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A Cooperative Model of In-Service Training in Statistics for Mathematics Teachers (Grades 5-10)

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THE INTERACTION BETWEEN TEACHING PRACTICE AND THEORETICAL CONCEPTIONS – A COOPERATIVE MODEL OF IN-SERVICE TRAINING IN STATISTICS FOR MATHEMATICS TEACHERS (GRADES 5-10)

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1. Introduction: Why is statistics so seldom taught?

For several years now, there have been many convincing reasons in favor of introducing probability and statistics into the junior secondary school (cf. Barnett 1883, Holmes 1880, Råde 1885). Statistics should provide possibilities to relate school mathematics to pupils' interests, to promote project work in mathematics teaching, to integrate other school subjects, to make meaningful applications, to construct simple models, to support pupils' concrete activities, to perform statistical simulations with the computer, etc. Moreover, the mathematical computations required for statistics in junior high school are not very difficult.

There are seemingly nothing but reasons favoring the introduction of statistics into schools. But the actual situation is quite different. "One would think that, with such support, probability and statistics should by now have a well-established place in school curriculum in most countries of the world. This seems, however, not to be the case." (Råde 1985, p.98). Råde then identifies a number of reasons for that. "1) The, foundations of statistics as a scientific discipline are under debate. 2) The uncertainty of where should be the suitable place for statistics in the school curriculum. 3) Very little is known about the didactics of statistics. 4) Lack of school teachers who are qualified to teach statistics. 5) Lack of appropriate teaching material for teaching statistics at the school level." (Råde 1986, p.24)

I think this list must be completed because most teachers are not willing to teach statistics as in their opinion probability and statistics form a completely different type of mathematics than they know. The epistemological status of indeterminism is strange to them and contradicts the "deterministic" mathematics they have learned. How is it possible to teach statistics with its specific nature in the classroom? The relation between the specific epistemological status of statistics and the necessary means of dealing with it in the teaching process is very important for teacher education and (presently especially in the F.R. of Germany) for the inservice training of teachers as well.

2. The specific nature of statistics in teaching processes

Statistics cannot be taught the same way as "conventional" school mathematics. If one intends to teach statistical concepts, methods and diagrams as mathematical techniques for building up a small coherent theory, the random character and specific nature of statistics is lost very quickly. It degenerates to a collection of rules and recipes without any explanatory power (cf. Holmes 1980, p.40). The consequence is that statistics must be taught in a broader context of meaningful situations which offer opportunities for statistical arguments, interpretations and decisions.

But this means, that the subject matter of teacher training and teacher in-service training is not simply the mathematical "theory" of statistics. For the teacher's teaching activity one has to distinguish three epistemological levels of statistical knowledge. First there is the technical structure of the statistical content, second one has at the same time to take into account the pupil which must learn statistics in an appropriate meaningful way, and third, the teacher has to plan, organise, support and develop this learning process. This leads to the following three epistemological levels:

1. the mathematical structure of statistics: concepts, methods and diagrams

2. pupils' contexts of activity and learning: special means of representation and of activity

3. the perspective from teachers' activity on the teaching process: planning, organisation, guidance and evaluation of the learning process.

The mathematical structure of statistics in junior high school is simple and it contains no serious difficulties. But this simplicity often causes harmful misinterpretations, as Holmes remarks: "Techniques should aid understanding, but too great a stress on techniques hinders understanding." (Holmes 1980, p.41) The real didactical problems for teaching statistics are not the mathematical techniques, but their use and the appropriate application and interpretation of concepts, methods and diagrams.

How is it possible to grasp randomness with the help of deterministic mathematical concepts? To answer this question one has to take a closer look at the central statistical concepts resp. the ways of their mathematical representations and their meaningful applications. In this respect, the concept of statistical proportion plays a fundamental role in junior high school statistics. Statistical proportions are e.g. relative frequencies, relative proportions, probabilities, expected values, different mean values and other characteristic values of distributions.

Statistical proportions are given by formal definitions, by rules of calculation or simply as mathematical fractions. But the exactly calculated values of statistical proportions alone cannot reflect the specific nature of randomness. One needs a frame of reference in which the random character of these values can be assessed. Normally, such frames of reference are concrete applications, real random situations, small projects or statistical polls. The meaningful context of the concrete characteristics which are brought in a relation in form of the mathematical proportion helps to introduce randomness into mathematical calculations.

This necessity of treating statistical concepts always in close connection with frames of reference is quite unique in school mathematics. Only this relation between formal concepts and frames of meaning can in fact accentuate the nature of randomness. At the same time, however, this relation requires mathematical means to estimate the degree of uncertainty. At the lower secondary level, the graphical representations and visualizations are the central means to treat randomness mathematically. The different possibilities to visualize empirical and theoretical distributions graphically (e.g. in the way of pie-charts, column graphs, histograms, stem-and-leaf displays etc.) are not mere illustrations but conceptual models of statistical relations between different statistical variables. These diagrams do not represent single values; the functional relation permits predictions about the uncertainty and its range of validity. Graphical representations provide an elementary frame for pupils to make evaluations and decisions without being forced to proceed simply according to a unique scheme of calculation which furnishes definite deterministic values.

Many mathematics teachers take applications as dispensable extra-mathematical motivations and graphical diagrams as simplifying illustrations. But if they are taken seriously, then this changes the way of concept development radically. Statistical concepts are not established in a universal manner by a formal mathematical definition, they have to be elaborated step by step in the classroom with the help of frames of reference.

The epistemology of randomness enforces the distinction between the mathematical structure of statistics and pupils' contexts of learning statistics. Generally, there is always such a distinction between the mathematical knowledge and the ways of interpreting, understanding and applying this knowledge. We have to distinguish knowledge itself from meta-knowledge about this knowledge. These two sides of scientific knowledge are fundamental in every mathematical learning process for pupils as well as for teachers.

3. Knowledge and meta-knowledge - A dual conception of teaching material and in-service training

It could be stated that there are two opposed reasons why many teachers do not like to teach statistics. If the subject is reduced to its mathematical structure, this seems to be mathematically trivial, and that could not satisfy the demands of mathematics teaching. On the other hand, if statistics is seen in close relation to frames of reference – as it is indispensable for the development of statistical thinking – then most teachers fear that the requirements of statistics teaching will be much too ambitious. Especially with regard to the new forms of teaching, e.g. by organizing and performing small statistical projects and in regard to extra-mathematical applications, teachers often feel uncomfortable and insufficiently prepared.

This points to the fact that in-service training and teaching materials have to reflect the whole complexity of the teaching/learning process. When learning a new school subject like "statistics", teachers need frames of reference, too, that is contexts of their professional activity wherein statistics becomes meaningful. According to the diversity of the teaching profession, the meta-knowledge teachers need refers to different levels:

statistical knowledge in relation to the

1. personal frame: the personal meaning for the teacher himself

2. didactical frame: the incorporation of statistics in the curriculum

3. teaching frame: the arrangement of statistics courses in the classroom

4. learning frame: tasks and activities for pupils' learning process

It is quite natural that these four frames are not totally independent from each other, but should be regarded in their own right. Besides they are helpful to emphasize the particularities of in-service training in comparison to teacher education. In their initial education, teacher students study at first mathematical knowledge as a scientific discipline, then they learn about didactics, and at the end they will acquire practical teaching experiences. They are educated in a top down manner from theoretical mathematics to practical teaching. In contrast to this in-service training has to treat these four levels of meta-knowledge simultaneously, because experienced teachers learn new knowledge in quite another way that is by always trying to relate aspects of these levels and to integrate them.

The main problem of our project was how to develop these different components of meta-knowledge in co-ordination? A fundamental relation between all these components is that between theory and practice, not simply in the way of how theory is practically useful and applicable, but in the shape of a genuine dialectic interrelation. According to this interrelation between theory and practice, our project group was composed of teachers and researchers, the cooperative work was organized and the material was constructed in a dual conception.

The project group consisted of six mathematics teachers from comprehensive schools, two researchers from the IDM/Bielefeld, and two members of the state institute for school and curriculum development (LSW/Soest), which organized and financed the in-service training and the teaching material. This group elaborated a first proposal of the material and conducted the in-service seminars later on. Before they took place, parts from the first version were tested by about 8 teachers. We observed and recorded some of these lessons on statistics; afterwards several interesting episodes of the recorded lessons were transcribed. These transcripts were also used as material for discussion and analysis during the in-service seminars. The first version of the teaching material served as a learning material for the participating teachers, but the project group also expected criticisms and suggestions for improvements.

The material for teaching statistics in grades 7/8 (Steinbring et al. 1986) contains two main parts: First a mathematical and didactical introduction into statistics and second, a number of concrete teaching proposals in the form of different statistical projects. The first part, that is the mathematical presentation of statistics for the teacher, subdivides again into two sections. The first section "Concepts and means of representation in statistics at the lower secondary level" does not simply provide the mathematical structure of statistics but it tries to relate it to a personal frame of reference for the teacher. In the form of a small "project for the teacher" some data on schools, pupils and mathematics teachers are presented and evaluated, which serves to give a personal context of interpretation. The second section "Statistics teaching – Didactical orientations and practical hints for teaching" presents a didactical frame of reference for the teacher. The place of statistics in the curriculum – especially with regard to probability – is analyzed; a didactical conception of statistics is developed, this means that among others the concept of statistical proportion and the role of graphical representations are explained; problems of realizing a statistics course are treated by presenting a possible succession for the central statistical concepts in the shape of projects. At the end, a list of concrete hints for teaching statistics is attached, as they result from systematical reflections and from practical experiences.

The second main part is also constructed according to the dual conception. On the right sides of the material there are listed specific tasks, project works and statistical evaluations for pupils. The left sides contain corresponding explanations and hints for the teacher, namely concerning the expected amount of time, the statistical concepts and their meaning in the specific project, possible teaching problems which might arise at this point, and possible variations or simplifications of tasks. The proposed projects all deal with questions pupils might have in their everyday life. The right sides in this part of the material on statistical projects refer to the learning frame and the left sides to the teaching frame.

This short description shows the dual conception of knowledge and meta-knowledge realised in the material. Even this improved version should not be considered waterproof; it is not a scientific book but a working material which teachers should use several times over many years, and which thereby should be improved by teachers' own experiences and be adopted to their purposes. The material could only encourage and support the first access to teaching statistics. The further teaching will only be stabilized when teachers critically use this or similar material while contrasting it with their teaching experiences. Teachers must try to learn to teach statistics in a step by step way of self-optimization, which should really be supported by a cooperative procedure among several teachers at the same school.

The project work has shown that there cannot exist a perfect material for teachers which they could study from the beginning to the end and which would enable them then to teach statistics perfectly. Especially the statistical subject matter with its particular epistemological difficulties emphasizes that the acquisition of a new mathematical subject must be supported by a variety of measures which refer to the mathematical knowledge, the problems of teaching and to the organization of teaching processes. Because there cannot be a simple automatical way to teaching statistics, it is understandable why statistics unfortunately is still seldom taught, but one can also imagine some basic problems which have to be overcome.

4. References

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