***	(0.081)	-0.007	(0.124)	0.40
GDZ	0.001	(0.008)	0.374**	(0.173)	0.226***	(0.081)	-0.062	(0.124)	0.23
RWE	-0.001	(0.007)	0.498***	(0.156)	0.149**	(0.073)	0.034	(0.112)	0.28
ENEL	-0.008	(0.006)	0.754***	(0.131)	0.054	(0.061)	-0.154	(0.094)	0.39
ELE	0.001	(0.012)	0.796***	(0.270)	-0.068	(0.127)	-0.076	(0.194)	0.13
IBE	0.003	(0.009)	0.738***	(0.208)	0.145	(0.097)	-0.196	(0.149)	0.23
IPR	0.012	(0.009)	1.328***	(0.205)	0.075	(0.096)	-0.031	(0.148)	0.47
ENBW	-0.003	(0.006)	0.224	(0.146)	0.145**	(0.068)	-0.041	(0.105)	0.14
CEZ	0.003	(0.007)	0.45***	(0.156)	0.222***	(0.073)	0.329***	(0.112)	0.49
FOT	0.006	(0.007)	0.641***	(0.151)	0.152**	(0.071)	0.401***	(0.109)	0.55
DEI	-0.005	(0.012)	0.864***	(0.270)	0.079	(0.127)	-0.229	(0.194)	0.17
EDN	-0.01	(0.010)	0.711***	(0.217)	0.138	(0.102)	-0.044	(0.156)	0.22
EDP	0.002	(0.007)	0.695***	(0.149)	0.141**	(0.070)	-0.318***	(0.107)	0.35
SSE	0.002	(0.006)	0.411***	(0.131)	0.023	(0.062)	0.057	(0.094)	0.20
VER	0.001	(0.009)	0.54***	(0.197)	0.256***	(0.092)	0.413***	(0.141)	0.46
DRX	-0.005	(0.011)	0.55**	(0.241)	-0.014	(0.113)	0.437**	(0.173)	0.24
CNA	0.007	(0.008)	0.384**	(0.171)	0.03	(0.080)	-0.081	(0.123)	0.09
ALPH	0.007	(0.009)	0.748***	(0.196)	0.122	(0.092)	0.096	(0.141)	0.31
MVV	0.007	(0.006)	0.216*	(0.127)	0.154**	(0.060)	-0.042	(0.091)	0.19
HNA	0.005	(0.010)	1.087***	(0.233)	0.073	(0.109)	0.119	(0.167)	0.35
IBR	-0.007	(0.017)	0.847**	(0.336)	-0.255	(0.222)	0.209	(0.258)	0.20
Mean	0.001	(0.009)	0.661	(0.192)	0.109	(0.093)	0.033	(0.139)	0.30

Tab. D.11: Four-factor model: 2005–2010 regression results with estimates for firm-specific equity coefficients $\lambda_{0,i}$, $\lambda_{m,i}$, $\lambda_{eua,i}$, $\lambda_{el,i}$, $\lambda_{es,i}$ with excess returns of the DJ Euro Stoxx Utilities index (ECB, 2011), EUA front year futures (EEX, 2011), Phelix base front year futures (EEX, 2011), and ifo Business Climate Index (IFO, 2011) as considered risk factors. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\lambda}_{0,i}$		$\hat{\lambda}^*_{m,i}$		$\hat{\lambda}^*_{eua,i}$		$\hat{\lambda}^*_{el,i}$		$\hat{\lambda}^*_{es,i}$		R^2_{adj}
EDF	0.002	(0.009)	1.081***	(0.216)	0.292***	(0.094)	-0.094	(0.150)	-0.289	(0.570)	0.48
EON	-0.001	(0.008)	0.616***	(0.192)	0.267***	(0.084)	-0.009	(0.132)	0.171	(0.506)	0.40
GDZ	0.001	(0.008)	0.435**	(0.182)	0.231***	(0.079)	-0.057	(0.125)	-0.368	(0.480)	0.24
RWE	-0.001	(0.007)	0.514***	(0.171)	0.15**	(0.075)	0.036	(0.118)	-0.096	(0.452)	0.28
ENEL	-0.007	(0.006)	0.675***	(0.140)	0.048	(0.061)	-0.16	(0.096)	0.48*	(0.369)	0.41
ELE	0.001	(0.012)	0.754***	(0.263)	-0.071	(0.114)	-0.08	(0.181)	0.259**	(0.693)	0.13
IBE	0.003	(0.009)	0.719***	(0.219)	0.143	(0.095)	-0.198	(0.151)	0.112	(0.578)	0.23
IPR	0.012	(0.009)	1.275***	(0.227)	0.071	(0.099)	-0.035	(0.156)	0.322	(0.598)	0.47
ENBW	-0.002	(0.006)	0.127	(0.155)	0.138**	(0.068)	-0.049	(0.107)	0.58**	(0.410)	0.18
CEZ	0.003	(0.007)	0.39**	(0.169)	0.217***	(0.074)	0.324***	(0.117)	0.366	(0.447)	0.49
FOT	0.006	(0.006)	0.51***	(0.162)	0.143**	(0.070)	0.391***	(0.112)	0.786*	(0.427)	0.58
DEI	-0.005	(0.012)	0.877***	(0.299)	0.08	(0.130)	-0.228	(0.206)	-0.073	(0.790)	0.17
EDN	-0.009	(0.010)	0.615**	(0.239)	0.131	(0.104)	-0.052	(0.164)	0.581	(0.630)	0.24
EDP	0.002	(0.007)	0.691***	(0.166)	0.14*	(0.072)	-0.318***	(0.114)	0.022	(0.437)	0.35
SSE	0.002	(0.006)	0.392***	(0.143)	0.022	(0.062)	0.055	(0.099)	0.118	(0.378)	0.20
VER	0.001	(0.009)	0.533**	(0.218)	0.256***	(0.095)	0.412***	(0.150)	0.041	(0.576)	0.46
DRX	-0.005	(0.011)	0.627**	(0.263)	-0.009	(0.114)	0.445**	(0.182)	-0.478	(0.695)	0.25
CNA	0.007	(0.008)	0.321*	(0.182)	0.026	(0.079)	-0.086	(0.125)	0.379	(0.479)	0.10
ALPH	0.007	(0.009)	0.839***	(0.212)	0.128	(0.092)	0.104	(0.146)	-0.553	(0.559)	0.32
MVV	0.007	(0.006)	0.215	(0.141)	0.154**	(0.061)	-0.042	(0.097)	0.007	(0.371)	0.19
HNA	0.005	(0.010)	1.151***	(0.258)	0.078	(0.112)	0.125	(0.178)	-0.388	(0.680)	0.35
IBR	-0.005	(0.018)	0.963**	(0.395)	-0.247	(0.226)	0.215	(0.268)	-0.493	(0.976)	0.21
Mean	0.001	(0.009)	0.651	(0.209)	0.109	(0.094)	0.032	(0.144)	0.068	(0.550)	0.31

Tab. D.12: Five-factor model: 2005–2010 regression results with estimates for firm-specific equity coefficients $\lambda_{0,i}$, $\lambda_{m,i}$, $\lambda_{eua,i}$, $\lambda_{el,i}$, $\lambda_{es,i}$, and $\lambda_{oil,i}$ with excess returns of the DJ Euro Stoxx Utilities index (ECB, 2011), EUA front year futures (EEX, 2011), Phelix base front year futures (EEX, 2011), ifo Business Climate Index (IFO, 2011), and WTI oil futures (EIA, 2011) as considered risk factors. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\lambda}_{0,i}$		$\hat{\lambda}^*_{m,i}$		$\hat{\lambda}^*_{eua,i}$		$\hat{\lambda}^*_{el,i}$		$\hat{\lambda}^*_{es,i}$		$\hat{\lambda}^*_{oil,i}$		R^2_{adj}
EDF	0.002	(0.009)	1.085***	(0.216)	0.3***	(0.094)	-0.067	(0.150)	-0.152	(0.570)	-0.085	(0.140)	0.48
EON	-0.001	(0.008)	0.617***	(0.192)	0.27***	(0.084)	0	(0.132)	0.218	(0.506)	-0.029	(0.124)	0.40
GDZ	0.003	(0.007)	0.445**	(0.182)	0.259***	(0.079)	0.031	(0.125)	0.096	(0.480)	-0.283**	(0.118)	0.31
RWE	-0.001	(0.007)	0.519***	(0.171)	0.164**	(0.075)	0.078	(0.118)	0.127	(0.452)	-0.136	(0.111)	0.30
ENEL	-0.007	(0.006)	0.68***	(0.140)	0.063	(0.061)	-0.115	(0.096)	0.719*	(0.369)	-0.146	(0.090)	0.44
ELE	0.005	(0.011)	0.779***	(0.263)	0.001	(0.114)	0.138	(0.181)	1.414**	(0.693)	-0.704***	(0.170)	0.34
IBE	0.005	(0.009)	0.732***	(0.219)	0.179*	(0.095)	-0.088	(0.151)	0.693	(0.578)	-0.354**	(0.142)	0.31
IPR	0.013	(0.009)	1.278***	(0.227)	0.079	(0.099)	-0.009	(0.156)	0.462	(0.598)	-0.085	(0.147)	0.47
ENBW	-0.002	(0.006)	0.133	(0.155)	0.153**	(0.068)	-0.002	(0.107)	0.827**	(0.410)	-0.15	(0.101)	0.21
CEZ	0.004	(0.007)	0.395**	(0.169)	0.232***	(0.074)	0.369***	(0.117)	0.609	(0.447)	-0.148	(0.110)	0.51
FOT	0.006	(0.007)	0.509***	(0.162)	0.14*	(0.070)	0.38***	(0.112)	0.731*	(0.427)	0.034	(0.105)	0.58
DEI	-0.004	(0.012)	0.88***	(0.299)	0.089	(0.130)	-0.2	(0.206)	0.074	(0.790)	-0.09	(0.194)	0.17
EDN	-0.01	(0.010)	0.611**	(0.239)	0.121	(0.104)	-0.081	(0.164)	0.424	(0.630)	0.096	(0.154)	0.24
EDP	0.002	(0.007)	0.692***	(0.166)	0.143*	(0.072)	-0.31***	(0.114)	0.063	(0.437)	-0.025	(0.107)	0.35
SSE	0.003	(0.006)	0.397***	(0.143)	0.036	(0.062)	0.096	(0.099)	0.337	(0.378)	-0.134	(0.093)	0.23
VER	0.002	(0.009)	0.535**	(0.218)	0.261***	(0.095)	0.43***	(0.150)	0.132	(0.576)	-0.056	(0.141)	0.47
DRX	-0.006	(0.011)	0.617**	(0.263)	-0.028	(0.114)	0.384**	(0.182)	-0.791	(0.695)	0.191	(0.170)	0.27
CNA	0.008	(0.007)	0.33*	(0.182)	0.052	(0.079)	-0.007	(0.125)	0.798	(0.479)	-0.255**	(0.117)	0.17
ALPH	0.008	(0.009)	0.847***	(0.212)	0.149	(0.092)	0.167	(0.146)	-0.216	(0.559)	-0.205	(0.137)	0.35
MVV	0.007	(0.006)	0.217	(0.141)	0.16**	(0.061)	-0.024	(0.097)	0.103	(0.371)	-0.058	(0.091)	0.19
HNA	0.005	(0.010)	1.152***	(0.258)	0.081	(0.112)	0.133	(0.178)	-0.341	(0.680)	-0.029	(0.167)	0.35
IBR	-0.003	(0.017)	0.983**	(0.395)	-0.204	(0.226)	0.297	(0.268)	0.04	(0.976)	-0.284	(0.237)	0.25
Mean	0.002	(0.009)	0.656	(0.209)	0.123	(0.094)	0.073	(0.144)	0.289	(0.550)	-0.133	(0.135)	0.34

Tab. D.13: Six-factor model: 2005–2010 regression results with estimates for firm-specific equity coefficients $\lambda_{0,i}$, $\lambda_{m,i}$, $\lambda_{eua,i}$, $\lambda_{el,i}$, $\lambda_{es,i}$, $\lambda_{oil,i}$, and $\lambda_{gas,i}$ with excess returns of the DJ Euro Stoxx Utilities index (ECB, 2011), EUA front year futures (EEX, 2011), Phelix base front year futures (EEX, 2011), ifo Business Climate Index (IFO, 2011), WTI oil futures (EIA, 2011), and German gas import prices (BAFA, 2010) as considered risk factors. Standard errors are provided in parentheses. Thereby, *** denotes significance of the coefficient at the 1% level, ** at the 5% level and * at the 10% level.

Company	$\hat{\lambda}_{0,i}$		$\hat{\lambda}^*_{m,i}$		$\hat{\lambda}^*_{eua,i}$		$\hat{\lambda}^*_{el,i}$		$\hat{\lambda}^*_{es,i}$		$\hat{\lambda}^*_{oil,i}$		$\hat{\lambda}^*_{gas,i}$		R^2_{adj}
EDF	0.003	(0.009)	1.08***	(0.217)	0.286***	(0.097)	-0.061	(0.151)	-0.222	(0.580)	-0.074	(0.141)	0.286***	(0.097)	0.49
EON	0	(0.008)	0.608***	(0.187)	0.235***	(0.083)	0.018	(0.129)	0.05	(0.499)	-0.005	(0.121)	0.235***	(0.083)	0.44
GDZ	0.003	(0.007)	0.444**	(0.184)	0.255***	(0.082)	0.033	(0.127)	0.075	(0.491)	-0.28**	(0.119)	0.255***	(0.082)	0.31
RWE	-0.001	(0.007)	0.52***	(0.173)	0.166**	(0.077)	0.077	(0.119)	0.135	(0.463)	-0.137	(0.112)	0.166**	(0.077)	0.30
ENEL	-0.006	(0.006)	0.673***	(0.136)	0.037	(0.060)	-0.102	(0.094)	0.596	(0.363)	-0.129	(0.088)	0.037	(0.060)	0.48
ELE	0.005	(0.011)	0.773***	(0.263)	-0.024	(0.117)	0.151	(0.181)	1.296*	(0.703)	-0.687***	(0.171)	-0.024	(0.117)	0.35
IBE	0.006	(0.009)	0.722***	(0.213)	0.139	(0.095)	-0.067	(0.147)	0.498	(0.569)	-0.327**	(0.138)	0.139	(0.095)	0.36
IPR	0.013	(0.009)	1.271***	(0.226)	0.054	(0.100)	0.004	(0.156)	0.34	(0.604)	-0.068	(0.147)	0.054	(0.100)	0.49
ENBW	-0.001	(0.006)	0.127	(0.154)	0.132*	(0.068)	0.009	(0.106)	0.723*	(0.411)	-0.136	(0.100)	0.132*	(0.068)	0.24
CEZ	0.004	(0.007)	0.386**	(0.163)	0.198***	(0.073)	0.387***	(0.113)	0.442	(0.436)	-0.125	(0.106)	0.198***	(0.073)	0.55
FOT	0.006	(0.007)	0.508***	(0.163)	0.134*	(0.073)	0.383***	(0.113)	0.705	(0.436)	0.037	(0.106)	0.134*	(0.073)	0.58
DEI	-0.003	(0.012)	0.87***	(0.296)	0.051	(0.132)	-0.181	(0.205)	-0.113	(0.793)	-0.064	(0.193)	0.051	(0.132)	0.20
EDN	-0.009	(0.009)	0.6**	(0.233)	0.079	(0.104)	-0.06	(0.161)	0.221	(0.623)	0.124	(0.151)	0.079	(0.104)	0.29
EDP	0.002	(0.007)	0.688***	(0.166)	0.129*	(0.074)	-0.303**	(0.115)	-0.003	(0.444)	-0.016	(0.108)	0.129*	(0.074)	0.36
SSE	0.003	(0.006)	0.397***	(0.144)	0.036	(0.064)	0.096	(0.100)	0.338	(0.387)	-0.134	(0.094)	0.036	(0.064)	0.23
VER	0.002	(0.009)	0.535**	(0.220)	0.258**	(0.098)	0.431***	(0.152)	0.117	(0.589)	-0.054	(0.143)	0.258**	(0.098)	0.47
DRX	-0.007	(0.011)	0.627**	(0.264)	-0.008	(0.117)	0.377**	(0.183)	-0.687	(0.708)	0.175	(0.171)	-0.008	(0.117)	0.28
CNA	0.008	(0.007)	0.331*	(0.183)	0.056	(0.081)	-0.009	(0.127)	0.817	(0.490)	-0.258**	(0.119)	0.056	(0.081)	0.17
ALPH	0.009	(0.009)	0.844***	(0.213)	0.138	(0.095)	0.173	(0.147)	-0.267	(0.570)	-0.198	(0.138)	0.138	(0.095)	0.35
MVV	0.007	(0.006)	0.219	(0.141)	0.17***	(0.063)	-0.029	(0.098)	0.151	(0.378)	-0.065	(0.092)	0.17***	(0.063)	0.20
HNA	0.006	(0.010)	1.145***	(0.257)	0.052	(0.114)	0.148	(0.177)	-0.479	(0.687)	-0.009	(0.167)	0.052	(0.114)	0.37
IBR	-0.004	(0.017)	0.943**	(0.389)	-0.319	(0.236)	0.354	(0.266)	-0.302	(0.988)	-0.219	(0.237)	-0.319	(0.236)	0.30
Mean	0.002	(0.009)	0.651	(0.208)	0.102	(0.095)	0.083	(0.143)	0.201	(0.554)	-0.120	(0.134)	0.102	(0.095)	0.36

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Declaration

Erklärung (gem. § 10, Abs. 2 der Promotionsordnung)

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und ohne Verwendung anderer als der angegebenen Hilfsmittel angefertigt habe. Alle Stellen, die wörtlich oder sinngemäß aus veröffentlichten oder unveröffentlichten Schriften entnommen wurden, sind als solche kenntlich gemacht. Die Arbeit ist in gleicher Form oder auszugsweise noch nicht im Rahmen anderer Prüfungen vorgelegt worden.

Essen, 15. August 2013

(Malte Sunderkötter)

Erklärung (gem. § 9, Abs. 7 der Promotionsordnung)

Die Artikel in Kapitel 3 und 4 dieser Arbeit sind in gemeinsamer Autorenschaft mit Prof. Dr. Christoph Weber entstanden. Die wissenschaftliche Einzelleistung von Malte Sunderkötter umfasst insbesondere die Grundidee und Konzeption der Artikel, die Literaturrecherche, die analytische Konzeption und Modellierung inkl. Entwicklung und Umsetzung der Beweise, die Entwicklung und quantitative Analyse der Beispiele inkl. Datenrecherche und Programmierung sowie die Auswertung und Ergebnisdiskussion. Ferner liegt die Erstellung und Überarbeitung der Manuskripte inkl. aller grafischen Darstellungen sowie die Kommunikation mit den Reviewern im Rahmen des Veröffentlichungsprozesses bei Malte Sunderkötter. Christoph Weber trug zur Konzeption der Artikel, der Formulierung der mathematischen Modelle und Beweise sowie der Ausgestaltung und Darstellung der Anwendungsbeispiele bei. Er beteiligte sich auch an der Texterstellung und übernahm das Korrekturlesen.

Der Artikel in Kapitel 5 dieser Arbeit ist in gemeinsamer Autorenschaft von Malte Sunderkötter, Prof. Dr. C. Weber und Daniel Ziegler entstanden. Die grundlegende Idee und Konzeption ist in der gemeinsamen Diskussion der Autoren entstanden. Die wissenschaftliche Einzelleistung von Malte Sunderkötter umfasst dabei insbesondere:

- die Einleitung und Literaturrecherche im Abschnitt 5.1,
- die analytische Konzeption und Modellierung des Marktgleichgewichts im perfekten Wettbewerb im Abschnitt 5.3,

- die Konzeption und Ausarbeitung wesentlicher Teile des numerischen Beispiels im Abschnitt 5.5, insbesondere die Teilabschnitte Modellkalibrierung und Parameterschätzung (5.5.1), die Programmierung in Maple und die Berechnung der Portfoliostrukturen (5.5.2), die Auswertung des numerischen Beispiels (5.5.3)-(5.5.4), sowie

- die Auswertung und Ergebnisdiskussion in Abschnitt 5.6.

Die wissenschaftliche Einzelleistung von Daniel Ziegler umfasst insbesondere:

- die Konzeption und Formulierung der allgemeinen Modellannahmen in Abschnitt 5.2,

- die analytische Konzeption und Modellierung des Marktgleichgewichts mit risiko-aversen Agenten im Abschnitt 5.4, sowie

- den Ansatz zur Bestimmung des Risikoaversionsparameters und dessen Berechnung im Abschnitt 5.5.1

Christoph Weber trug zur Konzeption des Artikels, zur Formulierung der mathematischen Modelle und Beweise sowie zur Ausgestaltung und Darstellung des Anwendungsbeispiels bei.

Die Prüfung der Berechnungen und Ergebnisse in den einzelnen Abschnitten erfolgte gegenseitig durch die drei Autoren. Die redaktionelle Verantwortung für die Überarbeitung des Manuskripts und die Kommunikation mit den Gutachtern im Rahmen des Veröffentlichungsprozesses trägt Daniel Ziegler in Abstimmung mit den Co-Autoren.

Essen, 15. August 2013

(Malte Sunderkötter) (Prof. Dr. Christoph Weber) (Daniel Ziegler)