Combinatorial Principles and $\aleph_k$-free Modules

Héctor Gabriel Salazar Pedroza

For a natural number $k > 1$ and a ring A with free additive structure, we realize two different constructions of arbitrarily large $\aleph_k$-free A-modules which are separable as abelian groups. The main tool to construct these modules is a new variant of Shelah's Black Box principle, which is provable in ZFC. In the first case, we take $A = \mathbb{Z}$ and construct an $\aleph_k$-free abelian group G with no epimorphisms onto a free abelian group of countably infinite rank. In the second case, we take A to be a separably realizable ring and construct an $\aleph_k$-free A-module G with the prescribed endomorphism ring $\operatorname{End} G = A \oplus \operatorname{Fin} G$, where $\operatorname{Fin} G$ is the ideal of all endomorphisms of G whose images have finite rank.