Observer-based fault diagnosis using multiple-model and LMI techniques

Von der Fakultät für Ingenieurwissenschaften

der Universität Duisburg-Essen

zur Erlangung des akademischen Grades eines

Doktor-Ingenieurs

genehmigte Dissertation

von

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aus

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Tag der mündlichen Prüfung:	09.07.2009

Acknowledgement

The present dissertation was performed during my work as scientific co-worker at the Institute for Automatic Control and Complex Systems (AKS) of the Faculty for Engineering in the University of Duisburg-Essen.

I would like to thank my supervisor Professor Dr.-Ing. Steven X. Ding (head of the Institute) for his continuous guidance and support during this work and also for numerous fruitful discussions to the treated topic. His constant encouragement has been a true inspiration and were conditions for the success of this work. I am grateful to him for leading me into the area of fault diagnosis.

I am grateful to Professor Didier Maquin for the acceptance as my co-advisor and for the careful revision of the work. As well as good indications and suggestions that were very favorable and helpful for the work.

I also owe my gratitude to all staff of AKS for their kindness, help and a friendly atmosphere. I am grateful to Dipl.-Ing. Amol Naik, M.Eng. Jedsada Saijai and Dipl.-Ing. Cristian Chihaia for all discussions, collaborative work, and proof reading of the manuscript.

My thanks also to the German Academic Exchange Service (DAAD) for the economic support and guidance during my stay in Germany.

I would like to thank to my brother Guillermo, and to my parents Guillermo and Ana Bertha for their support. Finally, I would like to thank specially my wife Alethya Deydree for the affectionate support, promotion, the proof reading of the work, and for the patience and encouragement, that she gives to me. I would like to dedicate this work to my wife Alethya Deydree.

Duisburg, July 2009 Alejandro Rodríguez Solis

To my family,
my daughter and
especially my wife Alethya

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Nomenclature

Aileron positioning system

Scalar	Units	Meaning
A_p	[m^2]	Piston area
c_a	[N/m]	Aerodynamic force coefficient
C_H	[m^3/Pa]	Hydraulic capacity
C_y	[m^2/s]	Flowrate gain
d_{lin}	[$N{\cdot}s/m$]	Lineal damping factor
δ	[$^\circ$]	Command input
Δ_p	[Pa]	Pressure difference
d_{sv}	[-]	Damping factor
f_v	[$N{\cdot}s/m$]	Viscose friction
F_L	[kN]	External air loads
i_{max}	[A]	Maximal input current
i_{sv}	[A]	Input current
k_p	[A/m]	Controller gain
k_{sv}	[m/A]	Servovalve gain
M_e	[$N{\cdot}m$]	Aerodynamic hinge moment
m_p	[kg]	Piston mass
p_A, p_B	[Pa]	Pressure in chambers A and B
p_S	[Pa]	Supply pressure
p_T	[Pa]	Tank pressure
p_V	[Pa]	System pressure
r_h	[m]	Reduced moment arm
ω_{sv}	[Hz]	Cut-off frequency
x_d	[m]	Desired piston position
x_{max}	[m]	Maximal extension movement
x_{min}	[m]	Maximal retraction movement
x_p	[m]	Piston position
$\dot{x}_p$	[m/s]	Piston velocity
$\dot{x}_{p_{max}}$	[m/s]	Maximal piston velocity
y_{sv}	[m]	Servovalve position
y_{max}	[m]	Maximal spool movement
$\dot{y}_{sv}$	[m/s]	Servovalve velocity
$\dot{y}_{max}$	[m/s]	Maximal servovalve velocity

Vehicle lateral dynamic model

Scalar	Units	Meaning
$C'_{\alpha V}$	[N/rad]	Front tire cornering stiffness
$C_{\alpha H}$	[N/rad]	Rear tire cornering stiffness
i_L	[-]	Steering transmission ratio
I_z	[$kg \cdot m^2$]	Moment of inertia (z-Axis)
l_V	[m]	Distance from the vehicle CG to the front axle
l_H	[m]	Distance from the vehicle CG to the rear axle
K_{ϕ_R}	[-]	Roll coefficient
m	[kg]	Total mass
m_R	[kg]	Rolling sprung mass
m_{NR}	[kg]	Non-rolling unsprung mass
r	[rad/s]	Vehicle yaw rate
v_{ref}	[m/s]	Vehicle longitude velocity
β	[rad]	Vehicle side slip angle
δ_L^*	[rad]	Vehicle steering angle
a_y	[m/s^2]	Lateral acceleration

Abbreviations

Acronym	Meaning
APS	Aileron positioning system
FDF	Fault detection filter
FDI	Fault detection and isolation
LMI	Linear matrix inequality
REF	Residual evaluation function
RFD	Robust fault detection
TS	Takagi-Sugeno
TSFO	Takagi-Sugeno fuzzy observer
TSFUIO	Takagi-Sugeno fuzzy unknown input observer
UIO	Unknown input observer

Abstract

The ever-increasing complexity of technical processes requires a higher performance, safety and reliability. For this reason, fault detection and isolation (FDI), which consists of residual generation and residual evaluation, has received more attention in the last years. Most technical processes are represented by a nonlinear system; however it is possible to apply FDI techniques only for a few classes of nonlinear systems.

In the last years, the idea of using an aggregation of local models (multiple-models), as a means to capture the global dynamic characteristics of nonlinear systems, has been successfully integrated in the field of FDI. These multiple-models have been used as an alternative for dealing with nonlinear systems. An advantage of using multiple-models for FDI is that the theory for linear systems can be used for nonlinear systems.

This thesis mainly focuses on the design of robust FDI schemes for nonlinear systems using multiple-model approaches. The considered approaches are (i) the Takagi-Sugeno (TS) fuzzy model (ii) linear systems with polytopic uncertainty.

Three robust FDI schemes based on TS fuzzy models are presented. The first scheme generalizes the linear unknown input observer to a class of nonlinear systems described by TS fuzzy models. The objective of this scheme is to decouple the unknown inputs for residual generation. The second scheme handles nonlinear systems affected by stochastic disturbances; this scheme minimizes the expected steady state estimation error using linear matrix inequality (LMI) techniques. The last one simultaneously enhances the robustness to unknown inputs without sacrificing the fault detection sensitivity.

For linear systems with polytopic uncertainty, a robust fault detection filter is designed considering a reference model. The residuals can be evaluated with a threshold based on this filter.

The effectiveness of each proposed robust FDI scheme is demonstrated with the help of four application examples.

Chapter 1

Introduction

Technical processes have become more and more complex. For this reason, an increasing level of automation is required.

Consequently, it is desired to have higher performance, availability, reliability and security in these processes. In order to fulfill these desired requirements, it is necessary to avoid malfunctions, which are normally caused by a fault in one of the process components.

To better understand security of processes, it is necessary to know the concept of "faults". A fault in a process is defined as an unpermitted deviation of a least one characteristic property or parameter of the system from the standard condition [42]. Faults can be detected and also isolated with the implementation of fault detection and isolation (FDI) approaches.

However, most technical processes are often represented as nonlinear systems due to their complexity, which leads to difficulties when FDI techniques are applied to the process. For this reason, only a few classes of nonlinear systems are considered in the literature of FDI [3, 4, 14, 16, 46].

Instead of using the nonlinear system for FDI, some simplifications and assumptions of a quantitative mathematical model are considered. Commonly, these refer to the reduction of the dynamic order and/or the linearization of the process behavior.

One of the most popular means to linearize a nonlinear system is Taylor series approximation [9, 56]. Once the linear model is obtained, it is possible to apply FDI approaches for linear systems [17, 18, 22, 26, 80].

Linearized systems only work properly around the operating point where the nonlinear system was linearized. For this reason, conventional analytical linear models are not accurate enough to achieve an effective FDI. For these reasons, considering multiple-models are gaining more attention in the field of FDI [40, 61].

Multiple-model approaches, as its name says, use multiple linear models to approximate the behavior of the nonlinear system. They provide a mathematical framework to analyze a complex nonlinear system using a set of simple models (generally linear or affine models) valid in different state space regions of the nonlinear system.

In this thesis, two multiple model approaches have been considered in order to construct a residual generator based on linear FDI theories. The first approach is an approximation of nonlinear systems, by means of the Takagi-Sugeno fuzzy model, the second approach considers linear systems with uncertainty of the polytopic type.

A Takagi-Sugeno (TS) fuzzy model uses multiple linearized models to approximate the behavior of nonlinear systems. These models are described by fuzzy IF-THEN rules which represent local linear input-output relations of a nonlinear system.

The main feature of a TS fuzzy model is that the local dynamics of each fuzzy implication (rule) is represented by a linear model. The overall fuzzy model of the system is achieved by a fuzzy “blending” of the linear models.

On the other hand, linear systems with polytopic uncertainties are basically constituted by two parts, the first part is given by the linearization of the nonlinear system around an operating point and the second part is constituted by the polytopic uncertainty of the system.

1.1 State of the art

In this section the development of fault detection and isolation (FDI) and the related methods in the past few years for TS fuzzy models and linear systems with polytopic uncertainty are introduced.

FDI based Takagi-Sugeno fuzzy model

The topic of TS fuzzy observer for nonlinear systems has received more attention in recent years because of its ability to estimate nonlinear systems using multiple-models [24, 74, 76]. They are very useful in the practice because it is possible to reach an estimation of the states despite the nonlinearities. This is because each model considered in the TS fuzzy model is a linear model, so that one can apply theory for linear systems.

In [76] the first work in the literature for TS fuzzy observers was reported. The TS fuzzy observer is developed by means of parallel distributed compensation (PDC) into a closed loop control. The implementation of a TS fuzzy controller together with a TS fuzzy observer, guarantees not only the stability of the fuzzy control system in the sense of Lyapunov, but also guarantees the convergence of the state estimation error to zero. Both designs for the TS fuzzy controller and observer are made together in an augmented system using an LMI algorithm.

Nonlinear systems affected by stochastic noise have been handled using the Extended Kalman Filter (EKF) based on fuzzy systems [70, 85]. This approach provides an efficient solution to the optimization of fuzzy membership function for both inputs and outputs of the fuzzy controller.

The use of Kalman filters for TS fuzzy systems is a relatively new approach [71]. Here, it is shown how to approximate the time-varying Kalman filter with a time-varying linear combination of steady state Kalman filters (for each linearized system is constructed a Kalman filter). The use of the TS Kalman filter gives an insignificant loss in estimation performance (in relation to the time-varying Kalman filter).

In [23, 49], a robust fault detection filter for TS fuzzy model is proposed. The purpose of the filter is to generate a residual as robust as possible to disturbances and at the same time as sensitive as possible to the presence of faults. The design procedure is provided in terms of LMIs. The performance index corresponding to fault sensitivity is considered constant and only the performance index corresponding to the disturbance attenuation is minimized.

FDI for linear systems with polytopic uncertainty

A topic that has gained tremendous attention in the field of FDI for multiple-models is the residual generation for linear systems with polytopic uncertainty [10, 11, 51, 52]. The principal idea here is to design a fault detection filter robust to disturbances considering the presence of polytopic uncertainty.

An improvement that has been made in FDI for linear systems with polytopic uncertainties is the incorporation of a reference model in the computation of the fault detection filter [32, 52, 86].

The reference model is derived without considering the existence of polytopic uncertainty in the system. The purpose of the fault detection filter with polytopic uncertainty is the approximation of the solution given by the reference model.

1.2 Motivation and objective of the work

It is a well-known fact that most technical processes exhibit a nonlinear behavior, and that only few classes of nonlinear systems can be treated with FDI approaches. In order to implement an FDI approach, it is required the design of a residual generator, which compares the measured output of the system against an estimated output given by an observer. For this purpose the design of a residual generator for the nonlinear system is not easy even if the mathematical model is known [28, 29, 30, 31].

For the design of the residual generator, the most adopted solution is to use a linearization of the nonlinear system. Unfortunately sometimes the linear model does not give good results for FDI, because the observer used in the FDI can not estimate the behavior of the nonlinear system correctly. Moreover, the generated residual differs from zero or takes too much time to converge to zero even if faults and disturbances are not affecting the system. This behavior indicates that the linear system utilized to construct the residual generator does not approximate the nonlinear system correctly.

In the last few years, the idea of using an aggregation of local models (multiple-models), as a means to capture the global dynamic characteristics of nonlinear systems, has been successfully incorporated in the field of FDI. These multiple-models are used as an alternative for dealing with nonlinear systems and applied in FDI generating the multiple-model approaches.

One of these multiple-model approaches is the TS fuzzy model, which approximates nonlinear systems. In this approach, local linear systems are used to represent the local dynamics in different state space regions.

The application of this TS fuzzy model has given a good solution to some problems in nonlinear systems and at the same time allows the use of FDI theories for linear systems to represent nonlinear systems.

An advantage of TS fuzzy models over a simple linear system is that a TS fuzzy model can work around multiple operating points, i.e. the TS fuzzy model operates on a state space region.

Another multiple-model approach is residual generation for linear systems with polytopic uncertainty. In this approach, the FDI works in a better way, because the polytopic uncertainty is considered in the design of the residual generator. Therefore a better convergence of the residual to zero in the absence of faults and disturbance is assured.

Both of these multiple-model approaches improve the performance of a residual generator for a nonlinear system, the first one considers multiple linearization, i.e. around a region and the second one considers the polytopic uncertainty enplicity in the system. **Objective of the work**

In this thesis, the TS fuzzy model is obtained from the approximation of the nonlinear model with a set of linear models. The polytopic uncertainty is assumed known and comes from the linearization in Taylor Series of the nonlinear equations.

The main objective of this thesis is to incorporate the TS fuzzy model for its use with linear FDI approaches. The principal objective is to make the residual generator as robust as possible to disturbances (could be deterministic or stochastic) and as sensitive as possible to the faults. Therefore, the disturbances are minimized and the detection of faults in an early stage is increased.

Three different schemes are introduced for TS fuzzy models:

- Unknown input observers for linear systems are generalized for a class of nonlinear systems described by TS fuzzy models.
- Nonlinear systems affected by stochastic disturbances are considered to design a TS fuzzy observer. This scheme minimizes the expected steady state estimation error using LMI techniques.
- A robust fault detection observer is extended for its use with TS fuzzy models based on iterative LMI schemes. This scheme simultaneously enhances the robustness against unknown inputs without sacrificing the fault detection sensitivity.

An FDI approach for linear systems with polytopic uncertainty from [17, 66] is applied to the aileron positioning system. Both multiple-model approaches aim for a better FDI for nonlinear systems.

1.3 Organization of the work

Chapter 2 addresses concepts referring to the fuzzy logic and fuzzy models, which are considered essential to understand the remainder of the work concerning TS fuzzy models.

The definition of TS fuzzy observer and stability conditions are given. Finally, some concepts concerning to fault detection and isolation are briefly defined.

Chapter 3 handles the unknown input observer (UIO) for TS fuzzy systems, the UIO for linear systems from [17] is generalized for a class of nonlinear systems described by TS fuzzy models. The UIO for TS fuzzy systemsis proposed.

The objective of this observer is to decouple the unknown inputs and to estimate the states, on the basis of the derivative of the output. A robust sensor fault isolation scheme [12] based on the TS fuzzy UIO is presented in order to detect and isolate faults.

Chapter 4 considers the discrete TS fuzzy model with stochastic disturbances in order to design a residual generator. The objective of this scheme is to minimize the expected value of the steady state estimation error with the use of LMI techniques.

Chapter 5 presents a robust fault detection observer for TS fuzzy models. The objective of this observer is to find a trade-off between maximizing the effect of faults and minimizing the effect of disturbances known as robust fault detection (RFD). For the RFD with TS fuzzy model two iterative LMI schemes for linear systems, taken from [79] and [81] are used.

Chapter 6 uses theory of FDI for linear systems with polytopic uncertainty from [17, 66] to design a fault detection filter, which is robust to disturbances and is sensible to faults and a threshold is designed to evaluate the generated residual.

This approach consists of three steps. First is the calculation of a reference model, follow the design of the FDF using the reference model to build an extended system. Finally, the obtained gain matrix from the previous step is used to calculate a threshold.

Chapter 7 concludes the results obtained from this thesis and the idea of future work is outlined.

Appendix A gives the formulas for signal norm computation, Schur complement, the relax stability condition for TS fuzzy models and the concept of LMI and convex optimization techniques (COT), which constitute the principal tools in the solution of the proposed optimization problems for both multiple-model approaches.

Appendix B shows the numerical values for the variables used in the application examples.

Chapter 2

TS fuzzy model and FDI Concepts

In this chapter, basic concepts regarding to Takagi-Sugeno (TS) fuzzy models and fault detection and isolation (FDI) are reviewed. It includes the description of a TS fuzzy model, the stability analysis of a TS fuzzy observer, the definition of linear matrix inequalities (LMI), convex optimization techniques (COT) and definitions on the field of fault detection and isolation.

2.1 Takagi-Sugeno fuzzy model

A Takagi-Sugeno (TS) fuzzy model is a fuzzy rule-based model approach suitable to approximate a large class of nonlinear dynamic systems [73]. Fig. 2.1 illustrates the model-based TS fuzzy observer used in this thesis.

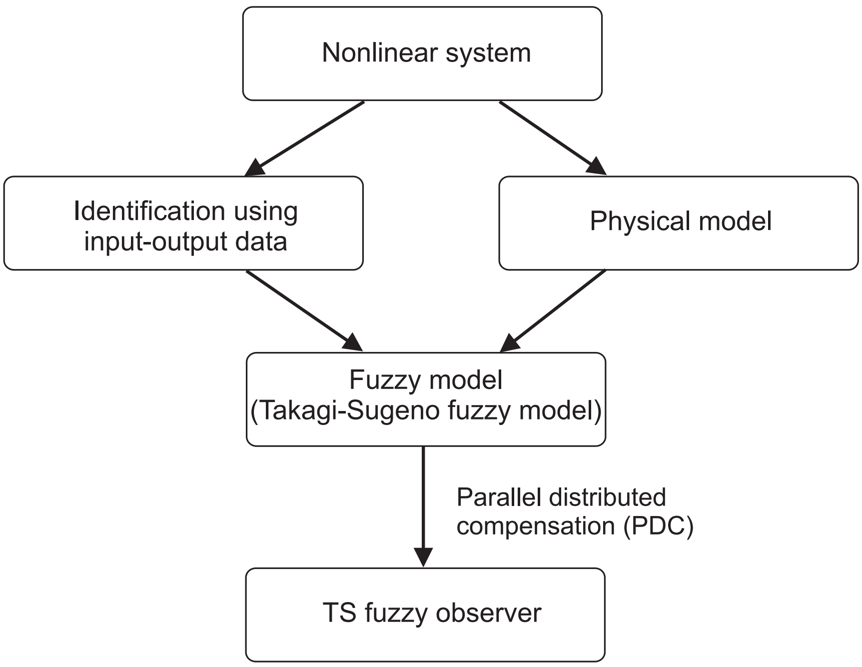

Fig. 2.1: Model-based TS fuzzy observer design

To design a TS fuzzy observer, a TS fuzzy model which approximates the nonlinear system is needed. Therefore the construction of a TS fuzzy model represents an important and basic procedure in this approach.

In general, there are two approaches for the construction of TS fuzzy models:

1. Identification (fuzzy modeling) using input-output data

2. Derivation from given nonlinear equations.

The identification approach is mainly constituted by two parts: structure identification and parameter identification [39, 72]. This approach is suitable for plants that are very complex or too difficult to be represented by analytical and/or physical models.

On the other hand, nonlinear dynamic models can be obtained by, e.g. the *Lagrange method* and the *Euler-Newton method*. In such cases, the second approach, which derives a TS fuzzy model from given nonlinear dynamic models is more appropriate [77].

In this thesis, the second approach is considered in order to generate a TS fuzzy model, which approximates the behavior of the nonlinear system. In the TS fuzzy model, local dynamics in different state space regions are represented by local linear systems [55, 57].

Unlike conventional modeling which uses a single model to describe the global behavior of a nonlinear system, fuzzy modelling is essentially a multiple-model approach, in which simple submodels (linear models) are combined to approximate the global behavior of the nonlinear system.

The TS fuzzy model proposed by Takagi and Sugeno in [73] is described by fuzzy IF-THEN rules, where local linear models are used to represent the dynamic behavior in different state space regions [77], i.e. the nonlinear trajectories are linearized over different state space regions.

A fuzzy IF-THEN rule represents a local relation input-output of the nonlinear system in a state space region. The set of linear models are used to calculate the overall model of the system by "blending" these linear models through fuzzy membership functions.

The TS fuzzy model makes possible the use of FDI theory for linear systems to obtain a TS fuzzy residual generator. Because of its better approximation of the behavior of a nonlinear system, the TS fuzzy model can be seen as a good alternative for an efficient residual generation.

The design of TS fuzzy models based on given nonlinear equations considers a class of nonlinear systems described by

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{2.1a}$$
$$y(t) = h(x(t)) \tag{2.1b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ is the input vector and $y(t) \in \mathbb{R}^m$ is the output vector and $f(x(t)), g(x(t))$ and $h(x(t))$ are functions of $x(t)$.

For each state space region there is a fuzzy IF-THEN rule describing the dynamics of the system in that region as follows

Model rule $\boldsymbol{i}$

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & \begin{cases} \dot{x}(t) = & A_i x(t) + B_i u(t) \\ y(t) = & C_i x(t) + D_i u(t) \end{cases} \end{array} \tag{2.2}$$

where $i = 1, \ldots, r$ and r is the number of fuzzy IF-THEN rules, M_{ij} are fuzzy sets, $z_1(t), \ldots z_p(t)$ are premise variables, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ and $y(t) \in \mathbb{R}^m$ are the input and output vectors respectively. Matrices A_i, B_i, C_i and D_i are known system matrices with appropriate dimension.

The premise variables can be functions of the measured state variables, inputs of the system and possibly on some varying parameter (which does not depend on the states).

The truth value of the proposition "$z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}" in the antecedent part is calculated by

$$M_{i1}(z_1(t)) \ \wedge \ldots \wedge \ M_{ip}(z_p(t))$$

where the symbol "$\wedge$" stands for a t-norm (usually min-operator or product), and "$z_p(t)$ is M_{ip}" is the grade of membership of $z_p(t)$ in M_{ip}. All fuzzy sets are associated with a membership function.

The choice of premise variables leads to different classes of models [1]. The following example of a nonlinear system is considered in order to explain this point

$$\dot{x_1}(t) = x_1(t)x_2^2(t) \tag{2.3a}$$

$$\dot{x_2}(t) = x_1(t) - x_2(t) \tag{2.3b}$$

The nonlinear system in eq. (2.3) can be represented in the following two forms

$$\dot{x}(t) = \begin{bmatrix} 0 & x_1(t)x_2(t) \\ 1 & -1 \end{bmatrix} x(t) \qquad \text{or} \qquad \dot{x}(t) = \begin{bmatrix} x_2^2(t) & 0 \\ 1 & -1 \end{bmatrix} x(t) \tag{2.4}$$

As can be seen in eq. (2.4), the premise variable can be defined as $z(t) = x_1(t)x_2(t)$ and also can be defined as $z(t) = x_2^2(t)$, therefore, there are two possible models. The linearized models are valid on a state space region and are calculated using the maximum and minimum value of these premise variables.

A membership function takes values between 0 and 1 , i.e. $M_{ip}(z_p(t)) \in [0,1]$. The value 0 means that $z_p(t)$ is not a member of the fuzzy set and the value 1 means that $z_p(t)$ is fully a member of the fuzzy set [73, 83].

The entire fuzzy model of the plant in eq. (2.2) is obtained with a fuzzy "blending" of all rule consequents, where each consequent part contains a locally valid linear model. For a given pair $(x(t),\, u(t))$, the final outputs of the TS fuzzy model are inferred as follows:

$$\dot{x}(t) = \frac{\sum\limits_{i=1}^{r} w_i(z(t))\Big[A_i x(t) + B_i u(t)\Big]}{\sum\limits_{i=1}^{r} w_i(z(t))} \tag{2.5a}$$

$$y(t) = \frac{\sum\limits_{i=1}^{r} w_i(z(t))\Big[C_i x(t) + D_i u(t)\Big]}{\sum\limits_{i=1}^{r} w_i(z(t))} \tag{2.5b}$$

where

$$\begin{aligned} z(t) &= [z_1(t)\ z_2(t)\ \ldots\ z_p(t)] \\ w_i(z(t)) &= \prod_{j=1}^{p} M_{ij}(z_j(t)) \\ h_i(z(t)) &= \frac{w_i(z(t))}{\sum\limits_{i=1}^{r} w_i(z(t))} \end{aligned}$$

for all t. The term $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Since

$$\begin{cases} \sum_{i=1}^{r} w_i(z(t)) > 0 \\ w_i(z(t)) \geq 0 \end{cases} \qquad \text{for } i = 1, 2, ..., r, \ \forall t. \tag{2.6}$$

the weighting functions $h_i(z(t))$ satisfy the following constraints

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1 \\ h_i(z(t)) \geq 0 \end{cases} \qquad \text{for } i = 1, 2, ..., r, \ \forall t. \tag{2.7}$$

Based on these constraints, one can also write eq. (2.8) instead of eq. (2.5)

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) + B_i u(t) \Big] \tag{2.8a}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[C_i x(t) + D_i u(t) \Big] \tag{2.8b}$$

The overall structure of a TS fuzzy model can be seen in fig. 2.2.

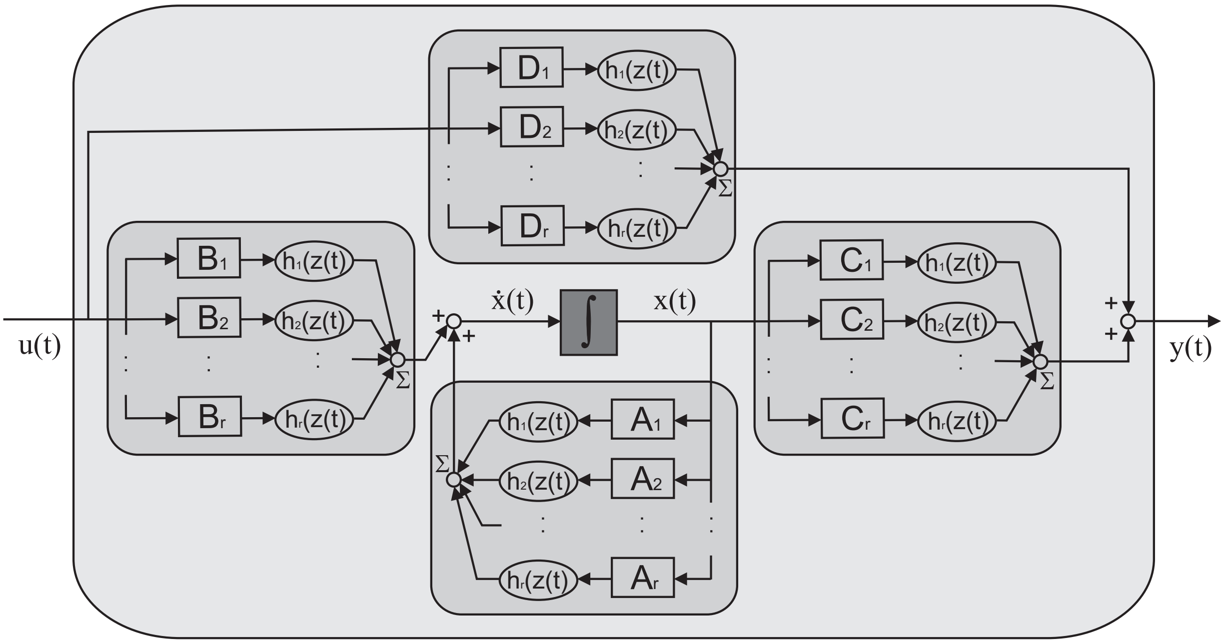

Fig. 2.2: Overall structure of a TS fuzzy model

2.2 Takagi-Sugeno fuzzy observer

For a nonlinear dynamic system approximated by a TS fuzzy model, a TS fuzzy observer can be designed in order to estimate the system state vector [6, 24, 47, 74, 76].

In the design of a TS fuzzy observer, it is assumed that the TS fuzzy model is locally observable, i.e. all pairs (A_i, C_i) are observable.

Using the same idea as in the TS fuzzy model, a TS fuzzy observer utilizes a number of local linear time-invariant (LTI) observers. Each local observer is associated with each fuzzy IF-THEN rule given below:

Observer rule *i*

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & \left\{ \begin{array}{rl} \dot{\hat{x}}(t) = & A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = & C_i\hat{x}(t) + D_iu(t) \end{array} \right. \end{array} \tag{2.9}$$

The concept of parallel distributed compensation (PDC) is used for the design of TS fuzzy observers [75, 82]. The idea is to design an observer for each rule of the fuzzy model. The concept of PDC is illustrated in fig. 2.3.

TS fuzzy models share the same fuzzy sets with the TS fuzzy observer, i.e. both use the same membership functions M_{ij} and the same weighting functions $h_i(z(t))$.

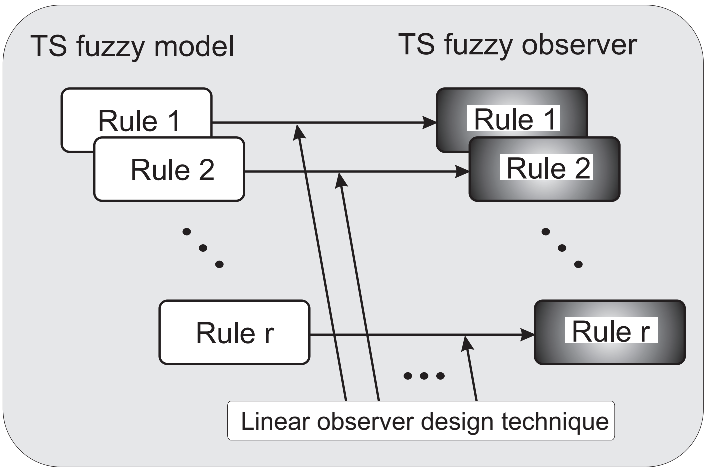

Fig. 2.3: PDC design

The overall state estimation is inferred as a weighted sum of individual local observers:

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[\; A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \; \Big] \\ \hat{y}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[\; C_i\hat{x}(t) + D_iu(t) \; \Big] \end{aligned} \tag{2.10}$$

where L_i is the observer gain matrix for each observer in the corresponding fuzzy IF-THEN rule.

Remark 2.1 *In the subsequent part of this thesis, the notation* $\mathbf{S} > \mathbf{0}$ *means that* $\mathbf{S}$ *is a positive definite matrix,* $\mathbf{S} > \mathbf{T}$ *means that* $\mathbf{S} - \mathbf{T} > \mathbf{0}$ *and* $\mathbf{W} = \mathbf{0}$ *means that* $\mathbf{W}$ *is a zero matrix, i.e. its elements are all zero.*

The following notation can also be used: $\sum_{\mathbf{i<j}}^{\mathbf{r}}, \sum_{\mathbf{i \neq j}}^{\mathbf{r}}$*, which means*

$$\sum_{i<j}^{3} a_{ij} \iff a_{12} + a_{13} + a_{23}$$
$$\sum_{i \neq j}^{3} a_{ij} \iff a_{12} + a_{13} + a_{21} + a_{23} + a_{31} + a_{32}$$

2.2.1 Stability analysis for TS fuzzy observers

For the stability analysis, TS fuzzy observers are required to satisfy the following requirement:

$$\lim_{t \to \infty} (x(t) - \hat{x}(t)) = 0 \tag{2.11}$$

where $\hat{x}(t)$ denotes the state vector estimated by a TS fuzzy observer. The condition in eq. (2.11) guarantees that the state estimation error $e(t)$ between the state vector $x(t)$ and the estimated state vector $\hat{x}(t)$ (estimated by the TS fuzzy observer) converges to zero as time approaches its steady state.

In order to analyze the convergence of the TS fuzzy observer, the state estimation error is defined as $e(t) = x(t) - \hat{x}(t)$ and its dynamics is given by

$$\dot{e}(t) = \dot{x}(t) - \dot{\hat{x}}(t) \tag{2.12}$$

By straight substitution, the dynamics of the state estimation error is given as

$$\begin{aligned}
\dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[\big[A_i x(t) + B_i u(t)\big] - \big[A_i \hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))\big] \Big] \\
&= \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) - A_i \hat{x}(t) - L_i \big(y(t) - \hat{y}(t)\big) \Big] \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \Big[A_i \big(x(t) - \hat{x}(t)\big) - L_i C_j \big(x(t) - \hat{x}(t)\big) \Big] \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \Big[A_i - L_i C_j \Big] e(t) \\
&= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) A_{ij} e(t)
\end{aligned} \tag{2.13}$$

where

$$A_{ij} = A_i - L_i C_j$$

Note that eq. (2.13) can also be written as follows

$$\dot{e}(t) = \sum_{i=1}^{r} h_i^2(z(t)) A_{ii} e(t) + 2 \sum_{i=1}^{r} \sum_{i<j} h_i(z(t)) h_j(z(t)) \left(\frac{A_{ij} + A_{ji}}{2} \right) e(t) \tag{2.14}$$

The stability of the dynamic eq. (2.14) can be proved by the Theorem 2.1.

Theorem 2.1 *[6, 74, 77]: The equilibrium of the system described by eq. (2.14) is asymptotically stable if there exists a common positive definite matrix P for $i = 1, ..., r$ such that*

$$A_{ii}^T P + P A_{ii} < 0 \tag{2.15}$$

$$\left(\frac{A_{ij} + A_{ji}}{2}\right)^T P + P\left(\frac{A_{ij} + A_{ji}}{2}\right) \leq 0 \qquad i < j \tag{2.16}$$

Proof: Consider a candidate of Lyapunov function $V(e(t)) = e^T(t)Pe(t)$, where $P > 0$. Then,

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\
&= e^T(t)\left(\sum_{i=1}^{r} h_i^2(z(t))A_{ii} + 2\sum_{i=1}^{r}\sum_{i<j} h_i(z(t))h_j(z(t))\left(\frac{A_{ij} + A_{ji}}{2}\right)\right)^T Pe(t) \\
&\quad + e^T(t)P\left(\sum_{i=1}^{r} h_i^2(z(t))A_{ii} + 2\sum_{i=1}^{r}\sum_{i<j} h_i(z(t))h_j(z(t))\left(\frac{A_{ij} + A_{ji}}{2}\right)\right)e(t) \\
&= \sum_{i=1}^{r} h_i^2(z(t))e^T(t)\left[A_{ii}^T P + PA_{ii}\right]e(t) \\
&\quad + 2\sum_{i=1}^{r}\sum_{i<j} h_i(z(t))h_j(z(t))e^T(t)\left[\left(\frac{A_{ij} + A_{ji}}{2}\right)^T P + P\left(\frac{A_{ij} + A_{ji}}{2}\right)\right]e(t)
\end{aligned}$$

Q.E.D.

The fuzzy observer design problem is to determine matrices L_i $(i = 1, \ldots, r)$ which satisfy the conditions of Theorem 2.1 with a common positive definite matrix P.

With the same strategy as in [8], it is possible to transform the conditions given by eq. (2.15)-(2.16) in LMIs and obtain directly the gain matrices L_i for the TS fuzzy observer.

For this purpose, let us substitute A_{ii} in eq. (2.15) and A_{ij} and A_{ji} in eq. (2.16), which results in

$$\begin{aligned}
A_i^T P + PA_i - C_i^T L_i^T P - PL_iC_i &< 0 \\
A_i^T P + PA_i + A_j^T P + PA_j - C_j^T L_i^T P - PL_iC_j - C_i^T L_j^T P - PL_jC_i &\leq 0 \qquad i < j
\end{aligned}$$

Defining $N_i = PL_i$ and $N_j = PL_j$ for $P > 0$, after substituting N_i and N_j in the above matrix inequalities, it results in

$$\begin{aligned}
A_i^T P + PA_i - C_i^T N_i^T - N_iC_i &< 0 \\
A_i^T P + PA_i + A_j^T P + PA_j - C_j^T N_i^T - N_iC_j - C_i^T N_j^T - N_jC_i &\leq 0 \qquad i < j
\end{aligned}$$

These LMI conditions, allow us to define a TS fuzzy observer design problem as

Problem 2.1 ***TS fuzzy observer design:*** *Find $P > 0$ and N_i $(i = 1, \ldots, r)$ satisfying*

$$A_i^T P + PA_i - C_i^T N_i^T - N_iC_i < 0 \tag{2.17a}$$

$$A_i^T P + PA_i + A_j^T P + PA_j - C_j^T N_i^T - N_iC_j - C_i^T N_j^T - N_jC_i \leq 0 \qquad i < j \tag{2.17b}$$

The above conditions are LMIs with respect to variables P and N_i. A positive definite matrix P and matrices N_i satisfying these LMIs can be found. In contrast, if this is not possible, then the feasibility problem is rendered as infeasible.

This feasibility problem can be solved efficiently using mathematical tools, e.g. MATLAB. The observer gain matrices L_i can be obtained as

$$L_i = P^{-1} N_i$$

In this sense, the stability analysis of TS fuzzy observers is reduced to a problem of finding a common matrix P.

Remark 2.2 *If the number of rules "r" is large, it might be difficult to find a common matrix P satisfying the conditions of theorem 2.1. In such cases relaxed stability conditions for the theorem 2.1, found in Appendix A.3, can be applied.*

2.3 Fault Detection and Isolation (FDI)

The objective of fault detection and isolation is to detect faults appearing in the system as early as possible, so that the failure of the whole system can be avoided.

The most important concepts in the field of FDI are **fault** and **disturbance**. Both represent a deviation of the process state from the required operating condition, but they are basically different.

Fault is defined as an unpermitted deviation of a least one characteristic property or parameter of the system from the standard condition [42], which results in an undesired behavior of the nominal system.

A fault can affect the system in an *unfavorable* (e.g. by reduced efficiency due to increasing friction losses) or in a *dangerous* (e.g. by danger of explosion in chemical reactors due to increasing temperature) way.

The detectable effect of the fault can manifest itself by constant off-sets, exceeding a range of values, modifying scaling factors or modifying dynamic behavior.

Disturbance is a tolerable (maybe inevitable) discrepancy from the ideal operating state, and can not have as a consequence an undesired behavior of the nominal system.

A disturbance represents therefore no potential danger, but describes "the completely normal" deviation of the real process from the ideal case. Disturbances are, e.g. inevitable friction and absorption losses, measuring and discretization noise.

The use of process models for fault detection in real systems incorporates another source of disturbance signal: the modeling noise due to the inevitable discrepancy between the process and the model.

However, it is desired not to detect these effects but to reduce them. Only if a disturbance changes into a fault (e.g. if the friction losses exceed a certain limit value "normal" friction), then the detection should take place.

As described in [27, 28], faults can be divided in: actuator, component and sensor faults. This classification is needed in order to be able to differentiate the arising faults according to the place of its occurrence, as depicted in fig. 2.4.

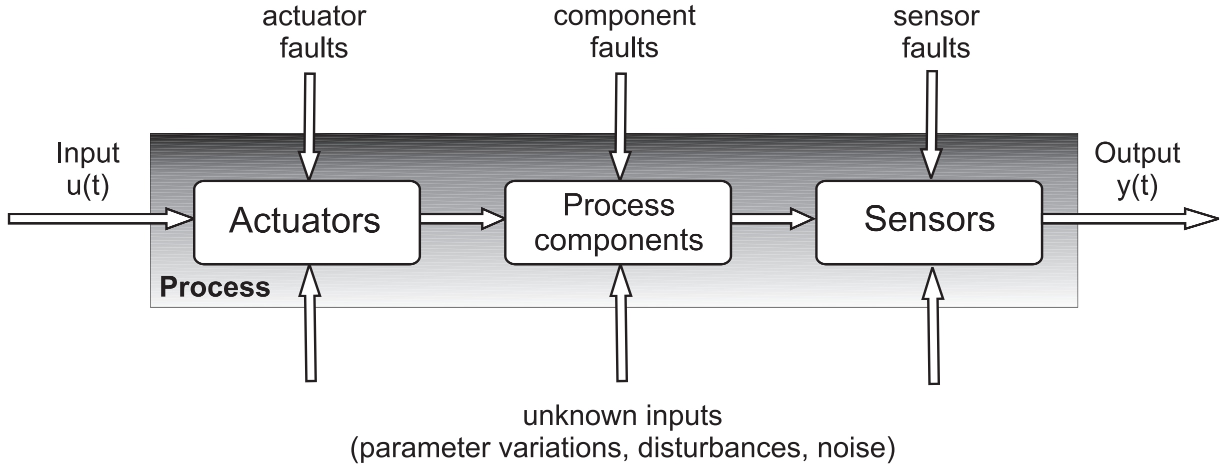

Fig. 2.4: Definition of faults in the plant of the process

An **actuator fault** is a fault that appears in an actuator of the process, e.g. defect in gears and aging effects. The faults that appear in the sensors are identified as **sensor fault**, e.g. scaling errors and contact failures.

Component faults produce critical parameter changes in the process itself, e.g. leakages and loose parts.

Actuator, component and sensor faults are *additive faults* because are unknown extra inputs acting on the system [35] while there exist also *multiplicative faults* which imply changes of some plant parameters.

In order to know if a fault is affecting the system, a compared signal between measured and estimated one known as *residual signal* is required.

Residuals are designed to be equal or to converge to zero in the fault-free case and diverge significantly from zero when fault occurs in the system. Therefore, the residual signals represent the effect of faults in the system.

Most model-based FDI approaches incorporate two sequential steps in order to achieve FDI. They are residual generation and residual evaluation [46, 58].

1. Residual generation: In this stage, the data taken (measured) from the actual process, which reflects the faults, are compared with the corresponding reference values of the fault-free (nominal) case.

The residual generation process can be interpreted as the evaluation of redundancy.

$$r(t) = y(t) - \hat{y}(t) \tag{2.18}$$

In order to detect and isolate faults, system redundancy is necessary. Redundancy is the relation among the measured variables. The system redundancy in FDI can be divided in two classes, i.e. *physical* and *analytical redundancy*:

- *Physical redundancy*: The process variables are measured by multiple (redundant) sensors. This approach is effective only for the detection of sensor failures, because any malfunction in the actuators or in the process itself will affect all the sensors simultaneously.

- *Analytical redundancy* are the procedures of using model information to generate additional signals, which are compared with the original measured signals. Analytical redundancy can be used to avoid the repetition of hardware in the alternative approach known as physical redundancy [58].

  Observer-based fault diagnosis is an example of analytical redundancy based-approach.

2. Residual evaluation: In this stage, the processing of the residual signal by threshold selection is performed. This threshold is utilized together with a residual evaluation function and it allows to establish a limit. This limit is the maximal value of the evaluated residual for the free-fault case.

The design of the threshold plays a very important role in the residual evaluation and it must be robust against disturbances affecting the system.

In the FDI approaches, signal norms (Appendix A.1) are used to evaluate the residual signal [21, 63]. In the signal norms, the size (in the sense of a norm) of the residual signal is calculated on-line and then compared with a given threshold.

The decision logic for the threshold is as follows:

$$\begin{aligned} \|r(t)\| &\leq \text{threshold} \Rightarrow \text{no alarm, (fault-free)} \\ \|r(t)\| &> \text{threshold} \Rightarrow \text{alarm, (a fault is detected)} \end{aligned} \tag{2.19}$$

where $\|\cdot\|$ stands for the norm of the residual signal.

Model-based FDI approaches are based on a mathematical model and as explained before, a precise and accurate model of a real system is not always possible to obtain.

This is due to different causes, e.g. disturbances, different noise effects and uncertain or time-varying system parameters.

FDI approaches that can be able to handle these kind of disturbances, are referred as *robust* FDI approaches.

The robustness problem in FDI is defined as the maximization of the detectability and isolability of faults together with the minimization of the effect of uncertainty and disturbance on the FDI procedure.

The optimization problems can be achieved using sensitivity theory, as long as due care has been paid to the robustness of the global system operation.

FDI using analytical redundancy (model-based) methods is currently a subject of extensive research [59]. The model-based FDI process is depicted in fig. 2.5.

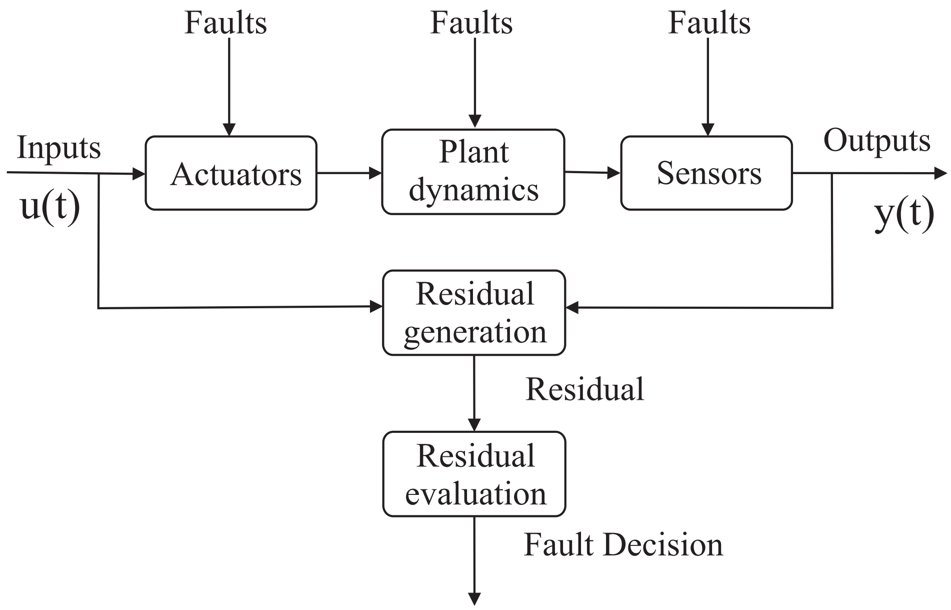

Fig. 2.5: Model-based FDI process

False alarms are another important concept in the FDI field. It is defined as a misinterpretation of the system, where a change in some variable is considered as a fault. False alarms can be activated by a large model uncertainty, by high detection sensitivity, particularly within the dynamic range, or by disturbances.

The sensitivity to faults and avoidance of false alarms due to disturbances leads to the optimization problem in the design of fault diagnosis systems. Since a robust FDI scheme is desired, the principal objective is to increase the robustness to unknown inputs and simultaneously to enhance the sensitivity to faults [19].

The next step is to evaluate the generated residual and to compare it with a threshold. The selection of the threshold plays an important role in FDI.

Chapter 3

Unknown input observer for TS fuzzy models

An unknown input observer (UIO) is a robust observer which can tolerate a degree of model uncertainty and hence increase the reliability of fault diagnosis [2, 12, 13, 69]. In this approach, the model-reality mismatch is represented by the so-called unknown input and hence the state estimate and, consequently, the output estimate are obtained by taking into account model uncertainty.

Unfortunately, the existing nonlinear extensions of the UIO as in [13, 60] require a relatively complex design procedure, even for simple laboratory systems [88]. Moreover, they are usually limited to a very restricted class of nonlinear systems.

On the other hand, it is well known that UIO-based solution works well for linear systems only when there is no large mismatch between the linearized model around the current state estimate and the nonlinear behavior of the system.

The use of a linear UIO allows the robust estimation of the states even if the system has unknown inputs (disturbances). The design of UIO for linear systems is well established but only works around the operating point were the nonlinear system was linearized.

TS fuzzy models consider a state space region and not only an operating point and they allow the use of linear theories, therefore they are used to make an extension of the UIO approach developed in [17] for its use with TS fuzzy models.

3.1 UIO approach for linear systems

One of the most important tasks in model-based fault diagnosis techniques is the generation of robust residuals. Disturbance decoupling approaches are a good option to generate these robust residuals. In these approaches, uncertain factors in system modeling are considered to affect the linear system via an unknown input (or disturbance) [12]. Despite the fact that the unknown input vector is unknown, its distribution matrix is assumed known.

Considering the information given by the distribution matrix, the unknown input (disturbance) can be decoupled from the residual. The decoupling of the unknown inputs can be achieved using unknown input observers (UIO). It also decouples state estimation error from disturbances.

For the design of UIOs a class of linear systems is considered. The system uncertainty can be summarized as an *additive* unknown disturbance term in the dynamic equation

$$\dot{x}(t) = Ax(t) + Bu(t) + E_d d(t) \tag{3.1a}$$

$$y(t) = Cx(t) \tag{3.1b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ is the known input vector, $d(t) \in \mathbb{R}^{k_d}$ is the unknown input (or disturbance) vector and $y(t) \in \mathbb{R}^m$ is the measurement or output vector. A, B, E_d and C are known system matrices with appropriate dimensions.

Remark:

There is no loss of generality in assuming that the unknown input distribution matrix E_d should be full column rank. When this is not the case, the following rank decomposition can be applied to the matrix E_d

$$E_d d(t) = E_{d_1} E_{d_2} d(t) \tag{3.2}$$

where E_{d_1} is a full column rank matrix and $E_{d_2} d(t)$ can now be considered as a new unknown input vector (for a proof refer to [12], page 301).

Definition 3.1 (Unknown Input Observer (UIO) [12]) *An observer designed for the system described by eq. (3.1) is considered as an unknown input observer, if its state estimation error vector $e(t)$ approaches to zero asymptotically, despite of the presence of the unknown input (disturbance) in the system.*

One can also interpret the UIO as a Luenberger type observer that delivers a state estimation $\hat{x}(t)$ independent of the unknown input (disturbance) $d(t)$ in the sense that :

$$\lim_{t\to\infty} \big(x(t) - \hat{x}(t)\big) = 0 \qquad \text{for all } u(t),\, d(t),\, x_0 \tag{3.3}$$

With the use of the state estimate $\hat{x}(t)$, it is possible to construct a residual signal as follows:

$$r(t) = y(t) - C\hat{x}(t) \tag{3.4}$$

3.1.1 UIO design

For the design of the UIO [15, 17], the derivative of the output signal $y(t)$ is given by

$$\begin{aligned} \dot{y}(t) &= C\dot{x}(t) \\ \dot{y}(t) &= C\big(Ax(t) + Bu(t) + E_d d(t)\big) \end{aligned} \tag{3.5}$$

From eq. (3.5), the term $CE_d d(t)$ is taken to the left

$$CE_d d(t) = \dot{y}(t) - CAx(t) - CBu(t) \tag{3.6}$$

Assume that

$$rank(CE_d) = rank(E_d) = k_d \tag{3.7}$$

and that CE_d is left invertible, i.e. there exists a Moore-Penrose pseudoinverse matrix [68] $(CE_d)^+$ of the product CE_d

$$(CE_d)^+ = \big[(CE_d)^T CE_d\big]^{-1}(CE_d)^T, \qquad (CE_d)^+ \in \mathbb{R}^{k_d \times m} \tag{3.8}$$

Multiplying both sides of eq. (3.6) by the Moore-Penrose pseudoinverse matrix results in

$$\begin{aligned}(CE_d)^+CE_d d(t) &= (CE_d)^+\big[\dot{y}(t) - CAx(t) - CBu(t)\big]\\ d(t) &= (CE_d)^+\big[\dot{y}(t) - CAx(t) - CBu(t)\big]\end{aligned} \tag{3.9}$$

the unknown input (disturbance) vector is obtained from the eq. (3.9). Therefore, using the output vector derivative $\dot{y}(t)$, the estimation of the state vector $\hat{x}(t)$ and the input vector $u(t)$, the unknown input vector $\hat{d}(t)$ can be constructed by

$$\hat{d}(t) = (CE_d)^+\big(\dot{y}(t) - CA\hat{x}(t) - CBu(t)\big) \tag{3.10}$$

Considering the estimate of the unknown input vector $\hat{d}(t)$, it is possible to construct a full order state observer, on the assumption that $\dot{y}(t)$ is available. The observer is given as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + E_d\hat{d}(t) + L\left(y(t) - C\hat{x}(t)\right) \tag{3.11}$$

substituting $\hat{d}(t)$ from eq. (3.10) in eq. (3.11) results in

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + E_d(CE_d)^+\big(\dot{y}(t) - CA\hat{x}(t) - CBu(t)\big) + L\left(y(t) - C\hat{x}(t)\right)\\ \dot{\hat{x}}(t) &= (A - LC - H_{ce}CA)\hat{x}(t) + (B - H_{ce}CB)u(t) + H_{ce}\dot{y}(t) + Ly(t)\end{aligned} \tag{3.12}$$

where

$$H_{ce} = E_d(CE_d)^+ \tag{3.13}$$

The state estimation error $e(t) = x(t) - \hat{x}(t)$ is governed by the equation:

$$\begin{aligned}\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t)\\ \dot{e}(t) &= Ax(t) + Bu(t) + E_d d(t) - A\hat{x}(t) - Bu(t) - E_d\hat{d}(t) - L\left(y(t) - \hat{y}(t)\right)\\ \dot{e}(t) &= (A - LC)e(t) + E_d\left(d(t) - \hat{d}(t)\right)\\ \dot{e}(t) &= \left(A - LC - H_{ce}CA\right)e(t)\end{aligned} \tag{3.14}$$

In case that there exists an observer gain matrix L, such that matrix $(A - LC - H_{ce}CA)$ is stabilizable, then the observer in eq. (3.12) fulfills eq. (3.3).

The observer in eq. (3.12) requires the knowledge of $\dot{y}(t)$, this fact may cause some problems in on-line implementation. To get over this difficulty, it is necessary to implement a modification. Therefore a new state vector $\psi(t)$ is introduced

$$\psi(t) = \hat{x}(t) - H_{ce}y(t) \tag{3.15}$$

then, it turns out that the derivative of eq. (3.15) is

$$\begin{aligned}
\dot{\psi}(t) &= \dot{\hat{x}}(t) - H_{ce}\dot{y}(t) \\
\dot{\psi}(t) &= \dot{\hat{x}}(t) - H_{ce}C\dot{x}(t) \\
\dot{\psi}(t) &= (A - LC - H_{ce}CA)\,\hat{x}(t) + (B - H_{ce}CB)\,u(t) + Ly(t) \\
\dot{\psi}(t) &= (TA - LC)\,\psi(t) + TBu(t) + \big((TA - LC)\,H_{ce} + L\big)y(t) \qquad (3.16)\\
\hat{x}(t) &= \psi(t) + H_{ce}y(t) \qquad (3.17)
\end{aligned}$$

where

$$T = I_{n\times n} - H_{ce}C \qquad (3.18)$$

It is clear that for all $d(t)$, $u(t)$ and x_o

$$\lim_{t\to\infty}(Tx(t) - \psi(t)) = 0, \qquad \lim_{t\to\infty}(x(t) - \hat{x}(t)) = 0 \qquad (3.19)$$

Setting $G = TA - LC$ and $H = TB$ allows to express the eq. (3.16) as

$$\dot{\psi}(t) = G\psi(t) + Hu(t) + (GH_{ce} + L)\,y(t) \qquad (3.20)$$

The system composed by eq. (3.17) and eq. (3.20) is an unknown input observer of the Luenberger type, and by substituting $\hat{x}(t)$ from eq. (3.17) in eq. (3.4) gives

$$\begin{aligned}
r(t) &= y(t) - C\hat{x}(t) \\
r(t) &= y(t) - C\big(\psi(t) + H_{ce}y(t)\big) \\
r(t) &= (I_{m\times m} - CH_{ce})y(t) - C\psi(t) \qquad (3.21)
\end{aligned}$$

a residual vector $r(t)$ free of unknown inputs $d(t)$ is obtained. It can be noticed that the essence of the UIO approach is the reconstruction of the unknown input $d(t)$, which requires the condition given in eq. (3.7).

The stability of observer in eq. (3.12) or equivalently in eq. (3.16) is ensured, if the pair (C, TA) is observable or at least detectable. In summary, the following theorem is obtained:

Theorem 3.1 *[17]: Given the system model in eq. (3.1) and suppose*

Condition I. $rank(CE_d) = rank(E_d) = k_d$

Condition II. the pair (C, TA) is detectable, where

$$T = I_{n\times n} - H_{ce}C$$

then there exists an UIO in the sense of eq. (3.3).

Remark 3.1 *It can be demonstrated that condition I and II are also necessary conditions for the existence of an UIO. It is interesting to notice that matrix T is singular. This can be readily seen by observing the fact*

$$TE_d = E_d - E_d H_{ce} C E_d = 0$$

Based on the linear approach for unknown input observers (UIO), it is introduced the extension of the UIO for its use with TS fuzzy models.

3.2 TS fuzzy UIO

The objective of the proposed UIO for TS fuzzy systems is the same as the one for UIOs in linear systems, i.e. it delivers a state estimate $\hat{x}(t)$ independent of the unknown input $d(t)$.

$$\lim_{t\to\infty} (x(t) - \hat{x}(t)) = 0 \qquad \text{for all } u(t),\ d(t),\ x_0 \tag{3.22}$$

In order to construct an UIO for TS fuzzy systems (TS fuzzy UIO) a class of nonlinear systems is considered. The unknown inputs (disturbance) can be summarized as an additive term in the dynamic equation described by

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) + E_d d(t) \tag{3.23a}$$
$$y(t) = Cx(t) \tag{3.23b}$$

where the distribution matrix for unknown inputs E_d and the output matrix C do not depend on the state vector $x(t)$, in other words, they are linear (constant) matrices. A TS fuzzy model that approximates the behavior of the nonlinear system given by eq. (3.23) is obtained as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) + B_i u(t) + E_d d(t) \Big] \tag{3.24a}$$
$$y(t) = Cx(t) \tag{3.24b}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ is the known input vector, $d(t) \in \mathbb{R}^{k_d}$ is the unknown input (disturbance) vector and $y(t) \in \mathbb{R}^m$ is the measurement or output vector. A_i, B_i, E_d and C are known system matrices with appropriate dimensions.

To this TS fuzzy model corresponds the following fuzzy IF-THEN rules

Model rule i

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & \left\{ \begin{array}{rl} \dot{x}(t) = & A_i x(t) + B_i u(t) + E_d d(t) \\ y(t) = & Cx(t) \end{array} \right. \end{array} \tag{3.25}$$

With the use of the state estimate $\hat{x}(t)$, it is possible to construct a residual signal as follows:

$$r(t) = y(t) - C\hat{x}(t) \tag{3.26}$$

3.2.1 Design of the TS fuzzy UIO

For the design of the TS fuzzy UIO, the derivative of the output signal $y(t)$ is given by

$$\begin{aligned}\dot{y}(t) &= C\dot{x}(t)\\ \dot{y}(t) &= C\left(\sum_{i=1}^{r} h_i(z(t))\big[A_i x(t) + B_i u(t) + E_d d(t)\big]\right)\end{aligned} \tag{3.27}$$

From eq. (3.27), the term $CE_d d(t)$ is taken to the left

$$CE_d d(t) = \dot{y}(t) - \sum_{i=1}^{r} h_i(z(t))\big[CA_i x(t) + CB_i u(t)\big] \tag{3.28}$$

Assume that

$$rank(CE_d) = rank(E_d) = k_d \tag{3.29}$$

and that CE_d is left invertible, i.e. there exists a Moore-Penrose pseudoinverse matrix $(CE_d)^+$ of the product CE_d

$$(CE_d)^+ = \big[(CE_d)^T CE_d\big]^{-1}(CE_d)^T, \qquad (CE_d)^+ \in \mathbb{R}^{k_d \times m} \tag{3.30}$$

Multiplying both sides of eq. (3.28) by the Moore-Penrose pseudoinverse matrix results in

$$\begin{aligned}(CE_d)^+ CE_d d(t) &= (CE_d)^+\left(\dot{y}(t) - \sum_{i=1}^{r} h_i(z(t))\Big[CA_i x(t) + CB_i u(t)\Big]\right)\\ d(t) &= (CE_d)^+\left(\dot{y}(t) - \sum_{i=1}^{r} h_i(z(t))\Big[CA_i x(t) + CB_i u(t)\Big]\right)\end{aligned} \tag{3.31}$$

the unknown input (disturbance) vector is obtained from the eq. (3.31). Therefore, using the output vector derivative $\dot{y}(t)$, the estimation of the state vector $\hat{x}(t)$ and the input vector $u(t)$, the unknown input vector $\hat{d}(t)$ can be constructed as

$$\hat{d}(t) = (CE_d)^+\left(\dot{y}(t) - \sum_{i=1}^{r} h_i(z(t))\Big[CA_i \hat{x}(t) + CB_i u(t)\Big]\right) \tag{3.32}$$

Considering the estimate of the unknown input vector $\hat{d}(t)$, it is possible to construct a full order TS fuzzy observer, on the assumption that $\dot{y}(t)$ is available. The TS fuzzy observer is given by the following equation

$$\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\left[A_i \hat{x}(t) + B_i u(t) + E_d \hat{d}(t) + L_i\big(y(t) - C\hat{x}(t)\big)\right] \tag{3.33}$$

with its correspondent fuzzy IF-THEN rules

Observer rule i

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & \left\{ \begin{array}{rl} \dot{\hat{x}}(t) = & A_i\hat{x}(t) + B_iu(t) + E_d\hat{d}(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = & C\hat{x}(t) \end{array} \right. \end{array} \tag{3.34}$$

substituting $\hat{d}(t)$ from eq. (3.32) in eq. (3.33) results in

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t)) \left[A_i\hat{x}(t) + B_iu(t) + E_d(CE_d)^+ \left(\dot{y}(t) - \sum_{i=1}^{r} h_i(z(t)) \left[CA_i\hat{x}(t) + CB_iu(t) \right] \right) + L_i \left(y(t) - C\hat{x}(t) \right) \right] \\
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[A_i\hat{x}(t) + B_iu(t) + E_d(CE_d)^+ \big(\dot{y}(t) - CA_i\hat{x}(t) - CB_iu(t) \big) + L_i \left(y(t) - C\hat{x}(t) \right) \Big] \\
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[(A_i - L_iC - H_{ce}CA_i)\hat{x}(t) + (B_i - H_{ce}CB_i)u(t) + H_{ce}\dot{y}(t) + L_iy(t) \Big] \qquad (3.35)
\end{aligned}$$

where

$$H_{ce} = E_d(CE_d)^+ \tag{3.36}$$

The state estimation error $e(t) = x(t) - \hat{x}(t)$ is governed by the equation

$$\begin{aligned}
\dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[A_ix(t) + B_iu(t) + E_dd(t) - A_i\hat{x}(t) - B_iu(t) - E_d\hat{d}(t) - L_i \left(y(t) - \hat{y}(t) \right) \Big] \\
\dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[A_i - L_iC - H_{ce}CA_i \Big] e(t)
\end{aligned}$$

In case that there exists observer gain matrices L_i, such that each matrix $(A_i - L_iC - H_{ce}CA_i)$ is stabilizable, then $e(t)$ will approach zero asymptotically, i.e. the condition given by eq. (3.22) is fulfilled. This means that the TS fuzzy observer in eq. (3.35) is an unknown input observer for the system in eq. (3.24) according to definition 3.1.

The TS fuzzy observer in eq. (3.35) requires the knowledge of $\dot{y}(t)$, this fact may cause some problems in on-line implementation. To get over this difficulty, it is necessary to implement a modification. Therefore a new state vector is introduced

$$\psi(t) = \hat{x}(t) - H_{ce}y(t) \tag{3.37}$$

then, it turns out that the derivative of eq. (3.37) is

$$
\begin{aligned}
\dot{\psi}(t) &= \dot{\hat{x}}(t) - H_{ce}\dot{y}(t) \\
\dot{\psi}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) - H_{ce}\big(CA_i\hat{x}(t) + CB_iu(t)\big) + L_i\left(y(t) - C\hat{x}(t)\right)\Big] \\
\dot{\psi}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(A_i - L_iC - H_{ce}CA_i\right)\hat{x}(t) + \left(B_i - H_{ce}CB_i\right)u(t) + L_iy(t)\Big] \\
\dot{\psi}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(TA_i - L_iC\right)\hat{x}(t) + TB_iu(t) + L_iy(t)\Big] \\
\dot{\psi}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(TA_i - L_iC\right)\left(\psi(t) + H_{ce}y(t)\right) + TB_iu(t) + L_iy(t)\Big] \\
\dot{\psi}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(TA_i - L_iC\right)\psi(t) + TB_iu(t) + \left(\left(TA_i - L_iC\right)H_{ce} + L_i\right)y(t)\Big] \qquad (3.38) \\
\hat{x}(t) &= \psi(t) + H_{ce}y(t) \qquad (3.39)
\end{aligned}
$$

where

$$T = I_{n\times n} - H_{ce}C \qquad (3.40)$$

It is clear that for all $d(t)$, $u(t)$ and x_o

$$\lim_{t\to\infty}\left(Tx(t) - \psi(t)\right) = 0, \qquad \lim_{t\to\infty}\left(x(t) - \hat{x}(t)\right) = 0 \qquad (3.41)$$

and furthermore, setting $G_i = TA_i - L_iC$ and $H_i = TB_i$ allows to express eq. (3.38) as

$$\dot{\psi}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[G_i\psi(t) + H_iu(t) + \left(G_iH_{ce} + L_i\right)y(t)\Big] \qquad (3.42)$$

The system constituted by eq. (3.38)-(3.39) is an unknown input observer of the Luenberger type for TS fuzzy systems, and by substituting $\hat{x}(t)$ from eq. (3.39) in eq. (3.26) gives

$$
\begin{aligned}
r(t) &= y(t) - C\hat{x}(t) \\
r(t) &= y(t) - C\big(\psi(t) + H_{ce}y(t)\big) \\
r(t) &= (I_{m\times m} - CH_{ce})y(t) - C\psi(t) \qquad (3.43)
\end{aligned}
$$

a residual vector free of unknown inputs $d(t)$ is obtained. The stability of the TS fuzzy observer in eq. (3.35) or equivalently in eq. (3.38) is ensured, if all pairs (C, TA_i) are observable or at least detectable. In summary, the following theorem is obtained:

Theorem 3.2 *Given the system model in eq. (3.24) and suppose*

Condition 1. $rank(CE_d) = rank(E_d) = k_d$

Condition 2. all pairs (C, TA_i) *are detectable, where*

$$T = I_{n\times n} - H_{ce}C$$

then there exists a TS fuzzy UIO in the sense of eq. (3.22).

3.2.2 Computation of observer gain matrices

To compute the observer gain matrices L_i, it is required to realize the convergence analysis of the TS fuzzy UIO. The state estimation error dynamics is given by

$$\begin{aligned}
\dot{e}(t) &= \dot{x}(t) - \dot{\hat{x}}(t) \\
\dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[A_i - L_i C - H_{ce} C A_i \Big] e(t) \\
\dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[T A_i - L_i C \Big] e(t)
\end{aligned} \tag{3.44}$$

The stability of the dynamic eq. (3.44) can be proved by the Theorem 3.3.

Theorem 3.3 *[77]: The equilibrium of the system described by eq. (3.44) is asymptotically stable if there exists a common positive definite matrix P for $i = 1, ..., r$ such that*

$$\bar{A}_i^T P + P \bar{A}_i < 0 \tag{3.45}$$

where $\bar{A}_i = T A_i - L_i C$.

Proof: Consider a candidate of Lyapunov function $V(e(t)) = e^T(t) P e(t)$, where $P > 0$. Then,

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^T(t) P e(t) + e^T(t) P \dot{e}(t) < 0 \\
&= e^T(t) \left(\sum_{i=1}^{r} h_i(z(t)) \bar{A}_i \right)^T P e(t) + e^T(t) P \left(\sum_{i=1}^{r} h_i(z(t)) \bar{A}_i \right) e(t) < 0 \\
&= \sum_{i=1}^{r} h_i(z(t)) e^T(t) \left(\bar{A}_i^T P + P \bar{A}_i \right) e(t) < 0 \\
&= \sum_{i=1}^{r} h_i(z(t)) e^T(t) \left[\Big(T A_i - L_i C \Big)^T P + P \Big(T A_i - L_i C \Big) \right] e(t) < 0
\end{aligned}$$

Q.E.D.

With the same strategy as in [8], it is possible to transform the conditions given by eq. (3.45) in linear matrix inequalities (LMIs) and use these LMIs to obtain the gain matrices L_i for the TS fuzzy UIO if and only if there exist a positive definite matrix P.

For this purpose, substitute $\bar{A}_i$ in eq. (3.45)

$$\begin{gathered}
(T A_i - L_i C)^T P + P (T A_i - L_i C) < 0 \\
A_i^T T^T P + P T A_i - C^T L_i^T P - P L_i C < 0
\end{gathered}$$

Define $N_i = P L_i$ so that for $P > 0$ results $L_i = P^{-1} N_i$, after substituting this in the above matrix inequality follows that

$$A_i^T T^T P + PTA_i - C^T N_i^T - N_i C < 0$$

The use of these LMI conditions allow us to define a stable TS fuzzy UIO design problem as follows:

Problem 3.1 ***TS fuzzy UIO design:*** *Find $P > 0$ and N_i $(i = 1, \ldots, r)$ satisfying*

$$A_i^T T^T P + PTA_i - C^T N_i^T - N_i C < 0 \tag{3.46}$$

Applying the relaxed stability conditions (given in the Appendix A.3) to the above TS fuzzy UIO design problem results in:

Problem 3.2 ***TS fuzzy UIO design using relaxed stability conditions:*** *Find $P > 0$, $Q \geq 0$ and N_i $(i = 1, \ldots, r)$ satisfying*

$$A_i^T T^T P + PTA_i - C^T N_i^T - N_i C + (s-1)Q < 0 \tag{3.47}$$

where $1 < s \leq r$ and

$$N_i = PL_i$$

The above conditions are LMIs with respect to variables P, Q and N_i. It can be found a positive definite matrix P, a positive semidefinite matrix Q and a matrix N_i satisfying the LMIs or determine that no such P, Q and N_i exist. The observer gain matrices L_i can be obtained as

$$L_i = P^{-1} N_i$$

The design problem given by eq. (3.47) is solved efficiently using mathematical tools as for example MATLAB. Following the procedure given in 3.2.1 is made an algorithm for the design of the TS fuzzy UIO as follows

Algorithm 3.1 *Takagi-Sugeno fuzzy UIO based residual generation*

Step 1. Check the rank condition for E_d and CE_d, if $rank(CE_d) = rank(E_d) = k_d$ is satisfied then go to the next step, otherwise it is not possible to find a TS fuzzy UIO for such system (STOP).

Step 2. Compute matrices $(CE_d)^+$, H_{ce} and T according to eq. (3.30), (3.36) and (3.40) respectively.

Step 3. Check the observability: If each pair (C, TA_i) is observable, then a TS fuzzy UIO exists and matrices L_i can be computed using LMI techniques.

Step 4. Find gain matrices L_i using eq. (3.47) that ensures the stability of each matrix $(TA_i - L_iC)$.

Step 5. Construct residual generator following eq. (3.38) and eq. (3.43).

3.3 Robust sensor fault isolation schemes based on TS fuzzy UIO

The main task of robust fault detection is to generate a residual signal which is robust to unknown inputs (disturbance). To detect a particular fault, the residual has to be sensitive to this fault. A TS fuzzy system with possible sensor fault can be described by

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) + B_i u(t) + E_d d(t) \Big] \tag{3.48a}$$

$$y(t) = Cx(t) + f_s(t) \tag{3.48b}$$

where $f_s(t) \in \mathbb{R}^m$ denotes the presence of sensor faults. To generate a robust (in the sense of unknown input decoupling) residual, a TS fuzzy UIO described by eq. (3.35) is required. As described before, when the state estimation is available, the residual can be generated as:

$$\begin{aligned} r(t) &= y(t) - C\hat{x}(t) \\ r(t) &= (I_{m\times m} - CH_{ce})y(t) - C\psi(t) \end{aligned} \tag{3.49}$$

When this TS fuzzy UIO based residual generator is applied to the system described in eq. (3.48), the residual and the state estimation error $e(t)$ result as

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} h_i(z(t)) \Big[(TA_i - L_i C)\, e(t) - L_i f_s(t) - H_{ce} \dot{f}_s(t) \Big] \\ r(t) &= Ce(t) + f_s(t) \end{aligned} \tag{3.50}$$

The residual has to be made sensitive to $f_s(t)$ in order to detect sensor faults. This is generally possible, since the sensor fault vector $f_s(t)$ has a direct effect on the residual signal $r(t)$.

The fault isolation problem has as main task the localization of the fault, i.e. to determine in which sensor the fault has occurred. One approach that facilitates fault isolation is to design a structured residual set. Each residual in the set is designed to be insensitive to a certain fault and sensitive to all other faults.

To design robust sensor fault isolation schemes, all actuators are assumed to be fault-free and the system equations can be expressed as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) + B_i u(t) + E_d d(t) \Big] \tag{3.51a}$$

$$y^k(t) = C^k x(t) + f_s^k(t) \tag{3.51b}$$

$$y_k(t) = C_k x(t) + f_{s_k}(t) \qquad \text{for } k = 1, \ldots, m \tag{3.51c}$$

where $C_k \in \mathbb{R}^{1\times n}$ is the k_{th} row of the matrix C, $C^k \in \mathbb{R}^{(m-1)\times n}$ is obtained from the matrix C by deleting k_{th} row C_k, $y_k(t)$ is the k_{th} component of $y(t)$ and $y^k(t) \in \mathbb{R}^{m-1}$ is obtained from the vector $y(t)$ by deleting k_{th} component $y_k(t)$.

Based on this description, m TS fuzzy UIO based residual generators can be constructed as

$$\begin{aligned}
\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t)) \left[A_i\hat{x}(t) + B_iu(t) + E_d\hat{d}(t) + L_i^k \left(y^k(t) - C^k\hat{x}(t)\right)\right] \\
&= \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) + E_d(C^kE_d)^{+}\big(\dot{y}^k(t) - C^kA_i\hat{x}(t) - C^kB_iu(t)\big) + L_i^k\left(y^k(t) - C^k\hat{x}(t)\right)\Big] \\
&= \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) + H_{ce}^k\big(\dot{y}^k(t) - C^kA_i\hat{x}(t) - C^kB_iu(t)\big) + L_i^k\left(y^k(t) - C^k\hat{x}(t)\right)\Big] \qquad (3.52)
\end{aligned}$$

where

$$H_{ce}^k = E_d(C^kE_d)^{+} \qquad (3.53)$$

As mentioned before, a modification is needed to avoid problems due to on-line computation of the TS fuzzy UIO based residual generators. For this reason a new state vector is introduced

$$\psi^k(t) = \hat{x}(t) - H_{ce}^k y^k(t) \qquad (3.54)$$

whose derivative is given as

$$\begin{aligned}
\dot{\psi}^k(t) &= \dot{\hat{x}}(t) - H_{ce}^k\dot{y}^k(t) \\
\dot{\psi}^k(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) - H_{ce}^k\left(C^kA_i\hat{x}(t) + C^kB_iu(t)\right) + L_i^k\left(y^k(t) - C^k\hat{x}(t)\right)\Big] \\
\dot{\psi}^k(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(A_i - L_i^kC^k - H_{ce}^kC^kA_i\right)\hat{x}(t) + \left(B_i - H_{ce}^kC^kB_i\right)u(t) + L_i^ky^k(t)\Big] \\
\dot{\psi}^k(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(T^kA_i - L_i^kC^k\right)\hat{x}(t) + T^kB_iu(t) + L_i^ky^k(t)\Big] \\
\dot{\psi}^k(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[\left(T^kA_i - L_i^kC^k\right)\left(\psi^k(t) + H_{ce}^ky^k(t)\right) + T^kB_iu(t) + L_i^ky^k(t)\Big] \\
\dot{\psi}^k(t) &= \sum_{i=1}^{r} h_i(z(t))\left[\left(T^kA_i - L_i^kC^k\right)\psi^k(t) + T^kB_iu(t) + \left(\left(T^kA_i - L_i^kC^k\right)H_{ce}^k + L_i^k\right)y^k(t)\right] \qquad (3.55) \\
\hat{x}(t) &= \psi^k(t) + H_{ce}^ky^k(t) \qquad (3.56)
\end{aligned}$$

where

$$T^k = I_{n\times n} - H_{ce}^kC^k \qquad (3.57)$$

and furthermore, setting $G_i^k = T^kA_i - L_i^kC^k$ and $H_i^k = T^kB_i$ allows to express the eq. (3.55) as

$$\dot{\psi}^k(t) = \sum_{i=1}^{r} h_i(z(t))\Big[G_i^k\psi^k(t) + H_i^ku(t) + \left(G_i^kH_{ce}^k + L_i^k\right)y^k(t)\Big] \qquad (3.58)$$

The system constituted by eq. (3.55)-(3.56) is an unknown input observer of the Luenberger type for TS fuzzy models, and by setting

$$
\begin{aligned}
r^k(t) &= y^k(t) - C^k\hat{x}(t) \\
r^k(t) &= y^k(t) - C^k\big(\psi^k(t) + H^k_{ce}y^k(t)\big) \\
r^k(t) &= (I_{(m-1)\times(m-1)} - C^kH^k_{ce})y^k(t) - C^k\psi^k(t)
\end{aligned}
\tag{3.59}
$$

Each residual generator is driven by all inputs and all outputs except one output. When all actuators are fault-free and a fault occurs in the k_{th} sensor, the residual will satisfy the following isolation logic

$$
\begin{cases} \|r^k(t)\| < T^k_{SFI} \\ \|r^l(t)\| \geq T^l_{SFI} \end{cases} \quad \text{for } l = 1,\ldots,k-1,k+1,\ldots,m \tag{3.60}
$$

where T^k_{SFI} $(k = 1,\ldots,m)$ are isolation thresholds. A robust and TS fuzzy UIO based sensor fault isolation scheme is shown in fig. 3.1.

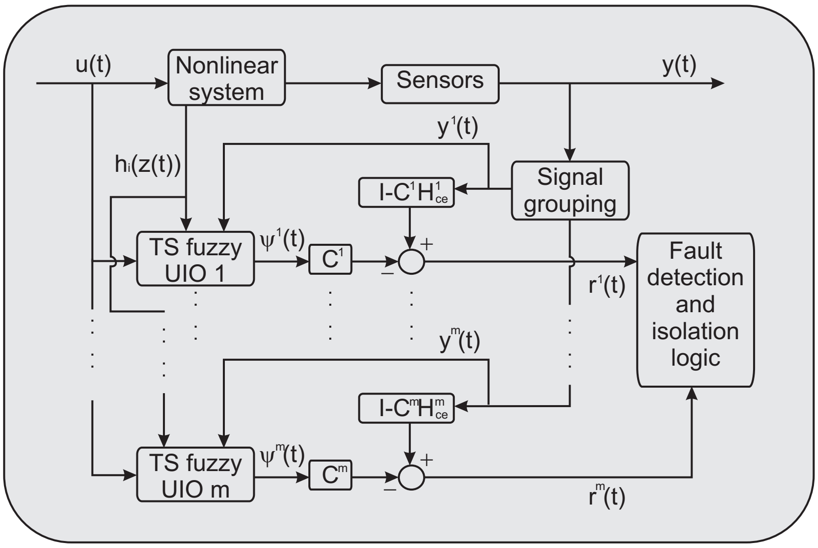

Fig. 3.1: A robust sensor fault isolation scheme

3.4 An application example

A nonlinear system is used to implement the TS fuzzy UIO based residual generator, the nonlinear system is described by

$$
\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\end{bmatrix} = \begin{bmatrix} -x_1(t) + x_1(t)x_2^3(t) \\ -x_2(t) + \big(3 + x_2(t)\big)x_1^3(t) \\ x_2(t) - x_3(t)\end{bmatrix} + \begin{bmatrix}1\\0.1\\0.2\end{bmatrix} u(t) + \begin{bmatrix}1\\-2.5\\0.1\end{bmatrix} d(t)
$$

$$
\begin{bmatrix}y_1(t)\\ y_2(t)\\ y_3(t)\end{bmatrix} = \begin{bmatrix}x_1(t)\\ x_2(t)\\ x_3(t)\end{bmatrix} + \begin{bmatrix}f_{s_1}(t)\\ f_{s_2}(t)\\ f_{s_3}(t)\end{bmatrix}
$$

it is considered that $x_1(t) \in [-1, 1]$ and $x_2(t) \in [-1, 1]$. The above system can be written in the following form:

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -1 & x_1(t)x_2^2(t) & 0 \\ \big(3 + x_2(t)\big)x_1^2(t) & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.1 \\ 0.2 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ -2.5 \\ 0.1 \end{bmatrix} d(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) + f_s(t) \end{aligned}$$

where $x_1(t)x_2^2(t)$ and $\big(3 + x_2(t)\big)x_1^2(t)$ are nonlinear terms. For the nonlinear terms are defined $z_1(t) = x_1(t)x_2^2(t)$ and $z_2(t) = \big(3 + x_2(t)\big)x_1^2(t)$ as premise variables. Substituting $z_1(t)$ and $z_2(t)$ in the above system results in

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -1 & z_1(t) & 0 \\ z_2(t) & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.1 \\ 0.2 \end{bmatrix} u(t) + \begin{bmatrix} 1 \\ -2.5 \\ 0.1 \end{bmatrix} d(t) \\ y(t) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x(t) + f_s(t) \end{aligned}$$

Next, calculate the minimum and maximum values of $z_1(t)$ and $z_2(t)$, these are obtained as:

$$\begin{array}{ll} \max\limits_{z_1(t), z_2(t)} z_1(t) = 1 & \max\limits_{x_1(t), x_2(t)} z_2(t) = 4 \\ \min\limits_{z_1(t), z_2(t)} z_1(t) = -1 & \min\limits_{x_1(t), x_2(t)} z_2(t) = 0 \end{array}$$

from the maximum and minimum values, $z_1(t)$ and $z_2(t)$ can be represented by

$$\begin{aligned} z_1(t) &= x_1(t)x_2^2(t) = F_{11}(z_1(t)) \cdot 1 + F_{12}(z_1(t)) \cdot -1 \\ z_2(t) &= \big(3 + x_2(t)\big)x_1^2(t) = F_{21}(z_2(t)) \cdot 4 + F_{22}(z_2(t)) \cdot 0 \end{aligned}$$

where:

$$F_{11}(z_1(t)) + F_{12}(z_1(t)) = 1 \qquad \text{and} \qquad F_{21}(z_2(t)) + F_{22}(z_2(t)) = 1$$

The membership functions can be calculated as follows

$$\begin{array}{ll} F_{11}(z_1(t)) = \dfrac{z_1(t) + 1}{2} & F_{12}(z_1(t)) = \dfrac{1 - z_1(t)}{2} \\ F_{21}(z_2(t)) = \dfrac{z_2(t)}{4} & F_{22}(z_2(t)) = \dfrac{4 - z_2(t)}{4} \end{array}$$

The membership functions are named “Positive”, “Negative”, “Big” and “Small”, respectively. Then, the nonlinear system is approximated by the following fuzzy IF-THEN rules

Model rule 1

$$\text{IF} \quad z_1(t) \text{ is “Positive” and } z_2(t) \text{ is “Big”}$$
$$\text{THEN} \begin{cases} \dot{x}(t) = & A_1x(t) + B_1u(t) + E_d d(t) \\ y(t) = & Cx(t) + f_s(t) \end{cases}$$

Model rule 2

$$\text{IF} \quad z_1(t) \text{ is “Positive” and } z_2(t) \text{ is “Small”}$$
$$\text{THEN} \begin{cases} \dot{x}(t) = & A_2x(t) + B_2u(t) + E_d d(t) \\ y(t) = & Cx(t) + f_s(t) \end{cases}$$

Model rule 3

$$\text{IF} \quad z_1(t) \text{ is “Negative” and } z_2(t) \text{ is “Big”}$$
$$\text{THEN} \begin{cases} \dot{x}(t) = & A_3x(t) + B_3u(t) + E_d d(t) \\ y(t) = & Cx(t) + f_s(t) \end{cases}$$

Model rule 4

$$\text{IF} \quad z_1(t) \text{ is “Negative” and } z_2(t) \text{ is “Small”}$$
$$\text{THEN} \begin{cases} \dot{x}(t) = & A_4x(t) + B_4u(t) + E_d d(t) \\ y(t) = & Cx(t) + f_s(t) \end{cases}$$

Here

$$A_1 = \begin{bmatrix} -1 & 1 & 0 \\ 4 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \ A_3 = \begin{bmatrix} -1 & -1 & 0 \\ 4 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \ A_4 = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$B_{1,2,3,4} = \begin{bmatrix} 1 \\ 0.1 \\ 0.2 \end{bmatrix}, \ E_d = \begin{bmatrix} 1 \\ -2.5 \\ 0.1 \end{bmatrix}, \ C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The defuzzification (that gives the TS fuzzy model) is carried out as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{4} h_i(z(t)) \Big[A_i x(t) + B_i u(t) + E_d d(t) \Big] \\ y(t) &= Cx(t) + f_s(t) \end{aligned}$$

where

$$\begin{aligned} h_1(z(t)) &= F_{11}(z_1(t)) \times F_{21}(z_2(t)) \\ h_2(z(t)) &= F_{11}(z_1(t)) \times F_{22}(z_2(t)) \\ h_3(z(t)) &= F_{12}(z_1(t)) \times F_{21}(z_2(t)) \\ h_4(z(t)) &= F_{12}(z_1(t)) \times F_{22}(z_2(t)) \end{aligned}$$

Following the steps given in Algorithm 3.1, the rank of CE_d and E_d are compared

$$rank(CE_d) = rank(E_d) = 1$$

The above condition is satisfied, and hence matrices $(CE_d)^+$, H_{ce} and T using eq. (3.30), (3.36) and (3.40) respectively are computed.

$$(CE_d)^+ = \begin{bmatrix} 0.1377 & -0.3443 & 0.0138 \end{bmatrix}, \qquad H_{ce} = \begin{bmatrix} 0.1378 & -0.3443 & 0.0138 \\ -0.3443 & 0.8608 & -0.0344 \\ 0.0138 & -0.0344 & 0.0014 \end{bmatrix},$$

$$T = \begin{bmatrix} 0.8622 & 0.3443 & -0.0138 \\ 0.3443 & 0.1391 & 0.0344 \\ -0.0138 & 0.0344 & 0.9986 \end{bmatrix}$$

The following gain matrices L_i are obtained using eq. (3.47) with the relaxed stability conditions.

$$L_1 = \begin{bmatrix} 2.015 & 0.358 & 0.082 \\ 0.358 & 1.739 & 0.458 \\ 0.082 & 0.458 & 0.501 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.638 & 0.079 & 0.014 \\ 0.079 & 1.739 & 0.458 \\ 0.014 & 0.458 & 0.501 \end{bmatrix}$$

$$L_3 = \begin{bmatrix} 2.015 & -0.504 & 0.082 \\ -0.504 & 1.051 & 0.472 \\ 0.082 & 0.472 & 0.501 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.638 & -0.782 & 0.014 \\ -0.782 & 1.051 & 0.471 \\ 0.014 & 0.472 & 0.501 \end{bmatrix}$$

Simulation results

The TS fuzzy UIO (TSFUIO) based residual generator is compared against a TS fuzzy observer (TSFO) in normal operation (without affectation of disturbances or faults). Their respective residuals are shown in fig. 3.3 and in fig. 3.2.

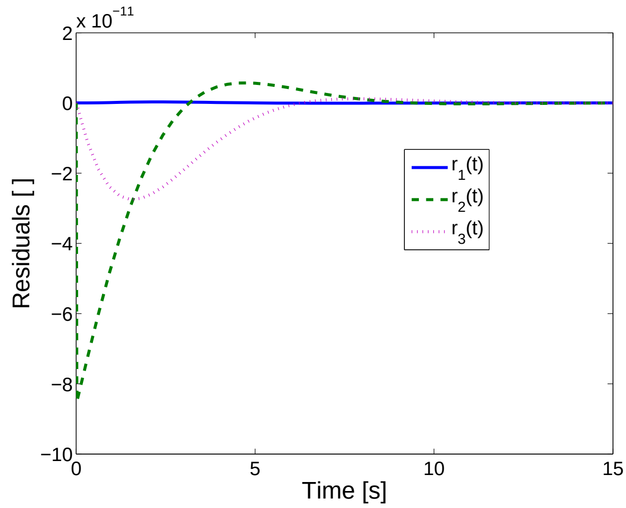

Fig. 3.2: Residuals for TSFO

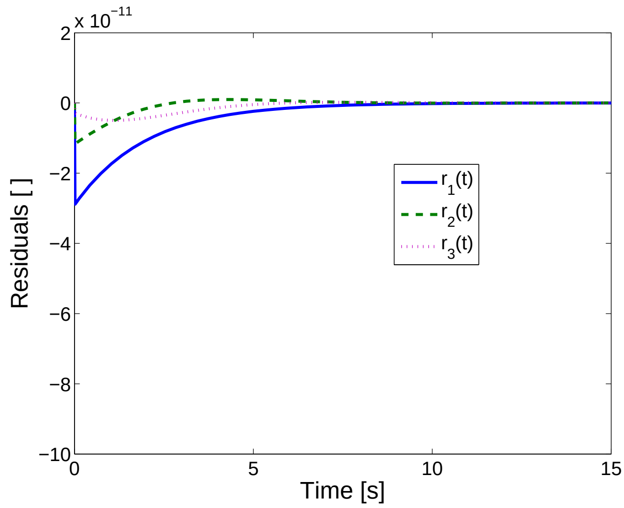

Fig. 3.3: Residuals for TSFUIO

It can be noticed that both observers converge to zero at $t \approx 12\ s$. The use of the relaxed stability conditions ($s = 3$) in the design of both observers allows to improve the convergence as can be seen in the following residuals:

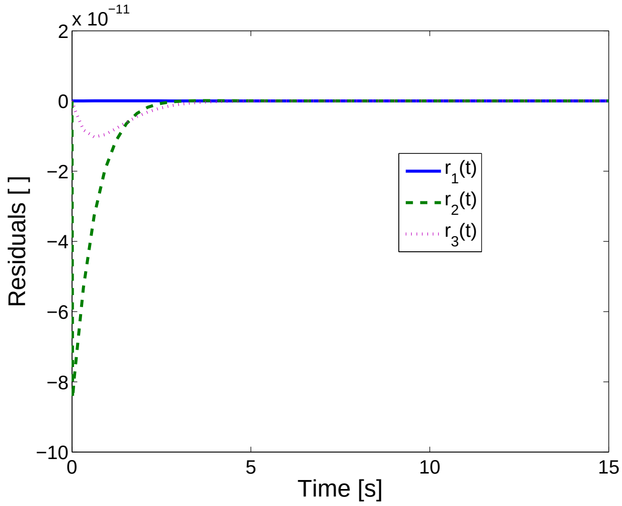

Fig. 3.4: Residuals for relaxed TSFO

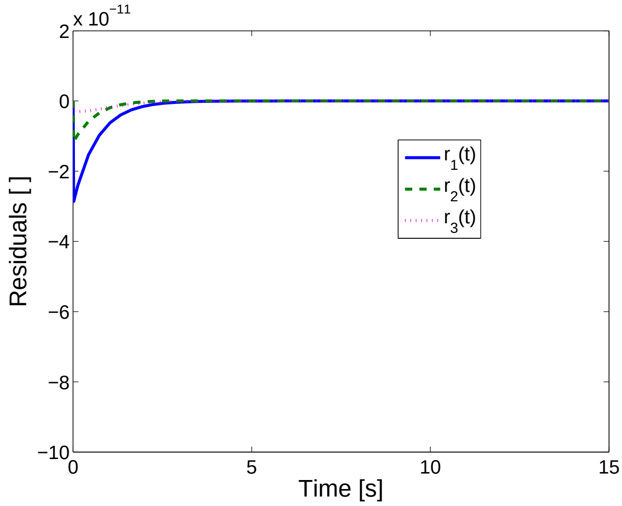

Fig. 3.5: Residuals for relaxed TSFUIO

The unknown input (disturbance) signal

$$d(t) = 0.3 \cos{(2t)} e^{-0.2t} \tag{3.61}$$

is applied to the system.

In fig. 3.6 and fig. 3.7 the residuals for both observers are shown, when the disturbance affects the system.

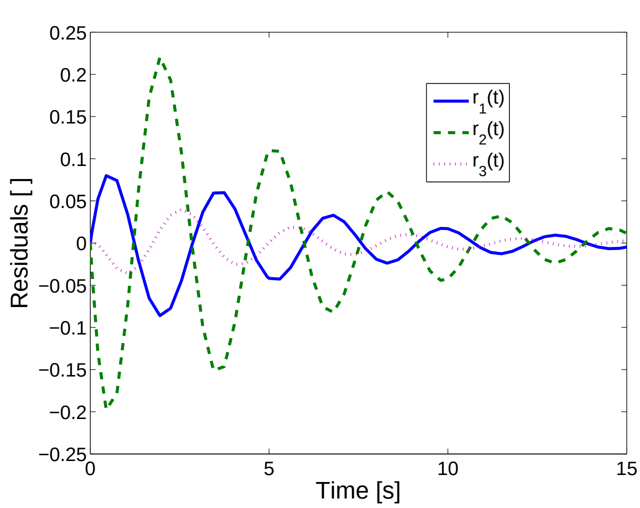

Fig. 3.6: Residuals for TSFO with disturbance

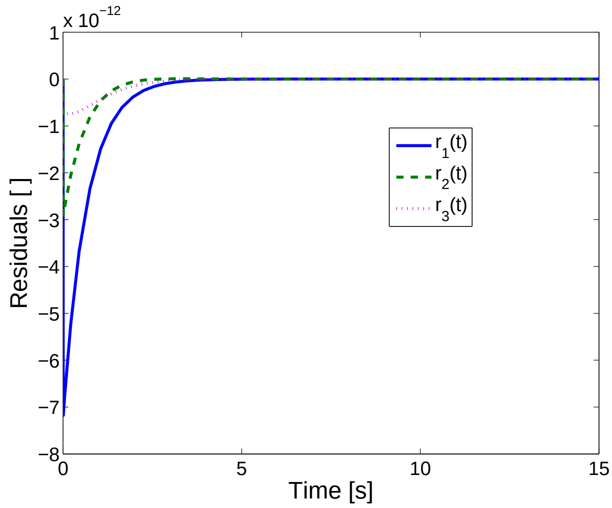

Fig. 3.7: Residuals for TSFUIO with disturbance

As can be seen in fig. 3.6, the TS fuzzy observer is clearly affected by the unknown input while the TS fuzzy UIO is decoupled from the unknown input as shown in fig. 3.7.

The procedure described in the subsection 3.3 is applied to build three TS fuzzy UIO based residual generator. Each observer is insensitive to one sensor fault but sensitive to the another two.

The rank condition $rank(C^k E_d) = rank(E_d)$ for $k = 1, 2, 3$ is satisfied. All three observers fulfill this condition.

The sensitivity and insensitivity of the observers to the faults is shown in the tab. 3.1

TS fuzzy UIO	Insensitive to	Sensitive to
1	f_{s1}	f_{s2} and f_{s3}
2	f_{s2}	f_{s1} and f_{s3}
3	f_{s3}	f_{s1} and f_{s2}

Tab. 3.1: Robust sensor fault isolation scheme

TS fuzzy UIO 1: The dynamic equation for the first TS fuzzy UIO is

$$\dot{\psi}^1(t) = \sum_{i=1}^{4} h_i(z(t)) \Big[G_i^1 \psi^1(t) + H_i^1 u(t) + \left(G_i^1 H_{ce}^1 + L_i^1 \right) y^1(t) \Big]$$

and the parameter matrices $(C^1 E_d)^+$, H_{ce}^1 and T^1 are computed using eq. (3.30), (3.36) and (3.40) respectively

$$(C^1 E_d)^+ = \begin{bmatrix} -0.3994 & 0.0159 \end{bmatrix}, \; H_{ce}^1 = \begin{bmatrix} -0.3994 & 0.0159 \\ 0.9984 & -0.0399 \\ -0.0399 & 0.0016 \end{bmatrix}, \; T^1 = \begin{bmatrix} 1 & 0.3994 & -0.0159 \\ 0 & 0.0016 & 0.0399 \\ 0 & 0.0399 & 0.9984 \end{bmatrix}$$

The following gain matrices L_i^1 are obtained using eq. (3.47) with the relaxed stability conditions $(s = 2)$:

$$L_1^1 = \begin{bmatrix} -9.555 & 20.774 \\ 1.044 & 2.091 \\ -0.931 & 2.183 \end{bmatrix}, \quad L_2^1 = \begin{bmatrix} -7.962 & 36.887 \\ 1.048 & 2.010 \\ -0.665 & 4.575 \end{bmatrix},$$
$$L_3^1 = \begin{bmatrix} -24.651 & 21.298 \\ 1.047 & 4.534 \\ -3.296 & 2.179 \end{bmatrix}, \quad L_4^1 = \begin{bmatrix} -16.73 & 37.158 \\ 1.049 & 3.273 \\ -1.887 & 4.574 \end{bmatrix}$$

The residual is generated by

$$r^1(t) = \left(I - C^1 H_{ce}^1 \right) y^1(t) - C^1 \psi^1(t)$$

TS fuzzy UIO 2: The dynamic equation for the second TS fuzzy UIO is

$$\dot{\psi}^2(t) = \sum_{i=1}^{4} h_i(z(t)) \Big[G_i^2 \psi^2(t) + H_i^2 u(t) + \left(G_i^2 H_{ce}^2 + L_i^2 \right) y^2(t) \Big]$$

and the parameter matrices $(C^2E_d)^+$, H^2_{ce} and T^2 are computed using eq. (3.30), (3.36) and (3.40) respectively

$$(C^2E_d)^+ = \begin{bmatrix} 0.9901 & 0.0990 \end{bmatrix}, \; H^2_{ce} = \begin{bmatrix} 0.9901 & 0.0990 \\ -2.4752 & -0.2475 \\ 0.0990 & 0.0099 \end{bmatrix}, \; T^2 = \begin{bmatrix} 0.0099 & 0 & -0.0990 \\ 2.4752 & 1 & 0.2475 \\ -0.0990 & 0 & 0.9901 \end{bmatrix}$$

The following gain matrices L^2_i are obtained using eq. (3.47) with the relaxed stability conditions $(s = 2)$:

$$L^2_1 = \begin{bmatrix} 1.025 & -0.233 \\ -1.558 & 29.896 \\ -0.294 & 3.740 \end{bmatrix}, \quad L^2_2 = \begin{bmatrix} 1.012 & 0.019 \\ -5.972 & 29.855 \\ -0.419 & 3.753 \end{bmatrix},$$
$$L^2_3 = \begin{bmatrix} 1.111 & -1.332 \\ -4.956 & 71.827 \\ -0.677 & 9.927 \end{bmatrix}, \quad L^2_4 = \begin{bmatrix} 1.057 & -0.261 \\ -10.719 & 71.65 \\ -1.211 & 9.981 \end{bmatrix}$$

The residual is generated by

$$r^2(t) = \left(I - C^2 H^2_{ce}\right) y^2(t) - C^2\psi^2(t)$$

<u>**TS fuzzy UIO 3:**</u> The dynamic equation for the third TS fuzzy UIO is

$$\dot{\psi}^3(t) = \sum_{i=1}^{4} h_i(z(t))\left[G^3_i\psi^3(t) + H^3_i u(t) + \left(G^3_i H^3_{ce} + L^3_i\right) y^3(t)\right]$$

and the parameter matrices $(C^3E_d)^+$, H^3_{ce} and T^3 are computed using eq. (3.30), (3.36) and (3.40) respectively

$$(C^3E_d)^+ = \begin{bmatrix} 0.1379 & -0.3448 \end{bmatrix}, \; H^3_{ce} = \begin{bmatrix} 0.1379 & -0.3448 \\ -0.3448 & 0.8620 \\ 0.0138 & -0.0345 \end{bmatrix}, \; T^3 = \begin{bmatrix} 0.8620 & 0.3448 & 0 \\ 0.3448 & 0.1379 & 0 \\ -0.0138 & 0.0345 & 1 \end{bmatrix}$$

The following gain matrices L^3_i are obtained using eq. (3.47) with the relaxed stability conditions $(s = 4)$:

$$L^3_1 = \begin{bmatrix} 2.517 & -0.197 \\ 0.921 & 2.207 \\ 0.152 & 0.952 \end{bmatrix}, \quad L^3_2 = \begin{bmatrix} 1.138 & -0.748 \\ 0.920 & 2.207 \\ 0.014 & 0.952 \end{bmatrix},$$
$$L^3_3 = \begin{bmatrix} 2.517 & -1.102 \\ 0.102 & 1.517 \\ 0.152 & 0.979 \end{bmatrix}, \quad L^3_4 = \begin{bmatrix} 1.138 & -1.399 \\ -0.152 & 1.517 \\ 0.014 & 0.979 \end{bmatrix}$$

The residual is generated by

$$r^3(t) = \left(I - C^3 H^3_{ce}\right) y^3(t) - C^3 \psi^3(t)$$

In order to show the robust sensor fault isolation schemes based on TS fuzzy UIO, the following sensor fault signal is applied to the system

$$f(t) = \begin{cases} -0.08 & 5 \le t \le 10 \\ 0 & \text{elsewhere.} \end{cases} \tag{3.62}$$

the correspondent simulation is shown in fig. 3.8.

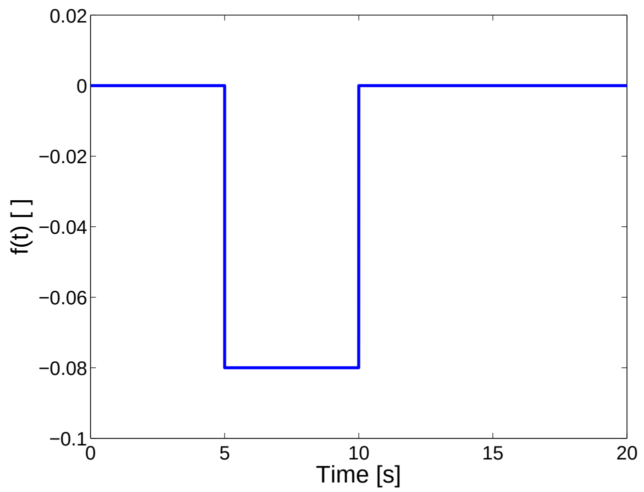

Fig. 3.8: Fault for sensor 1,2 and 3

The same sensor fault is applied to all the three sensors. In fig. 3.9 the three evaluated residuals without the sensor fault are shown.

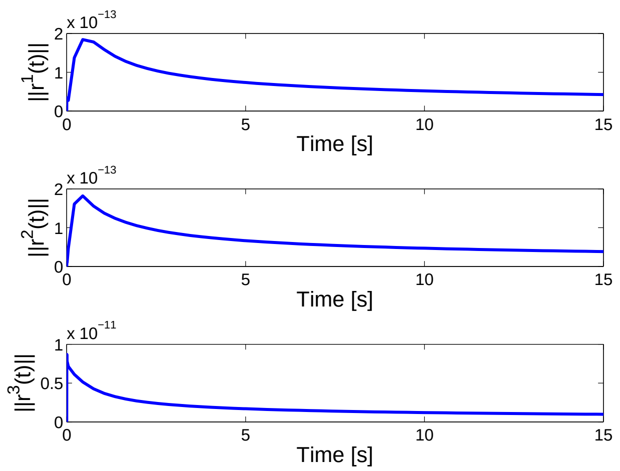

Fig. 3.9: Evaluated residuals

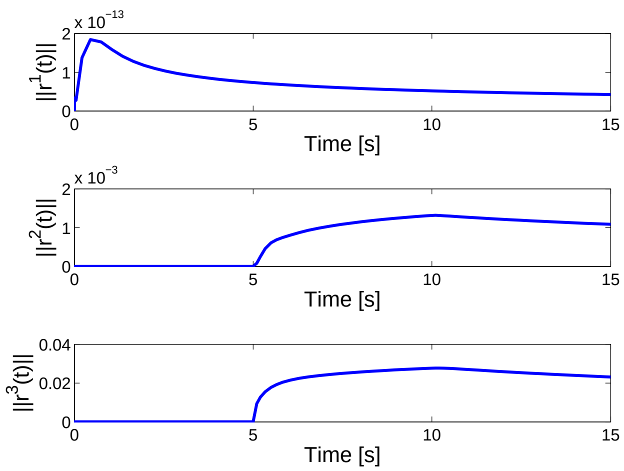

Fig. 3.10: Isolation of the fault in sensor 1

Fig. 3.10 shows that the fault in sensor 1 does not affect the residual 1 but affect the another two residuals, therefore this fault can be isolated.

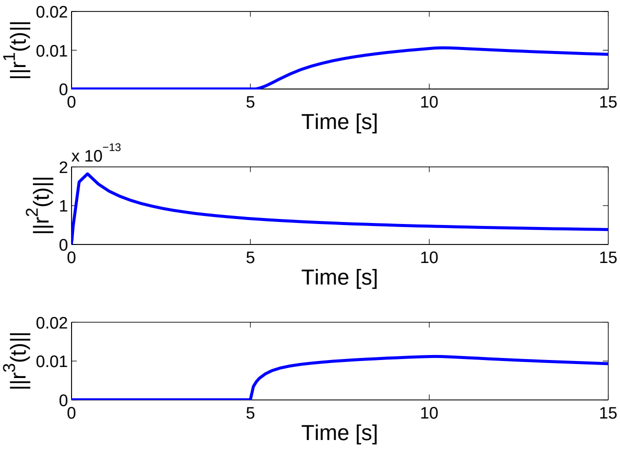

Fig. 3.11: Isolation of the fault in sensor 2

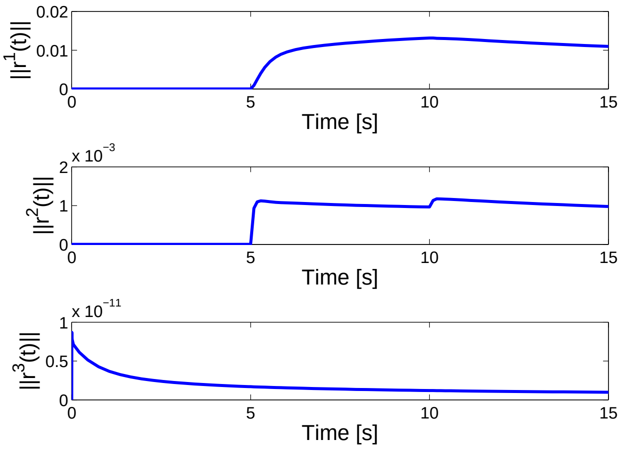

Fig. 3.12: Isolation of the fault in sensor 3

It can be seen in fig. 3.11 that the fault in sensor 2 does not affect the residual 2 but affects the another two residuals, therefore this fault can be isolated, too. The same result is shown in fig. 3.12 where the fault on sensor 3 can also be isolated.

The proposed unknown input observer for a class of nonlinear systems (described by the TS fuzzy model) makes possible to decouple the unknown input from teh residual signal. The robust sensor fault isolation scheme allows to isolate sensor faults using the TS fuzzy UIO theory.

Chapter 4

Attenuating stochastic disturbances based on TS fuzzy models

This chapter considers the discrete TS fuzzy model with stochastic noise (disturbance) in order to design a residual generator. An LMI optimization approach is proposed to minimize the expected value of the steady state estimation error, knowing the stochastic features of the noises.

4.1 Discrete TS fuzzy model

Consider the following discrete TS fuzzy model with influence of stochastic noise and faults. The model is represented by fuzzy IF-THEN rules

Model rule i

$$\begin{aligned}&\text{IF} \quad z_1(k) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(k) \text{ is } M_{ip}\\&\text{THEN} \left\{ \begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + E_{w_i} w(k) + E_{f_i} f(k) \\ y(k) &= C_i x(k) + D_i u(k) + F_{w_i} w(k) + v(k) + F_{f_i} f(k) \end{aligned} \right.\end{aligned} \tag{4.1}$$

where $i = 1, ..., r$, r is the number of IF-THEN rules, M_{ij} are fuzzy sets, $z_1(k), \ \dots \ , z_p(k)$ are the premise variables, $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^{k_u}$ is the input vector, $y(k) \in \mathbb{R}^m$ is the output vector, $w(k) \in \mathbb{R}^{k_w}$ is the system noise vector, $v(k) \in \mathbb{R}^{k_v}$ is the measurement noise vector and $f(k) \in \mathbb{R}^{k_f}$ is the fault vector. Matrices $A_i, B_i, E_{w_i}, E_{f_i}, C_i, D_i, F_{w_i}$ and F_{f_i} are known system matrices with appropriate dimension.

The defuzzified output of the discrete TS fuzzy model in eq. (4.1) is represented as

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k)) \Big[A_i x(k) + B_i u(k) + E_{w_i} w(k) + E_{f_i} f(k) \Big] \tag{4.2a}$$

$$y(k) = \sum_{i=1}^{r} h_i(z(k)) \Big[C_i x(k) + D_i u(k) + F_{w_i} w(k) + v(k) + F_{f_i} f(k) \Big] \tag{4.2b}$$

The above system description provides

$$E_{w_i} = B_i \qquad \text{and} \qquad F_{w_i} = D_i$$

for the influence of the system noise. It is assumed that noise signals $w(k)$ and $v(k)$ are uncorrelated, zero-mean, and Gaussian white noise vectors, i.e. its mean vector are

$$E\left[w(k)\right] = 0 \qquad \text{and} \qquad E\left[v(k)\right] = 0$$

where $E[\cdot]$ denotes the expectation and consequently, the covariance matrices for $w(k)$ and $v(k)$ are defined as

$$\begin{aligned} E\left[w(k)w^T(k)\right] &= \Sigma_w,\ \Sigma_w = diag(\sigma_{w,1},\dots,\sigma_{w,k_w})^2 \\ E\left[v(k)v^T(k)\right] &= \Sigma_v,\ \Sigma_v = diag(\sigma_{v,1},\dots,\sigma_{v,k_v})^2 \end{aligned}$$

The above assumptions on stochastic features of the noise are all reasonable from a practical point of view [25].

4.1.1 System reformulation

To get a more general description of the discrete TS fuzzy model described in eq. (4.1), the noise vector $n(k)$ is introduced

$$n(k) = \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \tag{4.3}$$

Thus, the fuzzy IF-THEN rules in eq. (4.1) can be written into

Model rule $\boldsymbol{i}$

$$\begin{array}{ll} \text{IF} & z_1(k) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(k) \text{ is } M_{ip} \\ \text{THEN} & \left\{ \begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + E_{n_i} n(k) + E_{f_i} f(k) \\ y(k) &= C_i x(k) + D_i u(k) + F_{n_i} n(k) + F_{f_i} f(k) \end{aligned} \right. \end{array} \tag{4.4}$$

where $n(k) \in \mathbb{R}^{k_n}$ is the vector of stochastic noise and matrices E_{n_i} and F_{n_i} are known system matrices with appropriate dimensions.

The defuzzified output of the discrete TS fuzzy model in eq. (4.4) is inferred as

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k)) \Big[A_i x(k) + B_i u(k) + E_{n_i} n(k) + E_{f_i} f(k)\Big] \tag{4.5a}$$

$$y(k) = \sum_{i=1}^{r} h_i(z(k)) \Big[C_i x(k) + D_i u(k) + F_{n_i} n(k) + F_{f_i} f(k)\Big] \tag{4.5b}$$

where

$$E_{n_i} = \begin{bmatrix} E_{w_i} & 0 \end{bmatrix} \qquad F_{n_i} = \begin{bmatrix} F_{w_i} & I \end{bmatrix} \tag{4.6}$$

Moreover, declaring

$$\bar{w}(k) = \sum_{i=1}^{r} h_i(z(k)) E_{n_i} n(k) \qquad \bar{v}(k) = \sum_{i=1}^{r} h_i(z(k)) F_{n_i} n(k) \tag{4.7}$$

allow us to obtain the standard system description

$$x(k+1) = \sum_{i=1}^{r} h_i(z(k)) \Big[A_i x(k) + B_i u(k) + \bar{w}(k) + E_{f_i} f(k)\Big] \tag{4.8a}$$

$$y(k) = \sum_{i=1}^{r} h_i(z(k)) \Big[C_i x(k) + D_i u(k) + \bar{v}(k) + F_{f_i} f(k)\Big] \tag{4.8b}$$

mostly used in the literature. The covariances matrices can be defined as

$$\begin{aligned} E\left[\bar{w}(k)\bar{w}^T(k)\right] &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))E_{n_i}\Sigma_n E_{n_j}^T \\ E\left[\bar{v}(k)\bar{v}^T(k)\right] &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))F_{n_i}\Sigma_n F_{n_j}^T \end{aligned}$$

where Σ_n means

$$\Sigma_n = \begin{bmatrix} \Sigma_w & 0 \\ 0 & \Sigma_v \end{bmatrix} = diag(\sigma_{w,1}, \ldots, \sigma_{w,k_w}, \sigma_{v,1}, \ldots, \sigma_{v,k_v})^2 \quad (4.9)$$

and the cross covariance matrices are given by

$$\begin{aligned} E\left[\bar{w}(k)\bar{v}^T(k)\right] &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))E_{n_i}\Sigma_n F_{n_j}^T \\ E\left[\bar{v}(k)\bar{w}^T(k)\right] &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))F_{n_i}\Sigma_n E_{n_j}^T \end{aligned}$$

4.2 Proposed approach for the TS fuzzy observer

Because of the stochastic noise, the state estimates given by a TS fuzzy observer are no longer accurate. Therefore, a TS fuzzy observer is proposed. The objective of the observer is to minimize the expected value of the steady state estimation error, knowing the stochastic features of the noises.

A TS fuzzy observer is constructed to estimate the states and is given by the following fuzzy IF-THEN rules

Observer rule i

$$\begin{aligned} &\text{IF} \quad z_1(k) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(k) \text{ is } M_{ip} \\ &\text{THEN} \begin{cases} \hat{x}(k+1) = & A_i\hat{x}(k) + B_iu(k) + L_i(y(k) - \hat{y}(k)) \\ \hat{y}(k) = & C_i\hat{x}(k) + D_iu(k) \end{cases} \end{aligned} \quad (4.10)$$

The defuzzified output of the TS fuzzy observer in eq. (4.10) is represented as

$$\hat{x}(k+1) = \sum_{i=1}^{r} h_i(z(k))\Big[A_i\hat{x}(k) + B_iu(k) + L_i\big(y(k) - \hat{y}(k)\big)\Big] \quad (4.11a)$$

$$\hat{y}(k) = \sum_{i=1}^{r} h_i(z(k))\Big[C_i\hat{x}(k) + D_iu(k)\Big] \quad (4.11b)$$

Based on the state equations (4.8a) and (4.11a), the state estimation error $e(k)$ is defined by

$$e(k) = x(k) - \hat{x}(k) \quad (4.12)$$

and has to be minimized in order to find the best estimation of $x(k)$. In order to analyze the convergence of the TS fuzzy observer, the dynamics of the state estimation error without the presence of faults is considered.

$$\begin{aligned} e(k+1) &= x(k+1) - \hat{x}(k+1) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))\Big[(A_i - L_iC_j)e(k) + \bar{w}(k) - L_i\bar{v}(k)\Big] \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))\Big[A_{ij}e(k) + \bar{w}(k) - L_i\bar{v}(k)\Big] \end{aligned} \tag{4.13}$$

where

$$A_{ij} = A_i - L_iC_j$$

Using the description of the noise vectors, especially the assumption that they are zero-mean, the following equation is given for the value of expectation

$$E\left[e(k+1)\right] = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(k))h_j(z(k))A_{ij}E\left[e(k)\right] \tag{4.14}$$

The error covariance matrix can be defined based on eq. (4.13) as

$$P(k+1) = E\left[e(k+1)e^T(k+1)\right]$$

$$\begin{aligned} P(k+1) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{o=1}^{r} h_i(z(k))h_j(z(k))h_l(z(k))h_o(z(k))\Bigg(A_{ij}E\left[e(k)e^T(k)\right]A_{lo}^T + A_{ij}E\left[e(k)\bar{w}^T(k)\right] - \\ &A_{ij}E\left[e(k)\bar{v}^T(k)\right]L_l^T + E\left[\bar{w}(k)e^T(k)\right]A_{ij}^T + E\left[\bar{w}(k)\bar{w}^T(k)\right] - E\left[\bar{w}(k)\bar{v}^T(k)\right]L_i^T - \\ &L_iE\left[\bar{v}(k)e^T(k)\right]A_{jl}^T - L_iE\left[\bar{v}(k)\bar{w}^T(k)\right] + L_iE\left[\bar{v}(k)\bar{v}^T(k)\right]L_j^T\Bigg) \end{aligned} \tag{4.15}$$

Under the assumption that the current error is independent of the current noise, it is provided

$$\begin{aligned} E\left[e(k)\bar{w}^T(k)\right] &= \Big(E\left[\bar{w}(k)e^T(k)\right]\Big)^T &= 0 \\ E\left[e(k)\bar{v}^T(k)\right] &= \Big(E\left[\bar{v}(k)e^T(k)\right]\Big)^T &= 0 \end{aligned}$$

Due to the fact, that the current error is independent of the current noise, the eq. (4.15) can be reduced to

$$\begin{aligned} P(k+1) = &\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{o=1}^{r} h_i(z(k))h_j(z(k))h_l(z(k))h_o(z(k))\Bigg(A_{ij}E\left[e(k)e^T(k)\right]A_{lo}^T + \\ &E\left[\bar{w}(k)\bar{w}^T(k)\right] - E\left[\bar{w}(k)\bar{v}^T(k)\right]L_i^T - L_iE\left[\bar{v}(k)\bar{w}^T(k)\right] + L_iE\left[\bar{v}(k)\bar{v}^T(k)\right]L_j^T\Bigg) \end{aligned}$$

substituting $P(k) = E\left[e(k)e^T(k)\right]$ and the correspondent values for the covariance matrices in the above equation results in

$$P(k+1)=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{o=1}^{r} h_i(z(k))h_j(z(k))h_l(z(k))h_o(z(k))\Bigg(A_{ij}P(k)A_{lo}^T+$$

$$E_{n_i}\Sigma_n E_{n_j}^T - E_{n_i}\Sigma_n F_{n_j}^T L_l^T - L_i F_{n_j}\Sigma_n E_{n_l}^T + L_i F_{n_j}\Sigma_n F_{n_l}^T L_o^T\Bigg)$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{o=1}^{r} h_i(z(k))h_j(z(k))h_l(z(k))h_o(z(k))\Bigg(A_{ij}P(k)A_{lo}^T + E_{n_{ij}}\Sigma_n E_{n_{lo}}^T\Bigg)$$

where

$$E_{n_{ij}} = E_{n_i} - L_i F_{n_j}$$

Assuming that $P(k+1)$=$P(k)$, the following equation is obtained for the steady state

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{o=1}^{r} h_i(z(k))h_j(z(k))h_l(z(k))h_o(z(k))\Bigg(A_{ij}PA_{lo}^T - P + E_{n_{ij}}\Sigma_n E_{n_{lo}}^T\Bigg) = 0 \qquad (4.16)$$

It is clear that

$$A_{ij}PA_{lo}^T - P + E_{n_{ij}}\Sigma_n E_{n_{lo}}^T = 0 \qquad (4.17)$$

has the form of a discrete algebraic Riccati equation (DARE). It is known from [7, 87], that the DARE in eq. (4.17) is solvable for a common matrix $P \geq 0$ if and only if $\exists P \geq 0$ such that

$$A_{ij}PA_{lo}^T - P + E_{n_{ij}}\Sigma_n E_{n_{lo}}^T \leq 0 \qquad (4.18)$$

In [20, 62] the relationship between the solution of a discrete algebraic Riccati equation and its associated LMI can be found. The following lemma from [62] is used to prove that the DARE in eq. (4.17) which is equivalent to the eq. (4.18).

Lemma 4.1 *Given the discrete algebraic Riccati equation*

$$A^TPA - P + Q - (C + B^TPA)^T(R + B^TPB)^{-1}(C + B^TPA) = 0 \qquad (4.19)$$

with $R > 0$, $P^T = P$, and let

$$\mathcal{Q}(P) = A^TPA - P + Q - (C + B^TPA)^T(R + B^TPB)^{-1}(C + B^TPA) \qquad (4.20)$$

Assume that there exists $P = P^T$ such that $\mathcal{Q}(P) \geq 0$. Then if (A, B) is stabilizable, there exists a minimal solution $P_- \geq 0$ to the Riccati eq. (4.19). Moreover,

$$P_- \leq P,\ \forall P \text{ such that } \mathcal{Q}(P) \geq 0 \qquad (4.21)$$

and $A - B(R + B^TP_-B)^{-1}(C + B^TP_-A)$ is stable.

In order to minimize the expected value of the steady state estimation error $e(k)$ [64], the following LMI optimization problem is formulated

$$\begin{aligned} &\min \ tr(P), \ \text{subject to } P \geq 0 \\ &A_{ij} P A_{lo}^T - P + E_{n_{ij}} \Sigma_n E_{n_{lo}}^T \leq 0 \end{aligned} \tag{4.22}$$

Considering that all pairs (A_i, C_i) are detectable and hence (A_i^T, C_i^T) are stabilizable, it follows from Lemma 4.1 that the minimal solution of eq. (4.17) is indeed the minimal solution of eq. (4.22).

The above matrix inequality can be expressed in the following equivalent form

$$-P + \begin{bmatrix} A_{ij} & E_{n_{ij}} \end{bmatrix} \begin{bmatrix} P & 0 \\ 0 & \Sigma_n \end{bmatrix} \begin{bmatrix} A_{ij}^T \\ E_{n_{ij}}^T \end{bmatrix} \leq 0 \tag{4.23}$$

According to the Schur complement, the eq. (4.23) is rearranged in the following matrix inequality

$$\begin{bmatrix} -P & A_{ij} & E_{n_{ij}} \\ A_{ij}^T & -P^{-1} & 0 \\ E_{n_{ij}}^T & 0 & -\Sigma_n^{-1} \end{bmatrix} \leq 0$$

Substituting A_{ij} and $E_{n_{ij}}$ in the above matrix inequality results in

$$\begin{bmatrix} -P & A_i - L_i C_j & E_{n_i} - L_i F_{n_j} \\ A_i^T - C_j^T L_i^T & -P^{-1} & 0 \\ E_{n_i}^T - F_{n_j}^T L_i^T & 0 & -\Sigma_n^{-1} \end{bmatrix} \leq 0$$

Both sides of the above matrix inequality are multiplied by block diagonal matrix $\{ \ P^{-1}, \ \ I, \ \ I \ \}$, and results in

$$\begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} -P & A_i - L_i C_j & E_{n_i} - L_i F_{n_j} \\ A_i^T - C_j^T L_i^T & -P^{-1} & 0 \\ E_{n_i}^T - F_{n_j}^T L_i^T & 0 & -\Sigma_n^{-1} \end{bmatrix} \begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} =$$

$$\begin{bmatrix} -I & P^{-1} A_i - P^{-1} L_i C_j & P^{-1} E_{n_i} - P^{-1} L_i F_{n_j} \\ A_i^T - C_j^T L_i^T & -P^{-1} & 0 \\ E_{n_i}^T - F_{n_j}^T L_i^T & 0 & -\Sigma_n^{-1} \end{bmatrix} \begin{bmatrix} P^{-1} & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} =$$

$$\begin{bmatrix} -P^{-1} & P^{-1} A_i - P^{-1} L_i C_j & P^{-1} E_{n_i} - P^{-1} L_i F_{n_j} \\ A_i^T P^{-1} - C_j^T L_i^T P^{-1} & -P^{-1} & 0 \\ E_{n_i}^T P^{-1} - F_{n_j}^T L_i^T P^{-1} & 0 & -\Sigma_n^{-1} \end{bmatrix} \leq 0$$

Let $X = P^{-1}$ and $N_i = X L_i$. Thus, the following LMI is obtained:

$$\begin{bmatrix} -X & X A_i - N_i C_j & X E_{n_i} - N_i F_{n_j} \\ A_i^T X - C_j^T N_i^T & -X & 0 \\ E_{n_i}^T X - F_{n_j}^T N_i^T & 0 & -\Sigma_n^{-1} \end{bmatrix} \leq 0$$

Substituting $E_{n_i} = \begin{bmatrix} E_{w_i} & 0 \end{bmatrix}$ and $F_{n_i} = \begin{bmatrix} F_{w_i} & I \end{bmatrix}$ in the above LMI results in

$$\begin{bmatrix} -X & XA_i - N_iC_j & XE_{w_i} - N_iF_{w_j} & -N_i \\ A_i^TX - C_j^TN_i^T & -X & 0 & 0 \\ E_{w_i}{}^TX - F_{w_j}{}^TN_i{}^T & 0 & -\Sigma_w{}^{-1} & 0 \\ -N_i^T & 0 & 0 & -\Sigma_v{}^{-1} \end{bmatrix} \leq 0$$

Therefore, the above LMI represents the optimization problem from eq. (4.22) as follows

$$\max\ tr(X),\ \text{subject to } X \geq 0$$

$$\begin{bmatrix} -X & XA_i - N_iC_i & XE_{w_i} - N_iF_{w_i} & -N_i \\ A_i^TX - C_i^TN_i^T & -X & 0 & 0 \\ E_{w_i}{}^TX - F_{w_i}{}^TN_i{}^T & 0 & -\Sigma_w{}^{-1} & 0 \\ -N_i^T & 0 & 0 & -\Sigma_v{}^{-1} \end{bmatrix} \leq\ 0 \tag{4.24}$$

$$\begin{bmatrix} -4X & \begin{bmatrix} XA_i - N_iC_j + \\ XA_j - N_jC_i \end{bmatrix} & \begin{bmatrix} XE_{w_i} - N_iF_{w_j} + \\ XE_{w_j} - N_jF_{w_i} \end{bmatrix} & -N_i - N_j \\ \begin{bmatrix} A_i^TX - C_j^TN_i^T + \\ A_j^TX - C_i^TN_j^T \end{bmatrix} & -X & 0 & 0 \\ \begin{bmatrix} E_{w_i}{}^TX - F_{w_j}{}^TN_i{}^T + \\ E_{w_j}{}^TX - F_{w_i}{}^TN_j{}^T \end{bmatrix} & 0 & -\Sigma_w{}^{-1} & 0 \\ -N_i^T - N_j^T & 0 & 0 & -\Sigma_v{}^{-1} \end{bmatrix} \leq\ 0 \tag{4.25}$$

$$\forall i < j$$

where

$$L_i = X^{-1}N_i \qquad \text{and} \qquad P = X^{-1}$$

It is clear, that in the formulation of eq. (4.24)-(4.25), the maximization of matrix X implies the minimization of matrix P in eq. (4.22).

4.2.1 Residual Evaluation

To evaluate the generated residual and based on [21, 48], the use of LMIs is the widely adopted approaches to calculate the threshold value $J_{th} > 0$ and based on this, the following logic relationship for fault detection is used:

$$\begin{aligned} \|r(k)\|_{2,N} \leq J_{th} \quad &\Rightarrow \quad \text{no alarm, fault-free} \\ \|r(k)\|_{2,N} > J_{th} \quad &\Rightarrow \quad \text{alarm, a fault is detected} \end{aligned}$$

where the so-called residual evaluation $\|r(k)\|_{2,N}$ is determined by:

$$\|r(k)\|_{2,N} = \sqrt{\sum_{k=0}^{N} r^T(k)r(k)} \tag{4.26}$$

N a is discrete-time window. Since an evaluation of the signal over the whole time range is impractical, it is desired that the fault will be detected as easy as possible. Based on eq. (4.13), it follows

$$\|r(k)\|_{2,N} = \|r_n(k) + r_f(k)\|_{2,N} \tag{4.27}$$

where $r_n(k)$ and $r_f(k)$ are defined as:

$$r_n(k) = r(k)|_{f=0} \tag{4.28}$$

$$r_f(k) = r(k)|_{n=0} \tag{4.29}$$

Moreover, the fault-free case residual evaluation function is

$$\|r(k)\|_{2,N} \leq \|r_n\|_{2,N} \leq J_{th,n} \tag{4.30}$$

where $J_{th,n} = \sup_{n \in L_2} \|r_n\|_{2,N}$. Therefore, the threshold J_{th} is chosen as $J_{th} = J_{th,n}$. Where J_{th} is constant and can be evaluated off-line.

To demonstrate the effectiveness of the proposed approach to minimize the expected value of the steady state estimation error, the approach is applied to the vehicle lateral dynamic model.

4.3 An application example

The vehicle lateral dynamic model, which is represented by the so-called bicycle model [41, 54], it is a linear parameter varying (LPV) system and it is approximated using the TS fuzzy model.

The continuous state space representation for the vehicle lateral dynamic model is given by

$$\begin{bmatrix} \dot{\beta}(t) \\ \dot{r}(t) \end{bmatrix} = \begin{bmatrix} -\frac{C_{\alpha H}+C'_{\alpha V}}{mv_{ref}} K_{\phi R} & \frac{l_H C_{\alpha H} - l_V C'_{\alpha V}}{m{v_{ref}}^2} K_{\phi R} - 1 \\ \frac{l_H C_{\alpha H} - l_V C'_{\alpha V}}{I_z} & -\frac{l_V^2 C'_{\alpha V} + l_H^2 C_{\alpha H}}{I_z v_{ref}} \end{bmatrix} \begin{bmatrix} \beta(t) \\ r(t) \end{bmatrix} + \begin{bmatrix} \frac{C'_{\alpha V}}{mv_{ref}} K_{\phi R} \\ \frac{l_V C'_{\alpha V}}{I_z} \end{bmatrix} (\delta_L^*(t) + n_{\delta_L}(t))$$

$$\begin{bmatrix} a_y(t) \\ r(t) \end{bmatrix} = \begin{bmatrix} -\frac{C_{\alpha H}+C'_{\alpha V}}{m} & \frac{l_H C_{\alpha H} - l_V C'_{\alpha V}}{mv_{ref}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta(t) \\ r(t) \end{bmatrix} + \begin{bmatrix} \frac{C'_{\alpha V}}{m} \\ 0 \end{bmatrix} (\delta_L^*(t) + n_{\delta_L}(t)) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_{a_y}(t) \\ n_r(t) \end{bmatrix}$$

where v_{ref} is the varying parameter, $x^T(t) = \begin{bmatrix} \beta^T(t) & r^T(t) \end{bmatrix}^T$, $u(t) = \delta_L^*(t)$, $w(t) = n_{\delta_L}(t)$, $v^T(t) = \begin{bmatrix} n_{a_y}^T(t) & n_r^T(t) \end{bmatrix}^T$ and $y^T(t) = \begin{bmatrix} a_r^T(t) & r^T(t) \end{bmatrix}^T$. Using the numerical values from Appendix B, this system can be written as follows:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -\frac{144.034}{v_{ref}} & \frac{58.896}{{v_{ref}}^2} - 1 \\ 29.859 & -\frac{170.981}{v_{ref}} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} \frac{52.802}{v_{ref}} \\ 40.939 \end{bmatrix} u(t) + \begin{bmatrix} \frac{52.802}{v_{ref}} & 0 & 0 \\ 40.939 & 0 & 0 \end{bmatrix} n(t)$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} -152.756 & \frac{62.463}{v_{ref}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 56 \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} 56 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} n(t)$$

where $n^T(t) = \begin{bmatrix} n_{\delta_L}^T(t) & n_{a_y}^T(t) & n_r^T(t) \end{bmatrix}^T$.

In tab. 4.1 the typical sensor noise data for the vehicle lateral dynamic model are listed.

Sensor	**Standard variation σ**	**Unit**
n_{δ_L}	$\sigma_{\delta_L} = 3.5 \times 10^{-3}$	$[rad]$
n_{a_y}	$\sigma_{a_y} = 0.2$	$[m/s^2]$
n_r	$\sigma_{n_r} = 3.5 \times 10^{-3}$	$[rad/s]$

Tab. 4.1: Typical sensor noise of vehicle lateral dynamic model

To obtain the TS fuzzy model, it is necessary to define two premise variables (each premise variable represent in this case a varying parameter). The premise variables are defined as follows:

$$z_1(t) = \frac{1}{v_{ref}} \qquad z_2(t) = \frac{1}{{v_{ref}}^2}$$

Matrices $A(z(t))$, $B(z(t))$, $E_n(z(t))$ and $C(z(t))$ are expressed as follows:

$$\begin{aligned}
A(z(t)) &= \begin{bmatrix} -144.034z_1(t) & 58.896z_2(t) - 1 \\ 29.859 & -170.981z_1(t) \end{bmatrix} & B(z(t)) &= \begin{bmatrix} 58.802z_1(t) \\ 40.939 \end{bmatrix} \\
E_n(z(t)) &= \begin{bmatrix} 58.802z_1(t) & 0 & 0 \\ 40.939 & 0 & 0 \end{bmatrix} & C(z(t)) &= \begin{bmatrix} -152.756 & 62.463z_1(t) \\ 0 & 1 \end{bmatrix}
\end{aligned}$$

The computation of the minimum and maximum values of $z_1(t)$ and $z_2(t)$ for $v_{ref} \in [5, 55]$ m/s are

$$\begin{aligned}
&\max_{v_{ref}} \; z_1(t) = z_1^+ = 0.2 & &\max_{{v_{ref}}^2} \; z_2(t) = z_2^+ = 0.04 \\
&\min_{v_{ref}} \; z_1(t) = z_1^- = 0.0182 & &\min_{{v_{ref}}^2} \; z_2(t) = z_2^- = 3.3 \times 10^{-4}
\end{aligned}$$

from the maximum and minimum values, $z_1(t)$ and $z_2(t)$ can be represented by

$$\begin{aligned}
z_1(t) &= F_{11}(z_1(t)) \cdot 0.2 + F_{12}(z_1(t)) \cdot 0.0182 \\
z_2(t) &= F_{21}(z_2(t)) \cdot 0.04 + F_{22}(z_2(t)) \cdot 3.3 \times 10^{-4}
\end{aligned}$$

where:

$$F_{11}(z_1(t)) + F_{12}(z_1(t)) = 1 \qquad \text{and} \qquad F_{21}(z_2(t)) + F_{22}(z_2(t)) = 1$$

the membership functions are calculated as follows

$$\begin{aligned}
F_{11}(z_1(t)) &= \frac{z_1(t) - 0.0182}{0.1818} & F_{12}(z_1(t)) &= \frac{0.2 - z_1(t)}{0.1818} \\
F_{21}(z_2(t)) &= \frac{z_2(t) - 3.3 \times 10^{-4}}{0.03967} & F_{22}(z_2(t)) &= \frac{0.04 - z_2(t)}{0.03967}
\end{aligned}$$

Each subsystem is discretized using 10 milliseconds as sample time, in order to have the TS fuzzy model in its discrete form. The vehicle lateral dynamic model is represented by the following discrete fuzzy IF-THEN rules:

Model rule 1

$$\text{IF} \quad z_1(k) \text{ is } F_{11} \text{ and } z_2(k) \text{ is } F_{21}$$
$$\text{THEN} \begin{cases} x(k+1) = & A_1 x(k) + B_1 u(k) + E_{n_1} n(k) \\ y(k) = & C_1 x(k) + D_1 u(k) + F_{n_1} n(k) \end{cases}$$

Model rule 2

$$\text{IF} \quad z_1(k) \text{ is } F_{11} \text{ and } z_2(k) \text{ is } F_{22}$$
$$\text{THEN} \begin{cases} x(k+1) = & A_2 x(k) + B_2 u(k) + E_{n_2} n(k) \\ y(k) = & C_2 x(k) + D_2 u(k) + F_{n_2} n(k) \end{cases}$$

Model rule 3

$$\text{IF} \quad z_1(k) \text{ is } F_{12} \text{ and } z_2(k) \text{ is } F_{21}$$
$$\text{THEN} \begin{cases} x(k+1) = & A_3 x(k) + B_3 u(k) + E_{n_3} n(k) \\ y(k) = & C_3 x(k) + D_3 u(k) + F_{n_3} n(k) \end{cases}$$

Model rule 4

$$\text{IF} \quad z_1(k) \text{ is } F_{12} \text{ and } z_2(k) \text{ is } F_{22}$$
$$\text{THEN} \begin{cases} x(k+1) = & A_4 x(k) + B_4 u(k) + E_{n_4} n(k) \\ y(k) = & C_4 x(k) + D_4 u(k) + F_{n_4} n(k) \end{cases}$$

Here

$$A_1 = \begin{bmatrix} 0.7512 & 0.0099 \\ 0.2181 & 0.7118 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0941 \\ 0.3598 \end{bmatrix}, \quad E_{n_1} = \begin{bmatrix} 0.0941 & 0 & 0 \\ 0.3598 & 0 & 0 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} 0.7486 & -0.0072 \\ 0.2178 & 0.7093 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.0901 \\ 0.3594 \end{bmatrix}, \quad E_{n_2} = \begin{bmatrix} 0.0901 & 0 & 0 \\ 0.3594 & 0 & 0 \end{bmatrix}$$
$$A_3 = \begin{bmatrix} 0.9761 & 0.0132 \\ 0.2904 & 0.9714 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.0122 \\ 0.4048 \end{bmatrix}, \quad E_{n_3} = \begin{bmatrix} 0.0122 & 0 & 0 \\ 0.4048 & 0 & 0 \end{bmatrix}$$
$$A_4 = \begin{bmatrix} 0.9727 & -0.0095 \\ 0.2900 & 0.9680 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.0075 \\ 0.4043 \end{bmatrix}, \quad E_{n_4} = \begin{bmatrix} 0.0075 & 0 & 0 \\ 0.4043 & 0 & 0 \end{bmatrix}$$
$$C_{1,2} = \begin{bmatrix} -152.76 & 12.49 \\ 0 & 1 \end{bmatrix}, \; C_{3,4} = \begin{bmatrix} -152.76 & 1.13 \\ 0 & 1 \end{bmatrix}$$
$$D_{1,2,3,4} = \begin{bmatrix} 56 \\ 0 \end{bmatrix}, \; F_{n_{1,2,3,4}} = \begin{bmatrix} 56 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Sigma_w = 1.2185 \times 10^{-5}, \; \Sigma_v = \begin{bmatrix} 0.04 & 0 \\ 0 & 1.2185 \times 10^{-5} \end{bmatrix}$$

The defuzzification (that give the discrete TS fuzzy model) is carried out as

$$
\begin{aligned}
x(k+1) &= \sum_{i=1}^{4} h_i(z(k))\Big[\ A_i x(k) + B_i u(k) + E_{n_i} n(k)\ \Big] \\
y(k) &= \sum_{i=1}^{4} h_i(z(k))\Big[\ C_i x(k) + D u(k) + F_n n(k)\ \Big]
\end{aligned}
$$

where

$$
\begin{aligned}
h_1(z(k)) &= F_{11}(z_1(k)) \times F_{21}(z_2(k)) \\
h_2(z(k)) &= F_{11}(z_1(k)) \times F_{22}(z_2(k)) \\
h_3(z(k)) &= F_{12}(z_1(k)) \times F_{21}(z_2(k)) \\
h_4(z(k)) &= F_{12}(z_1(k)) \times F_{22}(z_2(k))
\end{aligned}
$$

4.3.1 Simulation Results

The proposed approach to minimize the expected value of the steady state estimation error is applied to the vehicle lateral dynamic model, where eq. (4.24)-(4.25) are used to make the minimization of the expected value of the steady state estimation error for each output separately.

The following longitude velocity profile is considered for the $v_{ref}(k)$

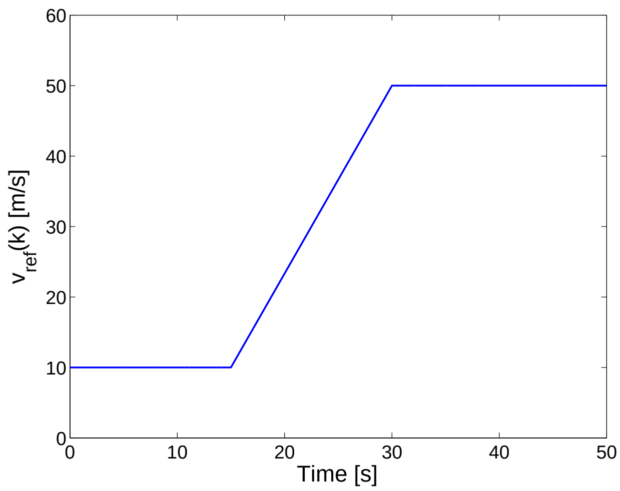

Fig. 4.1: Longitude velocity profile

4.3.1.1 Lateral acceleration output

The gain matrices obtained for the lateral acceleration $a_y(k)$ output are:

$$L_1 = \begin{bmatrix} -0.00091 \\ 0.00243 \end{bmatrix}, \; L_2 = \begin{bmatrix} -0.00096 \\ 0.00288 \end{bmatrix}, \; L_3 = \begin{bmatrix} -0.00194 \\ -0.00321 \end{bmatrix}, \; L_4 = \begin{bmatrix} -0.00170 \\ -0.00215 \end{bmatrix}.$$

An offset of 5 m/s^2 is considered as a sensor fault that appears from 48 to 50 s.

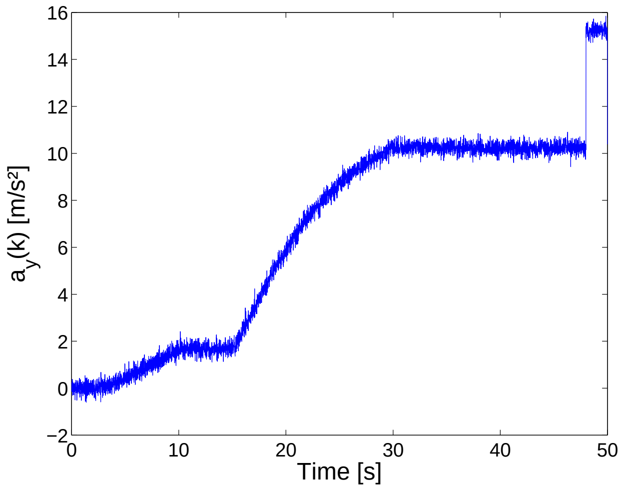

Fig. 4.2: Lateral acceleration output

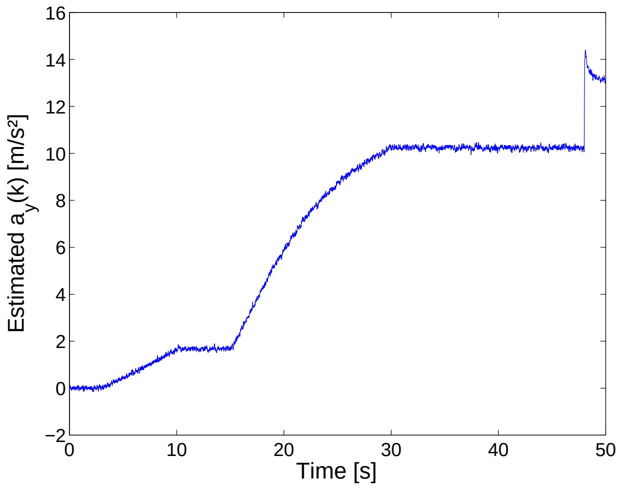

Fig. 4.3: Estimated lateral acceleration

It can be seen in fig. 4.3 that the estimated lateral acceleration $a_y(k)$ attenuates the effect of the stochastic noise. Using L_2 norm as evaluation function and a residual evaluation window of 20 s. for the lateral acceleration output, the obtained threshold value (J_{th}) is 1.608.

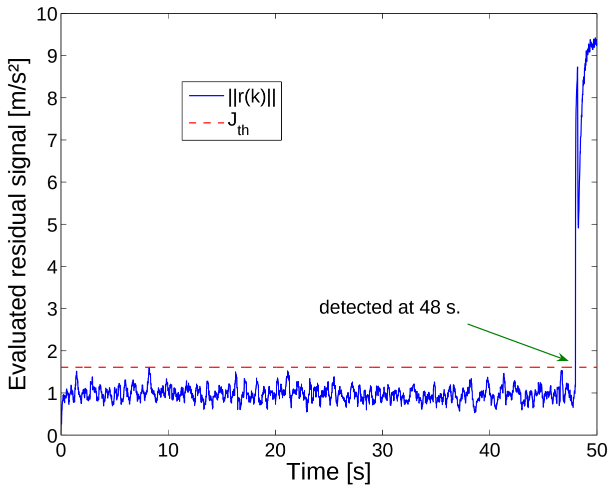

Fig. 4.4: Evaluated residual for the lateral acceleration sensor

In fig. 4.4, the evaluated residual has exceeded the threshold value at $t = 48$ s. Therefore, the sensor fault can be detected.

4.3.1.2 Yaw rate output

The gain matrices obtained for the yaw rate $r(k)$ output are:

$$L_5 = \begin{bmatrix} 0.02435 \\ 0.12236 \end{bmatrix}, \quad L_6 = \begin{bmatrix} 0.02304 \\ 0.11732 \end{bmatrix}, \quad L_7 = \begin{bmatrix} 0.06409 \\ 0.34173 \end{bmatrix}, \quad L_8 = \begin{bmatrix} 0.05863 \\ 0.34753 \end{bmatrix}.$$

An offset of 10 $°/s$ (0.1745 rad/s) is considered as a sensor fault that appears from 44 to 46 s.

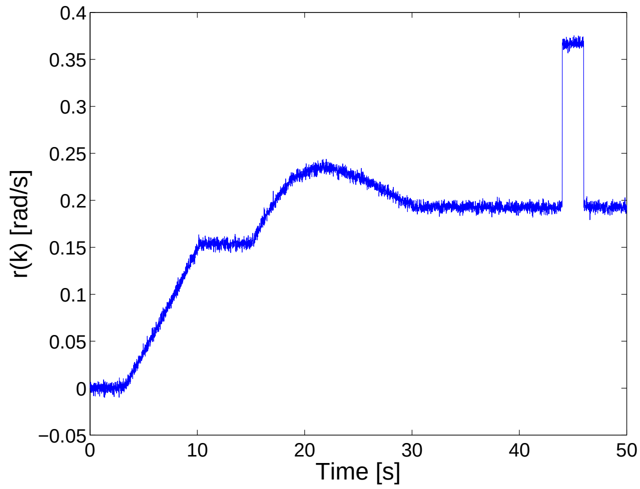

Fig. 4.5: Yaw rate output

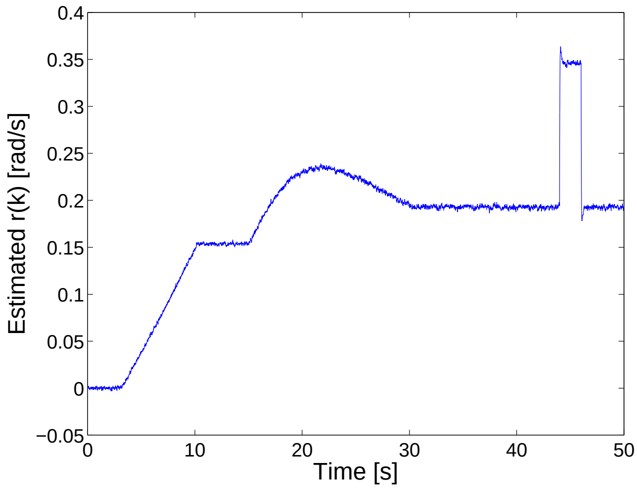

Fig. 4.6: Estimated yaw rate

As can be appreciated in fig. 4.6, the estimated yaw rate $r(k)$ attenuates the effect of the stochastic noise. Using L_2 norm as evaluation function and a residual evaluation window of 20 s. for the yaw rate output, the obtained threshold value (J_{th}) is 0.027.

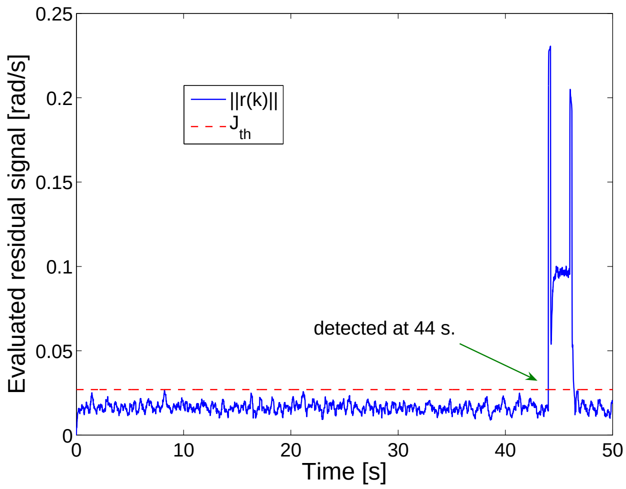

Fig. 4.7: Evaluated residual for the yaw rate sensor

It can be seen, that the evaluated residual has exceeded the threshold value at $t = 44$ s. Therefore, the sensor fault can be detected.

A scheme to minimize the expected value of the steady state estimation error for a class of nonlinear systems described by the TS fuzzy model has been presented. The minimization is made using LMI techniques for the solution of the problem.

The proposed scheme is applied to the vehicle lateral dynamic model. The simulation results for the estimated lateral acceleration $a_y(k)$ and the estimated yaw rate $r(k)$ show that the effect of stochastic noise is attenuated, and the applied faults can be easily detected.

Chapter 5

Fault detection observer for TS fuzzy systems

Robustness is the most fundamental problem in model-based fault detection. Based on this problem, the study of a robust fault detection problem, which aims at enhancing the robustness to disturbances without sacrificing the fault detection sensitivity has received attention in recent years [19, 79, 81].

In this chapter, the robust fault detection observer using iterative linear matrix inequality (LMI) algorithms [79, 81] is generalized for a class of nonlinear systems described by the TS fuzzy model.

These iterative LMI algorithms are implemented to design a robust TS fuzzy fault detection observer (FDO). The objective of the FDO is to find a trade-off between maximizing the effect of faults in order to increase the sensitivity to faults and minimizing the effect of disturbances in order to enhance the robustness to disturbances.

In this design, two performance indexes need to be found, one of them is used to minimize the effect of disturbances (γ_1) and the another one is used to maximize the effect of faults (γ_2). Both of them have a dependence on each other, in which, a gain ratio is established, it is given by γ_1/γ_2.

Consider the following TS fuzzy model with influence of disturbances and faults and the model is represented by fuzzy IF-THEN rules

Model rule *i*

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & \left\{ \begin{array}{rl} \dot{x}(t) = & A_i x(t) + B_i u(t) + E_d d(t) + E_f f(t) \\ y(t) = & C_i x(t) + D_i u(t) + F_d d(t) + F_f f(t) \end{array} \right. \end{array} \tag{5.1}$$

where $i = 1, \dots, r$ and r is the number of fuzzy IF-THEN rules, M_{ij} are fuzzy sets, $z_1(t), \dots z_p(t)$ are premise variables, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ and $y(t) \in \mathbb{R}^m$ are the input and output vectors respectively, $d(t) \in \mathbb{R}^{k_d}$ is the disturbance vector and $f(t) \in \mathbb{R}^{k_f}$ is the fault vector. Matrices A_i, B_i, E_d, E_f, C_i, D_i, F_d and F_f are known system matrices with appropriate dimension.

The defuzzified output of the TS fuzzy model in eq. (5.1) is represented as

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[A_i x(t) + B_i u(t) + E_d d(t) + E_f f(t) \Big] \tag{5.2a}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t)) \Big[C_i x(t) + D_i u(t) + F_d d(t) + F_f f(t) \Big] \tag{5.2b}$$

For this TS fuzzy model, there is a TS fuzzy observer given by fuzzy IF-THEN rules

Observer rule i

$$
\begin{array}{ll}
\text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \ldots \text{ and } z_p(t) \text{ is } M_{ip} \\
\text{THEN} & \begin{cases} \dot{\hat{x}}(t) = & A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t)) \\ \hat{y}(t) = & C_i\hat{x}(t) + D_iu(t) \end{cases}
\end{array} \tag{5.3}
$$

The defuzzified output of the TS fuzzy observer eq. (5.3) is represented as

$$
\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) + L_i\left(y(t) - \hat{y}(t)\right)\Big] \tag{5.4a}
$$

$$
\hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_i\hat{x}(t) + D_iu(t)\Big] \tag{5.4b}
$$

Define the state estimation error as $e(t) = x(t) - \hat{x}(t)$ and the residual vector as $r(t) = y(t) - \hat{y}(t)$, then it follows from eq. (5.2)-(5.4) that

$$
\dot{e}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_ie(t) + E_dd(t) + E_ff(t) - L_i\left(y(t) - \hat{y}(t)\right)\Big] \tag{5.5a}
$$

$$
r(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_ie(t) + F_dd(t) + F_ff(t)\Big] \tag{5.5b}
$$

The following sections show the design of a TS fuzzy observer for the disturbance attenuation problem and for the fault sensitivity problem. Then the TS fault detection observer is formulated. The objective of this FDO is to solve both optimization problems at the same time.

5.1 Disturbance attenuation for TS fuzzy observer

The effect of disturbances can be minimized by disturbance rejection with a TS fuzzy observer. For this purpose, the continuous TS fuzzy model given by eq. (5.2) without the effect of faults $f(t)$ is considered

$$
\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_ix(t) + B_iu(t) + E_dd(t)\Big] \tag{5.6a}
$$

$$
y(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_ix(t) + D_iu(t) + F_dd(t)\Big] \tag{5.6b}
$$

where $d(t)$ is the disturbance, the effect of disturbances on the residual signal need to be minimized. A TS fuzzy observer is given by

$$
\dot{\hat{x}}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) + L_i\big(y(t) - \hat{y}(t)\big)\Big]
$$

$$
\hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_i\hat{x}(t) + D_iu(t)\Big]
$$

The disturbance rejection can be realized by minimizing γ_1 subject to

$$\sup_{\|d(t)\|_2 \neq 0} \frac{\|r_d(t)\|_2}{\|d(t)\|_2} \leq \gamma_1 \tag{5.7}$$

Suppose there exists a candidate quadratic Lyapunov function $V_1(e(t)) = e^T(t)Pe(t)$, $P > 0$, and $\gamma_1 > 0$ such that, for all t,

$$\dot{V}_1(e(t)) + r_d^T(t)r_d(t) - \gamma_1^2 d^T(t)d(t) \leq 0 \tag{5.8}$$

for eq. (5.6a) and eq. (5.6b). The dynamics of the state estimation error is defined as follows

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\Big[\big(A_i - L_iC_j\big)e(t) + \big(E_d - L_iF_d\big)d(t)\Big] \tag{5.9}$$

$$r_d(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_ie(t) + F_dd(t)\Big] \tag{5.10}$$

By integrating eq. (5.7) from 0 to T, it is obtained

$$\int_0^T \Big(\dot{V}_1(e(t)) + r_d^T(t)r_d(t) - \gamma_1^2 d^T(t)d(t)\Big)dt \leq 0 \tag{5.11}$$

It is assumed that the initial condition for the state estimation error $e(0)$ is 0, then eq. (5.12) is obtained after the integration of eq. (5.11)

$$V_1(e(T)) + \int_0^T \Big(r_d^T(t)r_d(t) - \gamma_1^2 d^T(t)d(t)\Big)dt \leq 0 \tag{5.12}$$

Since $V_1(e(T)) \geq 0$, this implies

$$\frac{\|r_d(t)\|_2}{\|d(t)\|_2} \leq \gamma_1$$

Therefore the $\mathcal{L}_2$ gain of the TS fuzzy model is less than γ_1. Considering the eq. (5.8), a LMI condition is derived from this equation

$$\dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + r_d^T(t)r_d(t) - \gamma_1^2 d^T(t)d(t) \leq 0 \tag{5.13}$$

For the following part, $z(t)$, $e(t)$ and $d(t)$ are expressed as z, e and d respectively.

$$\begin{aligned}
&\dot{e}^TPe + e^TP\dot{e} + r_d^Tr_d - \gamma_1^2d^Td \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[e^T\bar{A}_{ij}^T + d^T\bar{E}_{d_i}^T\Big]Pe + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)e^TP\Big[\bar{A}_{ij}e + \bar{E}_{d_i}d\Big] \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[\big(e^TC_i^T + d^TF_d^T\big)\big(C_je + F_dd\big)\Big] - \gamma_1^2d^Td \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\begin{bmatrix} e^T & d^T \end{bmatrix}\begin{bmatrix} \bar{A}_{ij}^TP + P\bar{A}_{ij} + C_i^TC_j & P\bar{E}_{d_i} + C_i^TF_d \\ \bar{E}_{d_i}^TP + F_d^TC_i & -\gamma_1^2I + F_d^TF_d \end{bmatrix}\begin{bmatrix} e \\ d \end{bmatrix} \leq 0
\end{aligned} \tag{5.14}$$

where

$$\bar{A}_{ij} = A_i - L_i C_j \qquad \text{and} \qquad \bar{E}_{d_i} = E_d - L_i F_d$$

The following matrix inequality is obtained from eq. (5.14)

$$\begin{bmatrix} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[\bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j\Big] & \sum_{i=1}^{r} h_i(z)\Big[P\bar{E}_{d_i} + C_i^T F_d\Big] \\ \sum_{i=1}^{r} h_i(z)\Big[\bar{E}_{d_i}^T P + F_d^T C_i\Big] & -\gamma_1^2 I + F_d^T F_d \end{bmatrix} \leq 0 \tag{5.15}$$

The matrix inequality given by eq. (5.15) can be rewritten as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z) \begin{bmatrix} \frac{1}{2}\Big(\bar{A}_{ij}^T P + \bar{A}_{ji}^T P + P\bar{A}_{ij} + P\bar{A}_{ji} + C_i^T C_j + C_j^T C_i\Big) & \frac{1}{2}\Big(P\bar{E}_{d_{ij}} + P\bar{E}_{d_{ji}} + C_i^T F_{d_j} + C_j^T F_{d_i}\Big) \\ \frac{1}{2}\Big(\bar{E}_{d_{ij}}^T P + \bar{E}_{d_{ji}}^T P + F_{d_i}^T C_j + F_{d_j}^T C_i\Big) & -\gamma_1^2 + F_d^T F_d \end{bmatrix} \leq 0$$

Therefore, from the above inequality

$$\begin{bmatrix} \frac{1}{2}\Big(\bar{A}_{ij}^T P + \bar{A}_{ji}^T P + P\bar{A}_{ij} + P\bar{A}_{ji} + C_i^T C_j + C_j^T C_i\Big) & \frac{1}{2}\Big(P\bar{E}_{d_{ij}} + P\bar{E}_{d_{ji}} + C_i^T F_{d_j} + C_j^T F_{d_i}\Big) \\ \frac{1}{2}\Big(\bar{E}_{d_{ij}}^T P + \bar{E}_{d_{ji}}^T P + F_{d_i}^T C_j + F_{d_j}^T C_i\Big) & -\gamma_1^2 + F_d^T F_d \end{bmatrix} \leq 0 \tag{5.16}$$

The disturbance rejection can be achieved by solving the following optimization problem:

Problem 5.1 *The observer gain matrices L_i that minimize γ_1 in eq. (5.7) can be obtained by solving the following minimization problem based on LMIs*

$$\textit{minimize } \gamma_1^2 \textit{ subject to } P > 0 \textit{ and}$$

$$\begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i & P\bar{E}_{d_i} + C_i^T F_d \\ \bar{E}_{d_i}^T P + F_d^T C_i & -\gamma_1^2 I + F_d^T F_d \end{bmatrix} < 0 \tag{5.17}$$

$$\begin{bmatrix} \bar{A}_{ij}^T P + \bar{A}_{ji}^T P + P\bar{A}_{ij} + P\bar{A}_{ji} + C_i^T C_j + C_j^T C_i & P\bar{E}_{d_i} + P\bar{E}_{d_j} + C_i^T F_d + C_j^T F_d \\ \bar{E}_{d_i}^T P + \bar{E}_{d_j}^T P + F_d^T C_j + F_d^T C_i & -2\gamma_1^2 I + 2F_d^T F_d \end{bmatrix} \leq 0 \quad i < j \tag{5.18}$$

5.2 Fault sensitivity for TS fuzzy observer

Fault sensitivity can be achieved using a TS fuzzy observer in order to maximize the effect of faults in the residual signal $r(t)$. The continuous TS fuzzy model given by eq. (5.2) without the effect of disturbances $d(t)$ is considered

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\Big[A_i x(t) + B_i u(t) + E_f f(t)\Big] \tag{5.19a}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_i x(t) + D_i u(t) + F_f f(t)\Big] \tag{5.19b}$$

where $f(t)$ is the fault, the effect of faults on the residual signal need to be maximized. A TS fuzzy observer is given by

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[A_i\hat{x}(t) + B_iu(t) + L_i\big(y(t) - \hat{y}(t)\big)\Big] \\ \hat{y}(t) &= \sum_{i=1}^{r} h_i(z(t))\Big[C_i\hat{x}(t) + D_iu(t)\Big]\end{aligned}$$

The fault sensitivity can be realized by maximizing γ_2 subject to

$$\inf_{\|f(t)\|_2 \neq 0} \frac{\|r_f(t)\|_2}{\|f(t)\|_2} \geq \gamma_2 \tag{5.20}$$

Suppose there exists a candidate quadratic Lyapunov function $V_2(e(t)) = e^T(t)Qe(t)$, $Q > 0$, and $\gamma_2 > 0$ such that, for all t

$$\dot{V}_2(e(t)) - r_f^T(t)r_f(t) + \gamma_2^2 f^T(t)f(t) \leq 0 \tag{5.21}$$

for eq. (5.19a) and eq. (5.19b). The dynamics of the state estimation error is defined as follows

$$\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\Big[\big(A_i - L_iC_j\big)e(t) + \big(E_f - L_iF_f\big)f(t)\Big] \tag{5.22}$$

$$r_f(t) = \sum_{i=1}^{r} h_i(z(t))\Big[C_ie(t) + F_ff(t)\Big] \tag{5.23}$$

By integrating eq. (5.21) from 0 to T, it is obtained

$$\int_0^T \Big(\dot{V}_2(e(t)) - r_f^T(t)r_f(t) + \gamma_2^2 f^T(t)f(t)\Big)dt \leq 0 \tag{5.24}$$

It is assumed that the initial condition for the state estimation error $e(0)$ is 0, then eq. (5.25) is obtained after the integration of eq. (5.24)

$$V_2(e(T)) + \int_0^T \Big(-r_f^T(t)r_f(t) + \gamma_2^2 f^T(t)f(t)\Big)dt \leq 0 \tag{5.25}$$

Since $V_2(e(T)) \geq 0$, this implies

$$\frac{\|r_f(t)\|_2}{\|f(t)\|_2} \geq \gamma_2$$

Therefore the $\mathcal{L}_2$ gain of the TS fuzzy model is more than γ_2. Considering the eq. (5.20), a LMI condition is derived from this equation

$$\dot{e}^T(t)Qe(t) + e^T(t)Q\dot{e}(t) - r_f^T(t)r_f(t) + \gamma_2^2 f^T(t)f(t) \leq 0 \tag{5.26}$$

For the following part, $z(t)$, $e(t)$ and $f(t)$ are expressed as z, e and f respectively.

$$\begin{aligned}
&\dot{e}^T Q e + e^T Q \dot{e} - r_f^T r_f + \gamma_2^2 f^T f \\
&\quad = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[e^T \bar{A}_{ij}^T + f^T \bar{E}_{f_i}^T\Big]Qe + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)e^T Q\Big[\bar{A}_{ij}e + \bar{E}_{f_i}f\Big] \\
&\quad\quad - \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[\big(e^T C_i^T + f^T F_f^T\big)\big(C_j e + F_f f\big)\Big] + \gamma_2^2 f^T f \\
&\quad = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\begin{bmatrix} e^T & f^T \end{bmatrix}\begin{bmatrix} \bar{A}_{ij}^T Q + Q\bar{A}_{ij} - C_i^T C_j & Q\bar{E}_{f_i} - C_i^T F_f \\ \bar{E}_{f_i}^T Q - F_f^T C_i & \gamma_2^2 I - F_f^T F_f \end{bmatrix}\begin{bmatrix} e \\ f \end{bmatrix} \leq 0
\end{aligned} \tag{5.27}$$

where

$$\bar{A}_{ij} = A_i - L_i C_j \qquad \text{and} \qquad \bar{E}_{f_i} = E_f - L_i F_f$$

The following matrix inequality is obtained from eq. (5.27)

$$\begin{bmatrix} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[\bar{A}_{ij}^T Q + Q\bar{A}_{ij} - C_i^T C_j\Big] & \sum_{i=1}^{r} h_i(z)\Big[Q\bar{E}_{f_i} - C_i^T F_f\Big] \\ \sum_{i=1}^{r} h_i(z)\Big[\bar{E}_{f_i}^T Q - F_f^T C_i\Big] & \gamma_2^2 I - F_f^T F_f \end{bmatrix} \leq 0 \tag{5.28}$$

The matrix inequality given by eq. (5.28) can be rewritten as

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\begin{bmatrix} \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z)h_j(z)\Big[\bar{A}_{ij}^T Q + Q\bar{A}_{ij} - C_i^T C_j\Big] & \sum_{i=1}^{r} h_i(z)\Big[Q\bar{E}_{f_i} - C_i^T F_f\Big] \\ \sum_{i=1}^{r} h_i(z)\Big[\bar{E}_{f_i}^T Q - F_f^T C_i\Big] & \gamma_2^2 I - F_f^T F_f \end{bmatrix} \leq 0$$

Therefore, from the above inequality

$$\begin{bmatrix} \frac{1}{2}\Big(\bar{A}_{ij}^T Q + \bar{A}_{ji}^T Q + Q\bar{A}_{ij} + Q\bar{A}_{ji} - C_i^T C_j - C_j^T C_i\Big) & \frac{1}{2}\Big(Q\bar{E}_{f_i} + Q\bar{E}_{f_j} - C_i^T F_f - C_j^T F_f\Big) \\ \frac{1}{2}\Big(\bar{E}_{f_i}^T Q + \bar{E}_{f_j}^T Q - F_f^T C_j - F_f^T C_i\Big) & \gamma_2^2 - F_f^T F_f \end{bmatrix} \leq 0$$

The fault sensitivity can be achieved by solving the following optimization problem:

Problem 5.2 *The observer gain matrices L_i that maximize γ_2 in eq. (5.20) can be obtained by solving the following maximization problem based on LMIs*

$$\textit{maximize } \gamma_2^2 \textit{ subject to } Q > 0 \textit{ and}$$

$$\begin{bmatrix} \bar{A}_{ii}^T Q + Q\bar{A}_{ii} - C_i^T C_i & Q\bar{E}_{f_{ii}} - C_i^T F_f \\ \bar{E}_{f_{ii}}^T Q - F_f^T C_i & \gamma_2^2 I - F_f^T F_f \end{bmatrix} < 0 \tag{5.29}$$

$$\begin{bmatrix} \bar{A}_{ij}^T Q + \bar{A}_{ji}^T Q + Q\bar{A}_{ij} + Q\bar{A}_{ji} - C_i^T C_j - C_j^T C_i & Q\bar{E}_{f_i} + Q\bar{E}_{f_j} - C_i^T F_f - C_j^T F_f \\ \bar{E}_{f_i}^T Q + \bar{E}_{f_j}^T Q - F_f^T C_j - F_f^T C_i & 2\gamma_2^2 I - 2F_f^T F_f \end{bmatrix} \leq 0 \quad i < j \tag{5.30}$$

5.3 Robust TS fault detection observer

The TS fault detection observer aims to solve the disturbance attenuation and the fault sensitivity problem at the same time, i.e. it is necessary to solve both optimization problems simultaneously.

They can be solved using iterative LMI schemes. In the following part is shown the generalization of two iterative LMI schemes for linear systems for its use with TS fuzzy models. The first one is taken from [79] and the second one from [81].

5.3.1 Iterative LMI scheme 1

For the TS fuzzy model in eq. (5.2) with the TS fuzzy observer in eq. (5.4), determine observer gain matrices L_i such that

1. The state estimation error in eq. (5.5a) is asymptotically stable.
2. The fault detection "disturbance-signal" gain ratio

$$J_1 = \frac{\gamma_1}{\gamma_2}$$

   is made small where $\gamma_1 > 0$, $\gamma_2 > 0$ and

$$\|r_d(t)\|_2 < \gamma_1 \|d(t)\|_2 \tag{5.31}$$

$$\|r_f(t)\|_2 > \gamma_2 \|f(t)\|_2 \tag{5.32}$$

   where $d(t)$ and $f(t)$ are non-zero.

A solution scheme that leads to LMIs is that, by setting $Q = P$ in the fault sensitivity problem 5.2 given by eq. (5.29)-(5.30), the following optimization problem can be obtained

Problem 5.3 *For given $\gamma_1 > 0$, $\gamma_2 > 0$ and F_f of full column rank, state estimation error in eq. (5.5a) is asymptotically stable and satisfies*

$$\frac{\|r_d\|_2}{\|r_f\|_2} < \frac{\gamma_1}{\gamma_2}\frac{\|d\|_2}{\|f\|_2} \tag{5.33}$$

if $P > 0$ and N_i exists such that LMIs

$$\begin{bmatrix} A_i^T P + PA_i - C_i^T N_i^T - N_i C_i + C_i^T C_i & PE_d - N_i F_d + C_i^T F_d \\ E_d^T P - F_d^T N_i^T + F_d^T C_i & -\gamma_1^2 I + F_d^T F_d \end{bmatrix} < 0 \tag{5.34}$$

$$\begin{bmatrix} A_i^T P + PA_i - C_i^T N_i^T - N_i C_i - C_i^T C_i & PE_f - N_i F_f - C_i^T F_f \\ E_f^T P - F_f^T N_i^T - F_f^T C_i & \gamma_2^2 I - F_f^T F_f \end{bmatrix} < 0 \tag{5.35}$$

$$\begin{bmatrix} \begin{bmatrix} A_i^T P + PA_i - C_j^T N_i^T - N_i C_j + C_i^T C_j + \\ A_j^T P + PA_j - C_i^T N_j^T - N_j C_i + C_j^T C_i \end{bmatrix} & \begin{bmatrix} PE_d - N_i F_d + C_i^T F_d + \\ -N_j F_d + C_j^T F_d \end{bmatrix} \\ \begin{bmatrix} E_d^T P - F_d^T N_i^T + F_d^T C_i + \\ -F_d^T N_j^T + F_d^T C_j \end{bmatrix} & -2\gamma_1^2 + 2F_d^T F_d \end{bmatrix} \leq 0 \tag{5.36}$$

$$\begin{bmatrix} \begin{bmatrix} A_i^TP + PA_i - C_j^TN_i^T - N_iC_j - C_i^TC_j + \\ A_j^TP + PA_j - C_i^TN_j^T - N_jC_i - C_j^TC_i \end{bmatrix} & \begin{bmatrix} PE_f - N_iF_f - C_i^TF_f + \\ -N_jF_f - C_j^TF_f \end{bmatrix} \\ \begin{bmatrix} E_f^TP - F_f^TN_i^T - F_f^TC_i + \\ -F_f^TN_j^T - F_f^TC_j \end{bmatrix} & 2\gamma_2^2 - 2F_f^TF_f \end{bmatrix} \leq 0 \tag{5.37}$$

hold, where $N_i = PL_i$ and $N_j = PL_j$ and gain matrices are obtained as $L_i = P^{-1}N_i$.

Based on this optimization problem, it is possible to construct an iterative LMI algorithm to obtain a TS fault detection observer, given in the following schematic form.

Algorithm 5.1 *Given system matrices A_i, B_i, E_d, E_f, C_i, D_i, F_d, F_f and let $\mu_1 \geq 0$ and $\mu_2 \geq 0$ be sufficiently small adjustable parameters. Set $k = 0$.*

Step 1. Choose a sufficiently large γ_1 and let $\gamma_2 = 0$ and solve LMIs in eq. (5.34)-(5.37) to find a feasible solution for P and N_i where $N_i = PL_i$. Compute $L_i = P^{-1}N_i$ and store it as L_{0_i}. If L_{0_i} cannot be found , then this algorithm does not give a feasible solution to the problem. STOP.

Step 2. (Main iterative steps)

(a) Put $k = k + 1$ with

$$\gamma_1 := \gamma_1 - \mu_1 > \|F_d\|,\ \gamma_2 := \gamma_2 + \mu_2 < \|F_f\|$$

Find a feasible solution for P and N_i for LMIs in eq. (5.34)-(5.37). Store $L_{i_k} = P^{-1}N_i$ and $J_k = \gamma_1/\gamma_2$. Repeat step 2(a). If a feasible solution can not be found, then $L_{i_k} = L_{i_{k-1}}$.

(b) If the performance γ_1/γ_2 is less than some desired level, then a desired observer gain $L_i = L_{i_k}$ is found. STOP.

LMIs in eq. (5.34) and eq. (5.36) are always feasible for sufficiently large $\gamma_1 > \|E_d\|$. Furthermore, the feasibility problems in step 2 are always solvable provided that step 1 is feasible and μ_1 and μ_2 are sufficiently small.

5.3.2 Iterative LMI scheme 2

For the TS fuzzy model in eq. (5.2) with the TS fuzzy observer in eq. (5.4), determine observer gain matrices L_i such that

1. The state estimation error in eq. (5.5a) is asymptotically stable.
2. The fault detection “disturbance-signal” gain ratio

$$J_1 = \frac{\gamma_1}{\gamma_2}$$

is made small where $\gamma_1 > 0$, $\gamma_2 > 0$ and

$$\|r_d(t)\|_2 < \gamma_1 \|d(t)\|_2 \tag{5.38}$$

$$\|r_f(t)\|_2 > \gamma_2 \|f(t)\|_2 \tag{5.39}$$

where $d(t)$ and $f(t)$ are non-zero.

A solution scheme that leads to LMIs is that, the solution of both optimization problems allows to obtain the following optimization problem

Problem 5.4 *For given $\gamma_1 > 0$, $\gamma_2 > 0$ and F_f of full column rank, state estimation error in eq. (5.5a) is asymptotically stable and satisfies*

$$\frac{\|r_d\|_2}{\|r_f\|_2} < \frac{\gamma_1}{\gamma_2}\frac{\|d\|_2}{\|f\|_2} \tag{5.40}$$

if $P > 0$, $Q > 0$ and L_i exists such that LMIs

$$\begin{bmatrix} A_i^T P + PA_i - C_i^T L_i^T P - PL_iC_i + C_i^T C_i & PE_d - PL_iF_d + C_i^T F_d \\ E_d^T P - F_d^T L_i^T P + F_d^T C_i & -\gamma_1^2 I + F_d^T F_d \end{bmatrix} \leq 0 \tag{5.41}$$

$$\begin{bmatrix} A_i^T Q + QA_i - C_i^T L_i^T Q - QL_iC_i - C_i^T C_i & QE_f - QL_iF_f - C_i^T F_f \\ E_f^T Q - F_f^T L_i^T Q - F_f^T C_i & \gamma_2^2 I - F_f^T F_f \end{bmatrix} \leq 0 \tag{5.42}$$

$$\begin{bmatrix} \begin{bmatrix} A_i^T P + PA_i - C_j^T L_i^T P - PL_iC_j + C_i^T C_j + \\ A_j^T P + PA_j - C_i^T L_j^T P - PL_jC_i + C_j^T C_i \end{bmatrix} & \begin{bmatrix} PE_d - PL_iF_d + C_i^T F_d + \\ -PL_jF_d + C_j^T F_d \end{bmatrix} \\ \begin{bmatrix} E_d^T P - F_d^T L_i^T P + F_d^T C_i + \\ -F_d^T L_j^T P + F_d^T C_j \end{bmatrix} & -2\gamma_1^2 + 2F_d^T F_d \end{bmatrix} \leq 0 \tag{5.43}$$

$$\begin{bmatrix} \begin{bmatrix} A_i^T Q + QA_i - C_j^T L_i^T Q - QL_iC_j - C_i^T C_j + \\ A_j^T Q + QA_j - C_i^T L_j^T Q - QL_jC_i - C_j^T C_i \end{bmatrix} & \begin{bmatrix} QE_f - QL_iF_f - C_i^T F_f + \\ -QL_jF_f - C_j^T F_f \end{bmatrix} \\ \begin{bmatrix} E_f^T Q - F_f^T L_i^T Q - F_f^T C_i + \\ -F_f^T L_j^T Q - F_f^T C_j \end{bmatrix} & 2\gamma_2^2 - 2F_f^T F_f \end{bmatrix} \leq 0 \tag{5.44}$$

hold.

Based on this optimization problem, it is possible to construct an iterative LMI algorithm to obtain a TS fault detection observer, given in the following schematic form.

Algorithm 5.2 *Given system matrices A_i, B_i, E_d, E_f, C_i, D_i, F_d, F_f and let $\mu_1 \geq 0$ and $\mu_2 \geq 0$ be sufficiently small adjustable parameters. Set $k = 0$, $l = 0$ and $m \in Z^+$ to control the number of computational loops.*

Step 1. *Choose a sufficiently large $\gamma_1 = \zeta$ and solve LMIs in eq. (5.41) and eq. (5.43) to find a feasible solution for P and N_i where $N_i = PL_i$. Compute $L_i = P^{-1}N_i$ and let $\gamma_1 = \zeta$ and $\gamma_2 = 0$.*

Step 2. *(Main iterative steps)*

(a) Substitute L_i into eq. (5.41)-(5.44) and find a feasible solution set of variables P, Q.

(b) Put $k = k + 1$. With P, Q obtained in step 2(a) and with

$$\gamma_1 := \gamma_1 - \mu_1 > \|F_d\|, \ \gamma_2 := \gamma_2 + \mu_2 < \|F_f\|$$

find a feasible solution L_i for LMIs in eq. (5.41)-(5.44). Store $L_{i_k} = L_i$ and $J_k = \gamma_1/\gamma_2$. Repeat step 2(b). If a feasible solution can not be found, then $L_{i_k} = L_{i_{k-1}}$.

(c) If the performance γ_1/γ_2 is less than some desired level, then a desired observer gain $L_i = L_{i_k}$ is found. STOP.

Step 3. *Set $l = l + 1$. If $l < m$, repeat step 2, else STOP (the feasible solution can not be found).*

Step 1 *is always feasible for sufficiently large $\gamma_1 > \|E_d\|$. Furthermore, for given P and Q, matrix inequalities in eq. (5.41)-(5.44) become LMIs and a feasible solution L_i can always be obtained provided that μ_1 and μ_2 are sufficiently small. Therefore, the feasibility problems in step 2 can always provide a local improvement through each iteration.*

5.4 Design of the threshold

After designing the TS fuzzy FDO, the remaining important task for robust fault diagnosis is the evaluation of the generated residual. One of the widely adopted approaches is to choose a threshold $J_{th} > 0$ and, based on this, use the following logical relationship for fault detection

$$\begin{array}{lll} \|r(t)\|_{2,\tau} \leq J_{th} & \Rightarrow & \text{no faults} \\ \|r(t)\|_{2,\tau} > J_{th} & \Rightarrow & \text{with faults} \quad \Rightarrow \textit{alarm} \end{array} \tag{5.45}$$

where the residual evaluation function (REF) $\|r(t)\|_{2,\tau}$ is determined by

$$\|r(t)\|_{2,\tau} = \left[\int_{t_1}^{t_2} r^T(t)r(t)dt\right]^{\frac{1}{2}}, \ \tau = t_2 - t_1 \tag{5.46}$$

$\tau \in (t_1, t_2]$ is the finite-time window. Note that the length of the time window is finite, (i.e. τ instead of ∞) because it does not make sense to detect faults over the whole time range. It is assumed that the faults could be detected, if occurred, over the finite time interval.

By selecting eq. (5.46) as the residual evaluation function results in

$$\|r(t)\|_{2,\tau} = \|r_d(t) + r_f(t)\|_{2,\tau}$$

where $r_d(t)$ and $r_f(t)$ are defined as

$$r_d(t) = r(t)|_{f(t)=0} \qquad r_f(t) = r(t)|_{d(t)=0}$$

Furthermore, the fault-free case residual evaluation function is defined as

$$\|r(t)\|_{2,\tau} \leq \|r_d(t)\|_{2,\tau} \leq J_{th,d}$$

where

$$J_{th,d} = \sup_{d \in L_2} \|r_d(t)\|_{2,\tau}$$

The threshold is selected as $J_{th} = J_{th,d}$ and $J_{th,d}$ is constant and can be evaluated off-line.

5.5 An application example

A nonlinear system [77] is used to implement the TS fault detection observer, the nonlinear system is described by

$$\begin{aligned}
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \begin{bmatrix} -x_1(t) + x_1(t)x_2^3(t) \\ -x_2(t) + \big(3 + x_2(t)\big)x_1^3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.8 \\ -2.4 \end{bmatrix} d(t) + \begin{bmatrix} 4 \\ 4 \end{bmatrix} f(t) \\
\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} &= \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} d(t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} f(t)
\end{aligned}$$

it is considered that $x_1(t) \in [-1, 1]$ and $x_2(t) \in [-1, 1]$. The above system can be written as

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -1 & x_1(t)x_2^2(t) \\ \big(3 + x_2(t)\big)x_1^2(t) & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.8 \\ -2.4 \end{bmatrix} d(t) + \begin{bmatrix} 4 \\ 4 \end{bmatrix} f(t) \\
y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} d(t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} f(t)
\end{aligned}$$

where $x_1(t)x_2^2(t)$ and $\big(3 + x_2(t)\big)x_1^2(t)$ are nonlinear terms. For the nonlinear terms are defined $z_1(t) = x_1(t)x_2^2(t)$ and $z_2(t) = \big(3 + x_2(t)\big)x_1^2(t)$ as premise variables. It follows

$$\begin{aligned}
\dot{x}(t) &= \begin{bmatrix} -1 & z_1(t) \\ z_2(t) & -1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} 0.8 \\ -2.4 \end{bmatrix} d(t) + \begin{bmatrix} 4 \\ 4 \end{bmatrix} f(t) \\
y(t) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix} d(t) + \begin{bmatrix} 2 \\ -1 \end{bmatrix} f(t)
\end{aligned}$$

Next, calculate the minimum and maximum values of $z_1(t)$ and $z_2(t)$:

$$\max_{z_1(t),z_2(t)} z_1(t) = z_1^+(t) = 1 \qquad \max_{x_1(t),x_2(t)} z_2(t) = z_2^+(t) = 4$$
$$\min_{z_1(t),z_2(t)} z_1(t) = z_1^-(t) = -1 \qquad \min_{x_1(t),x_2(t)} z_2(t) = z_2^-(t) = 0$$

From the maximum and minimum values of $z_1(t)$ and $z_2(t)$

$$\begin{aligned} z_1(t) &= x_1(t)x_2^2(t) = F_{11}(z_1(t)) \cdot 1 + F_{12}(z_1(t)) \cdot -1 \\ z_2(t) &= \big(3 + x_2(t)\big)x_1^2(t) = F_{21}(z_2(t)) \cdot 4 + F_{22}(z_2(t)) \cdot 0 \end{aligned}$$

where

$$F_{11}(z_1(t)) + F_{12}(z_1(t)) = 1 \qquad \text{and} \qquad F_{21}(z_2(t)) + F_{22}(z_2(t)) = 1$$

The membership functions can be calculated as:

$$F_{11}(z_1(t)) = \frac{z_1(t)+1}{2} \qquad F_{12}(z_1(t)) = \frac{1-z_1(t)}{2}$$
$$F_{21}(z_2(t)) = \frac{z_2(t)}{4} \qquad F_{22}(z_2(t)) = \frac{4-z_2(t)}{4}$$

The membership functions are named “Positive”, “Negative”, “Big” and “Small”, respectively. Then, the nonlinear system is approximated by the following TS fuzzy model:

<u>**Model rule** 1</u>

IF $z_1(t)$ is “Positive” and $z_2(t)$ is “Big”

$$\text{THEN} \begin{cases} \dot{x}(t) = & A_1x(t) + B_1u(t) + E_dd(t) + E_ff(t) \\ y(t) = & C_1x(t) + F_dd(t) + F_ff(t) \end{cases}$$

<u>**Model rule** 2</u>

IF $z_1(t)$ is “Positive” and $z_2(t)$ is “Small”

$$\text{THEN} \begin{cases} \dot{x}(t) = & A_2x(t) + B_2u(t) + E_dd(t) + E_ff(t) \\ y(t) = & C_2x(t) + F_dd(t) + F_ff(t) \end{cases}$$

<u>**Model rule** 3</u>

IF $z_1(t)$ is “Negative” and $z_2(t)$ is “Big”

$$\text{THEN} \begin{cases} \dot{x}(t) = & A_3x(t) + B_3u(t) + E_dd(t) + E_ff(t) \\ y(t) = & C_3x(t) + F_dd(t) + F_ff(t) \end{cases}$$

<u>**Model rule** 4</u>

IF $z_1(t)$ is “Negative” and $z_2(t)$ is “Small”

$$\text{THEN} \begin{cases} \dot{x}(t) = & A_4x(t) + B_4u(t) + E_dd(t) + E_ff(t) \\ y(t) = & C_4x(t) + F_dd(t) + F_ff(t) \end{cases}$$

Here

$$A_1 = \begin{bmatrix} -1 & 1 \\ 4 & -1 \end{bmatrix}, \ A_2 = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}, \ A_3 = \begin{bmatrix} -1 & -1 \\ 4 & -1 \end{bmatrix}, \ A_4 = \begin{bmatrix} -1 & -1 \\ 0 & -1 \end{bmatrix}$$

$$B_{1,2,3,4} = \begin{bmatrix} 1 \\ 0.01 \end{bmatrix}, \ E_d = \begin{bmatrix} 1 \\ -2.5 \end{bmatrix}, \ E_f = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

$$C_{1,2,3,4} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \ F_d = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, \ F_f = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

The defuzzification (that give the TS fuzzy model) is carried out as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{4} h_i(z(t))\Big[A_i x(t) + B_i u(t) + E_d d(t) + E_f f(t)\Big] \\ y(t) &= \sum_{i=1}^{4} h_i(z(t))\Big[C_i x(t) + F_d d(t) + F_f f(t)\Big] \end{aligned}$$

where

$$\begin{aligned} h_1(z(t)) &= F_{11}(z_1(t)) \times F_{21}(z_2(t)) \\ h_2(z(t)) &= F_{11}(z_1(t)) \times F_{22}(z_2(t)) \\ h_3(z(t)) &= F_{12}(z_1(t)) \times F_{21}(z_2(t)) \\ h_4(z(t)) &= F_{12}(z_1(t)) \times F_{22}(z_2(t)) \end{aligned}$$

For the above example, the TS fault detection observer is applied. The system was simulated with a disturbance

$$d(t) = 0.3 \cos(2t) e^{-0.2t} \tag{5.47}$$

and an actuator fault $f(t)$ such that

$$f(t) = \begin{cases} -0.08 & 5 \leq t \leq 10 \\ 0 & \text{elsewhere.} \end{cases} \tag{5.48}$$

In fig. 5.1 and fig. 5.2 are shown the simulated disturbance and the actuator fault respectively.

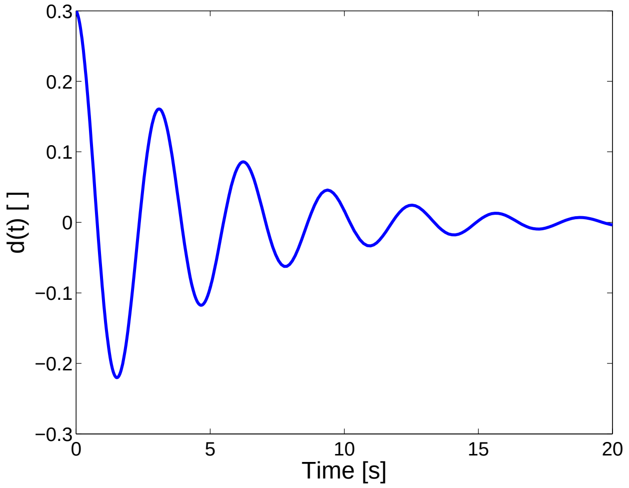

Fig. 5.1: Disturbance signal

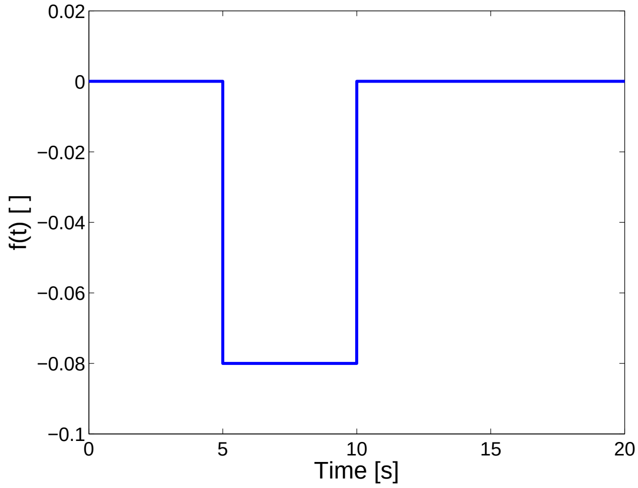

Fig. 5.2: Actuator fault signal

5.5.1 Iterative LMI scheme 1

A numerical simulation for the iterative algorithm 1 was carried out using LMI tools from MATLAB 7.0, where $\gamma_1 = 0.762$ and $\gamma_2 = 2.183$ so that $J = \gamma_1/\gamma_2 = 0.349$ was achieved. The following gain matrices L_i were obtained

$$L_1 = \begin{bmatrix} -635.96 & -839.05 \\ 2501.8 & 3289.3 \end{bmatrix} \qquad L_2 = \begin{bmatrix} -623.87 & -842.51 \\ 2454.4 & 3302.9 \end{bmatrix}$$
$$L_3 = \begin{bmatrix} -658.4 & -963.62 \\ 2590.1 & 3778.3 \end{bmatrix} \qquad L_4 = \begin{bmatrix} -696.3 & -918.3 \\ 2738.5 & 3600.2 \end{bmatrix}$$

Fig. 5.3 shows a residual signal designed with a TS fuzzy observer that aims only to make the disturbance attenuation and, a residual signal design with a TS fuzzy observer, that realizes the fault sensitivity is shown in fig. 5.4.

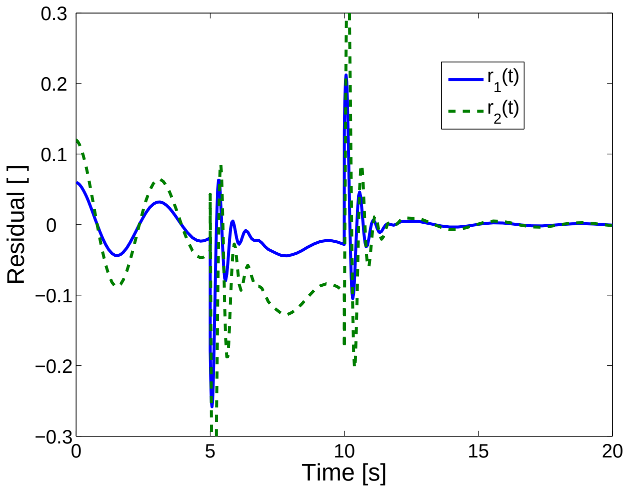

Fig. 5.3: Disturbance attenuation

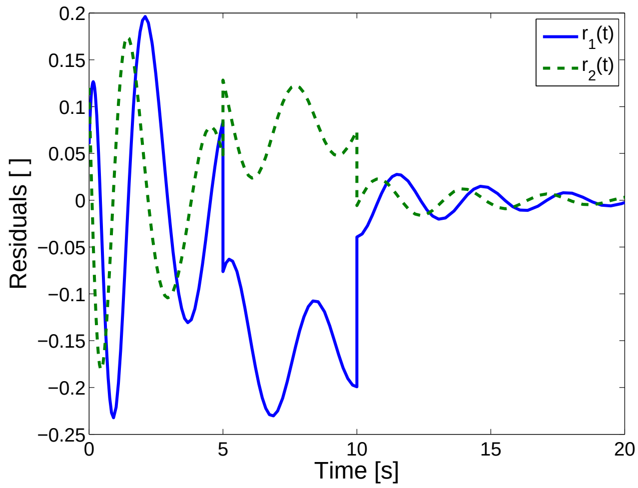

Fig. 5.4: Fault sensitivity

As can be seen from fig. 5.3, in the presence of faults and disturbances in the system, the TS fuzzy observer can not detect the fault. In the case for a TS fuzzy observer that aims to achieve only teh fault sensitivity, the effect of disturbances is difficult to differenciate from the fault in fig. 5.4.

The residual signal generated with a TS fuzzy fault detection observer for iterative LMI scheme 1 is shown in fig. 5.5

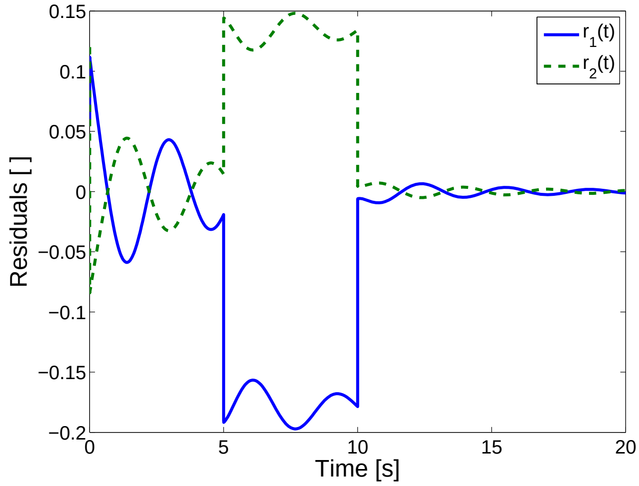

Fig. 5.5: TS fault detection observer for the iterative LMI scheme 1

In fig. 5.5 a desirable fault detection behavior is achieved, i.e. despite the influence of an unknown input, it is much easier to detect faults in comparison with the separated objectives in fig. 5.3 and fig. 5.4. And for the design of the threshold was obtained $J_{th_d} = 0.1088$

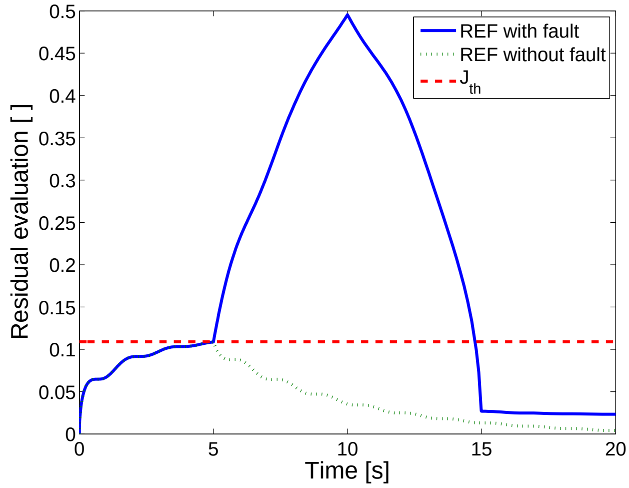

Fig. 5.6: Residual evaluation for the iterative LMI scheme 1

Using the threshold for the evaluated residual allows to detect the fault in fig. 5.6 at 5 s.

5.5.2 Iterative LMI scheme 2

A numerical simulation for the iterative algorithm 2 was carried out using LMI tools from MATLAB 7.0, where $\gamma_1 = 0.671$ and $\gamma_2 = 1.595$ so that $J = \gamma_1/\gamma_2 = 0.4207$ was achieved. The following gain matrices L_i were obtained

$$L_1 = \begin{bmatrix} 1.8993 & -0.3783 \\ -2.8515 & 8.1397 \end{bmatrix} \qquad L_2 = \begin{bmatrix} 2.1667 & -0.6479 \\ -5.6639 & 9.9531 \end{bmatrix}$$
$$L_3 = \begin{bmatrix} 8.5931 & -5.2915 \\ -22.547 & 22.68 \end{bmatrix} \qquad L_4 = \begin{bmatrix} 1.5492 & -0.8554 \\ -8.6154 & 11.789 \end{bmatrix}$$

The residual signal for iterative LMI scheme 2 is shown in fig. 5.7

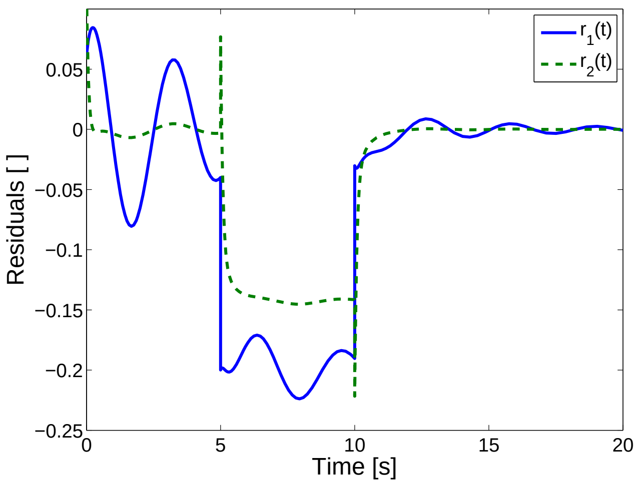

Fig. 5.7: TS fault detection observer for the iterative LMI scheme 2

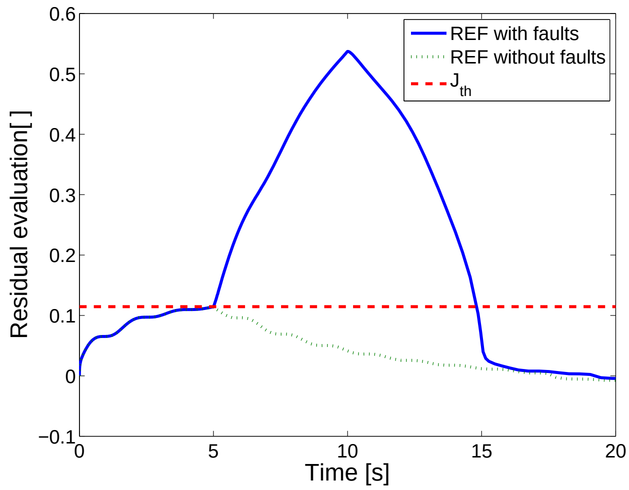

Fig. 5.8: Residual evaluation for the iterative LMI scheme 2

In fig. 5.7 a desirable fault detection behavior is achieved, i.e. despite the influence of an unknown input, it is easier to detect faults. For the design of the threshold was obtained $J_{th_d} = 0.1145$. The fault in fig. 5.8 can be detected at 5 s.

Chapter 6

Fault diagnosis for systems with polytopic uncertainties

A nonlinear system can be represented by a linearization around some operating points, in this form, a linear model for the nonlinear system is obtained. Through this linearization, part of the dynamic of the nonlinear system is not considered due to assumptions that are necessary to make in order to linearize the nonlinear system.

The use of polytopic uncertainty allows to use the unmodeled dynamic into the linear model. That means, the design of the residual generator will contain more information about the nonlinear system thanks to the polytopic uncertainty and therefore the performance of the residual generator will be improved.

6.1 Problem formulation

Linear systems that consider polytopic uncertainties are normally described by the following state space representation:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + (B + \Delta B)u(t) + (E_d + \Delta E_d)d(t) + E_f f(t) \\ y(t) &= (C + \Delta C)x(t) + (D + \Delta D)u(t) + (F_d + \Delta F_d)d(t) + F_f f(t) \end{aligned} \tag{6.1}$$

where polytopic uncertainties are defined as:

$$\begin{bmatrix} \Delta A & \Delta B & \Delta E_d \\ \Delta C & \Delta D & \Delta F_d \end{bmatrix} = \sum_{i=1}^{l} \beta_i \begin{bmatrix} A_i & B_i & E_{d_i} \\ C_i & D_i & F_{d_i} \end{bmatrix}$$
$$\sum_{i=1}^{l} \beta_i = 1,\ \beta_i \geq 0,\ i = 1, \ldots, l.$$

and $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^{k_u}$ is the input vector, $d(t) \in \mathbb{R}^{k_d}$ is the disturbance vector, $f(t) \in \mathbb{R}^{k_f}$ is the fault vector and $y(t) \in \mathbb{R}^m$ is the measurement or output vector. A, B, E_d, E_f, C, D, F_d,F_f and the matrices for the polytopic uncertainty are known system matrices with appropriate dimensions.

The dynamic of a residual generator using FDF theory, and for systems with polytopic uncertainties can be described by:

$$\begin{aligned} \begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} &= \begin{bmatrix} A + \Delta A & 0 \\ \Delta A - L\Delta C & A - LC \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B + \Delta B \\ \Delta B - L\Delta D \end{bmatrix} u(t) + \begin{bmatrix} E_d + \Delta E_d \\ (E_d + \Delta E_d) - L(F_d + \Delta F_d) \end{bmatrix} d(t) + \begin{bmatrix} E_f \\ E_f - LF_f \end{bmatrix} f(t) \\ r(t) &= V \left(\begin{bmatrix} \Delta C & C \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \Delta D u(t) + (F_d + \Delta F_d)d(t) + F_f f(t) \right) \end{aligned}$$

where the matrix L is called the observer gain matrix, and the matrix V is a post-filter. In order to compute matrices L and V, it is used a reference residual model together with the above dynamic equation.

A reference residual model is an ideal solution for robust FDI under the assumption that no disturbance or model uncertainty are present on the system [17, 32, 52, 86]. In such a form, that an augmented system is obtained, where the dynamic of the reference model together with the dynamic of the FDF is considered.

6.1.1 Reference residual model

The reference model is made under the assumption that there is no model uncertainty apart from disturbances affecting the system. The basic idea behind such a reference model is the trade-off between the robustness and fault detectability. The unified solution in [17], due to its optimal trade-off, is adopted as reference model.

Consider the following linear system, which has no affectation of polytopic uncertainty and is described by

$$\dot{x}(t) = Ax(t) + Bu(t) + E_d d(t) + E_f f(t) \tag{6.2a}$$
$$y(t) = Cx(t) + Du(t) + F_d d(t) + F_f f(t) \tag{6.2b}$$

A FDF in its state space representation form is given by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L_{opt}(y(t) - \hat{y}(t)) \tag{6.3a}$$
$$\hat{y}(t) = C\hat{x}(t) + Du(t) \tag{6.3b}$$
$$r(t) = V_{opt}(y(t) - \hat{y}(t)) \tag{6.3c}$$

The dynamics of the FDF in the frequency domain is described by

$$r(s) = \hat{N}_d(s)d(s) + \hat{N}_f(s)f(s) \tag{6.4}$$
$$\hat{N}_d(s) = V_{opt}\Big(F_d + C(sI - A + L_{opt}C)^{-1}(E_d - L_{opt}F_d)\Big) \tag{6.5}$$
$$\hat{N}_f(s) = V_{opt}\Big((F_f + C(sI - A + L_{opt}C)^{-1}(E_f - L_{opt}F_f)\Big) \tag{6.6}$$

The main objective is to find an observer gain matrix L_{opt} and matrix V_{opt} such that the FDF is stable and the robustness of $r(s)$ against $d(s)$ and the sensitivity of $r(s)$ against $f(s)$ are enhanced at the same time. The unified solution is given by the following theorem from [17, 20]

Theorem 6.1 *(the unified solution): Given the system described by eq. (6.2a)-(6.2b) and suppose that the following assumptions are fulfilled*

A1. The pair (C, A) is detectable;

A2. The matrix F_d has full row rank with $F_d F_d^T = I$;

A3. rank $\begin{bmatrix} A - jwI & E_d \\ C & F_d \end{bmatrix} = n + m,$

then, the unified solution

$$L_{opt} = (E_d F_d^T + YC^T)(F_d F_d^T)^{-1}, \qquad V_{opt} = (F_d F_d^T)^{-\frac{1}{2}} \tag{6.7}$$

with $Y \geq 0$ *as the stabilizing solution to the following Riccati equation*

$$AY + YA^T + E_d E_d^T - \left(E_d F_d^T + YC^T\right)\left(F_d F_d^T\right)^{-1}\left(F_d E_d^T + CY\right) = 0 \tag{6.8}$$

delivers an optimal FDF in the sense of $\forall w,\ \sigma_i(\hat{N}_f(jw)), i = 1, \cdots, k_f$

$$\sup_{L_{opt}, V_{opt}} \frac{\sigma_i(\hat{N}_f(jw))}{\|\hat{N}_d(s)\|_\infty} = \sigma_i(\hat{N}_{f,opt}(jw)) \tag{6.9}$$

with

$$\hat{N}_{f,opt}(s) = V_{opt}\Big(F_f + C(sI - A + L_{opt}C)^{-1}(E_f - L_{opt}F_f)\Big)$$

The reference residual model, obtained from the unified solution [17], is shown below:

$$\begin{aligned} \dot{x}_{ref}(t) &= A_{ref}x_{ref}(t) + E_{f_{ref}}f(t) + E_{d_{ref}}d(t) \\ r_{ref}(t) &= C_{ref}x_{ref}(t) + F_{f_{ref}}f(t) + F_{d_{ref}}d(t) \end{aligned} \tag{6.10}$$

where

$$\begin{array}{lll} A_{ref} = A - L_{opt}C, & E_{f_{ref}} = E_f - L_{opt}F_f, & E_{d_{ref}} = E_d - L_{opt}F_d \\ C_{ref} = V_{opt}C, & F_{f_{ref}} = V_{opt}F_f, & F_{d_{ref}} = V_{opt}F_d. \end{array}$$

6.1.2 Design of the augmented system

The augmented system given in eq. (6.11) includes the dynamics of the FDF for systems with polytopic uncertainties, and the dynamics of the reference residual model.

$$\begin{aligned} \dot{x}_o(t) &= (A_o + \Delta A_o)x_o(t) + (E_{o_d} + \Delta E_{o_d})\bar{d}(t) \\ r_{ref}(t) - r(t) &= (C_o + \Delta C_o)x_o(t) + (F_{o_d} + \Delta F_{o_d})\bar{d}(t) \end{aligned} \tag{6.11}$$

with

$$\begin{aligned} x_o(t) &= \begin{bmatrix} x_{ref}(t) \\ x(t) \\ e(t) \end{bmatrix},\ \bar{d}(t) = \begin{bmatrix} u(t) \\ d(t) \\ f(t) \end{bmatrix},\ A_o = \begin{bmatrix} A_{ref} & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A - LC \end{bmatrix},\ C_o = \begin{bmatrix} C_{ref} & 0 & -VC \end{bmatrix} \\ E_{o_{\bar{d}}} &= \begin{bmatrix} 0 & E_{d_{ref}} & E_{f_{ref}} \\ B & E_d & E_f \\ 0 & E_d - LF_d & E_f - LF_f \end{bmatrix},\ F_{o_{\bar{d}}} = \begin{bmatrix} 0 & F_{d_{ref}} - VF_d & F_{f_{ref}} - VF_f \end{bmatrix} \\ \Delta A_o &= \sum_{i=1}^{l} \beta_i \bar{A}_i,\ \bar{A}_i = \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_i & 0 \\ 0 & A_i - LC_i & 0 \end{bmatrix},\ \Delta C_o = \sum_{i=1}^{l} \beta_i \bar{C}_i,\ \bar{C}_i = -\begin{bmatrix} 0 & VC_i & 0 \end{bmatrix} \\ \Delta E_{o_d} &= \sum_{i=1}^{l} \beta_i \bar{E}_i,\ \bar{E}_i = \begin{bmatrix} 0 & 0 & 0 \\ B_i & E_{d_i} & 0 \\ B_i - LD_i & E_{d_i} - LF_{d_i} & 0 \end{bmatrix},\ \Delta F_{o_d} = \sum_{i=1}^{l} \beta_i \bar{F}_i,\ \bar{F}_i = \begin{bmatrix} -VD_i & -VF_{d_i} & 0 \end{bmatrix} \end{aligned}$$

The residual generator design is formulated as

Find matrices $L,\ V$ such that $\gamma > 0$ is minimized, where γ is given by

$$\int_0^\infty (r_{ref}(t) - r(t))^T (r_{ref}(t) - r(t))dt < \gamma^2 \int_0^\infty \bar{d}^T(t)\bar{d}(t)dt \qquad (6.12)$$

The optimization problem given by eq. (6.12) as

$$\min_{L,V} \gamma \text{ subject to}$$

$$\begin{bmatrix} (A_o + \bar{A}_i)^T P + P(A_o + \bar{A}_i) & P(E_{o_{\bar{d}}} + \bar{E}_i) & (C_o + \bar{C}_i)^T \\ (E_{o_{\bar{d}}} + \bar{E}_i)^T P & -\gamma I & (F_{o_{\bar{d}}} + \bar{F}_i)^T \\ (C_o + \bar{C}_i) & (F_{o_{\bar{d}}} + \bar{F}_i) & -\gamma I \end{bmatrix} < 0 \qquad (6.13)$$

For some $P > 0$. In order to solve the optimization problem given by eq. (6.13), let

$$P = \begin{bmatrix} P_{11} & P_{12} & 0 \\ P_{21} & P_{22} & 0 \\ 0 & 0 & P_{33} \end{bmatrix} > 0,\ L = P_{33}^{-1}Y \qquad (6.14)$$

then the eq. (6.13) becomes a LMI regarding to matrices $P > 0,\ V$ and Y, as described by

$$N_i = N_i^T = [N_{jk}]_{7\times 7} < 0,\ i = 1, ..., l \qquad (6.15)$$

where

$$N_{11} = \begin{bmatrix} A_{ref} & 0 \\ 0 & A + A_i \end{bmatrix}^T \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} A_{ref} & 0 \\ 0 & A + A_i \end{bmatrix},\ N_{12} = \begin{bmatrix} 0 \\ A_i^T P_{33} - C_i^T Y^T \end{bmatrix}$$

$$N_{13} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} 0 \\ B + B_i \end{bmatrix},\ N_{14} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} E_{d_{ref}} \\ E_d + E_{d_i} \end{bmatrix},\ N_{15} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} E_{f_{ref}} \\ E_f \end{bmatrix}$$

$$N_{16} = \begin{bmatrix} C_{ref}^T \\ -C_i^T V^T \end{bmatrix},\ N_{22} = A^T P_{33} - C^T Y^T + P_{33}A - YC,\ N_{23} = P_{33}B_i - YD_i$$

$$N_{24} = P_{33}(E_d + E_{d_i}) - Y(F_d + F_{d_i}),\ N_{25} = P_{33}E_f - YF_f,\ N_{26} = -C^T V^T$$

$$N_{33} = -\gamma I,\ N_{34} = 0,\ N_{35} = 0,\ N_{36} = -D_i^T V^T,\ N_{44} = -\gamma I,\ N_{45} = 0$$

$$N_{46} = F_{d_{ref}}^T - (F_d + F_{d_i})^T V^T,\ N_{55} = -\gamma I,\ N_{56} = F_{f_{ref}}^T - F_f^T V^T,\ N_{66} = -\gamma I$$

Based on this result, the optimal design of residual generators for systems with polytopic uncertainties can be achieved using the following algorithm

Algorithm 6.1 *[17]: LMI solution of eq. (6.12)*

Step 1. Form a matrix $N_i = [N_{jk}]_{7\times 7} < 0,\ i = 1, ..., l$

Step 2. Given $\gamma > 0$*, find* $P > 0$ *,* Y *and* V *so that*
$N_i < 0.$

Step 3. Decrease γ *and repeat step 2 until the tolerance value for the LMI algorithm is reached.*

Step 4. Set L *according to eq. (6.14).*

6.2 Threshold computation

Once the residual generator is obtained, the next task is to design a threshold in order to evaluate the residual signal. For this purpose, consider the linear system with polytopic uncertainties, disturbances and faults described by

$$\dot{x}_r(t) = (A_r + \Delta A_r)x_r(t) + (E_{r_d} + \Delta E_r)d_r(t) + E_{r_f}f(t) \tag{6.16a}$$

$$r(t) = (C_r + \Delta C_r)x_r(t) + (F_{r_d} + \Delta F_r)d_r(t) + F_{r_f}f(t) \tag{6.16b}$$

where

$$\begin{aligned}
x_r(t) &= \begin{bmatrix} x(t) \\ e(t) \end{bmatrix},\ d_r(t) = \begin{bmatrix} u(t) \\ d(t) \end{bmatrix},\ A_r = \begin{bmatrix} A & 0 \\ 0 & A-LC \end{bmatrix},\ C_r = \begin{bmatrix} 0 & C \end{bmatrix} \\
E_{r_d} &= \begin{bmatrix} B & E_d \\ 0 & E_d - LF_d \end{bmatrix},\ E_{r_f} = \begin{bmatrix} E_f \\ E_f - LF_f \end{bmatrix},\ F_{r_d} = \begin{bmatrix} 0 & F_d \end{bmatrix},\ F_{r_f} = F_f \\
\Delta A_r &= \sum_{i=1}^{l} \beta_i A_{r_i},\ A_{r_i} = \begin{bmatrix} A_i & 0 \\ A_i - LC_i & 0 \end{bmatrix},\ \Delta C_r = \sum_{i=1}^{l} \beta_i C_{r_i},\ C_{r_i} = \begin{bmatrix} C_i & 0 \end{bmatrix} \\
\Delta E_r &= \sum_{i=1}^{l} \beta_i E_{r_i},\ E_{r_i} = \begin{bmatrix} B_i & E_{d_i} \\ B_i - LD_i & E_{d_i} - LF_{d_i} \end{bmatrix},\ \Delta F_r = \sum_{i=1}^{l} \beta_i F_{r_i},\ F_{r_i} = \begin{bmatrix} D_i & F_{d_i} \end{bmatrix}
\end{aligned}$$

where the matrix L is the one obtained by solving the optimization problem in eq. (6.13).

Theorem 6.2 *[17] Given system in eq. (6.16) considering the polytopic uncertainties and $\gamma > 0$, and suppose that $x_r(0) = 0$, then*

$$\|r(t)\|_2 < \gamma \|d_r(t)\|_2 \tag{6.17}$$

if there exists $P > 0$ so that $\forall i = 1, \ldots, l$,

$$\begin{bmatrix} (A_r + A_{r_i})^T P + P(A_r + A_{r_i}) & P(E_{r_d} + E_{r_i}) & (C_r + C_{r_i})^T \\ (E_{r_d} + E_{r_i})^T P & -\gamma I & (F_{r_d} + F_{r_i})^T \\ (C_r + C_{r_i}) & (F_{r_d} + F_{r_i}) & -\gamma I \end{bmatrix} < 0 \tag{6.18}$$

setting the matrix P as

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0 \tag{6.19}$$

yields

$$eq.\ (6.18) \Longleftrightarrow N_i = N_i^T = [N_{jk}]_{5\times 5} < 0,\ i = 1, ..., l \tag{6.20}$$

with

$$\begin{aligned}
&N_{11} = \left(A^T + A_i^T\right) P_1 + P_1 \left(A + A_i\right),\ N_{12} = A_i^T P_2 - C_i^T L^T P_2,\ N_{13} = P_1 \left(B + B_i\right) \\
&N_{14} = P_1 \left(E_d + E_{d_i}\right),\ N_{15} = C_i^T,\ N_{22} = A^T P_2 - C^T L^T P_2 + P_2 A - P_2 LC \\
&N_{23} = P_2 B_i - P_2 LD_i,\ N_{24} = P_2(E_d + E_{d_i}) - P_2 L(F_d + F_{d_i}),\ N_{25} = C^T \\
&N_{33} = -\gamma I,\ N_{34} = 0,\ N_{35} = D_i^T,\ N_{44} = -\gamma I,\ N_{45} = F_d^T + F_{d_i}^T,\ N_{55} = -\gamma I
\end{aligned}$$

Suppose that $d_r(t)$ is bounded by and in the sense of $\|d_r(t)\|_2 \leq \delta_{u,2} + \delta_{d,2}$. The root mean square (RMS) value of the residual r is defined by

$$\|r(t)\|_{RMS} = \left(\frac{1}{T} \int_{t}^{t+T} \|r(\tau)\|^2 d\tau \right)^{1/2} \tag{6.21}$$

$\|r(t)\|_{RMS}$ calculates the average energy of r over the time interval $(t, t+\tau)$. The RMS of a signal is related to its $\mathcal{L}_2$ norm. In fact, it holds

$$\|r(t)\|_{RMS} \leq \frac{1}{\sqrt{T}} \|r(t)\|_2 \tag{6.22}$$

Define

$$J_{th,RMS} = \sup_{fault-free} \|r(t)\|_{RMS} \tag{6.23}$$

as the threshold, then the detection logic becomes

$$\begin{aligned} \|r(t)\|_{RMS} &\leq J_{th,RMS} \Rightarrow \text{no alarm, fault-free} \\ \|r(t)\|_{RMS} &> J_{th,RMS} \Rightarrow \text{alarm, a fault is detected} \end{aligned}$$

Based on the Theorem 6.2 as well as the relation between the $\mathcal{L}_2$ norm and the RMS eq. (6.22), the following algorithm can be formulated:

Algorithm 6.2 *[17]: Computation of $J_{th,RMS,2}$ for systems with polytopic uncertainties*

Step 1. Solve the optimization problem

$$\textit{min } \gamma \textit{ subject to eq. (6.18).}$$

for $P > 0$ and set $\gamma^* = arg(min\ \gamma)$

Step 2. Set $J_{th,RMS,2} = \frac{\gamma^*(\delta_{d,2}+\delta_{u,2})}{\sqrt{T}}$

6.3 Application to the aileron positioning system

The mathematical model of a civil aircraft primary flight control actuation system (Aileron positioning system) has been often discussed [5, 53, 78] as challenge to design FDI strategies.

6.3.1 Nonlinear model of the APS

The actuation system in an active-standby configuration behaves no linear [65]. Its dynamics is represented in the block diagram of the fig. 6.1.

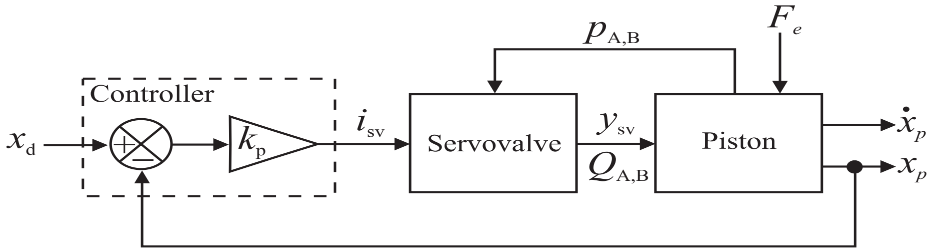

Fig. 6.1: Block diagram of the actuation system

6.3.1.1 Electrohydraulic Servovalve

The modeled servovalve is formed by two stages, to transform the electric input signal in a hydraulic output signal. The first stage transforms the current i_{sv} received from the ACE into a spool displacement y_{sv} and its mathematical model is represented by a second order differential equation

$$\ddot{y}_{sv} + 2\delta_{sv}\omega_{sv}\dot{y}_{sv} + \omega_{sv}^2 y_{sv} = k_{sv}\omega_{sv}^2 i_{sv} \tag{6.24}$$

with δ_{sv} as damping coefficient, ω_{sv} as natural frequency and k_{sv} as the servovalve gain and $\dot{y}_{sv}$ and $\ddot{y}_{sv}$ are the servovalve spool velocity and acceleration respectively.

The second stage is formed by a spool-sleeve assembly (fig. 6.2) with ideal zero-lapped control edges which, with the aid of the supply pressure p_S, the tank pressure p_T, the direction of the spool movement y_{sv} and the pressures generated in the piston p_A and p_B, generate the flow rates Q_A and Q_B which move the piston.

$$Q_1 = \begin{cases} B_{sv}|y_{sv}|\sqrt{|p_S - p_A|}\text{sign}(p_S - p_A) & \text{for } y_{sv} > 0 \\ 0 & \text{for } y_{sv} \leq 0 \end{cases} \tag{6.25}$$

$$Q_2 = \begin{cases} B_{sv}|y_{sv}|\sqrt{|p_A - p_T|}\text{sign}(p_A - p_T) & \text{for } y_{sv} < 0 \\ 0 & \text{for } y_{sv} \geq 0 \end{cases} \tag{6.26}$$

$$Q_3 = \begin{cases} B_{sv}|y_{sv}|\sqrt{|p_S - p_B|}\text{sign}(p_S - p_B) & \text{for } y_{sv} > 0 \\ 0 & \text{for } y_{sv} \leq 0 \end{cases} \tag{6.27}$$

$$Q_4 = \begin{cases} B_{sv}|y_{sv}|\sqrt{|p_B - p_T|}\text{sign}(p_B - p_T) & \text{for } y_{sv} < 0 \\ 0 & \text{for } y_{sv} \geq 0 \end{cases} \tag{6.28}$$

with B_{sv} as the servovalve orifice constant. The system pressure $p_P = p_S - p_T$.

$$B_{sv} = \alpha_D \pi \mathrm{d}\sqrt{\frac{2}{\rho}} \tag{6.29}$$

where α_D is the flow rate coefficient, πd is the control edge length and ρ is the density of the hydraulic fluid.

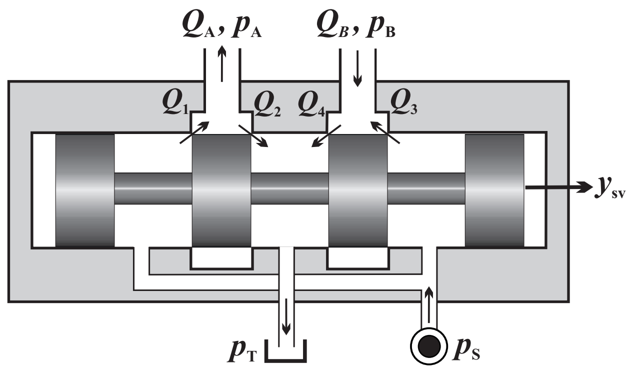

Fig. 6.2: Servovalve spool-sleeve assembly

The flows Q_A and Q_B, going to the cylinder chambers A and B fig. 6.2, are calculated by:

$$Q_A = Q_1 - Q_2, \tag{6.30}$$

$$Q_B = Q_4 - Q_3. \tag{6.31}$$

The sign function is described by:

$$sign(\varepsilon) = \begin{cases} -1 & \text{for } \varepsilon < 0 \\ 0 & \text{for } \varepsilon = 0 \\ 0 & \text{for } \varepsilon > 0 \end{cases} \tag{6.32}$$

6.3.1.2 Cylinder dynamics

The pressure in the chamber of the cylinders in the *active* mode depends on the applied volume flow Q_A and Q_B, on the external loads and on the movement in the piston. The movement of the piston in *standby* mode have effect through the volume flow of the damping force.

The generation of the pressure in the active cylinder, without consider the internal leaks, is described in the following continuity equations:

$$\dot{p}_A = E \frac{Q_A - A_p \dot{x}_p}{V_D + \mid A_p x_{min} \mid + A_p x_p} \tag{6.33}$$

$$\dot{p}_B = E \frac{A_p \dot{x}_p - Q_B}{V_D + \mid A_p x_{max} \mid - A_p x_p} \tag{6.34}$$

where E is the oil bulk modulus, V_D is the dead volume of the cylinder, Q_A and Q_B are the flow rates in the control edges, $\dot{x}_p$ is the piston speed, x_p is the piston position and A_P is the piston area, p_A and p_B are the pressure generated in the chambers A and B.

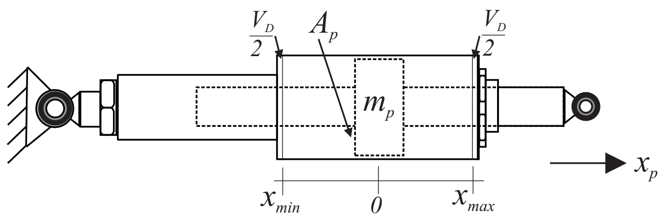

Fig. 6.3: Cylinder

Under consideration of rigid fixation [36, 45], the Newton movement equation of the piston is given by eq. (6.35).

$$m_p \ddot{x}_p = A_p(p_A - p_B) - F_f - F_d - F_a \tag{6.35}$$

with m_p as the piston mass, F_f are the friction forces, F_d the force of the effect reflected in the active actuator caused by the parallel actuator in damping mode and F_e represents the external forces affecting the control surface.

The friction forces F_f can be modeled according to the Stribeck-curve [43]. The curve is described by the superposition of three friction parts, static friction (f_e), dynamic friction (f_d) and viscose friction (f_v), shown in fig. 6.4.

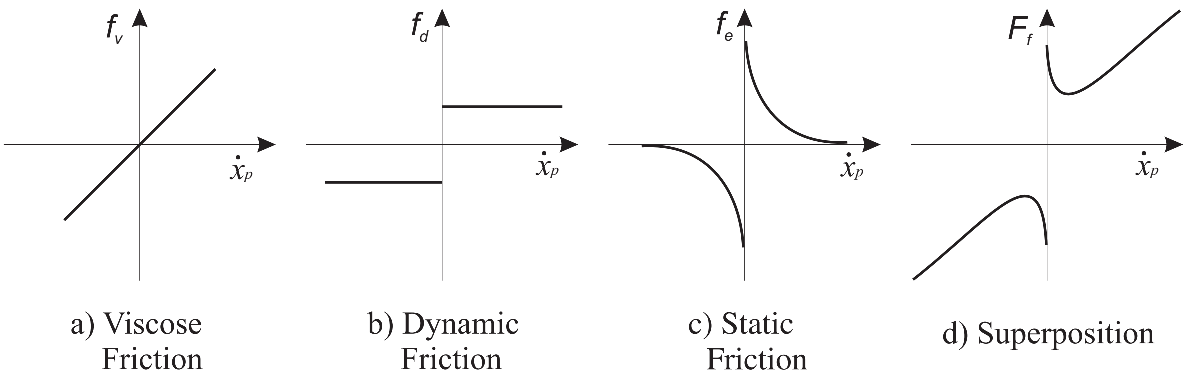

Fig. 6.4: Types of friction

The following equation is obtained from the friction combination

$$F_f = f_d sign(\dot{x}_p) + f_e e^{-\tau_H |\dot{x}_p|} sign(\dot{x}_p) + f_v \dot{x}_p \tag{6.36}$$

The dynamic friction (f_d) depends on the sign of the piston velocity. The viscose friction (f_v) depends on the piston velocity. The static friction (f_e) depends on the sign of the piston velocity and will be constructed with growing piston velocity with the decrement τ_H.

At rest ($\dot{x}_p = 0$), only the static friction affects the system. For low $\dot{x}_p$, this friction is reduced with the diminution of τ_H. The total friction for low velocities will be dominated by the dynamic friction. As the velocity increases, the friction will be proportional to the viscose friction. For the generation of the system only the viscose friction will be considered [37].

With the assumption of the incompressibility of the fluid used in the actuation system [45], the influence of the standby actuator can be modeled by a quadratic damping equation:

$$F_d(\dot{x}_p) = d_t \dot{x}_p \mid \dot{x}_p \mid = \frac{A_p^3}{C_q^2 A_D^2} \dot{x}_p \mid \dot{x}_p \mid = \frac{A_p^3}{C_q^2 A_D^2} \dot{x}_p^2 sign(\dot{x}_p) \tag{6.37}$$

where C_q is the flow coefficient of the standby actuator and A_D is the cross section of the damping valve. The value of the turbulent damping d_t is given by the manufacturer system description.

6.3.2 Linearization of the APS

In this subsection the linearization of the nonlinear model for the aileron positioning system is considered. In order to make the linearization is considered that, for the servovalve, it is only necessary to linearize the mechanic to hydraulic transformation of energy in the servovalve.

$$Q_{sv} = Q_A = Q_B = B_{sv} y_{sv} \sqrt{\frac{1}{2}(P_v - \Delta_p sign(y_{sv})} \tag{6.38}$$

The Taylor's series expansion for the flow Q_{sv} is described below.

$$Q_{sv} = Q_{sv}\Big|_{(y_{sv_{op}}, \Delta_{pop})} + \frac{\partial Q_{sv}}{\partial y_{sv}}\Big|_{\Delta_{pop}} \cdot (y_{sv} - y_{sv_{op}}) + \frac{\partial Q_{sv}}{\partial \Delta_p}\Big|_{y_{sv_{op}}} \cdot (\Delta_p - \Delta_{pop}) + NL_{terms}(y_{sv}, \Delta_p) \tag{6.39}$$

The piston centered position, i.e. hydraulic null, is chosen as operating point (op), so that $x_0 = y_{sv_{op}} = \Delta_{pop} = 0$. Neglecting the nonlinear terms of eq. (6.39), the linearized flow equation is presented below.

$$Q_{sv_{lin}} = C_y y_{sv} + C_p \Delta_p \tag{6.40}$$

where C_y is the flow rate gain and C_p is the pressure gain, $\Delta_p = p_A - p_B$. The values of C_y, and C_p are described below:

$$C_y = \frac{\partial Q_{sv}}{\partial y_{sv}}\Big|_{\Delta_{pop}} = B_{sv}\sqrt{\frac{p_V}{2}} \tag{6.41}$$

$$C_p = \frac{\partial Q_{sv}}{\partial \Delta_p}\Big|_{y_{sv_{op}}} = 0 \tag{6.42}$$

Assuming that both cylinder chambers have the same volumes $V_A = V_B = V$ around the piston initial condition x_0 and that $|A_p x_{max}| = |A_p x_{min}|$, then they have the same hydraulic capacities C_H, given by:

$$C_H = \frac{|A_p x_{max}| + V_D}{E} = \frac{V}{E} \tag{6.43}$$

Applying the Bernoulli's continuity equation, it is possible to obtain $\dot{\Delta}_p = \dot{p}_A - \dot{p}_B$ by subtracting eq. (6.33) and eq. (6.34), and substituting eq. (6.43), so that:

$$\dot{\Delta}_p = \frac{1}{C_H}\left[2Q_{sv_{lin}} - 2A_p \dot{x}_p\right] \tag{6.44}$$

Substituting eq. (6.40) into eq. (6.44), the linearized equation for the pressure difference is obtained as:

$$\dot{\Delta}_p = \frac{2C_y}{C_H} y_{sv} - \frac{2A_p}{C_H}\dot{x}_p \tag{6.45}$$

According to the Newton's movement equation for the piston position

$$m_p \ddot{x}_p = A_p \Delta_p - F_r - F_e - F_p \tag{6.46}$$

In order to make it linear, it is necessary to linearize the terms F_r, F_e, and F_p. $F_{p_{lin}}$ is set to zero if the parallel actuator is in active mode. From F_r, given in eq. (6.36), only the viscose friction f_v is considered [37]. It is now represented as a linear function, so that:

$$F_{r_{lin}} = f_v \dot{x}_p \tag{6.47}$$

The quadratic law function, shown in eq. (6.37), can be linearized [38] by:

$$F_{p_{lin}} = d_t \dot{x}_{max} \dot{x}_p = d_{lin} \dot{x}_p \tag{6.48}$$

The complete system is represented by the following linearized differential equations

$$\begin{aligned} \ddot{y}_{sv} &= -\omega_{sv}^2 y_{sv} - 2\delta_{sv}\omega_{sv}\dot{y}_{sv} + k_{sv}\omega_{sv}^2 i_{sv} \\ \dot{\Delta}_p &= \tfrac{2C_y}{C_H} y_{sv} - \tfrac{2A_p}{C_H}\dot{x}_p \\ \ddot{x}_p &= \tfrac{A_p}{m_p}\Delta_p - \tfrac{c_a}{m_p} x_p - \tfrac{(f_v + d_{lin})}{m_p}\dot{x}_p \end{aligned}$$

6.3.3 Model Uncertainties for the APS

When a nonlinear system is linearized, some information is lost through it. This lack of information can be represented as uncertainties in the system. For the aileron positioning system two main uncertainties can be considered. The first uncertainty appears in the linearization of the standby actuator which is represented by a quadratic damping equation.

$$F_d = d_t |\dot{x}_p| \dot{x}_p \tag{6.49}$$

According to [38], the quadratic damping equation can be linearized by

$$F_{d_{lin}} = d_t \dot{x}_{max} \dot{x}_p = d_{lin} \dot{x}_p \tag{6.50}$$

The linear and nonlinear response of the damping actuator are shown in fig. 6.5.

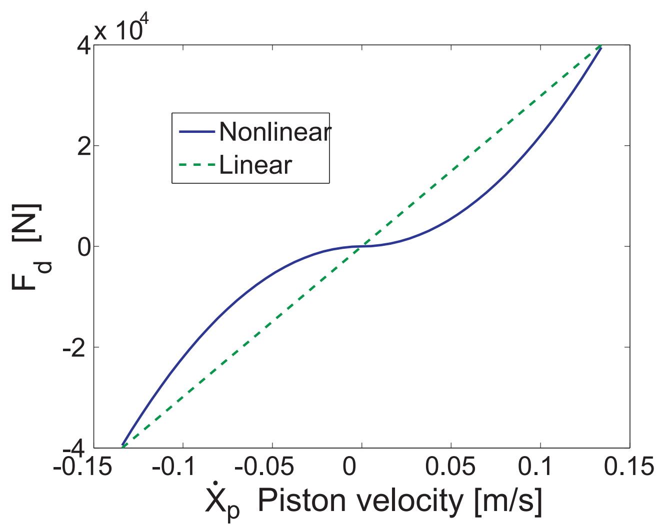

Fig. 6.5: Damping response

It can be seen that the linear and nonlinear response coincide only in the origin and in its extremes, which means that between this points there is an uncertainty. The second uncertainty comes from the nonlinear equation for the flow Q_{sv}

$$Q_{sv}(y_{sv}, \Delta_p) = C_y y_{sv} \sqrt{1 - \left(\frac{\Delta_p}{P_v} sign(y_{sv}) \right)} \tag{6.51}$$

The linearization of the flow rate depends on the operating points used by the linearization.

$$Q_{sv}(y_{sv}, \Delta_p)_{lin} = C_y y_{sv} + C_p \Delta_p = C_y y_{sv} \tag{6.52}$$

However for the purpose of linearization, an operating point is chosen. The linearization will touch the nonlinear response only in the point where it is linearized. For this work an operating point of $\Delta_p = 0$ is chosen. It means that the linearization will touch the nonlinear function only at the beginning and from there it will be linearized as a straight horizontal line. This can seen in the fig. 6.6.

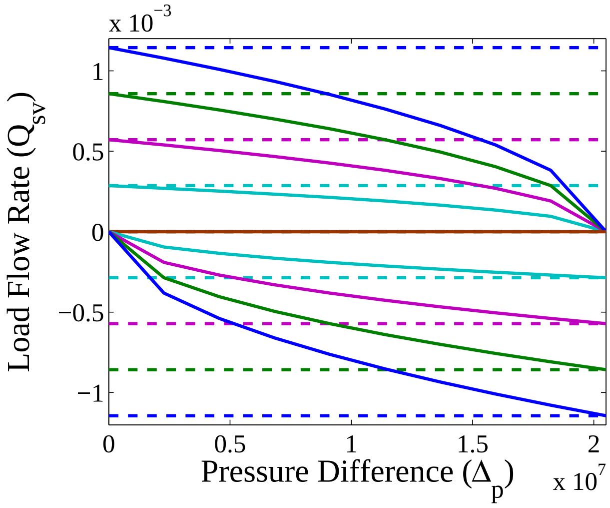

Fig. 6.6: Flow rate

The uncertainties presented above affect the system matrix A and consequently the uncertainty matrix Δ_A is defined as:

$$\Delta_A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\Delta_{1_i}}{m_p} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\Delta_{2_i}}{C_H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.53}$$

This kind of uncertainties are of the polytopic type because they are denoted by a convex set that depends of different operating points.

$$\begin{bmatrix} \Delta A & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \sum_{i=1}^{l} \beta_i \begin{bmatrix} A_i & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \sum_{i=1}^{l} \beta_i = 1,\ \beta_i \geq 0 \tag{6.54}$$

For the polytopic uncertainties were chosen 5 operating points. The corresponding values for Δ_1 and Δ_2 in each operating point are shown in tab. 6.1.

i	Δ_1	Δ_2
1	−14227	0.09794
2	−78533	0.05084
3	−128614	0.02524
4	−185418	0.00714
5	−229773	0.00074

Tab. 6.1: Polytopic uncertainties

The state space representation of the linearized model is given by

$$\begin{bmatrix} \ddot{y}_{sv} \\ \ddot{x}_p \\ \dot{y}_{sv} \\ \dot{\Delta}_p \\ \dot{x}_p \end{bmatrix} = \begin{bmatrix} -2\delta_{sv}\omega_{sv} & 0 & -\omega_{sv}^2 & 0 & 0 \\ 0 & -\frac{f_v + d_{lin}}{m_p} & 0 & \frac{A_p}{m_p} & -\frac{c_a}{m_p} \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -\frac{2A_p}{C_H} & \frac{2C_y}{C_H} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{y}_{sv} \\ \dot{x}_p \\ y_{sv} \\ \Delta_p \\ x_p \end{bmatrix} + \begin{bmatrix} k_{sv}\omega_{sv}^2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} i_{sv} + \begin{bmatrix} 0 & 0 & 0 \\ -\frac{1}{m_p} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} d + \begin{bmatrix} k_{sv}\omega_{sv}^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} f$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{y}_{sv} \\ \dot{x}_p \\ y_{sv} \\ \Delta_p \\ x_p \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} d + \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} f$$

where $\dot{y}_{sv}$ and y_{sv} are the servovalve velocity and position respectively, Δ_p the pressure difference, $\dot{x}_p$ the piston velocity, and x_p the piston position. There are two sensors available, one sensor measures the piston position x_p, and the other one measures the pressure difference Δ_p. The input $u(t)$ is constituted by a current i_{sv}, which changes according to a command input. A variable and unknown but bounded disturbance $d(t)$ affect the system all the time. The fault vector $f(t) = [f_A^T\ f_{\Delta_p}^T\ f_{x_p}^T]^T$ is formed by additive faults that can occur in the actuator f_A, or in each of the available sensors, f_{x_p} and f_{Δ_p}.

The matrices for the linear mathematical model of the aileron positioning system are calculated with the numerical values given in appendix B and they are

$$A = \begin{bmatrix} -884.7 & 0 & -3.06\times 10^{5} & 0 & 0 \\ 0 & -36244 & 0 & 12.19\times 10^{-4} & -26.28 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & -3.29\times 10^{10} & 4.96\times 10^{12} & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 33973 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \; E_d = \begin{bmatrix} 0 & 0 & 0 \\ -0.143 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E_f = \begin{bmatrix} 33973 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \; F_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \; F_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

6.3.4 Simulation results

Solving the *algorithm* 6.1 give us the solution of the Riccati equation eq. (6.8). The values of L_{opt} and V_{opt} are

$$L_{opt} = \begin{bmatrix} 0 & 0 \\ -6.79\times 10^{-2} & 0 \\ 0 & 0 \\ 66889 & -2.03\times 10^{-6} \\ -2.03\times 10^{-6} & 0 \end{bmatrix}, \qquad V_{opt} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6.55}$$

The matrices for the solution of the optimization problem given in the step 2 of *Algorithm* 6.1 are

$$V = \begin{bmatrix} 1.0932 & -2.62\times 10^{-5} \\ -2.62\times 10^{-5} & 1.1672 \end{bmatrix}, \; L = \begin{bmatrix} 3.78\times 10^{6} & 113.09 \\ -95.114 & -0.154 \\ 230.58 & 5.47\times 10^{-3} \\ 5.57\times 10^{11} & 1.68\times 10^{7} \\ 24.676 & 199.98 \end{bmatrix} \tag{6.56}$$

and $\gamma = 1000$.

In order to show the performance improvement of the residual generator with polytopic uncertainties, this residual generator is compared against a residual generator without the polytopic uncertainty.

First, the residuals for the pressure difference Δ_p sensor are shown. Fig. 6.7 shows the residual signal without considering the polytopic uncertainty and fig. 6.8 shows the residual generator considering the polytopic uncertainty.

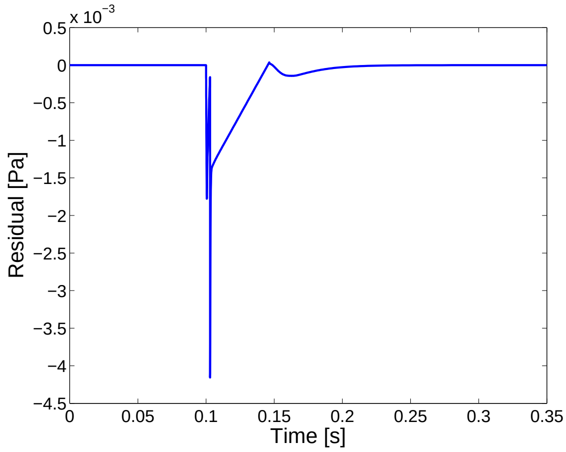

Fig. 6.7: r_{Δ_p} without polytopic uncertainties

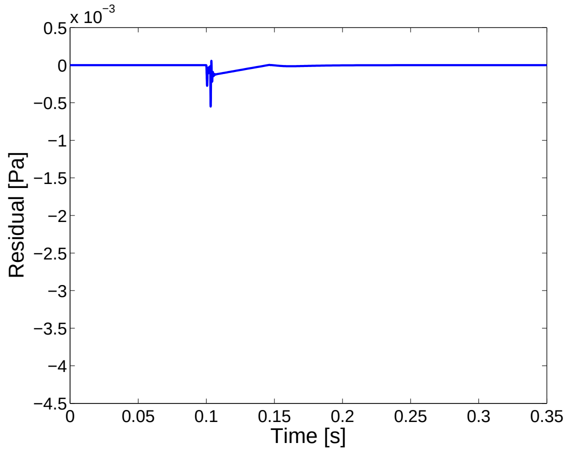

Fig. 6.8: r_{Δ_p} with polytopic uncertainties

The residuals for the piston position x_p sensor are shown below, fig. 6.9 shows the residual signal without considering the polytopic uncertainty and fig. 6.10 shows the residual generator considering the polytopic uncertainty.

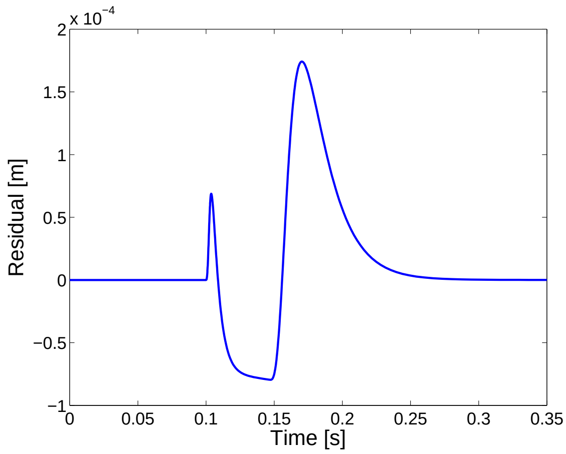

Fig. 6.9: r_{x_p} without polytopic uncertainties

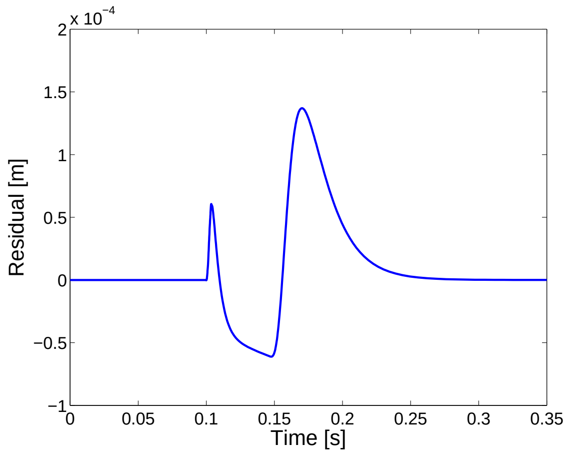

Fig. 6.10: r_{x_p} with polytopic uncertainties

It can be seen that the residual signals, which considers polytopic uncertainties deliver a smaller transient in comparison to the one that does not consider the polytopic uncertainty.

Threshold design

The observer gain matrix L (from eq. (6.56)) is used for the computation of the threshold. It is assumed that $\delta_{d,2}$ is 0.225 because the disturbance is unknown but bounded and the evaluation window (T) is 5 s. The computed values that solves the *Algorithm* 6.2 are:

$$\gamma^* = 0.9$$

and for the step 2

$$J_{th,RMS,2} = \frac{0.9\ (0.225 + \delta_{u,2})}{\sqrt{5}}$$

The value of $\delta_{u,2}$ is calculated on-line, because it depends on the characteristics of the input. In fig. 6.11 both the RMS value of the residual and the corresponding threshold are shown, where an actuator fault f_A occurred at $t = 3\ s$.

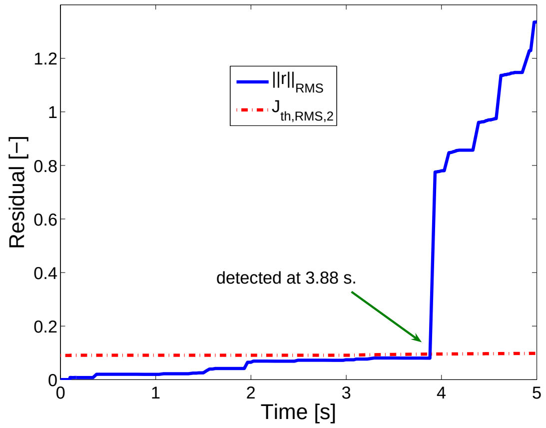

Fig. 6.11: Evaluated residual for the actuator fault

As can be seen, the RMS value of the evaluated residual surpasses the corresponding threshold at $t = 3.85\ s$. Thus, the actuator fault f_A is detected.

Fig. 6.12 shows the RMS evaluation of the residual signal and the corresponding threshold, where a fault in Δ_p sensor occurred at $t = 3\ s$.

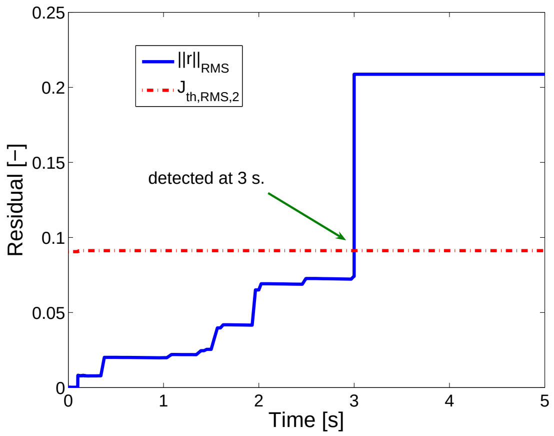

Fig. 6.12: Evaluated residual for fault in Δ_p sensor

It can be seen that the computed threshold contains the disturbances but allows the detection of the sensor fault f_{Δ_p} at $t = 3\ s$.

Fig. 6.13 shows the RMS evaluation of the residual signal and the corresponding threshold, where a fault in x_p sensor occurred at $t = 3\ s$.

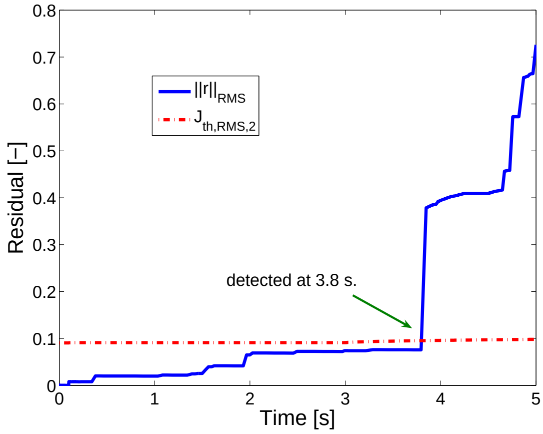

Fig. 6.13: Evaluated residual for fault in x_p sensor

As can be seen, the RMS value of the evaluated residual surpasses the corresponding threshold at $t = 3.8\ s$. Thus, the sensor fault f_{x_p} is detected.

Chapter 7

Conclusions and future work

Two multiple-model approaches have been studied in this thesis in order to give a better performance in fault detection and isolation for nonlinear systems. Multiple-model approaches have an advantage over linear approaches. They incorporate more information about the nonlinear system in comparison to one linearization. The first approach of this scheme is the TS fuzzy model and the second is the linear system with polytopic uncertainties.

In chapter 3, the unknown input observer for TS fuzzy systems (TS fuzzy UIO) for a class of nonlinear systems is presented. This observer is an extension from the linear case studied in [17]. A robust sensor fault isolation scheme [12] based on the TS fuzzy UIO is also considered.

An example is used to demonstrate the functionality of the developed TS fuzzy UIO. The goal of this observer is to decouple unknown inputs from the nonlinear system. The simulation results show that the unknown inputs are decoupled from the system by delivering a residual signal free of unknown inputs. Moreover, the robust fault sensor isolation scheme makes possible to isolate the sensor faults appearing in the system.

Chapter 4 considers the discrete version of the TS fuzzy model with the influence of stochastic noise in order to design a residual generator. The design of the residual generator is made using a LMI optimization approach, in order to minimize the expected value of the steady state estimation error and the effect of the noise is reduced in the residual signal.

To demonstrate the effectiveness of this approach, the vehicle lateral dynamic model is considered, and the results show that the stochastic disturbance is indeed reduced. Therefore, the proposed approach attenuates the effect of the stochastic disturbance and increases teh detection rate of faults.

In chapter 5 the robust fault detection observer for TS fuzzy systems has been applied. In this design two performance indexes were found. The first one is used to minimize the effect of disturbances and the another one to maximize the effect of faults. Both optimization problems are solved simultaneously using iterative LMI.

Both performance indexes have a dependence on each other, in which, a gain ratio is established. The gain ratio is the division of the performance index for disturbances between the performance index for faults.

Two schemes are proposed in order to solve the problem of robust fault detection. The first scheme consider that both optimization objectives are considered to have the same stability matrix in the sense of Lyapunov. In contrast, stability matrix of each optimization objective is considered individually for the second scheme. Simulation results of the proposed schemes have shown that a desirable fault detection behavior is obtained. Moreover, it is much effective to detect the fault despite the influence of the unknown inputs.

Chapter 6 presents the use of polytopic uncertainty for the design of a residual generator and its correspondent threshold. In this approach, the design of the residual generator will contain more information about the nonlinear system in the form of the polytopic uncertainty and therefore the performance of the residual generator will be improved. A reference model is considered in order to construct an augmented system, where the generated residual is compared with a reference residual.

This approach has been applied to the aileron positioning system, and simulation results shown that this fault detection scheme improves the generated residual signals, by reducing the transient magnitude compared with one without polytopic uncertainty.

Future work

Problems related with varying matrices C and E_d (they depend on the states) in the TS fuzzy UIO should be studied in the future work. This will allow to implement also robust actuator fault isolation schemes for the TS fuzzy UIO. Another topic for further research is an integrated solution for nonlinear systems represented by TS fuzzy model, which are affected by deterministic and stochastic disturbances.

Another point is to consider stability in the sense of Lyapunov for each linear system in TS fuzzy model instead of the common Lyapunov stability. One of the approaches that considers this topic is the Lyapunov function described by fuzzy IF-THEN rules.

Each TS fuzzy rule has fuzzy sets in the antecedent part and quadratic Lyapunov functions in the consequent part. A generic rule for the Lyapunov function can be written as follows:

Rule i for the Lyapunov function

$$\begin{array}{ll} \text{IF} & z_1(t) \text{ is } M_{i1} \text{ and } \dots \text{ and } z_p(t) \text{ is } M_{ip} \\ \text{THEN} & V(x(t)) = x^T(t)P_i x(t) \end{array} \tag{7.1}$$

This can be expressed as

$$V(x(t)) = \sum_{i=1}^{r} h_i(z(t))x^T(t)P_i x(t) \tag{7.2}$$

This approach has been done recently in some FDI approaches for TS fuzzy systems but only in the discrete case, the extension to continuous cases can be considered. Actually, the continuous version for this fuzzy Lyapunov function implies the derivative of the membership function and this is not straightforward to obtain. This option could be a good alternative because the conservatism for TS fuzzy models can be reduced.

The topic for residual generation in linear systems with polytopic uncertainty is very interesting. It can be also extended to other fault detection and isolation problems, considering the reference model proposed by [17].

Appendix A

Mathematical tools

A.1 Norms for continuous and discrete systems

Norms for continuous and discrete systems are shown in the table given below

	System type	
Norm	Continuous	Discrete
$\mathcal{L}_1$	$\sum_{i=1}^{n} \int_0^{\infty} \lvert r_i(t)\rvert \, dt$	$\sum_{i=1}^{n} \left(\sum_{k=0}^{\infty} \lvert r_i(k)\rvert \right)$
$\mathcal{L}_2$	$\left(\int_0^{\infty} r^T(t) r(t) dt \right)^{1/2}$	$\left(\sum_{k=0}^{\infty} r^T(k) r(k) \right)^{1/2}$
$\mathcal{L}_\infty$	$\sup_{T\to\infty} \max_i \lvert r_i(t)\rvert$	$\max_i \lvert r_i(k)\rvert$
$\mathcal{L}_{RMS}$	$\left(\frac{1}{T} \int_0^{T} r^T(t) r(t) dt \right)^{1/2}$	$\left(\frac{1}{N} \sum_{i=1}^{N} {r_i}^T(k) r_i(k) \right)^{1/2}$

Tab. A.1: Norms for continuous and discrete systems

A.2 Schur complement

The Schur complement of a block of a matrix within a larger matrix is defined as follows [87]. Suppose that $A_{11} \in \mathcal{R}^{n_1 \times n_1}$, $A_{12} \in \mathcal{R}^{n_1 \times n_2}$, $A_{21} \in \mathcal{R}^{n_2 \times n_1}$, $A_{22} \in \mathcal{R}^{n_2 \times n_2}$ and A_{22} is nonsingular. Let

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{A.1}$$

so that $A \in \mathcal{R}^{(n_1+n_2)\times(n_1+n_2)}$. Then A has the following decomposition:

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & A_{12}A_{22}^{-1} \\ 0 & I \end{bmatrix} \begin{bmatrix} \Delta & 0 \\ 0 & A_{22} \end{bmatrix} \begin{bmatrix} I & 0 \\ A_{22}^{-1}A_{21} & I \end{bmatrix} \tag{A.2}$$

with $\Delta = A_{11} - A_{12}A_{22}^{-1}A_{21}$, and A is nonsingular if and only if Δ is nonsingular. Dually, if A_{11} is nonsingular, then

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ A_{21}A_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} A_{11} & 0 \\ 0 & \hat{\Delta} \end{bmatrix} \begin{bmatrix} I & A_{11}^{-1}A_{12} \\ 0 & I \end{bmatrix} \tag{A.3}$$

with $\hat{\Delta} = A_{22} - A_{21}A_{11}^{-1}A_{12}$, and A is nonsingular if and only if $\hat{\Delta}$ is nonsingular. The matrix $\Delta(\hat{\Delta})$ is called the Schur complement [84] of $A_{22}(A_{11})$ in A.

A.3 Relaxed stability analysis for TS fuzzy observer

As has been shown in subsection 2.2.1, the stability analysis of a TS fuzzy observer is reduced to a problem of finding a common P. If the number of rules (r) is large, it might be difficult to find a common P satisfying the conditions of Theorem 2.1. This subsection presents new stability conditions from [74, 77] by relaxing the conditions of Theorem 2.1.

Theorem A.1 contains the relaxed stability conditions. But first, the following lemmas are needed to prove Theorem A.1.

Lemma A.1

$$\sum_{i=1}^{r} h_i^2(z(t)) - \frac{1}{r-1}\sum_{i=j}^{r}\sum_{i<j} 2h_i(z(t))h_j(z(t)) \geq 0$$

where

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \qquad \textit{and} \qquad h_i(z(t)) \geq 0 \qquad \forall\ i$$

Proof. It holds since

$$\begin{aligned}
\sum_{i=1}^{r} h_i^2(z(t)) \ &- \ \frac{1}{r-1}\sum_{i=j}^{r}\sum_{i<j} 2h_i(z(t))h_j(z(t)) \ \geq \ 0 \\
&= \frac{1}{r-1}\sum_{i=1}^{r}\sum_{i<j}\Big\{h_i(z(t)) - h_j(z(t))\Big\}^2 \ \geq \ 0
\end{aligned}$$

Q.E.D.

Lemma A.2 *If the number of rules r that fire for all t is less than or equal to s, where $1 < s \leq r$, then*

$$\sum_{i=1}^{r} h_i^2(z(t)) - \frac{1}{s-1}\sum_{i=1}^{r}\sum_{i<j} 2h_i(z(t))h_j(z(t)) \geq 0$$

where

$$\sum_{i=1}^{r} h_i(z(t)) = 1 \qquad \textit{and} \qquad h_i(z(t)) \geq 0 \qquad \forall\ i$$

Theorem A.1 *[74]: Assume that the number of rules r that fire for all t is less than or equal to s, where $1 < s \leq r$. The equilibrium of the continuous fuzzy system described by eq. (2.14) is* *globally asymptotically stable if there exist a common positive definite matrix P and a common positive semidefinite matrix Q such that*

$$\begin{aligned} A_{ii}^T P + P A_{ii} + (s-1)Q &< 0 && && (A.4) \\ \left(\frac{A_{ij}+A_{ji}}{2}\right)^T P + P\left(\frac{A_{ij}+A_{ji}}{2}\right) - Q &\leq 0 && i<j && (A.5) \end{aligned}$$

for all i and j with the exception of the pairs (i, j) so that $h_i(z(t))h_j(z(t)) = 0$, for all t and $s > 1$.

Proof: Consider a candidate of Lyapunov function $V(e(t)) = e^T(t)Pe(t)$, where $P > 0$. Then,

$$\begin{aligned} \dot{V}(e(t)) &= \dot{e}^T Pe(t) + e^T(t)P\dot{e}(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))e^T(t)\left[(A_i - L_iC_j)^T P + P(A_i - L_iC_j)\right]e(t) \\ &= \sum_{i=1}^{r} h_i^2(z(t))e^T(t)\left[A_{ii}^T P + PA_{ii}\right]e(t) \\ &+ \sum_{i=1}^{r}\sum_{i<j} 2h_i(z(t))h_j(z(t))e^T(t)\left[\left(\frac{A_{ij}+A_{ji}}{2}\right)^T P + P\left(\frac{A_{ij}+A_{ji}}{2}\right)\right]e(t) \end{aligned}$$

From eq. (A.5) and Corollary A.2, it follows

$$\begin{aligned} \dot{V}(e(t)) &\leq \sum_{i=1}^{r} h_i^2(z(t))e^T(t)\left[A_{ii}^T P + PA_{ii}\right]e(t) + \sum_{i=1}^{r}\sum_{i<j} 2h_i(z(t))h_j(z(t))e^T(t)Qe(t) \\ &\leq \sum_{i=1}^{r} h_i^2(z(t))e^T(t)\left[\ A_{ii}^T P + PA_{ii}\ \right]e(t) + (s-1)\sum_{i=1}^{r} h_i^2(z(t))e^T(t)Qe(t) \\ &= \sum_{i=1}^{r} h_i^2(z(t))e^T(t)\left[\ A_{ii}^T P + PA_{ii} + (s-1)Q\ \right]e(t) \end{aligned}$$

Q.E.D.

if eq. (A.4) holds, then $\dot{V}(e(t)) < 0$ at $e(t) \neq 0$. Then, from the relaxed stability conditions of Theorem A.1, the design problem to determine the gain matrices L_i can be defined as follows

Find $P > 0$, $Q \geq 0$ and N_i $(i = 1,\ 2,\ \ldots,\ r)$ satisfying

$$\begin{aligned} A_i^T P + PA_i - C_i^T N_i^T - N_iC_i + (s-1)Q &< 0 \\ A_i^T P + PA_i + A_j^T P + PA_j - C_j^T N_i^T - N_iC_j - C_i^T N_j^T - N_jC_i - 2Q &\leq 0 \qquad \forall\, i<j \end{aligned}$$

where

$$N_i = PL_i \qquad \text{and} \qquad N_j = PL_j$$

The above conditions are LMI with respect to variables P, Q and N_i. It can be find a positive definite matrix P, a semi positive definite matrix Q and a matrix N_i satisfying the LMI's or determine that no such P, Q and N_i exist.

A.4 LMI and convex optimization techniques

Linear matrix inequalities (LMI) and convex optimization techniques (COT) are basic tools utilized not only for stability analysis of Takagi-Sugeno fuzzy systems but also for the computation of gain matrices and other performance indexes for Takagi-Sugeno fuzzy observers.

A.4.1 Convex optimization techniques

Many important problems for fault detection and isolation theory can lately be solved numerically by reformulating them as convex optimization problems with a linear objective function and LMI constraints [8].

LMIs are an important class of convex constraints. For their solution, the so-called interior-point methods are applied. Nowadays, there are software toolboxes available to solve numerically many FDI problems such as LMI Lab for MATLAB [33, 34].

The main strength of LMI formulations is the ability to combine diverse design constraints or objectives in a numerically tractable manner.

A.4.2 Linear Matrix Inequalities

A linear matrix inequality has the form

$$A(p) = A_0 + \sum_{i=1}^{m} p_i A_i < 0 \tag{A.6}$$

where

- $p = [p_1, p_2, \ldots, p_m]$ is a vector of m variables or parameters, called also decision or optimization variables.
- $A_i = A_i^T \in \mathbb{R}^{n \times n}$ for $i = 0, 1, \ldots, m$ are given constant symmetric matrices.
- the inequality "< 0" in eq. (A.6) means that $A(p)$ is a "negative definite matrix". That is, $u^T A(p) u < 0$ for all non-zero real vectors u. Because all eigenvalues of a real symmetric matrix are real, the eq. (A.6) is equivalent to say that all eigenvalues $\lambda(A(p))$ are negative. Equivalently, the maximal eigenvalue $\lambda_{max}(A(p)) < 0$ [67].
- its solution set, called the *feasibility set*, is a convex subset of $\mathbb{R}^m$, and
- finding a solution p to eq. (A.6), if any exists, is a convex optimization problem.

Convexity has an important consequence: despite the fact that eq. (A.6) has no analytical solution in general, it can be solved numerically with guarantees of finding a solution when one exists. If no solution can be found, the corresponding optimization problem is referred as *infeasible* [44].

A.4.3 Standard LMI-problems

Some standard problems with respect to solving LMI-constraints in order to solve the optimization problems in this work are listed below [44].

1. Finding a solution p to the LMI system

$$A(p) < 0 \tag{A.7}$$

   is called the **feasibility problem**. Given the LMI in eq. (A.7), the corresponding feasibility problem is to find p^{feas} such that $A(p^{feas}) < 0$ or to determine that the problem is infeasible.

2. Minimizing a convex objective under LMI constraints is also a convex problem. In particular, the linear objective minimization problem:

   minimize $c^T p$ over p subject to $A(p) < 0$.

   plays an important role in the LMI-based design.

   These LMI problems allow us to determine whether the problem is either infeasible or to obtain a feasible solution with the corresponding optimal objective values having prescribed accuracy.

In this thesis, all LMI-related computations have been solved using the MATLAB LMI Lab [50].

Appendix B

System parameters

Aileron positioning system

Scalar	Value	Units
A_p	8.54×10^{-3}	$[m^2]$
c_1	90×10^6	$[N/m]$
c_2	78.3×10^6	$[N/m]$
F_{max}	170.7×10^3	$[N]$
p_S	205×10^5	$[Pa]$
p_T	5×10^5	$[Pa]$
p_V	200×10^5	$[Pa]$
$x_{p_{max}}$	0.038	$[m]$
x_r	$[-x_{p_{max}}, x_{p_{max}}]$	$[m]$

Vehicle lateral dynamic model

Scalar	Value	Units
$C'_{\alpha V}$	103600	$[\ N/rad\ ]$
$C_{\alpha H}$	179000	$[\ N/rad\ ]$
g	9.81	$[\ m/s^2\ ]$
i_L	18	[-]
I_z	3870	$[\ kg \cdot m^2\ ]$
l_V	1.52931	$[\ m\ ]$
l_H	1.53069	$[\ m\ ]$
K_{ϕ_R}	0.9429	[-]
m	1850	$[\ kg\ ]$
m_{NR}	220	$[\ kg\ ]$
m_R	1630	$[\ kg\ ]$

Bibliography

[1] Akhenak, A., Chadli, M., Ragot, J., and Maquin, D. *Unknown input multiple observer based approach. Application to secure communications.* 1st IFAC Conference on Analysis and Control of Chaotic Systems, CHAOS06, (2006).

[2] Alcorta García, E., and Frank, P. M. *Deterministic nonlinear observer-based approaches to fault diagnosis.* Control Engineering Practice, vol. 5 (1997), pages 663–670.

[3] Alcorta García, E., and Frank, P. M. *Fault Detection and Isolation in Nonlinear Systems.* 5th European Control Conference, Paper ID: F246, (1999).

[4] Alcorta García, E., Seliger, R., and Frank, P. M. *Nonlinear Decoupling Approach to Fault Isolation in Linear Systems.* Proceedings of the American Control Conference, (June 1998), pages 2867–2871.

[5] Backé, W. Servohydraulik - umdruck zur vorlesung, 6. auflage. Institut für fluidtechnische Antriebe und Steuerungen der RWTH Aachen, Aachen, Germany, 1992.

[6] Bergsten, P., Palm, R., and Driankov, D. *Fuzzy Observers.* The 10th IEEE International Fuzzy Systems Conference, vol. 3 (December 2001), pages 700–703.

[7] Bittanti, S., Laub, A. J., and Willems, J. C. *The Riccati Equation.* Communications and Control Engineering. Springer, 1991.

[8] Boyd, S., Ghaoui, L., Feron, E., and Balakrishnan, V. *Linear Matrix Inequalities in System and Control Theory.* SIAM Studies in Applied Mathematics, 1994.

[9] Brogan, W. L. *Modern Control Theory.* Prentice Hall, 1991.

[10] Casavola, A., Famularo, D., and Franzè, G. *A robust deconvolution scheme for fault detection and isolation of uncertain linear systems: an LMI approach.* Automatica, vol. 41 (2005), pages 1463–1472.

[11] Casavola, A., Famularo, D., and Franzè, G. *Robust fault detection of uncertain linear systems via quasi-LMIs.* Automatica, vol. 44 (2008), pages 289–295.

[12] Chen, J., and Patton, R. J. *Robust Model-Based Fault Diagnosis for Dynamic Systems.* Kluwer Academic Publishers, 1999.

[13] Chen, J., Patton, R. J., and Zhang, H. Y. *Design of unknown input observers and robust fault-detection filters.* International Journal of Control, vol. 63 (1996), pages 85–105.

[14] Chen, M., Zhao, Q., and Zhou, D. *A Robust Fault detection Approach for Nonlinear Systems.* International Journal of Automation and Computing, vol. 1 (2006), pages 23–28.

[15] Darouach, M., Zasadzinski, M., and Xu, S. J. *Full-Oder Observers for Linear Systems with Unknown Inputs.* IEEE Transactions on Automatic Control, vol. 39 (March 1994), pages 606–609.

[16] De Persis, C., and Isidori, A. *A Geometric Approach to Nonlinear Fault Detection and Isolation.* IEEE Transaction on automatic control, vol. 46 (June 2001), pages 853–865.

[17] Ding, S. X. *Model-Based Fault Diagnosis Techniques: Design schemes, algorithms and tools.* Springer, 2008.

[18] Ding, S. X., Ding, E. L., and Jeinsch, T. *A characterization of parity space and its application to robust fault detection.* IEEE Transactions on Automatic Control, vol. 44 (February 1999), pages 337–343.

[19] Ding, S. X., Jeinsch, T., Frank, P. M., and Ding, E. L. a. *A unified approach to the optimization of fault detection systems.* International Journal on Adaptive Control and signal processing, vol. 14 (2000), pages 725–745.

[20] Ding, S. X., Zhang, P., and Ding, E. L. *On the unified solution for fault detection filter design: a generalization and the LMI version.* To be published.

[21] Ding, S. X., Zhang, P., and Frank, P. M. *Threshold calculation using LMI-technique and its integration in the design of fault detection systems.* Proceedings of the 42nd IEEE Conference on Decision and Control, (December 2003), pages 469–474.

[22] Ding, X., and Frank, P. M. *Comparison of Observer-based Fault Detection Approaches.* Proceedings of the IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes SAFEPROCESS 94, vol. 1 (June 1994), pages 1–6.

[23] El-ghatwary, M., Ding, S. X., and Gao, Z. *Robust fuzzy fault detection for continuous-time nonlinear dynamic systems.* Sixth IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, vol. 6, part 1 (2006).

[24] Fayaz, A. M. *On the Sugeno-type fuzzy observers.* Proceedings of 38th IEEE Conference on Decision and Control, vol. 5 (December 1999), pages 4828–4833.

[25] Feng, L. *Robust Control Design: An optimal control approach.* John Wiley & Sons, Ltd., 2007.

[26] Frank, P. M. *Fault Diagnosis in Dynamic Systems Using Analytical and Knowledge-based Redundancy - A Survey and Some New Results.* Automatica, vol. 26 (1990), pages 459–474.

[27] Frank, P. M. *Analytical and Qualitative Model-based Fault Diagnosis.* 2nd International Conference on Intelligent Systems Engineering, (1994), pages 1–39.

[28] Frank, P. M. *Diagnoseverfahren in der Automatisierungstechnik.* Automatisierungstechnik, vol. 42 (1994), pages 47–64.

[29] Frank, P. M. *Enhancement of robustness in observer-based fault detection.* International Journal of Control, vol. 59 (1994), pages 955–981.

[30] Frank, P. M. *Advances in Observer-Based Fault Diagnosis in Dynamic Systems.* Engineering Simulation, vol. 13 (June 1996), pages 717–760.

[31] Frank, P. M., and Ding, X. *Survey of robust residual generation and evaluation methods in observer-based fault detection systems.* Journal of Process Control, vol. 7 (1997), pages 403–424.

[32] Frisk, E., and Nielsen, L. *Robust Residual Generation for Diagnosis Including a Reference Model for Residual Behavior.* 14th Triennial World Congress of IFAC, Beijing, P. R. China, (1999), pages 55–60.

[33] Gahinet, P., Nemirovskii, A., Laub, A. J., and Chilali, M. *The LMI Control Toolbox.* Proceedings of the 33rd Conference on Decision and Control, (December 1994), pages 2038–2041.

[34] Gahinet, P., Nemirovskii, A., Laub, A. J., and Chilali, M. *LMI Control Toolbox.* The Mathworks, 1995.

[35] Gertler, J. J. *Fault detection and diagnosis in engineering systems.* Marcel Dekker, 1998.

[36] Gojny, M. H. Approximation von zustandsgrößen zur vermeidung zusätzlicher sensoren zum einsatz an elektrohydraulischen ruderstellsystemen. Technische Universität Hamburg-Harburg, Hamburg, Germany, 1997.

[37] Gojny, M. H. *Systemtechnische Untersuchungen zur aktiven Regelung aeroelastischer Schwingungen bei Transportflugzeugen.* PhD thesis, Institute of Aircraft Systems Engineering, Germany, 2005.

[38] Guillon, M. *Hydraulische Regelkreise und Servosteuerungen.* Hanser, 1968.

[39] Hatanaka, T., Uosaki, K., and Manabe, N. *Structure identification of Takagi-Sugeno fuzzy modeling.* Proceedings of the 2002 IEEE International Conference on fuzzy systems, vol. 1 (August 2002), pages 79–84.

[40] Ichtev, A., Hellendoorn, J., and Babuska, R. *Fault detection and isolation using multiple Takagi-Sugeno fuzzy models.* The 10th IEEE International Conference on Fuzzy Systems, vol. 3 (December 2001), pages 1498–1508.

[41] Isermann, R. *Fahrdynamik-Regelung: Modellbildung, Fahrerassistenzsysteme, Mechatronik.* Springer Verlag, 2006.

[42] Isermann, R., and Ballé, P. *Trends in the Application of Model Based Fault Detection and Diagnosis of Technical Processes.* Proceedings of the 13th IFAC World Congress, San Francisco, vol. N (1996), pages 1–12.

[43] Ivantysyn, J., and Ivantysynova, M. *Hydrostatische Pumpen und Motoren.* Vogel Verlag, 1993.

[44] Korba, P. *A Gain-scheduling Approach to Model-Based Fuzzy Control.* PhD thesis, University Duisburg-Essen, Germany, 1999.

[45] Kordt, M. *Nichtlineare Zustandsregelung elektrohydraulischer Ruderstellsystem moderner Großraumflugzeuge-Anwendung der Direkten Methode von Ljapunow.* Technische Universität Hamburg-Harburg, 1996.

[46] Lootsma, T. F. *Observer-based Fault Detection and Isolation for Nonlinear Systems.* PhD thesis, Aalborg University, Denmark, 2001.

[47] Ma, X., Sun, Z., and He, Y. *Analysis and design of fuzzy controller and fuzzy observer.* IEEE Transactions on fuzzy systems, vol. 6 (February 1998), pages 41–51.

[48] Ma, Y. *Integrated Design of Observer-based Fault Diagnosis Systems and its Application to Vehicle Lateral Dynamic Control Systems.* PhD thesis, University of Duisburg-Essen, Germany, 2005.

[49] Magdy, E. *Robust Fuzzy Observer-based Fault Detection for Nonlinear Systems.* PhD thesis, University of Duisburg-Essen, Germany, 2007.

[50] Matlab. Robust control toolbox. The Mathworks, 2008.

[51] Mazars, E. *Fault detection and isolation filter design for systems subject to polytopic uncertainty.* Proceedings of the 15th Mediterranean Conference on Control and Automation, (July 2007).

[52] Mazars, E., Jaimoukha, I., and Li, Z. *Computation of a reference model for robust fault detection and isolation residual generation.* Journal of Control Science and Engineering, vol. 2008 (2008), pages 1–12.

[53] Merkel, M. *Modellierung und regelungstechnisches Konzept eines Höhenflossenaktuators mit verstellbaren Hydraulikmotoren.* Technische Universität Hamburg-Harburg, 1999.

[54] Mitschke, M. *Dynamik der Kraftfahrzeuge.* Springer Verlag, 2004.

[55] Nguang, S., Zhang, P., and Ding, S. *Parity Based Fault Estimation for Nonlinear Systems: An LMI Approach.* Proceedings of the 2006 American Control Conference, (June 2006), pages 5141–5146.

[56] Ogata, K. *Modern Control Engineering,*, third ed. Prentice Hall International, 1997.

[57] Passino, K., and Yurkovich, S. *Fuzzy Control.* Adisson-Wesley, 1998.

[58] Patton, R. J. *Robust model-based fault diagnosis: the state of the art.* In IFAC Fault Detection, Supervision and Safety for Technical Processes, (September 1994), pages 1–24.

[59] Patton, R. J., and Chen, J. *A review of parity space approaches to fault diagnosis.* Fault detection, supervision and safety for technical processes SAFEPROCESS 91, (September 1991), pages 239–255.

[60] Patton, R. J., and Chen, J. *Observer-based fault detection and isolation: robustness and applications.* Control Engineering Practice, vol. 5 (1997), pages 671–682.

[61] Patton, R. J., Lopez, Toribio, C. J., and Simani, S. *Robust Fault Diagnosis in a Chemical Process Using Multiple-Model Approach.* Proceedings of the 40th IEEE Conference on Decision and Control, (December 2001), pages 149–154.

[62] Ran, A. C., and Vreugdenhil, R. *Existence and comparison theorems for algebraic Riccati equations for continuous- and discrete time systems.* Linear Algebra and its Applications, vol. 99 (1988), pages 63–83.

[63] Rank, M., and Niemann, H. *Norm based Threshold Selection for Fault Detectors.* Proceedings of the American Control Conference, (1998), pages 2027–2031.

[64] Rodriguez-Solis, A., El-ghatwary, M., and Ding, S. X. *TS Kalman Filter Based Fault Detection Schemes.* IAR ICD 4th Workshop on Advanced Control and Diagnosis, (2006).

[65] Salas, A. *Observer-based fault tolerant strategy towards enhancing the availability of flight control systems in safety-critical applications.* PhD thesis, University of Duisburg-Essen, Germany, 2008.

[66] Salas, A., Rodríguez, A., and Ding, S. X. *Fault Diagnosis of Electrohydraulic Servoactuators - A Benchmark study-.* 16th Mediterranean Conference on control and Automation, (2008).

[67] Scherer, C., and Weiland, S. Linear matrix inequalities in control. Delft Center for Systems and Control (DCSC), Delft, The Netherlands, 2005.

[68] Schmidt, K., and Trenkler, G. *Einführung in die Moderne Matrix-Algebra.* Springer Verlag, 2006.

[69] Seliger, R., and Frank, P. M. *Robust observer-based fault diagnosis in non-linear uncertain systems.* Issues of Fault Diagnosis for Dynamic Systems, (2000).

[70] Simon, D. *Fuzzy membership optimization via the Extended Kalman Filter.* North American Fuzzy Information Processing Society Conference, (2000), pages 311–315.

[71] Simon, D. *Kalman filtering for fuzzy discrete time dynamic systems.* Applied Soft Computing, vol. 3 (2003), pages 191–207.

[72] Sugeno, M., and Tang, G. T. *Structure identification of fuzzy model.* Fuzzy sets and systems, vol. 28 (1986), pages 329–346.

[73] Takagi, T., and Sugeno, M. *Fuzzy identification of systems and its applications to modeling and control.* IEEE Transactions on Systems, Man, and Cybernetics, vol. 15 (January 1985), pages 116–132.

[74] Tanaka, K., Ikeda, T., and Wang, H. *Fuzzy regulators and fuzzy observers: Relaxed stability conditions and LMI-Based designs.* IEEE Transactions on Fuzzy Systems, vol. 6 (May 1998), pages 250–265.

[75] Tanaka, K., and Sano, M. *A Robust Stabilization Problem of Fuzzy Control Systems and Its Application to Backing up Control of a Truck-Trailer.* IEEE transactions on fuzzy systems, vol. 2 (May 1994), pages 119–134.

[76] Tanaka, K., and Sano, M. *On the concepts of Regulator and Observer of fuzzy control systems.* IEEE World congress on computational intelligence, Proceedings of the third conference on fuzzy systems, vol. 2 (June 1994), pages 767–772.

[77] Tanaka, K., and Wang, H. O. *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach.* John Wiley & sons, INC., 2001.

[78] Wachendorf, N. *Monitorkonzepte für die primären Stellsysteme eines Flugzeuges mit aktiver Strukturdämpfung.* Technische Universität Hamburg-Harburg, 2003.

[79] Wang, H., Ding, S. X., and Zhong, M. *Iterative linear matrix inequality algorithms for fault detection with unknown inputs.* Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering, vol. 219 (2005), pages 161–172.

[80] Wang, H., Wang, J., Liu, J., and Lam, J. *Iterative LMI Approach for Robust Fault Detection Observer Design.* Proceedings of the 42th IEEE Conference on Decision and Control, (2003), pages 1974–1979.

[81] Wang, H. B., Wang, J. L., and Lam, J. *Robust Fault Detection Observer Design: Iterative LMI Approaches.* Journal of Dynamic Systems, Measurement and Control, vol. 129 (January 2007), pages 77–82.

[82] Wang, H. O., Tanaka, K., and Griffin, M. *Parallel Distributed Compensation of Nonlinear Systems by Takagi-Sugeno Fuzzy Model.* Fourth IEEE International Conference on Fuzzy Systems and The Second International Fuzzy Engineering Symposium, vol. 2 (March 1995), pages 531–538.

[83] Zadeh, L. A. *Fuzzy sets.* Information and Control, vol. 8 (1965), pages 338–353.

[84] Zhang, F. *The Schur complement and its applications.* Springer, 2005.

[85] Zhang, N., and Wunsch, D. D. *An Extended Kalman Filter (EKF) approach on fuzzy system optimization problem.* The IEEE international conference on Fuzzy Systems, (2003), pages 1465–1470.

[86] Zhong, M., Ding, S. X., Lam, J., and Wang, H. *An LMI approach to design robust fault detection filter for uncertain LTI systems.* Automatica, vol. 39 (2003), pages 543–550.

[87] Zhou, K., Doyle, J. C., and Glover, K. *Robust and Optimal Control.* Prentice Hall, 1996.

[88] Zolghadri, A., Henry, D., and Monision, M. *Design of nonlinear observers for fault diagnosis. A case study.* Control Engineering Practice, vol. 4 (1996), pages 1535–1544.