Robust Fuzzy Observer-based Fault Detection for Nonlinear Systems

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Dedication

to
my parents,
my wife,
my kids Umar and youssef,
my brothers and sisters,
and
all my family

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Abstract

With the increasing demand for higher performance, safety and reliability of dynamic systems, fault diagnosis has received more and more attention. The observer-based strategy is one of the active research fields, which is widely used to construct model-based fault detection systems for technical processes which can be well modelled as linear time invariant systems. Fault diagnosis for nonlinear system is an active area of research.

Observer-based fault detection includes two stages, residual generation and residual evaluation. The residual generation problems and residual evaluation problems for systems with only deterministic disturbances or stochastic disturbances have been widely separately studied. Recently some efforts have been made in the integrated design of fault detection systems for systems with deterministic disturbances and stochastic disturbances.

Recently, successful results of applying Takagi-Sugeno (TS) fuzzy model-based technique to solve fault detection and isolation problems met in the nonlinear system have been achieved. With TS model, a nonlinear dynamic system can be linearised around a number of operating points. Each linear model represents the local system behaviour around the operating point. The global system behaviour is described by a fuzzy IF-THEN rules which represent local linear input/output relations of the nonlinear system. Applying the Takagi-Sugeno fuzzy model based technique to solve fault detection and isolation problems in the nonlinear systems is active area of research.

The main contribution of this thesis is the design of robust fault detection systems based on Takagi-Sugeno fuzzy filters. There are a number of schemes to achieve robustness problem in fault detection. One of them is to introduce a performance index. It is function of unknown input signal and fault signal. For continuous time system, first, robust fault detection system will be designed for nonlinear system with only deterministic disturbance as unknown inputs. Second, robust fault detection system will be designed for nonlinear system with deterministic disturbance as unknown inputs and parameter uncertainties. Finally, robust fault detection system will be designed for nonlinear system with deterministic disturbance as unknown inputs and stated delay. Sufficient conditions for solving robustness problem are given in terms of Linear Matrix Inequalities (LMIs). For discrete time system, kalman filter design for nonlinear system is difficult. In this thesis new fault detection approach will be presented for nonlinear system with only stochastic disturbance. Fault Detection (FD) system for each local subsystem is design by solving the corresponding Discrete-time Algebraic Riccati Equation (DARE). Optimisation algorithm based on minimizing the residual covariance matrix is used to obtain a robust FD system optimised for global system behaviour. The optimisation algorithm is established in terms of LMIs.

The different robust fault diagnosis system are developed to detect sensor faults of vehicle lateral dynamic control systems.

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Notation and symbols

General notations

Symbols	:	*Description*
R^n	:	n dimension real vector
$\|.\|$	:	Norm
sup	:	Supremum
inf	:	Infimum
A_i, B_i	:	System matrix and input matrix
C_i, D_i	:	Output matrix and input-to-output matrix
$E_{d,i}, F_{d,i}$	:	The distribution matrices of deterministic disturbance
$E_{f,i}, F_{f,i}$	:	The distribution matrices of fault
$E_{n,i}, F_{n,i}$	:	The distribution matrices of stochastic disturbances
J_{th}	:	The threshold for the fault decision
Σ_n	:	The variance matrix of random vector
min	:	Minimum
p	:	Number of fuzzy rules
M_{ij}	:	Fuzzy set
z	:	Number of premise variables
$\Delta A_i, \Delta E_{d,i}$	:	Time-varing matrices
J	:	Performance index
X^T	:	The transpose of matrix X
ϵ_1, ϵ_3	:	Scalar numbers greater than zero
h_{1i}, h_{2i}	:	Time varying bounded time delays
r	:	Residual signal
G_{rd}	:	Transfer function from disturbance to residual vector
G_{rf}	:	Transfer function from fault to residual vector

Specific symbols for the vehicle models

Symbols	:	*Description*	*Unit*
a_x, a_y, a_z	:	The acceleration in x, y, z direction	$[m/s^2]$
c	:	Distance from CG of unsprung mass to CG of vehicle	$[m]$
c_α	:	tire cornering stiffness	$[N/rad]$
$c'_{\alpha V}$	:	Front tire cornering stiffness	$[N/rad]$
$c_{\alpha H}$	:	Rear tire cornering stiffness	$[N/rad]$
$c_{\gamma V}$	:	Camber thrust coefficient at the front axle	$[N/rad]$
C_R	:	Roll damping coefficient	$[Nm/rad]$
e	:	Distance from CG of sprung mass to CG of vehicle	$[m]$
F_y	:	Lateral force	$[N]$
g	:	Gravity constant	$[m/s^2]$
h	:	Distance from CG of sprung mass to the roll axis	$[m]$

Symbols	:	Description	Unit
n_{ay}	:	Lateral acceleration sensor noise	$[m/s^2]$
i_L	:	Steering transmission ratio	$[-]$
n_r	:	Yaw rate sensor noise	$[rad/s]$
$n_{\delta L}$	:	Steering angle noise	$[rad]$
I_{zzN}	:	Moment of inertia of unsprung mass about the z axis	$[kg.m^2]$
I_{xz}	:	Moment of inertia about x-z axis	$[kg.m^2]$
I_Z	:	Moment of inertia about the z axis	$[kg.m^2]$
K_R	:	Roll stiffness	$[N.m]$
l_V	:	Distance from the CG to the front axle	$[m]$
l_H	:	Distance from the CG to the rear axle	$[m]$
l	:	Distance from the front axle to rear axle	$[m]$
m	:	Total mass	$[kg]$
m_R	:	Sprung mass	$[kg]$
m_{NR}	:	Unsprung mass	$[kg]$
M_x	:	Moment about the x axis	$[Nm]$
M_z	:	Moment about the z axis	$[Nm]$
p_c	:	Vehicle roll rate	$[rad/s]$
r_c	:	Vehicle yaw rate	$[rad/s]$
v_{ref}	:	Vehicle longitudinal reference velocity	$[m/s]$
α	:	Lateral tire side slip angle	$[rad]$
α_x	:	Road bank angle	$[rad]$
β	:	Vehicle side slip angle	$[rad]$
δ_L^*	:	Vehicle steering angle	$[rad]$

Abbreviations

Symbols	:	Description
ABS	:	Anti-lock Braking System
CG	:	Center of Gravity
DARE	:	Discrete-time Algebraic Riccati Equation
DLE	:	Discrete-time Lyapunov Equation
DOF	:	Degree of Freedom
ESP	:	Electronic Stability Program
FAR	:	False Alarm Rate
FDA	:	Frequency Domain Approach
FDF	:	Fault Detection Filter
FDI	:	Fault Detection and Isolation
FFDF	:	Fuzzy Fault Detection Filter
FIS	:	Fuzzy Inference System
FLS	:	Fuzzy Logic System
ILMIs	:	Iterative Linear Matrix Inequalities
LMIs	:	Linear Matrix Inequalities
LTI	:	Linear Time-Invariant

Symbols	:	*Description*
MI	:	Matrix Inequality
MIs	:	Matrix Inequalities
$MISO$	:	Multi-Input-Single-Output
OBA	:	Observer Based Approach
PDC	:	Parallel Distributed Compensation
PEM	:	Parameter Estimation Method
PSA	:	Parity Space Approaches
$QDEs$	:	Qualitative Differential Equations
TCS	:	Traction Control System
$TSFM$	:	Takagi-Sugeno Fuzzy Model

1 Introduction

1.1 Motivation

Modern control systems are becoming more and more complex and control algorithms more and more sophisticated. Consequently, the demand for higher performance, quality, availability, cost, efficiency, reliability, operating safety and environment protection are of major important. These issues are not only important for normally accepted safety-critical systems such as nuclear reactors, but also for other advanced systems such as employed in cars.

For safety-critical systems, fault diagnosis has received more attention. A fault must be diagnosed as early as possible to prevent the system from wrong situations and bad performance.

In [45], a ”fault” is defined as an unexpected change of system function. Such a fault disturbs the normal operation of a system, thus causing an unacceptable deterioration of performance of the system or even leading to dangerous situations. One of the active research fields is the development of the model-based fault detection systems. Figure 1.1 illustrates the conceptual structure of model-based fault detection systems.

This system can be used to detect, isolate and characterize faults in components of system from the comparison of the system’s available measurements, with a prior information represented by the system’s mathematical model. The generated signal is called ”residual”.

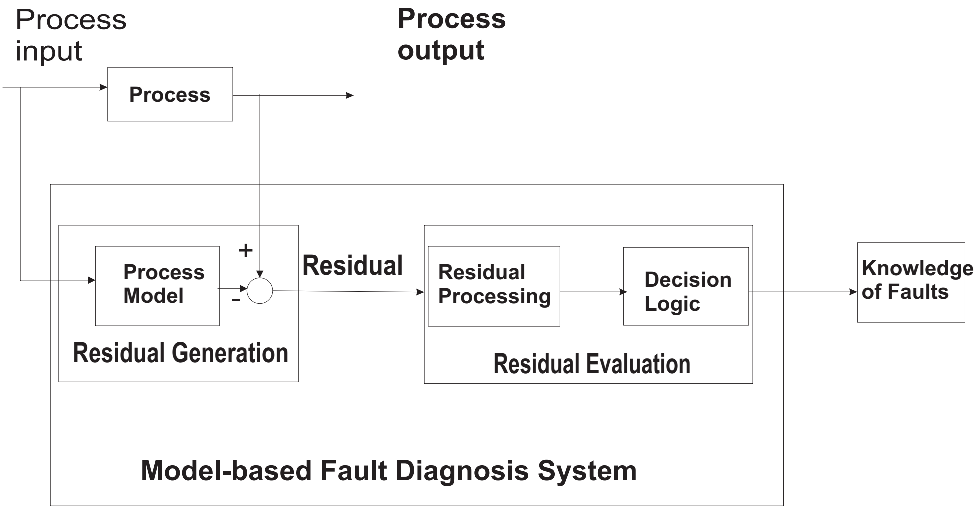

Figure 1.1: Model-based Fault Diagnosis

Faults are detected by comparing a (fixed or variable) threshold with a residual signal. In [18], a monitoring system which is used to detect faults and diagnosis their location is studied. It is called a " fault diagnosis system ". Such a system has normally the following tasks:

1. **Residual Generation**: Its purpose is to generate the residual signal using available input and output information from the monitored system. The residual signal should be normally zero or close to zero when no fault is present, but it should be different from zero when a fault occurs. This means that the residual signal is characteristically independent of the system inputs and outputs. The residual signal should ideally carry only fault information. To ensure reliable Fault Detection and Isolation (FDI), the loss of fault information in residual generation should be as small as possible.
2. **Residual Evaluation**: In residual evaluation, a mathematical feature of the residual signal called residual evaluation function is studied. After that, a threshold value is computed. At the end, the feature of the residual signal is compared with the corresponding threshold. The decision for successfully detecting a fault is finally made based on the comparison result.

Over the past two decades, the problem of Fault Detection (FD) in dynamic systems has attracted considerable attention of many researchers. The residual generation and residual evaluation problems are often studied separately. For linear systems, various residual generation and residual evaluation approaches have been proposed, see [10], [29], [43], [56], [81] and [95]. In [30] and [75], fault detection approaches have been developed to improve robustness against unknown disturbances. Robust fault detection filter for uncertain linear time-invariant systems is designed in [61]. Fault detection filter for time-delay Linear Matrix Inequality (LMI) systems with unknown inputs is designed in [82]. Filter design for linear systems with state delay and parameter uncertainty is studied in [113]. The problem of nonlinear systems remains as an open research area.

One of the main difficulties in designing a fault detection filter for nonlinear dynamic systems is that a rigorous mathematical model may be very difficult to obtain. However, many physical systems can be expressed either in some form of mathematical model locally or as an aggregation of a set of mathematical models. Fuzzy system theory enables us to utilize qualitative, linguistic information from a highly complex nonlinear system to construct a mathematical model for it. Recent studies [5], [6], [7], [51], [65], [66], [67], [99] and [100] have shown that a fuzzy linear model can be used to approximate the global behavior of a highly complex nonlinear system.

In this fuzzy linear model, local dynamic in different state space regions is represented by local linear systems. The overall model of the system is obtained by "blending" these linear models through nonlinear fuzzy membership functions. Unlike conventional modeling which uses a single model to describe the global behavior of the system, fuzzy modeling is essentially a multi-model approach in which simple sub-models (linear models) are fuzzily combined to describe the global behavior of the system.

Vehicle lateral dynamic has a great effect on the vehicle maneuverability, stability and driving safety. With the development of electronics and computer techniques, many important vehicle lateral dynamic control systems have been developed and widely fitted in the vehicles, such as Electronic Stability Program (ESP), Anti-lock Braking System (ABS), Traction Control system (TCS), X-by-Wire systems (drive-by-wire, brake-by-wire). As an information provider for controllers, the performance of sensors embedded in lateral dynamic control systems plays a key role in the vehicle stabilization. To meet the demand for high reliability of the embedded sensors, FDI systems are integrated in the electronic control systems. They ensure an automatic early detection and isolation of possible faults in the sensors.

Recently, it is reported that a new generation of fault diagnosis system based on model-based FDI technology has been successfully developed and integrated into ESP as a series component [24]. Driven by the strong demand from the practice, research on the development of advanced fault diagnosis strategies for vehicle dynamics control systems has received more attention. In the reported results, applications of advanced model-based FDI include: robust/adaptive observer, parity space methods, and computation intelligent technology mark the state of the art in the research field [44]. In this thesis, vehicle lateral dynamic model is considered as a nonlinear system in vehicle longitudinal reference velocity from which Takagi-Sugeno Fuzzy Model (TSFM) can be obtained. The proposed robust fault diagnosis approaches will be applied to detect the sensor faults of the vehicle lateral dynamic control systems. It was pointed out that a more efficient way to design robust fault detection is to integrate together residual generation and residual evaluation [22], [30]

1.2 State of the art

In this section, basic approaches of the fault diagnosis will be briefly reviewed.

1.2.1 Linear dynamic system fault detection

Fault algorthms for linear systems are: Fault Detection Filters (FDF) see [58], [61], [85], [107] and [113], Parity Space Approach (PSA) [19], [37], [75] and [83] Frequency Domain Approach (FDA) [32] as well as Observer Based Approach (OBA) [73], [110] and Parameter Estimation Method (PEM) [43].

1.2.2 Nonlinear dynamic system fault detection

Traditionally, the FD problem for nonlinear dynamic systems has been approached in two steps. The model is linearized at an operating point, and then robust techniques are applied to generate residual signals which are insensitive to model parameter variations within a small neighborhood of the operating point. The robustness is tracked using techniques developed for linear system models. This method only works well when the linearization does not cause a large mismatch between linear and nonlinear behavior and the system operates close to the operating point. However, for systems with high nonlinearity and wide dynamic operating range, the linearized approach fails to give satisfactory results.

One solution is to use a large number of linearized models corresponding to a range of operating points. However, this would involve a large number of FD systems corresponding to all operating points. This is not very practical for real-time applications. It is necessary to develop fault diagnosis methods which tackle nonlinear dynamic system models directly. There have been attempts to use nonlinear observers to solve nonlinear system FD problems [41] and [49]. An adaptive filter based FD approach for time-varying nonlinear systems was proposed in [31]. There have been also some studies on extending the parity relations approach to nonlinear system [102].

Unlike linear systems, there is no direct link between parity relation and Observer-based Approach (OBA). Sometimes, the system cannot be modeled by explicit mathematical models. Without a model the observer-based FD is impossible. To overcome this problem, it is desirable to find a " universal " approximate model which can be used to represent any nonlinear system approximately. Moreover, there should be a mechanism which can automatically identify this universal model.

The neural networks are exactly such a powerful tool for handling nonlinear problems. One of the most important advantage of neural networks is their ability to implement nonlinear transformation for functional approximation problems, given suitable weighting factors and a network architecture. Neural networks have been widely used in many engineering domains and FD applications [96], [97].

In the use of neural networks for fault diagnosis, there are two major problems: the first problem is that most studies only deal with steady-state processes. To achieve online fault diagnosis in the presence of transient behaviors, the system dynamics have to be considered. The second problem is that the neural network is only used as a fault classifier. In these applications, neural networks are used to examine the possibility of a fault in the system outputs and give a fault classification signal to declare whether or not the system is faulty. It may be valid to use only system outputs to diagnose faults for some static systems. However, this is not the case for diagnosing faults in dynamic systems because the change in system inputs can also affect certain features of system outputs. A diagnosis method which only utilizes output information could give incorrect information about faults in the system when the system has been changed. Recently, the residual generation and evaluation concepts have been combined with neural networks to form a powerful FD tool for nonlinear dynamic systems [8], [78].

To overcome the neural network problem of in FD, important approaches based on fuzzy-logic have been developed. In [91], Takagi and Sugeno prove that the fuzzy logic can be used to form the fuzzy model which is very powerful in modeling nonlinear dynamic systems, this model is called TSFM. Recently, successful results of applying the TS fuzzy model based technique to solve FD problems met in the nonlinear systems have been reported [99]. The most convincing and promising arguments for applying TS fuzzy model based technique to deal with nonlinear FD problems are:

1. It has been demonstrated that a TSFM, composed of a number of local sub-models, can well describe the global behavior of a highly complex nonlinear process [99].
2. In the last decade, a framework of designs of TSFM based controllers and observers has been well established [14], [39], [52] and [77].
3. Many methods in this framework have been successfully applied in practice.

Recently, there have been some studies about combining neural networks with fuzzy logic to form the so-called ”neuro-fuzzy approach ” for nonlinear dynamic system FD.

1.3 Robustness of model-based fault diagnosis

Model-based FD makes use of mathematical models of the supervised system. However, a perfectly accurate and complete mathematical model of the physical system is never available. Usually, the parameters of the system may vary with time in an uncertain manner. Also, the characteristic of the disturbance and noise are unknown so that they cannot be modeled. Hence, there is always a mismatch between the actual process and its mathematical model even if there is no process fault.

Apart from the modeling used for the purpose of control, such discrepancies cause fundamental methodology difficulties in FD applications. They constitute a source of false alarms which can corrupt the FD system performance to such an extent that it may even become totally useless. The effect of modeling uncertainties is therefore the most crucial point in the model-based FD concept.

To overcome the difficulties introduced by modeling uncertainties, a model based FD has to be made robust, i.e. insensitive or even invariant to modeling uncertainty. Sometimes, a more reduction of the sensitivity to modeling uncertainties does not solve the problem because such a sensitivity reduction may be associated with a reduction of the sensitivity to fault. A more meaningful formulation of the robust FD problem is to increase the robustness against modeling uncertainties without losing fault sensitivity.

An FD scheme designed to provide satisfactory sensitivity to faults associated with the necessary robustness with respect to modeling uncertainties, is called a robust FD scheme. The development of robust model-based FD methods has been a key research topic. A number of methods have been proposed to tackle this problem, for example, the unknown input observer, eigenstructure assignment and optimally robust parity relation method. The generally used optimization approach is to design residual generators under a certain performance index. Since the goal of residual generation is to enhance the robustness of residuals to the model uncertainties without loss of the sensitivity to the fault, the minimization of performance index is generally formulated as:

$$\min \frac{\text{influence of model uncertainties}}{\text{influence of the faults}}$$

which is widely recognized as a suitable design objective for robust fault detection design. According to the norm used, the type of residual generator and mathematical tool are adapted. A number of optimization approaches have been developed [21], [25], [76], [83] and [110]. Most recently, [84] has derived a unified solution for a number of optimization problems and provided an elegant solution to the above-defined optimization problem.

1.4 Linear matrix inequalities tools

Linear Matrix Inequality (LMI) formulation has become more important in the control theory [35], [46] and [57], especially in dealing with the robust optimization problems.

The papers [36], [64], [79], [80] and [94] provide an interesting history of LMI and application in control theory.

Based on the definition in [80], [103], a general form of LMI is given as

$$F(x) = F_0 + \sum_{i=1}^{m} x_i F_i > 0, \tag{1.4.1}$$

where $x = [x_1, \ldots, x_m]$ is the decision variable, $F_i = F_i^T \in \mathbb{R}^{n\times n}$, $i = 0, 1, \ldots, m$. The inequality means that $F(x)$ is a positive definite matrix, which means

$$z^T F(x) z > 0, \forall z \neq 0, z \in \mathbb{R}^n$$

One of the most distinguished features of LMI is that the inequality (1.4.1) is a convex set. The convexity of the LMI plays a crucial role in optimization because it is well known that a convex matrix has a global optimum over a convex set. Many optimization problems in control design, identification and signal processing can be formulated (or reformulated) using LMIs

The standard form of the LMI optimization problem, which is widely used in control optimization, is defined as: Let $f : \delta \to \mathbf{R}$ and suppose that $\delta = \{x|F(x) > 0\}$. The problem to determine

$$V_{opt} = \inf_{x\in\delta} f(x),$$

is called an optimization problem with LMI constrain. This problem involves the determination of the infimum V_{opt} and for arbitrary $\varepsilon > 0$ the calculation of an almost optimal solution x which satisfies $x \in \delta$ and $V_{opt} \leq f(x) \leq V_{opt} + \varepsilon$. where $x \in \mathbb{R}^m$. Many performance analysis tests, such as computing the H_∞ norm can be formulated as such LMI optimization problem.

LMI problems may be solved using the ellipsoid algorithm or the interior-point methods [91]. In addition, several commercial packages for solving LMIs are available, such as the most popular Matlab LMI control toolbox.

A great number of papers on the application of LMI techniques have been published including papers on eigenvalue minimization [26], [27], calculation of the structured singular value [47], mixed H_2/H_∞ control and observer approaches [9], [13], multi-objective output-feedback control [13], and the application in FD [23], [59], [82] and [90]. In these papers, the optmization problems are represented as set of LMIs, which easly solved in matlab toolbox.

1.5 Purpose of the thesis

Nonlinear observer and control approaches based on the TS fuzzy model have been successfully developed in the framework of LMI. The approaches mainly consist of three stages: the first stage is fuzzy modeling for nonlinear objects. There are two major ways in fuzzy modeling. One is fuzzy model identification that determines structures and parameters of fuzzy models from input-output data [50], [99], [104] and [105]. This method is valid for the case where a physical model for a nonlinear system is not available. Using this method in FD is shown in [69]. On the other hand, if the physical model for nonlinear system is available, the fuzzy model construction is employed to exactly represent the nonlinear dynamics of the model. In this case, the complicated system is represented by a set of IF-THEN rules. The second stage is realized by defuzzification process the so-called Parallel Distributed Compensation (PDC) [39] and [100]. The third stage is the design of fuzzy controller and fuzzy filter. The powerful LMI-based designs play an important role in this stage. The main objects of this thesis are:

- The mathematical model of nonlinear dynamic model is available, the TF fuzzy model for nonlinear dynamic model is designed .
- Robust FD system for TSFM with unknown inputs is designed. This approach attempts to optimize two contradictory objectives: disturbance attenuation and fault sensitivity. This approach is based on H_∞/H_- optimisation problem, which allows optimizing the attenuation of the worst-case effects of disturbances on the residual, in the same time maximizes the fault sensitivity.
- Robust FD system for an uncertain TSFM with unknown inputs is designed. The existence of a robust fault detection guarantees the L_2-gain from unknown inputs to a residual signal is less than a prescribed value and the L_2-gain from a fault signal to a residual signal is greater than a prescribed value.
- Robust FD system for time delay TSFM with unknown inputs is to be designed. The aim of this study is to design a delay dependent fuzzy filter. This filter is robust against the time delay and unknown inputs and sensitive to the fault.
- Robust FD system for TSFM with stochastic noise signal is designed.
- Test and evaluate the developed fault diagnosis systems with TS fuzzy model developed from a vehicle lateral dynamic model. Based on the test and evaluation results, the comparison between those fault diagnosis systems is made. The validity and performance of the proposed approaches are also verified.

1.6 Organization of the thesis

The thesis is organized as follows:

In chapter 2, the construction of fuzzy system is introduced. Fuzzy methods used in fuzzy model-based fault detection are defined. Obtaining TSFM from nonlinear dynamic model is shown. At the end, the problems for robust fuzzy FD systems are formulated.

In chapter 3, robust FD system for nonlinear system with unknown inputs is designed. This system is represented by TS fuzzy model. A fault detection filter guarantees the following requirements: (1) The asymptotic stability of the closed-loop system. (2) The minimization of disturbance effects. (3) The maximization of faults effects. These conditions are regularly used to determine a robust filter. They will be interpreted as an H_∞/H_- optimization problem. This problem is solved by LMI. But there are some critical cases for which the optimal solution cannot be obtained, so an Iterative Linear Matrix Inequalities (ILMIs) algorithm will be used.

In chapter 4, robust FD system for TSFM with unknown inputs and parameters uncertainties is designed. The existence of a robust fault detection guarantees: (1) The L_2-gain from an unknown inputs and parameters uncertainty to a residual signal is less than a prescribed value (2) The L_2-gain from a fault signal to a residual signal is greater than a prescribed value. The solution is given in terms of the solvability of (ILMIs).

In chapter 5, robust FD system for time delay TSFM with unknown inputs is designed. The aim of this study is to design a fuzzy filter with delay. This filter is robust against the time delay and unknown inputs and sensitive to the fault. Sufficient conditions for the existence of a robust fault detection system are given in terms of ILMIs.

In chapter 6, robust FD system for TSFM with stochastic noises is designed. FD system for each local subsystem is design by solving the corresponding DARE. Optimization algorithm based on minimizing the residual covariance matrix is used to obtain a robust FD system optimized for global system behavior. The optimization algorithm is established in terms of LMIs.

In chapter 7, the dynamic model and TSFM for vehicle lateral dynamic system are obtained. The model unknown inputs, uncertainties, sensor fault types and sensor noise for this model are discussed. The developed robust FD systems have been tested and evaluated TS fuzzy model developed from a vehicle lateral dynamic model.

Finally, in chapter 8, conclusions and future work are discussed.

2 Fuzzy Logic in Fault Diagnosis

In this chapter, the construction of fuzzy system is introduced. Fuzzy methods used in fuzzy model-based fault detection are defined. The obtained TSFM from nonlinear dynamic model is shown. At the end, the problems for robust fuzzy FD systems are formulated.

2.1 Fuzzy observer-based fault diagnosis

The structure of a Fuzzy Logic System (FLS) which is widely used in engineering applications is depicted in Figure 2.1. It contains three components: fuzzifier, Fuzzy Inference System (FIS) and defuzzifier. The fuzzifier converts the input from physical (crisp) domain into fuzzy domain. The FIS represents the core of the FLS. FIS is built out of two conceptual components: rule base and data base, where both constitute the knowledge base and inference engine. The defuzzifier converts the output signal from the fuzzy domain to the physical output (crisp) domain. Since the FIS operates with fuzzy sets, it must be interfaced with a numerical environment by means of a fuzzifier and a defuzzifier, respectively. Such a framework is based on the well-established theory of fuzzy reasoning.

The application of FLS is to the design of FDI system consists of fuzzy system identification for residual generation and fuzzy reasoning evaluation. Such fuzzy systems provide a rather transparent representation of the system under study even if it is nonlinear, based on the linguistic interpretation in the form of rules. Moreover, the rules extracted from data can be validated by experts and combined with their prior knowledge. A more or less complete system model which describes the real process can be thus obtained.

Fuzzy models make use of heuristic knowledge instead of differential equations. Linguistic variables specify the input and output signals using linguistic terms. This enhances the robustness of the model unknowns or time-dependent parameters of the system. The relationship between the input and output variables may be described in different ways, including data-based approaches:

1. Fuzzy qualitative models, using a rule base [93], [116].
2. Fuzzy relational models, using a set of parameters that are determined during an identification stage based on a learning data set [3], [70].
3. Fuzzy function models, using several local sub-models to describe the system behavior in the environment of different operating points [20], [63], [71]. This type of models called TS fuzzy model. This model is presented in details.

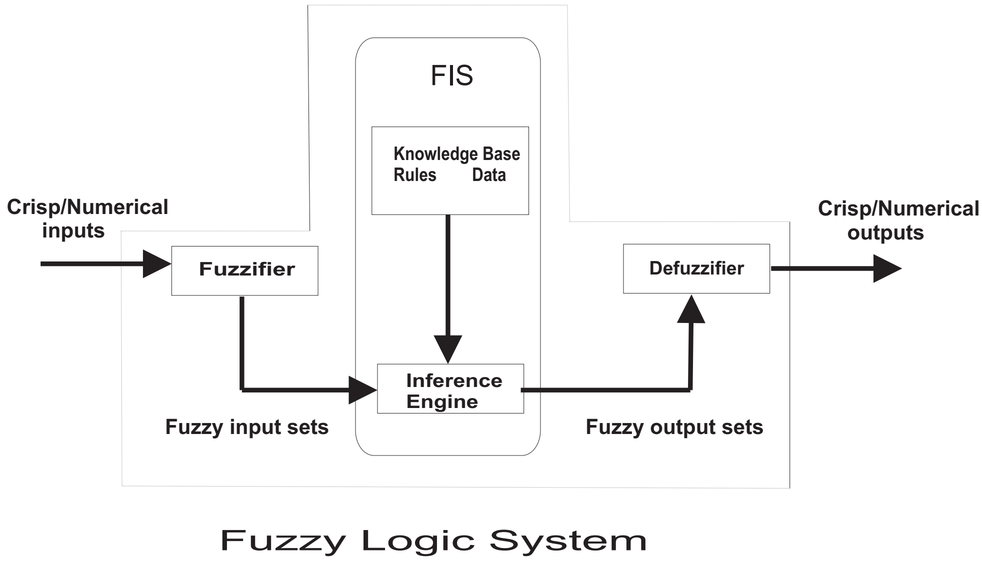

Figure 2.1: Fuzzy Logic System

2.2 Ordinary TS fuzzy systems

The TSFM [99] is of the following form:
Rule i
IF $z_1(t)$ is M_{i1} and ... and $z_\theta(t)$ is $M_{i\theta}$ THEN

$$\dot{x}(t) = A_i x(t) + B_i u(t), \tag{2.2.1}$$

where $z_j(t)$ are premise variables, M_{ij} are fuzzy sets for $i = 1, \ldots, p, j = 1, \ldots, \theta$, $x(t) \in \mathcal{R}^n$ is state vector, $u(t) \in \mathcal{R}^{k_u}$, $A_i \in R^{n\times n}$ and $B_i \in R^{n\times k_u}$. Premise variables may be functions of the measurable states, external disturbances, and/or time. $z(t)$ is used to denote the vector containing all the individual elements $z_1(t) \sim z_\theta(t)$. Given a pair of $[x(t), u(t), z(t)]$, the final output of the fuzzy system is inferred by using the center of gravity method for defuzzification:

$$\dot{x}(t) = \sum_{i=1}^{p} \mu_i(z(t))[A_i x(t) + B_i u(t)], \tag{2.2.2}$$

where $\mu_i(z(t)) = \frac{h_i(z(t))}{\sum\limits_{i=1}^{p} h_i(z(t))}$, $h_i(z(t)) = \prod\limits_{j=1}^{\theta} M_{ij}(z_j(t))$. $M_{ij}(z_j(t)) \geq 0$ is the grade of membership of $z_j(t)$ in M_{ij}. Assume that $\sum_{i=1}^{p} \prod_{j=1}^{\theta} M_{ij}(z_j(t)) \geq 0$. We have

$$\forall k \sum_{i=1}^{p} \mu_i(z(t)) = 1$$

2.2.1 A generalized form of TS fuzzy systems

As in [98], a class of nonlinear systems is represented as

$$\dot{x}_i(t) = \sum_{j=1}^{\theta} f_{ij}(z(t))x_j(t) + \sum_{k=1}^{k_u} g_{ik}(z(t))u_k(t) \quad for \quad i = 1, \dots, n \ , \tag{2.2.3}$$

where n and k_u denotes the number of states and inputs, respectively. $x_1(t) \dots x_n(t)$ are states and $u_1(t) \dots u_{k_u}(t)$ are inputs. $f_{ij}(z(t))$ and $g_{ik}(z(t))$ are functions of $z(t)$, where $z(t) = [z_j(t) \dots z_\theta(t)]$ are known variables, may be functions of the states, external variables and/or time.
To obtain a generalized form, new variables are defined

$$a_{ij1} \equiv \max_{z(t)}\{f_{ij}(z(t))\}, \qquad a_{ij2} \equiv \min_{z(t)}\{f_{ij}(z(t))\}$$
$$b_{ik1} \equiv \max_{z(t)}\{g_{ik}(z(t))\}, \qquad b_{ik2} \equiv \min_{z(t)}\{g_{ik}(z(t))\}$$

The derivation of the generalized form begins with transforming $f_{ij}(z(t))$ and $g_{ik}(z(t))$ into fuzzy model representation. By utilizing the new variables, $f_{ij}(z(t))$ and $g_{ik}(z(t))$ can be represented as:

$$f_{ij}(z(t)) = \sum_{l^a_{(i,j)=1}}^{2} h_{ijl^a_{(i,j)}}(z(t))a_{ijl^a_{(i,j)}}, \qquad g_{ik}(z(t)) = \sum_{l^b_{(i,k)=1}}^{2} v_{ikl^b_{(i,k)}}(z(t))b_{ikl^b_{(i,k)}},$$

where

$$\sum_{l^a_{(i,j)=1}}^{2} h_{ijl^a_{(i,j)}}(z(t)) = 1, \qquad \sum_{l^a_{(i,k)=1}}^{2} v_{ikl^b_{(i,k)}}(z(t)) = 1$$

The membership functions are assigned as follows:

$$h_{ij1}(z(t)) = \frac{f_{ij}(z(t)) - a_{ij2}}{a_{ij1} - a_{ij2}}, \qquad h_{ij2}(z(t)) = \frac{a_{ij1} - f_{ij}(z(t))}{a_{ij1} - a_{ij2}}$$
$$v_{ik1}(z(t)) = \frac{g_{ik}(z(t)) - b_{ik2}}{b_{ik1} - b_{ik2}}, \qquad v_{ik2}(z(t)) = \frac{b_{ik1} - g_{ik}(z(t))}{b_{ik1} - b_{ik2}}$$

By using the fuzzy model, equation (2.2.3) is rewritten as

$$\begin{aligned} \dot{x}_i(t) &= \sum_{j=1}^{\theta} f_{ij}(z(t))x_j(t) + \sum_{k=1}^{k_u} g_{ik}(z(t))u_k(t) \\ &= \sum_{j=1}^{\theta} \sum_{l^a_{(i,j)=1}}^{2} h_{ijl^a_{(i,j)}}(z(t))a_{ijl^a_{(i,j)}}x_j(t) + \sum_{k=1}^{k_u} \sum_{l^b_{(i,k)=1}}^{2} v_{ikl^b_{(i,k)}}(z(t))b_{ikl^b_{(i,k)}}u_k(t) \end{aligned} \tag{2.2.4}$$

By transforming equation (2.2.4) to a matrix form, the generalized form of TS fuzzy systems is

$$\dot{x}(t) = \sum_{i=1}^{n}\sum_{j=1}^{\theta}\sum_{l^a_{(i,j)}=1}^{2} h_{ijl^a_{(i,j)}}(z(t))a_{ijl^a_{(i,j)}}U^A_{ij}x(t) + \sum_{i=1}^{n}\sum_{k=1}^{k_u}\sum_{l^b_{(i,k)}=1}^{2} v_{ikl^b_{(i,k)}}(z(t))b_{ikl^b_{(i,k)}}U^B_{ik}u(t) \tag{2.2.5}$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{\theta}\sum_{l^a_{(i,j)}=1}^{2} h_{ijl^a_{(i,j)}}(z(t))A_{ijl^a_{(i,j)}}x(t) + \sum_{i=1}^{n}\sum_{k=1}^{k_u}\sum_{l^b_{(i,k)}=1}^{2} v_{ikl^b_{(i,k)}}(z(t))B_{ikl^b_{(i,k)}}u(t),$$

where

$$A_{ijl^a_{(i,j)}} = \begin{bmatrix} 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & a_{ijl^a_{(i,j)}} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, B_{ikl^b_{(i,k)}} = \begin{bmatrix} 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & b_{ikl^b_{(i,k)}} & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & & \vdots & \vdots & \vdots & & \vdots \\ 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

2.3 Problem formulation for robust fuzzy fault detection design

In this thesis, a continuous time TS fuzzy model for nonlinear dynamic system is obtained. Then robust fault detection for TSFM with only deterministic disturbance is designed. After that, robust fault detection for TSFM with deterministic disturbance and parameters uncertainties is designed. Then a robust fault detection for TSFM with deterministic disturbance and state time delay is designed. At the end, discrete-time TSFM is obtained from continuous TSFM after the discretization of each subsystem, using 10 millisecond as sample time. After that fault detection system for TSFM with stochastic disturbance is designed. In general, system faults can be modeled as additive and multiplicative fault. The main focus in this thesis is on the detection of additive deterministic faults.

2.3.1 A brief review of used observer-based fault detection models

A typical observer-based FD system consists of a residual generation and residual evaluation [17], [29], [30] and [37]. Residual evaluation stage including an evaluation function and a threshold . In the following the residual generation and residual evaluation are shortly introduced.

1- **Residual generation** : For the purpose of residual generation, based on a TS fuzzy model of nonlinear dynamic system, a fuzzy filter can be designed to estimate the system state vector. For the fuzzy filter design, it is assumed that the fuzzy system model is locally observable i.e., all (A_i, C_i), $(i = 1, \dots, p)$ pairs are observable. In this case, the filter gain matrix L_i is the design parameter. There are a number of schemes to achieve robustness in FD systems. One of them is to introduce a performance index and formulate the Fault Detection Filter (FDF) design optimization problem as in [72].

$$\min_{L_i} J = \min_{L_i} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_-} \tag{2.3.1}$$

where $\|G_{rd}\|_\infty$ is the H_∞ norm of the transfer function from disturbance to residual vector, this norm represent the maximum infulance of disturbances and unknown inputs on residual signal and $\|G_{rf}\|_-$ is the H_- index of the transfer function from fault to residual vector, this index represent the minimum infulance of faults on residual signal.

2- **Residual evaluation** : For the residual evaluation purpose, there exist three well-developed strategies, the statistical testing-based [11], [37], [112], the norm-based residual evaluation [1], [33], [89] and fuzzy logic residual evaluation [56] and [95].

For the norm-based residual evaluation approach, the residual is evaluated with some special function. Generally the evaluation function is expressed as

$$\|r(t)\|_e \quad \text{and} \quad \|r(t)\|_e \in \mathbb{R}^+,$$

where r is the residual signal and the above equation is some kind of norm or norm-like function of the residual in the evaluation window [89], since evaluation over the whole time domain is usually unrealistic. In this thesis, the L_2 norm of residual signal in evaluation window is taken as the norm-based evaluation function, this norm study the energy in residual signala, L_2 is represented as:

$$\|r(t)\|_{e,T} = \left(\int_{t1}^{t2} r^T(t) r(t) dt \right)^{\frac{1}{2}}, \tag{2.3.2}$$

for continuous time system and T the length of the evaluated window it is defined as $T = t_2 - t_1$. For fault decision, the corresponding decision logic should be used to decide whether faults in the system exist. The general used decision logic is formulated as:

$$\begin{cases} \|r(t)\|_e < J_{th} & \text{no fault} \\ \|r(t)\|_e > J_{th} & \text{fault,} \end{cases} \tag{2.3.3}$$

where $J_{th} \in \mathbb{R}^+$ is the threshold.

For the norm-based approaches, the computation of the threshold J_{th} is based on the following equation

$$J_{th} = \sup_{d(t), f(t)=0} \|r(t)\|_e,$$

the threshold is set equal to the maximum influence of $d(t)$ on $r(t)$ in fault-free case.

In statistical evaluation approach, the TS fuzzy system is represented in discrete form, consider that the system contains only stochastic noise. The L_2 norm of residual signal in evaluated window is represented as follows

$$\|r\|_e = \left(\sum_{i=k-s} r^T(i) r(i) \right)^{\frac{1}{2}}, \tag{2.3.4}$$

The fault decision is based on some statistical testing of the residual [11]. The used statistical test is exactly similar as the evaluation function in norm-based methods. Such as in [108], the statistical test for the integrated residual evaluation is

$$\sum_{i=k-s}^{k} r^T(i) \phi_r^{-1} r(i),$$

where ϕ_r is the covariance of residual signal $r(k)$. Since the normalized residual is

$$\bar{r}(i) = V r(i), \tag{2.3.5}$$

where $\phi_r^{-1} = V^T V$, therefore the test statistic can also be expressed as

$$\sum_{i=k-s}^{k} r^T(i) \phi_r^{-1} r(i) = \sum_{i=k-s}^{k} \bar{r}^T(i) \bar{r}(i),$$

which is exactly the square of L_2 norm of the normalized residual $\bar{r}(k)$ in the evaluated window.

For the purpose of residual evaluation, the evaluation window is introduced, therefore the residual is generally reformulated into a new vector $r_{k-s,k}$ in the evaluated window $[k-s, k]$, and defined as $r_{k-s,k} = [r^T(k-s), \ldots, r^T(k)]$, see [37] and [81]. The residual signal based on fault signal and stochastic is represented as

$$r_{k-s,k} = r_{f,k-s,k} + r_{n,k-s,k}, \tag{2.3.6}$$

where $r_{f,k-s,k} = \left[r_f^T(k-s), \ldots, r_f^T(k) \right]^T$, $r_{n,k-s,k} = \left[r_n^T(k-s), \ldots, r_n^T(k) \right]^T$. For fault decision equation (2.3.3) is used and the computation of the threshold J_{th} is based on the following equation

$$J_{th} = \sup_{n(k), f(k)=0} \|r(k)\|_e,$$

that is, the threshold is set equal to the maximum influence of $n(k)$ on $r(k)$ in fault-free case.

For the statistic testing, the threshold is computed according to some statistics distribution for a given False Alarm Rate (FAR) α_0, such as normal distribution in [108], and central Chi-squared distribution in [11].

$$\alpha_0 = P_{\theta_0} \left\{ \|r(k)\|_e > J_{th} \right\},$$

where α_0 is the false alarm rate, θ_0 is the assumed distribution of the test statistic $\|r(k)\|_e$ at a fault-free case.

2.3.2 Problem formulation

From the above description of the considered FD system, the design problem of the FD system consists of the following parts:

The first part of FD system is the design of a residual generator. Based on the given Fuzzy Fault Detection Filter (FFDF), the filter gain L_i has to be chosen, when there exists deterministic disturbance only and the system is in continuous time case, the widely adopted robust FD scheme is to maximize the influence of fault in residual signal and minimize the effect of disturbance simultaneously. That is the robustness the disturbance, and sensitivity to the fault.

The second part is to design a robust FD system for systems with deterministic disturbance and parametric uncertainty.

The third part is to design robust FD system for systems with deterministic disturbance and state delays.

The fourth part is to design robust FD system for systems with stochastic disturbance, In this case the discrete time TSFM is considered.

The final part is the design of the residual evaluation, that is, to select evaluation function $\|r\|_e$ and to calculate threshold J_{th}.

Therefore the design problems for robust and integrated FD system are formulated as follows:

- Given nonlinear system with deterministic disturbance represented in TS fuzzy model with (FFDF), robust FD system is designed so that performance index (2.3.1) is satisfied.
- Given nonlinear system with deterministic disturbance and parametric uncertainty, this system represented as TS fuzzy system with (FFDF), robust FD system is designed so that the performance index (2.3.1) is satisfied.
- Given nonlinear system with deterministic disturbance and state delayed, this system represented as TS fuzzy system with (FFDF), robust FD system is designed so that the performance index (2.3.1) is satisfied.
- Given nonlinear system with stochastic disturbance, this system represented as TS fuzzy system with (FFDF), robust FD system is designed so that the covariance matrix of residual signal is minimized.
- To verify the detection performance of above proposed approaches based on a practical example.

3 Robust Fuzzy Fault Detection for a System with Deterministic Disturbances

This chapter presents a robust fault detection scheme for nonlinear dynamic systems. The residual signal that is generated by a fuzzy filter which is based on TSFM, also attempts to optimize two objectives: disturbance attenuation and fault sensitivity. This approach allows optimizing the attenuation of the worst-case effects of the disturbance on the residual while in the same time enhances the sensitivity of this residual to faults. The robust fuzzy fault detection is presented and solved in the linear matrix inequality framework.

3.1 TS fuzzy model construction

The TSFM with faults and deterministic disturbance is described by the following fuzzy IF-THEN rules :
Rule i
IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + E_{d,i} d(t) + E_{f,i} f(t) \\ y(t) &= C_i x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t), \end{aligned} \tag{3.1.1}$$

where $M_{ij}(i = 1, \cdots, p, j = 1, \ldots, \theta)$ are fuzzy sets, $z = [z_1, \ldots, z_\theta]$ are premise variables, $x(t) \in \mathcal{R}^n$ is state vector, $u(t) \in \mathcal{R}^{k_u}$ and $y(t) \in \mathcal{R}^m$ are the input and measure output vectors respectively, $d(t) \in \mathcal{R}^{k_d}$ is the deterministic disturbance; $f(t) \in \mathcal{R}^{k_f}$ is unknown fault vector acting on system. The matrices $A_i, B_i, C_i, D_i, E_{d,i}, E_{f,i}, F_{d,i}, F_{f,i}$ are of appropriate dimension.

The defuzzified output of TS fuzzy system (3.1.1) is represented as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{p} \mu_i(t)[A_i x(t) + B_i u(t) + E_{d,i} d(t) + E_{f,i} f(t)] \\ y(t) &= \sum_{i=1}^{p} \mu_i(t)[C_i x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t)], \end{aligned} \tag{3.1.2}$$

$$\mu_i(z(t)) = \frac{h_i(z(t))}{\sum_{i=1}^{p} h_i(z(t))}, \; h_i(z(t)) = \prod_{j=1}^{\theta} M_{ij}(z_j(t)).$$

$M_{ij}(z_j(t)) \geq 0$ is the grade of membership of $z_j(t)$ in M_{ij}. Assume that $\sum_{i=1}^{p} \prod_{j=1}^{\theta} M_{ij}(z_j(t)) \geq 0$. We have

$$\forall k \sum_{i=1}^{p} = 1$$

In this thesis, for simplifying notation $\mu_i(z(t))$ or $\mu_i(z(k))$ is replaced by μ_i.

3.2 Residual generation

The first step to achieve a successful FD is to generate a residual signal which is decoupled from the known input signal $u(t)$. In this thesis, TS fuzzy filter is described as follows:

3.2.1 Fuzzy filter design

For a nonlinear dynamic system described by TSFM (3.1.1) a fuzzy filter [111] can be designed to estimate the system state vector. For fuzzy filter design, it is assumed that the fuzzy model is locally observable for each (A_i, C_i) pair with $(i = 1, 2, \ldots, p)$. Using the same TS model, a fuzzy filter uses a number of local time invariant filters. Each filter is associated with the fuzzy rule given below:
Rule i
IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_i\hat{x}(t) + B_iu(t) + L_i[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= C_i\hat{x}(t) + D_iu(t) \\ r(t) &= y(t) - \hat{y}(t), \end{aligned} \tag{3.2.1}$$

where L_i is the filter gain matrix for sub-model i and $r(t)$ is residual signal. Using the idea of PDC [51], the overall state estimation is a nonlinear fuzzy combination of individual local observer output. The overall filter dynamics will be a weighted sum of individual linear filters.

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{p} \mu_i \left[A_i\hat{x}(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))\right] \\ \hat{y}(t) &= \sum_{i=1}^{p} \mu_i \left[C_i\hat{x}(t) + D_iu(t)\right] \\ r(t) &= y(t) - \hat{y}(t), \end{aligned} \tag{3.2.2}$$

where μ_i is the same weight in TS model (3.1.2). Using fuzzy filter in residual generation is shown in Figure 3.1. To analyze the convergence of the filter, the state error vector

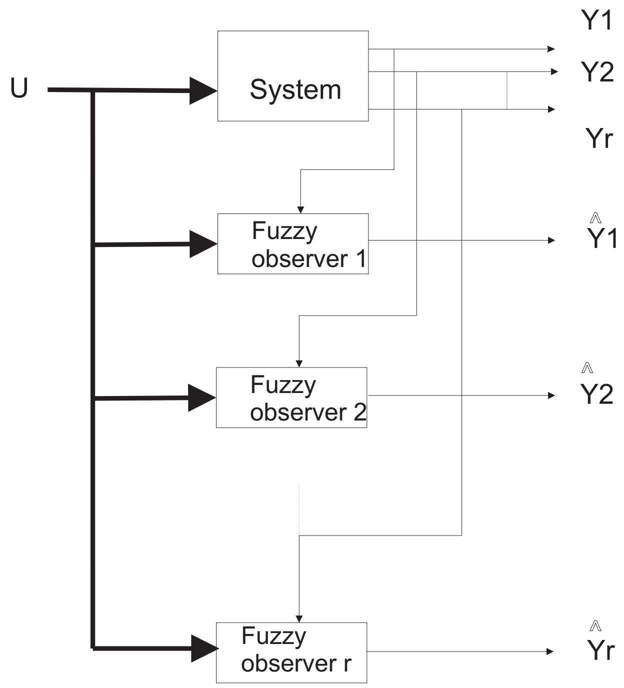

Figure 3.1: Fuzzy Functional Observer Based on Multi-model Approach

$e(t) = x(t) - \hat{x}(t)$ is given by the following differential equation.

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j \left[(A_i - L_iC_j)e(t) + (E_{d,i} - L_iF_{d,j})d(t) + (E_{f,i} - L_iF_{f,j})f(t)\right] \tag{3.2.3}$$

$$r(t) = \sum_{i=1}^{p} \mu_i \left[C_ie(t) + F_{d,i}d(t) + F_{f,i}f(t)\right]$$

The dynamic of residual signal depends on $f(t)$ and $d(t)$, the dynamics of the fuzzy residual generator (3.2.3) can be expressed by

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j \left[\bar{A}_{ij}e(t) + \bar{E}_{d,ij}d(t) + \bar{E}_{f,ij}f(t)\right] \tag{3.2.4}$$

$$r(t) = \sum_{i=1}^{p} \mu_i \left[C_ie(t) + F_{d,i}d(t) + F_{f,i}f(t)\right],$$

where $\bar{A}_{ij} = A_i - L_iC_j$, $\bar{E}_{d,ij} = E_{d,i} - L_iF_{d,j}$ and $\bar{E}_{f,ij} = E_{f,i} - L_iF_{f,j}$. Thus, the problem of designing TS fuzzy fault detection filter can be described as designing the filter gain matrix L_i such that the following conditions are simultaneously filled.

- $\bar{A}_{ij}$ is asymptotically stable for all subsystems A_i for $i, j = 1, \ldots, p$.
- The generated residual $r(t)$ is as sensitive as possible to fault $f(t)$ and as robust as possible to deterministic disturbance $d(t)$.

There are a number of schemes to achieve robustness in FDI. One of them is to introduce a performance index and formulate the Fault Detection Filter (FDF) design optimization problem as in [72].

$$\min_{L_i} J = \min_{L_i} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_-}, \tag{3.2.5}$$

The robust fault diagnosis design problem can be formulated as finding fuzzy filter gain matrix L_i such that the system (3.2.4) is asymptotically stable and the performance index (3.2.5) is made as small as possible in the feasibility of $\|G_{rd}\|_\infty < \gamma$, $\|G_{rf}\|_- > \beta$ for $\gamma > 0$ and $\beta > 0$.

3.2.2 Robust fault detection filter design

In this section, performance index (3.2.5) will be satisfied for system (3.2.4). The following lemma is important in this approach.

Lemma 1 *(Schur Complements) Given constant matrices Ω_1, Ω_2 and Ω_3, where $\Omega_1 = \Omega_1^T$ and $\Omega_2 = \Omega_2^T > 0$, then $\Omega_1 + \Omega_3^T \Omega_2^{-1} \Omega_3 < 0$ if and only if*

$$\begin{bmatrix} \Omega_1 & \Omega_3^T \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad , or \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^T & \Omega_1 \end{bmatrix} < 0$$

The following theorem gives the LMIs formulation of H_∞ estimation problem. This problem can be defined as follows: determine L_i such that the H_∞ norm of the transfer function from disturbances to the residual vector is bounded by a given $\gamma > 0$, γ being as small as possible

Theorem 1 *System (3.2.4), with $f(t) = 0$ is asymptotically stable and satisfies $\|G_{rd}\|_\infty < \gamma$, if for $\gamma > 0$ there exists a positive definite matrix $P > 0$ such that the following Matrix Inequalties (MIs) are satisfied for $1 \le i \le p$ and $1 \le i < j \le p$ respectively at the same time:*

$$\begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i & P\bar{E}_{d,ii} + C_i^T F_{d,i} \\ * & -\gamma^2 I + F_{d,i}^T F_{d,i} \end{bmatrix} < 0 \tag{3.2.6}$$

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j + \bar{A}_{ji}^T P \\ +P\bar{A}_{ji} + C_j^T C_i \end{bmatrix} & P\bar{E}_{d,ij} + C_i^T F_{d,j} + P\bar{E}_{d,ji} + C_j^T F_{d,i} \\ * & -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} \end{bmatrix} < 0, \tag{3.2.7}$$

$*$ denotes the transpose elements in the symmetric position.

Proof of theorem 1 Based on system (3.2.4) with $f(t) = 0$, the following equation is obtained which is only a function of deterministic disturbance.

$$\dot{e}(t) = \sum_{i=1}^{p} \sum_{j=1}^{p} \mu_i \mu_j [\bar{A}_{ij} e(t) + \bar{E}_{d,ij} d(t)] \tag{3.2.8}$$

$$r(t) = \sum_{i=1}^{p} \mu_i [C_i e(t) + F_{d,i} d(t)]$$

The above system is stable and the disturbance rejection can be realized by minimizing γ subject to

$$\sup_{\|d(t)\|_2 \neq 0} \frac{\|r(t)\|_2}{\|d(t)\|_2} < \gamma \tag{3.2.9}$$

Suppose that there exists a quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t),$$

and the derivative of Lyapunov function is

$$\dot{V}(e(t)) = \dot{e}^T(t)Pe(t) + e^T P\dot{e}(t)$$

The stability of system (3.2.8) is ensured if for the given Lyapunov function, the derivative of the Lyapunov function is lower than zero. Based on equation (3.2.9), this can be written like

$$\dot{V}(e(t)) + r^T(t)r(t) - \gamma^2 d^T(t)d(t) < 0. \tag{3.2.10}$$

The LMI conditions are derived from equations (3.2.8) and (3.2.10) to give

$$\begin{aligned} &\dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + \sum_{i=1}^{p} \mu_i \left[C_i e(t) + F_{d,i} d(t)\right]^T \times \sum_{j=1}^{p} \mu_j \left[C_j e(t) + F_{d,j} d(t)\right] \\ &- \gamma^2 d^T(t)d(t) < 0. \end{aligned} \tag{3.2.11}$$

$$\begin{aligned} &\sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j \left[e^T(t)\bar{A}_{ij}^T + d^T(t)\bar{E}_{d,ij}^T\right] Pe(t) + e^T(t)P\sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j \left[\bar{A}_{ij}e(t) + \bar{E}_{d,ij}d(t)\right] + \\ &\sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j [e^T(t)C_i^T C_j e(t) + e^T(t)C_i^T F_{d,j} d(t) + d^T(t)F_{d,i}^T C_j e(t) + d^T(t)F_{d,i}^T F_{d,j} d(t)] \\ &-\gamma^2 d^T(t)d(t) < 0. \end{aligned}$$

The above equation can be rewritten as

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[e^T(t)\bar{A}_{ij}^T Pe(t)+d^T(t)\bar{E}_{d,ij}^T Pe(t)+e^T(t)P\bar{A}_{ij}e(t)+e^T(t)P\bar{E}_{d,ij}d(t)+ \tag{3.2.12}$$

$$+e^T(t)C_i^T C_j e(t)+e^T(t)C_i^T F_{d,j}d(t)+d^T(t)F_{d,i}^T C_j e(t)+d^T(t)F_{d,i}^T F_{d,j}d(t)-$$
$$\gamma^2 d^T(t)d(t)]<0$$
$$=\sum_{i=1}^{p}\mu_i^2[e^T(t)\bar{A}_{ii}^T Pe(t)+d^T(t)\bar{E}_{d,ii}^T Pe(t)+e^T(t)P\bar{A}_{ii}e(t)+e^T(t)P\bar{E}_{d,ii}d(t)$$
$$+e^T(t)C_i^T C_i e(t)+e^T(t)C_i^T F_{d,i}d(t)+d^T(t)F_{d,i}^T C_i e(t)+d^T(t)F_{d,i}^T F_{d,i}d(t)$$
$$-\gamma^2 d^T(t)d(t)]$$
$$+\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}[e^T(t)\bar{A}_{ij}^T Pe(t)+e^T(t)\bar{A}_{ji}^T Pe(t)+d^T(t)\bar{E}_{d,ij}^T Pe(t)+d^T(t)\bar{E}_{d,ji}^T Pe(t)$$
$$+e^T(t)P\bar{A}_{ij}e(t)+e^T P\bar{A}_{ji}e(t)+e^T(t)P\bar{E}_{d,ij}d(t)+e^T(t)P\bar{E}_{d,ji}d(t)+e^T C_i^T C_j e(t)$$
$$+e^T C_j^T C_i e(t)+e^T(t)C_i^T F_{d,j}d(t)+e^T(t)C_j^T F_{d,i}d(t)+d^T(t)F_{d,i}^T C_j e(t)$$
$$+d^T(t)F_{d,j}^T C_i e(t)+d^T(t)F_{d,i}^T F_{d,j}d(t)+d^T(t)F_{d,j}^T F_{d,i}d(t)-2\gamma^2 d^T(t)d(t)]<0$$

Equation (3.2.12) is negative definite if each sum is negative definite.

First, assume that the first sum of equation (3.2.12) is negative definite then:

$$\sum_{i=1}^{p}\mu_i^2[e^T(t)\bar{A}_{ii}^T Pe(t)+d^T(t)\bar{E}_{d,ii}^T Pe(t)+e^T(t)P\bar{A}_{ii}e(t)+e^T(t)P\bar{E}_{d,ii}d(t)+ \tag{3.2.13}$$
$$e^T(t)C_i^T C_i e(t)+e^T(t)C_i^T F_{d,i}d(t)+d^T(t)F_{d,i}^T C_i e(t)+d^T(t)F_{d,i}^T F_{d,i}d(t)$$
$$-\gamma^2 d^T(t)d(t)]<0,$$

putting equation (3.2.13) in matrix to give

$$\sum_{i=1}^{p}\mu_i^2\begin{bmatrix} e(t)\\ d(t)\end{bmatrix}^T\begin{bmatrix}\bar{A}_{ii}^T P+P\bar{A}_{ii}+C_i^T C_i & P\bar{E}_{d,ii}+C_i^T F_{d,i}\\ \bar{E}_{d,ii}^T P+F_{d,i}^T C_i & -\gamma^2 I+F_{d,i}^T F_{d,i}\end{bmatrix}\begin{bmatrix} e(t)\\ d(t)\end{bmatrix}<0, \tag{3.2.14}$$

then Matrix Inequality (MI) (3.2.6) is obtained for $i \le i \le p$.

Second, consider the second sum of (3.2.12) is negative definite then

$$\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}[e^T(t)\bar{A}_{ij}^T Pe(t)+e^T(t)\bar{A}_{ji}^T Pe(t)+d^T(t)\bar{E}_{d,ij}^T Pe(t)+d^T(t)\bar{E}_{d,ji}^T Pe(t) \tag{3.2.15}$$

$$+e^T(t)P\bar{A}_{ji}e(t)+e^T(t)P\bar{A}_{ij}e(t)+e^T(t)P\bar{E}_{d,ji}d(t)+e^T(t)P\bar{E}_{d,ij}d(t)+e^T(t)C_j^T C_i e(t)$$
$$+e^T(t)C_i^T C_j e(t)+e^T(t)C_j^T F_{d,i}d(t)+e^T(t)C_i^T F_{d,j}d(t)+d^T(t)F_{d,j}^T C_i e(t)$$
$$+d^T(t)F_{d,i}^T C_j e(t)+d^T(t)F_{d,i}^T F_{d,j}d(t)+d^T(t)F_{d,j}^T F_{d,i}d(t)-2\gamma^2 d^T(t)d(t)]<0,$$

putting equation (3.2.15) in matrix form. Then we obtain the following MI:

$$\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}\begin{bmatrix} e(t) \\ d(t) \end{bmatrix}^T \tag{3.2.16}$$

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j + \\ \bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i \end{bmatrix} & \begin{bmatrix} P\bar{E}_{d,ij} + C_i^T F_{d,j} \\ P\bar{E}_{d,ji} + C_j^T F_{d,i} \end{bmatrix} \\ \begin{bmatrix} \bar{E}_{d,ij}^T P + F_{d,i}^T C_j \\ \bar{E}_{d,ji}^T P + F_{d,j}^T C_i \end{bmatrix} & -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} \end{bmatrix} \begin{bmatrix} e(t) \\ d(t) \end{bmatrix} < 0,$$

the MI (3.2.7) is obtained for $i \leq i < j \leq p$. Then the proof is therefore complete.

The following theorem gives the LMIs formulation of H_- estimation problem. The main requirement of the H_- is to maximise fault sensitivity on residual signal $r(t)$

Theorem 2 *System (3.2.4) with $d(t) = 0$ is asymptotically stable and satisfies $\|G_{rf}\|_- > \beta$, if for $\beta > 0$ there exist matrix $Q > 0$ such that the following MIs are satisfied for $1 \leq i \leq p$ and $1 \leq i < j \leq p$ respectively at the same time:*

$$\begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\ * & -\beta^2 I + F_{f,i}^T F_{f,i} \end{bmatrix} > 0 \tag{3.2.17}$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} + C_i^T C_j \\ -\bar{A}_{ji}^T Q - QA_{ji} + C_j^T C_i \end{bmatrix} & -Q\bar{E}_{f,ij} + C_i^T F_{f,j} - Q\bar{E}_{f,ji} + C_j^T F_{f,i} \\ * & -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} \end{bmatrix} > 0 \tag{3.2.18}$$

Proof of theorem 2 Based on system (3.2.4) with $d(t) = 0$, the following equation is obtained.

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[\bar{A}_{ij}e(t) + \bar{E}_{f,ij}f(t)] \tag{3.2.19}$$

$$r(t) = \sum_{i=1}^{p}\mu_i[C_i e(t) + F_{f,i}f(t)]$$

The output is sensitive to fault if

$$\inf_{\|f(t)\|_2 \neq 0}\frac{\|r(t)\|_2}{\|f(t)\|_2} > \beta \tag{3.2.20}$$

Suppose that there exists a quadratic Lyapunov function

$$V(e(t)) = e^T(t)Qe(t),$$

for $\gamma > 0$, and $Q > 0$, the stability of system (3.2.19) is ensured if the derivative of Lyapunov function is lower than zero. With respect to (3.2.20), the condition can be written like:

$$r^T(t)r(t) - \beta^2 f^T(t)f(t) - \dot{V}(e(t)) > 0 \tag{3.2.21}$$

Insert with equation (3.2.19) in (3.2.21) then:

$$\begin{aligned}&\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[-e^T(t)\bar{A}_{ij}^T - f^T(t)\bar{E}_{f,ij}^T]Qe(t) + e^T(t)Q\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[-\bar{A}_{ij}e(t)\\&-\bar{E}_{f,ij}f(t)] + \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[e^T(t)C_i^TC_je(t) + e^T(t)C_i^TF_{f,j}f(t)\\&+f^T(t)F_{f,i}^TC_je(t) + f^T(t)F_{f,i}^TF_{f,j}f(t) - \beta^2 f^T(t)f(t)] > 0\end{aligned} \tag{3.2.22}$$

Equation (3.2.22) can be rewritten as

$$\begin{aligned}&\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[-e^T(t)\bar{A}_{ij}^TQe(t) - f^T(t)\bar{E}_{f,ij}^TQe(t) - e^T(t)Q\bar{A}_{ij}e(t) - e^T(t)Q\bar{E}_{f,ij}f(t)\\&+ e^T(t)C_i^TC_je(t) + e^T(t)C_i^TF_{f,j}f(t) + f^T(t)F_{f,i}^TC_je(t) + f^T(t)F_{f,i}^TF_{f,j}f(t) - \beta^2 f^T(t)f(t)]\\&= \sum_{i=1}^{p}\mu_i^2[-e^T(t)\bar{A}_{ii}^TQe(t) - f^T(t)\bar{E}_{f,ii}^TQe(t) - e^T(t)Q\bar{A}_{ii}e(t) - e^T(t)Q\bar{E}_{f,ii}f(t)\\&+ e^T(t)C_i^TC_ie(t) + e^T(t)C_i^TF_{f,i}f(t) + f^T(t)F_{f,i}^TC_ie(t) + f^T(t)F_{f,i}^TF_{f,i}f(t) - \beta^2 f^T(t)f(t)]\\&+ \sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}[-e^T(t)\bar{A}_{ij}^TQe(t) - e^T(t)\bar{A}_{ji}^TQe(t) - f^T(t)\bar{E}_{f,ij}^TQe(t) -\\&f^T(t)\bar{E}_{f,ji}^TQe(t) - e^T(t)Q\bar{A}_{ij}e(t) - e^T(t)Q\bar{A}_{ji}e(t) - e^T(t)Q\bar{E}_{f,ij}f(t) - e^T(t)Q\bar{E}_{f,ji}f(t)\\&+ e^T(t)C_j^TC_ie(t) + e^T(t)C_i^TC_je(t) + e^T(t)C_i^TF_{f,j}f(t) + e^T(t)C_j^TF_{f,i}f(t)\\&+ f^T(t)F_{f,i}^TC_je(t) + f^T(t)F_{f,j}^TC_ie(t) + f^T(t)F_{f,j}^TF_{f,i}f(t) + f^T(t)F_{f,i}^TF_{f,j}f(t)\\&- 2\beta^2 f^T(t)f(t)] > 0\end{aligned} \tag{3.2.23}$$

Equation (3.2.23) is positive definite if each sum of both terms are positive definite.

The first sum is satisfied if:

$$\begin{aligned}&\sum_{i=1}^{p}\mu_i^2[-e^T(t)\bar{A}_{ii}^TQe(t) - f^T(t)\bar{E}_{f,ii}^TQe(t) - e^T(t)Q\bar{A}_{ii}e(t) - e^T(t)Q\bar{E}_{f,ii}f(t) + e^T(t)C_i^TC_ie(t)\\&+e^T(t)C_i^TF_{f,i}f(t) + f^T(t)F_{f,i}^TC_ie(t) + f^T(t)F_{f,i}^TF_{f,i}f(t) - \beta^2 f^T(t)f(t)] > 0,\end{aligned} \tag{3.2.24}$$

putting equation (3.2.24) in matrix form then

$$\sum_{i=1}^{p}\mu_i^2\begin{bmatrix} e(t) \\ f(t) \end{bmatrix}^T\begin{bmatrix} -\bar{A}_{ii}^TQ-Q\bar{A}_{ii}+C_i^TC_i & -Q\bar{E}_{f,ii}+C_i^TF_{f,i} \\ -\bar{E}_{f,ii}^TQ+F_{f,i}^TC_i & -\beta^2I+F_{f,i}^TF_{f,i} \end{bmatrix}\begin{bmatrix} e(t) \\ f(t) \end{bmatrix}>0, \tag{3.2.25}$$

from which MI (3.2.17) is obtained for $1 \leq i \leq p$.

The second sum is satisfied if

$$\begin{aligned}&\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}[-e^T(t)\bar{A}_{ij}^TQe(t)-e^T(t)\bar{A}_{ji}^TQe(t)-f^T(t)\bar{E}_{f,ij}^TQe(t)-f^T(t)\bar{E}_{f,ji}^TQe(t)\\&-e^T(t)Q\bar{A}_{ij}e(t)-e^T(t)Q\bar{A}_{ji}e(t)-e^T(t)Q\bar{E}_{f,ij}f(t)-e^T(t)Q\bar{E}_{f,ji}f(t)+e^T(t)C_j^TC_ie(t)\\&+e^T(t)C_i^TC_je(t)+e^T(t)C_i^TF_{f,j}f(t)+e^T(t)C_j^TF_{f,i}f(t)+f^T(t)F_{f,i}^TC_je(t)+f^T(t)F_{f,j}^TC_ix(t)\\&+f^T(t)F_{f,j}^TF_{f,i}f(t)+f^T(t)F_{f,i}^TF_{f,j}f(t)-2\beta^2f^T(t)f(t)]>0,\end{aligned} \tag{3.2.26}$$

putting equation (3.2.26) in matrix form then

$$\begin{aligned}&\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}\begin{bmatrix} e(t) \\ f(t) \end{bmatrix}^T \\ &\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ij}^TQ-Q\bar{A}_{ij}+C_i^TC_j- \\ \bar{A}_{ji}^TQ-Q\bar{A}_{ji}+C_j^TC_i \end{bmatrix} & \begin{bmatrix} -Q\bar{E}_{f,ij}+C_i^TF_{f,j} \\ -Q\bar{E}_{f,ji}+C_j^TF_{f,i} \end{bmatrix} \\ \begin{bmatrix} -\bar{E}_{f,ij}^TQ+F_{f,i}^TC_j \\ -\bar{E}_{f,ji}^TQ+F_{f,j}^TC_i \end{bmatrix} & -2\beta^2I+F_{f,i}^TF_{f,j}+F_{f,j}^TF_{f,i} \end{bmatrix}\begin{bmatrix} e(t) \\ f(t) \end{bmatrix}>0,\end{aligned} \tag{3.2.27}$$

for $1 \leq i < j \leq p$. So we can obtain MI (3.2.18). The proof is therefore complete.

Performance index (3.2.5) is satisfied if there exists a gain matrix L_i such that MIs (3.2.6), (3.2.7), (3.2.17) and (3.2.18) can be simultaneously solved. The optimal solution is given for γ minimal and β maximal. Isolability of faults is then ensured when γ and β can be found such that $\gamma < \beta$. Unfortunately, this requirement can not be satisfied. So iterative linear matrix inequality is used.

3.2.3 Iterative linear matrix inequality approach

In this section, the problem of the H_∞/H_- estimation is studied. A new LMI formulation is proposed to ensure disturbance attenuation and fault sensitivity.

Theorem 3 *For given $\beta > 0$, $\gamma > 0$ system (3.2.4) with L_i is asymptotically stable and satisfies (3.2.5), if there exist $P > 0$, $Q > 0$, P_0, Q_0, L_i and L_{i0} such that the following LMIs*

$$\begin{bmatrix} M_{11} & PE_{d,i}+C_i^TF_{d,i} & (P-L_iC_i)^T & P \\ * & M_{22} & 0 & -F_{d,i}^TL_i^T \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix}<0 \tag{3.2.28}$$

$$\begin{bmatrix} N_{11} & -QE_{f,i}+C_i^TF_{f,i} & (Q-L_iC_i)^T & Q \\ * & N_{22} & 0 & -F_{f,i}^TL_i^T \\ * & * & I & 0 \\ * & * & * & I \end{bmatrix} > 0, \tag{3.2.29}$$

for $1 \le i \le p$ *hold.*

$$\begin{aligned} M_{11} &= A_i^TP + PA_i + C_i^TC_i + 2(P_0P_0 - P_0P - PP_0) + C_i^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_i \\ M_{22} &= -\gamma^2 I + F_{d,i}^TF_{d,i} + F_{d,i}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{d,i} \\ N_{11} &= -A_i^TQ - QA_i + C_i^TC_i - 2(-Q_0Q - QQ_0 + Q_0Q_0) - C_i^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_i \\ N_{22} &= -\beta^2 I + F_{f,i}^TF_{f,i} - F_{f,i}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{f,i}, \end{aligned}$$

and

$$\begin{bmatrix} \phi_{11} & \begin{bmatrix} PE_{d,i}+PE_{d,j}+ \\ C_i^TF_{d,j}+C_j^TF_{d,i} \end{bmatrix} & (P-L_iC_j)^T & P & (P-L_jC_i)^T & P \\ * & \phi_{22} & 0 & -F_{d,j}^TL_i^T & 0 & -F_{d,i}^TL_j^T \\ * & * & -I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{3.2.30}$$

$$\begin{bmatrix} \psi_{11} & \begin{bmatrix} C_i^TF_{f,j}-QE_{f,i} \\ +C_j^TF_{f,i}-QE_{f,j} \end{bmatrix} & (Q-L_iC_j)^T & Q & (Q-L_jC_i)^T & Q \\ * & \psi_{22} & 0 & -F_{f,j}^TL_i^T & 0 & -F_{f,i}^TL_j^T \\ * & * & I & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 \\ * & * & * & * & I & 0 \\ * & * & * & * & * & I \end{bmatrix} > 0, \tag{3.2.31}$$

for $1 \le i < j \le p$. *where*

$$\begin{aligned} \phi_{11} &= A_i^TP + PA_i + A_j^TP + PA_j + C_i^TC_j + C_j^TC_i + 4(P_0P_0 - P_0P - PP_0) \\ &\quad + C_i^T(L_{j0}^TL_{j0} - L_{j0}^TL_j - L_j^TL_{j0})C_i + C_j^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_j \\ \phi_{22} &= -2\gamma^2 I + F_{d,i}^TF_{d,j} + F_{d,j}^TF_{d,i} + F_{d,i}^T(L_{j0}^TL_{j0} - L_{j0}^TL_j - L_j^TL_{j0})F_{d,i} \\ &\quad + F_{d,j}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{d,j} \\ \psi_{11} &= C_i^TC_j - A_i^TQ - QA_i - A_j^TQ - QA_j + C_j^TC_i - 4(-Q_0Q - QQ_0 + Q_0Q_0) \\ &\quad - C_i^T(L_{j0}^TL_{j0} - L_{j0}^TL_j - L_j^TL_{j0})C_i - C_j^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_j \\ \psi_{22} &= -2\beta^2 I + F_{f,i}^TF_{f,j} + F_{f,j}^TF_{f,i} - F_{f,i}^T(L_{j0}^TL_{j0} - L_{j0}^TL_j - L_j^TL_{j0})F_{f,i} \\ &\quad - F_{f,j}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{f,j} \end{aligned}$$

Proof of theorem 3 (i) For the system (3.2.4) and based on Theorem 1 and Theorem 2, for $1 \le i \le p$ so we can obtain the following MIs:

$$\begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i & P\bar{E}_{d,ii} + C_i^T F_{d,i} \\ * & -\gamma^2 I + F_{d,i}^T F_{d,i} \end{bmatrix} < 0, \tag{3.2.32}$$

and

$$\begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\ * & -\beta^2 I + F_{f,i}^T F_{f,i} \end{bmatrix} > 0, \tag{3.2.33}$$

for any P_0, Q_0 and L_0 MIs, (3.2.32) and (3.2.33) can be represented as

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i \\ +2(P-P_0)(P-P_0) \\ +C_i^T (L_i - L_{i0})^T (L_i - L_{i0}) C_i \end{bmatrix} & P\bar{E}_{d,ii} + C_i^T F_{d,i} \\ * & \begin{bmatrix} -\gamma^2 I + F_{d,i}^T F_{d,i} + \\ F_{d,i}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{d,i} \end{bmatrix} \end{bmatrix} < 0 \tag{3.2.34}$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i \\ -2(Q-Q_0)(Q-Q_0) \\ -C_i^T (L_i - L_{i0})^T (L_i - L_{i0}) C_i \end{bmatrix} & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\ * & \begin{bmatrix} -\beta^2 I + F_{f,i}^T F_{f,i} - \\ F_{f,i}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{f,i} \end{bmatrix} \end{bmatrix} > 0 \tag{3.2.35}$$

The above MIs can be rewritten in the following form

$$\begin{bmatrix} \begin{bmatrix} M_{11}(ii) - C_i^T L_i^T P - PL_i C_i + \\ 2PP + C_i^T L_i^T L_i C_i \end{bmatrix} & M_{12}(ii) - PL_i F_{d,i} \\ * & M_{22}(ii) + F_{d,i}^T L_i^T L_i F_{d,i} \end{bmatrix} < 0 \tag{3.2.36}$$

$$\begin{bmatrix} \begin{bmatrix} N_{11}(ii) + C_i^T L_i^T Q + QL_i C_i \\ -2QQ - C_i^T L_i^T L_i C_i \end{bmatrix} & N_{12}(ii) + QL_i F_{f,i} \\ * & N_{22}(ii) - F_{f,i}^T L_i^T L_i F_{f,i} \end{bmatrix} > 0, \tag{3.2.37}$$

where

$$\begin{aligned}
M_{11}(ii) &= A_i^T P + PA_i + C_i^T C_i + 2(P_0 P_0 - PP_0 - P_0 P) + C_i^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_i \\
M_{12}(ii) &= PE_{d,i} + C_i^T F_{d,i} \\
M_{22}(ii) &= -\gamma^2 I + F_{d,i}^T F_{d,i} + F_{d,i}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{d,i} \\
N_{11}(ii) &= C_i^T C_i - A_i^T Q - QA_i - 2(Q_0 Q_0 - Q_0 Q - QQ_0) - C_i^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_i \\
N_{12}(ii) &= -QE_{f,i} + C_i^T F_{f,i} \\
N_{22}(ii) &= -\beta^T I + F_{f,i}^T F_{f,i} - F_{f,i}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{f,i}.
\end{aligned}$$

Remark 1 *Notice that if $P_0 = P$, $Q_0 = Q$ and $L_{i0} = L_i$ then MIs (3.2.34) and (3.2.35) are the same as (3.2.32) and (3.2.33), respectively.*

MIs (3.2.36) and (3.2.37) can be represented as

$$\begin{bmatrix} M_{11}(ii) & M_{12}(ii) \\ * & M_{22}(ii) \end{bmatrix} + \begin{bmatrix} -C_i^T L_i^T P - PL_iC_i + 2PP + C_i^T L_i^T L_i C_i & -PL_iF_{d,i} \\ * & F_{d,i}^T L_i^T L_i F_{d,i} \end{bmatrix} < 0 \quad (3.2.38)$$

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) \\ * & N_{22}(ii) \end{bmatrix} + \begin{bmatrix} C_i^T L_i^T Q + QL_iC_i - 2QQ - C_i^T L_i^T L_i C_i & QL_iF_{f,i} \\ * & -F_{f,i}^T L_i^T L_i F_{f,i} \end{bmatrix} > 0 \quad (3.2.39)$$

Using Schur complements Lemma then MIs (3.2.38) and (3.2.39) can represented as:

$$\begin{bmatrix} M_{11}(ii) & M_{12}(ii) \\ * & M_{22}(ii) \end{bmatrix} + \begin{bmatrix} (P - L_iC_i)^T & P \\ 0 & -F_{d,i}^T L_i^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} (P - L_iC_i) & 0 \\ P & -L_iF_{d,i} \end{bmatrix} < 0 \quad (3.2.40)$$

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) \\ * & N_{22}(ii) \end{bmatrix} + \begin{bmatrix} (Q - L_iC_i)^T & Q \\ 0 & -F_{f,i}^T L_i^T \end{bmatrix} \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} (Q - L_iC_i) & 0 \\ Q & -L_iF_{f,i} \end{bmatrix} > 0, \quad (3.2.41)$$

putting MIs (3.2.40) and (3.2.41) in matrix form then we obtain MIs (3.2.28) and (3.2.29).

(ii) Based on theorem1, theorem2 and for $1 \leq i < j \leq p$, system (3.2.4) is asymptotically stable and satisfy (3.2.5), if there exist matrices $P > 0$, $Q > 0$ and L_i such that:

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j \\ +\bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i \end{bmatrix} & P\bar{E}_{d,ij} + P\bar{E}_{d,ji} + C_j^T F_{d,i} + C_i^T F_{d,j} \\ * & -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} \end{bmatrix} < 0 \quad (3.2.42)$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} + C_i^T C_j \\ -\bar{A}_{ji}^T Q - Q\bar{A}_{ji} + C_j^T C_i \end{bmatrix} & -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} + C_i^T F_{f,j} + C_j^T F_{f,i} \\ * & -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} \end{bmatrix} > 0 \quad (3.2.43)$$

For any P_0, Q_0 and L_0 MIs (3.2.42) and (3.2.43) can be represented as

$$\begin{bmatrix} M_{11}(ij) & P\bar{E}_{d,ij} + P\bar{E}_{d,ji} + C_i^T F_{d,j} + C_j^T F_{d,i} \\ * & M_{22}(ij) \end{bmatrix} < 0 \quad (3.2.44)$$

$$\begin{bmatrix} N_{11}(ij) & -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} + C_i^T F_{f,j} + C_j^T F_{f,i} \\ * & N_{22}(ij) \end{bmatrix} > 0, \quad (3.2.45)$$

where

$$
\begin{aligned}
M_{11}(ij) &= \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j + \bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i + 4(P-P_0)(P-P_0) \\
&\quad + C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i + C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j \\
M_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T (L_j - L_{j0})^T (L_j - L_{j0}) F_{d,i} \\
&\quad + F_{d,j}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{d,j} \\
N_{11}(ij) &= -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} - \bar{A}_{ji}^T Q - Q\bar{A}_{ji} + C_i^T C_j + C_j^T C_i \\
&\quad - 4(Q-Q_0)(Q-Q_0) - C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j \\
&\quad - C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i \\
N_{22}(ii) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T (L_{j0} - L_j)^T (L_{j0} - L_j) F_{f,i} \\
&\quad - F_{f,j}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{f,j}
\end{aligned}
$$

Remark 2 *Notice that if $P_0 = P$, $Q_0 = Q$, $L_{i0} = L_i$ and $L_{j0} = L_j$, then MIs (3.2.44) and (3.2.45) are the same as (3.2.42) and (3.2.43), respectively.*

MIs (3.2.44) and (3.2.45) can be represented as

$$
\begin{bmatrix}
\begin{bmatrix}
\phi_{11}(ij) - C_j^T L_i^T P - PL_iC_j \\
+4PP - C_i^T L_j^T P - PL_jC_i \\
+C_j^T L_i^T L_i C_j + C_i^T L_j^T L_j C_i
\end{bmatrix} & \phi_{12}(ij) - PL_jF_{d,i} - PL_iF_{d,j} \\
* & \begin{bmatrix}
\phi_{22}(ij) + F_{d,j}^T L_i^T L_i F_{d,j} \\
+F_{d,i}^T L_j^T L_j F_{d,i}
\end{bmatrix}
\end{bmatrix} < 0 \tag{3.2.46}
$$

$$
\begin{bmatrix}
\begin{bmatrix}
\psi_{11}(ij) + C_j^T L_i^T Q + QL_iC_j \\
+C_i^T L_j^T Q + QL_jC_i - 4QQ- \\
C_j^T L_i^T L_i C_j - C_i^T L_j^T L_j C_i
\end{bmatrix} & \psi_{12}(ij) + QL_jF_{f,i} + QL_iF_{f,j} \\
* & \begin{bmatrix}
\psi_{22}(ij) - F_{f,j}^T L_i^T L_i F_{f,j} \\
-F_{f,i}^T L_j^T L_j F_{f,i}
\end{bmatrix}
\end{bmatrix} < 0, \tag{3.2.47}
$$

where

$$
\begin{aligned}
\phi_{11}(ij) &= A_i^T P + PA_i + C_i^T C_j + A_j^T P + PA_j + C_j^T C_i + 4(P_0P_0 - P_0P - PP_0) \\
&\quad + C_i^T (L_{j0}^T L_{j0} - L_{j0} L_j - L_j^T L_{j0}) C_i + C_j^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_j \\
\phi_{12}(ij) &= PE_{d,j} + PE_{d,i} + C_i^T F_{d,j} + C_j^T F_{d,i} \\
\phi_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T (L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0}) F_{d,i} \\
&\quad + F_{d,j}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{d,j} \\
\psi_{11}(ij) &= -A_i^T Q - QA_i - A_j^T Q - QA_j + C_i^T C_j + C_j^T C_i - 4(Q_0Q_0 - Q_0Q - QQ_0) \\
&\quad - C_j^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_j - C_i^T (L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0}) C_i \\
\psi_{12}(ij) &= -QE_{f,j} - QE_{f,i} + C_i^T F_{f,j} + C_j^T F_{f,i} \\
\psi_{22}(ii) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,j} - F_{f,i}^T (L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0}) F_{f,i} \\
&\quad - F_{f,j}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{f,j}
\end{aligned}
$$

MIs (3.2.46) and (3.2.47) can be represented as some of matrices as:

$$\begin{bmatrix} \phi_{11}(ij) & \phi_{12}(ij) \\ * & \phi_{22}(ij) \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} (P - L_iC_j)^T(P - L_iC_j) \\ (P - L_jC_i)^T(P - L_jC_i) \\ PP + PP \end{bmatrix} & -PL_jF_{d,i} - PL_iF_{d,j} \\ * & F_{d,j}^T L_i^T L_i F_{d,j} + F_{d,i}^T L_j^T L_j F_{d,i} \end{bmatrix} < 0 \tag{3.2.48}$$

$$\begin{bmatrix} \psi_{11}(ij) & \psi_{12}(ij) \\ * & \psi_{22}(ij) \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} -(Q - L_iC_j)^T(Q - L_iC_j) \\ -(Q - L_jC_i)^T(Q - L_jC_i) \\ -QQ - QQ \end{bmatrix} & QL_jF_{f,i} + QL_iF_{f,j} \\ * & -F_{f,j}^T L_i^T L_i F_{f,j} - F_{f,i}^T L_j^T L_j F_{f,i} \end{bmatrix} < 0 \tag{3.2.49}$$

Using Schur complements lemma, MIs (3.2.30) and (3.2.31) are obtained, then the proof is therefore complete.

Remark 3 *If P_0, Q_0 and L_{i0} are fixed and known, then MIs (3.2.28)- (3.2.31) become LMIs in $P > 0$, $Q > 0$ and L_i, which can be solved via Mat lab LMI Tool Box.*

For given $\gamma > 0$, the solving algorithm of LMIs is represented in the following.

Algorithm 1 *Given $\beta > 0$, a small constant $\delta > 0$ and the iteration number L_n.*
***Step 1** : Set $L_i = 0$. Solve LMIs (3.2.32), (3.2.33), (3.2.42) and (3.2.43) for P and Q by choosing γ is big. Assign $P_0 = P$, $Q_0 = Q$, $L_{i0} = L_i$.*
***Step 2**: With obtained P_0, Q_0 and L_{i0}, solve LMIs (3.2.28)- (3.2.31) for new solutions P, Q and L_i by minimizing γ. Again, assign, $L_{i0} = L_i$, $P_0 = P$ and $Q_0 = Q$. Denote the jth iterative γ as γ^j.*
***Step 3**: Repeat the operation in step 2 till $|\gamma^{j+1} - \gamma^j| < \delta$, finally L_i and γ are obtained.*

Iterative linear matrix inequalities algorithm is represented by the following flowchart

3.3 Residual evaluation

After the designing of the fault generator, the remaining important task for robust fault detection is the evaluation of the generated residual. Based on LMI technique in [89], one can calculate the threshold value $J_{th} > 0$. Furthermore, we can use the following logic relationship for fault detection

$$\|r(t)\|_{2,T} \begin{cases} \leq J_{th} \rightarrow & \text{no fault} \\ > J_{th} \rightarrow & \text{alarm,} \end{cases} \tag{3.3.1}$$

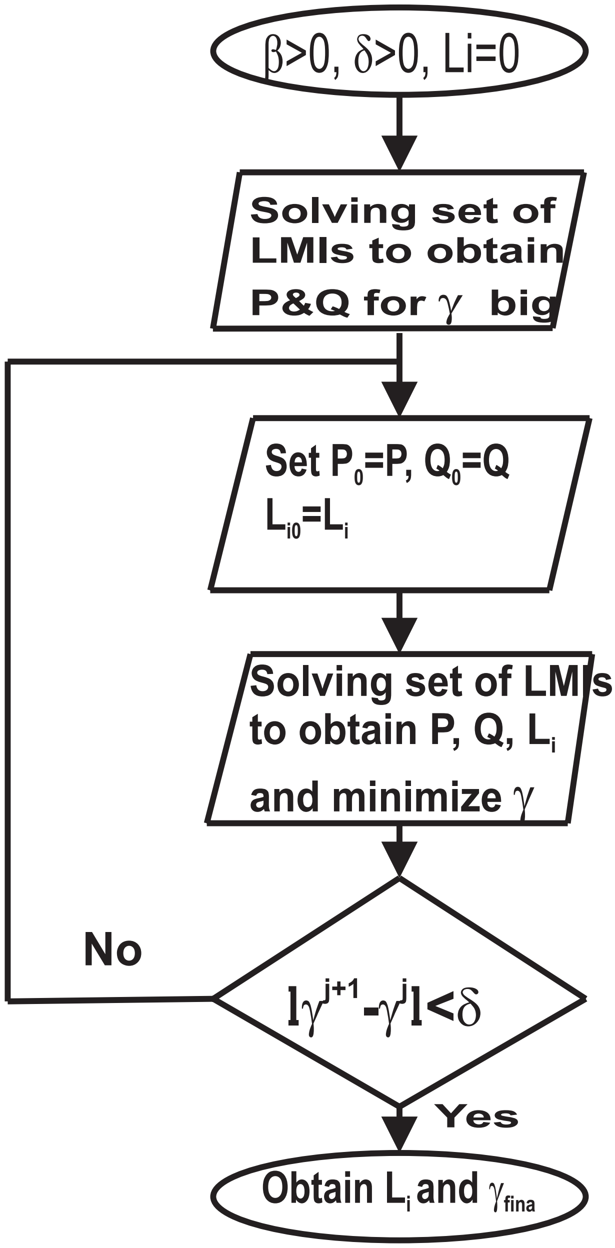

Figure 3.2: Iterative Linear Matrix Inequality Algorithm

where

$$\|r(t)\|_{2,T} = \left[\int_{t_1}^{t_2} r^T(t)r(t)dt\right]^{\frac{1}{2}}, \tag{3.3.2}$$

$T = t_2 - t_1$ and $t \in [t_1, t_2]$ is the finite-time window. Note that the length of the time is finite (i.e. T instead of ∞). Since an evaluation of the signal over the whole time range is impractical, it is desired that the fault will be detected as easy as possible. Based on (3.2.4) we have

$$\|r(t)\|_{2,T} = \|r_{d(t)}(t) + r_{f(t)}(t)\|_{2,T},$$

where $r_{d(t)}(t) = r(t)|_{f(t)=0}$, $r_{f(t)}(t) = r(t)|_{d(t)=0}$. Moreover, the fault-free case residual evaluation function is defined as:

$$\|r(t)\|_{2,T} \leq \|r_{d(t)}(t)\|_{2,T} \leq J_{th,d(t)},$$

where $J_{th,d(t)} = \sup_{d(t)\in L_2} \|r_{d(t)}(t)\|_{2,T}$.

We choose the threshold J_{th} as $J_{th} = J_{th,d(t)}$. This value is constant and can be evaluated off-line.

3.4 Example

Consider the following nonlinear system

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(0.67x_1^2 + 1) & -1.726 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3.4.1}$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Assume that $x_1 = [-1, 1]$ and $x_2 = [-1, 1]$. In order to obtain the TS fuzzy model, it is necessary to define one premise variables. This variable represents a nonlinear term

$$z_1(t) = (1 + 0.67x_1^2)$$

Insert with premise variable, (3.4.1) represented as

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -z_1(t) & -1.726 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{3.4.2}$$

$$y(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The minimum and maximum values of $z_1(t)$ is

$$max(z_1) = z_1^+ = 1.67, \quad min(z_1) = z_1^- = 1$$

From the maximum and minimum values, the membership functions for $z_1(t)$ are calculated as follows:

$$F_{11}(z_1^+) = \mu_1 = \frac{z_1 - z_1^-}{z_1^+ - z_1^-} = \frac{z_1 - 0.67}{0.67}, \quad F_{12}(z_1^-) = \mu_2 = \frac{z_1^+ - z_1}{z_1^+ - z_1^-} = \frac{1.67 - z_1}{0.67}$$

System (3.4.1) is represented by the following fuzzy rules:

Model Rule 1
If $z_1(t)$ is F_{11}

$$THEN \begin{cases} \dot{x}(t) & = A_1 x(t) + B_1 u(t) \\ y(t) & = C_1 x(t) \end{cases}$$

Model Rule 2
If $z_1(t)$ is F_{12}

$$THEN \begin{cases} \dot{x}(t) & = A_2 x(t) + B_2 u(t) \\ y(t) & = C_2 x(t) \end{cases}$$

TS fuzzy model used in fault detection filter design with deterministic disturbances and faults is represented as:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i [A_i x(t) + B_i u(t) + E_{d,i} d(t) + E_{f,i} f(t)]$$
$$y(t) = \sum_{i=1}^{2} \mu_i [C_i x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t)],$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.726 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -1.67 & -1.726 \end{bmatrix}, \quad E_{d,1} = E_{d,2} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad E_{f,1} = E_{f,2} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$$

$$D_1 = D_2 = 0.5 \qquad F_{d,1} = F_{d,2} = 0.5, \qquad F_{f,1} = F_{f,2} = 0.3$$

$$\mu_1 = 1 - x^2, \qquad \mu_2 = x^2$$

Applying the procedure of iterative linear matrix inequality algorithm, the following values for gain matrix L_i, P and Q are given:

$$L_1 = \begin{bmatrix} -0.0837 \\ 0.0877 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0832 \\ 0.0502 \end{bmatrix}, \quad P = \begin{bmatrix} 14.1409 & 4.1154 \\ 4.1154 & 6.4959 \end{bmatrix}, \quad Q = \begin{bmatrix} 11.1032 & 3.358 \\ 3.358 & 6.0011 \end{bmatrix}$$

With $\gamma = 1.6901$, $\beta = 2.5$, based on the unknown input and model uncertainty as shown in figure 3.3(a), the threshold value is $J_{th} = 0.1726$. The displacement sensor fault occurred at $t = 15$ seconds with offset 10% as shown in Figure 3.3(b). In Figure 3.3(c), it is noticed that, the evaluation function is greater than the threshold from $t = 15$ seconds. In the fault evaluation a function time window with 5 seconds is used.

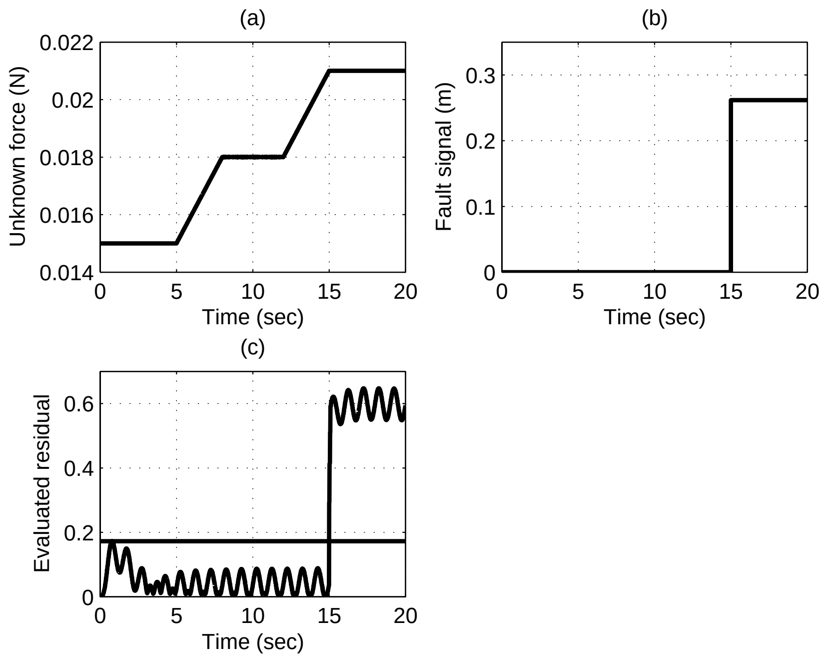

Figure 3.3: Fault Detection for a System with Unknown Inputs

3.5 Summary

In this chapter, a method for robust fault detection system for nonlinear systems has been investigated. It applies to systems corrupted by deterministic disturbances. This FD system ensures simultaneously the disturbances attenuation and sensitivity to faults. To obtain robust FD system based on H_∞/H_- performance index, ILMIs algorithm is used.

4 Robust fuzzy fault detection for a nonlinear system with parametric uncertainty

In this chapter, a fault detection system for continuous-time nonlinear dynamic system with deterministic disturbances and parametric uncertainties is studied. The fault detection process consists of residual generation and residual evaluation. In residual generation robust fuzzy filter is designed to produce the residual signal. The generated residual signal is as sensitive as possible to fault and as robust as possible to deterministic disturbance and parameter uncertainty. In residual evaluation the evaluation function and threshold calculation are also studied.

4.1 TS fuzzy model

Proposed by Takagi and Sugeno [99], a TSFM consists of a number of fuzzy rules and corresponding local models. Let p be the number of the fuzzy rules. Suppose that the i-th rule is described by

Rule i: IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} \dot{x}(t) &= (A_i + \triangle A_i)x(t) + B_i u(t) + (E_{d,i} + \triangle E_{d,i})d(t) + E_{f,i}f(t) \\ y(t) &= C_i x(t) + D_i u(t) + F_{d,i}d(t) + F_{f,i}f(t), \end{aligned} \tag{4.1.1}$$

where $M_{ij}(i = 1, 2, \ldots, p, j = 1, \cdots, \theta)$ are fuzzy sets; $z = [z_1, \ldots, z_\theta]$ are premise variables, $x(t) \in \mathcal{R}^n$ is state vector; $u(t) \in \mathcal{R}^{k_u}$ and $y(t) \in \mathcal{R}^m$ are the input and measured output vectors respectively; $d(t) \in \mathcal{R}^{k_d}$ is the deterministic disturbance; $f(t) \in \mathcal{R}^{k_f}$ is unknown fault vector acting on system. The matrices $A_i, B_i, E_{d,i}, E_{f,i}, C_i, D_i, F_{d,i}, F_{f,i}$ are of appropriate dimension, $\triangle A_i$ and $\triangle E_{d,i}$ are time-varying matrices with appropriate dimensions, which represent parametric uncertainties in the process. The uncertainties are assumed to be norm-bounded and are given by:

$$\begin{bmatrix} \triangle A_i(t) & \triangle E_{d,i}(t) \end{bmatrix} = E_i F_i(t) \begin{bmatrix} H_{1i} & H_{3i} \end{bmatrix}, \tag{4.1.2}$$

where E_i, H_{1i} and H_{3i} are known constant matrices of appropriate dimension and $F_i(t) \in R^{n_{f_1} \times n_{f_2}}$ are unknown nonlinear time-varying matrix functions satisfying

$$\|F_i(t)\|_2 \le 1$$

This type of uncertainties is an effective representation of nonlinear uncertainties see [106]. The defuzzified output of TS fuzzy system (4.1.1) is represented as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{p} \mu_i[(A_i + \triangle A_i)x(t) + B_i u(t) + (E_{d,i} + \triangle E_{d,i})d(t) + E_{f,i}f(t)] \\ y(t) &= \sum_{i=1}^{p} \mu_i[C_i x(t) + D_i u(t) + F_{d,i}d(t) + F_{f,i}f(t)] \end{aligned} \tag{4.1.3}$$

4.2 Residual generation

4.2.1 Fuzzy filter design

The first step to achieve a robust FD system is to generate a residual signal which is decoupled from the known input signal $u(t)$. In this study, a TSFM based fault detection filter is considered. This filter is described as follows:
Rule i : IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} \dot{\hat{x}}(t) &= A_i\hat{x}(t) + B_i u(t) + L_i[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= C_i\hat{x}(t) + D_i u(t) \\ r(t) &= y(t) - \hat{y}(t), \end{aligned} \tag{4.2.1}$$

where L_i is the filter gain matrix for sub-model i and $r(t)$ is the residual signal. Using the idea of PDC [51], the overall state estimation is a nonlinear fuzzy combination of the individual local filter output. The overall filter dynamics will be a weighted sum of individual linear filters.

$$\begin{aligned} \dot{\hat{x}}(t) &= \sum_{i=1}^{p} \mu_i[A_i\hat{x}(t) + B_i u(t) + L_i(y(t) - \hat{y}(t))] \\ \hat{y}(t) &= \sum_{i=1}^{p} \mu_i[C_i\hat{x}(t) + D_i u(t)] \\ r(t) &= y(t) - \hat{y}(t), \end{aligned} \tag{4.2.2}$$

where μ_i is the same weight function used in TS model (4.1.3). Using fuzzy filter in residual generation as shown in Figure 3.1. To analyze the stability of the filter, the state error vector $e(t) = x(t) - \hat{x}(t)$ is given by the following differential equation.

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j[\triangle A_i x(t) + (A_i - L_i C_j)e(t) + \triangle E_{d,i}d(t) \\ &\quad + (E_{d,i} - L_i F_{d,j})d(t) + (E_{f,i} - L_i F_{f,j})f(t)] \\ r(t) &= \sum_{i=1}^{p} \mu_i[C_i e(t) + F_{d,i}d(t) + F_{f,i}f(t)] \end{aligned} \tag{4.2.3}$$

Equation (4.2.3) can be simplified as

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[\triangle A_i x(t) + \bar{A}_{ij}e(t) + \triangle E_{d,i}d(t) + \bar{E}_{d,ij}d(t) + \bar{E}_{f,ij}] \tag{4.2.4}$$
$$r(t) = \sum_{i=1}^{p}\mu_i[C_i e(t) + F_{d,i}d(t) + F_{f,i}f(t)],$$

where $\bar{A}_{ij} = A_i - L_iC_j$, $\bar{E}_{d,ij} = E_{d,i} - L_iF_{d,j}$ and $\bar{E}_{f,ij} = E_{f,i} - L_iF_{f,j}$. The dynamic of residual signal depends not only on $f(t)$ and $d(t)$ but also on the state $x(t)$. Thus, the problem of designing robust TS fuzzy fault detection filter can be described as designing the filter gain matrix L_i such that the following conditions are simultaneously fulfilled.

- $\bar{A}_{ij}$ is asymptotically stable for all subsystems A_i with $i, j = 1, \ldots, p$.
- The generated residual $r(t)$ is as sensitive as possible to fault $f(t)$ and as robust as possible to $d(t)$ (deterministic disturbance), and $\triangle A_i, \triangle E_{d,i}$ (process uncertainties).
- system (4.2.4) is robust stable, while the influence of modeling errors, deterministic disturbance and uncertainty on control output decreases. The aim of a robust fuzzy fault detection system is to satisfy the following performance index

$$\min_{L_i} J = \min_{L_i} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_-} \tag{4.2.5}$$

The robust fault detection design problem can thus be formulated as finding a fuzzy filter gain matrix L_i such that system (4.2.4) is asymptotically stable and the performance index (4.2.5) is made as small as possible in the feasibility of $\|G_{rd}\|_\infty < \gamma$, $\|G_{rf}\|_- > \beta$, $\beta > 0$ and $\gamma > 0$.

4.2.2 Robust fault detection filter design

The following theorem gives the LMIs formulation of H_∞ estimation problem. The main requirement of the H_∞ is to minimize disturbances and parameter uncertainties on residual signal $r(t)$.

Theorem 4 *System (4.2.4) with $f(t) = 0$, is asymptotically stable and satisfies $\|G_{rd}\|_\infty < \gamma$, if for $\gamma > 0$, and for $\epsilon_1 > 0$ and $\epsilon_3 > 0$, there exist a positive definite matrix $P > 0$ such that the following MIs are hold for $1 \le i \le p$ and $1 \le i < j \le p$, respectively*

$$\begin{bmatrix} M_{11}(ii) & P\bar{E}_{d,ii} + C_i^T F_{d,i} & PE_i & PE_i \\ * & M_{22}(ii) & 0 & 0 \\ * & * & -\epsilon_1 I & 0 \\ * & * & * & -\epsilon_3 I \end{bmatrix} < 0 \tag{4.2.6}$$

$$\begin{bmatrix} M_{11}(ij) & \begin{bmatrix} P\bar{E}_{d,ij} + C_i^T F_{d,j} \\ P\bar{E}_{d,ji} + C_j^T F_{d,i} \end{bmatrix} & PE_i & PE_i & PE_j & PE_j \\ * & M_{22}(ij) & 0 & 0 & 0 & 0 \\ * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & -\epsilon_3 I & 0 & 0 \\ * & * & * & * & -\epsilon_1 I & 0 \\ * & * & * & * & * & -\epsilon_3 I \end{bmatrix} < 0, \qquad (4.2.7)$$

where

$$M_{11}(ii) = P\bar{A}_{ii} + \bar{A}_{ii}^T P + \epsilon_1 H_{1i}^T H_{1i} + C_i^T C_i$$
$$M_{22}(ii) = -\gamma^2 I + \epsilon_3 H_{3i}^T H_{3i} + F_{d,i}^T F_{d,i}$$
$$M_{11}(ij) = P\bar{A}_{ij} + \bar{A}_{ij}^T P + P\bar{A}_{ji} + \bar{A}_{ji}^T P + \epsilon_1 H_{1i}^T H_{1i} + \epsilon_1 H_{1j}^T H_{1j} + C_i^T C_j + C_j^T C_i$$
$$M_{22}(ij) = -2\gamma^2 I + \epsilon_3 H_{3i}^T H_{3i} + \epsilon_3 H_{3j}^T H_{3j} + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i}$$

In order to prove this theorem, in addition to lemma 1, the following lemma is important, see [114] :

Lemma 2 *Let E, H and F be real matrices of appropriate dimensions with F satisfies $\|F\| \leq I$. Then for any real number $\epsilon > 0$ we have*

$$EFH + H^T F^T E^T \leq \epsilon^{-1} E E^T + \epsilon H^T H \qquad (4.2.8)$$

Proof of theorem 4. System (4.2.4) with $f(t) = 0$ is represented as:

$$\dot{e}(t) = \sum_{i=1}^{p} \sum_{j=1}^{p} \mu_i \mu_j [\bar{A}_{ij} e(t) + \triangle A_i x(t) + (\bar{E}_{d,ij} + \triangle E_{d,i}) d(t)] \qquad (4.2.9)$$
$$r(t) = \sum_{i=1}^{p} \mu_i [C_i e(t) + F_{d,i} d(t)]$$

The disturbance rejection can be realized by minimizing γ such that (3.2.9) is satisfied. Based on (3.2.10) the following inequality is obtained.

$$\dot{e}^T(t) P e(t) + e^T(t) P \dot{e}(t) + \sum_{i=1}^{p} \mu_i [C_i e(t) + F_{d,i} d(t)]^T \qquad (4.2.10)$$
$$\times \sum_{j=1}^{p} \mu_j [C_j e(t) + F_{d,j} d(t)] - \gamma^2 d^T(t) d(t) < 0$$

Insert with system (4.2.9) in equation (4.2.10), the following equation is obtained

$$\sum_{i=1}^{p} \sum_{j=1}^{p} \mu_i \mu_j [e^T(t) \bar{A}_{ij}^T P e(t) + x^T \triangle A_i^T P e(t) + d^T(t) \bar{E}_{d,ij}^T P e(t) + d^T(t) \triangle E_{d,i}^T P e(t) \qquad (4.2.11)$$
$$+ e^T(t) P \bar{A}_{ij} e(t) + e^T P \triangle A_i x(t) + e^T(t) P \bar{E}_{d,ij} d(t) + e^T(t) P \triangle E_{d,i} d(t)$$
$$+ e^T(t) C_i^T C_j e(t) + e^T(t) C_i^T F_{d,j} d(t) + d^T(t) F_{d,i}^T C_j e(t) + d^T(t) F_{d,i}^T F_{d,j} d(t)$$
$$- \gamma^2 d^T(t) d(t) < 0$$

Based on Lemma 2, we have

$$x^T(t)\triangle A_i^T Pe(t) + e^T(t)P\triangle A_i x(t) = x^T(t)H_{1i}^T F^T E_i^T Pe(t) + e^T(t)PE_iFH_{1i}x(t) \leq \\ x^T(t)\epsilon_1^{-1}PE_iE_i^TPx(t) + e^T(t)\epsilon_1 H_{1i}^T H_{1i}e(t) \tag{4.2.12}$$

$$d^T(t)\triangle E_{d,i}^T Pe(t) + e^T(t)P\triangle E_{d,i}d(t) = d^T(t)H_{3i}^T F^T E_i^T Pe(t) + e^T(t)PE_iFH_{3i}d(t) \leq \\ e^T(t)\epsilon_2^{-1}PE_iE_i^TPe(t) + d^T(t)\epsilon_2 H_{3i}^T H_{3i}d(t) \tag{4.2.13}$$

Insert with equations (4.2.12) and (4.2.13) in equation (4.2.11). Using the same sequence in chapter 3, LMIs (4.2.6) and (4.2.7) are obtained.

The following theorem gives the LMIs formulation of H_- estimation problem. This problem can be defined as follows: determine L_i such that the H_- norm of the transfer function from fault to the residual vector is maximize by a given $\beta > 0$.

Theorem 5 *System (4.2.4) with $d(t) = 0$ is asymptotically stable and satisfies $\|G_{rf}\|_- > \beta$ if for $\beta > 0$ and $\epsilon_1 > 0$, there exists a matrix $Q > 0$ such that the following MIs are satisfied for $1 \leq i \leq p$ and $1 \leq i < j < p$ respectively.*

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) & QE_i \\ * & -\beta^2 I + F_{f,i}^T F_{f,i} & 0 \\ * & * & \epsilon_1 I \end{bmatrix} > 0 \tag{4.2.14}$$

$$\begin{bmatrix} N_{11}(ij) & N_{12}(ij) & QE_i & QE_j \\ * & N_{22}(ij) & 0 & 0 \\ * & * & \epsilon_1 I & 0 \\ * & * & * & \epsilon_1 I \end{bmatrix} > 0, \tag{4.2.15}$$

where

$$\begin{aligned}
N_{11}(ii) &= -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i - \epsilon_1 H_{1i}^T H_{1i} \\
N_{12}(ii) &= -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\
N_{11}(ij) &= -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} - \bar{A}_{ji}^T Q - Q\bar{A}_{ji} + C_i^T C_j + C_j^T C_i - \epsilon_1 H_{1i}^T H_{1j} - \epsilon_1 H_{1j}^T H_{1i} \\
N_{12}(ij) &= -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} + C_j^T F_{f,i} + C_i^T F_{f,j} \\
N_{22}(ij) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i}
\end{aligned}$$

Proof of theorem 5 System (4.2.4) with $d(t) = 0$ is represented as

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j[\bar{A}_{ij}e(t) + \triangle A_i x(t) + \bar{E}_{f,ij}f(t)] \tag{4.2.16}$$

$$r(t) = \sum_{i=1}^{p} \mu_i[C_i e(t) + F_{f,i}f(t)]$$

System (4.2.16) is stable and is sensitive to fault if condition (3.2.20) is satisfied. Based on condition (3.2.21) and system (4.2.16) and using the same sequence in theorem 2, LMIs (4.2.14) and (4.2.15) are obtained.

Performance index (4.2.5) is satisfied if there exists a gain matrix L_i such that MIs (4.2.6), (4.2.7), (4.2.14) (4.2.15) can be simultaneously solved. Optimal solution is given for γ minimal and β maximal. Isolability of faults is then ensured when γ and β can be found such that $\gamma < \beta$. Unfortunately, this requirement can not be satisfied so iterative linear matrix inequality is used.

4.2.3 Iterative linear matrix inequality approach

In this section, ILMIs algorithm is used to solve H_∞/H_- problem. A new LMI formulation is proposed to ensure disturbance attenuation and fault sensitivity.

Theorem 6 *For given $\alpha > 0$ and $\gamma > 0$, system (4.2.4) with L_i is asymptotically stable and satisfies (4.2.5), if there exist $\epsilon_1 > 0$, $\epsilon_3 > 0$, $P > 0$, $Q > 0$, P_0, Q_0, L_i and L_{i0} such that the following LMIs are satisfied for $1 \le i \le p$ and for $1 \le i < j \le p$ respectively at the same time.*

$$\begin{bmatrix} \Lambda_{11}(ii) & \Lambda_{12}(ii) & PE_i & PE_i & (P-L_iC_i)^T & P \\ * & \Lambda_{22}(ii) & 0 & 0 & 0 & -F_{d,i}^T L_i^T \\ * & * & -\epsilon_1 I & 0 & 0 & 0 \\ * & * & * & -\epsilon_3 I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0 \tag{4.2.17}$$

$$\begin{bmatrix} \Pi_{11}(ii) & \Pi_{12}(ii) & QE_i & (Q-L_iC_i)^T & Q \\ * & \Pi_{22}(ii) & 0 & 0 & -F_{f,i}^T L_i^T \\ * & * & \epsilon_1 I & 0 & 0 \\ * & * & * & I & 0 \\ * & * & * & * & I \end{bmatrix} > 0, \tag{4.2.18}$$

where

$$\begin{aligned}
\Lambda_{11} &= A_i^T P + PA_i + C_i^T C_i + 2(P_0P_0 - P_0P - PP_0) + C_i^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_i \\
&\quad + \epsilon_1 H_{1i}^T H_{1i} \\
\Lambda_{12} &= C_i^T F_{d,i} + PE_{d,i} \\
\Lambda_{22} &= -\gamma^2 I + \epsilon_3 H_{3i}^T H_{3i} + F_{d,i}^T F_{d,i} + F_{d,i}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,i} \\
\Pi_{11} &= C_i^T C_i - A_i^T Q - QA_i - 2(Q_0Q_0 - Q_0Q - QQ_0) - C_i^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_i \\
&\quad - \epsilon_1 H_{1i}^T H_{1i} \\
\Pi_{12} &= C_i^T F_{f,i} - QE_{f,i} \\
\Pi_{22} &= -\beta^2 I + F_{f,i}^T F_{f,i} - F_{f,i}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{f,i}
\end{aligned}$$

$$
\begin{bmatrix}
\Theta_{11}(ij) & \Theta_{12}(ij) & PE_i & PE_i & PE_j & PE_j & (P-L_iC_j)^T & P & (P-L_jC_i)^T & P \\
* & \Theta_{22}(ij) & 0 & 0 & 0 & 0 & 0 & -F_{d,j}^T L_i^T & 0 & -F_{d,i}^T L_j^T \\
* & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\epsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\epsilon_3 I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & -I & 0 & 0 \\
* & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & -I \\
* & * & * & * & * & * & * & * & * & *
\end{bmatrix}
$$

$$
\begin{bmatrix}
\Theta_{11}(ij) & \Theta_{12}(ij) & PE_i & PE_i & PE_j & PE_j & (P-L_iC_j)^T & P & (P-L_jC_i)^T & P \\
* & \Theta_{22}(ij) & 0 & 0 & 0 & 0 & 0 & -F_{d,j}^T L_i^T & 0 & -F_{d,i}^T L_j^T \\
* & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & -\epsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -\epsilon_1 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\epsilon_3 I & 0 & 0 & 0 & 0 \\
* & * & * & * & * & * & -I & 0 & 0 & 0 \\
* & * & * & * & * & * & * & -I & 0 & 0 \\
* & * & * & * & * & * & * & * & -I & 0 \\
* & * & * & * & * & * & * & * & * & -I
\end{bmatrix} < 0, \tag{4.2.19}
$$

where

$$
\begin{aligned}
\Theta_{11} &= A_i^T P + PA_i + A_j^T P + PA_j + C_i^T C_j + C_j^T C_i + 4(P_0P_0 - P_0P - PP_0) \\
&\quad + C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i + C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_j \\
&\quad + \epsilon_1 H_{1i}^T H_{1i} + \epsilon_1 H_{1j}^T H_{1j} \\
\Theta_{12} &= PE_{d,i} + PE_{d,j} + C_i^T F_{d,j} + C_j^T F_{d,i} \\
\Theta_{22} &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{d,i} \\
&\quad + F_{d,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,j} + \epsilon_3 H_{3i}^T H_{3i} + \epsilon_3 H_{3j}^T H_{3j}
\end{aligned}
$$

$$
\begin{bmatrix}
\Upsilon_{11}(ij) & \Upsilon_{12}(ij) & QE_i & QE_j & (Q-L_iC_j)^T & Q & (Q-L_jC_i)^T & Q \\
* & \Upsilon_{22}(ij) & 0 & 0 & 0 & -F_{f,j}^T L_i^T & 0 & -F_{f,i}^T L_j^T \\
* & * & \epsilon_1 I & 0 & 0 & 0 & 0 & 0 \\
* & * & * & \epsilon_1 I & 0 & 0 & 0 & 0 \\
* & * & * & * & I & 0 & 0 & 0 \\
* & * & * & * & * & I & 0 & 0 \\
* & * & * & * & * & * & I & 0 \\
* & * & * & * & * & * & * & I
\end{bmatrix} > 0, \tag{4.2.20}
$$

where

$$\begin{aligned}
\Upsilon_{11} &= -A_i^T Q - QA_i - A_j^T Q - QA_j + C_i^T C_j + C_j^T C_i - 4(-Q_0 Q - QQ_0 + Q_0 Q_0) \\
&\quad - C_i^T (L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0}) C_i - C_j^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_j \\
&\quad - \epsilon_1 H_{1i}^T H_{1i} - \epsilon_1 H_{1j}^T H_{1j} \\
\Upsilon_{12} &= C_i^T F_{f,j} + C_j^T F_{f,i} - QE_{f,i} - QF_{f,j} \\
\Upsilon_{22} &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T (L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0}) F_{f,i} \\
&\quad - F_{f,j}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{f,j}
\end{aligned}$$

Proof of theorem 6 (i) To obtain LMIs (4.2.17) and (4.2.18), for given $\gamma > 0$ and $\beta > 0$, from Theorem 4 and Theorem 5 system (4.2.4) is asymptotically stable and satisfy (4.2.5), if there exist matrices $P > 0$, $Q > 0$ and L_i such that the following MIs are satisfied.

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i + \epsilon_1^{-1} PE_i E_i^T P \\ +\epsilon_3^{-1} PE_i E_i^T P + \epsilon_1 H_{1i}^T H_{1i} \end{bmatrix} & P\bar{E}_{d,ii} + C_i^T F_{d,i} \\ * & -\gamma^2 I + F_{d,i}^T F_{d,i} + \epsilon_3 H_{3i}^T H_{3i} \end{bmatrix} < 0 \qquad (4.2.21)$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i \\ -\epsilon_1^{-1} QE_i E_i^T Q - \epsilon_1 H_{1i}^T H_{1i} \end{bmatrix} & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\ * & -\beta^2 I + F_{f,i}^T F_{f,i} \end{bmatrix} > 0 \qquad (4.2.22)$$

For any P_0, Q_0 and L_0 MIs (4.2.21) and (4.2.22) can be represented as:

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i \\ +2(P - P_0)(P - P_0) + \epsilon_1 H_{1i}^T H_{1i} \\ +C_i^T (L_i - L_{i0})^T (L_i - L_{i0}) C_i + \\ \epsilon_1^{-1} PE_i E_i^T P + \epsilon_3^{-1} PE_i E_i^T P \end{bmatrix} & P\bar{E}_{d,ii} + C_i^T F_{d,i} \\ * & \begin{bmatrix} -\gamma^2 I + F_{d,i}^T F_{d,i} + \epsilon_3 H_{3i}^T H_{3i} \\ +F_{d,i}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{d,i} \end{bmatrix} \end{bmatrix} < 0 \qquad (4.2.23)$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i \\ -2(Q - Q_0)(Q - Q_0) - \epsilon_1 H_{1i}^T H_{1i} \\ -C_i^T (L_i - L_{i0})^T (L_i - L_{i0}) C_i \\ -\epsilon_1^{-1} PE_i E_i^T P \end{bmatrix} & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} \\ * & \begin{bmatrix} -\beta^2 I + F_{f,i}^T F_{f,i} - \\ F_{f,i}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{f,i} \end{bmatrix} \end{bmatrix} > 0 \qquad (4.2.24)$$

Remark 4 *Notice that if $P_0 = P$, $Q_0 = Q$, $L_{i0} = L_i$ and $L_{j0} = L_j$, then MIs (4.2.23) and (4.2.24) are the same as (4.2.21) and (4.2.22), respectively.*

MIs (4.2.23) and (4.2.24) can be represented as

$$\begin{bmatrix} \begin{bmatrix} M_{11}(ii) - C_i^T L_i^T P - P L_i C_i + \\ \epsilon_1^{-1} P E_i E_i^T P + \epsilon_3^{-1} P E_i E_i^T P \\ +2PP + C_i^T L_i^T L_i C_i \end{bmatrix} & M_{12}(ii) - P L_i F_{d,i} \\ * & M_{22}(ii) + F_{d,i}^T L_i^T L_i F_{d,i} \end{bmatrix} < 0 \quad (4.2.25)$$

$$\begin{bmatrix} \begin{bmatrix} N_{11}(ii) + C_i^T L_i^T Q + Q L_i C_i - \epsilon_1^{-1} Q E_i E_i Q \\ -2QQ - C_i^T L_i^T L_i C_i \end{bmatrix} & N_{12}(ii) + Q L_i F_{f,i} \\ * & N_{22}(ii) - F_{f,i}^T L_i^T L_i F_{f,i} \end{bmatrix} > 0, \quad (4.2.26)$$

where

$$\begin{aligned} M_{11}(ii) &= A_i^T P + P A_i + C_i^T C_i + 2(P_0 P_0 - P P_0 - P_0 P) + C_i^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_i \\ &\quad + \epsilon_1 H_{1i}^T H_{1i} \\ M_{12}(ii) &= P E_{d,i} + C_i^T F_{d,i} \\ M_{22}(ii) &= -\gamma^2 I + F_{d,i}^T F_{d,i} + F_{d,i}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{d,i} + \epsilon_3 H_{3i}^T H_{3i} \\ N_{11}(ii) &= -A_i^T Q - Q A_i - 2(Q_0 Q_0 - Q_0 Q - Q Q_0) - C_i^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) C_i \\ &\quad + C_i^T C_i - \epsilon_1 H_{1i}^T H_{1i} \\ N_{12}(ii) &= -Q E_{f,i} + C_i^T F_{f,i} \\ N_{22}(ii) &= -\beta^T I + F_{f,i}^T F_{f,i} - F_{f,i}^T (L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0}) F_{f,i} \end{aligned}$$

MIs (4.2.25) and (4.2.26) can be represented as

$$\begin{aligned} &\begin{bmatrix} M_{11}(ii) & M_{12}(ii) \\ * & M_{22}(ii) \end{bmatrix} + \begin{bmatrix} \epsilon_1^{-1} P E_i E_i^T P + \epsilon_3^{-1} P E_i E_i P & 0 \\ 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} \begin{bmatrix} -C_i^T L_i^T P - P L_i C_i \\ +2PP + C_i^T L_i^T L_i C_i \end{bmatrix} & -P L_i F_{d,i} \\ * & F_{d,i}^T L_i^T L_i F_{d,i} \end{bmatrix} < 0 \end{aligned} \quad (4.2.27)$$

$$\begin{aligned} &\begin{bmatrix} N_{11}(ii) & N_{12}(ii) \\ * & N_{22}(ii) \end{bmatrix} + \begin{bmatrix} -\epsilon_1^1 Q E_i E_i^T Q & 0 \\ 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} C_i^T L_i^T Q + Q L_i C_i - 2QQ - C_i^T L_i^T L_i C_i & Q L_i F_{f,i} \\ * & -F_{f,i}^T L_i^T L_i F_{f,i} \end{bmatrix} > 0, \end{aligned} \quad (4.2.28)$$

then (4.2.27) and (4.2.28) can be rewritten as:

$$\begin{aligned} &\begin{bmatrix} M_{11}(ii) & M_{12}(ii) \\ * & M_{22}(ii) \end{bmatrix} + \begin{bmatrix} P E_i & P E_i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_1^{-1} I & 0 \\ 0 & \epsilon_3^{-1} I \end{bmatrix} \begin{bmatrix} E_i^T P & 0 \\ E_i^T P & 0 \end{bmatrix} \\ &+ \begin{bmatrix} (P - L_i C_i)^T & P \\ 0 & -F_{d,i}^T L_i^T \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} (P - L_i C_i) & 0 \\ P & -L_i F_{d,i} \end{bmatrix} < 0 \end{aligned} \quad (4.2.29)$$

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) \\ * & N_{22}(ii) \end{bmatrix} + \begin{bmatrix} QE_i \\ 0 \end{bmatrix} [-\epsilon_1^{-1}] \begin{bmatrix} E_i^T Q & 0 \end{bmatrix}$$
$$+ \begin{bmatrix} (Q - L_iC_i)^T & Q \\ 0 & -F_{f,i}^T L_i^T \end{bmatrix} \begin{bmatrix} -I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} (Q - L_iC_i) & 0 \\ Q & -L_iF_{f,i} \end{bmatrix} > 0 \quad (4.2.30)$$

Using Schur complements lemma for (4.2.29) and (4.2.30), then LMIs (4.2.17) and (4.2.18) are obtained.

(ii) To obtain LMIs (4.2.19) and (4.2.20), for given $\gamma > 0$ and $\beta > 0$, from Theorem 4 and Theorem 5, we know that system (4.2.4) is asymptotically stable and satisfy (4.2.5), if there exist matrices $P > 0$, $Q > 0$ and L_i such that the following MIs are satisfied:

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j \\ +\bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i \\ +\epsilon_1^{-1} PE_jE_j^T P + \epsilon_1^{-1} PE_iE_i^T P \\ +\epsilon_1 H_{1i}^T H_{1i} + \epsilon_1 H_{1j}^T H_{1j} \\ +\epsilon_3^{-1} PE_iE_i^T P + \epsilon_3^{-1} PE_jE_j^T P \end{bmatrix} & P\bar{E}_{d,ij} + P\bar{E}_{d,ji} + C_j^T F_{d,i} + C_i^T F_{d,j} \\ * & \begin{bmatrix} -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} \\ +\epsilon_3 H_{3i}^T H_{3i} + \epsilon_3 H_{3j}^T H_{3j} \end{bmatrix} \end{bmatrix} < 0 \quad (4.2.31)$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} + C_i^T C_j \\ -\bar{A}_{ji}^T Q - Q\bar{A}_{ji} + C_j^T C_i \\ -\epsilon_{1i}^{-1} PE_iE_i^T - \epsilon_1^{-1} QE_jE_j^T Q \\ -\epsilon_1 H_{1i}^T H_{1i} - \epsilon_1 H_{1j}^T H_{1j} \end{bmatrix} & -Q\bar{E}_{f,ij} - QE_{f,ji} + C_i^T F_{f,j} + C_j^T F_{f,i} \\ * & -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} \end{bmatrix} > 0 \quad (4.2.32)$$

For any P_0, Q_0 and L_0 MIs (4.2.31) and (4.2.32) can be represented as

$$\begin{bmatrix} M_{11}(ij) & P\bar{E}_{d,ij} + P\bar{E}_{d,ji} + C_i^T F_{d,j} + C_j^T F_{d,i} \\ * & M_{22}(ij) \end{bmatrix} < 0 \quad (4.2.33)$$

$$\begin{bmatrix} N_{11}(ij) & -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} + C_i^T F_{f,j} + C_j^T F_{f,i} \\ * & N_{22}(ij) \end{bmatrix} > 0, \quad (4.2.34)$$

where

$$\begin{aligned} M_{11}(ij) = {}& \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j + \bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i + 4(P - P_0)(P - P_0) \\ & + C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i + C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j + \epsilon_1^{-1} PE_jE_j^T P \\ & + \epsilon_1^{-1} PE_iE_i^T P + \epsilon_1 H_{1i}^T H_{1i} + \epsilon_1 H_{1j}^T H_{1j} + \epsilon_3^{-1} PE_iE_i^T P + \epsilon_3^{-1} PE_jE_j^T P \\ M_{22}(ij) = {}& -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T (L_j - L_{j0})^T (L_j - L_{j0}) F_{d,i} + F_{d,j}^T (L_i - L_{i0})^T \\ & (L_i - L_{i0}) F_{d,j} + \epsilon_3 H_{3i}^T H_{3i} + \epsilon_3 H_{3j}^T H_{3j} \\ N_{11}(ij) = {}& -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} - \bar{A}_{ji}^T Q - Q\bar{A}_{ji} - 4(Q - Q_0)(Q - Q_0) \\ & - C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j - C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i - \epsilon_1^{-1} PE_iE_i^T P \\ & - \epsilon_1^{-1} PE_jE_j^T P - \epsilon_1 H_{1i}^T H_{1i} - \epsilon_{1j} H_{1j}^T H_{1j} + C_i^T C_j + C_j^T C_i \\ N_{22}(ii) = {}& -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,j} - F_{f,i}^T (L_j - L_{j0})^T (L_j - L_{j0}) F_{f,i} - \\ & F_{f,j}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{f,j} \end{aligned}$$

Remark 5 *Notice that if $P_0 = P$, $Q_0 = Q$, $L_{i0} = L_i$ and $L_{j0} = L_j$, then MI (4.2.33) and (4.2.34) are the same as (4.2.31) and (4.2.32), respectively.*

MIs (4.2.33) and (4.2.34) can be represented as

$$\begin{bmatrix} \begin{bmatrix} \phi_{11}(ij) - C_j^T L_i^T P - PL_iC_j \\ +4PP - C_i^T L_j^T P - PL_jC_i \\ +C_j^T L_i^T L_iC_j + C_i^T L_j^T L_jC_i \\ +\epsilon_1^{-1} PE_iE_i^T P + \epsilon_1^{-1} PE_jE_j^T P \\ +\epsilon_3^{-1} PE_iE_i^T P + \epsilon_3^{-1} PE_jE_j^T P \end{bmatrix} & \phi_{12}(ij) - PL_jF_{d,i} - PL_iF_{d,j} \\ * & \begin{bmatrix} \phi_{22}(ij) + F_{d,j}^T L_i^T L_iF_{d,j} \\ +F_{d,i}^T L_j^T L_jF_{d,i} \end{bmatrix} \end{bmatrix} < 0 \tag{4.2.35}$$

$$\begin{bmatrix} \begin{bmatrix} \psi_{11}(ij) + C_j^T L_i^T Q + QL_iC_j \\ +C_i^T L_j^T Q + QL_jC_i - 4QQ- \\ C_j^T L_i^T L_iC_j - C_i^T L_j^T L_jC_i \\ -\epsilon_1^{-1} QE_iE_i^T Q - \epsilon_1^{-1} QE_jE_j^T Q \end{bmatrix} & \psi_{12}(ij) + QL_jF_{f,i} + QL_iF_{f,j} \\ * & \begin{bmatrix} \psi_{22}(ij) - F_{f,j}^T L_i^T L_iF_{f,j} \\ -F_{f,i}^T L_j^T L_jF_{f,i} \end{bmatrix} \end{bmatrix} < 0, \tag{4.2.36}$$

where

$$\begin{aligned}
\phi_{11}(ij) &= A_i^T P + PA_i + C_i^T C_j + A_j^T P + PA_j + C_j^T C_i + 4(P_0P_0 - P_0P - PP_0) \\
&\quad + C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i + C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{j0})C_j + \\
&\quad \epsilon_1 H_{1i}^T H_{1i} + \epsilon_1 H_{1j}^T H_{1j} \\
\phi_{12}(ij) &= PE_{d,j} + PE_{d,i} + C_i^T F_{d,j} + C_j^T F_{d,i} \\
\phi_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{d,i} \\
&\quad + F_{d,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,j} + \epsilon_3 H_{3i}^T H_{3i} + \epsilon_3 H_{3j}^T H_{3j} \\
\psi_{11}(ij) &= -A_i^T Q - QA_i - A_j^T Q - QA_j + C_i^T C_j + C_j^T C_i - 4(Q_0Q_0 - Q_0Q - QQ_0) \\
&\quad - C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_j - C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i - \\
&\quad \epsilon_1 H_{1i}^T H_{1i} - \epsilon_1 H_{1j}^T H_{1j} \\
\psi_{12}(ij) &= -QE_{f,j} - QE_{f,i} + C_i^T F_{f,j} + C_j^T F_{f,i} \\
N_{22}(ii) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{f,i} \\
&\quad - F_{f,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{f,j}
\end{aligned}$$

MIs (4.2.35) and (4.2.36) respectively, can be represented as:

$$\begin{bmatrix} \phi_{11}(ij) & \phi_{12}(ij) \\ * & \phi_{22}(ij) \end{bmatrix} + \begin{bmatrix} \begin{bmatrix} \epsilon_1^{-1} PE_iE_i^T P + \epsilon_1^{-1} PE_jE_j^T P \\ +\epsilon_3^{-1} PE_iE_i^T P + \epsilon_3^{-1} PE_jE_j^T P \end{bmatrix} & 0 \\ * & 0 \end{bmatrix} + \tag{4.2.37}$$

$$\begin{bmatrix} \begin{bmatrix} (P - L_iC_j)^T(P - L_iC_j) \\ (P - L_jC_i)^T(P - L_jC_i) \\ PP + PP \end{bmatrix} & -PL_jF_{d,i} - PL_iF_{d,j} \\ * & F_{d,j}^T L_i^T L_iF_{d,j} + F_{d,i}^T L_j^T L_jF_{d,i} \end{bmatrix} < 0$$

$$\begin{bmatrix} \psi_{11}(ij) & \psi_{12}(ij) \\ * & \psi_{22}(ij) \end{bmatrix} + \begin{bmatrix} -\epsilon_1^{-1}QE_iE_i^TQ - \epsilon_1^{-1}QE_jE_j^TQ & 0 \\ * & 0 \end{bmatrix} + \tag{4.2.38}$$
$$\begin{bmatrix} \begin{bmatrix} -(Q-L_iC_j)^T(Q-L_iC_j) \\ (Q-L_jC_i)^T(Q-L_jC_i) \\ -QQ-QQ \end{bmatrix} & QL_jF_{f,i} + QL_iF_{f,j} \\ * & -F_{f,j}^TL_i^TL_iF_{f,j} - F_{f,i}^TL_j^TL_jF_{f,i} \end{bmatrix} < 0$$

Using Schur complement lemma, LMIs (4.2.19) and (4.2.20) are obtained, then the proof is therefore complete.

Remark 6 *If P_0, Q_0 and L_{i0} are fixed and known, then MIs (4.2.17)-(4.2.20) become LMIs in $P > 0$, $Q > 0$ and L_i, which can be solved via Mat lab LMI Tool Box.*

For given $\gamma > 0$, the solving algorithm of LMIs is represented in the following. Given $\beta > 0$, a small constant $\delta > 0$ and the iteration number L_n.

Algorithm 2 ***Step 1*** *: Set $L_i = 0$. Solve LMIs (4.2.21), (4.2.22), (4.2.31), (4.2.32) for P and Q by choosing γ. Assign $P_0 = P$, $Q_0 = Q$.*
Step 2*: With obtained P_0, Q_0 and L_{i0}, solve LMIs (4.2.17)-(4.2.20) for new solutions P, Q and L_i by minimizing γ. Again, assign, $L_{i0} = L_i$, $P_0 = P$ and $Q_0 = Q$. Denote the jth iterative γ as γ^j.*
Step 3*: Repeat the operation in step 2 till $|\gamma^{j+1} - \gamma^j| < \delta$, finally L_i is obtained.*

Iterative linear matrix inequality algorithm is shown in figure (3.2).

4.3 Residual evaluation

After the designing of the fault generator, the remaining important task for robust fault detection is the evaluation of the generated residual. Based on LMI technique in [89], one can calculate the threshold value $J_{th} > 0$. Furthermore, we can use the following logic relationship for fault detection

$$\|r(t)\|_{2,T} \begin{cases} \leq J_{th} \rightarrow & \text{no fault} \\ > J_{th} \rightarrow & \text{alarm,} \end{cases}$$

where the residual evaluation $\|r(t)\|_{2,T}$ is determined by :

$$\|r(t)\|_{2,T} = \left[\int_{t_1}^{t_2} r^T(t)r(t)dt \right]^{1/2}, \tag{4.3.1}$$

where $T = t_2 - t_1$ and $t \in [t_1, t_2]$ is the finite-time window. Note that the length of the time is finite (i.e. T instead of ∞). Since an evaluation of the signal over the whole time range is impractical, it is desired that the fault will be detected as easy as possible. Based on (4.2.4) and H_2 norm we have

$$\|r(t)\|_{2,T} = \|r_d(t) + r_f(t)\|_{2,T}, \tag{4.3.2}$$

where $r_d(t) = r(t)|_{f(t)=0}$, $r_f(t) = r(t)|_{d(t)=0}$. Moreover, the fault-free case residual evaluation function is

$$\|r_d(t)\|_{2,T} \leq \|r_d(t)\|_{2,T} \leq J_{th,d(t)}, \tag{4.3.3}$$

where $J_{th,d(t)} = \sup_{\triangle A_i \in \Omega_1, \triangle E_{d,i} \in \Omega_2, d \in L_2} \|r_d(t)\|_{2,T}$. we choose the threshold J_{th} as $J_{th} = J_{th,d(t)}$ where $J_{th,d(t)}$ is constant and can be evaluated off-line.

4.4 Example

Based on TS fuzzy model shown in chapter 3, TS fuzzy model with parameters uncertainties is represented as:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{2} \mu_i[(A_i + \triangle A_i)x(t) + B_i u(t) + (E_{d,i} + \triangle E_{d,i})d(t) + E_{f,i} f(t)] \\
y(t) &= \sum_{i=1}^{2} \mu_i[Cx(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t)],
\end{aligned}$$

with

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.726 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} 0.0 \\ 1 \end{bmatrix}, \qquad C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0 & 1 \\ -1.67 & -1.726 \end{bmatrix}, \quad E_{d,1} = E_{d,2} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad E_{f,1} = E_{f,2} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$$

$$D_1 = D_2 = 0.5 \qquad F_{d,1} = F_{d,2} = 0.5, \qquad F_{f,1} = F_{f,2} = 0.3$$

$$E = \begin{bmatrix} -0.03 & 0 \\ 0 & 0.03 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} 0.1 & 0 \\ 0.1 & 0 \end{bmatrix}, \qquad H_3 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

$$\mu_1 = 1 - x^2, \qquad \mu_2 = x^2$$

Applying the procedure of iterative linear matrix inequality algorithm, the following values for gain matrix L_i, P and Q are given:

$$L_1 = \begin{bmatrix} -0.03 \\ -0.0006 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0668 \\ -0.0001 \end{bmatrix}, \quad P = \begin{bmatrix} 3.3056 & 1.9435 \\ 1.9435 & 4.0639 \end{bmatrix}, \quad Q = \begin{bmatrix} 58.1374 & 48.5227 \\ 48.5227 & 97.8353 \end{bmatrix}$$

In this algorithm we choose $\beta = 2$ as constant and big value for gamma. At the end of iteration the final value of gamma is obtained as $\gamma = 1.0909$. For signal evaluation time window with 5 second is used, the deterministic disturbance and the sensor fault are shown in Figure 4.1(a, b), respectively. The fault occurred at $t = 15$ second with offset 5%. When no fault occurred, the evaluated residual signal is obtained due to deterministic disturbance and parameters uncertainty. Based on (4.3.3), the threshold value due to deterministic disturbance and parameters uncertainties is $J_{th} = 1.4027$. Comparing the threshold value with evaluated signal, it is noticed that the evaluation signal is greater than the threshold value when fault occurred as shown in Figure 4.1(c).

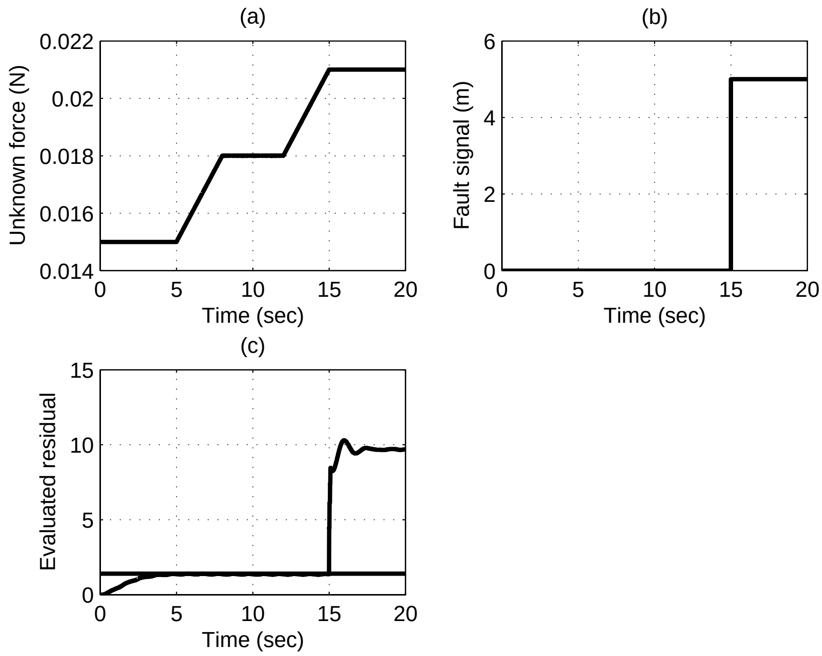

Figure 4.1: Fault Detection for a System with Unknown Inputs and Parameters Uncertainties

4.5 Summary

The problem of designing a robust fault detection system for an uncertain TSFM has been studied. Sufficient conditions for the existence of a robust FD system have been derived. The proposed FD system not only guarantees the L_2-gain from deterministic disturbance and parameter uncertainty to a residual is less than a prescribed value, but also ensures the L_2-gain from a fault signal to residual signal is greater than a prescribed value. Robust fault detection has been formulated in terms of ILMIs which can be easy solved.

5 Robust Fuzzy Fault Detection for a State-delayed Nonlinear Dynamic Systems

A robust fault detection scheme for continuous-time state delayed nonlinear dynamic system with deterministic disturbance is studied. The nonlinear system is modeled by Takagi-Sugeno fuzzy model. Generated residual signal is robust with respect to undesirable effects of deterministic disturbance and modeling errors but sensitive to faults. By applying H_∞ optimization techniques, a sufficient condition to solvability of the formulated problem is established in terms of LMIs. The algorithm of the residual evaluation is also presented.

5.1 TS fuzzy model

The TS fuzzy model with fault, deterministic disturbance and delay is represented as:
Rule i : IF z_1 is M_{i1} and $\ldots$ and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + A_{di} x(t - h_{d1i}(t)) + B_i u(t) + B_{di} u(t - h_{d2i}(t)) + E_{d,i} d(t) + E_{f,i} f(t) \\
y(t) &= C_i x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t),
\end{aligned} \tag{5.1.1}$$

where $M_{ij}(i = 1, 2, \ldots, p, j = 1, \cdots, \theta)$ are fuzzy sets, $z = [z_1, \ldots, z_\theta]$ are premise variables, $x(t) \in \mathcal{R}^n$ is state vector, $u(t) \in \mathcal{R}^{k_u}$ and $y(t) \in \mathcal{R}^m$ are the input and measure output vectors respectively, $d(t) \in \mathcal{R}^{k_d}$ is the deterministic disturbance, $f(t) \in \mathcal{R}^{k_f}$ is the fault to be detected. $h_{d1i}(t)$ and $h_{d2i}(t)$ are time-varying bounded time delays. The time delays satisfies $0 < h_{di}(t) \leq \bar{h}_{di}(t) < \infty$, $0 \leq \dot{h}_{di}(t) \leq \bar{m} < 1$. The matrices $A_i, A_{di}, B_i, B_{di}, E_{d,i}, E_{f,i}, C_i, D_i, F_{d,i}, F_{f,i}$ are of appropriate dimension. The final output of fuzzy system is inferred as a weighted sum of the sub-models and is written as

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{p} \mu_i [A_i x(t) + A_{di} x(t - h_{d1i}(t)) + B_i u(t) + B_{di} u(t - h_{d2i}(t)) + E_{d,i} d(t) + E_{f,i} f(t)] \\
y(t) &= \sum_{i=1}^{p} \mu_i [C_i x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t)]
\end{aligned} \tag{5.1.2}$$

5.2 Residual generation

5.2.1 Fuzzy filter design

The first step to achieve fault detection is to generate a residual signal. In this study, we consider the TS fuzzy filter is described as follows:
Rule i : IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned}\dot{\hat{x}}(t) &= A_i\hat{x}(t) + A_{di}\hat{x}(t-h_{d1i}(t)) + B_iu(t) + B_{di}u(t-h_{d2i}(t)) + L_i[y(t)-\hat{y}(t)] \\ \hat{y}(t) &= C_i\hat{x}(t) + D_iu(t) \\ r(t) &= y(t)-\hat{y}(t),\end{aligned} \tag{5.2.1}$$

where L_i is the filter gain matrix for sub-model i and $r(t)$ is residual signal. The fuzzy filter based residual generator is inferred as the weighted sum

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{p}\mu_i[A_i\hat{x}(t) + A_{di}\hat{x}(t-h_{d1i}(t)) + B_iu(t) + B_{di}u(t-h_{d2i}(t)) + L_i(y(t)-\hat{y}(t))] \\ \hat{y}(t) &= \sum_{i=1}^{p}\mu_i[C_i\hat{x}(t) + D_iu(t)] \\ r(t) &= y(t)-\hat{y}(t),\end{aligned} \tag{5.2.2}$$

where μ_i is the same weight function used in TS model (5.1.2).

Remark 7 *It can be seen that TS fuzzy model of nonlinear system and TS fuzzy filter can be further simplified if the subsystem in each of fuzzy plant model possesses a common time-delay terms $h_{d1}(t)$ and $h_{d2}(t)$, namely $h_{d1i}(t) = h_{d1}$ and $h_{d2i}(t) = h_{d2}$ for all i.*

To analyze the convergence of the filter, the state error vector $e(t) = x(t) - \hat{x}(t)$ is given by the following differential equation.

$$\begin{aligned}\dot{e}(t) &= \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[(A_i-L_iC_j)e(t) + A_{di}e(t-h_{d1}(t)) + (E_{d,i}-L_iF_{d,j})d(t) \\ &\quad + (E_{f,i}-L_iF_{f,j})f(t)] \\ r(t) &= \sum_{i=1}^{p}\mu_i[C_ie(t) + F_{d,i}d(t) + F_{f,i}f(t)]\end{aligned} \tag{5.2.3}$$

Equation (5.2.3) can be simplified as

$$\begin{aligned}\dot{e}(t) &= \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[\bar{A}_{ij}e(t) + A_{di}e(t-h_{d1}(t)) + \bar{E}_{d,ij}d(t) + \bar{E}_{f,ij}f(t)] \\ r(t) &= \sum_{i=1}^{p}\mu_i[C_ie(t) + F_{d,i}d(t) + F_{f,i}f(t)],\end{aligned} \tag{5.2.4}$$

where $\bar{A}_{ij} = A_i - L_iC_j$, $\bar{E}_{d,ij} = E_{d,i} - L_iF_{d,j}$, $\bar{E}_{f,ij} = E_{f,i} - L_iF_{f,j}$. Note that the dynamic of residual signal depends on $f(t)$ and $d(t)$. Thus, the problem of designing TS fuzzy fault detection filter can be described as designing the filter gain matrix L_i such that the following conditions are simultaneously filled.

- $\bar{A}_{ij}$ is asymptotically stable for all subsystems A_i with $(i, j = 1, .., p)$.
- The generated residual $r(t)$ is as sensitive as possible to fault $f(t)$ and as robust as possible to deterministic disturbance $d(t)$.

There are a number of schemes to achieve robustness in FDI. One of them is to introduce a performance index and formulate the (FDF) design optimization problem as in [72].

$$\min_{L_i} J = \min_{L_i} \frac{\|G_{rd}\|_\infty}{\|G_{rf}\|_-} \tag{5.2.5}$$

The robust fault diagnosis design problem can be formulated as finding an fuzzy filter gain matrix L_i such that system (5.2.4) is asymptotically stable and the performance index (5.2.5) is made as small as possible in the feasibility of $\|G_{rd}\|_\infty < \gamma$, $\|G_{rf}\|_- > \beta$, $\beta > 0$ and $\gamma > 0$.

5.2.2 Robust fault detection filter design

In FDI problem the residual is expected to be insensitive to deterministic disturbance and modeling error, whilst sensitive to fault. In the following theorem, H_∞ estimation problem is represented in LMI form. This problem can be defined as determining gain matrix L_i such that the H_∞ norm of the transfer function from disturbances to the residual vector is bounded by a given $\gamma > 0$, γ being as small as possible.

Theorem 7 *System (5.2.4), with $f(t) = 0$ is asymptotically stable and satisfies $\|G_{rd}\|_\infty < \gamma$, if for $\gamma > 0$ there exists a positive definite matrix $P > 0$ and $S_i > 0$ such that the following MIs are satisfied for $1 \le i \le p$ and $1 \le i < j \le p$, respectively :*

$$\begin{bmatrix} M_{11}(ii) & P\bar{E}_{d,ii} + C_i^T F_{d,i} & PA_{di} \\ * & -\gamma^2 I + F_{d,i}^T F_{d,i} & 0 \\ * & * & -S_i(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.6}$$

$$\begin{bmatrix} M_{11}(ij) & M_{12}(ij) & PA_{di} + PA_{dj} \\ * & M_{22}(ij) & 0 \\ * & * & -(S_i + S_j)(1-\bar{m}) \end{bmatrix} < 0, \tag{5.2.7}$$

where

$$M_{11}(ii) = \bar{A}_{ii}^T P + P\bar{A}_{ii} + S_i + C_i^T C_i$$
$$M_{11}(ij) = C_i^T C_j + C_j^T C_i + \bar{A}_{ij}^T P + P\bar{A}_{ij} + \bar{A}_{ji}^T P + P\bar{A}_{ji} + S_j + S_i$$
$$M_{12}(ij) = P\bar{E}_{d,ij} + P\bar{E}_{d,ji} + C_i^T F_{d,j} + C_j^T F_{d,i}$$
$$M_{22}(ij) = -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i}$$

Proof of theorem 7 System (5.2.4) with $f(t) = 0$ is represented as

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[\bar{A}_{ij}e(t) + A_{di}e(t-h_{d1}(t)) + \bar{E}_{d,ij}d(t)] \tag{5.2.8}$$
$$y(t) = \sum_{i=1}^{p}\mu_i[C_ix(t) + F_{d,i}d(t)]$$

The disturbance rejection can be realized by minimizing γ subject to

$$\sup_{\|d(t)\|_2\neq 0}\frac{\|r(t)\|_2}{\|d(t)\|_2} < \gamma \tag{5.2.9}$$

Suppose there exists a quadratic Lyapunov function

$$V(e(t)) = e^T(t)Pe(t) + \sum_{i=1}^{p}\int_{t-h_{d1}}^{t} e^T(\tau)S_ie(\tau)d\tau, \tag{5.2.10}$$

the derivative of (5.2.10) is represented as

$$\dot{V}(e(t)) = \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) + \sum_{i=1}^{p}[e^TS_ie(t) - e^T(t-h_{d1})S_ie(t-h_{d1})(1-\dot{h}_{d1})] \tag{5.2.11}$$

The stability of (5.2.8) is ensured if there exists a quadratic Lyapunov function $V(e(t))$, $P = P^T > 0$ such that $\dot{V}(e(t)) < 0$, with respect of condition (5.2.9). This can be written like

$$\dot{V}(e(t)) + r^T(t)r(t) - \gamma^2d^T(t)d(t) < 0 \tag{5.2.12}$$

Based on (5.2.8) and (5.2.12), we obtain the following equation

$$\begin{aligned}
&\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[e^T(t)\bar{A}_{ij}^T + e^T(t-h_{d1})A_{di}^T + d^T(t)\bar{E}_{d,ij}^T]Pe(t) \\
&+ e^T(t)P\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[\bar{A}_{ij}e(t) + A_{di}e(t-h_{d1}) + \bar{E}_{d,ij}d(t)] + \\
&\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[e^T(t)C_i^TC_je(t) + e^T(t)C_i^TF_{d,j}d(t) + d^T(t)F_{d,i}^TC_je(t) - \gamma^2d^T(t)d(t) \\
&+ d^T(t)F_{d,i}^TF_{d,j}d(t)] + \sum_{i=1}^{p}[e^T(t)S_ie(t) - e^T(t-h_{d1})S_ie(t-h_{d1})(1-\dot{h}_{d1})] < 0
\end{aligned} \tag{5.2.13}$$

Equation (5.2.13) can be rewritten as

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j[e^T(t)\bar{A}_{ij}^T Pe(t) + d^T(t)\bar{E}_{d,ij}^T Pe(t) + e^T(t)P\bar{A}_{ij}e(t) \tag{5.2.14}$$
$$+ e^T(t)P\bar{E}_{d,ij}d(t) + e^T(t)C_i^T C_j e(t) + e^T(t)C_i^T F_{d,j}d(t) + d^T(t)F_{d,i}^T C_j e(t)+$$
$$d^T(t)F_{d,i}^T F_{d,j}d(t) - \gamma^2 d^T(t)d(t) + e^T(t-h_{d1})A_{di}^T Pe(t) + e^T(t)PA_{di}e(t-h_{d1})$$
$$+ e^T(t)S_i e(t) - e^T(t-h_{d1})S_i e(t-h_{d1})(1-\dot{h}_{d1})] < 0$$

Equation (5.2.14) can be represented as

$$\sum_{i=1}^{p}\mu_i^2[e^T(t)\bar{A}_{ii}^T Pe(t) + d^T(t)\bar{E}_{d,ii}^T Pe(t) + e^T(t)P\bar{A}_{ii}e(t) + e^T(t)P\bar{E}_{d,ii}d(t) \tag{5.2.15}$$
$$+ e^T(t)C_i^T C_i e(t) + e^T(t)C_i^T F_{d,i}d(t) + d^T(t)F_{d,i}^T C_i e(t) + d^T(t)F_{d,i}^T F_{d,i}d(t)$$
$$- \gamma^2 d^T(t)d(t) + e^T(t)(t-h_{d1})A_{di}^T Pe(t) + e^T(t)PA_{di}e(t-h_{d1}) + e^T(t)S_i e(t)$$
$$- e^T(t-h_{d1})S_i e(t-h_{d1})(1-\dot{h}_{d1})]$$
$$+ \sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\frac{1}{2}[e^T(t)\bar{A}_{ij}^T Pe(t) + e^T(t)\bar{A}_{ji}^T Pe(t) + d^T(t)\bar{E}_{d,ij}^T Pe(t)$$
$$+ d^T(t)\bar{E}_{d,ji}^T Pe(t) + e^T(t)P\bar{A}_{ij}e(t) + e^T(t)P\bar{A}_{ji}e(t) + e^T(t)P\bar{E}_{d,ji}d(t)$$
$$+ x^T(t)P\bar{E}_{d,ij}d(t) + e^T(t)C_j^T C_i e(t) + e^T(t)C_i^T C_j e(t) + e^T(t)C_j^T F_{d,i}d(t)$$
$$+ e^T(t)C_i^T F_{d,j}d(t) + d^T(t)F_{d,j}^T C_i e(t) + d^T(t)F_{d,i}^T C_j e(t) + d^T(t)F_{d,i}^T F_{d,j}d(t)$$
$$+ d^T(t)F_{d,j}^T F_{d,i}d(t) - 2\gamma^2 d^T(t)d(t) + e^T(t-h_{d1})A_{di}^T Pe(t) + e^T(t)PA_{di}e(t-h_{d1})+$$
$$e^T(t)S_i e(t) - e^T(t-h_{d1})S_i e(t-h_{d1})(1-\dot{h}_{d1}) + e^T(t-h_{d1})A_{dj}^T Pe(t)$$
$$+ e^T(t)PA_{dj}e(t-h_{d1}) + e^T(t)S_j e(t) - e^T(t-h_{d1})S_j e(t-h_{d1})(1-\dot{h}_{d1})] < 0$$

Equation (5.2.15) is negative definite if each sum is negative definite.

First, assume that the first sum of (5.2.15) is negative definite then.

$$\sum_{i=1}^{p}\mu_i^2[e^T(t)\bar{A}_{ii}^T Pe(t) + d^T(t)\bar{E}_{d,ii}^T Pe(t) + e^T(t)P\bar{A}_{ii}e(t) + e^T(t)P\bar{E}_{d,ii}d(t)+ \tag{5.2.16}$$
$$e^T(t)C_i^T C_i e(t) + e^T(t)C_i^T F_{d,i}d(t) + d^T(t)F_{d,i}^T C_i e(t) + d^T(t)F_{d,i}^T F_{d,i}d(t) - \gamma^2 d^T(t)d(t)$$
$$+ x^T(t-h_{d1})A_{di}^T Pe(t) + e^T(t)PA_{di}e(t-h_{d1}) + e^T(t)S_i e(t)$$
$$- e^T(t-h_{d1})S_i e(t-h_{d1})(1-\dot{h}_{d1})] < 0,$$

putting equation (5.2.16) in matrix for so

$$\sum_{i=1}^{r}\mu_i^2\begin{bmatrix} e(t) \\ d(t) \\ e(t-h_{d1})\end{bmatrix}^T \tag{5.2.17}$$
$$\begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i + S_i & P\bar{E}_{d,ii} + C_i^T F_{d,i} & PA_{di} \\ \bar{E}_{d,ii}^T P + F_{d,i}^T C_i & -\gamma^2 I + F_{d,i}^T F_{d,i} & 0 \\ A_{di}^T P & 0 & -S_i(1-\dot{h}_{d1})\end{bmatrix}\begin{bmatrix} e(t) \\ d(t) \\ e(t-h_{d1})\end{bmatrix} < 0,$$

then MI (5.2.6) is obtained for $1 \leq i \leq p$.

Second, consider the second sum of (5.2.15) is negative definite then

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j\frac{1}{2}[e^T(t)\bar{A}_{ij}^T Pe(t) + e^T(t)\bar{A}_{ji}^T Pe(t) + d^T(t)\bar{E}_{d,ij}^T Pe(t) + d^T(t)\bar{E}_{d,ji}^T Pe(t) \tag{5.2.18}$$

$$\begin{aligned}
&+ e^T(t)P\bar{A}_{ji}e(t) + e^T(t)P\bar{A}_{ij}e(t) + e^T(t)P\bar{E}_{d,ji}d(t) + e^T(t)P\bar{E}_{d,ij}d(t) + e^T(t)C_j^T C_i e(t)\\
&+ e^T(t)C_i^T C_j e(t) + e^T(t)C_j^T F_{d,i}d(t) + e^T(t)C_i^T F_{d,j}d(t) + d^T(t)F_{d,j}^T C_i e(t)\\
&+ d^T(t)F_{d,i}^T C_j e(t) + d^T(t)F_{d,i}^T F_{d,j}d(t) + d^T(t)F_{d,j}^T F_{d,i}d(t) - 2\gamma^2 d^T(t)d(t)\\
&+ e^T(t-h_{d1})A_{di}^T Pe(t) + e^T(t)PA_{di}e(t-h_{d1}) + e^T(t)S_i e(t)\\
&- e^T(t-h_{d1})S_i e(t-h_{d1})(1-\dot{h}_{d1}) + e^T(t-h_{d1})A_{dj}^T Pe(t) + e^T(t)PA_{dj}e(t-h_{d1})\\
&+ e^T(t)S_j e(t) - e^T(t-h_{d1})S_j e(t-h_{d1})(1-\dot{h}_{d1})] < 0
\end{aligned}$$

Putting equation (5.2.18) in matrix form.

$$\sum_{i=1}^{p}\sum_{j=1}^{p}\mu_i\mu_j \begin{bmatrix} e(t)\\ d(t)\\ e(t-h_{d1})\end{bmatrix}^T \begin{bmatrix} \phi_{11}(ij) & \phi_{12}(ij) & P(A_{di}+A_{dj})\\ * & \phi_{22}(ij) & 0\\ * & * & \phi_{33}(ij)\end{bmatrix} \begin{bmatrix} e(t)\\ d(t)\\ e(t-h_{d1})\end{bmatrix} < 0 \tag{5.2.19}$$

where

$$\begin{aligned}
\phi_{11}(ij) &= C_i^T C_j + C_j^T C_i + \bar{A}_{ij}^T P + \bar{A}_{ji}^T P + P\bar{A}_{ij} + P\bar{A}_{ji} + S_i + S_j\\
\phi_{12}(ij) &= P\bar{E}_{d,ij} + C_i^T F_{d,j} + P\bar{E}_{d,ji} + C_j^T F_{d,i}\\
\phi_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i}\\
\phi_{33}(ij) &= -(S_i + S_j)(1-\dot{h}_1)
\end{aligned}$$

then MI (5.2.7) is obtained for $1 \leq i < j \leq p$. Then the proof is therefore complete.

In the following theorem, the fault sensitivity is expressed by maximisation of H_- norm of the transfer between residuals and faults so $\|G_{rf}\|_- > \beta$.

Theorem 8 *System (5.2.4) with $d(t) = 0$, is asymptotically stable and satisfies $\|G_{rd}\|_- > \beta$, if for $\beta > 0$ there exist matrix $Q > 0$ and $R_i > 0$ such that the following MIs are satisfied for $1 \leq i \leq p$ and $1 \leq i < j \leq p$, respectively*

$$\begin{bmatrix} N_{11}(ii) & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} & -QA_{di}\\ * & -\beta^2 I + F_{f,i}^T F_{f,i} & 0\\ * & * & (1-\bar{m})R_i\end{bmatrix} > 0 \tag{5.2.20}$$

$$\begin{bmatrix} N_{11}(ij) & N_{12}(ij) & -QA_{di} - QA_{dj}\\ * & N_{22}(ij) & 0\\ * & * & (1-\bar{m})(R_i+R_j)\end{bmatrix} > 0, \tag{5.2.21}$$

where

$$N_{11}(ii) = -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} - R_i + C_i^T C_i$$
$$N_{11}(ij) = C_i^T C_j + C_j^T C_i - \bar{A}_{ij}^T Q - Q\bar{A}_{ij} - \bar{A}_{ji}^T Q - Q\bar{A}_{ji} - R_i - R_j$$
$$N_{12}(ij) = C_i^T F_{f,j} + C_j^T F_{f,i} - Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji}$$
$$N_{22}(ij) = -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i}$$

Proof of theorem 8 System (5.2.4) with $d(t) = 0$ is represented as

$$\dot{e}(t) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j[\bar{A}_{ij}e(t) + A_{di}e(t - h_{d1}(t)) + \bar{E}_{f,ij}f(t)] \tag{5.2.22}$$
$$r(t) = \sum_{i=1}^{p} \mu_i[C_i e(t) + F_{f,i}f(t)]$$

The output is sensitive to fault if

$$\inf_{\|f(t)\|_2 \neq 0} \frac{\|r(t)\|_2}{\|f(t)\|_2} > \beta \tag{5.2.23}$$

Suppose there exists a quadratic Lyapunov function

$$V(e(t)) = e^T(t)Qe(t) + \sum_{i=1}^{p} \int_{t-h_1(t)}^{t} e^T(\tau)R_i e(\tau)d\tau \tag{5.2.24}$$

The derivative of (5.2.24) is represented as

$$\dot{V}(e(t)) = \dot{e}^T(t)Qe(t) + e^T(t)Q\dot{e}(t) + \sum_{i=1}^{p}[e^T R_i e(t) - e^T(t - h_{d1})R_i e(t - h_{d1}(t))(1 - \dot{h}_{d1})(t)] \tag{5.2.25}$$

The stability of (5.2.22) is ensured if there exists a quadratic Lyapunov function $V(e(t))$, $Q = Q^T > 0$ such that $\dot{V}(e(t)) < 0$ with respect of condition (5.2.23). This can be written like

$$r^T(t)r(t) - \beta^2 f^T(t)f(t) - \dot{V}(e(t)) > 0 \tag{5.2.26}$$

Using the same sequence in chapter 3, MIs (5.2.20) and (5.2.21) are obtained.

Performance index (5.2.4) is satisfied if there exists a gain matrix L_i such that MIs (5.2.6), (5.2.7), (5.2.20) and (5.2.21) can be simultaneously solved. Optimal solution is given for γ minimal and β maximal. Isolability of faults is then ensured when γ and β can be found such that $\gamma < \beta$. Unfortunately, this requirement can not be satisfied. So iterative linear matrix inequality is used.

5.2.3 Iterative linear matrix inequality approach

H_∞/H_- problem is studied in the following theorem. The solution is represented in term of ILMIs. A new LMI formulation is proposed to ensure disturbance attenuation and fault sensitivity.

Theorem 9 *For given $\alpha > 0$, $\gamma > 0$ system (5.2.4) with L_i is asymptotically stable and satisfies (5.2.5), if there exist $P > 0$, $Q > 0$, P_0, Q_0, L_i, $R_i > 0$, $S_i > 0$ and L_{i0} such that the following LMIs are satisfied for $1 \leq i \leq p$ and $1 \leq i < j \leq p$ respectively, at the same time.*

$$\begin{bmatrix} \phi_{11}(ii) & PE_{d,i} + C_i^T F_{d,i} & PA_{di} & (P - L_iC_i)^T & P \\ * & \phi_{22}(ii) & 0 & 0 & -F_{d,i}^T L_i^T \\ * & * & -(1-\bar{m})S_i & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{5.2.27}$$

$$\begin{bmatrix} \psi_{11}(ii) & -QE_{f,i} + C_i^T F_{f,i} & -QA_{di} & (Q - L_iC_i)^T & Q \\ * & \psi_{22}(ii) & 0 & 0 & -F_{f,i}^T L_i^T \\ * & * & (1-\bar{m})R_i & 0 & 0 \\ * & * & * & I & 0 \\ * & * & * & * & I \end{bmatrix} > 0 \tag{5.2.28}$$

$$\begin{bmatrix} \phi_{11}(ij) & \phi_{12}(ij) & PA_{di} + PA_{dj} & (P - L_iC_j)^T & P & (P - L_jF_{d,i})^T & P \\ & \phi_{22}(ij) & 0 & 0 & -F_{d,j}^T L_i^T & 0 & -F_{d,i}^T L_j^T \\ * & * & -(S_i + S_j)(1-\bar{m}) & 0 & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0 \tag{5.2.29}$$

$$\begin{bmatrix} \psi_{11}(ij) & \psi_{12}(ij) & -QA_{di} - QA_{dj} & (Q - L_iC_j)^T & Q & (Q - L_jC_i)^T & Q \\ * & \psi_{22}(ij) & 0 & 0 & -F_{f,j}^T L_i^T & 0 & -F_{f,i}^T L_j^T \\ * & * & (R_i + R_j)(1-\bar{m}) & 0 & 0 & 0 & 0 \\ * & * & * & I & 0 & 0 & 0 \\ * & * & * & * & I & 0 & 0 \\ * & * & * & * & * & I & 0 \\ * & * & * & * & * & * & I \end{bmatrix} > 0, \tag{5.2.30}$$

where

$$\begin{aligned}
\phi_{11}(ii) &= A_i^T P + PA_i + C_i^T C_i + 2(P_0P_0 - P_0P - PP_0) + C_i^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_i \\
&\quad + S_i \\
\phi_{22}(ii) &= -\gamma^2 I + E_{d,i}^T F_{d,i} + F_{d,i}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,i} \\
\psi_{11}(ii) &= -A_i^T Q - QA_i + 2(Q_0Q + QQ_0 - Q_0Q_0) - C_i^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_i \\
&\quad + C_i^T C_i - R_i \\
\psi_{22}(ii) &= -\beta^2 I + F_{f,i}^T F_{f,i} - F_{f,i}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{f,i} \\
\phi_{11}(ij) &= A_i^T P + PA_i + A_j^T P + PA_j + C_i^T C_j + C_j^T C_i + 4(P_0P_0 - P_0P - PP_0) \\
&\quad + C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i + C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_j + S_i + S_j \\
\phi_{12}(ij) &= C_i^T F_{d,j} + C_j^T F_{d,i} + PE_{d,i} + PE_{d,j}
\end{aligned}$$

$$\begin{aligned}
\phi_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})E_{d,i} \\
&\quad + F_{d,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,j} \\
\psi_{11}(ij) &= -A_i^T Q - QA_i - A_j^T Q - QA_j + C_i^T C_j + C_j^T C_i + 4(Q_0Q + QQ_0 - Q_0Q_0) \\
&\quad - C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i - C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_j \\
&\quad - (R_i + R_j) \\
\psi_{12}(ij) &= C_i^T F_{f,j} - QE_{f,i} + C_j^T F_{f,i} - QE_{f,j} \\
\psi_{22}(ij) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{f,i} \\
&\quad - F_{f,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{f,j}
\end{aligned}$$

Proof of theorem 9 (i) To obtain (5.2.27) and (5.2.28), for given $\gamma > 0$ and $\beta > 0$, from theorem 7 and theorem 8 system (5.2.4) is asymptotically stable and satisfy (5.2.5), if there exist matrices $P > 0$, $Q > 0$ and L_i such that the following MIs are satisfied:

$$\begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i + S_i & P\bar{E}_{d,ii} + C_i^T F_{d,i} & PA_{di} \\ * & -\gamma^2 I + F_{d,i}^T F_{d,i} & 0 \\ * & * & -S_i(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.31}$$

$$\begin{bmatrix} -\bar{A}_{ii}^T Q - Q\bar{A}_{ii} + C_i^T C_i - R_i & -Q\bar{E}_{f,ii} + C_i^T F_{f,i} & -QA_{di} \\ & -\beta^2 I + F_{f,i}^T F_{f,i} & 0 \\ * & * & R_i(1-\bar{m}) \end{bmatrix} > 0 \tag{5.2.32}$$

For any P_0, Q_0 and L_0 MIs (5.2.31) and (5.2.32) can be represented as

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ii}^T P + P\bar{A}_{ii} + C_i^T C_i \\ +2(P-P_0)(P-P_0) + S_i \\ +C_i^T(L_i - L_{i0})^T(L_i - L_{i0})C_i \end{bmatrix} & P\bar{E}_{d,ii} + C_i^T F_{d,i} & PA_{di} \\ * & \begin{bmatrix} -\gamma^2 I + F_{d,i}^T F_{d,i} + \\ F_{d,i}^T(L_i - L_{i0})^T(L_i - L_{i0})F_{d,i} \end{bmatrix} & 0 \\ * & * & -S_i(1-\bar{m}) \end{bmatrix} < 0$$

(5.2.33)

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ii}^TQ - Q\bar{A}_{ii} + C_i^TC_i \\ -2(Q-Q_0)(Q-Q_0) - R_i \\ -C_i^T(L_i - L_{i0})^T(L_i - L_{i0})C_i \end{bmatrix} & -Q\bar{E}_{f,ii} + C_i^TF_{f,i} & -QA_{di} \\ * & \begin{bmatrix} -\beta^TI + F_{f,i}^TF_{f,i} - \\ F_{f,i}^T(L_i - L_{i0})^T(L_i - L_{i0})F_{f,i} \end{bmatrix} & 0 \\ * & * & R_i(1-\bar{m}) \end{bmatrix} > 0 \tag{5.2.34}$$

MIs (5.2.33) and (5.2.35) can be rewritten as

$$\begin{bmatrix} \begin{bmatrix} M_{11}(ii) - C_i^TL_i^TP - PL_iC_i + \\ 2PP + C_i^TL_i^TL_iC_i \end{bmatrix} & M_{12}(ii) - PL_iF_{d,i} & PA_{di} \\ * & M_{22}(ii) + F_{d,i}^TL_i^TL_iF_{d,i} & 0 \\ * & * & -S_i(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.35}$$

$$\begin{bmatrix} \begin{bmatrix} N_{11}(ii) + C_i^TL_i^TQ + QL_iC_i \\ -2QQ - C_i^TL_i^TL_iC_i \end{bmatrix} & N_{12}(ii) + QL_iF_{f,i} & -QA_{di} \\ * & N_{22}(ii) - F_{f,i}^TL_i^TL_iF_{f,i} & 0 \\ * & * & R_i(1-\bar{m}) \end{bmatrix} > 0, \tag{5.2.36}$$

where

$$\begin{aligned} M_{11}(ii) &= A_i^TP + PA_i + 2(P_0P_0 - PP_0 - P_0P) + C_i^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_i \\ &\quad + C_i^TC_i + S_i \\ M_{12}(ii) &= PE_{d,i} + C_i^TF_{d,i} \\ M_{22}(ii) &= -\gamma^2I + F_{d,i}^TF_{d,i} + F_{d,i}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{d,i} \\ N_{11}(ii) &= -A_i^TQ - QA_i - 2(Q_0Q_0 - Q_0Q - QQ_0) - C_i^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})C_i \\ &\quad + C_i^TC_i - R_i \\ N_{12}(ii) &= -QE_{f,i} + C_i^TF_{f,i} \\ N_{22}(ii) &= -\beta^TI + F_{f,i}^TF_{f,i} - F_{f,i}^T(L_{i0}^TL_{i0} - L_{i0}^TL_i - L_i^TL_{i0})F_{f,i} \end{aligned}$$

MIs (5.2.35) and (5.2.36) can be represented as

$$\begin{bmatrix} M_{11}(ii) & M_{12}(ii) & PA_{di} \\ * & M_{22}(ii) & 0 \\ * & * & -S_i(1-\bar{m}) \end{bmatrix} + \tag{5.2.37}$$

$$\begin{bmatrix} -C_i^TL_i^TP - PL_iC_i + 2PP + C_i^TL_i^TL_iC_i & -PL_iF_{d,i} & 0 \\ * & F_{d,i}^TL_i^TL_iF_{d,i} & 0 \\ * & * & 0 \end{bmatrix} < 0$$

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) & -QA_{di} \\ * & N_{22}(ii) & 0 \\ * & * & R_i(1-\bar{m}) \end{bmatrix} + \tag{5.2.38}$$
$$\begin{bmatrix} C_i^T L_i^T Q + QL_iC_i - 2QQ - C_i^T L_i^T L_i C_i & QL_iF_{f,i} & 0 \\ * & -F_{f,i}^T L_i^T L_i F_{f,i} & 0 \\ * & * & 0 \end{bmatrix} > 0$$

Using Schure complement Lemma then MIs (5.2.37) and (5.2.38) can represented as:

$$\begin{bmatrix} M_{11}(ii) & M_{12}(ii) & PA_{di} \\ * & M_{22}(ii) & 0 \\ * & * & S_i(1-\bar{m}) \end{bmatrix} + \begin{bmatrix} (P-L_iC_i)^T & P & 0 \\ 0 & -F_{d,i}^T L_i^T & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ * & * & I \end{bmatrix} \tag{5.2.39}$$
$$\begin{bmatrix} (P-L_iC_i) & 0 & 0 \\ P & -L_iF_{d,i} & 0 \\ * & * & 0 \end{bmatrix} < 0$$

$$\begin{bmatrix} N_{11}(ii) & N_{12}(ii) & -QA_{di} \\ * & N_{22}(ii) & 0 \\ * & * & R_i(1-\bar{m}) \end{bmatrix} + \begin{bmatrix} (Q-L_iC_i)^T & Q & 0 \\ 0 & -F_{f,i}^T L_i^T & 0 \\ * & * & 0 \end{bmatrix} \begin{bmatrix} -I & 0 & 0 \\ 0 & -I & 0 \\ * & * & -I \end{bmatrix} \tag{5.2.40}$$
$$\begin{bmatrix} (Q-L_iC_i) & 0 & 0 \\ Q & -L_iF_{f,i} & 0 \\ * & * & 0 \end{bmatrix} > 0,$$

putting MIs (5.2.39) and (5.2.40) in matrix form then LMIs (5.2.27) and (5.2.28) are obtained for $1 \leq i \leq p$.

(ii) To obtain (5.2.29), (5.2.30), for given $\gamma > 0$ and $\beta > 0$, from theorem 7 and theorem 8, system (5.2.4) is asymptotically stable and satisfy (5.2.5), if there exist matrices $P > 0$, $Q > 0$ and L_i such that:

$$\begin{bmatrix} \begin{bmatrix} \bar{A}_{ij}^T P + P\bar{A}_{ij} \\ +C_i^T C_j + S_j + S_i + C_j^T C_i \\ +\bar{A}_{ji}^T P + P\bar{A}_{ji} \end{bmatrix} & \begin{bmatrix} P\bar{E}_{d,ij} + P\bar{E}_{d,ji} \\ +C_j^T F_{d,i} + C_i^T F_{d,j} \end{bmatrix} & PA_{di} + PA_{dj} \\ * & -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} & 0 \\ * & * & -(S_i + S_j)(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.41}$$

$$\begin{bmatrix} \begin{bmatrix} -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} \\ +C_i^T C_j + C_j^T C_i - R_i - R_j \\ -\bar{A}_{ji}^T Q - Q\bar{A}_{ji} \end{bmatrix} & \begin{bmatrix} -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} \\ +C_i^T F_{f,j} + C_j^T F_{f,i} \end{bmatrix} & -QA_{di} - QA_{dj} \\ * & -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} & 0 \\ * & * & (R_i + R_j)(1-\bar{m}) \end{bmatrix} > 0 \tag{5.2.42}$$

For any P_0, Q_0 and L_0 MIs (5.2.41) and (5.2.42) can be represented as

$$\begin{bmatrix} M_{11}(ij) & \begin{bmatrix} P\bar{E}_{d,ij} + P\bar{E}_{d,ji} \\ +C_i^T F_{d,j} + C_j^T F_{d,i} \end{bmatrix} & PA_{di} + PA_{dj} \\ * & M_{22}(ij) & 0 \\ * & * & -(S_i + S_j)(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.43}$$

$$\begin{bmatrix} N_{11}(ij) & \begin{bmatrix} -Q\bar{E}_{f,ij} - Q\bar{E}_{f,ji} \\ +C_i^T F_{f,j} + C_j^T F_{f,i} \end{bmatrix} & -QA_{di} - QA_{dj} \\ * & N_{22}(ij) & 0 \\ * & * & (R_i + R_j)(1-\bar{m}) \end{bmatrix} > 0 \tag{5.2.44}$$

where

$$\begin{aligned}
M_{11}(ij) &= \bar{A}_{ij}^T P + P\bar{A}_{ij} + C_i^T C_j + \bar{A}_{ji}^T P + P\bar{A}_{ji} + C_j^T C_i + 4(P-P_0)(P-P_0) \\
&\quad + C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i + C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j + (S_i + S_j) \\
M_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T (L_j - L_{j0})^T (L_j - L_{j0}) F_{d,i} \\
&\quad + F_{d,j}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{d,j} \\
N_{11}(ij) &= -\bar{A}_{ij}^T Q - Q\bar{A}_{ij} - \bar{A}_{ji}^T Q - Q\bar{A}_{ji} - 4(Q-Q_0)(Q-Q_0) \\
&\quad - C_j^T (L_i - L_{i0})^T (L_i - L_{i0}) C_j - C_i^T (L_j - L_{j0})^T (L_j - L_{j0}) C_i - (R_i + R_j) \\
&\quad + C_i^T C_j + C_j^T C_i \\
N_{22}(ii) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T (L_j - L_{j0})^T (L_j - L_{j0}) F_{f,i} \\
&\quad - F_{f,j}^T (L_i - L_{i0})^T (L_i - L_{i0}) F_{f,j}
\end{aligned}$$

MIs (5.2.43) and (5.2.44) can be represented as

$$\begin{bmatrix} \begin{bmatrix} \phi_{11}(ij) - C_j^T L_i^T P - PL_i C_j \\ +4PP - C_i^T L_j^T P - PL_j C_i \\ +C_j^T L_i^T L_i C_j + C_i^T L_j^T L_j C_i \end{bmatrix} & \phi_{12}(ij) - PL_j F_{d,i} - PL_i F_{d,j} & PA_{di} + PA_{dj} \\ * & \begin{bmatrix} \phi_{22}(ij) + F_{d,j}^T L_i^T L_i F_{d,j} \\ +F_{d,i}^T L_j^T L_j F_{d,i} \end{bmatrix} & 0 \\ * & * & -(S_i + S_j)(1-\bar{m}) \end{bmatrix} < 0 \tag{5.2.45}$$

$$\begin{bmatrix} \begin{bmatrix} \psi_{11}(ij) + C_j^T L_i^T Q + QL_i C_j \\ +C_i^T L_j^T Q + QL_j C_i - 4QQ- \\ C_j^T L_i^T L_i C_j - C_i^T L_j^T L_j C_i \end{bmatrix} & \psi_{12}(ij) + QL_j F_{f,i} + QL_i F_{f,j} & -QA_{di} - QA_{dj} \\ * & \begin{bmatrix} \psi_{22}(ij) - F_{f,j}^T L_i^T L_i F_{f,j} \\ -F_{f,i}^T L_j^T L_j F_{f,i} \end{bmatrix} & 0 \\ * & * & (R_i + R_j)(1-\bar{m}) \end{bmatrix} < 0, \tag{5.2.46}$$

where

$$\begin{aligned}
\phi_{11}(ij) &= A_i^T P + PA_i + A_j^T P + PA_j + C_j^T C_i + 4(P_0P_0 - P_0P - PP_0) + (S_i + S_j) \\
&\quad + C_i^T(L_{j0}^T L_{j0} - L_{j0}L_j - L_j^T L_{j0})C_i + C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{j0})C_j + C_i^T C_j \\
\phi_{12}(ij) &= PE_{d,j} + PE_{d,i} + C_i^T F_{d,j} + C_j^T F_{d,i} \\
\phi_{22}(ij) &= -2\gamma^2 I + F_{d,i}^T F_{d,j} + F_{d,j}^T F_{d,i} + F_{d,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{d,i} \\
&\quad + F_{d,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{d,j} \\
\psi_{11}(ij) &= -A_i^T Q - QA_i - A_j^T Q - QA_j + C_j^T C_i - 4(Q_0Q_0 - Q_0Q - QQ_0) - (R_i + R_j) \\
&\quad - C_j^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})C_j - C_i^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})C_i + C_i^T C_j \\
\psi_{12}(ij) &= -QE_{f,j} - QE_{f,i} + C_i^T F_{f,j} + C_j^T F_{f,i} \\
N_{22}(ii) &= -2\beta^2 I + F_{f,i}^T F_{f,j} + F_{f,j}^T F_{f,i} - F_{f,i}^T(L_{j0}^T L_{j0} - L_{j0}^T L_j - L_j^T L_{j0})F_{f,i} \\
&\quad - F_{f,j}^T(L_{i0}^T L_{i0} - L_{i0}^T L_i - L_i^T L_{i0})F_{f,j}
\end{aligned}$$

Using the same sequence as in LMIs (5.2.37) and (5.2.38), LMIs (5.2.29) and (5.2.30) are obtained, then the proof is therefore complete.

Remark 8 *Notice that if $P = P_0$, $Q = Q_0$ and $L_i = L_{i0}$ the MIs (5.2.33), (5.2.34) (5.2.43) and (5.2.44) are the same as MIs (5.2.31), (5.2.32),(5.2.41) and (5.2.42) respectively. If P_0, Q_0 and L_{i0} are fixed and known, then MIs (5.2.27) - (5.2.30) become LMIs in $P > 0$, $Q > 0$ and L_i, which can be solved via Matlab LMI Tool Box.*

For given $\gamma > 0$, the solving algorithm of LMIs is represented in the following.

Algorithm 3 *Given $\beta > 0$, a small constant $\delta > 0$ and the iteration number L_n.*
Step 1 *: Set $L_i = 0$. Solve LMIs (5.2.31), (5.2.32), (5.2.41) and (5.2.42) for P and Q by choosing initial value for γ. Assign $P_0 = P$, $Q_0 = Q$.*
Step 2*: With the obtained P_0, Q_0 and L_{i0}, solve LMIs (5.2.27)-(5.2.30) for new solutions P, Q and L_i by minimizing γ. Again, assign, $L_{i0} = L_i$, $P_0 = P$ and $Q_0 = Q$. Denote the jth iterative γ as γ_j.*
Step 3*: Repeat the operation in step 2 till $|\gamma_{j+1} - \gamma_j| < \delta$, finally L_i is obtained.*

Iterative linear matrix inequality algorithm is shown in figure 3.3.

5.3 Residual evaluation

After designing of fault generator, the remaining important task for robust fault detection is the evaluation of the generated residual. Based on LMI technique in [89], one can calculate the threshold value $J_{th} > 0$. Furthermore, we can use the following logic relationship for fault detection

$$\|r(t)\|_{2,T} \begin{cases} \leq J_{th} \rightarrow & \text{no fault} \\ > J_{th} \rightarrow & \text{alarm,} \end{cases} \tag{5.3.1}$$

where

$$\|r(t)\|_{2,T} = \left[\int_{t_1}^{t_2} r^T(t) r(t) dt \right]^{\frac{1}{2}}, \tag{5.3.2}$$

$T = t_2 - t_1$ and $t \in [t_1, t_2]$ is the finite-time window. Note that the length of the time is finite (i.e. T instead of ∞). Since an evaluation of the signal over the whole time range is impractical, it is desired that the fault will be detected as easy as possible. Based on (5.2.4) we have

$$\|r(t)\|_{2,T} = \|r_{d(t)}(t) + r_{f(t)}(t)\|_{2,T},$$

where $r_{d(t)}(t) = r(t)|_{f(t)=0}$, $r_{f(t)}(t) = r(t)|_{d(t)=0}$. Moreover, the fault-free case residual evaluation function is defined as:

$$\|r(t)\|_{2,T} \leq \|r_d(t)\|_{2,T} \leq J_{th,d(t)},$$

where $J_{th,d(t)} = \sup_{d(t) \in L_2} \|r_{d(t)}\|_{2,T}$. We choose the threshold J_{th} as $J_{th} = J_{th,d(t)}$. This value is constant and can be evaluated off-line.

5.4 Example

Based on TS fuzzy model shown in chapter 3, consider the following TS time-delay system as follows:

$$\dot{x}(t) = \sum_{i=1}^{2} \mu_i [A_i x(t) + A_{di} x(t - h_{d1}(t)) + B_i u(t) + B_{di} u(t - h_{d2}(t)) + E_{d,i} d(t) + E_{f,i} f(t)]$$
$$y(t) = \sum_{i=1}^{2} \mu_i [C x(t) + D_i u(t) + F_{d,i} d(t) + F_{f,i} f(t)],$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & -1.726 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0.0 \\ 1 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0 & 1 \\ -1.67 & -1.726 \end{bmatrix}, \quad E_{d,1} = E_{d,2} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad E_{f,1} = E_{f,2} = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$$

$$D_1 = D_2 = 0.5, \quad F_{d,1} = F_{d,2} = 0.5, \quad F_{f,1} = F_{f,2} = 0.3$$

$$A_{d1} = A_{d2} = \begin{bmatrix} 0.1 & 0 \\ 0.0 & 0.02 \end{bmatrix}, \quad B_{d1} = B_{d2} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \quad \mu_1 = 1 - x^2$$

$$\mu_2 = x^2$$

Applying the procedure of iterative linear matrix inequality algorithm, the following values for gain matrix L_i, P and Q are given:

$$L_1 = \begin{bmatrix} 0.003 \\ -0.0293 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.0045 \\ -0.1233 \end{bmatrix}, \quad P = \begin{bmatrix} 3.224 & 1.8189 \\ 1.8189 & 3.6274 \end{bmatrix}, \quad Q = \begin{bmatrix} 36.7525 & 24.3053 \\ 24.3053 & 45.9746 \end{bmatrix}$$

For $\beta = 2.5$, accuracy value $\delta = 0.00001$, $gamma(initial) = 5$ and time delay $h_{d1} = h_{d2} = 2second$, $gamma(final) = 1.2773$. Based on the deterministic disturbance, the threshold value is $J_{th} = 0.0422$. The unknown input is shown in Figure 5.1(a). The sensor fault occurred at $t = 15$ second with offset 2% as shown in Figure 5.1(b). It is notice that, Comparing the threshold with evaluated signal, the evaluation signal is greater than the threshold value when fault occurred as shown in Figure 5.1(c).

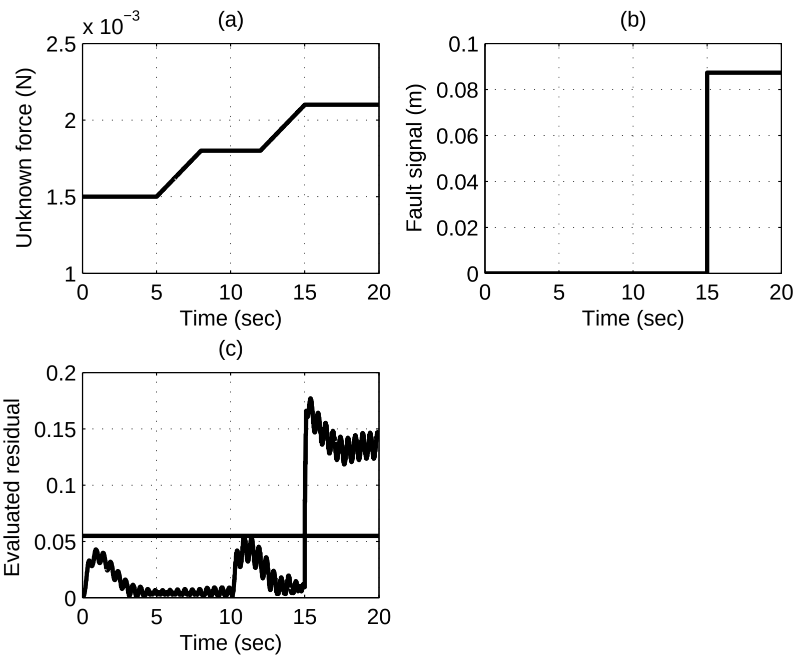

Figure 5.1: Fault Detection for a System with State Delay

5.5 Summary

In this chapter the fault detection problem for time delay Takagi-Sugeno fuzzy model with deterministic disturbance has addressed. LMI-based sufficient conditions for the existence of a robust fault detection filter have been provided. The proposed fuzzy filter is robust against the deterministic disturbance based on H_∞-norm. In the same time is senstive to the fault based on H_--index. This algorithm has been solved in terms of ILMIs.

6 Robust fuzzy fault detection for a nonlinear stochastic dynamic systems

In this chapter, robust fault detection approach for TSFM with stochastic disturbance studied. Based on stochastic signal, the residual signal is also a stochastic process. The characteristics of the residual signal such as the mean value and the covariance value are very important for the residual evaluation. The computation of covariance of residual signal generated by kalman filter-based residual generator is studied. The FD system for each local subsystem is designed by solving the corresponding DARE. Optimization algorithm based on minimizing the residual covariance matrix is used to obtain a robust FD system optimized for global system behavior. The optimization algorithm is established in terms of LMIs.

6.1 TS fuzzy model construction

The TS fuzzy model with faults and stochastic noises is described by the following fuzzy IF-THEN rules:
Rule i : IF z_1 is M_{i1} and $\ldots$ and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} x(k+1) &= A_i x(k) + B_i u(k) + E_{n,i} n(k) + E_{f,i} f(k) \\ y(k) &= C_i x(k) + D_i u(k) + F_{n,i} n(k) + F_{f,i} f(k), \end{aligned} \tag{6.1.1}$$

where $M_{ij}(i = 1, 2, \ldots, p, j = 1, \cdots, \theta)$ are fuzzy sets, $z = [z_1, \ldots, z_\theta]$ are premise variables, $x(k) \in \mathcal{R}^n$ is state vector, $u(k) \in \mathcal{R}^p$ and $y(k) \in \mathcal{R}^q$ are the input and measured output vectors respectively, $n(k) \in \mathcal{R}^m$ vector of zero mean white Gaussian noises with positive definite covariance matrix Σ_n, $f(k) \in \mathcal{R}^l$ is the fault to be detected. $A_i, B_i, E_{n,i}, E_{f,i}, C_i, D_i, F_{n,i}, F_{f,i}$ are known matrices with appropriate dimension. The defuzzified output of TS fuzzy system (6.1.1) is represented as:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{p} \mu_i [A_i x(k) + B_i u(k) + E_{n,i} n(k) + F_{f,i} f(k)] \\ y(k) &= \sum_{i=1}^{p} \mu_i [C_i x(k) + D_i u(k) + F_{n,i} n(k) + F_{f,i} f(k)] \end{aligned} \tag{6.1.2}$$

Under the assumption that the current error is independent of the current noise, we provide

$$E[e_k n_i^T] = E[n_k e_i^T] = 0,$$

for all i,k. In this chapter, the fault detection problem can be formulated as design fault detection system which is robust with respect stochastic noises and sensitive with respect to faults.

6.2 Residual generation

The first step to achieve robust FD system is to generate residual signal which is decoupled from the input signal $u(k)$. In this case, we consider the so-called TS fuzzy filter which is described as follows:
Rule i: IF z_1 is M_{i1} and ... and z_θ is $M_{i\theta}$ THEN

$$\begin{aligned} \hat{x}(k+1) &= A_i\hat{x}(k) + B_iu(k) + (L_i^* + \triangle L_i)[y(k) - \hat{y}(k)] \\ \hat{y}(k) &= C_i\hat{x}(k) + D_iu(k), \end{aligned} \tag{6.2.1}$$

where L_i^* is the filter gain matrix for sub-model i obtained from solving DARE for each local system, $\triangle L_i$ is increment in gain matrices obtained from reducing covariance matrix of residual signal. The fuzzy filter based residual generator is inferred as the weighted sum

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^{p} \mu_i[A_i\hat{x}(k) + B_iu(k) + (L_i^* + \triangle L_i)(y(k) - \hat{y}(k))] \\ \hat{y}(k) &= \sum_{i=1}^{p} \mu_i[C_i\hat{x}(k) + D_iu(k)], \end{aligned} \tag{6.2.2}$$

where μ_i is the same weight function used in TS model (6.1.2). To analyze the convergence of the filter, the state error vector $e(k) = x(k) - \hat{x}(k)$ is given by the following differential equation.

$$\begin{aligned} e(k+1) &= \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j[(A_i - (L_i^* + \triangle L_i)C_j)e(k) \\ &\quad + (E_{n,i} - (L_i^* + \triangle L_i)F_{n,j})n(k) + (E_{f,i} - (L_i^* + \triangle L_i)F_{f,j})f(k)], \\ r(k) &= \sum_{i=1}^{p} \mu_i[C_ie(k) + F_{n,i}n(k) + F_{f,i}f(k)], \end{aligned} \tag{6.2.3}$$

where $r(k)$ is residual signal. System (6.2.3) can be represented as:

$$\begin{aligned} e(k+1) &= \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i\mu_j[(\bar{A}_{ij} - \triangle L_iC_j)e(k) + (\bar{E}_{n,ij} - \triangle L_iF_{n,j})n(k) \\ &\quad + (\bar{E}_{f,ij} - \triangle L_iF_{f,j})f(k)] \\ r(k) &= \sum_{i=1}^{p} \mu_i[C_ie(k) + F_{n,i}n(k) + F_{f,i}f(k)], \end{aligned} \tag{6.2.4}$$

where $\bar{A}_{ij} = A_i - L_i^* C_j$, $\bar{E}_{n,ij} = E_{n,i} - L_i^* F_{n,j}$ and $\bar{E}_{f,ij} = E_{f,i} - L_i^* F_{f,j}$. (6.2.4) can be more simplified and represented as

$$e(k+1) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i \mu_j [\tilde{A}_{ij} e(k) + \tilde{E}_{n,ij} n(k) + \tilde{E}_{f,ij} f(k)] \tag{6.2.5}$$
$$r(k) = \sum_{i=1}^{p} \mu_i [C_i e(k) + F_{n,i} n(k) + F_{f,i} f(k)],$$

where $\tilde{A}_{ij} = \bar{A}_{ij} - \triangle L_i C_j$, $\tilde{E}_{n,ij} = \bar{E}_{n,ij} - \triangle L_i F_{n,j}$ and $\tilde{E}_{f,ij} = \bar{E}_{f,ij} - \triangle L_i F_{f,j}$.

6.2.1 Gain matrix design based on DARE

In this section, the gain matrix for each local sub-system is obtained. The computation of covariance of residual signal generated by kalman filter-based residual generator and fault detection filter is shown. Consider system (6.2.5) with $\triangle L_i = 0$, the following system is obtained.

$$e(k+1) = \sum_{i=1}^{p}\sum_{j=1}^{p} \mu_i \mu_j [\bar{A}_{ij} e(k) + \bar{E}_{n,ij} n(k) + \bar{E}_{f,ij} f(k)], \tag{6.2.6}$$
$$r(k) = \sum_{i=1}^{p} \mu_i [C_i e(k) + F_{n,i} n(k) + F_{f,i} f(k)]$$
$$r(k) = r_n(k) + r_f(k)$$

Based on [60], the following theorem provides a solution to obtain L_i^*, the proof for linear system is given in [59]

Theorem 10 *Each sub-system is stable and satisfy $H_\infty-$norm if*

$$L_i^* = (S + A_i P C_i^T)(C_i P C_i^T + R)^{-1}, \tag{6.2.7}$$

where $Q = E_{n,i} \Sigma_n E_{n,i}^T$, $R = F_{n,i} \Sigma_n F_{n,i}^T$, $S = E_{n,i} \Sigma_n F_n^T$ and $P \geq 0$ is the covariance of the estimation error, it is given as a solution of the following DARE

$$P = A_i P A_i^T + Q - (S + A_i P C_i^T)(C_i P C_i^T + R)^{-1}(S + A_i P C_i^T)^T \tag{6.2.8}$$

6.2.2 Covariance of residual generated by Kalman filter

For FD of the dynamic system with only stochastic noise, the steady-state one-step predictive kalman filter is often used as residual generator [11], [81]. In this case, the generated residual is a zero-mean white Gaussian signal with minimal covariance in the fault-free case, and the residual covariance can be easily calculated. Based on the statistical characteristic of residual signal, the covariance matrix $\phi_r(i)$ of residual $r(k)$ is equal to the covariance matrix of noise induced residual signal $r_n(k)$, therefore

$$\phi_r(l) = \phi_{r_n}(l) \begin{cases} C_i P C_i^T + R & l = 0 \\ 0 & l \neq 0 \end{cases} \tag{6.2.9}$$

Since the residual vector $r_{k-s,k}$ in the evaluated window is defined as $r_{k-s,k} = [r^T(k-s), \ldots, r^T(k)]^T$, thus the covariance matrix of residual vector $r_{k-s,k}$ is

$$\begin{aligned} \Sigma = E\{r_{k-s,k} r_{k-s,k}^T\} &= \begin{bmatrix} \phi_r(0) & \phi_r^T(1) & \ldots & \phi_r^T(s) \\ \phi_r(1) & \phi_r(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \phi_r(s) & \ldots & 0 & \phi_r(0) \end{bmatrix}_{(s+1)\times(s+1)} \\ &= \begin{bmatrix} C_i P C_i^T + R & 0 & \ldots & 0 \\ \phi_r(1) & \phi_r(0) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \phi_r(s) & \ldots & 0 & \phi_r(0) \end{bmatrix}_{(s+1)\times(s+1)} \end{aligned} \tag{6.2.10}$$

Since the generated residual $r(k)$ is un-correlated, it can be found from above expression, the covariance matrix of residual signal $r_{k-s,s}$ is a block diagram matrix, therefore a statistical residual for the residual vector $r_{k-s,k}$ can be easily carried out based on this property.

6.2.3 Increment gain matrix design based on LMI

In this section, incremented gain matrix $\triangle L_i$ is designed for (6.2.5). For this purpose, the residual covariance for system (6.2.5) will be firstly analyzed. If the residual dynamic is stable, the unique stabilizing solution of following DARE denoted by ϕ is the covariance of estimated error

$$\begin{aligned} \lim_{k=\infty} E\{e(k+1)e^T(k+1)\} = \phi = & \sum_{i=1}^{p} \mu_i^4 \{(\bar{A}_{ii} - \triangle L_i C_i)\phi(\bar{A}_{ii} - \triangle L_i C_i)^T \\ & + (\bar{E}_{n,ii} - \triangle L_i F_{n,i})\Sigma_n(\bar{E}_{n,ii} - \triangle L_i F_{n,i})^T\} \\ & + \frac{1}{4}\sum_{i=1}^{p}\sum_{i<j}^{p} \mu_i^2\mu_j^2\{(\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)\phi \\ & (\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)^T \\ & + (\bar{E}_{n,ij} - \triangle L_i F_{n,j} + \bar{E}_{n,ji} - \triangle L_j F_{n,i})\Sigma_n \\ & (\bar{E}_{n,ij} - \triangle L_i F_{n,j} + \bar{E}_{n,ji} - \triangle L_j F_{n,i})^T\} \end{aligned} \tag{6.2.11}$$

Since the estimated error $e(k+1)$ is independent of $n(k)$, the covariance of residual signal ϕ_r is

$$\lim_{k=\infty} E\{r(k)r^T(k)\} = \phi_r = \sum_{i=1}^{p} \mu_i^2\{\bar{C}_i\phi\bar{C}_i^T + \bar{F}_{n,i}\Sigma_n\bar{F}_{n,i}^T\} + \frac{1}{4}\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\{(\bar{C}_i+\bar{C}_j)\phi(\bar{C}_i+\bar{C}_j)^T + (\bar{F}_{n,i}+\bar{F}_{n,j})\Sigma_n(\bar{F}_{n,i}+\bar{F}_{n,j})^T\} \tag{6.2.12}$$

Therefore,

$$\begin{aligned} tr(\phi_r) &= tr(\sum_{i=1}^{p}\mu_i^2 C_i\phi C_i^T + \sum_{i=1}^{p}\mu_i^2 F_{n,i}\Sigma_n F_{n,i}^T \\ &+ \frac{1}{4}\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(C_i+C_j)\phi(C_i+C_j)^T \\ &+ \frac{1}{4}\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(F_{n,i}+F_{n,j})\Sigma_n(F_{n,i}+F_{n,j})^T) \\ &= tr(\sum_{i=1}^{p}\mu_i^2 C_i\phi C_i^T) + tr(\sum_{i=1}^{p}\mu_i^2 F_{n,i}\Sigma_n F_{n,i}^T) \\ &+ \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(C_i+C_j)\phi(C_i+C_j)^T) \\ &+ \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(F_{n,i}+F_{n,j})\Sigma_n(F_{n,i}+F_{n,j})^T), \end{aligned} \tag{6.2.13}$$

where $tr(\sum_{i=1}^{p}\mu_i^2 F_{n,i}\Sigma_n F_{n,i}^T) + \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(F_{n,i}+F_{n,j})\Sigma_n(F_{n,i}+F_{n,j})^T)$ is only decided by noise and is a positive scalar constant. As $tr(AB) = tr(BA)$ then,

$$\begin{aligned} tr(\phi_r) &= tr(\sum_{i=1}^{p}\mu_i^2 \phi C_i^T C_i) + tr(\sum_{i=1}^{p}\mu^2 F_{n,i}\Sigma_n F_{n,i}^T) \\ &+ \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\phi(C_i+C_j)^T(C_i+C_j)) \\ &+ \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j(F_{n,i}+F_{n,j})\Sigma_n(F_{n,i}+F_{n,j})^T) \end{aligned} \tag{6.2.14}$$

Based on the above results, the optimization of FD design can be expressed as: Find $\triangle L_i$ such that, the residual dynamic (6.2.5) is stable and

$$tr(\sum_{i=1}^{p}\mu_i^2\phi C_i^T C_i) + \frac{1}{4}tr(\sum_{i=1}^{p}\sum_{i<j}^{p}\mu_i\mu_j\phi(C_i+C_j)^T(C_i+C_j)) \rightarrow \min$$

Based on [112], the following lemma is obtained

Lemma 3 *Assume that the matrices L_i stabilizes the residual dynamics (6.2.5) then*

$$tr(\psi V) = tr(\sum_{i=1}^{p} \mu_i^2 \phi C_i^T C_i) + \frac{1}{4} tr(\sum_{i=1}^{p} \sum_{i<j}^{p} \mu_i \mu_j \phi (C_i + C_j)^T (C_i + C_j)), \tag{6.2.15}$$

where

$$\begin{aligned} V = & \sum_{i=1}^{p} \mu_i^4 (\bar{E}_{n,ii} - \triangle L_i F_{n,i}) \Sigma_n (\bar{E}_{n,ii} - \triangle L_i F_{n,i})^T \\ & + \frac{1}{4} \sum_{i=1}^{p} \sum_{i<j}^{p} \mu_i^2 \mu_j^2 (\bar{E}_{n,ij} - \triangle L_i F_{n,j} + \bar{E}_{n,ji} - \triangle L_j F_{n,i}) \\ & \Sigma_n (\bar{B}_{n,ij} - \triangle L_i F_{n,j} + \bar{E}_{n,ji} - \triangle L_j F_{n,i})^T, \end{aligned}$$

and $\psi > 0$ is the unique stable solution of DARE

$$\begin{aligned} \psi = & \sum_{i=1}^{p} \mu_i^4 [(\bar{A}_{ii} - \triangle L_i C_i)^T \psi (\bar{A}_{ii} - \triangle L_i C_i) + C_i^T C_i] \\ & + \frac{1}{4} \sum_{i=1}^{p} \sum_{i<j}^{p} \mu_i^2 \mu_j^2 [\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i]^T \psi \\ & [\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i] + (C_i + C_j)^T (C_i + C_j) \end{aligned} \tag{6.2.16}$$

Proof of Lemma 3 : From the solution of DARE (6.2.11) and (6.2.16), we know that

$$\begin{aligned} & tr(\sum_{i=1}^{p} \mu_i^2 \phi C_i^T C_i) + \frac{1}{4} tr(\sum_{i<j}^{p} \mu_i \mu_j \phi (C_i + C_j)^T (C_i + C_j)) \\ & = tr(\sum_{l=0}^{\infty} \sum_{i=1}^{p} \mu^4 (\bar{A}_{ii} - \triangle L_i C_i)^l V (\bar{A}_{ii} - \triangle L_i C_i)^{lT} C_i^T C_i) \\ & + \frac{1}{4} tr(\sum_{i=1}^{p} \sum_{i<j}^{p} \mu_i^2 \mu_j^2 (\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)^l \\ & V (\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)^{lT} (C_i + C_j)^T (C_i + C_j)) \\ & = tr(\sum_{l=0}^{\infty} \sum_{i=1}^{p} \mu^4 (\bar{A}_{ii} - \triangle L_i C_i)^{lT} C_i^T C_i (\bar{A}_{ii}^l - \triangle L_i C_i)) \\ & + \frac{1}{4} tr(\sum_{i=1}^{p} \sum_{i<j}^{p} \mu_i^2 \mu_j^2 (\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)^{lT} \\ & (C_i + C_j)^T (C_i + C_j) (\bar{A}_{ij} - \triangle L_i C_j + \bar{A}_{ji} - \triangle L_j C_i)^l) V = tr \Psi V \end{aligned} \tag{6.2.17}$$

Based on lemma 3, the following theorem is used to obtain the change in gain matrix $\triangle L_i$.

Theorem 11 *Assume that $\triangle L_i$ is given and there exists a symmetric matrix $X > 0$ and $\gamma > 0$ such that :*

1. $\gamma^2 I - \triangle L_i^T X \triangle L_i > 0$
2. $4\gamma^2 I - (\triangle L_i + \triangle L_j)^T X(\triangle L_i + \triangle L_j) > 0$
3. $-X + \tilde{A}_{ii}^T X \tilde{A}_{ii} + C_i^T C_i + \tilde{A}_{ii}^T X \triangle L_i(\gamma^2 I - \triangle L_i^T X \triangle L_i)^{-1} \triangle L_i^T X \tilde{A}_{ii} < 0$
4. $-4X + (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) + (C_i + C_j)^T(C_i + C_j) + (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\triangle L_i + \triangle L_j)$
$(4\gamma^2 I - (\triangle L_i + \triangle L_j)^T X(\triangle L_i + \triangle L_j))^{-1}(\triangle L_i + \triangle L_j)^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) < 0,$

are satisfied for $\forall i, j$. *Then:*

i) system (6.2.5) is stable.

ii) $\psi \leq X$ *consequently* $tr(\sum_{i=1}^p \mu_i^2 \phi C_i^T C_i) + \frac{1}{4} tr(\sum_{i=1}^p \sum_{i<j}^p \mu_i \mu_j \phi (C_i + C_j)^T (C_i + C_j)) \leq tr(XV)$.

Proof of theorem 11 The results (i) follows directly from the bounded real lemma [115]. For the results (ii), based on the schur complements lemma 1, (3) is equivalent to the following LMI:

$$\begin{bmatrix} -\gamma^2 I + \triangle L_i^T X \triangle L_i & \triangle L_i^T X \tilde{A}_{ii} \\ \tilde{A}_{ii}^T X \triangle L_i & -X + \tilde{A}_{ii}^T X \tilde{A}_{ii} + C_i^T C_i \end{bmatrix} < 0 \quad for \quad 1 \leq i \leq p, \tag{6.2.18}$$

hold if $X > \tilde{A}_{ii}^T X \tilde{A}_{ii} + C_i^T C_i$. And (4) is equivalent to the following LMI:

$$\begin{bmatrix} -4\gamma^2 I + (\triangle L_i + \triangle L_j)^T X(\triangle L_i + \triangle L_j) & (\triangle L_i + \triangle L_j)^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) \\ (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\triangle L_i + \triangle L_j) & \begin{bmatrix} -4X + (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\tilde{A}_{ij} \\ +\tilde{A}_{ji}) + (C_i + C_j)^T(C_i + C_j) \end{bmatrix} \end{bmatrix} < 0 \tag{6.2.19}$$

for $1 \leq i < j \leq p$ hold if $4X > (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) + (C_i + C_j)^T(C_i + C_j)$. Compering above LMIs with DARE (6.2.17), based on the monotonicity of the DARE [84], we know $\Psi \leq X$. And based on Lemma 9, we have that

$$tr(\sum_{i=1}^p \mu_i^2 \phi C_i^T C_i) + \frac{1}{4} tr(\sum_{i=1}^p \sum_{i<j}^p \mu_i \mu_j \phi (C_i + C_j)^T (C_i + C_j)) = tr(\Psi V) \leq tr(XV)$$

Based on Theorem 10, the previous problem can be reformulated as: for a given $\gamma > 0$ and symmetric $X > 0$, find $\triangle L_i$ such that:

$$-X + \tilde{A}_{ii}^T X \tilde{A}_{ii} + C_i^T C_i + \tilde{A}_{ii}^T X \triangle L_i(\gamma^2 I - \triangle L_i^T X \triangle L_i)^{-1} \triangle L_i^T X \tilde{A}_{ii} < 0, \tag{6.2.20}$$

and $tr(XV) \rightarrow \min$ for $1 \leq i \leq p$

$$\begin{aligned} &-4X + (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) + (C_i + C_j)^T(C_i + C_j) + (\tilde{A}_{ij} + \tilde{A}_{ji})^T X(\triangle L_i + \triangle L_j) \\ &(4\gamma^2 I - (\triangle L_i + \triangle L_j)^T X(\triangle L_i + \triangle L_j))^{-1}(\triangle L_i + \triangle L_j)^T X(\tilde{A}_{ij} + \tilde{A}_{ji}) < 0, \end{aligned} \tag{6.2.21}$$

and $tr(XV) \rightarrow$ min. for $1 \leq i < j \leq p$. It is known that

$$tr(XV) = tr\Sigma_n^{1/2}(\bar{E}_{n,ii} - \triangle L_i F_{n,i})^T X(\bar{E}_{n,ii} - \triangle L_i F_{n,i})\Sigma_n^{1/2},$$

for $1 \leq i \leq p$ and

$$4tr(XV) = tr\Sigma_n^{1/2}([\bar{E}_{n,ij} - \triangle L_i F_{n,j}] + [\bar{E}_{n,ji} - \triangle L_j F_{n,i}])^T X([\bar{E}_{n,ij} - \triangle L_i F_{n,j}] + [\bar{E}_{n,ji} - \triangle L_j F_{n,i}])\Sigma_n^{1/2},$$

for $1 \leq i < j \leq p$. Therefore the minimization of $tr(XV)$, can be realized with the following method,

$$\Sigma_n^{1/2}(\bar{E}_{n,ii}^T - \triangle L_i F_{n,i})X(\bar{E}_{n,ii} - \triangle L_i F_{n,i})\Sigma_n^{1/2} < \bar{\Phi},$$

for $1 \leq i \leq p$, and

$$\Sigma_n^{1/2}\left([\bar{E}_{n,ij} - \triangle L_i F_{n,j}] + [\bar{E}_{n,ji} - \triangle L_j F_{n,i}]\right)^T X\left([\bar{E}_{n,ij} - \triangle L_i F_{n,j}] + [\bar{E}_{n,ji} - \triangle L_j F_{n,i}]\right)\Sigma_n^{1/2} < 4\bar{\Phi},$$

for $1 \leq i < j \leq p$. This formulation can be represented as LMI as:

$$\begin{bmatrix} \bar{\Phi} & S(\bar{E}_{n,ii}^T X - F_{n,i}^T Y_i^T) \\ * & X \end{bmatrix} > 0 \quad for \quad 1 \leq i \leq p \tag{6.2.22}$$

$$\begin{bmatrix} 4\bar{\Phi} & S(\bar{E}_{n,ij}^T X - F_{n,j}^T Y_i^T + \bar{E}_{n,ji}^T X - F_{n,i}^T Y_j^T) \\ * & X \end{bmatrix} > 0 \quad for \quad 1 \leq i < j \leq p \ , \tag{6.2.23}$$

where $S = \sum_n^{1/2}$, $Y_i = X\triangle L_i$. Therefore, this problem can be reformulated as the following optimization problem:

For a given $\gamma > 0$, find symmetric matrices $X > 0$ and matrix Y, so that the following LMIs:

$$\begin{bmatrix} -X & X\bar{A}_{ii} - Y_i C_i & Y_i & 0 \\ * & -X & 0 & C_i^T \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \tag{6.2.24}$$

$$\begin{bmatrix} \bar{\Phi} & S(\bar{E}_{n,ii}^T X - F_{n,i}^T Y_i^T) \\ * & X \end{bmatrix} > 0 \quad for \quad 1 \leq i \leq p \ , \tag{6.2.25}$$

and

$$\begin{bmatrix} -4X & M(ij) & Y_i + Y_j & 0 \\ * & -X & 0 & (C_i + C_j)^T \\ * & * & -I & 0 \\ * & * & * & -4\gamma^2 I \end{bmatrix} < 0, \tag{6.2.26}$$

$$\begin{bmatrix} 4\bar{\Phi} & S(\bar{E}_{n,ij}^T X - F_{n,j}^T Y_i^T + \bar{E}_{n,ji}^T X - F_{n,i}^T Y_j^T) \\ * & X \end{bmatrix} > 0 \quad for \quad 1 \le i < j \le p \ , \tag{6.2.27}$$

where

$$M(ij) = X\bar{A}_{ij} - Y_i C_j + X\bar{A}_{ji} - Y_j C_i$$

Then the problem can be solved and the solution for $\triangle L_i$ is $\triangle L_i = X^{-1} Y_i$.

6.3 Residual evaluation

After the design of the residual generator, the remaining important task for robust fault detection is the evaluation of the generated residual. Based on [89] and LMI approach, threshold value $J_{th} > 0$ can be calculated. Using the following logic relationship for fault detection:

$$\begin{cases} \|r(k)\|_{2,N} < J_{th} & \text{no fault} \\ \|r(k)\|_{2.N} > J_{th} & \text{fault,} \end{cases} \tag{6.3.1}$$

where the so-called residual evaluation $\|r(k)\|_{2,N}$ is determined by

$$\|r(k)\|_{2,N} = \left[\sum_{k=0}^{N} r^T(k) r(k) \right]^{\frac{1}{2}} , \tag{6.3.2}$$

with N is length of the evaluated window. Since an evaluation of the signal over the whole time range is impractical, it is desired that the fault will be detected as easy as possible. Based on (6.2.5), we have

$$\|r(k)\|_{2,N} = \|r_n(k) + r_f(k)\|_{2,N},$$

where $r_n(k)$ and $r_f(k)$ are defined as: $r_n(k) = r(k)|_{f(k)=0}$, $r_f(k) = r(k)|_{n=0}$. Moreover, the fault-free case residual evaluation function is

$$\|r(k)\|_{2,N} \le \|r_n(k)\|_{2,N} \le J_{th,n},$$

where $J_{th,n} = \sup_{n \in L_2} \|r_n(k)\|_{2,N}$. We choose the threshold J_{th} as $J_{th} = J_{th,n}$. Where J_{th} is constant and can be evaluated off-line.

6.4 Example

Based on TS fuzzy model shown in chapter 3, discrete TS fuzzy model system with stochastic noises represented as follows:

$$x(k+1) = \sum_{i=1}^{2} \mu_i[A_i x(k) + B_i u(k) + E_{n,i} n_1(k)] \tag{6.4.1}$$
$$y(k) = \sum_{i=1}^{2} \mu_i[C_i x(k) + D_i u(k) + F_{n,i} n_2(k)],$$

$n(k)$ is represented by

$$\begin{bmatrix} n_1(k) & n_2(k) \end{bmatrix}^T,$$

where

$$A_1 = \begin{bmatrix} 1.0 & 0.0099 \\ -0.0099 & 0.9828 \end{bmatrix}, \quad B_1 = B_2 = \begin{bmatrix} 0.0 \\ 0.0099 \end{bmatrix}, \quad C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.9999 & 0.0099 \\ -0.0166 & 0.9828 \end{bmatrix}, \quad E_{n,1} = E_{n,2} = \begin{bmatrix} 0 & 0 \\ 0.0099 & 0 \end{bmatrix}, \quad E_{f,1} = E_{f,2} = \begin{bmatrix} 0 \\ 0.0099 \end{bmatrix}$$

$$D_1 = D_2 = 1, \quad F_{n,1} = F_{n,2} = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad F_{f,1} = F_{f,2} = 1$$

$$\mu_1 = 1 - x^2, \quad \mu_2 = x^2$$

Applying the procedure given in Section 6.2, the following results are obtained: Based on section 6.2.1, the gain matrices obtained from solving the DARE are

$$L_1^* = \begin{bmatrix} 0.0002 \\ 0.009 \end{bmatrix}, \quad L_2^* = \begin{bmatrix} 0.0001 \\ 0.009 \end{bmatrix}$$

Based on section 6.2.2 the covariance matrix for each sub system with $t_s = 5$ (sampling time) is represented as:

$$\Sigma_1 = \begin{bmatrix} 4.009 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.009 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.009 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.009 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.009 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.009 \end{bmatrix},$$

$$\Sigma_2 = \begin{bmatrix} 4.0061 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.0061 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.0061 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4.0061 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4.0061 & 0 \\ 0 & 0 & 0 & 0 & 0 & 4.0061 \end{bmatrix}$$

and the increment in gain matrices are

$$\triangle L_1 = \begin{bmatrix} 0.008 \\ -0.0209 \end{bmatrix}, \quad \triangle L_2 = \begin{bmatrix} 0.0065 \\ -0.0188 \end{bmatrix}$$

Using L_2-norm as evaluation function with the length of evaluation window $N = 5$.The stochastic signal is shown in figure 6.1(a).The sensor fault occurred at $t = 15$ second with offset 5% as shown in figure 6.1.(b). Based on stocastic signal only, the threshold value in this case is $J_{th} = 1.8855$. In figure 6.1(c), from $t = 15$ second the evaluated signal has exceeded the threshold value.

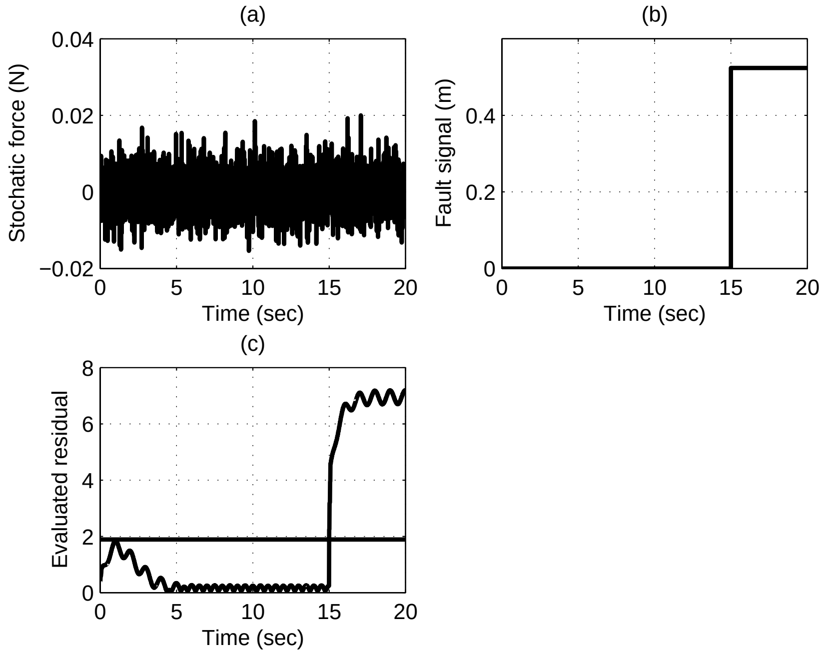

Figure 6.1: Fault Detection for a Nonlinear System with Stochastic Signal

6.5 Summary

In this chapter, robust FD design approach for TSFM with measurement noises has been developed. The generated algorithm consists of two parts, in the first part, the fault detection for local subsystem is obtained by solving DARE, in the second part, the incremented fault detection is obtained from reducing covariance matrix of residual signal. The generated FD system is robust against stochastic noises and sensitive to the fault. The design procedure has been provided in term of LMIs,

7 Application of fuzzy model to FD of vehicle lateral dynamic system

The vehicle lateral dynamic is a very important factor of the vehicle maneuverability, stability and driving safety. With the development of the electronic and computer techniques and their application in the vehicle system, many important vehicle lateral dynamic control systems have been developed and widely equipped in the vehicle. Among them the central ESP system, and some related systems are ABS, TCS and recently developed Drive-by-Wire systems (Steer-by-Wire, Brake-by Wire)

For the ESP system, the central functionality is to improve the active safety by stabilizing the vehicle in extreme driving situation [54]. As soon as a critical driving situation is identified, the controller will be activated until the vehicle returns to normal situation see Figure 7.1. For the Drive-by Wire system, the mechanical or hydraulic linkage has been replaced by the electronic connection to achieve many advantages, on the other side the system becomes more complex, and without fail-safe behavior by mechanical system [44]. Due to this fact, failures of components, which are integrated in those control loops, may strongly affect the system stability and safety. Therefore the high reliability is an essential requirement from these electronic control systems [28], [24]. It is very important to prevent these systems from wrong decisions caused by faulty sensors or actuators, wrong decision can cause very dangerous situations.

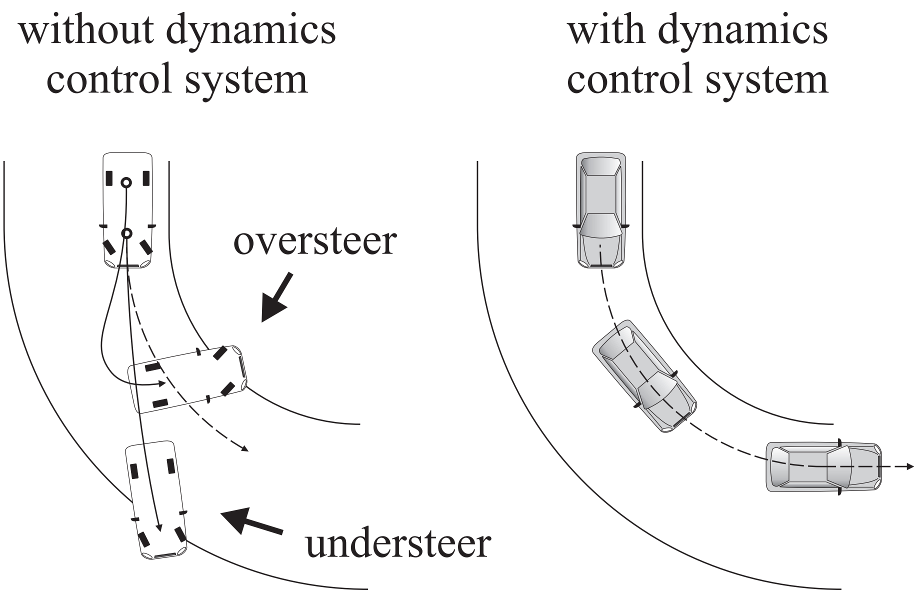

Figure 7.1: Improved driving safty with EPS

In order to apply the fault detection algorithms proposed in the last chapters to detect the sensor faults, The modeling and analysis of vehicle lateral dynamic are firstly presented.

7.1 Modeling of the vehicle lateral dynamic

In recent years many research have been done in the field of vehicle dynamics, many achievements have been fulfilled [62], [38], [12]. And in many applications different vehicle dynamic models have been achieved, [48], [34], [40], [54], [55], [101], [15]. The derivation of the vehicle dynamic model is based on the physical motion equations, therefore the different models can be classified according to the quality of model's freedom. The general used one-track model (or bicycle model) is a 3 DOF model [62], [53], [48], [34], [54], [101], [12], for the vehicle is simplified as a whole mass with the center of gravity on the ground, which can only move in x axis, y axis, and yaw around z axis. The coordinate system is shown in Figure 7.2, which is fixed to the CG. For the

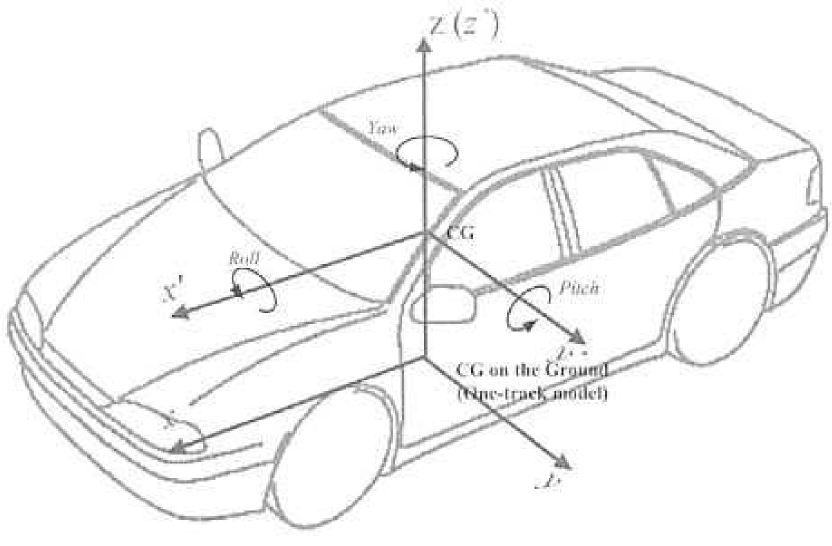

Figure 7.2: Coordinate System of Vehicle Model

purpose of studying the roll motion of the vehicle, the CG is not assumed on the ground. Comparing with one track model, the roll motion around the $x-$axis is introduced, so it is called a 4 DOF model, such as in [40], [55], [15]. For a more precise description of the vehicle dynamic, the vehicle is modeled as a multi-body system. Some large DOF models have been constructed, such as the vehicle simulation software Trucksim which includes a 14 DOF model. But such kind of model is too complicated to be used for the on-line application, only suitable for some off-line or simulation application.

In IFATIS project [4], in order to establish a design framework of model based monitorinng system for vehicle lateral dynamics control systems, the 4 DOF model and one-track model have been studied. The 4 DOF model proposed in [16] has been extended to include the road bank [2], [88]. For the purpose of online application, the extended 4 DOF model has been simplified to one track model or track model with roll motion [88], [86], [92]. In the following, the extended 4 DOF model and one track model will be introduced.

7.1.1 One-track model

The one-track model (or named as bicycle model) proposed in [62] is one of the most widely used models for purposes of vehicle longitudinal and lateral on-line control design [53], [48], [34], [54], [101], [12]. It has been proved that it can describe the vehicle behavior very well when the lateral acceleration is under 0.4g on normal dry asphalt roads [62]. The assumptions for the one-track model are:

1. The height of center of gravity is zero, therefore the four wheels can be simplified as front axle and rear axle.
2. Small longitudinal acceleration, $\dot{v_x} \approx 0$, and no pitch and roll motion.
3. The equations of motion are described according to the force balances and torque balances at the center of gravity.
4. Linear tire model,

$$F_y = C_\alpha.\alpha, \tag{7.1.1}$$

   where F_y is the lateral force, C_α is the cornering stiffness, α is the side slip angle.

5. Small angles simplification

$$\begin{cases} \alpha_H &= -\beta + l_H \frac{r_c}{v_{ref}} \\ \alpha_V &= -\beta + \delta_L^* - l_V \frac{r_c}{v_{ref}} \end{cases} \tag{7.1.2}$$

For driving the lateral dynamics, a coordinate system is fixed to the center of gravity see in Figure 7.3. The derivation of the model expression is according to the force balances in x,y direction and torque balances around z axis. The details of the derivation can be found in [62]. Here the state space form of the second order model is given:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r}_c \end{bmatrix} = \begin{bmatrix} -\frac{c'_{\alpha V}+c_{\alpha H}}{mv_{ref}} & \frac{l_H c_{\alpha H}}{mv_{ref}^2} - 1 \\ \frac{l_H c_{\alpha H} - l_V c'_{\alpha V}}{I_z} & -\frac{l_V^2 c'_{\alpha V} + l_H^2 c_{\alpha H}}{I_z v_{ref}} \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V}}{mv_{ref}} \\ \frac{l_V c'_{\alpha V}}{I_z} \end{bmatrix} \delta_L^* \tag{7.1.3}$$

$$\begin{bmatrix} a_y \\ r_c \end{bmatrix} = \begin{bmatrix} -\frac{c'_{\alpha V}+c_{\alpha H}}{m} & \frac{l_H c'_{\alpha H} - l_V c'_{\alpha V}}{mv_{ref}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V}}{m} \\ 0 \end{bmatrix} \delta_L^*$$

7.1.2 4 DOF model

The 4 DOF model introduced in [4] is exactly a modified one-track model, in which the influence of the roll motion has been considered to the one-track model. Therefore the model feature in high lateral acceleration can be improved. But the model in [4] is only valid for even road without bank angle. In IFATIS project [42], this model has been studied and extended for the road with bank angle [16], [2]. Therefore the extended 4 DOF model is called as IFATIS model.

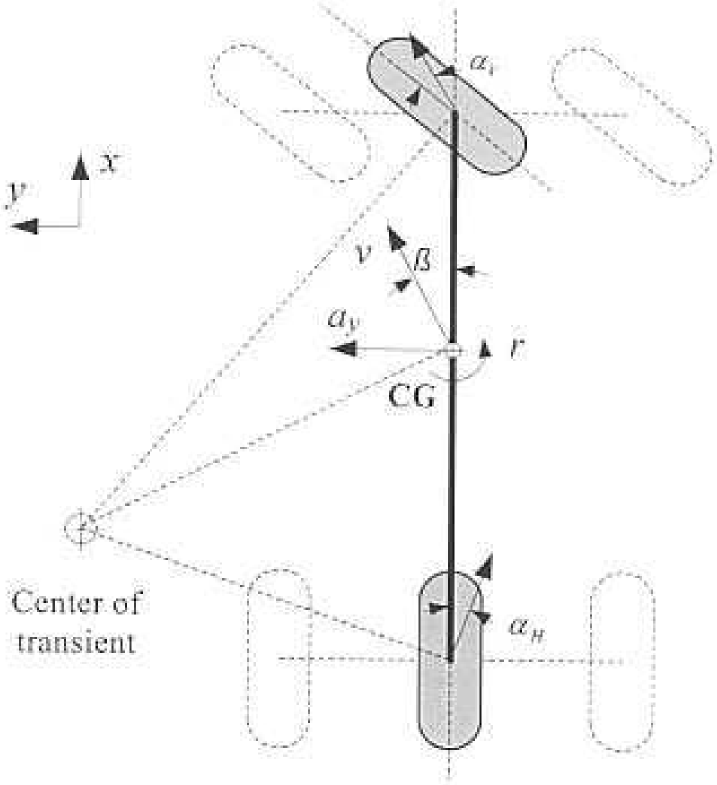

Figure 7.3: Kinematics of One-track Model

Comparing with the one track model, it is assumed that the vehicle includes sprung mass and usprung mass, the CG of sprung mass and unsprung mass are distributed symmetrically on the $x - z$ surface, and it is assumed that there exist only the roll motion for the sprung mass, the lateral force is proportional to the tire slip angle; the acceleration in longitudinal direction is very small, that is $\dot{v}_x \approx 0$; the pitch motion has been neglected.

The motion equation of the force balances and torque balances for the center gravity are

$$\sum F_y = ma_y - m_R h\dot{p} = c'_{\alpha V}\alpha_V + c_{\alpha H}\alpha_H + c_{\gamma V}\frac{\partial \gamma V}{\partial \phi_R}\phi_R - mg.\sin\alpha_x \tag{7.1.4}$$

$$\sum M_z = I_z\dot{r}_c + I_{xz}\dot{p} = l_V(c'_{\alpha V}\alpha_V + c_{\gamma V}\frac{\partial \gamma V}{\partial \phi_R}\phi_R) - l_H c_{\alpha H}\alpha_H$$

$$\sum M_x = I_{xz}\dot{r}_c + I_x\dot{p} = m_R h a_y - C_{Rp} + (m_R gh - K_R)\phi_R + m_R gh.\sin\alpha_x$$

Based on the relationship (7.1.1), (7.1.2) mentioned in one track model, the IFATIS model is described in the state-space for as:

$$\begin{bmatrix} mv_{ref} & 0 & -m_R h & 0 \\ 0 & I_z & I_{xz} & 0 \\ -m_R h v_{ref} & I_{xz} & I_x & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\beta} \\ \dot{r}_c \\ \dot{p} \\ \dot{\phi}_R \end{bmatrix} = \begin{bmatrix} Y_\beta & mv_{ref} - Y_r & 0 & Y_\phi \\ N_\beta & N_r & 0 & N_\phi \\ 0 & m_R h v_{ref} & L_p & L_\phi \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \\ p \\ \phi_R \end{bmatrix} \tag{7.1.5}$$

$$\begin{bmatrix} Y_\delta \\ N_\delta \\ 0 \\ 0 \end{bmatrix} \delta_L^* + \begin{bmatrix} -mg \\ 0 \\ m_R g h \\ 0 \end{bmatrix} \sin \alpha_x$$

where

$$\begin{aligned}
&Y_\beta = -(c'_{\alpha V} + c_{\alpha H}), Y_r = \frac{l_H c_{\alpha H} - l_V c'_{\alpha V}}{v_{ref}} \\
&N_\beta = (l_H c_{\alpha H} - l_V c'_{\alpha V}), N_r = -\frac{(l_V^2 c'_{\alpha V} + l_H^2 c_{\alpha H})}{v_{ref}} \\
&Y_\phi = (c_{\alpha H} \frac{\partial \delta_H}{\partial \phi_R} + c_{\gamma V} \frac{\partial \gamma V}{\partial \phi_R}), N_\phi = (l_V c_{\gamma V} \frac{\partial \gamma_V}{\partial \phi_R} - l_H c_{\alpha H} \frac{\partial \delta_H}{\partial \phi_R}) \\
&Y_\delta = c'_{\alpha V}, N_\delta = l_V c'_{\alpha V}, L_\phi = m_R g h - K_R, L_p = -C_R \\
&I_z = I_{zzR} + I_{zzN} + m_R c^2 + m_{NR} e^2, I_{xz} = I_{xzR} + m_R h c, \\
&I_x = I_{xxR} + m_R h^2
\end{aligned}$$

And the sensor models for the yaw rate sensor and lateral acceleration sensor are

$$\begin{aligned}
&r_c = r_c \\
&a_y = v_{ref}(\dot{\beta} + r_c) - \frac{m_R}{m} h \dot{p} + g \sin \alpha_x
\end{aligned} \tag{7.1.6}$$

7.1.3 Simplified 2. order model

In the ESP system, there is no roll angle sensor available. And for the purpose of on-line application, the IFATIS model has been simplified as a 2. order model.

The 2. order model is to take the vehicle side slip angle β and yaw rate r_c as the state variable. the steering angle δ_L^* as the input signal, and the lateral acceleration sensor signal α_y and the yaw rate sensor signal r_c as the output signal, the other signals in the IFATIS model (road bank angle α_x, vehicle boday roll angle ϕ_R and roll rate p_c) are considered as unknown signals. therefore, the model is represented as:

$$\begin{bmatrix} \dot{\beta} \\ \dot{r}_c \end{bmatrix} = \begin{bmatrix} -\frac{c'_{\alpha V} + c_{\alpha H}}{m v_{ref}} & \frac{l_H c_{\alpha H}}{m v_{ref}^2} - 1 \\ \frac{l_H c_{\alpha H} - l_V c'_{\alpha V}}{I_z} & -\frac{l_V^2 c^j_{\alpha V} + l_H^2 c_{\alpha H}}{I_z v_{ref}} \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V}}{m v_{ref}} \\ \frac{l_V c'_{\alpha V}}{I_z} \end{bmatrix} \delta_L^* + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} d_i \tag{7.1.7}$$

$$\begin{bmatrix} a_y \\ r_c \end{bmatrix} = \begin{bmatrix} -\frac{c'_{\alpha V} + c_{\alpha H}}{m} & \frac{l_H c'_{\alpha H} - l_V c'_{\alpha V}}{m v_{ref}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c'_{\alpha V}}{m} \\ 0 \end{bmatrix} \delta_L^* + d_o,$$

where

$$d_i = \begin{bmatrix} -\frac{g}{v_{ref}} sin\alpha_x + \frac{Y_\phi}{mv_{ref}}\phi_R + \frac{m_R h}{mv_{ref}}\dot{p} \\ \frac{N_\phi}{I_z}\phi_R - \frac{I_{xz}}{I_z}\dot{p} \end{bmatrix}, \quad d_o = \begin{bmatrix} \frac{Y_\phi}{m}\phi_R \\ 0 \end{bmatrix}$$

Assume that $p_c = 0$, $\dot{p}_c = 0$, $\phi_R = 0$, $\alpha_x = 0$ then $d_i = 0$ and $d_o = 0$ in this case model (7.1.4) is same as (7.1.3)

7.1.4 Model uncertainties analysis

The model uncertainties for IFATIS model and one track model are analyzed in the following

Unknown input signal In IFATIS model, there is one main unknown input signal, the road bank angel α_x (see Figure 7.4). Generally, this signal can not be measured directly in the normal vehicle control system, so it is taken as an unknown input signal. For the 2 order model in (7.1.7), the unknown input signals include also the roll angle and roll rate of the sprung mass.

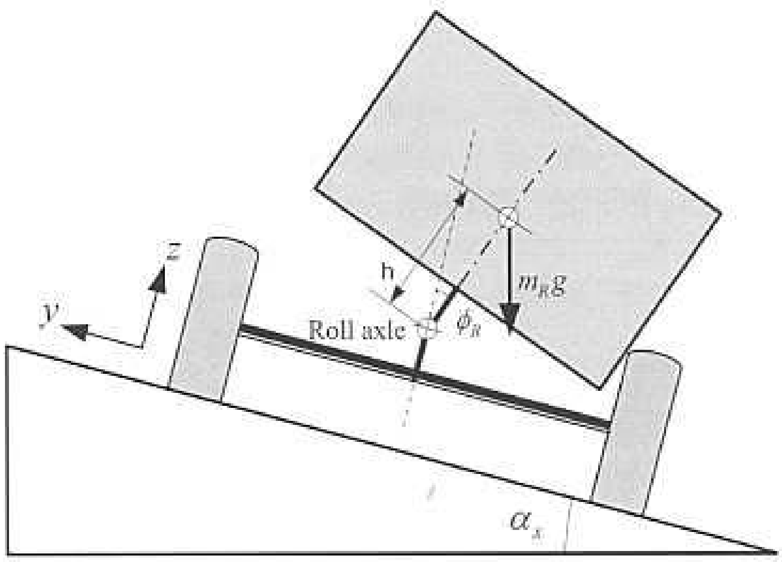

Figure 7.4: Road Bank Angle (real view)

Model parameter variation

- **Vehicle reference velocity**

  The system matrices of IFATIS model and one track model are function of the vehicle reference velocity, such as in A, B, C matrices, therefore the system is exactly an LTV system. For the purpose of the vehicle lateral dynamic system, the variation of the longitudinal vehicle velocity is comparably slow, so it can be considered as a constant during one observation interval.

- **Vehicle mass**

  When the load of the vehicle varies, accordingly the vehicle sprung mass and the inertia will be changed. Especially the load variation are very large for the truck, but for the personal car, comparing to the large total mass, the change caused by the number of passengers can be neglected normally.

- **Vehicle cornering stiffness**

  Cornering stiffness is the change in lateral force per unit slip angle change at a specified normal load in the linear range of the tire. For derivation of the one track model and IFATIS model, the linear model is used as (7.1.1), and the definition of the cornering stiffness is

  $$c_\alpha = \frac{F_y}{\alpha}, \tag{7.1.8}$$

  where F_y is the lateral tire force, α is the slip angle of the tire.

  Actually, the tire cornering stiffness c_α depends on road-tire friction coefficient, wheel load, camber, toe-in, wheel pressure etc. [62]. The problem of this fact is the number of the unknown parameters and functions are very large and very complex. There are some precise functions for nonlinear tire model, such as the well known HSRI (Dugoff) model [109] and Magic tire model [74], which are generally used in tire or vehicle off-line simulation.

  The general simple way to linearize the nonlinear tire model is to linearize its characteristics at the origin, so the cornering stiffness is taken as a constant. However this assumption is only valid in small side slip angle and constant road adhesion efficient.

  In some papers [12], [87], based on the stiffness of the steering mechanism (steering column, gear, etc.), the following assumption has been used,

  $$c_{\alpha H} = k\acute{c}_{\alpha V} \tag{7.1.9}$$

As it is analyzed in [2], the main source of the uncertainty comes from the linear tire model, such as large slip angle, low road adhesion coefficient, or load variation can cause the linear tire model to be invalid, and the direct influence on the model is the variation of the cornering stiffness ($\acute{c}_{\alpha V}, c_{\alpha H}$). The second uncertainty caused by the simplification in the force and moment balance equations, such as the force difference caused by the longitudinal or lateral load transfer, and the roll motion of the unsprung mass. And the third model uncertainty is caused by unknown input signal (Road bank angle α_x) and the small angle assumption.

7.1.5 The sensor faults and noise of vehicle lateral dynamic control systems

For the lateral dynamic control systems, the main used sensors are the speed sensor in four wheels, lateral acceleration, yaw rate and steering wheel angle sensor. In this study, the sensor fault for the four speed sensor at wheels is not considered. The typical failure types and values are given in Table 7.1. The given values show the most possible realistic range for the faults. For the steering angle, because of the sensor type, the ramp fault is impossible, so no fault value is given. For the critical working condition, the sensor noises are inevitable for the vehicle lateral dynamic control systems. Generally the sensor noises can be modeled as steady stochastic process, which follows zero means Gaussian distribution. But in vehicle control systems, the variance or standard variation of sensor noise can not be modeled as a constant value, since at different driving situations, the sensor noises are not only caused by the sensor own physical or electronic characteristic, but also strongly perturbed by the vibration of vehicle chassis. Such as nominal value of yaw rate sensor determined by sensor physical or electronic characteristic is $0.2°/s$, but when vehicle is braking with ABS on the uneven road surface, the standard variation is $0.9°/s$. In the following table, the standard variations at different test conditions are given. All sensor noise data are tested by Bosch Company [68]

Sensor	Offset faults	Ramp
Yaw rate	$\pm 2°/s$, $\pm 5°/s$, $\pm 10°/s$	$\pm 10°/s$
Lateral acceleration	$\pm 2m/s^2$, $\pm 5m/s^2$	$\pm 4m/s^2/s$, $\pm 10m/s^2/s$
Steering angle	$\pm 15°$,$\pm 30°$	-

Table 7.1: Typical failures of lateral dynamic control systems

7.2 TS fuzzy model for vehicle lateral dynamic model

Using physical parameters of the vehicle lateral dynamic model shown in table 2, equation (7.1.3) is represented as follows

$$\begin{bmatrix} \dot{\beta} \\ \dot{r_c} \end{bmatrix} = \begin{bmatrix} -\frac{144.03434}{v_{ref}} & \frac{58.8965}{v_{ref}^2} - 1 \\ 29.8596 & -\frac{170.9813}{v_{ref}} \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{52.8024}{v_{ref}} \\ 40.9396 \end{bmatrix} \delta_L^* \tag{7.2.1}$$

$$\begin{bmatrix} a_y \\ r_c \end{bmatrix} = \begin{bmatrix} -152.7567 & \frac{62.46324}{v_{ref}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c_{\alpha V}^j}{m} \\ 0 \end{bmatrix} \delta_L^*$$

In order to obtain the TS fuzzy model, it is necessary to define two premise variables. Each premise variable represents a nonlinear term; these are

$$z_1(t) = \frac{1}{v_{ref}}, \quad z_2(t) = \frac{1}{v_{ref}^2}$$

Insert with premise variables, (7.2.1) represented as

$$\begin{bmatrix} \dot{\beta} \\ \dot{r_c} \end{bmatrix} = \begin{bmatrix} -(144.03434)z_1 & (58.8965)z_2 - 1 \\ 29.8596 & -(170.9813)z_1 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} (52.8024)z_1 \\ 40.9396 \end{bmatrix} \delta_L^* \tag{7.2.2}$$

$$\begin{bmatrix} a_y \\ r_c \end{bmatrix} = \begin{bmatrix} -152.7567 & (62.46324)z_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \beta \\ r_c \end{bmatrix} + \begin{bmatrix} \frac{c_{\alpha V}^j}{m} \\ 0 \end{bmatrix} \delta_L^*$$

Physical constant	**Value**	**Unit**
g	9.80665	$[m/s^2]$
vehicle parameters		
i_L	18.0	$[-]$
m_R	1630	$[kg]$
m_{NR}	220	$[kg]$
$m = m_R + m_{NR}$	1850	[kg]
l_v	1.52931	$[m]$
l_H	1.53069	$[m]$
l_Z	3870	$[kg - m^2]$
$K_{\phi R}$	0.9429	$[-]$
tire model parameters		
$c'_{\alpha V}$	103600	$[N/rad]$
$c_{\alpha H}$	179000	$[N/rad]$
sensor noise data		
-	**Standard variation**	
n_{ay}	$\sigma_{ay} = (0.2, 2.4)$	$[m/s^2]$
n_r	$\sigma_r = (0.2, 0.9)$	$[rad/s]$
$n_{\delta L}$	$\sigma_{\delta_L^*} = 2$	$[rad]$

Table 7.2: The physical parameters of the vehicle lateral dynamic model

For $v_{ref} = [5, 55]m/s$, the calculation of the minimum and maximum values of $z_1(t)$ and $z_2(t)$ are

$$\begin{array}{ll} \max(z_1) = z_1^+ = 0.2, & \max(z_2) = z_2^+ = 0.04 \\ \min(z_1) = z_1^- = 0.0182, & \min(z_2) = z_2^- = 3.305810^{-4} \end{array}$$

From the maximum and minimum values of $z_1(t)$ and $z_2(t)$, the membership functions for each variables are calculated as follows:

$$\begin{array}{ll} F_{11}(z_1^+) = \frac{z_1 - z_1^-}{z_1^+ - z_1^-} = \frac{z_1 - 0.0182}{0.181}, & F_{12}(z_1^-) = \frac{z_1^+ - z_1}{z_1^+ - z_1^-} = \frac{0.2 - z_1}{0.1818} \\ F_{21}(z_2^+) = \frac{z_2 - z_2^-}{z_2^+ - z_2^-} = \frac{z_2 - 3.305810^{-4}}{0.0396}, & F_{22}(z_2^-) = \frac{z_2^+ - z_2}{z_2^+ - z_2^-} = \frac{0.04 - z_2}{0.0396} \end{array}$$

The vehicle lateral dynamic model is represented by the following continuous fuzzy rules:

Model Rule 1
If $z_1(t)$ is F_{11} and $z_2(t)$ is F_{21}

$$THEN \begin{cases} \dot{x}(t) & = A_1 x(t) + B_1 \delta_L^*(t) \\ y(t) & = C_1 x(t) + D_1 \delta_L^*(t) \end{cases}$$

Model Rule 2
If $z_1(t)$ is F_{11} and $z_2(t)$ is F_{22}

$$THEN \begin{cases} \dot{x}(t) & = A_2 x(t) + B_2 \delta_L^*(t) \\ y(t) & = C_2 x(t) + D_2 \delta_L^*(t) \end{cases}$$

Model Rule 3
If $z_1(t)$ is F_{12} and $z_2(t)$ is F_{21}

$$THEN \begin{cases} \dot{x}(t) & = A_3x(t) + B_3\delta_L^*(t) \\ y(t) & = C_3x(t) + D_3\delta_L^*(t) \end{cases}$$

Model Rule 4
If $z_1(t)$ is F_{12} and $z_2(t)$ is F_{22}

$$THEN \begin{cases} \dot{x}(t) & = A_4x(t) + B_4\delta_L^*(t) \\ y(t) & = C_4x(t) + D_4\delta_L^*(t) \end{cases}$$

where

$$A_1 = \begin{bmatrix} -28.8069 & 1.3559 \\ 29.8597 & -34.1963 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 10.5605 \\ 40.9397 \end{bmatrix}, \quad C_1 = \begin{bmatrix} -152.7568 & 12.4926 \\ 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -28.8069 & -0.9805 \\ 29.8597 & -34.1963 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 10.5605 \\ 40.9397 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -152.7568 & 12.4926 \\ 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -2.6188 & 1.3559 \\ 29.8597 & -3.1088 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0.96 \\ 40.9397 \end{bmatrix}, \quad C_3 = \begin{bmatrix} -152.7568 & 1.1357 \\ 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -2.6188 & -0.9805 \\ 29.8597 & -3.1088 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0.96 \\ 40.9397 \end{bmatrix}, \quad C_4 = \begin{bmatrix} -152.7568 & 1.1357 \\ 0 & 1 \end{bmatrix}$$

$$D_1 = D_2 = D_3 = D_4 = \begin{bmatrix} 56 \\ 0 \end{bmatrix}$$

The continuous time TS fuzzy model, given by the defuzzification, it is carried out as

$$\dot{x}(t) = \sum_{i=1}^{4} \mu_i[A_ix(t) + B_i\delta_L^*(t)]$$
$$y(t) = \sum_{i=1}^{4} \mu_i[C_ix(t) + D_i\delta_L^*(t)],$$

where

$$\mu_1(z(t)) = F_{11}(z_1(t)) \times F_{21}(z_2(t))$$
$$\mu_2(z(t)) = F_{11}(z_1(t)) \times F_{22}(z_2(t))$$
$$\mu_1(z(t)) = F_{12}(z_1(t)) \times F_{21}(z_2(t))$$
$$\mu_1(z(t)) = F_{12}(z_1(t)) \times F_{22}(z_2(t))$$
$$x(t) = \begin{bmatrix} \beta \\ r_c \end{bmatrix}$$
$$y(t) = \begin{bmatrix} a_y \\ r_c \end{bmatrix}$$

7.3 Robust fault detection for lateral vehicle dynamic model with unknown inputs

In this section, a FD system to detect the lateral acceleration sensor fault, yaw rate sensor fault and steering angle sensor fault for lateral dynamic control systems with unknown deterministic disturbances is studied. The system TS model with sensor faults and unknown deterministic disturbances is described as follows,

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{4} \mu_i [A_i x(t) + B_i \delta_L^*(t) + E_{d,i} d(t) + E_{f,i} f(t)] \\ y(t) &= \sum_{i=1}^{4} \mu_i [C_i x(t) + D_i \delta_L^*(t) + F_{d,i} d(t) + F_{f,i} f(t)],\end{aligned} \tag{7.3.1}$$

where

$$E_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad F_d = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad F_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_{f,1} = \begin{bmatrix} 10.5605 & 0 & 0 \\ 40.9397 & 0 & 0 \end{bmatrix}, \quad E_{f,2} = \begin{bmatrix} 10.5605 & 0 & 0 \\ 40.9397 & 0 & 0 \end{bmatrix}, \quad E_{f,3} = \begin{bmatrix} 0.96 & 0 & 0 \\ 40.9397 & 0 & 0 \end{bmatrix}$$

$$E_{f,4} = \begin{bmatrix} 0.96 & 0 & 0 \\ 40.9397 & 0 & 0 \end{bmatrix}, \qquad f(t) = \begin{bmatrix} f_{\delta_L} \\ f_{ay} \\ f_r \end{bmatrix}, \qquad d(t) = \begin{bmatrix} d_{\delta_L^*} \\ d_{ay} \\ d_r \end{bmatrix}$$

7.3.1 Residual generator design

As introduced above, the residual for nonlinear system is represented by TS fuzzy filter of the form like

$$\begin{aligned}\dot{\hat{x}}(t) &= \sum_{i=1}^{4} \mu_i [A_i \hat{x}(t) + B_i \delta_L^*(t) + L_i (y(t) - \hat{y}(t))] \\ \hat{y}(t) &= \sum_{i=1}^{4} \mu_i [C_i \hat{x}(t) + D_i \delta_L^*(t)] \\ r(t) &= y(t) - \hat{y}(t),\end{aligned} \tag{7.3.2}$$

where L_i is the filter gain matrix. The following are the details of the sub models and the corresponding filter-based residual generators.

The first sub model In this case, the steering angle is taken as input signal, and lateral acceleration as output signal. The residual generated is

$$r_1 = a_y - \hat{a}_y \tag{7.3.3}$$

Based on robust fault detection algorithm in chapter 3,

Algorithm 4 *Given $\beta = 9 > 0$, a small constant $\delta = 0.0001 > 0$ and $\gamma_{int} = 350$*
Step 1 *: Set*

$$L_{1,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{2,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{3,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{4,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve LMI (3.2.32), (3.2.33), (3.2.42) and (3.2.43) for P and Q by choosing γ

$$P = 10^4 \times \begin{bmatrix} 1.8261 & 0.0289 \\ 0.0289 & 0.0364 \end{bmatrix}, \quad Q = 10^3 \times \begin{bmatrix} 4.8351 & -0.3008 \\ -0.3008 & 0.1079 \end{bmatrix}$$

Assign $P_0 = P$, $Q_0 = Q$.

Step 2*: Given P_0, Q_0 and $L_{i,0}$, solve LMIs (3.2.28)- (3.2.31) for new solutions P, Q and L_i by minimizing γ. Again, assign, $L_{i0} = L_i$, $P_0 = P$ and $Q_0 = Q$. Denote the jth iterative γ as γ^j.*

Step 3*: Repeat the operation in step 2 till $|\gamma^{j+1} - \gamma^j| < \delta$, finally L_i is obtained. The final values of L_i are*

$$L_1 = \begin{bmatrix} -0.0106 \\ -0.0097 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0124 \\ -0.0127 \end{bmatrix}, \quad L_3 = \begin{bmatrix} -0.0248 \\ -0.0298 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.0017 \\ -0.0154 \end{bmatrix}$$

and

$$P = 10^4 \times \begin{bmatrix} 1.8257 & 0.0289 \\ 0.0289 & 0.0362 \end{bmatrix}, \quad Q = 10^3 \times \begin{bmatrix} 4.8363 & -0.3023 \\ -0.3023 & 0.1068 \end{bmatrix}$$

The second sub model In this case, the steering angle is adopted as input signal, yaw rate as output signal, the residual generated is

$$r_2 = r_c - \hat{r}_c \tag{7.3.4}$$

Based on robust fault detection algorithm in chapter 3,

Algorithm 5 *Given $\beta = 9 > 0$, a small constant $\delta = 0.0001 > 0$ and $\gamma_{int} = 350$*
Step 1 *: Set*

$$L_{1,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{2,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{3,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{4,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve LMI (3.2.32), (3.2.33), (3.2.42) and (3.2.43) for P and Q by choosing γ

$$P = 10^4 \times \begin{bmatrix} 1.9987 & 0.0463 \\ 0.0463 & 0.0353 \end{bmatrix}, \quad Q = 10^4 \times \begin{bmatrix} 1.9377 & -0.1245 \\ -0.1245 & 0.0517 \end{bmatrix}$$

Assign $P_0 = P$, $Q_0 = Q$

Step 2*: With obtained P_0, Q_0 and $L_{i,0}$, solve LMIs (3.2.28)- (3.2.31) for new solutions P, Q and L_i by minimizing γ. Again, assign, $L_{i0} = L_i$, $P_0 = P$ and $Q_0 = Q$. Denote the jth iterative γ as γ^j.*

Step 3: *Repeat the operation in step 2 till* $|\gamma^{j+1} - \gamma^j| < \delta$, *finally* L_i *is obtained. The final values of* L_i *are*

$$L_1 = \begin{bmatrix} 0.0067 \\ 0.0119 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.0071 \\ 0.0137 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0.0018 \\ -0.0004 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.0004 \\ 0.0163 \end{bmatrix}$$

and

$$P = 10^4 \times \begin{bmatrix} 1.9992 & 0.0462 \\ 0.0462 & 0.0348 \end{bmatrix}, \quad Q = 10^4 \times \begin{bmatrix} 1.9379 & -0.1246 \\ -0.1246 & 0.0515 \end{bmatrix}$$

7.3.2 Residual evaluation

After the design of the residual generator, the remaining important task for robust fault detection is the residual evaluator. The residual evaluation consists of evaluation function and threshold value. Using L_2-norm as evaluation function with the length of evaluation window $N = 20$. The threshold value is calculated at fault free case.

The first sub model The known input (steering angle) is shown in figure 7.5 (a). The unknown input signal is shown in figure 7.5 (b). The data with an offset sensor fault of $5m/s^2$ occured at $t = 48$ seconds is used to validate the designed robust FD system. The threshold value is $J_{th} = 0.109$. In figure 7.5 (c), from $t = 48$ seconds the evaluated signal has exceeded the threshold value.

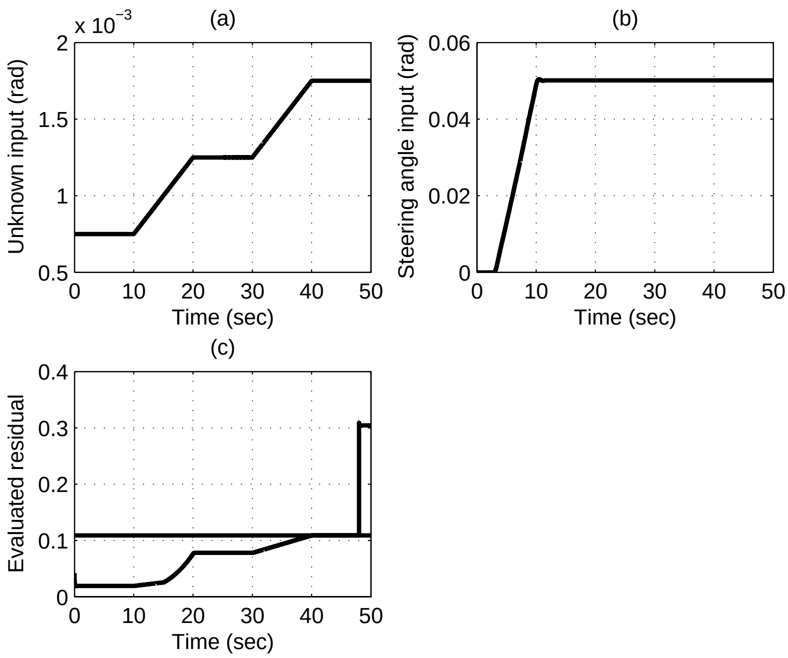

Figure 7.5: Robust Fault Detection for Lateral Acceleration with Unknown Inputs

The second sub model The known input (steering angle) is shown in figure 7.6 (a). The unknown input signal is shown in figure 7.6 (b). The data with an offset sensor fault of $5m/s^2$ occured at $t = 44$ seconds is used to validate the designed robust FD system. The threshold value is $J_{th} = 0.0029$. In figure 7.6 (c), at $t = 44$ seconds the evaluated signal has exceeded the threshold value

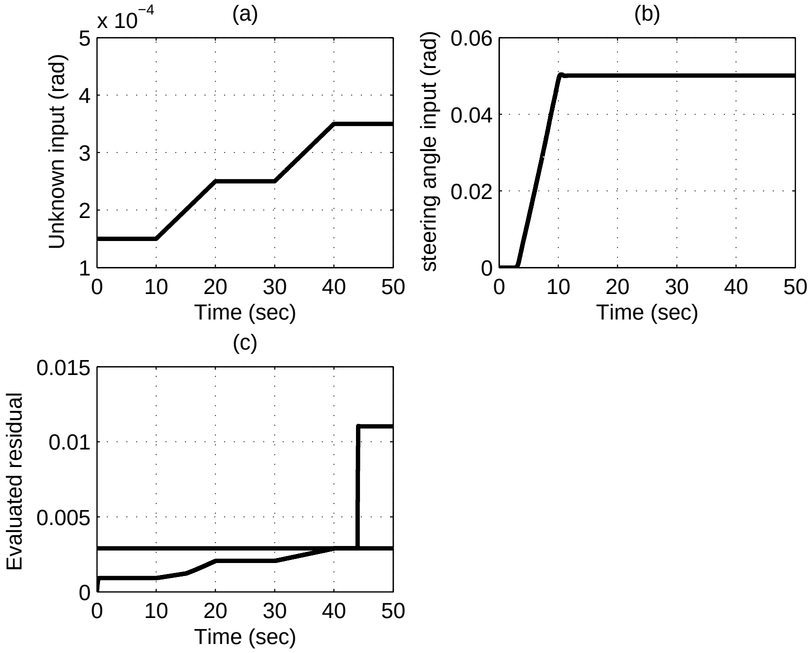

Figure 7.6: Robust Fault Detection for Yaw Rate with Unknown Inputs

7.4 Robust fault detection for lateral vehicle dynamic model with unknown inputs and parameters uncertainties

In this section, a FD system to detect the lateral acceleration a sensor fault, yaw rate sensor fault and steering angle sensor fault for lateral dynamic control systems with unknown deterministic disturbances and parameters uncertainties is studied. The system TS model with sensor faults, unknown deterministic disturbances and parameters uncertainties is described as follows,

$$\dot{x}(t) = \sum_{i=1}^{4} \mu_i[(A_i + \Delta A_i)x(t) + B_i\delta_L^*(t) + (E_{d,i} + \Delta E_{d,i})d(t) + E_{f,i}f(t)] \qquad (7.4.1)$$

$$y(t) = \sum_{i=1}^{4} \mu_i[C_i x(t) + D_i\delta_L^*(t) + F_{d,i}d(t) + F_{f,i}f(t)],$$

where $\Delta A_i, \Delta E_{d,i}$ satisfy (4.1.2) and

$$E = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0.1 & 0 & 0 \end{bmatrix}$$

7.4.1 Residual generator design

As introduced above, the residual for nonlinear system is represented by TS fuzzy filter of the form like

$$\dot{\hat{x}}(t) = \sum_{i=1}^{4} \mu_i[A_i\hat{x}(t) + B_i\delta_L^*(t) + l_i(y(t) - \hat{y})] \tag{7.4.2}$$
$$\hat{y}(t) = \sum_{i=1}^{4} \mu_i[C_i\hat{x}(t) + D_i\delta_L^*]$$
$$r(t) = y(t) - \hat{y}(t),$$

where L_i is the filter gain matrix. The following are the details of the sub models and corresponding filter-based residual generators.

The first sub model In this case, the steering angle is taken as input signal, and lateral acceleration as output signal. The residual generated is

$$r_1 = a_y - \hat{a}_y \tag{7.4.3}$$

Based on robust fault detection algorithm in chapter 4,

Algorithm 6 *Given* $\beta = 4 > 0$, *a small constant* $\delta = 0.00001 > 0$ *and* $\gamma_{int} = 500$
Step 1 *: Set*

$$L_{1,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{2,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{3,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{4,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve LMI (4.2.21), (4.2.22), (4.2.31), (4.2.32) for P *and* Q *by choosing* γ

$$P = 10^4 \times \begin{bmatrix} 3.1005 & 0.0973 \\ 0.0973 & 0.0648 \end{bmatrix}, \quad Q = 10^3 \times \begin{bmatrix} 3.9991 & -0.2429 \\ -0.2429 & 0.0897 \end{bmatrix}$$

Assign $P_0 = P$, $Q_0 = Q$

Step 2*: With obtained* P_0, Q_0 *and* $L_{i,0}$, *solve LMIs (4.2.17)- (3.2.20) for new solutions* P, Q *and* L_i *by minimizing* γ. *Again, assign,* $L_{i0} = L_i$, $P_0 = P$ *and* $Q_0 = Q$. *Denote the jth iterative* γ *as* γ^j.

Step 3*: Repeat the operation in step 2 till* $|\gamma^{j+1} - \gamma^j| < \delta$, *finally* L_i *is obtained. The final values of* L_i *are*

$$L_1 = \begin{bmatrix} -0.0399 \\ -0.0087 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0409 \\ -0.0134 \end{bmatrix}, \quad L_3 = \begin{bmatrix} -0.0263 \\ 0.0292 \end{bmatrix}, \quad L_4 = \begin{bmatrix} -0.026 \\ -0.0059 \end{bmatrix}$$

and

$$P = 10^4 \times \begin{bmatrix} 3.0907 & 0.0963 \\ 0.0963 & 0.0641 \end{bmatrix}, \quad Q = 10^3 \times \begin{bmatrix} 5.0388 & -0.3063 \\ -0.3063 & 0.1168 \end{bmatrix}$$

The second sub model In this case, the steering angle is adopted as input signal, yaw rate as output signal, the residual generated is

$$r_2 = r - \hat{r} \tag{7.4.4}$$

Based on robust fault detection algorithm in chapter 3,

Algorithm 7 *Given* $\beta = 4 > 0$, *a small constant* $\delta = 0.0001 > 0$ *and* $\gamma_{int} = 500$
Step 1 *: Set*

$$L_{1,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{2,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{3,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad L_{4,0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve LMI (4.2.21), (4.2.22), (4.2.31), (4.2.32) for P *and* Q *by choosing* γ

$$P = 10^4 \times \begin{bmatrix} 3.5893 & 0.1181 \\ 0.1181 & 0.0669 \end{bmatrix}, \quad Q = 10^4 \times \begin{bmatrix} 1.9452 & -0.1075 \\ -0.1075 & 0.0526 \end{bmatrix}$$

Assign $P_0 = P$, $Q_0 = Q$
Step 2*: With obtained* P_0, Q_0 *and* $L_{i,0}$, *solve LMIs (4.2.17)- (4.2.20) for new solutions* P, Q *and* L_i *by minimizing* γ. *Again, assign,* $L_{i0} = L_i$, $P_0 = P$ *and* $Q_0 = Q$. *Denote the jth iterative* γ *as* γ^j.
Step 3*: Repeat the operation in step 2 till* $|\gamma^{j+1} - \gamma^j| < \delta$, *finally* L_i *is obtained. The final values of* L_i *are*

$$L_1 = \begin{bmatrix} -0.0192 \\ -0.0058 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.0193 \\ -0.0065 \end{bmatrix}, \quad L_3 = \begin{bmatrix} -0.0101 \\ -0.0059 \end{bmatrix}, \quad L_4 = \begin{bmatrix} 0.0052 \\ 0.0261 \end{bmatrix}$$

and

$$P = 10^4 \times \begin{bmatrix} 3.5881 & 0.1174 \\ 0.1174 & 0.068 \end{bmatrix}, \quad Q = 10^4 \times \begin{bmatrix} 1.9458 & -0.1079 \\ -0.1079 & 0.0519 \end{bmatrix}$$

7.4.2 Residual evaluation

After the design of the residual generator, the remaining important task for robust fault detection is the residual evaluator. The residual evaluation consists of evaluation function and threshold value. Using L_2-norm as evaluation function with the length of evaluation window $N = 20$. The threshold value is calculated at fault free case.

The first sub model The known input (steering angle) is shown in figure 7.7 (a). The unknown input signal is shown in figure 7.7 (b). The data with an offset sensor fault of $5m/s^2$ occured at $t = 48$ second is used to validate the designed robust FD system. The threshold value in this case is represented as in (4.3.3) $J_{th} = 0.0276$. In figure 7.7 (c), from $t = 48$ second the evaluated signal has exceeded the threshold value.

The second sub model The known input (steering angle) is shown in figure 7.8 (a). The unknown input signal is shown in figure 7.8 (b) The data with an offset sensor fault of $5m/s^2$ occured at $t = 44$ second is used to validate the designed robust FD system. The threshold value is represented as in (4.3.3) with $J_{th} = 0.0297$. In figure 7.8 (c), from $t = 44$ second the evaluated signal has exceeded the threshold value

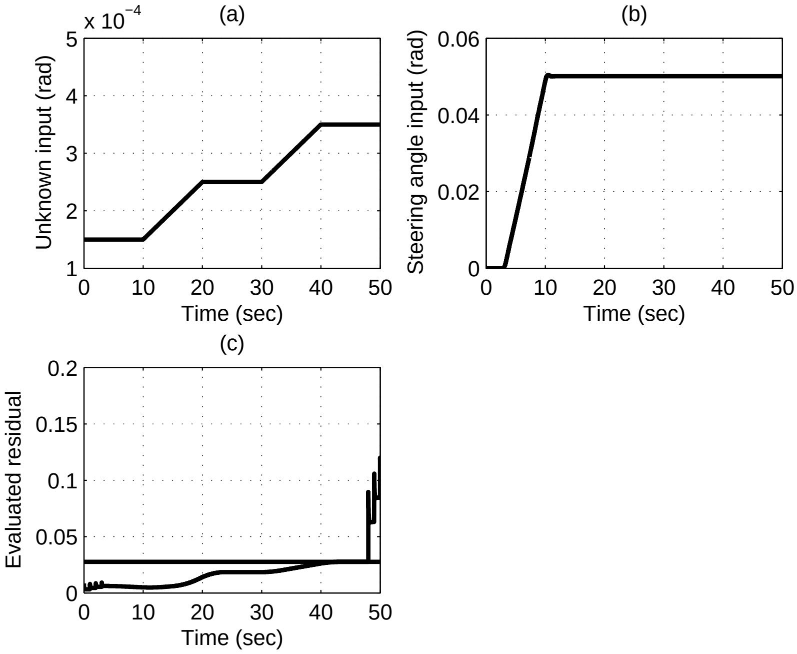

Figure 7.7: Robust Fault Detection for Lateral Acceleration with Unknown Inputs and Parameters Uncertainties

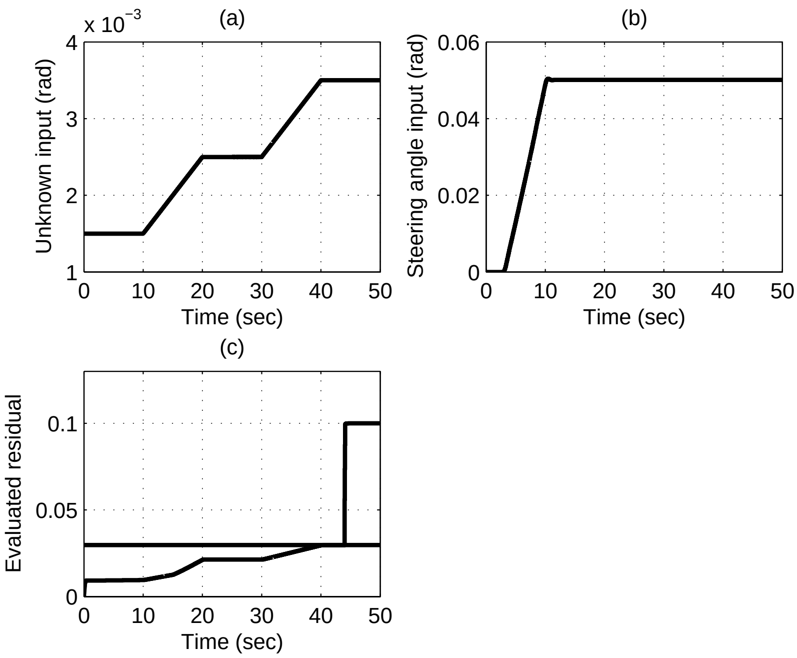

Figure 7.8: Robust Fault Detection for Yaw Rate with Unknown Inputs and Parameters Uncertainties

7.5 Robust fault detection for lateral vehicle dynamic model with stochastic noises

In this section, we deal with the discrete TS fuzzy model for lateral vehicle dynamic model. FD system to detect the lateral acceleration a sensor fault, yaw rate sensor fault and steering angle sensor fault for lateral dynamic control systems with stochastic noises is studied. After the discretization of each sub system , using 10 milliseconds as sampling time, the vehicle lateral dynamic model is represented ba the following:

$$\begin{aligned} x(k+1) &= \sum_{i=1}^{4} \mu_i [A_i x(k) + B_i(\delta_L^*(k) + n_{\delta_L}(k)) + E_{f,i} f(t)] \qquad (7.5.1) \\ y(k) &= \sum_{i=1}^{4} \mu_i [C_i x(k) + D_i \delta_L^*(k) + v(k) + F_{f,i} f(k)] \\ V(k) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} n_{ay}(k) \\ n_r(k) \end{bmatrix}, \end{aligned}$$

where

$$A_1 = \begin{bmatrix} 0.7512 & 0.0099 \\ 0.2181 & 0.7118 \end{bmatrix}, \quad B_1 = E_{f,1} \begin{bmatrix} 0.0941 \\ 0.3598 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.7486 & -0.0072 \\ 0.2178 & 0.7093 \end{bmatrix}, \quad B_2 = E_{f,2} \begin{bmatrix} 0.0901 \\ 0.3594 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.9761 & 0.0132 \\ 0.2904 & 0.9714 \end{bmatrix}, \quad B_3 = E_{f,3} = \begin{bmatrix} 0.0122 \\ 0.4048 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0.9727 & -0.0095 \\ 0.29 & 0.968 \end{bmatrix}, \quad B_4 = E_{f,4} = \begin{bmatrix} 0.0075 \\ 0.4043 \end{bmatrix}$$

7.5.1 Residual generator design

As introduced above, the residual for nonlinear system is represented by TS fuzzy filter of the form like

$$\begin{aligned} \hat{x}(k+1) &= \sum_{i=1}^{4} \mu_i [A_i x(\hat{k}) + B_i u(k) + (L_i^* + \triangle L_i)(y(k) - \hat{y}(k))] \qquad (7.5.2) \\ \hat{y}(k) &= \sum_{i=1}^{4} \mu_i [C_i \hat{x}(k) + D_i u(k)], \end{aligned}$$

where L_i and ΔL_i are defined as in (6.2.3). The following are the details of the sub models and corresponding filter-based residual generators.

The first sub model In this case, the steering angle is taken as input signal, and lateral acceleration as output signal. The residual generated is

$$r_1 = a_y - \hat{a}_y \tag{7.5.3}$$

The gain matrices obtained from solving the DARE (6.2.8) are

$$L_1^* = \begin{bmatrix} 0.0012 \\ 0.0067 \end{bmatrix}, \quad L_2^* = \begin{bmatrix} 0.0012 \\ 0.0067 \end{bmatrix}, \quad L_3^* = \begin{bmatrix} -0.0003 \\ -0.0027 \end{bmatrix}, \quad L_4^* = \begin{bmatrix} 0.0001 \\ 0.0072 \end{bmatrix}$$

The increment in gain matrices are obtained by solving (6.2.14)-(6.2.27)

$$\Delta L_1 = \begin{bmatrix} -0.0053 \\ 0.0056 \end{bmatrix}, \quad \Delta L_2 = \begin{bmatrix} -0.0055 \\ 0.0056 \end{bmatrix}, \quad \Delta L_3 = \begin{bmatrix} -0.0054 \\ -0.0192 \end{bmatrix}, \quad \Delta L_4 = \begin{bmatrix} -0.0067 \\ -0.072 \end{bmatrix}$$

The covariance matrices for each sub-system based on (6.2.10) are

$$\Sigma_{n,1} = \begin{bmatrix} 390.1355 & 0 & \cdots \\ 0 & 390.1355 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}, \quad \Sigma_{n,2} = \begin{bmatrix} 343.8649 & 0 & \cdots \\ 0 & 343.8649 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}$$

$$\Sigma_{n,3} = \begin{bmatrix} 8.0357 & 0 & \cdots \\ 0 & 8.0357 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}, \quad \Sigma_{n,4} = \begin{bmatrix} 6.4026 & 0 & \cdots \\ 0 & 6.4026 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}$$

The second sub model In this case, the steering angle is adopted as input signal, yaw rate as output signal, the residual generated is

$$r_2 = r - \hat{r} \tag{7.5.4}$$

The gain matrices obtained from solving the DARE (6.2.8) are

$$L_1^* = \begin{bmatrix} 0.1298 \\ 0.4805 \end{bmatrix}, \quad L_2^* = \begin{bmatrix} 0.1107 \\ 0.4756 \end{bmatrix}, L_3^* = \begin{bmatrix} -0.0998 \\ 0.6853 \end{bmatrix}, \quad L_4^* = \begin{bmatrix} 0.0034 \\ 0.677 \end{bmatrix}$$

The increment in gain matrices are obtained by solving (6.2.14)-(6.2.27)

$$\Delta L_1 = \begin{bmatrix} -0.0998 \\ 0.2443 \end{bmatrix}, \quad \Delta L_2 = \begin{bmatrix} -0.0975 \\ 0.2509 \end{bmatrix}, \quad \Delta L_3 = \begin{bmatrix} 0.0079 \\ 0.2417 \end{bmatrix}, \quad \Delta L_4 = \begin{bmatrix} 0.0151 \\ 0.2598 \end{bmatrix}$$

The covariance matrices for each sub-system based on (6.2.10) are

$$\Sigma_1 = \begin{bmatrix} 0.5798 & 0 & \cdots \\ 0 & 0.5798 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}, \quad \Sigma_2 = \begin{bmatrix} 0.5784 & 0 & \cdots \\ 0 & 0.5784 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}$$

$$\Sigma_3 = \begin{bmatrix} 0.7317 & 0 & \cdots \\ 0 & 0.7317 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}, \quad \Sigma_4 = \begin{bmatrix} 0.7297 & 0 & \cdots \\ 0 & 0.7297 & \ddots \\ 0 & 0 & \cdots \end{bmatrix}_{21\times 21}$$

7.5.2 Residual evaluation

After the design of the residual generator, the remaining important task for robust fault detection is the residual evaluator. The residual evaluation consists of evaluation function and threshold value. Using L_2−norm as evaluation function with the length of evaluation window $N = 20$. The threshold value is calculated at fault free case.

The first sub model The known input (steering angle) with noise is shown in figure 7.9 (a). The data with an offset sensor fault of $5m/s^2$ occured at $t = 48$ second is used to validate the designed robust FD system. The threshold value in this case is $J_{th} = 207.1923$. In figure 7.9 (b), from $t = 48$ second the evaluated signal has exceeded the threshold value.

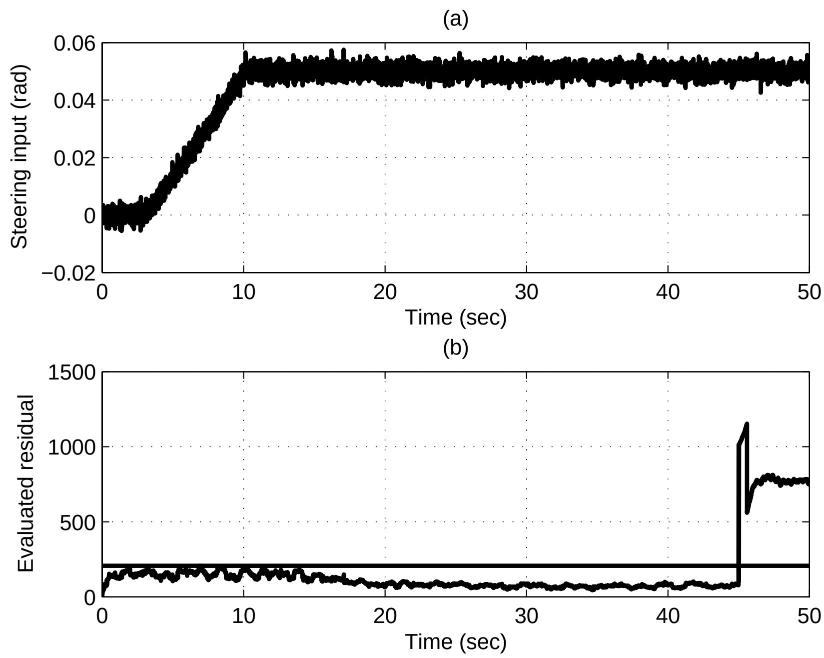

Figure 7.9: Robust Fault Detection for Lateral Acceleration with Stochastic Noises

The second sub model The known input (steering angle) with noise is shown in figure 7.10 (a). The data with an offset sensor fault of $5m/s^2$ occured at $t = 44$ second are used to validate the designed robust FD system. The threshold value in this case is $J_{th} = 172.3031$. In figure 7.10 (b), from $t = 44$ second the evaluated signal has exceeded the threshold value

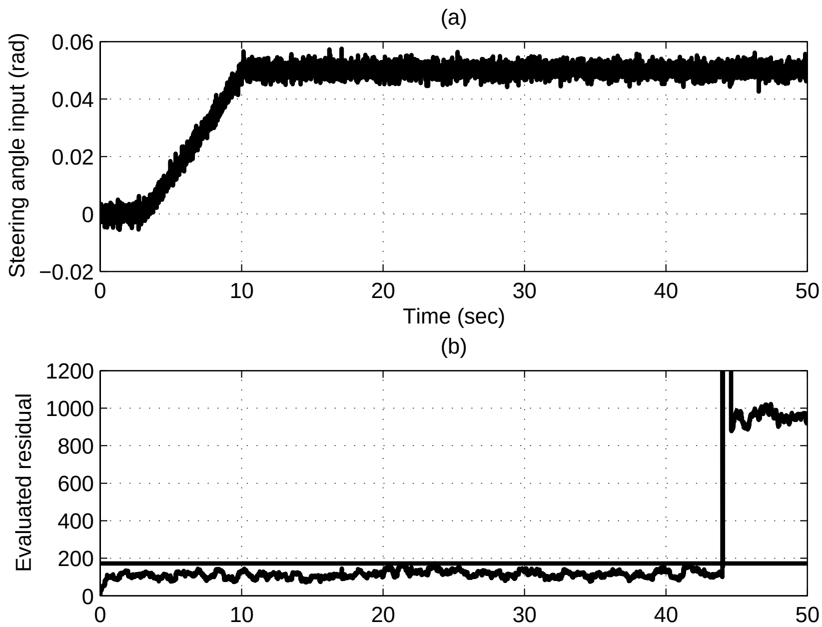

Figure 7.10: Robust Fault Detection for Yaw Rate with Stochastic Noises

8 Conclusions and future research

The global system behavior is described by a fuzzy fusion of linear model outputs. The model is described by fuzzy IF-Then rules, which represent local linear input-output relation of the nonlinear system. This type of model is called TS-fuzzy model. The investigation of the robustness problem in residual generation for FD system is studied while considerly different fuzzy observer-based residual generators for fault detection in dynamic systems.

In provious studies, robust FD systems have been designed for linear time invarian systems and robustness problems were delt with by reducing the deterministic disturbance and modeling errors. In this study, robust FD systems are studied for nonlinear dynamic systems. The robust FD problems are solved by reducing the effect of deterministic disturbance and increasing the effect of faults in the same time. In continuous time case, robust fuzzy FD system for TS fuzzy model first with deterministic disturbance, Second, with deterministic disturbance and parameters uncertainties, third, with deterministic disturbance and state delay are studied. In robust FD systems, the generated residual signal is found robust against deterministic disturbance, modeling errors and parameter uncertainties whilst remaining sensitive to the fault. The optimal fuzzy fault detection observer represents the result of a new optimally robust design that is based on an appropriately chosen performance index, which has function in filter gain matrices. The solution of optimization problem has been formulated in Linear Matrix Inequality.

In discrete time case, fuzzy FD system for TS fuzzy system with stochastic disturbances is studied. As Kalman filter design for nonlinear system is difficult, in this work, a new FD approach is presented. According to this approach, filter gain matrix consists of sum of two parts, in the first part, filter gain matrix for local subsystem is obtained by solving the crossponding DARE. In the second part, the incremented gain matrix is obtained from reducing covariance matrix of residual signal. The design procedure is provided in terms of LMIs.

The theoretical results are verified via simulated model obtained from vehicle lateral dynamic model and the sensor faults are detected. In this model some signals such as road bank angle could not be measured due to technical difficulties and high costs. These signals are modeled as deterministic disturbance. The variation of the cornering stiffness, the simplification in the force and moment balance equation are modeled as parameter uncertainties. Finally, this study allows to design robust FD system for a vehicle lateral dynamic system with deterministic disturbance, stochastic disturbance, parameter uncertainty and sensitive to sensor faults

8.1 Future work

The main task for further research is to study problems related to the integrated design of FD systems for nonlinear dynamic systems with deterministic and stochastic disturbances. The basic of this study would be to consider the design problems for the residual generation and residual evaluation under two well-developed model-based FD strategies, i.e. deterministic and stochastic strategies. Based on the properties of integrated residual evaluation, the design of residual generators is to be formulated as multi-objective optimization problems.

Another type of integration is needed to be studied for desiging FD systems for nonlinear dynamic system with deterministic disturbance, parameter uncertainties and time delay.

Finally, FD system can be considered as first step for designing fault tolerace control system for nonlinear dynamic system

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