Abstract

A huge amount of data needs to be processed these days. In many fields one wishes to interpret given datasets, which are often corrupted by noise. The development of efficient methods of denoising therefore is a challenging area of research. The need also arises in connection with many applications, e.g. signal processing in measurement and control technique, medical image analysis, spectroscopy and sensors in digital cameras.

This thesis is concerned with a new denoising method. We use a nonparametric approach where no prior information on the distribution of the data is assumed, and essentially focus on smooth datasets.

In the first chapter we describe some nonparametric regression methods and discuss the problems concerning the selection of the smoothing parameters. In case of datasets with varying smoothness, estimators with a local smoothing parameter are preferred, naturally, to those with one global smoothing parameter. The smoothing parameter of the Nadaraya-Watson kernel estimator for instance can be localized. However it is not suitable for denoising two-dimensional datasets since it takes relatively long to compute it. Similar drawbacks of other known methods are pointed out in Chapter 1, to show that our method can be utilized with advantage. Indeed, not only the computing time is reduced considerably by the use of our method, but also smoother results can be obtained.

We introduce the novel diffusion estimator $\hat{f}_\tau$ and its localized version $\hat{f}_a$ in the second chapter for the one-dimensional setting. We give a brief description of the finite differences method, which we use to compute $\hat{f}_\tau$ and $\hat{f}_a$ by solving particular differential equations numerically. The local smoothing parameter is selected with an iterative algorithm using the so called multiresolution criterion. In each iteration step, a statistical analysis of the residuals is made. The smoothing parameter is adapted such that eventually the residuals contain only the noise, which is to be removed. A balance between the smoothness of the solution and the closeness to the data has to be achieved. It is due to this iterative algorithm that the computational speed is significant. A numerical comparison of our method to other nonparametric regression methods is also presented at the end of Chapter 2.

The third chapter is of main interest. It deals with the two-dimensional denoising problem. As the ingredients of our algorithm – the inhomogeneous diffusion process, its numerical solution and the choice of the smoothing parameter – are described in detail in the previous chapter, here the explanation is brief. In the two-dimensional setting, we additionally need a partition to be combined with the multiresolution criterion. This partition is also required to ensure reasonable computing time. Here we present two possible partitions, namely the partition into dyadic squares and the wedge partition. The results are compared, also to other

two-dimensional smoothing methods.
The rest of the work is of more theoretical nature. In the fourth chapter we show that the diffusion estimator $\hat{f}_\tau$ achieves the optimal rate of convergence. Chapter 5 provides the theoretical background for the multiresolution criterion. It deals with the modulus of continuity for the Brownian motion and the Brownian sheet. As Gaussian white noise can be embedded into the Brownian motion, respectively into the Brownian sheet, the modulus of continuity justifies the multiresolution criterion.
We close in Chapter 6 with a brief outlook on further research ideas linked to the work, presented here.
Appendix A gives a collection of all the datasets that have been used in this thesis. The implementation of the diffusion estimator is realized in C and the statistical software R. The source code for an exemplary version of the diffusion estimator $\hat{f}_a$ can be found in Appendix B. The whole source code including different variations can be found on the webpage http://www.stat-math.uni-essen.de/∼stichtenoth.