Abstract

We investigate the problem of existence and flow invariance of mild solutions to non-autonomous partial differential delay equations of the general form

$$\text{(FDE)} \quad \begin{cases} \dot{u}(t) + B(t)u(t) \ni F(t, u_t), \quad 0 \leq s \leq t \\ u_s = \varphi. \end{cases}$$

Here $B(t)$ is a family of nonlinear multivalued, α-accretive operators with $D(B(t))$ possibly depending on t. For $I = \mathbb{R}^-$ or $I = [-R, 0]$, $R > 0$, $u_t : I \to X$ is the history of u up to t defined by $u_t(\xi) = u(t + \xi)$, $\xi \in I$, $\varphi : I \to X$ is a given initial history out of a space E of functions from I to X, and the operators $F(t, .)$ being defined – and Lipschitz continuous – possibly only on "thin" subsets of the initial history space E.
In this thesis, we shall present two approaches to flow invariance of solutions to (FDE): (a) under range conditions on $(B(t))_{t \geq 0}$; (b) under subtangential conditions.
In both cases, we associate with (FDE) a family of nonlinear operators $A(t)$ defined by:

$$\begin{cases} D(A(t)) = \{\varphi \in \hat{E}(t) \mid \ \varphi' \in E, \varphi(0) \in D(B(t)), \varphi'(0) \in F(t, \varphi) - B(t)\varphi(0)\} \\ A(t)\varphi := -\varphi' \ , \varphi \in D(A(t)). \end{cases}$$

Then, our analysis will be based on the evolution operator associated to the Cauchy problem $\dot{\Phi}(t) + A(t)\Phi(t) = 0$, $\Phi(s) = \varphi$ in the initial history space E.

We also investigate the asymptotic properties (such as asymptotic stability, compactness of the range of solutions, and asymptotic almost periodicity) of the solutions to (FDE).

Key words and phrases. Non–autonomous partial differential delay equations, flow invariance, accretive operators, nonlinear evolution operators.