Abstract

Given a graded ideal I in a polynomial ring, there are several other graded ideals associated to it e.g. graded reduction ideals or generic initial ideals. These ideals play a fundamental role in investigating several homological, algebraic, geometric and combinatorial properties of I. One main aim of this thesis is to understand and explore some of such relations. Another problem that we address in this thesis is the multiplicity conjecture.

Let I be a monomial ideal in a standard graded polynomial ring A. Using the convex-geometric properties, we prove that there exists a unique minimal monomial reduction ideal J of I and we show that the maximum degree of a monomial generator of J determines the slope p of the linear function $\operatorname{reg}(I^t) = pt + c$ for $t \gg 0$. We determine the structure of the reduced fiber ring $\mathcal{F}(J)_{\mathrm{red}}$ of J and show that $\mathcal{F}(J)_{\mathrm{red}}$ is isomorphic to the inverse limit of an inverse system of semigroup rings determined by convex geometric properties of J.

Another aim is to consider the homological properties of geaded ideals. Let K be a field, S a polynomial ring and E an exterior algebra over K, both in a finite set of variables. We study rigidity properties of the graded Betti numbers of graded ideals in S and E when passing to their generic initial ideals.

Through combinatorial properties of squarefree monomial ideals, we study and prove the multiplicity conjecture for a class of spheres. A linear ball is a simplicial complex whose geometric realization is homeomorphic to a ball and whose Stanley–Reisner ring has a linear resolution. It turns out that the Stanley–Reisner ring of the sphere which is the boundary complex of a linear ball satisfies the multiplicity conjecture. A class of shellable spheres arising naturally from commutative algebra whose Stanley–Reisner rings satisfy the multiplicity conjecture will be presented.