Study of principles and methods of proportion

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Abstract

The methodology instruments to proportion objects in art design and architecture have been used as tools for thousands of years. Our western culture assumes by default that the use of these tools is useful, necessary and recommended.

This thesis is a detailed analysis of these instruments, their uses and possibilities throughout western history. We will also examine the causes that led to the identification of these tools as useful in the design process, deeply analyzing their effectiveness as design strategies.

Finally, the potential future uses of the older instruments is detailed as well as an analysis of the possible new techniques which are profiled to become the design tools of the new century.

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1. Review of important manifestations in the proportion theory:

1.1 Introduction to part one.

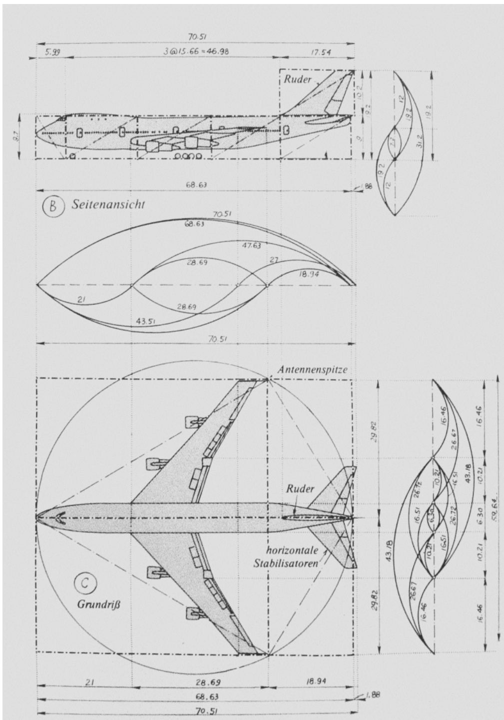

Figure 1.1. Analysis practiced on a Boeing Jet 747 cited by György Doczi.

This first part of the study reviews the manifestations of the theory of proportion in different cultures and times, which influenced in some way the concept of proportion, as we know it today in our western culture.

First of all, it is important to highlight that there are countless collections and anthologies on this topic throughout time and cultures. Libraries have numerous volumes that contain samples of how works of arts in our cultures are proportioned.

Along the same lines, there are countless books with proportional analysis of natural objects: ranging from live beings such as animals and plants to the most unimaginable manifestations in nature like galaxies and nebula. This type of study with respect to proportion in nature has generated a type of taboo.

The golden proportion, for example, has been “adjusted” many times to all kinds of analysis. This omnipresence has granted it a status of myth; it is as if we had accepted that it is magical and that it is present in our natural and cultural surroundings.

It is even possible to find proportional analysis in our modern material culture. These are not just limited to art objects and designs but to all kinds of productions, including the more technical and rigorous manifestations of our society.

The example in Figure 1.1, (Figure 1.1. Analysis practiced on a Boeing Jet 747 cited by György Doczi [1]), shows an analysis practiced on a Boeing Jet 747, an example that shows an extreme case where the proportional analysis is sought and justified even in objects where the object’s dimensions have been defined by the most rigorous technical and physical requirements. Doczi states: “One would think that an airplane’s design is defined by aerodynamic calculations given the hardiness of the materials and motors and consideration to its efficient use; however, the Boeing has the same harmonious proportions as any art and nature that has marveled us”[2]

This almost obsessive need to find and justify the **existence,** in the natural cases, and the **use,** in the case

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[1] D. György, (1981). **The Power of Limits**. Proportional harmonies in Nature, Art and Architecture. 5. Ed. Shambhala Publications Inc. Boston, U.S.A., Page 151

[2] D. György, (1981). Page 150

of material culture, of certain proportions has reached almost pathological levels.

It is for this reason that this work does not intend to gather yet more information on the use or discovery of proportion. It is more an analysis of the possible conscious use of the proportion systems throughout cultures and times; furthermore, it is an interpretation of how these could have influenced our present concept of proportion. The idea is to try and identify the method of thought of some manifestations of material culture with respect to the proportion systems. In this manner, the intention is to define which manifestations are based on **explicit** uses of the proportion system (or specific canons) and which possess proportions for any other reasons, including by chance.

That is why for the present study, it resulted more important to review which cultures have developed systems that handle proportions, canons or methods to help “measure” artistic works and material culture. It is also necessary to emphasize the type of proportions or numerical sequences used and how they influenced our present conception of art and design.

This first part of our study is not a recollection of works of art or manifestations in nature that lead to this or that proportion; it is more of a list of notes that describe the methods of thought and instruments used throughout time in the application of proportion; methods that gave way to the cult-myth of proportion as an instrument of design.

This first analysis could surmise which cultures used conscious measuring systems in their art and which did not. And even more importantly: what influences are obvious in the material inheritance of these societies that have formed what we now know as “proportion”.

1.2 *Horror vacui*.

It is natural to begin our analysis in the prehistoric period, not just because it is the beginning of art but because there are clear insinuations that prehistoric art was proportioned on purpose. So, beginning to analyze this possibility seems reasonable.

As we know, the first artistic manifestations known as Homo Sapiens Sapiens date back some 35, 000 years, the so-called glacial art.

If we want to go further than that, perhaps the first manifestations of some sort of recognizable aesthetic sentiment would be the funeral practices of the Homo Sapiens Neanderthalensis. A good example is the Shanidar cave in the mountains of Zagros, Irak[3] where 60,000 years ago, a person was buried amidst a combination of white, yellow and blue flowers with green foliage, definitely showing an interest for scenic beauty.

Further on in our analysis, we have the known golden age of glacial art. "Prehistoric art flourished towards the end of the last glacial period: it began some 35,000 years ago, reached its maximum splendor in the Lascaux age 15,000 years ago and ended forever when the ice caps retreated, approximately 10,000 years ago."[4]

These artistic manifestations expose clear artistic styles which are relatively consistent throughout the years and the different areas discovered up to know, although there is a marked difference between the Western European artistic expressions in the area surrounding the Alps and the Eastern European and Russian art[5]. Within each zone, the coherence is very marked and clearly recognizable. This coherence throughout thousands of years and thousands of kilometers speaks of a homogenous art and the conscious and extended "use" of these cultural manifestations.

The greater part of primitive art consists of representations of animals whose modern interpretations have changed drastically throughout the XX century. The first classical explanation, well recalled today, and widely overcome, was the mythical-religious representation of Abbé Breuil, where the artist painted the hunt, and magically, possesses the desired animal. The Freudian interpretation of André Leori-Gourhan and Annette Laming-Emperaire (not necessarily together) came later

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[3] R. Leakey, (1981). **La Formación de la Humanidad**. Ediciones del Serval. Barcelona. Page 154

[4] R. Leakey, (1981). Op. Cit. Page 166

[5] J. Alcázar and N. Bayo, (1997**). El Hombre Fósil**. Penthalon Ediciones. Madrid. Page. 113

where the duality of the male and female are represented by stereotypical figures such as the American buffalo and the horse; although sometimes the symbolic association changes from author to author. More recently, authors such as Margaret Conkey[6] state the possibility that these pictorial works are a symbolic representation of the social organization experienced by the people who lived at that time. According to this author, these images could represent a congregation of different social units.

With respect to its iconography, one can say that in Paleolithic two dimensional art (paintings and engravings) we can classify two great thematic groups: (1) the great majority of the ideomorphs and the (2) the zoomorphic. The ideomorphs are represented mainly by lines and points. The lines can be from simple isolated strokes up to complex and geometric groups. Its form can be straight or rolling. The points can also form greater groups. The zoomorphic group basically represents great mammals such as wild horses, stags, urus, American buffaloes, and ibis and to a lesser degree, pisiforms. Many of the artistic beast species are already extinct.

In our case, the main theme focuses perhaps on the idea of space in Paleolithic art. In all of the phases of its history, art has been accompanied by a conception of space. Prehistory is, to a certain degree, the pre-architectural stadium of human evolution. The spatial conception existent throughout the countless centuries of this first art changed with the first civilizations, with the appearance of architecture and its gradual conquest of painting and sculpture. However, the spatial conception of this primary art is very different to the one that would be created with the passing of years.

“The absolute freedom and independence of the vision of prehistoric art is something that we have been unable to reach since then. It was its distinctive character. There was no up and down, in the sense we know it. The fact that an animal appeared in one position or another lacked importance. Since it lacked importance, there was no clear separation between one object and another, or scale norms or proportion of sizes. The violent juxtaposition, in size as well as time, was acceptable and normal. Everything was displayed within an everlasting present, the perpetual fusion of today, yesterday and tomorrow.”[7]

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[6] M. Conkey, O. Soffer, and more, (1997). **Beyond Art Pleistocene Image and Symbol**. California Academy of Sciences. San Francisco, USA.

[7] G. Sigfrid, (1981**). El presente eterno: comienzos del arte**. Alianza Forma. Madrid. Page. 574

Evidently, according to present authors, the order, orientation and of course, “proportion”, were not a priority for those artists.
The sense of spatial composition in the work and with this, of any kind of canon or theory of proportion, appears to be absent. It seems to be that its basic desire to decorate, for whatever social purpose it represented, was perceived in the known *horror vacui,* very characteristic of primitive arts.

1.2.1 Anthropometric Images

Paleolithic art is filled with colorful and superimposed images from the bulls of Lacaux on to the Altamira bison to the horse of Pech-Merle. In contrast, the images of man are scarce and less celebrated. “The more famous prehistoric representations of the human form are, however, the so-called Venus statues, with overflowing buttocks and breasts which supposedly enclose the image of fertility of the mother goddess”[8]

"There are few samples of her. The most famous one is possible the Venus of Willendorf (Austria, figure 1.4), found in 1908 in a late Aurignacian layer, and reproduced in almost every history of art. Of all of the prehistoric figures, it is the one that more clearly shows plasticity. Its stunted proportions and the absence of neck seem to be rather strange in this period, but its exuberance complies with the figure’s purpose with undeniable force. (…) Beyond the differences in treatment, all of the representations of this period have one thing in common: a strong emphasis on the pelvic region, which is the most significant part of the body” [9].

“The prehistoric artist that sculpted them exaggerated terribly, deforming body parts of these “Venus’”, as if wanting to prove something, perhaps expecting precisely more from those body parts than others. The breasts, pelvis and buttock regions are very disproportionate in reference to the rest of the body, and the head, arms and legs are small in relation to the totality of the figure. These voluminous breasts, fallen from their weight…remind us of goddesses of abundance or fortune, and therefore, are associated with the rites of fertility, which would explain the bulky pelvis they possess.”[10]

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[8] Leakey, R., (1981). E. Op. Cit. Page 184
[9] V. Bozal, (1992), **La escultura**. Tomo 2 de la Historia del Arte. Carroggio S.A. de Ediciones. Barcelona
[10] L. Pericot yana R. Martín, (1973). **La prehistoria**. Ed. Salvat, Col GT. Barcelona. Pages 39-42

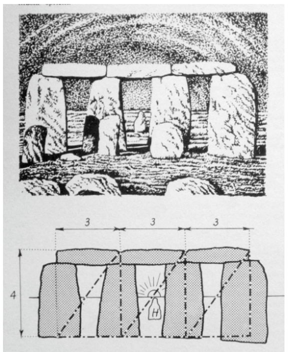

Figure 1.2: Proportional analysis prepared by Doczi showing the 3,4,5 triangle or Pythagorean in the ruins of Stonehenge.

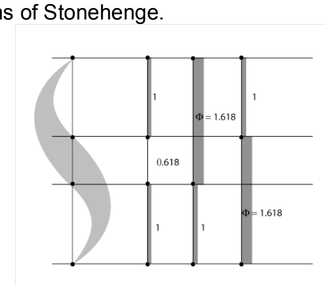

Figure 1.3: Golden scale: Making use of a general convention, we will the graphic representation of the double curve in this study to express the intertwined golden sections. As the figure shows, this graphic curve shows two intertwined golden sections. Its presence along with a work of art is due to the correspondence of their proportions.

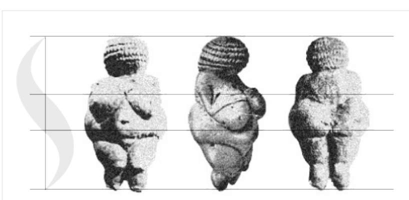

Figure 1.4: Venus of Willendorf : sculpted in oolite limestone. Height 11,1 cm. It was found in 1908 by the archeologist Josef Szombathy in an Aurignacian loess deposit near the city of Willendorf (Austria). The sculpture has brown paint residues.

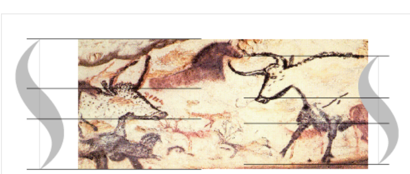

Figure 1.5: Proportional analysis of rupestrian paintings of ancient Lascaux cave, 15.000 years before our era. Painting made with mineral oxide on rock. Length of the bulls, 5,5 m. Lascaux, Montignac cave. As one can observe, it is fairly simple to find a specific proportion in an artistic work.

1.2.2 Proportion in the glacial age

As observed in figure 1.2[11], it is a common practice to analyze primitive artistic works; this type of analysis is relatively easy to prepare. Authors, such as the one cited, and other as well, let their imaginations fly analyzing all kinds of artistic works.

To show how easy it is to conduct these analyses, figures 1.4 and 1.5, developed by the author, exemplify this practice. It is possible, and very used, to seek specific proportions (in this case: the golden proportion) in samples of Paleolithic art as in the famous Venus of Willendorft, even finding interesting coincidences as in the following case: where the height from where the chest grows out is cut in an aureate manner towards the end of the buttocks. However, as we have already studied, the spatial conception of the first art doesn't even have format restrictions, a minimum characteristic to establish some kind of composition or proportion.

It would appear to be that these types of analyses are mere speculations. If one cannot find a reference to composition, such as the horizontal or vertical line; if there is an absolute absence, at least until to the present, of samples of use of some kind of conscious scale, order or proportion; the existence of a conscious notion of proportion used as a guideline for the creation of rupestrian art would be, therefore, less likely. There is no record of the deliberate and conscious use of proportion systems in this beginning stage of the human art.

Given this situation, I believe it is possible to affirm, without going out on a limb, that there was no explicit canon in the glacial art.

As we have said, this first part of our study is based on the search of explicit canons used in the different ages and regions of history. As a first observation, we can say that although some manifestations of rupestrian art could have specific proportions, it is very unlikely that its presence was more than plain coincidence.

The fact that we have found specific proportions in these artistic manifestations, as we have done ourselves in figures 1.4 and 1.5, cannot be explained as the conscious use of a convention or canon. This tendency to analyze a work of art in retrospective seems more a desire by the authors who carry out the analysis to do so than the existence of any evidence to affirm it.

[11] G. Doczi. Op. Cit. Page 58

1.3 The Sacred Cord.

The nature of this part of our study is the identification of the conscious use of the proportion systems throughout history and cultures; uses that have influenced how we consider this aspect in the design today. Continuing on this line of thought, the next step would be to analyze the Egyptian culture.

Many authors understand this culture as the summary of the most important advances and techniques in the production of material culture from prehistoric times until its appearance. Furthermore, the Egyptian culture is well understood as the melting pot where the general techniques that will be used later are summarized as the base of western culture.

For this reason, we continue with our analysis in Egypt. Unlike prehistoric art, Egypt is a highly documented culture and our knowledge of their customs and practices in the development of artistic works or design is very superior to previous periods.

Countless studies detail the types of tools that were used to “proportion” art in ancient Egypt; in this study, we will concentrate on describing general methods of thought in this respect.

The Egyptian culture appears towards the end of the neolithic age, along with other fluvial cultures, such as the Mesopotamian, Sumerians, Babylonians and Acadians. Egypt is a longitudinal country, following the course of the Nile River. The abundance and security provided by its valleys, amongst mountains or deserts, favored the settlement of Neolithic agricultural communities.

There are two geographical regions: the higher Nile, from its source located among mountains and the lower Nile: the delta. The former is very rich due to the constant overflowing of the river which brings fertilizing sediments. Here is where the primitive culture became organized: the constant flooding of the river followed a regular frequency with respect to the seasons. This frequency provoked the needs that would give rise to an incipient culture. The calendar, the registry of property and above all, the constant need to define territorial limits, generates in Egypt geometry, a science that will structure all of the Egyptian culture.

“The Neolithic population had become farmers. Their lives depended on the flooding of the river which, along with the sunny climate, ensured good crops and the picking of which was possible thanks to an industrious and organized community work. All of these factors – the isolation, the slow and predictable rhythm of the yearly activities, the safety of the crops, besides the excellent communication offered by the river – provided the basis for the early formation of the state towards the end of the fourth millennium."[12]

On the other hand, the richness of the land makes it possible for one part of the population to dedicate itself almost exclusively to the development of knowledge. It became necessary and important for this culture to know, for example, how to predict precisely when the river would flood or how to build grain lofts; the establishment of a priestly or monarchical caste was just a matter of time.

Art in primitive Egypt (before the pharaohs) is born around 4.000 b. Christ and develops until 3.200 b. Christ. Its beginnings are very similar to the last prehistoric painting with instrumental theoretic characteristics as those already discussed.

However, in 3.200 b. Christ a new period begins, the so-called Ancient Empire. The first pharaoh from the first dynasty was chosen and with him, the first aesthetic codes were established in painting and bas-relief (main techniques that mixed in pictorial art). During this period, the figure's frontal image is established, a technique that would accompany Egyptian art until the end of the last empire.

After the ancient empire, a period of instability occurred in which the pharaoh's power dissolved in the feudal territories from 2.258 to 2.134 b. Christ. At this moment, Mentuhotep was able to reunify the empire. New cults also appeared, such as the one to Isis, Osiris and Seth, and with them, the introduction of new iconographic themes; however, the Middle Empire also managed to maintain the basic frontal aesthetic and a very consistent aesthetic ideal.

The arrival of a new empire, from 1.570 to 1.085 b. Christ, marked a new route. The new imperialism put its art in contact with foreign forms, which were adopted to a lesser degree. The maximum relaxation of the style arrived with the religious revolution: pharaoh Amenophis IV resigned to his name and changed it to Akhenaton, resigning also to all of his gods in favor of one only God, "Aton", creating initiating the first case of monotheism in history.

Without any significant variations, by the end of the new empire, the decadence intensified and Egypt suffered successive invasions from its enemies.

[12] E. Wilson, (1986). **Diseños del antiguo Egipto**. Ediciones G.Gili. Barcelona. Page 7

During these 3000 years, despite the emergence of three empires and antagonist changes in the political organization, Egyptian art remained surprisingly consistent. "The key to understanding the Egyptians formal visual culture, in architecture as in art and the extraordinary homogeneity during 3000 years, lies in the concept of the type of ideal. Depending on our own cultural experience, we all have our own idea of, for example, what makes a traditional monarch, a convenient home or an adequate place of cult…"[13]

These pre-concepts generate what every culture understands as correct or adequate; in reality, for most of us, to see beyond our time and culture is practically impossible; "many of our decisions are not ours; they have been suggested; we have managed to persuade ourselves that they are a result of our initiative, whereas in reality, we have limited ourselves to adjust to other people's expectations"[14]

Well, the Egyptians were not the exception, and what their visual culture considered beautiful and correct is one of the most characteristic examples of this phenomenon. Perhaps because of its antagonism with our existing canons, it is also one of the hardest to fully understand.

"Egyptian art is basically two dimensional and its basis is drawing – even sculptors begin their sketches by outlining them in the corners of the block. The Egyptians tried to seek the way to express the essence of their objects and not the one-time impressions or the vision from a concrete point of view…objects were expressed more from the point of view of how much was known of it then its observation. Several points of view from one object were combined to create clear and simple images. The term simultaneous perspective was created to designate this type of representation…There is no intention of creating an illusion of space."[15]

"It seems pretty easy to describe the style of pharaonic art…we can choose three fundamental elements. Each composition had a marked lineal order through subdivisions outlined with horizontal lines…The second element, which is also related to the global composition, is the close relationship between the figures and the hieroglyphic writing that accompanies it... The third element is in regards to the artistic conventions of the figures, either human, animals or pieces of furniture. Each figure or each one of the main parts that make it up is reduced to a characteristic profile"[16]

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[13] B. Kemp, Barry J., (1992**). El antiguo Egipto. Anatomía de una civilización**. Editorial Crítica S.A. Barcelona.
[14] E. Fromm, (1993**). El miedo a la libertad.** Barcelona. Page 195
[15] E. Wilson, (1986). **Diseños del antiguo Egipto**. Ediciones G.Gili. Barcelona. Page 12
[16] B. Kemp, (1992)

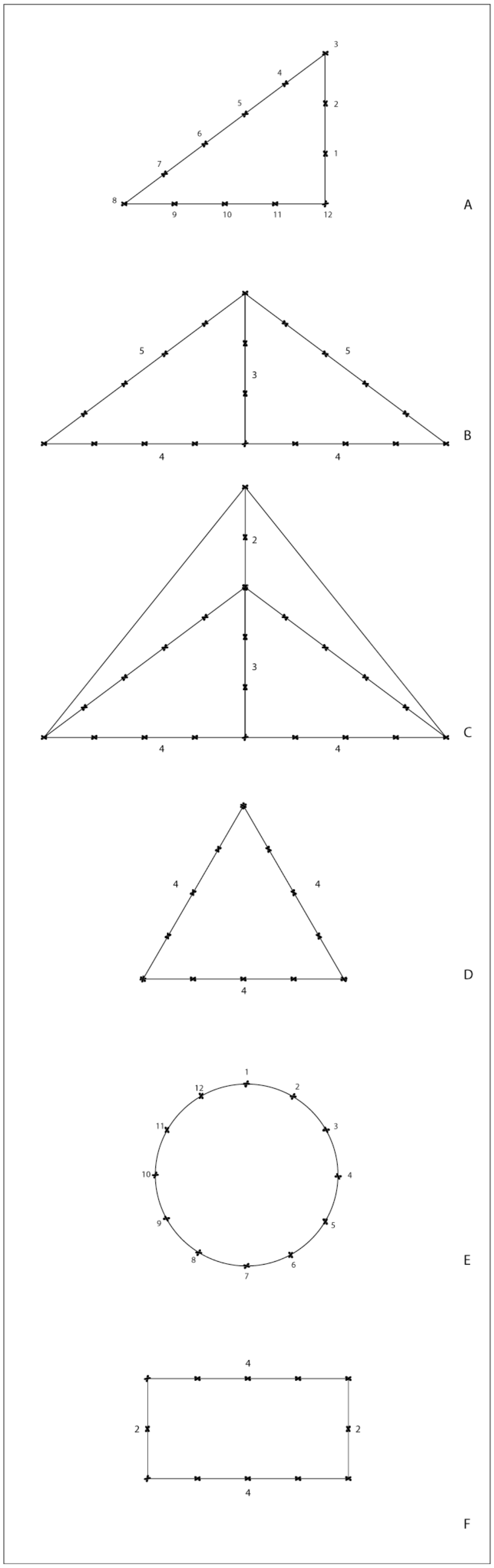

Figure 1.6: Basic constructions of the Egyptian cord.

This last characteristic cited by Kemp, is also the most stereotypical of this culture and the less understood.

How can an ideal design be so ugly and unnatural? How can this be considered beautiful? Not only is that true, but the more the canon was adhered to, the greater its acceptance and distinction within the culture.

These complex standards were established as an alphabet, and the mere indication of deviation implied a fault. They were applied to the representation of gods and the royal family; the lesser the rank of the represented figure, the greater freedom was allowed with his or her image. Consequently, the slaves and peasants were painted in a more naturalist manner, in heterodox positions; whereas the high-ranking figures were submitted to rigorous formal and proportional frames.

This sole characteristic reflects the importance given to the rigourosity with which the proportional canon was followed. In this case, as opposed to the rupestrian period, there is no doubt whatsoever of the conscious use of detailed proportional canons. “My conclusions, (writes Robins), will be based on surviving grids and grid traces where they exist, using direct measurements made on site.”[17]

In other words, the Egyptians have left countless evidence of their rigorous treatment of proportion.

1.3.1 Tools for Proportion

In order to learn more about the fundamental concepts of the Egyptian canons, it is worth the while to begin with the basic tools. Possibly, the more generally used tool was the Sacred Cord, used as a tool of design and construction in all empires.
“The origin of the historic building layout was the setting out of the 3:4:5 triangle with the Egyptian rope, wound about three pegs so that it formed three sides measuring three, four and five units, which provides a 90º angle between its 3 and 4 sides”[18] (figure 1.6.a)

This simple tool made it possible to trace almost exactly, several basic figures of the Egyptian material culture. As figure 1.6.b shows, the cord not only helps to determine the straight angle, but it makes it possible to trace triangles in relation to height: base 4:3, 8:5 and even circles. It is worthy to highlight that the length between

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[17] G. Robins, (1994). **Proportion and style in ancient Egyptian art**. University of Texas Press. Texas, USA.
[18] M. Gadalla, (2000). **Egyptian Harmony**. **The visual music**. Tehuti Research Foundation. Greensboro, NC, USA.

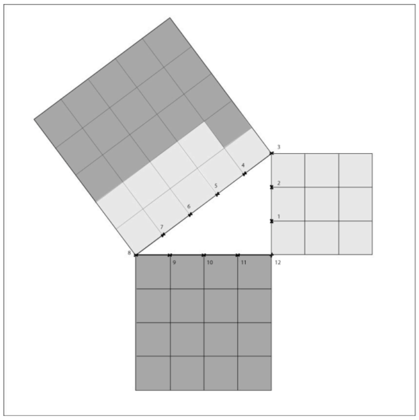

Figure 1.7: Graphic explanation of the Pythagorean theory using the Egyptian cord.

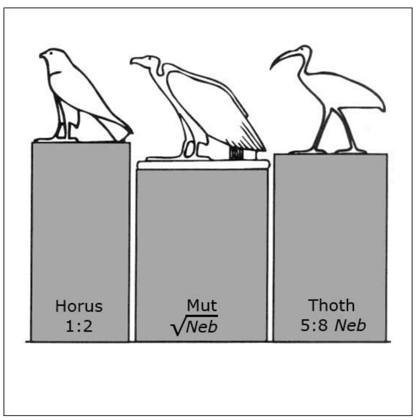

Figure 1.8: Proportions used as ceremonial pedestals for three Egyptian gods.

two knots in the cord was equal to one Egyptian cubit, around 0.5236 m.

Another interesting aspect of the Egyptian cord is that in its triangle rectangle form it exemplifies the clearest way to of represents the Pythagorean Theorem. As figure 1.7 shows, the sum of the square of the right angles evidently equals the square of the hypotenuse. In this respect, Gadalla notes "It must be noted that the Rhind Papyrus[19] shows that calculation of the slope of the pyramid [Rhind Nos. 56-60] employs the principles of a quadrangle triangle, which is called the Pythagoras Theorem."[20]

Furthermore, Plutarch in his *Moralia*, Vol IV, also refers to this relation: "The Egyptians hold in high honor the most beautiful of the triangles, since they, like the nature of the Universe most closely to it, as Plato in the Republic seems to have made use of it in formulating his figure of marriage. This triangle has its upright of three units, its base of four and its hypotenuse of five, whose power to that of the other two sides."[21]

Undoubtedly, another form used would be the 5:8 rectangle, also derived from the use of the Egyptian cord (figure 1.6.c). This rectangle has a 0.625 relation and was known as the *Neb* proportion. This relation is interestingly close to the relation we know today as θ (phi) 0.618., also known as the golden proportion. This also coincides with the fact that the word *Neb* means "gold or divinity" in Ancient Egypt. In other words, at least the beginnings of the most known proportion systems were already in the making in these Egyptian constructions.

The same can be said of the double square or √4 (Figure 1.6.f). Significant constructions from the Zoser Complex (2630-2611 b. Christ) in Saqqara up to the Festival Hall in the Karnak temple, in the New Empire, use this obvious proportion.

"An interesting observation regarding the significance of the differently proportioned rectangles is found on the pylon at the Temple of Khonsu, at Karnak. This pylon shows the falcon, vulture, and ibis[22], each on a different proportioned rectangle"[23]. (See figure 1.8)

It is also known that written texts on the walls of certain temples were summaries of books that were kept in the

[19] The Rhind Mathematical Papyrus (now in the British Museum) is a copy of an older document during King Nemara (1849-1801 BCE), 12th Dynasty. It contains a number of examples to which academic Egyptologists have given the serial number 1 - 84
[20] M. Gadalla, (2000). Page 64
[21] M. Gadalla, (2000). Page 88
[22] A bird with long and downward curved beak and black and white feathers.
[23] M. Gadalla, (2000). Page 98

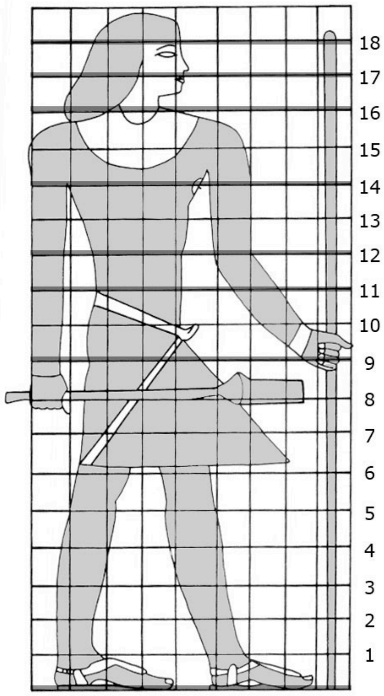

Figure 1.9: Typical canon of the Middle Empire Period.

temple's library. A group of them are construction treaties: "The new temple was being built, where a series of ideal types materialized. We were able to enjoy its existence thanks to the descriptions and allusions of the same texts. These even include the dimensions of the ideal buildings expressed in elbows (1 elbow = 41,8 cm)" [24].

1.3.2 Human Figure.

In reference to the representation of the human figure, the Egyptians were as rigorous as in everything else. Despite a few small differences in the conventions among the three empires, the use of similar canons is extraordinarily coherent for millenniums. Figure 1.9 shows a summary of the proportions of the standing masculine figure in the Old and Middle Kingdoms.

"A close examination of any sample of figures still on their original grids shows that the artists did not have to conform exactly to the system. Nevertheless, it is possible to abstract from such a sample an ideal relationship between the figure and the grid. It is this "typical" figure that I shall describe in the following sections.

Standing figures consisted of 18 squares from their soles on the baseline to their hairlines. As Lepsius pointed out, the hairline was used rather than the top of the head presumably because the latter might be obscured in the case of the king's figure by his various crowns. Typically, horizontal 18 of the grid ran through the hairline; horizontal 17 through or near the bottom of the nose; 16 through or near the junction of the neck and shoulders; 14 through or near the nipple; 12 through the bottom of the ribcage and through or near the elbow of the hanging arm; 11 often through or near the navel when shown, the small of the back, and sometime the top of the belt al the back; 9 through or near the lower border of the buttocks; 6 through the knees; and 0 below the soles… There is no fixed point for the top of the head, which lies somewhere between horizontals 18 and 19.

A vertical grid line, which can be termed the axial line usually passes through some part of the ear and divides the neck and upper torso roughly in half; the vertical line immediately in front runs through the eye. Often when a major figure stands with the arms hanging vertically by the side, the armpits lie on verticals 2 squares to the left and right of the axis line, so that they are 4 squares apart. The width of the upper arm is usually 1 square, so that the body from the outer edge of one arm to that

[24] B. Kemp, (1992). Page 129

of the other, must conveniently measure along horizontal 15, approximately 6 squares wide. The distance across the body at the level of the small of the back is usually 2.25 to 2.5 squares. The feet are most often 3 squares in length." [25]

The standing feminine figures differ from the masculine in some details that are proper of gender differences but its canons are as strict as those previously explained. The figures in sitting positions also possess their own canons, which are clearly defined. With the beginning of the new empire in Thutmose's III reign, one can also observe some changes in the basic grids; however, these changes can be described as minor.

It is noteworthy that the grids discussed are not just limited to dimension the human figures but also to frame all of the work's composition. The hieroglyphics, the furniture and other objects represented in the works are subject to the same strict grids that define the management of space in Egyptian art.

The establishment of the sculptures is the same as the paintings and the architecture. The statue is engraved in a cube. The papyrus shows drawings in the typical grid, and there are unfinished sculptures that present the blueprints and the lines of the grid with which the human figure is measured. It is clear that this is the beginning of the modular system of representation. The grid makes it possible to fix the module with precision, which is the square.

In summary, we receive the first conscious manifestations of the use of controlled proportions in Western art culture from the mix in the proportions 3:4:5 (triangle), 5:8 (rectangle), 2:4 (rectangle) and a strict canon in the representation of human figures. The explicit observance of the canon, and the mistake it meant to draw away from it, are artistic guidelines that are widely documented.

Note, also, that although the canon is a strong representation of reality, it is not its end purpose. More than the search of purist realism, it seems we are in the presence of a specific aesthetic profile with which a culture, religion and cultural expression became identified. This will mark a difference with new cultures, such as the Hellenic, where the faithful representation of reality is the productive nucleus of the artistic process.

Obviously, the handling of proportion in the Egyptian culture strongly and strictly defined the concept of space in art of that culture. The coherence of this art throughout three millenniums is perhaps the main

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[25] G. Robins, (1994). Page 73

reason for this immediate and unequivocal recognition of that culture.

Its strict rigorousness and the forcible affirmation of the “correct” ideal, were widely cited centuries later by authors such as Pluto and Plutarch; undoubtedly showing its influence in our conception of art and design in our history.

1.4 The Canon

The logical route to take to continue our analysis of the development of the methodological tools, to work on the proportion of art and design in the West, is to continue on to Greece.

"*And it was then that all these kinds of things thus established received their shapes from Ordering One, through the action of Ideas and Numbers*

It is not generally suspected how much the above pronouncement of Plato –or in more general way, his conception of Aesthetics- has influenced European (or, lets say, Western) Thought and Art, especially in Architecture" [26]

Without a doubt, ancient Greece is the birthplace of our Western society and its influence in our way of thinking and design is enormous.

"(The) …two different classes of proportion, both derived from the Pythagorean-Platonic world of ideas, were used during the long history of European art… the Middle Ages favored Pythagorean-Platonic geometry, while the Renaissance and Classical periods preferred the numerical, i.e. the arithmetical side of that tradition."[27]

In the case of ancient Greece, the information that has survived is greater than in the case of the Egyptian and, of course, that of prehistoric art.

The conscious use of proportions, and the need to study and fully understand them, is widely documented in Greek texts. There are, however, two main aspects where proportion is worked on in Greece: (1) sculpture and (2) geometry.

Despite what people accept as a given, there is not conclusive proof that proportion was used consciously in architecture.

Let's not confuse this affirmation with the thousands of books that speak of proportion in ancient Greek temples. It is true that it is possible to find complex proportions in different systems of ancient buildings; but it is also true that there is no actual information about the construction or use of said buildings where their proportion is planned, commented or critiqued.

"In the almost total absence of written documents that could help to determine the issue (the proportions of the

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[26] M. Ghyka, (1977). **The Geometry of Art and Life**. Dove Publication. New York. Page IX

[27] R. Wittkower, (1978). **The changing concept of proportion. In idea and Image**, Thames & Hudson. London. Page 116

Greek Architecture), the most varied speculations have been built up upon the conjectural analysis of measurements of buildings. The problem with this is that when one already has a theory one tends to find confirmation of it wherever one looks. Determined researchers, if they have the patience, can ‘prove’ almost anything they want about the proportion of a building, though this sometimes means basing the analysis on lines and shapes invisible in the building itself and existing only in the researcher’s mind or paper.”[28]

In this manner, the main objection to the thousands of proportional analyses of the classical buildings, and equal number of books, is the total absence of textual evidence about the methods that were really used by ancient Greeks.

As Theodore Cook tells us: “Because we can draw a spiral line through a series of developing members, it does not follow what a plant or a shell is attempting to make a spiral, or that a spiral would be any advantage to it… Geometrical constructions do not, in fact, give any clue to the causes which produce them, but only *express* what is seen, and the subjective connection of the leaves of a plant by a spyral curve does not at all imply any *inherent* tendency in the plant to such a construction.”[29] Obviously, the same can be said of Greek buildings.

Since our study wishes to base itself on the conscious use of proportion, and as we can observe, there is only circumstantial evidence of this use in Greek architecture, we will touch upon this aspect later when it is possible to establish an unequivocal connection between the use of proportion and the explicit desire to use it in architecture.

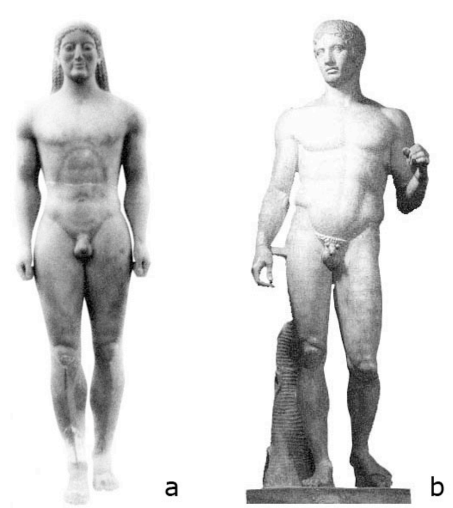

Figure 1.10: A: Typical sculpture of the Archaic period. Note the rigidity and frontality, as its similarity with the Egyptian style. B: Policleto, Doriforo (ca 450 a.C.) Roman copy, marble; height. m 2,12, Archeological Museum, Naples.

1.4.1 Proportion in sculpture

In sculpture, however, the case is opposed in diameters. The use of a proportional system to improve the quality of sculpture is, without a doubt, documented.

As is known, Greek sculpture goes through three clearly defined ages:

Archaic: The feminine and masculine figure appears. At the beginning, these figures were typically Egyptian, no movement and with a marked frontality (Figure 1.10.a). Then, certain characteristics of movement appeared, arms were disjoined from the body and the face shows a curious smile. The sculptures were offered to sportsmen.

[28] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Page 81
[29] T. Cook, (1979**). The Curve of life**. Dove Publications. New York. Page 23

Classical: This age was a period characterized by a greater boom in all of the artistic and literary manifestations. The sculptors are able to perfect their techniques, as well as the best sculptor pieces, where the magnificence of the human form is observed (Figure 1.10.b). The most well-known sculptors of this age are: Miron, Fidias, Policleto, Scopas, Praxiteles and Lisipo; all with styles and characteristics of their own.

Hellenist: This age corresponds to the end of Greek art; the works now take on former models, perfecting them, showing a great capacity of realization. The art of sculpturing takes on monumental characteristics. The portrait takes a forefront role.

The explicit canons developed during the classical age. It is well known that the problem was born as the solution of Greek sculptors for the inexperience of the artists. Even the definition of the canon as a tool for Greek sculpture was going through the Archaic age, and used the so-called “geometric style” (Figure 1.10.a). With it, the first sculpted forms arose, defined by triangular torsos, small heads over long necks and skinny waist, with a clear frontality and primitive rigidity.

Towards the middle of the V century b. Christ, during the times of Pericles, Greek classicism takes shape thanks to the works of three great sculptors: Miron, Policleto and Fidias; with them, the concept of beauty was defined as an archetype to imitate. The artists who paid the most attention to the mathematic study of the proportions of the human anatomy were Policleto, and a century later, Lisipo.

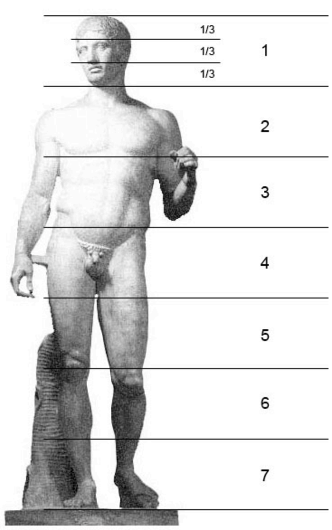

Figura 1.11: Policleto’s Canon, Doriforo (c 450 b.C.)

Policleto (450-420, a.C.) made a careful and detailed study of the proportions of the human body, a canon of the ideal masculine beauty based on strict mathematic proportions. His canon is found in the Doriforo (Figure 1.11). He had dozens of important followers during the following century. The well known “Canon” of Policleto is the materialization of the theoreric principles formulated by Policleto in a book, which unfortunately, was lost, named precisely: **Canon**.

Policleto stated that the head should be a seventh part of the figure’s height. The head itself is divided into three equal parts, which correspond to the front, the nose and the distance from the former to the chin; the foot would be three times the width of the palm of the hand. The leg, from the foot to the knee, must measure six palms; the same measurement is used between the knee and the center of the abdomen. As one can see, we are before a mathematical conception of human beauty.

It is important to highlight that the canon’s objective is to improve sculpting and serve as an instrument to create a more faithful “copy” of the model; in this case, the human body.

Note that this first canon does not pretend to possess beauty in itself, as would happen later. Beauty is already in the model and the canon allows us to reproduce this beauty in a more faithful manner than what would have been possible.

The use of this canon has some very particular characteristics, one of which is that it is more a technical problem rather than a problem of artistic creation. In this manner, success is guaranteed if by using a canon, a faithful copy of nature is made (in this case, the human form). Nature is beautiful *ergo* the result is beautiful. In other words, beauty was already in the model and the problem is reduced practically to making a reliable repetition of it. By having a methodology, as the theory of proportion that guarantees we are making a faithful copy, the matter is well resolved.

A century later, it was Lisipo (IV century b. Christ), a Greek sculptor who modified the ideal proportions in the representation of the human body. The new canon proposed by Lisipo consisted in a change of the body proportions: a greater height and a reduction of the volume of the head, part which was also chosen to serve as a proportion measurement: the body’s height must be equal to 8 times the head’s size.

As we can observe, once again, these proportional systems which took the head as a unit, already had the basic characteristics which many of today’s proportion theories conserve; for example:

- A certain unit is defined. In the case of Policleto and Lisipo: the head.
- This unit is defined as a sample and a factor or numerical sequence is repeated. In the aforementioned cases, they are simple arithmetic successions like 3 heads or 7 heads.
- Other dimensions are therefore produced that could be used in the parts of the works that need them.

These basic canon characteristics are present in the great majority of them, from ancient Egyptians to Le Corbusier.

1.4.2 Geometry

A second aspect where the study of proportion is highly documented in ancient Greece is geometry. With greater detail than in sculpture, geometry takes the study of proportion and proportional systems to unimaginable levels; so much that would have to wait two thousand years to be revised. The theoretic jump between what has occurred until now with respect to proportion in the West and what the Greeks managed to do is enormous. While the proportional studies of ancient Egypt is practically reduced to grids (simple arithmetic proportional systems), the Greek studies on geometry are summarized in books such as *Elements* by Euclide which is still subject today to consultation with respect to geometric proportions or mathematical demonstrations.

“The ancient Greeks did not discover geometry. They converted it into an instrument of obligatory rationality for the world’ knowledge.”[30]

It is precisely this level of depth that could have provoked the historical idolatry of the Greek proportional systems and its indiscriminate interpretation. As was mentioned before, there is no evidence that these elaborate processes and thoughts were used in architecture, but its depth almost categorically suggests that it is not possible they were not.

This tendency of the later theoretics of not being able to deny the use of the profound systems of structured proportion by the ancient Greeks in an important field such as architecture, even in the absence of evidence, is a well-known phenomenon in perception. Let’s just say that, when something is easy to imagine, people will assume it as possible.

As an example of this perceptual phenomenon, we can say that if an airplane is a heavy machine that can fly, it is easy to imagine that it will crash; *ergo* many people fear flying. Not so with a car, because it is less obvious that a car can crash even though statistics show that one is seven times less likely to crash while on an airplane than in a car.

For this reason, and as our argument has tried to make clear, that it can be taken as a fact and that the affirmations are merely circumstantial, we cannot use Greek architecture as an example of proportion in material culture.

[30] K. Gerstner, (1986). **Die Formen der Farben.** Verlag Athenäum. Frankfurt. Page 28

Even without this evidence of concrete use, the study of the proportions and the progress of geometric use is molded in Greek history.

We could affirm that Plato is the first to summarize in his *Timaeus*[31] many of the thoughts that finally converge in the fundamental basis of the Western spatial thought.

"Plato's cosmology, set out principally in the Timaeus, weaves together strands inherited from previous Greek thinkers – the mathematics of Pythagoras, the finite, spherical universe of Parmenides, the flux and the tuning of opposites of Heraclitus, the four elements of Empedocles, the primeval mixture of Anaxagoras, the atoms of Leucippus and Democritus, and the latter's distinction between the image and the thing itself – as well as, inherited from his own teacher Socrates (c.470-399 BC) and the sophist Protagoras (c.485-420 BC) the new emphasis on the human."[32]

From this array of thoughts, geometry cannot escape and with it the concept of proportion gathered by Plato.

"(Plato) who chose geometry as the new basis, and the geometrical method of proportion as the new method; who drew up a programme for a geometrization of mathematics, including arithmetic, astronomy, and cosmology; and who became the founder of the geometrical picture of the world, and thereby also the founder of modern science – of the science of Copernicus, Galileo, Kepler, and Newton."[33]

To the question asked in the former sections of this study: "when and how is the concept of proportion coined?" We find in Plato the correct answer: "But two things cannot be satisfactorily united without a third: for there must be some between them tying together."[34]

Here, for the first time, we find a clear definition of proportion, speculating on the relation between parts and how the components must "adjust together". This definition fits perfectly in the three basic proportions used in Western history: arithmetic, geometric and harmonic, which will be detailed in the second part of this study.

Besides this very clear allusion of Plato to proportions, we also have the platonic discussion of the elements.

Let us recall that Plato was mainly a philosopher and not a mathematician; therefore, his comments arise from a

[31] Plato, (1965**). Timaeus and Critias**. Penguin Books. London
[32] R. Padovan, (1999). Page 100
[33] R.K. Popper, (1966). **The Open Society and its Enemies**. Routledge. Vol I. Page 319
[34] Plato, (1965**).** In Op Cit. Page 40

philosophical point of view. The truth is that his definition of the four elements includes the universe: earth, water, air and fire and the fifth one added later, known as *ether.*

Plato built a directed association with the five regular polyhedron of geometry: tetrahedron, cube, octahedron, dodecahedron and icosahedron and with it, he formally includes the series of irrational numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and ϕ which will accompany the history of the study of proportions forever.

Chronologically, we should now bring into discussion the fact that Aristotle, perhaps due to his encyclopedical formation, is the first to explicitly declare the link between natural beauty and proportions. Aristotle wrote mainly about metaphysics, theory of knowledge, chemistry, physics, mechanics and mathematics; therefore, it is easy to image that this encyclopedical formation and thought led this thinker to presume a close relation between the perception of beauty and the mathematical configuration of nature.

"… we should approach the study of every form of life without disgust, knowing that in every one there is something of Nature and beauty. For it is in the works of Nature above all that design, in contras with random chance, is manifest; and the perfect from which anything born or made is designed to realize, holds the rank of beauty."[35]

Note that we are using the modern concept of the perception of beauty to make this Aristotelian relation.

The concept of the perception of beauty, which differs from the concept of aesthetics or beauty, refers to the isolated and unequivocal fact in which the human brain perceives the phenomenon called beauty. "Beauty as such is a convention. Just like the length of a meter, a whimsical measurement that was defined historically… beauty is a convention defined by the evolutionary process for our species' interests...beauty as such, just like something red or green, doesn't actually exist; it was just a translation of one or several physical conditions that help manipulate the universe on behalf of our survival"[36]

"As Leda Cosmides, John Tooby and Jerome Barkow point out: "Culture is not causeless and disembodied. It is generated in rich and intricate ways by information-processing mechanisms situated in human minds. These mechanisms are in turn the elaborately sculpted product of the evolutionary process". Clearly, culture cannot just

[35] Aristotle, (1960). **On the Parts of Animals**, **quoted in F.M. Conford**. Before and after Socrates. Cambridge University Press. Pages 93-94
[36] F. Hernández-Castro, (1995). **Estética Artificial.** Editorial MithOz. San José, Costa Rica. Pages 36-37

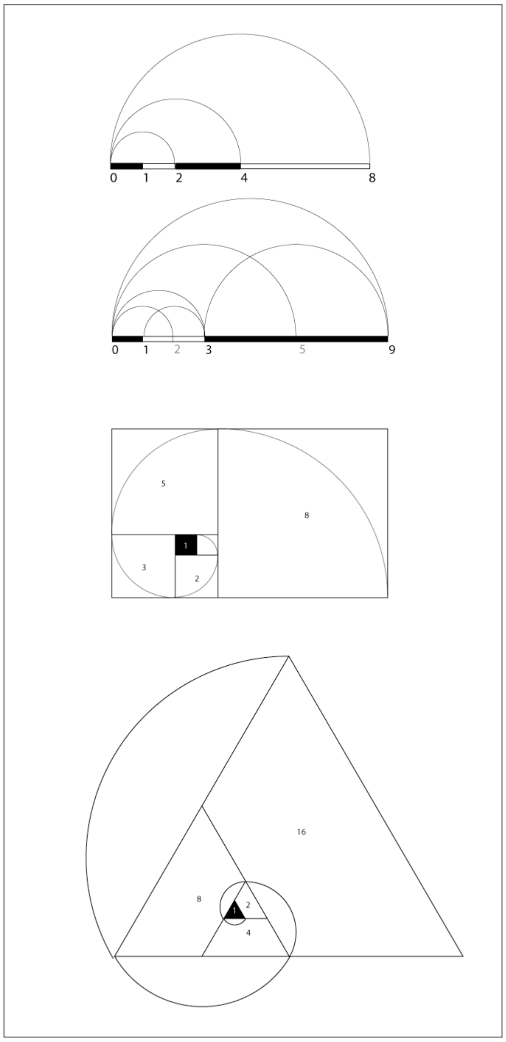

Figure 1.12: The two basic sequences defined by Aristotle and Euclides with their uses of *ad quadratum* and *ad triangulum.*

spring forth from nowhere; it must be shaped by, and be responsive to, basic human instincts and innate preference."[37]

This concept of beauty is what we will analyze to try and clear the existing relation between the use and study of proportion and the desire to generate beauty in the produced material culture.

Returning to Aristotle, it appears that it was he who first associated beauty with the order of nature; a concept that will be used, and abused, until our times. "The link Aristotle makes between beauty and order in nature, particularly in biology, is interesting in the light of the later history of the theory of proportion"[38]

Aristotle also worked on the concept of infinite and finite with a close relation between the discreet and continuous mathematics. The obvious relation is carried out with the Pythagorean proportion of the arithmetic division of the octave (better known by its later use by Alberti and Palladio in the Renaissance) and the geometric proportion based on the square roots and irrational numbers such as ϕ. By defining two types of infinite, the additive infinitive obtained from indefinitely adding units to a group and the subtractive infinitive by infinitely dividing a unit, Aristotle sets the philosophical frame for Euclides' deep formality.

"In the third century before Christ, Euclides summarizes all ancient knowledge in his volumes, *Elements*"[39]

As a matter of fact, Euclides introduces at the beginning of his fifth book of his *Elements*: "Let magnitudes which have the same ratio be called proportional."[40]

"If we accept Euclide´s definition, it follows that in order to obtain a proportional sequence the initial ratio of 1:2 between the unit and the first larger measure must be repeated each time, producing the geometric progression 1,2,4,8,… If instead we make each increment double the preceding measure we get the complementary series 1,3,9,27,…"[41]

As can be observed in Figure 1.12, these two sequences already defined by Euclides form the fundamental basis of what we know today as arithmetic and geometric proportions, which will be used *ad quadratum* and *ad triangulum* to obtain the harmonious proportions and

[37] N. Etcoff, (1999**). Survival of the pretties, the science of beauty.** Doubleday editions**.** New York. Page 22
[38] R. Padovan, (1999). Page 117
[39] K. Gerstner, (1986). **Die Formen der Farben**. Verlag Athenäum. Frankfurt. Page 29
[40] Euclid, (1956). **The Thirteen Books of the Elements**. Dove Publications. Vol II. Page 114
[41] R. Padovan, (1999). Page 133

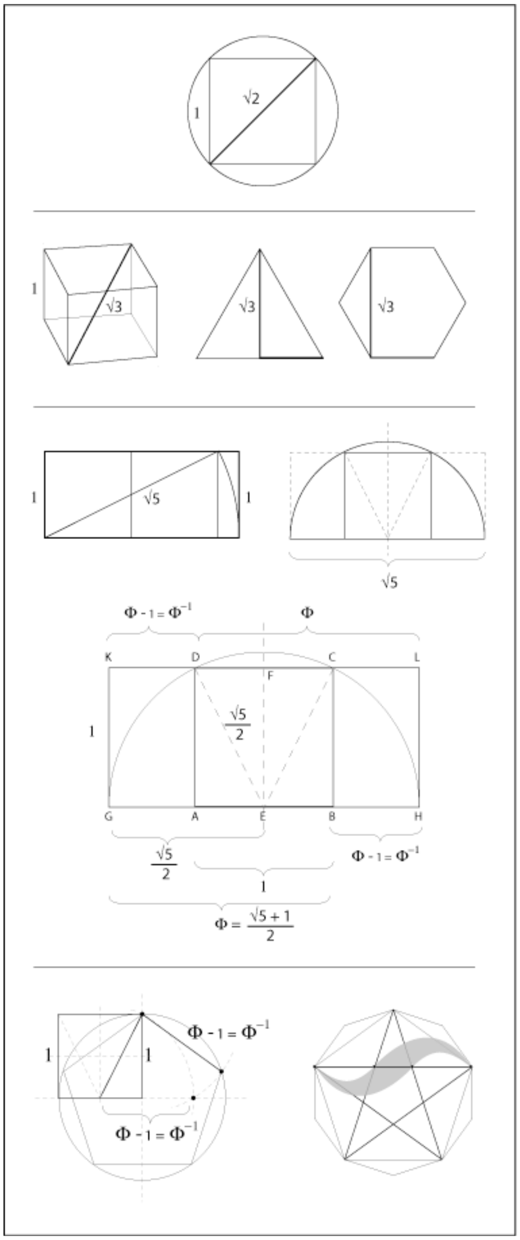

Figure 1.13: Description of the irrational number √2, √3, √5 and ϕ, according to Euclides. The double curve in the last figure was added to clearly show the relation.

sustain all of the history of the proportional methods in our culture. In the second part of this study, we will observe in more detail the generation of these proportions.

But this was only the beginning, since the thirteen *Elements* books of Euclides are developed to end with the thirteenth book dedicated to the golden section and to the five regular solids. Here Euclides presents the golden section, obviously the official introduction of ϕ and the five regular solids. With the presentation of the irrational numerical series, √2, √3, √5, the main numerical sequences are made official as well as the main irrational quantities, which will form the basis of all of the proportional studies, from ancient Greece until our days.

Furthermore, in his first book, Euclides shows with complete accuracy and elegance the Pythagorean Theorem, clearing up all doubts that had surrounded this concept from Egyptian times. (See Figure 1.7)

In order to introduce these concepts, Euclides resorts to diagrams in his thirteenth book (of his *Elements)* that have been cited and used as the basis of comparison for countless times.

Figure 1.13 describes and comments the constructions that illustrate the √2, √3, √5 y ϕ concepts. The first part of the figure shows how √2 is the diagonal of the square, holding down the circle's ratio where the square is circumscribed and its side.

In the following analysis, Euclides describes what we know today as the diagonal three dimensional of the square[42] and defines for the first time the relation between the side, the face's diagonal and the diagonal between two vertex opposed to the cube; in other words, 1: √2: √3. Furthermore, the relation √3 found in the equilateral triangle and regular hexagon is described.

In following, one can observe the two constructions that are found in the thirteenth book of *Elements;* first of all, the construction of the √5 rectangle and its relation to the golden section or ϕ; immediately, the construction of the pentagon inscribed in a circle and its internal golden relations. These constructions clearly demonstrated the most famous irrational relations in history.

However, Euclides's contribution does not end here. The proposals from 13 to 18 of the thirteenth book talk about the five regular polyhedrons (Figure 1.14). The analysis of these solids rounds out the relations between the four

[42] F. Hernández-Castro, (1995). **Fundamentos del Diseño Tridimensional**. Editorial Tecnológica de Costa Rica, Cartago. Page 74

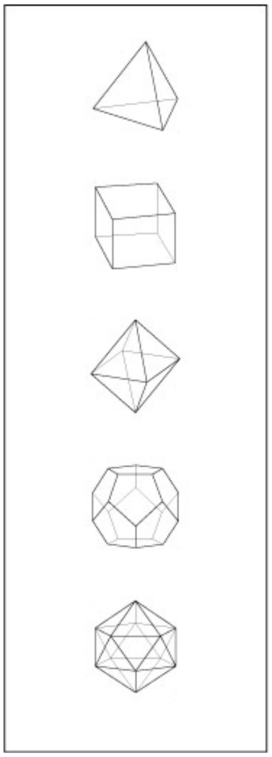

Figure 1.14: The five regular solids cited by Euclides.

irrationals. The root of two is the diagonal of the square, and of course, the diagonal of the cube's face. The root of three is not only the three-dimensional diagonal of the square; it is also the relation between the height of the equilateral triangle and half of the base, clearly related with the faces of the tetrahedron, the octahedron and the icosahedrons. Finally, the inclusion of the dodecahedron and its pentagonal face includes the golden relation or ϕ according to the former diagram (Figure 1.13).

The same relations are reported by Euclides in the inside of the solids and the relations between the diameters of the midspheres and circumspheres.

The following table from Padovan's[43] book summarizes these relations:

Form	edge	circumsphere	midsphere
tetrahedron	1	$1 / \sqrt{2}$	$\sqrt{2} / \sqrt{3}$
cube	1	$\sqrt{2}$	$\sqrt{3}$
octahedron	1	1	$\sqrt{2}$
icosahedrons	1	ϕ	$\sqrt{(\phi \sqrt{5})}$
dodecahedrons	1	ϕ^2	$\phi \sqrt{3}$

Table 1.1: Relations between platonic solids**.**

With these relations, elegant demonstrations and unquestionable constructions clearly relating the solids we know today as Pythagorean, Euclides concludes his *Elements* leaving a historical legacy that would have to wait two thousand years to be enlarged (never corrected) by Newton´s Mathematical Principles.

[43] R. Padovan, (1999). Page 151

1.5 Eurythmia

Roman architecture possesses independent and innovative characteristics when compared to Greek architecture. While Greek architecture conforms to simple structures such as the dintel or arquitrabe, the Roman, on the other hand, creates spaces that are vaulted and dome-shaped; introducing also new materials, new techniques and a monumentality that the Greek architecture did not have.

In the Greek world, the Temple is the architectural space *par excellence.* The Greek temple is designed to be observed from outside; therefore, its internal configuration is not important, it is less functional. For this reason, the Greek architecture and as a result, the classical temple, are defined mainly by its constructive harmony.

By comparison, an ever-present objective in the Roman world is the need for order and planning, a tendency to the megalomania, and the prevalence of regularity and symmetry, characteristics that, among others, lead to the functional conception of architecture. This functional character explains the great variety of public, political, commemorative, administrative and religious buildings and, of course, the Roman house.

Very similar to what happens in Greek architecture, no explicit proportional analysis in Roman architecture has been carried out during the time of construction or use of this architecture. The majority of sources that show us “a Roman Pantheon circumscribed in a perfect sphere”, for example, are no more than Renaissance interpretations of these works as in the case of Palladio in his *I Quatro Libri Dell’Architectura*[44].

It is important to highlight that often students of architecture or design observe an analysis by Palladio on the Roman Pantheon and assume it is contemporary to the building’s time; however, it is also important to point out that the historical distance between these analyses and his work of reference is more than 15 centuries. This situation is no less speculative if we decided today, in this book, to conduct an analysis completely ours, of a work built in the IV century, but with the extra difficulty of not having the actual sources of information.

This entire situation, however, has an exception. Whereas Greece has no historical reference in the use of proportion in architecture, Rome has a theoretic architect: Vitruvius.

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[44] A. Palladio, (1570**). I Quatro Libri Dell’Architectura**. Domenico D’Franceschi. Venezia quoted in K.Gerstner (1986). Die Formen der Farben. Verlag Athenäum. Frankfurt. Page 28

So, three hundred years after Euclides, the first direct reference to the use of the proportion systems in architecture arises.

In this manner the first documented theoretic on this specific topic is the Roman architect, Marcus Vitruvius Pollio. “Vitruvius advised that the architecture of temples should be based on the likeness of the perfectly proportioned human body where a harmony exists among all parts.”[45]

Here is where the relation is laid down for the first time which has been used since then: copy “the beauty of human proportions” and transport them to the design of buildings or material cultures. Before Vitruvius, canons had been used as technical assistance to improve sculptures and the writings that existed described proportions as complex geometric descriptions.

Aristotle had already insinuated that proportions of “natural order” possessed an intrinsic beauty. But now Vitruvius presents a clear intention to design architecture “according to natural orders”.

Obviously, there already exists a fallacy in the use of Aristotle’s syllogism – which we will analyze in more detail in the third part of the study; however, it is important to highlight that it was possibly Vitruvius who was the first to accept that if natural things, in this specific case, the human body, are beautiful, then, somehow, this beauty is transferred by the proportional system to artificial designs and constructions.

To tell the truth, this is not the only inconsistency in the teachings of Vitruvius, beginning with the “Fundamental Principles of Architecture”, presented in the second chapter of his book, “The Ten Books of Architecture.”[46]

These principles are filled with inconsistencies or, at least, there are yet not fully understood.

The five principles are:

1. *ordinatio*, taxis or ordinance
2. *dispositio*, *diathesis* or disposition
3. *eurythmia*
4. *symmetria*
5. *decor*, or decorum
6. *distribuito*, *oikonomia* or economy[47]

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[45] K. Elam, (2001). **Geometry of design, studies in proportion and composition**. Princeton Architectural Press. New York. Page 12

[46] Vitruvius, (1960). **The ten Books on Architecture**, M.H.Morgan trans. Dover Publications. Dove Publication. New York. Page 47

[47] R. Padovan, (1999**). Proportion. Science, philosophy, architecture**. Spon Press. New York. Page 158

As an example, let's analyze the concept of symmetry defined by Vitruvius himself:

"Symmetry resides in the correlation by measurement between the various elements of the plan, and between each of these elements and the whole… As in the human body… it proceeds from proportion –the proportion which the Greeks called analogy- consonance between every part and the whole…"[48]

As one can observe in the definition of symmetry given to us by Vitruvius, it is more related to our modern definition of proportion than with the concept of symmetry in force.

But this is not a matter of translation or definition of terms, since the solution would be a more careful translation. No, Vitruvius himself defines *ordinatio* as:

"The balance agreement of the measures of the building's members in each separately; and the relation of the proportion of the whole building with a view to symmetry"[49]

In other words, more or less the same as symmetry, at least, as previously defined by himself. Let's read about *eurhythmia* now:

"When every important part of the building is thus conveniently set in proportion by the right correlation between height and width, between width and depth, and when all these parts have also their place n the total symmetry of the building, we obtain *eurythmia*."[50]

Now we have the concept of *euritmia* as the correct application of *symmetria* or *ordinatio* as desired and not as an independent characteristic; therefore, the fundamental principles of architecture indicated by Vitruvius do not have morphological consistency. In other words, some are characteristic and others are appraisals of how these characteristics are used.

Or do we not have trustworthy translations? Or do we not understand the concepts that were used in those days? Or was it here with Vitruvius where we began to use a vague and confusing language in a failed attempt to organize the variables that intervene in the process of design?

As we said in another book: "The academic attempt to clear the nature of beauty through a vague and imprecise language, far from fathoming the causes, it

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[48] M. Ghyka, (1977). **The Geometry of Art and Life**. Dove Publication. New York. Page X
[49] R. Padovan, (1999). Page 158
[50] M. Ghyka, (1977). Page X

eludes them, trying to justify itself before the facts…this attempt to transmit knowledge (in design) as if it were philosophical material, leads us to the false illusion of dominating a field which, upon the first opportunity, leaves clear its deficiency in focus."[51]

We could continue with the three remaining principles, which are also redundant and incoherent, but that is not the matter at hand right now.

The truth is, as was mentioned before, that we have no doubts that Vitruvius associated proportions of architecture with the proportions of the human body; as a matter of fact, the first chapter of this third book was named: "Of the symmetry of temples and the human body".

In order to get closer to this focus, Vitruvius advises the use of one unit:

"Symmetry is the proper mutual agreement between the members of the building, and the relation, in accordance with certain parts selected as standard, of separate parts to the figure of the building as a whole… The human body is so designed by the nature that the face, from the chin to the top of the forehead and the lowest roots of the hair, is a tenth part of the whole height; the open hand from the wrist to the tip of the middle finger is the same"[52]

As we can see, Vitruvius spoke of a proportion of one tenth of the total height with respect to the height of the face. This type of relation used a specific type of canon: exactly as Policleto's style three centuries back but, instead of applying it to sculpture, it was applied to architecture. This practice became the *modus operandi* of all later thinkers in the matter of proportions, influencing Renaissance figures of the likes of Leonardo and Palladio until modern theoretics of design such as Le Corbusier or Van der Laan.

As a matter of fact, this was Vitruvius' greatest influence on his followers: his observation and procedure to circumscribe the human proportion in a circle and a square is perhaps the most famous of his images. Unfortunately, we must declare that the original illustration from the first century b.C. has been lost forever and what we have from Leonardo to Corbusier is no more than each author's interpretation of what he supposed Vitruvius wrote:

"For if a man be placed flat on his back, with his hands and feet extended, and a pair of compasses centered at

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[51] F. Hernández-Castro (1995). **Estética Artificial**. Editorial MithOz. San José, Costa Rica. Page 15
[52] R. Padovan, (1999). Page 162

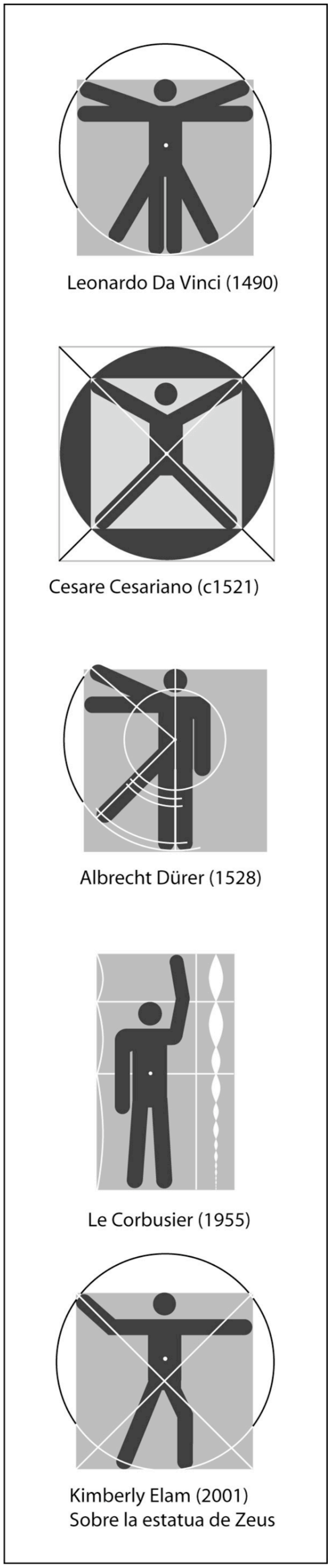

Figure 1.15: Comparative diagrams of the different interpretations throughout history of Vitruvius' analysis of proportion.

his navel, the fingers and toes of his hands and feet will touch the circumference of a circle described there from. And just as the human body yields a circular outline, so too a square figure may be found from it. For it we measure the distance from the soles of the feet to the top of the head, and then apply that measure to the outstretched arms, the breadth will be found to be the same height…"[53]

Multiple thinkers were able to interpret this description from the lacking diagram. Figure 1.15 shows diagrams of some of the more famous interpretations in this respect.

As we can observe, there are important differences in the interpretations of this description, whereas Cesare Cesariano[54] completely circumscribes the square in the circle, Leonardo and Dürer allow themselves to decentralize, ignoring the inconsistency that the diagonal of the square √2 is not the diameter of the circle, and with that, losing much of the original spirit of Vitruvius.

This is actually because it is not possible to accommodate a man in a circle and a square without giving himself permission to deform the man (as Cesariano) or decentralize the figures (as Leonardo)." There is no way in which, without anatomical distortion, the circle and square concept can be combined with the early breakdown of the measures into aliquot parts"[55]

Furthermore, Vitruvius describes all types of objects with his proportions; from war machines, such as catapults, to temples. Once more, it appears that Vitruvius initiates this tendency, which will become an obsession throughout history.

It is also interesting to add that Vitruvius was also the first to explicitly expose (or at least the first we know of), a methodology of design based on the system of proportion; but not just a methodology to evaluate proportions as is practiced today. It is a system where proportions define the dimensions of design from the beginning. This system is based on the decrease or increase of the primary dimensions of design according to specific proportional factors, controlling the increase or decrease of the *symmetria* understood as a type of proportional harmony among the parts, as was clarified before. In this manner, the design is modified in its *symmetria* in a constant manner and not with focus *a posteriori* of the work, considered today a normal practice.

53 R. Padovan, (1999). Page 167
54 C. Pedretti, (1999**). Leonardo, le machine**. Giunti Gruppo Editoriale. Firenze. Page 14
55 R. Padovan, (1999). Page 168

This advice from Vitruvius is definitely ahead of its time, considering that the relation between growth and decline of proportion in integral and infinite manner are concepts that are delved into until the beginning of the second half of the XXth century with the works of Mandelbrot[56] in fractal theory, but this is a topic for the third part of this study. Vitruvius, once again, expresses for the first time an intention that will become common practice for thousands of years.

With Vitruvius, and his legacy, the chapter on the explicit and thoughtful study of proportions in Roman art is opened and closed. Other types of art, besides architecture, were less explicit with respect to this and were kept in the shadow of Greek art, at least where proportion is concerned.

In sculpture, for example, it was thought for a long time that Roman sculpture was well behind the Greek; this was due to the fact that the many Greek works were copied without its characteristic perfection; however, many historians believe today that the perfection was abandoned deliberately for more realistic and more human works. In general, Roman sculpture is marked by a realistic character due to a practical sense (already commented in architecture). This tendency characterizes sculpture and makes it represent people and things as such. The Roman sculpture emphasizes historical events and public persons, reinforcing in this manner, its propagandistic sense. It is characterized by a greater realism, where it has a privileged place and not a proportional canon.

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[56] B.B. Mandelbrot, (1977). **The fractal geometry of nature**. W.H. Freeman and Company. New York.

1.6 Sacred Geometry.

It would be pretentious to jump over the Gothic or *Medium aevum* without dedicating some time to the history of the tools of the proportion of design; therefore, we will dedicate a few lines to this aspect.

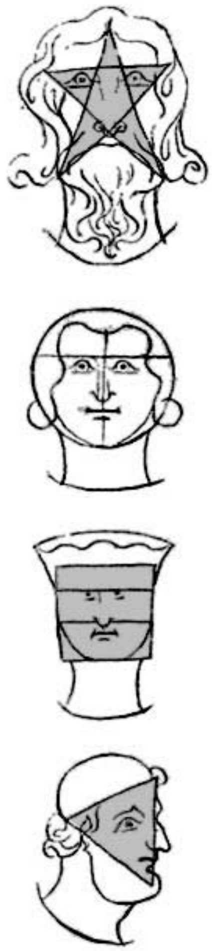

Figure 1.16: Proportional analysis by Villard de Honnecourt made circa 1235

Despite modern efforts to revindicate medievals, it is a worthy point to say that, in respect to proportional tools, they were not prolific.

Many of the Greek writings were not translated into Latin until the second millennium was well underway and the texts of the epoch such as the *German Geometry* of Mathes Roriczer not only did not contribute anything to what had been done until the Imperial Age (Rome) but often times contained mistakes already overcome since Euclides. "For many centuries after the fall of Rome the study of mathematics was largely confined to arithmetic"[57]

In the same manner, the rigourosity of the age, at least, where the treatment of the proportional systems is concerned, was very inferior to its predecessors. Figure 1.16 shows the proportional analysis of Villard de Honnecourt carried out around 1235, as an example of this affirmation.

Perhaps this age's contribution (if it can be called that) to our way of thinking about geometry and proportion in design is the quality of "sacred". Very much in tune with the times, a deficient interpretation of the methodological instruments of the Greek-Roman legacy was called "sacred geometry" for the first time.

"(about the symbolic importance in twelfth century of the Vitruvius' ideas)… According to the theory of *homo quadratus*, number is the principle of the universe, and numbers posses symbolical meanings which are grounded in correspondences at once numerical and aesthetics…" [58]

Therefore, geometry's association with certain mysticism became rooted in our culture. Since then, the lack of depth or rationality of its sources, typical of these practices, has accompanied much of the research on proportion or geometry.

"It seems to be the basic assumption of traditional philosophies that human intellectual powers are for the purpose of accelerating our own evolution beyond the restraints of the biological determinism which binds all

[57] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Page 175

[58] U. Eco, (1986). **Art and Beauty in the Middle Ages**. Yale University Press. New Haven. Pages 36-36

other living organisms. Methods such as yoga, meditation, concentration, the arts, the crafts, are psychophysical techniques to further this fundamental goal. The practice of Sacred Geometry is one of these essential techniques of self-development"[59]

"Numbers, Music, Geometry and Cosmology are the four Liberal Arts of the ancient world. Theses are simple universal languages, as relevant today as they have always been, and still found in all known science and cultures without disagreement."[60]

Two quotations from the two books with the same title "Sacred Geometry" leave a clear example of this type of mystical halo assumed by geometry and proportion from medieval times.

Possibly the lack of scientific strictness of the time in favor of "more spiritual" reasons led to this type of treatment given to geometry in art. "An almost sacred importance was attached by the medieval architects to standard measures and to the use of whole units"[61]

There have been many discussions with regard to the proportions in Gothic cathedrals. These discussions are full of incoherencies. Detailed analyses are carried out of a building in arithmetic systems of proportions, and immediately, another author details the same building's geometric proportion systems. Some are based on the Roman foot (294.45 mm); others in the Egyptian foot of 299.6 mm and others in the English which is equal to 305 mm; with the obvious consequences in the real calculus of dimensional modules over which they justify their proportional analysis. In summary, there is no evidence of a specific unit, of a defined proportional system or of a previous intention.

There is, however, a clear exception to this rule: Leonardo de Pisa (1170-1240), better known as Leonardo Fibonacci.

Leonardo was the most original and skilled mathematician of all of the Middle Ages, but his contemporaries had a hard time understanding a good part of his works due to its degree of difficulty. He made contributions to arithmetic, algebra and geometry.

Fibonacci was lucky enough to be educated in the mathematic systems used by the Arabs and Hindu, including the Arab-decimal system and the concept of the zero still unknown in Europe. In contact with these cultures, he realized the advantages of his numerical

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[59] R. Lawlor, (1982). **Sacred Geometry**. Philosophy and Practice. Thames & Hudson. London

[60] M. Lundy, (1998**). Sacred Geometry**. Walker & Company. Ontario. Page 1

[61] R. Padovan, (1999). Page 190

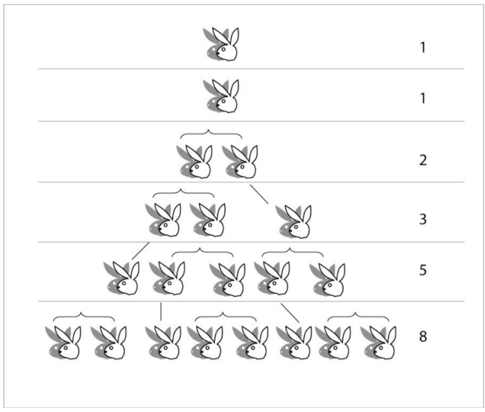

Figure 1.17: Graphic analysis of the Fibonacci sequence beginning with the birth of rabbits.

methods. In 1202, he published his most famous work known as *Liber Abaci*, which constitutes, fundamentally, a collection of arithmetic and algebraic problems with different calculation methods and the solution of first and second-degree equations. This work also contains an important defense of the superiority of the methods of Arab numeration and the positional notation with the nine figures 1, 2, 3, 4, 5, 6, 7, 8, 9 plus the 0, a concept known to Arabs as *céfiro* where our words for cero and figure originate.

Furthermore, Leonardo de Pisa offers clear rules in his works to carry out operations with these figures with whole numbers as well as fractions; he also provides the rule of three (simple and composed), norms to calculate the square root of a number, as well as instructions to solve first degree equations and some of the second degree.

The most famous problem that appears in the *Liber Abaci* and which brings Leonardo into history is the Problem of the Rabbits, which goes, more or less, as follows:

We start with a couple of rabbits that have just been born. In two months' time, this couple is ready to reproduce, producing, from now on, a new pair of rabbits, each month.

This new pair of rabbits that has just been born will, in two months' time, reproduce, breeding another couple of rabbits per month. What is the total number of rabbit pairs in a specific number of months?

The solution to this problem is the so-called "Fibonacci Succession" (Figure 1.17): 0, 1, 1, 2, 3, 5, 8, 13, 21, 34..., where each term in the succession is the sum of the previous; in mathematical terms, we have:

$$f(x_n) = \begin{cases} x & \text{if } x = 1 \text{ o } x = 2 \\ (x_{n-1} + x_{n-2}) & \text{if } x > 2 \end{cases}$$

The Fibonacci succession has very curious and interesting properties; for example, any two consecutive terms are prime numbers among them. Furthermore, the consecutive prime x / x – 1 in greater numbers tend to Φ the most used irrational in the history of art. We will observe this in greater detail in the second part of our study.

The numerical sequence can be found in countless vegetable growth and helped generate the myth of which we have spoken; however, it is a fact that Fibonacci, without meaning to, had found the key to growth in

Nature, which later would be widely delved into by authors such as Theodore Cook.

Fibonacci also publishes in 1220 the "*De practica geometriae*", in which he applied the new arithmetic system for the solution of geometric problems: a treaty of Geometry and Trigonometry. In 1225, the "*Liber quadratorum*" constitutes a brilliant work on the second degree undetermined equations.

With the passing of time, his *Liber Abaci* became the work of maximum influence among all that contributed to introduce Hindu-Arabic notation to the Western culture.

In this manner, the Middle Ages contributed to the theory of proportion indirectly, since the Fibonacci series will become centuries later one of the fundamental cornerstones over which authors in the category of Le Corbusier will base their work.

It would not be fair to finish our recollection of the Middle Ages without mentioning St. Thomas Aquinas who tried to rescue at least the Aristotelian tradition; this fact, however, did not fully influence the artistic manifestations of the times; we can only say that many of his main works date to the decade of 1260, moment in which the majority of great cathedrals were already finished.

Our conclusion is that besides the contribution of Fibonacci (never used during his time with artistic purpose), the main contribution of the Middle Ages to the theory of proportions was not a theoretic progress but more the association of it with some divine intention, association which in one way or another, is still kept in our days.

1.7 *Homo quadratus*.

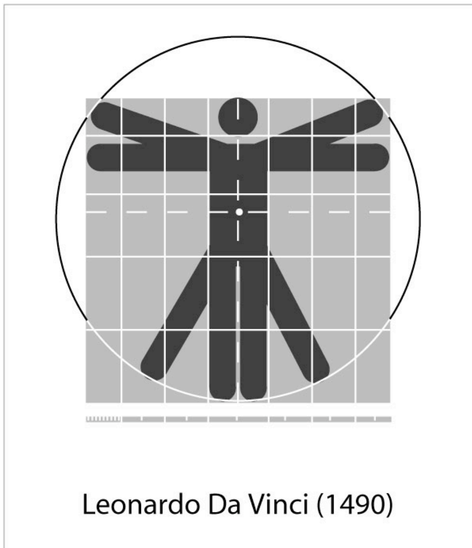

Figure 1.18.a: Graphic analysis of the proportions of the *homo cuadratus* proposed by Leonardo for an illustration of the book "*De Divina Proportione*¨ over the Vitruvian concept.

To summarize the new Renaissance spirit, it should suffice to mention Leonardo da Vinci's *Homo cuadratus* (Figure 1.18.a), "To say that in the Renaissance man was placed at the center of the universe… One may consider the well-know Vitruvian Man, draw by Leonardo in the last decade of the XV century, as the first evidence of this tendency."[62]

In the methodology of design, this is interpreted as the Vitruvian tendency to use the human body as model, its proportions and its relations, as a model to follow in the definition of buildings, its parts and relations. Once more, we cannot find in the Renaissance an increase of knowledge with respect to proportions or the proportional theory.

As its name indicates, the Renaissance is the rediscovery of classic texts after being left on the backburner for a millennium. There would be no advancement in this matter until the beginning of the XIX century. The works in these 20 centuries will concentrate in such variations such as the how and why these discoveries of and the Greek philosophers need to be used.

So, the Renaissance humanism takes design and humanizes it, interpreting its structure as human. The *ad quadratum* scheme influenced by the Vitruvian *Homo quadratus* is used often in cathedrals such as S.Maria degli Angeli by Bruneleschi in Florence or the S.Sebastiano in Mantua by Alberti. (Figure 1.18.b)

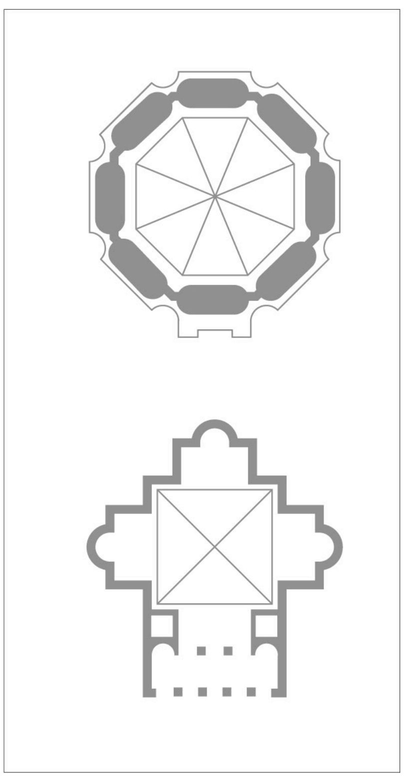

Figure 1.18.b: Graphic analysis of the Santa María degli Angeli (1434) by Brunelleschi floors and San Sebastiano (c1460) by Alberti showing the clear tendency toward the square plant which had not occurred until then in constructions.

In other words, influenced by the squaring of the circle, with the inscription of the Vitruvian human figure – which we have already detailed as inexact – the artists take this proportional scheme and builds from it cathedral floor as if hoping that some sort of mysterious divinity will be added to it. It is an age filled with mysticism, the heritage of a medieval millennium fills Western culture with mythical-religious thought. Even today it is difficult for modern man to separate dust from straw, the rational from the mystic; the Renaissance is the beginning of our age.

There is yet another manner of humanizing design, not just with the relation of the parts of the human body as a system or with the proportions of the Vitruvian *Homo quadratus*; it is a particular point of view. For the first time, one can pinpoint the point of view of a piece of work; with extreme precision one can reproduce a particular angle. It is the birth of perspective.

[62] M. Cianchi, (1998). **Leonardo The Anatomy**. Giunti Gruppo Editoriale. Florence. Page 4

The rebirth of perspective is attributed to Filippo Brunelleschi (1377-1446). The procedure had results that were so real that illusions were created with dark cameras and small openings that compared the paintings with reality[63]. But more than popular curiosity, the new technique expressed that the world operated much like a machine, that there were no unreachable dark mysteries and that man could “understand” how it worked.

At first, the new technique solves a little the theory of proportions. If everything is viewed with the perspective from where all the planes are located, made on the façade and floors of the previous artists, one can say that for the first time, duality is moved to the forefront: the two dimensions of the project versus the object’s three dimensions – a duality whose discussion continues today.

Lucky for the Renaissance figures, Alberti (Leon Batista Alberti, c. 1404-72) appears with his book *Della pinttura* (1435) with the first formal description of perspective. This is the reason why Brunelleschi’s discovery came to be known as Albertian perspective.

Brunelleschi and Alberti simplified the vision to one eye and imagined that rays of light entered through it. Each line trespasses the plane of the painting through one only point, interposed between the three dimensional objects and the eye, which creates a process of projection that is orderly and coherent.

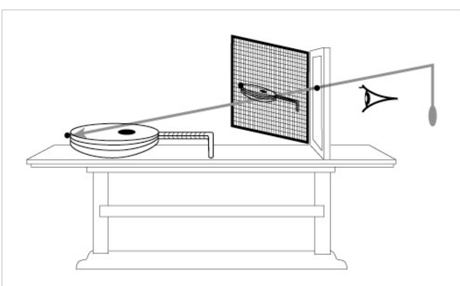

Figure 1.19: Graphic analysis of a print by , Albrecht Dürer explaining the process of perspective.

One of the most famous demonstrations that were built to demonstrate the Albertian perspective appears in a textbook print (See Figure 1.19) published by the German artist, Albrecht Dürer “ he rode in the autumn of 1506 from Venice to Bologna, residence of Luca Pacioli, in order to be initiated in the mysteries of a ‘secret perspective’”[64] in an effort to take back to Germany the progress initiated by the Italians.

Notice the use of the lead pencil to trace the straight lines from the edges of the musical instrument to a sole visual point located on the wall. The lines, corresponding to rays of light, pass through a transparent mesh, marked by series of dots that form an image of the instrument on the plane.

Piero de la Francesca (c. 1416-92) and Leonardo da Vinci (1452-1519) took this concept to mathematical precision and with their work, a progression of

[63] R. Wittkower, (1953). **Brunelleschi and Proportion in Perspective in Idea and Image**, Thames and Hudson. Quoted in R. Padovan (1999). Page 214

[64] M. Ghyka, (1977). **The Geometry of Art and Life**. Dove Publication. New York. Page 87

perspective. Both studied proportions and/or mathematical progressions that explain the nature of the perspective projection.

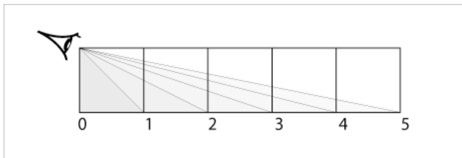

Figure 1.20: Graphic analysis of the numerical sequence use by Leonardo to explain perspective.

In the case of Leonardo, he leaves clear in his notebooks toward the beginning of the decade of 1490, that if several objects have the same size and separate from the spectator in equal distances, their apparent size in perspective would be the following sequence: 1, 1/2, 1/3 1/4,… (Figure 1.20)

This mathematical sequence is the so-called “sequence of the chord” – for musical chords- already cited by Plato and known as the “sequence of the four integers”. It is one of the most used in the ancient world.

The sequence of the four integers will become a characteristic of the Renaissance. Just like Brunelleschi, many architects of the 15th century used this simple proportion to organize their buildings.

But Alberti sets out to write not only about the Brunelleschi perspective; in his book, *De re aedificatoria*, he also explains some classics of his times such as Vitruvius’ *De architectura*. Unlike Vitruvius, who tried to describe how classical buildings were built, Alberti tried to teach how future buildings should be built. In this sense, there is no doubt of the adoption of the transposition method of proportions between nature and the methodology of design.

For the first time, the importance of the methodology of proportion as a key instrument of design is made evident. If (1) Policleto created the “Canon” as a technical instrument to guarantee the fidelity of copies; if (2) Aristotle had already insinuated that the natural order possessed beauty *par excellence*; and (3) Vitruvius had observed that, according to him, the ancient temples were made according to the proportions of the human body; then, now Alberti accepts everything as a given, and **explicitly** recommends that in order to design correctly, the proportions of nature and the human body must be followed.

Note, that from now on, it is an academic and written fact that the beauty of nature will be automatically transported – almost magically – to the artistic work.

Furthermore, the center and creator of the building is the user, a concept so modern that it sounds like ergonomics and usability. “The human occupant of the building is united with it through the proportions between his own limbs… Body, house, city and cosmos are all representations of each other, each ordered according to the same hierarchical plan, with head, body, limbs

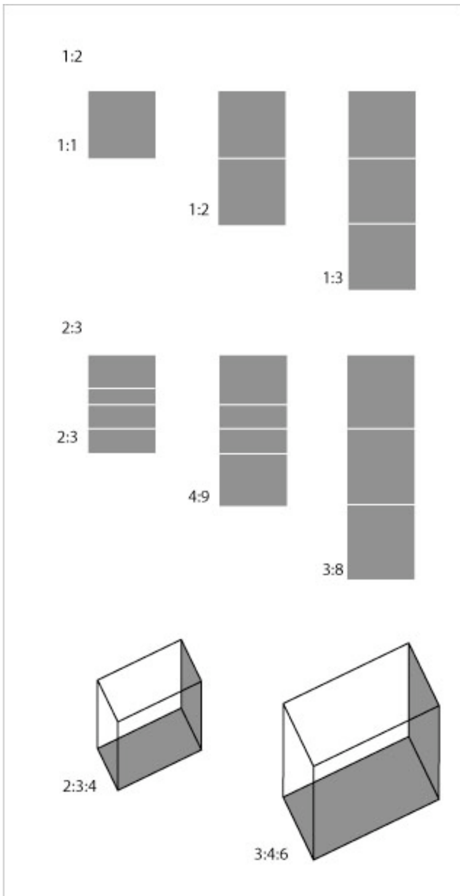

Figure 1.21: Graphic analysis of the logic of generation for Alberti´s Proportional Analysis.

and so on."[65] With it, Alberti introduces the concept of urban planning, with humanistic proportions, at the end of the sequence.

Alberti also offers us the first proportional vocabulary for design, an idea that would be used again five centuries later by Le Corbusier in his "Modulor".

Alberti, based in the sequence of the four integers, and, of course, in musical harmony, generates a series of proportions that "should" be used for both the floor of the building as well as its heights. By grouping and combining the possible relations in the basic sequence, he generates and recommends a series of proportions to define the dimensions of designs. (Figure 1.21)

1.7.1 *De Divina Proprotione*

After the work of Alberti, which constitutes the first formal treaty in Architecture since Vitruvius, a new title appears on scene: *De Divina Proprotione*

The first thing we need to specify about the book written by Luca Pacioli (1445-1517) is that it is not a book written as a treaty of design. It is a book about the golden proportion, which was not used explicitly in the design of that age except for the analysis of perspective.

Pacioli, a pupil of Piero de la Francesca, (versed in perspective from the mathematician's point of view), is the one who seals the name "Divine Proportion" for the golden section. With this, he reaffirms the sacred character, which had already become generalized in medieval times with the general denomination of Sacred Geometry; a character used even today.

This work is a mix of science and mysticism that pretends to summarize the knowledge of that age in geometry and proportion. Written more as a treaty of occult science, it was filled with very unscientific annotations. Very famous in its time, Albrecht Dürer himself, as we mentioned earlier, traveled from Bologne to be initiated as an "*apprenti*" of this secret science. Dürer confirmed this influence later in important works such as the "Melancolía I" print elaborated in 1514.

The most important theoretic sources of the treaty are: Plato's Timeo, The Elements of Euclides and the Vitruvian work.

Another important aspect of the work is, of course, the appearance of the sixty illustrations made by Leonardo da Vinci, and among these, the *Homo quadratus* as an

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[65] R. Padovan, (1999). Page 213

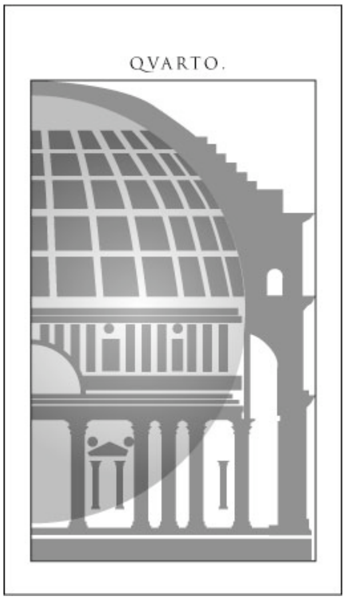

Figure 1.22: Graphic analysis of the sphere inscribed in the Roman Pantheon, described by Palladio in his *"I quattro libri della Architettura"*

allegory of the Vitruvian concept of proportion in the human body.

The book, *per se*, doesn't add anything to what is known until now with respect to proportions or methodology; if anything, it adds to the aforementioned mysticism, which this theory had received to date. Nonetheless, it is obligatory to cite for several reasons: (1) the title itself uses "The Divine Proportion", which represents the introduction of the term in history, (2) the intention of conciliating Plato with Christianity and, of course, (3) da Vinci's famous illustration.

The next important work in the matter of proportions was: *"I quattro libri della Architettura*" (1570, second edition in 1580)," by Andrea Palladio (1508-80), Italian architect from the late Renaissance. His treaty was so important that it became a canon for Western architecture in the following centuries. This influence extended over all of England, where the Palladianism style appears which faithfully followed its precepts and combination of rules. Among the followers of this tendency were architects of the caliber of Inigo Jones or Sir Christopher Wren, predecessors of some neoclassic styles, such as Neopalladianism or English Georgian and the American neoclassic.

The works are filled with proportional analysis of the classical works. These analyses, like Vitruvius', are based on Palladio's own speculations with no textual base evidence.

Figure 1.22[66] describes one of the typical diagrams. Palladio, in Vitruvius' and Alberti's style, speculates that the Roman Pantheon was designed circumscribing a perfect sphere and that its vertical dimensions followed defined proportional sequences.

Palladio, like Alberti, recommended in his treaty, forms with specific proportions, besides the 1:1, 1:2, 3:4, 3:5 already cited and used by his predecessors. Palladio added a √2 rectangle.
This fact cannot pass inadvertently since it adds an irrational proportion to the already known whole proportions by Alberti; however, as Wittkower affirms, it is not likely that neither Alberti nor Palladio, nor any other Renaissance architect, had used – in practice – irrational proportions.[67]

Palladio also adds the concept of interrelated proportion; not only proportioning the dimensions of the room or

[66] A. Palladio, (1570). **I Quatro Libri Dell'Architectura**. Domenico D'Franceschi. Venezia quoted in K.Gerstner (1986). Die Formen der Farben. Verlag Athenäum. Frankfurt. Page 28
[67] R. Wittkower, (1952). **Architectural Principles in the Age of Humanism**. Alec Tiranti. London. Page 95

façade but also taking into account the building's whole; in other words, the different rooms would be in proportional relation among them. This concept had already been introduced by Vitruvius in one of his six principles of architecture, *eurythmia*, but it had not been mentioned until this latter part of the Renaissance.

1.7.2 **Renaissance Solids**

One cannot finish recounting what was rescued in the Renaissance with respect to proportion without mentioning the work on solid geometrics elaborated by Johannes Kepler (1571-1630).

Kepler studied the five regular solids where inevitably, he would find the four irrationals $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and θ. By studying how they were related and their inscribed, circumscribed and half inscribed relations, he reveals the conclusions already reached by Euclides almost two thousand years ago. As a matter of fact, the discovery of three of his famous laws of planetary movement were based on the mythical and equivocal intent of relating the five platonic solids with the six planets discovered at that time. As it happens many times in history, a "happy" error leads to a great discovery.

As Euclides, 20 centuries back, the proportions studied by Kepler were not recognized or used consciously by his peers and remained theoretic elaborations, which were purely mathematic.

As has been already mentioned, no evidence has been found that suggests the use of geometric mathematical sequences or the use of the four irrationals in the Renaissance design except for in the well-intentioned speculations of some later analysts.

1.8 The Pan-Proportionalism

1. The first was never to accept anything for true which I did not clearly know to be such; that is to say, carefully avoid precipitancy and prejudice, and to comprise nothing more in my judgment than what was presented to my mind so clearly and distinctly as to exclude all ground of doubt.

2. The second, to divide each of the difficulties under examination into as many parts as possible, and as might be necessary for its adequate solution.

3. The third, to conduct my thoughts in such order that, by commencing with objects the simplest and easiest to know, I might ascend by little and little, and, as it were, step by step, to the knowledge of the more complex; assigning in thought a certain order even to those objects which in their own nature do not stand in a relation of antecedence and sequence.

4. And the last, in every case to make enumerations so complete, and reviews so general that I might be assured that nothing was omitted.[68]

With these words in his *Discours de la méthode,* René Descartes initiates a new era in Western thought in the XVII century. The scientific thought enters a new period and the scientific method takes on a new form.

Newton's laws appear to divest a mechanical universe susceptible to be analyzed and understood; the value scale until now defended, enters a crisis which will last until our time.

Consequently, the concepts such as harmony, proportion and beauty –until those days considered almost divine– loose position in favor of other more "mechanical" studies and scientifically rebuilt. We are witnesses for the first time of a divorce between the world of facts and that of beliefs.

The Renaissance canons on proportions decline in the Baroque Mannerism and the associations of a world explained by intuition, like Plato's five elements and their corresponding regular solids, appear to be childish in light of the new hard and abstract science.

In this mechanical world, the studies of proportion would have to wait the development of chemistry, physics and astronomy. A lot had to take place before taking a look at the "icing on the cake" again.

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[68] R. Descartes, (1637). **Discurso del método**. Meditaciones Metafísicas. Translation, M. García Morente (1980). Ed.Espasa-Calpe, Madrid. Page 49

Finally, in the XIX century, without previous notice, the golden section is rediscovered. Despite the fact that for some authors this affirmation is incorrect, in reality, it is here where the divine proportion is used for the first time as an instrument for design.
As we have seen, the so mentioned and analyzed proportion in numerous studies about classic or renaissance works is not based on any conclusive evidence that any architect had used proportion in a clear, reliable and explicit manner.

" A fairly good case could be made out for the view that nineteenth century actually discovered the golden section as instrument of architectural proportion…"[69]

As a matter of fact, the name of the golden section or *goldene schnitt* appears around 1835 in the treaty *Die reine Elementar Matematik* by Ohms.[70], retaking the term affixed by the Roman poet, Horace: *aurea mediocritas.*

However, inarguably, the credit for the reappearance of the golden proportion as an instrument of analysis goes to the German philosopher and mathematician, Adolf Zeising (1810-76), who publishes his research on the proportions in art and nature, in his book, *"Neue Lehre von den Proportionen des menschlichen Korpers"* (1854)

An important addition to the studies reported until that moment on the golden section is the treatment of it not only as the proportional section of a segment but as an infinite numerical sequence –as Fibonacci had done before him in his own sequence–, but generating in it a spiral logarithmic growth that develops proportionally to factor Φ.

Armed with this new interpretation of the "golden rule", Zeising throws himself to search frenziedly for this growth in botany, zoology, geometry of crystals, light physics and, of course, the human body. In this manner, Zeising declares the operation of a universal law: the golden section.

Zeising, therefore, develops his own concept of aesthetics, born out of tradition but based on his studies of application of the "golden sequence", now sequence and not just proportion; he is convinced that this specific constant Φ plays an important role in nature.
Furthermore, this proportion contains the fundamental principle of the beauty of nature, in art and in all organic and inorganic form, culminating with the maximum

[69] P.H. Scholfield, (1958). **The Theory of Proportion in Architecture**, Cambridge University Press. Page 98
[70] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Page 305

Figure 1.23: Anthropometric analysis by Adolf Zeising published in *"Neue Lehre von den Proportionen des menschlichen Korpers"* (1854) (the double curves are from me)*"*

expression of it in the human body.

Figure 1.23 shows a graphical analysis of Zeising's proportional study that divides the body into 4 sections, all of them related by golden proportions.

With these passionate ideas, perhaps it was Zeising who managed to convince the West of what we would call "pan-proportionalism". His are the first extensive studies of proportion in all fields, studies that would proliferate as Fibonacci's rabbits.

Furthermore, he is also the first to speak of a universal principle and of its obligated application in the arts and architecture to find the much desired harmony.

1.8.1 Experimental Aesthetics

This focus allows us to clearly see the *Zeit Geist* of the moment, a more scientific attempt to face a problem, which up until now had been a matter of gods. This focus would be delved upon 20 years later by the German psychologist, Gustav Fechner (1834-87), and his famous studies.

Fechner practically founds the experimental aesthetic concept because he had, for the first time, a scientific approximation to the problem of aesthetic preferences. His books, *Zur experimentalen Aesthetik* (1871) and *Vorschule der Aesthetik* (1876) clearly show a focus, unknown until now.

"Curior about the Golden Section… Fechner, late in the nineteenth century, investigated the human response to the especial aesthetic qualities of golden section rectangle. Fechner's curiosity was due to the documental evidence of a cross-cultural archetypal aesthetic preference for golden section proportion.

Fechner limited his experiment to the man-made world… He found that the average rectangle ratio was close to a ratio known as the golden section, 1: 1.1618…, and that the majority of people prefer a rectangle whose proportions are closed to the golden section."[71] (Figure 1.24)

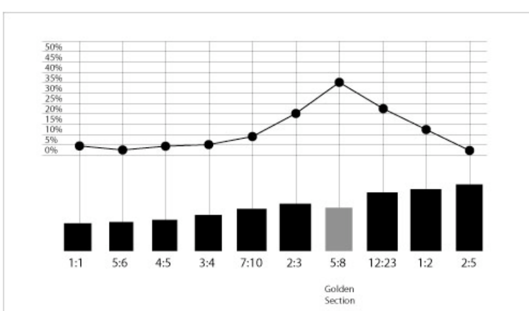

Figure 1.24: Results of the experiment of Gustav Fechner (1876) on the preferences in different proportions of rectangles.

It is important to highlight that Fechner provides several new elements to the study of proportion as a tool of design:

For the first time, the problem is faced with a scientific approach

[71] K. Elam, (2001), **Geometry of design, studies in proportion and composition. Princeton Architectural Press**. New York. Pages 6-7

It works with artificial objects of daily material culture–something that had not been attempted before

He worries that his conclusions are transcultural and not a lucky guess generated by some kind of regional culturalization process.

It is definitely an innovative focus and a result of the material positivism of the XIX century.

Figure 1.24 shows some of the results of the experiments carried out by Fechner. A series of rectangles with different proportions are shown to different human groups to carry out a survey. Surprisingly so, the results favor the golden section.

Of course, this procedure is open to criticism, which it has, of course, received. For example, the method extremely simplifies a phenomenon that covers natural complexities as well as manmade to the maximum degree. It could also be that people's preferences don't necessarily reflect the best solution; or, as was objected more recently, that the design of the statistic instrument can influence in a serial manner the results, etc.

But despite all of the objections that could be exposed, we finally have a scientific approach to a problem, which had not been exposed until now: the problem of generalized preferences in the matter of proportion.

With this, the study of proportion, the study of aesthetics and of beauty take an important turn by adding yet another topic of the study of "perception of beauty", a topic which wil be retaken later on in the XX century.

Toward the beginning of the XX century, Fechner's focus had been well received and some authors had begun to work on it. It is important to note that the strict scientific focus of collective preferences was not reused until the end of the XX century as was mentioned, but the idea of a universal law based on the golden section and its pan-proportionality in the natural and human world had arrived to stay.

In 1914, the English author, Sir Theodore Andrea Cook (1867-1928) published his famous work "The curves of life" where he delves into the use of the spiral based on the golden growth. One more time, materialism triggers Cook's works and that of a new perspective to the problem; in his own words: " Modern research is gradually becoming free both from accidental prejudices and from meretricious standards. There is a new spirit at work upon constructive philosophy which has never been so urgent, so creative, or so strong; and knowledge has become much more accessible at the very time when its transmission over the whole world has been enormously facilitated by new methods of communication." [72]

In this manner, making reference to the already acclaimed presence of the golden section and its spiral growth in multiple examples in natural life, Cook tries to give an explanation in this respect.

"We do not want mere catalogue. If every generation of great thinkers had not thirsted for explanations also we should never have evolved the complexity and beauty of modern science at all…

I do not ask you to believe that the occurrence of similar curvilinear formations in various organic and inorganic phenomena is a proof of 'conscious design'. I only suggest that it indicates a community of process imposed by the operation of universal laws. I am, in fact, not so much concerned with origins or reasons as with relations or resemblances… But we must not imagine that 'Nature' is ever 'mathematical' or that any natural object 'knows what a spiral means' "[73]

This focus of the observations reported until now by many authors, shows Cook's great contribution, despite the fact that his peers have not given him the attention he deserves, it is the first time we see a rationalist approximation to the presence of the golden proportions in nature. Instead of insinuating – with mythical-religious thought – that nature (earlier God) somehow "designs" with golden rules, Cook confronts the problem of considering proportion like some type of secondary effect which coincidentally helps us to observe nature with mathematical precision. This focus will be touched upon further on in the third part of this study.

Besides the already mentioned spiral of golden growth, Cook also contributes with the relation that exists between exponential progression of Φ and Fibonacci's series, widening forever the concept of Φ as an infinite series.

"The most significant fact, for our purpose, about the Φ series is that it gives a *double* Fibonacci series, and in the Φ series the ratio of any two successive numbers is not merely approximate but exact, for the constant ratio is $\Phi = 1.618034$.

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[72] T. Cook, (1979**). The curve of life.** Dove Publications. New York. Page VII
[73] T. Cook,(1979). Page 4

The double Fibonacci series in Φ is exhibited thus:

$\Phi^2 = 1 + \Phi$
$\Phi^3 = 1 + 2\Phi$
$\Phi^4 = 2 + 3\Phi$
$\Phi^5 = 3 + 5\Phi$
$\Phi^6 = 5 + 8\Phi$

and so on.” [74]

Armed with these analyses and discoveries, Cook makes the hypothesis that the spiral of the Φ growth is the law of natural growth.

“We have, therefore, reached a point at which it is possible, and even probable, to conclude, not merely that the Φ spiral (a new mathematical conception) is the best formula for the hypothesis of Perfect Growth, and better instrument than has yet been published for kindred forms of scientific research; but also that it suggests an underlying reason for artistic proportions, and provides an exquisitely dedicated standard by which to appreciate divergences and variations of different kinds.”[75]

And with this thought, he nears to what is possibly the best explanation of this entire phenomenon: proportion as a collateral effect of organic and inorganic growth at the natural level; however, this is a topic for another part of this study.

[74] T. Cook, (1979). Page 420
[75] T. Cook, (1979). Page 427

1.9 *Le Modul d'or.*

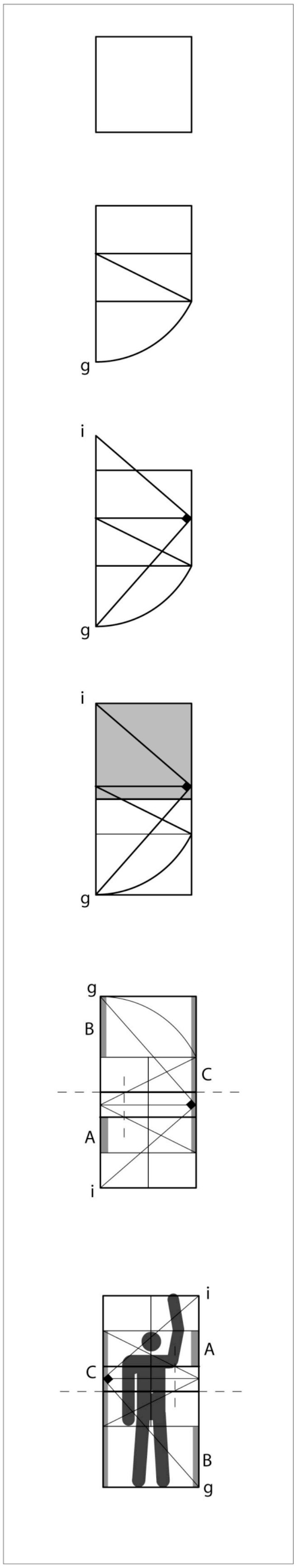

Figure 1.25: Proportional analysis that leads to the development of Le Corbusier's *Modulor* (1950)

The last of the theoretics with a wide influence and recognition in the field of proportional methodological instruments in the theory of design is, without a doubt, Le Corbusier (1887-1965). Le Corbusier designed his system of proportions based on the golden proportion using two alternate sequences, which he called blue and red.

Actually, this idea had already been introduced by Cook some decades past with a concept of proportional scale where the numerical progression based on Φ is multiplied by factors to proportion practically all of the possible units, including the human body[76].

However, Le Corbusier does not indicate that he had any knowledge of this, although in his time it was obligatory reading for architects and designers.

Le Corbusier's *Modulor* is a method based on the height of the human body, from which, at the proportional level, two overlapping dimensional scales are removed.

With these scales, Palladio style, Le Corbusier tries to create a proportional vocabulary to offer architects and designers a basic tool in all of design. Its name comes from the *Le Modul d'or* in French or "the golden model".

The origin of the idea is to circumscribe the human figure in two vertically-placed squares, the center of the construction coinciding with the bellybutton of the human figure and its maximum height with that of a man extended upwards. Obviously, he is paraphrasing Vitruvius and Leonardo's famous illustration of the same concept.

The original elaboration of the proportional analysis is described in detail in his book with the same name "*Le Modulor*". As Figure 1.25 shows, Corbusier parts from a square, cutting down the medium diagonal. This construction generates the golden rectangle. From the vertex g of this rectangle, Corbusier traces a diagonal to the square's medium and from this diagonal projects, in a straight angle, a new diagonal that cuts the extension of the original square's side[77]. The result of this construction is a √4 rectangle or a double square. From this construction, Corbusier finds three main measurements in golden progression: A, B and C (see figure 1.25) from which measurement scales are generated in the human body.

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[76] T. Cook, (1979). Op cit. Plate XI. A scale of Phi Proportions. Page 469

[77] Le Corbusier, (1950). **Il Modulor**. Fondation Le Corbusier. Paris. (1974) Per l'edizione italiana Gabrielle Mazzota editore. Milano. Page 36

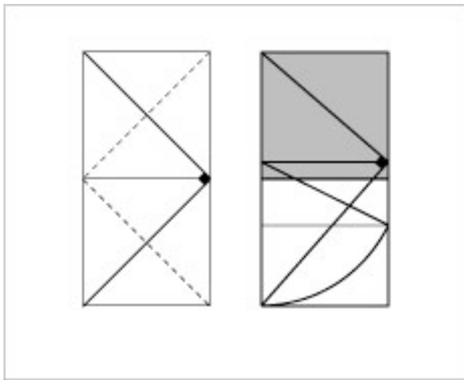

Figure 1.26: Geometric analysis of Le Corbusier's deduction error in the development of his *Modulor*

A geometric problem already exists in Le Corbusier's original construction. It turns out that a √4 rectangle is generated by two squares whose diagonals are located in the mid-point forming a 90º angle (see figure 1.26); therefore, Le Corbusier's construction is, beforehand, not correct.

Corbusier was already aware of the situation since an author of his time referred him to it. With his customary arrogance, he consulted with a mathematician called Taton, who stated that for his squares to have diagonals in straight angle, they would have to be in a 1:1.006 proportion. Le Corbusier adds this discussion as an appendix in his *Modular* and he excuses it in the following manner. Referring to the two sheets of calculations, which he had received as a response, Le Corbusier says:

"The answer of the mathematician is interpreted in the following manner: the beginning hypothesis (1942) is confirmed: you take two equal and joined squares, install inside of these in the adequate place 'the straight angle' of a third square equal to the others.

But…
But the mathematician adds: your two beginning squares are not squares; one of its sides is *6 thousandth times larger than the other.*

In everyday practice, six thousandth of a value is what is called a negligible quantity, which is not in terms of content…

But in philosophy (and I have no access to this severe science), I presume that the six thousandth of anything makes sense…"[78]

It is clear that there exists a contradiction in Le Corbusier's excuse. He begins by saying that the initial hypothesis is correct and that if two equal squares are taken, their scale can be built, but immediately, he says that the squares are not squares but rectangles with 1:1.006 proportion on its sides.

Not that we are wiser than Le Corbusier in the "severe science of philosophy", but this demonstration would have been unacceptable for Euclides, 23 centuries before. In geometry, as in logic, a reasoning is either correct or not; there is no in between. The construction of the Euclidian pentagon is used today inside the most sophisticated and exact software in computer aid design and the undeniable reasoning of the genius will continue to be valid no matter the precision with which it

[78] Le Corbusier, (1950). **Il Modulor.** Page 230. (italics from Le Corbusier)

will be used. This, however, is not the case in the Corbusian construction.

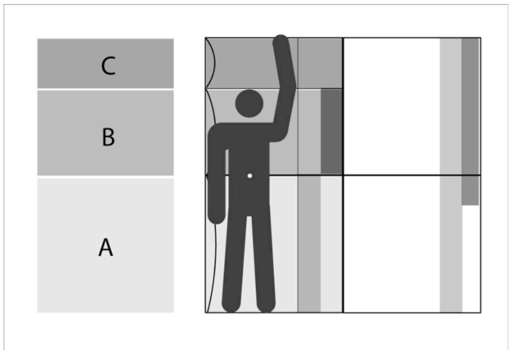

Figure 1.27: Base proportions for Le Corbusier's *Modulor*

In any case, and despite Le Corbusier's knowledge of these incoherencies, this was the construction, which gave birth to the sequences used by Le Corbusier in two scales. Figure 1.27 shows the interpretation discussed and the A, B and C areas; A a square; B a golden rectangle and C a golden rectangle plus a square. Corbusier used these areas as the basis of his work. These areas themselves represent a sequence of 1,Φ^{-1}, $\Phi^{-2.}$

Le Corbusier also tried to translate all of this theory into whole numbers that would be easy to handle. This has been a problem since Vitruvius since golden relations are by definition irrational numbers, so any practical approximation to these would have to go forcibly go through a more manageable interpretation of the recommended dimensions.

Corbusier, on his part, tries at first to take a height of 1.78 meters – the supposed height of a man – as a basis for his scale. From there, with two scales and using the Fibonacci sequence as a multiplier factor, he develops his two progressions.

The basic unit is defined in the bellybutton with a height of 1.10 meters, where it also defines half of the √4 rectangle and the double height of 2.20 meters which is the height of man with his arm upward.

According to Corbusier, space should have human dimensions; the height of the rooms was, generally, defined by its 2.2 meter dimension, which also represented 4 times the number 55 of the Fibonacci series. The height of the bellybutton, halfway to the total height, would be twice 55; or, 1.10 meters.

This tendency to define the height of rooms as a fixed and predefined number could have been the reason that today's architecture has lost many of the methodological advice of other theoretics such as Vitruvius and Alberti, those who defined the height of rooms as another parameter to proportion.

For regular architecture at present, the building is designed almost exclusively in plant. Another non-written rule automatically defines the height. This is normally fixed according to what the law defines as a legal minimum depending on the type of building being constructed. It is as if it didn't matter the height's proportion with respect to the width and height of the room. In other words, Le Corbusier could have well influenced the badly achieved two dimensionality of the architecture of our times.

In a second attempt, le Corbusier tries to conciliate the metric system with the imperial, changing the scales to feet based on a foot of 330 mm. The purpose of this was to make his scale international. Actually, this change only provoked more noise in his already unclear development of internal measurements, which should be measurable but also coincide with the mentioned proportions and progressions.

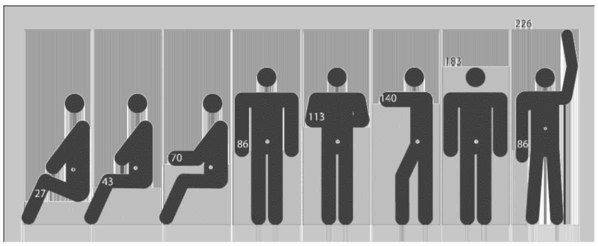

Figure 1.28: Analysis of the measurements generated from the human body Le Corbusier's *Le Modulor*(1950)

Furthermore, Le Corbusier developed a whole scale of measurements based on the proportions of the human body in different positions (figure 1.28). His intention was to open up the possibilities that would give a response to the majority of the needs that could present themselves in future designs.

Of course, as Padovan explains "because its measures are *identical* with those of the body (or mean to be), the system lacks some of the most necessary functional dimensions, such as the height of a normal door or the length of a bed, which must exceed the normal of a man by a comfortable margin"[79].

This would not be the only criticism in his system; let's start by saying that:

Le Corbusier takes for granted *a priori* that human proportions are defined by Fibonacci progressions.

The human figure is circumscribable in a double square, overlooking the huge discrepancies found throughout history when trying to interpret this ancient Vitruvian idea.

The construction of the internal straight angle in construction on which the *Modulor* is based has a basic geometric mistake.

His definitions on the height of man used as a module are conceptually wrong because no human is a prototype.

By defining his systems to "embellish" architecture, he takes it for granted that man's proportion will magically transport nature's beauty to the architectural work, but this aspect deserves a whole section of our study.

In this manner, Le Corbusier's Anthropocentricity is stigmatized with the ancient *umbilicus mundi*; literally taking the bellybutton as a unit, he makes a revealing allegory, more mythological than scientific that, as in many works of this type, that lack the scientific rigurosity to be taken seriously.

[79] R. Padovan, (1999). Page 331. (italics from Padovan)

However, this is one of the most famous and respected scales of proportion in the history of the methodology of proportions and despite its clear incoherencies, it is one of the more cited.

1.10 Conclusion of the first part.

We would like to conclude this part of our study with a summary of the more relevant aspects of the history in this topic of proportion by trying to summarize for the first time how and when methodological instruments are generated with respect to proportion. However, unlike other studies, as was mentioned before, we try to mention only those facts that have been proven with textual evidence and eliminate any speculation; speculations that have been countless in this field.

Although it is true that it is not possible to prove that these theories were not used explicitly because the only thing that can be said is that there is no conclusive evidence of its use, it is also true that, in general, it is best to be careful in this respect and not speculate in an over imaginative manner that they used one or another proportion method with no other basis than the analysis conducted by the same authors who fiercely defend the theory that they indeed did use said methods.

It is not this work's purpose to prove the possibility that with time and patience one can find almost any geometric or proportional justification to any work or classic building.

As was cited before, Padovan tells us:

"In the almost total absence of written documents that could help to determine the issue, the most varied speculations have been built up upon the conjectural analysis of measurements of buildings. The problem with this is that when one already has a theory one tends to find confirmation of it wherever one looks. Determined researchers, if they have the patience, can 'prove' almost anything they want about the proportion of a building, though this sometimes means basing the analysis on lines and shapes invisible in the building itself and existing only in the researcher's mind or paper."[80]

Therefore, with that in mind, we went through the different stages of the history of art and Western design.

As a first observation, we can affirm that although some manifestations of rupestrian art could have specific proportions, it is very likely that such presence is purely coincidental. The nature of glacial art and its concept of composition possess a spatial and temporary freedom that has not been seen since in history. We could say that, even adhering to the most controversial

[80] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Page 81

interpretations of this first art, it is not possible to find any element that speaks to us of a conscious use of the proportional systems.

The next step is without a doubt Egyptian art. Here, one can find a clear evidence of the use of proportional systems and ideal aesthetics. Note that both concepts often go hand in hand, because the most credible reason for the use of a specific canon is to reach a certain aesthetic ideal.

In the case of Egypt, the aesthetic ideals are particularly rigid, so rigid, as a matter of fact, that a high percentage remained consistent for three thousand years. Frontality, simultaneous points of view and proportion were zealously respected.

The Egyptian grids, as the first proportional system found, are an unequivocal proof that such rigurosity lay on a very elaborate plan. These, joined to the recognized “Sacred Cord”, speak to us of a clear awareness of proportions, at least, arithmetic.

These are the beginnings of the modular system of representation. The grid fixes the module with precision, which is the square.

Therefore, the mix of proportions generated by the “Sacred Cord”: 3:4:5 (triangle), 5:8 (rectangle), 2:4 (rectangle) and the strict canon in the representation of the human figure, gives us the first conscious manifestations of the use of controlled proportions in art in Western culture.

Following the time line, we find ourselves in Greece. It is not necessary to repeat the reasons why we consider Greece the fundamental basis of Western culture; suffice it to say that there is no other thought which has influenced our perception of the world to such a high degree than Greek thought.

In Greece, more so than in Egypt, there is documental evidence of the use of proportional systems, mainly in sculpture and geometry.

Practically, one can affirm that the canon as concept is definitely joined by Policleto (c.450 b.C). In his book of the same name (Canon), this sculptor not only proportioned the human body but he also invents a methodology in art and design. He defines concepts such as the selection of a unit – in this case, the head – and the use of this unit in some proportion - in its case, the arithmetic repetition of the head’s height – to define the dimensions of the other parts that need to be proportioned.

Furthermore, the canon represents a revolutionary thought, the mathematical concept of the human beauty. It is important to emphasize that the canon's objective is to improve sculpture and serve as a tool to "copy" a model in the most faithful manner; in this case, the human body.

Note that this first canon has no pretension of possessing beauty in itself, as would occur later. Beauty is already in the model and the canon allows us to reproduce this beauty in a much more faithful manner than before. It is more a problem of technique than artistic creation.

Success is, therefore, guaranteed if, by using a canon, a faithful copy of nature is obtained (in this case the human body), that nature is beauty *ergo* the result is beautiful. In other words, for Policleto beauty was already in the model and the problem is practically reduced to a faithful copy of it.

Besides this wonderful progress in matter of proportions, the Greeks also give us other elements in the field of geometry, which will become the fundamental basis of the study of proportions to our days.

The Greek mathematicians give us a clear theoretic jump, taking mathematics and geometry to eternal frontiers through the discovery of fundamental principles that govern the universe.

On the other hand, Plato is the first to elaborate the axiom of proportionality: "one cannot proportion two things without depending on a third", and with this, leaves clear the relation of the parts with everything. This philosophical expression clears the road for the discovery of the irrational ϕ, which will be identified as the only way to proportion the parts with the whole in a coherent manner.

Aristotle, on his part, is the first to expose that there is beauty in the natural order, and therefore, a living being or mineral formation is by definition more beautiful than chaos.

To conclude these contributions, Euclides in his *Elements* manages to synthesize (in his demonstrations), the four irrationals within the geometry of solids, including in this manner forever, the relations $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and ϕ in the fundamental basis on which Western spatial thought will be built.

Two centuries after Vitruvius makes his own contributions, it is significant to point out that it is Vitruvius and none other before him, who alludes to the direct use of systems of proportions in architecture.

It doesn’t matter the number of proportional analyses that we have observed in Greek and Roman buildings, there is no documental evidence of this type of work until Vitruvius.

With this practice, Vitruvius adds an ingredient to the methodology exposed until now. He decides that Greek architects used natural proportions – especially those of the human body – in artificial design; in his case, in architecture. Note that until now, Aristotles had only said that the natural order was beautiful and Euclides had developed geometric knowledge practically in abstract, but it is Vitruvius who in his “Ten Books of Architecture” clearly proposes that these proportions were used in classical architecture.

For the first time, the relation that has been used since then is made explicit: copying the “beauty” of the human or natural proportions and transport them to the design of buildings or cultural material in a mythical-religious desire to transport this beauty – like a stowaway – in this “transportation means” which should be proportions.

Vitruvius leaves us his badly defined methodologies for the five principles of Architecture and with them; he holds the doubtful honor of initiating the use of a vague and inexact language in the world of art and design. This language would not have had a chance of surviving in the strict world of Greek logic.

Vitruvius is also the first to proportion other objects besides buildings: tools, machines and furniture.
In a way, he initiates the pan-proportionalism of the XIX century.

Of course, we must mention his study of human proportions, the *Homo Quadratus,* one of the most famous, reinterpreted, and unfortunately, lost illustrations in all of the history of Western art.

From these Roman contributions, we can continue on to medieval times. As can be expected, the specific contributions to proportions are few in this age, but in the history of Western thought, there is a great change. Proportions and geometry receive, during this period, the title of “sacred” held to date. Euclidean geometry now has the degree of divine: The Sacred Geometry.

The other significant contribution of Medieval Ages is, of course, the Fibonacci series. Leonardo de Pisa, a mathematician too progressive for the age, leaves us history’s most famous succession. Deriving from his study of rabbit’s reproduction, Leonardo Fibonacci – as he was known later – develops a recurrent numerical succession, which is constantly used in the history of proportion. We must also add that Leonardo Fibonacci never related his mathematical studies to the theory of design.

With the beginning of the Renaissance, of course, Vitruvius' anthropocentric tendency takes on an unsuspected importance: the *Homo Quadratus* proportions are used in the floor of cathedrals and buildings. In this manner, design is humanized in several ways: (1) with Vitruvius' proportions, (2) with the relations between the parts and, of course, (3) with perspective.

The latter allows Renaissance thinkers to delve into the matter of proportions and numerical sequences. Brunelleschi's work is theorized in mathematical form by Piero de la Francesa and Leonard da Vinci and interpreted in Alberti's book "*Della pinttura"* (1435).

This is not Alberti's only book, who for the first time, tries to teach how buildings need to be designed: using the harmony of the human body as basis. In this case, as opposed to Vitruvius, who only speculated that classical Greek buildings must have been designed this way, Alberti dares to say for the first time that this is the correct manner to do it and calls upon architects to use this method. In other words, in some way, Vitruvius' speculation that classical architecture had been developed taking the human body as model, was now made official by the Renaissance thinkers as an instrument of design.

Note that the concept of canon, initiated by Policleto as a tool to make faithful copies, was misinterpreted and now it appears that the canon possesses beauty in itself, without a model.

It could be the Aristotle's comments on the beauty of nature and Vitruvius' speculations on the possible methods of classical architecture make Alberti take the proportional system, deliberately and unconsciously, and interpret it as a means to reach beauty and not as an instrument to copy it.

Alberti also offers the first proportional language in design and proposes specific proportional forms, like a palette, from which dimensions and proportions are chosen. Le Corbusier would propose a similar idea centuries later.

One century later, supported by the works of Kepler and Descartes, a new method of facing the problem of proportion was being developed. Therefore, Adolf Zeising (1810-76), for the first time, takes on a scientific approach to the problem of proportion. Treating the golden section as a numerical sequence, and not just a proportion among parts as had been done until then, he created the spiral of golden growth in 1854.

With this idea, Zeising analyzes the infinity of organic forms and, of course, the human body, generating studies for the first time that cover the proportional analysis of a great variety of biological growths, beginning the pan-proportionalism with which we still suffer today.

Two decades later, another German, Gustav Fechner (1834-87), designates the term “Experimental Aesthetics” and develops sampling experiments of aesthetic preferences, especially in relation to the golden proportion. These studies lead us closer to the science of the perception of beauty which will be developed later toward the end of the XX century.

These two theoretics prepare the way toward the XX century. In 1914, Sir Theodore Andrea Cook publishes his book, “The curves of life” where he delves into the use of the spiral based on the golden growth. In his study, Cook considers proportion like a secondary effect of biological growth; this focus would be considered a century later. Cook’s contribution also includes the relation, which exists between the exponential progression Φ and the Fibonacci series, widening forever the concept Φ as an infinite series.

Finally, it is important to cite the intervention of Le Corbusier who, in 1946, publishes his *Modulor,* which proposes a new proportional vocabulary for the design of architecture. Following Alberti’s style, Le Corbusier develops two proportional sequences and with them, generates dimensions related among themselves, which meant to serve as standard for architectural design of the XX century. Le Corbusier based his system on the height of the average man, at first 1.78 cm and later, 1.83 cm, trying unsuccessfully to match Fibonacci’s sequence with a practical and manageable measuring system in the decimal system as well as the imperial.

So, Le Corbusier’s anthropocentrism is stigmatized with the ancient *umbilicus mundi*; by literally taking the height of the bellybutton as a unit, makes a revealing allegory, falling into a more mythical than scientific tendency which lacks scientific rigurosity and therefore, cannot be taken seriously.

With this review, we end the historical background of the development of historical methodology instruments with respect to proportion.

In the third part of this study, other focus are explained in relation to the knowledge of proportions and the behavior of numerical sequences in nature, those which developed in the second half of the XX century but which have not been used extensively as a methodological instrument of design.

Second part
2. Nature of proportion

2.1 Introduction to the Second Part:

This second part of our study details the theoretical nature of proportion; without including complex definitions or theoretical structures, this part is intended to review how proportion is understood in our culture. For this purpose, it is required to approach the problem based on two different points of view: (1) the mathematical point of view and (2) the phenomenon of proportions in nature.

Thus, a careful review will be conducted, in the first place, in relation to the types of mathematical structures used as design tools.

In our western culture, there is a group of mathematical progressions and geometrical structures which have been used repeatedly throughout history; they constitute the basis for our systems of proportion. It is necessary to have a clear idea about these tools and the differences that exist among them, mainly to understand its historical uses and limitations.

In this part of our study, we are going to make a simple and clear description of the mathematics used. It is not the objective of this work to delve in this subject's mathematical branch; this work is aimed at including and clearly understanding the possibilities and mechanisms that such tools of analysis and design may offer.

Later, this second part also provides the proportional classification of patterns and progressions that are more frequently found in nature. The objective of the aforementioned is to provide information about the most natural phenomenon that involves progressions and mathematical proportions.

Through this review, the mechanisms behind these patterns of natural behavior are exposed. By clearly and simply explaining the occurrence of such patterns, it is intended to demystify the fact that certain proportions are omnipresent and almost magical in the universe. Perhaps, if it is included/understood how and why these phenomena occur in nature, the basis for a more scientific point of view may be found in relation to proportion phenomenon in arts and design.

2.2 Progressive proportions

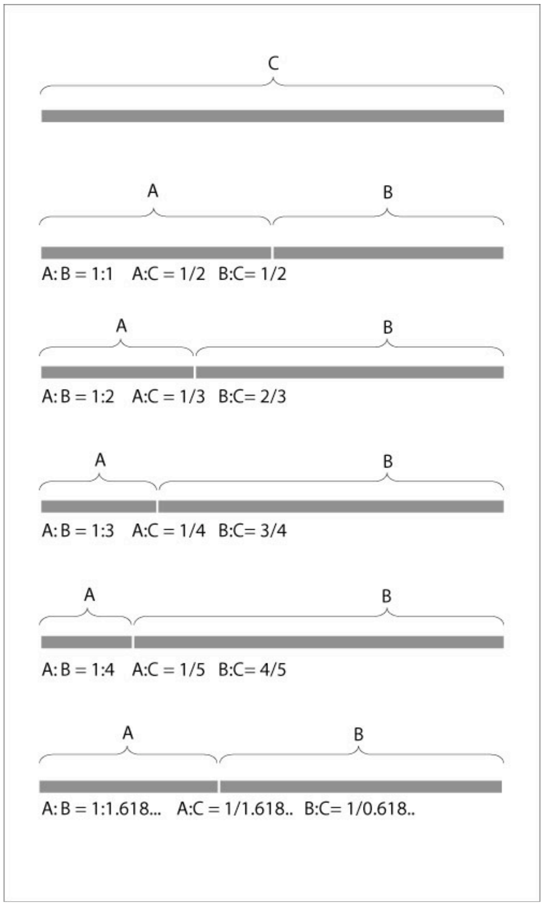

Figure 2.1. Different ways of cutting the unit and its corresponding proportions among the three resulting parts.

The minimum proportional expression is the division of the unit, which derives in three values: the two parts and the total.

According to Plato's words:

"But two things cannot be satisfactorily united without a third: for there must be some between them tying together."[81]

This minimum expression of proportion is the base for every proportional system; mathematically it is known as the mean.

The division of the unit implies three ratios or mathematical proportions (Figure 2.1):

(1). The first part: second part (A: B)
(2). The first part: the unit (A:C)
(3). The second part: the unit (B:C)

The proportion between these two parts determines the type of mean to be used.

Let us take a minute to analyze this phenomenon; by cutting a line in a specific point, two elements are created, which, in turn, inevitably generate three different proportions among them. (Figure 2.1)

Let us say that we begin by cutting the unit in the middle; in this case, the two dimensions created (A and B) are equal between them (proportion 1:1) and any of them, with respect to the whole will have double ratio (1:2).

Now, if we cut the line in another point, which is relatively close to the middle (but not exactly in the middle), the proportion between the two parts created is no longer 1 and becomes a ratio that is almost 1, let's say 1 : 0.8. In turn, the two new dimensions generate two different proportions with respect to the whole; one of them is closer to the unit (which increases) and the other moves away from the unit (the one decreasing).

In this way, a sequence of dimensions is generated from only one cut of the unit; in such sequence, we have a small part (A), an intermediate part (B) and the largest part (C, the whole). (See Figure 2.1)

Based on the aforementioned, if we test many cuts of the unit starting from the middle towards the end, we obtain several proportions among the parts. The small part distances itself from the dimension of the

[81] Plato, (1965). **Timaeus and Critias**. Penguin Books. London. Page 40

intermediate part, meanwhile the intermediate part closes in on the dimension of the whole. These dimensional increase and decrease factors are an individual characteristic of each cut or proportion.

In order to decide which cut should be used (and with them, the proportions they define), several methods have been generated throughout history. These methods are based on the procedure of cutting the basic unit, as mentioned before, and therefore, they are based on the type of proportion this cut involves.

In relation to the unit, there are many types of cuts (or means) which have been used more than other ones throughout history; the most popular means used as proportionality tools, in arts and design, were already known since ancient Greece; as mentioned before, they have not changed much since their origins.

The Pythagorean accept three types of means:
1. The arithmetic mean
2. The geometrical mean
3. The harmonic mean

2.2.1 The arithmetic mean

The arithmetic mean is the one where the interval between the largest term and the medium one is the same one that exists between the latter and the smallest. In this way, the arithmetic mean is the half of the addition between the two initial means; let us say that we have two numbers, 100 and 50, the arithmetic mean would be 75, since 100 + 50 equals 150, whose half is 75.

Mathematically, it may be defined that, between a and b, the arithmetic mean will be equal to:

$m = (a+b)/2$

Thus, it is the most common mean, which we know as average.

2.2.1.1 Arithmetic progression

Progression is the succession of numbers that keep a certain relationship among them. In this way, between the first one and the second one, the relationship that exists is equal to the one existing between the second and the third one, and the same one existing between the third and the fourth one, and so on.

Just as averages, three types of progressions exist analogically. In our case, a succession of numbers in which each term, except for the first one, is obtained by adding a constant number to the previous one, results in an arithmetic progression. This fixed added constant is called difference. It is easy to demonstrate that the general term is:

$A_n = A_1 + \boldsymbol{d}\,(A_{n-1})$

Where terms **A** are the terms of the succession, sub-indexes n represent the position of the term in the succession and the ***d*** factor is the constant known as difference.

By using this formula, and based on 1 as the first element, we may generate different arithmetic progressions depending on the constants ***d*** we use, for example:

1 : 2 : 3 : 4 : 5 : 6 : 7 … ***d*** = 1
1 : 4 : 7 : 10 : 13 : 16 : 19 … ***d*** = 3
1 : 6 : 11 : 16 : 21 : 26 : 31 … ***d*** = 5

In addition, it may be said that in an arithmetic progression, the addition of n terms is:

$S = ((A_1 + A_n) * n) / 2$

In terms of design, these mathematical successions are used generally to define the dimensions of the work. In this way, a building could be 13-meters wide, 16-meters high and 19 in length provided an arithmetic progression with difference ***d*** = 3 was used as a proportional system.

This is exactly the system used by Andrea Palladio (1508-80) in his book: "I quattro libri della Architettura" (1570)[82] in the late Renaissance. This system works on relatively low scales; however, we wonder what would happen if we were dealing with a work with many proportions and we wanted to proportion them with a similar system. In this case, for example, we would have to look in the succession from the width of the door to the length of the facade of a large public building.

Based on our arithmetic progression with ***d*** = 3, as specified for the example above, we would have a progression with the following measures:

1 : 4 : 7 : 10 : 13 : 16 : 19 … ***d*** = 3

Whenever we wanted to use this progression in global measures, such as the total height or total width of the works, we should use the highest sections of the progression, such as:

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[82] A. Palladio, (1570**). I Quatro Libri Dell'Architectura**. Domenico D'Franceschi. Venezia

112 : 115 : 118 : 121 : 124 : 127 : 130 : …

As we can see, in this case, the differences between the numbers become, among them, proportionally smaller and the idea of mathematical progression as an ordering axiom of the dimensions of the work is lost completely.

This is because if the dimensions that the progression dictates are alike among them, the final effect is that any dimension could be used. That is, if we can choose between 1030, 1033 or 1036, the difference between our options is hardly 0,003 %; therefore, almost insignificant. In other words, we can choose any dimension freely.

This is the main disadvantage of the arithmetic progressions, especially because the proportions between an element of the succession and the next element thereof may vary, drastically, in relation to small dimensions or may vary very little when speaking of large dimensions.

For a purist, this fact makes it very difficult to proportion the work, since if the small dimensions of the work show completely different proportional ratios than the greater dimensions; the system is not sufficiently reliable or appropriate.

This and other disadvantages of the arithmetic systems resulted in the loss of popularity after the Renaissance, which favored other progression systems.

2.2.2 Geometrical mean

The geometrical mean can be defined as the cut of the unit where the relationship between the greatest term and the medium is the same as the one existing between the medium and the smallest terms.

Therefore:

A:B = B:C

Reconsidering our example, if we have our two numbers 100 and 50, the result obtained would be approximately 70.71 as the geometrical mean among them:

50 / 70.71= 0.7071 … and
70.71 / 100 = 0.7071 …

Mathematically, it could be defined that between **a** and **b**, the arithmetic mean **m** will be equal to:

$m = \sqrt{(a*b)}$

in our example:

m = √(50*100)

m = √(5000)

m = 70.71…

2.2.2.1 Geometrical Progressions

As in the previous example, a geometrical progression can also be constructed with numbers.

In this specific case, each term is obtained by multiplying the previous one by a constant. This fixed constant is called factor.

In general terms, it could be said that:

$A_n = \boldsymbol{r} (A_{n-1})$

Where the terms **A** are the terms of the succession, the sub-indexes **n** represent the position of the term within the succession and the ***r*** factor is the constant called *factor*.

Using this formula and starting with 1, as first element, we can generate different geometric progressions depending on the constants r we use, for example:

1 : : 4 : : 8 : : 16 : : 32 : : 64 : : 128 … ***r*** = 2
1 : : 3 : : 9 : : 27 : : 81 : : 243 : : 729 … ***r*** = 3
1 : : 5 : : 25 : : 125 : : 625 : : 3125 … ***r*** = 5

This type of progression has been used vastly in western history, with some famous variations; for example, during the Renaissance the crossed combination of two sequences was used to increase the range of possibilities.

A table of progressions can be made using the axis X as a type of progression and axis Y as the other one, as shown in the following example:

1	3	9	27	81
2	6	18	54	162
4	12	36	108	324
8	24	72	216	648
16	48	144	432	1296
32	96	288	864	2592

Table 2.1: Combination of two progressions during the Renaissance.

Where the horizontal growth ratio is 3 and the vertical growth ratio is 2.

With this technique, the range of possibilities for the proportional system is significantly increased, thus providing the designer or artist a larger range of possibilities.

However, there are some details which must be resolved; one of the most common problems, concerning proportions, is the division of the unit because, according to the aforementioned, most of them begin with 1. Therefore, in order to provide spaces between 0 and 1, there are no terms in the progressions. One of the solutions applied in geometrical progression is the division of the unit in ratios whose denominator is a geometrical progression; in this way, an internal progression is created within the range [0,1]; for instance:

1/2 :: 1/4 :: 1/8 :: 1/16 :: 1/32 …

Nevertheless, this is not the greatest obstacle related to the use of such progressions; the most prominent disadvantage of geometrical progressions is that it is not possible to combine two dimensions in the system without getting out of the progression.

For example, if you take the following progression:

1 : : 4 : : 8 : : 16 : : 32 : : 64 : : 128 … ***r*** = 2

and add 4 + 8 (two numbers of the progression), the result is 12, which is a number that is not within the range of the progression. This seems an insignificant fact until we add the width of the window to the distance separating it from the corner of the building; the resulting dimension is “unbalanced” in the system of proportions that is used.

This small but important detail voids the efforts made in order to obtain a unified proportion in relation to a complete work. In other words, it disqualifies the systems of proportion to arrange the work’s dimensions harmonically.

The problem, of course, was already present in the arithmetic progressions; let us take one of them as an example:

1 : 4 : 7 : 10 : 13 : 16 : 19 … ***d*** = 3

where the addition of both numbers gives us necessarily one number that is out of the series.

However, there are two exceptions to the rule, or, maybe only one, since both numbers tend to be the same: Fibonacci's succession and the geometric progression, whose factor is the golden number or ϕ. Based on their significance, we will review these cases separately.

2.2.3 The harmonic mean

The third and last important mean in the history of proportions is the harmonic mean.

The harmonic mean is the medium term between two other terms that divide their difference in the same proportion the two terms have to each other.

According to mathematical terms, the definition of the harmonic mean H is:

$H = 2ab / a+b$

In our aforementioned example, if we had our two numbers 100 and 50, we would learn that:

$H = 2 * 100 * 50 / 100+ 50$
$H = 1000 / 150$
$H = 66.6666\ldots$

Besides, it can be affirmed that the arithmetic and harmonic mean are related to each other, according to the following formula:

$H = 2ab / a+b$

It can also be written as follows:

$1 / H = (1/a + 1/b) / 2$

That is, the arithmetic mean of the inverse of **a** and **b**.

2.2.3.1 Harmonic progressions

A harmonic progression is a numeric succession where their reciprocals constitute an arithmetic progression.

According to mathematical terms, it could be said that a harmonic succession is:

$H_1=1/A_1$, $H_2=1/(A_1+\boldsymbol{d})$, $H_3=1/(A_1+2\boldsymbol{d})$,...$H_n= 1 / (A_1+n\boldsymbol{d})$

Where the terms **H** are the terms of the harmonic succession and the terms **A** are the terms of the arithmetic succession and the sub-indexes **n** represent the position of the term within the succession and factor

d is the constant named difference of the arithmetic succession.

Therefore, each arithmetic progression can generate its corresponding harmonic progression, as follows:

1 : 2 : 3 : 4 : 5 : 6 : 7 … ***d*** = 1
1 : 1/2 : 1/3 : 1/4 : 1/5 : 1/6 : 1/7 …

1 : 4 : 7 : 10 : 13 : 16 : 19 … ***d*** = 3
1 : 1/4 : 1/7 : 1/10 : 1/13 : 1/16 : 1/19 …

1 : 6 : 11 : 16 : 21 : 26 : 31 … ***d*** = 5
1 : 1/6 : 1/11 : 1/16 : 1/21 : 1/26 : 1/31 …

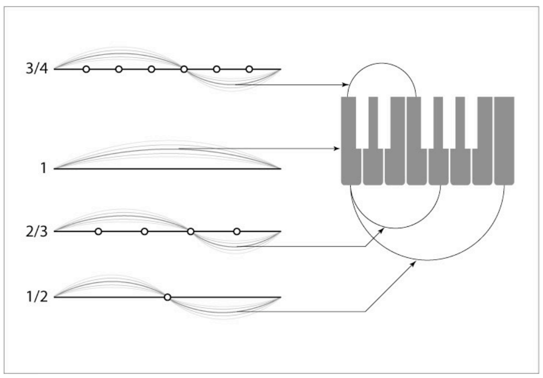

Figure 2.2. Correspondence among the monochord's notes and western musical scale.

2.2.4 Harmonic – musical progressions

Also, we should say that the arithmetic and harmonic means are also called musical means.
The Pythagorean's contribution to music is closely related to the harmonic mean. They demonstrated that the intervals between musical notes could be represented by means of whole numerical ratios. According to the aforementioned, they used a single-cord musical instrument, called a monochord. It had a self-mobile bridge that when moved, produced, in certain positions, notes that, compared with those produced by the whole cord, were more harmonious than others. The most basic of such intervals is the eighth interval; in the monochord, the octave is the interval between the note produced by the whole cord and the one produced by another length equal to half of the same one. That is, when the cord has a length of 1/2 of a certain base note, it sounds one octave higher than the original note. If its length is 3/4 of the primitive, the cord produces the fourth of the base note, and if its length is 2/3 of the initial one, the note that sounds is the fifth of the base note[83] (see Figure 2.2).
In this way, starting with a DO note, the following structure is achieved:

DO(base) RE MI FA(fourth) SOL(fifth) LA SI DO
The harmonic and arithmetic mean are denominated musical means because of this relationship between the division of the cord in equal proportions and its corresponding increase in the note produced.

This was interpreted by the Pythagoreans as proof that these proportions were an integral part of the universe and therefore its reach and universality were accepted as a given. The search of evidence in nature that may

[83] G. Doczi, (1981). **The Power of Limits. Proportional Harmonies in Nature, Art and Architecture**. 5. Ed. Shambhala Publications Inc. Boston, U.S.A. Page 22

certify the use of proportions in design has existed for thousands of years. As we will see, nature offers several possibilities of proportional systems that have been adopted in the projectual methodology, most of them partially successful. Nevertheless, their existence in a natural context has been considered as proof of their value and universality. Later, we will see that, in fact, none of the systems historically –used as a methodology of design– has been able to reproduce the natural proportional behavior faithfully. This behavior has been successfully structured at the theoretical level; however, these tools have not been used in the projectual practice.

2.2.5 Fibonacci's Succession and Progression Φ

The first part of our study introduced Fibonacci's theory. Leonardo de Pisa (1170-1240) — better known as Leonardo Fibonacci because of his father Bonacci (Fibonacci means "figlio di Bonacci", that is, Bonacci's son) — introduced the succession that bears his name in his most famous work (the Liber Abaci, 1202),

The Fibonacci succession, as mentioned, is defined as follows:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34...,

where each term of the succession is the addition of the two previous terms; in mathematical terms, we would obtain the following:

$$f(x_n) = \begin{cases} x & \text{if } x = 1 \text{ o } x = 2 \\ x_{n-2} + x_{n-1} & \text{if } x > 2 \end{cases}$$

This is the historical solution provided by Leonardo de Pisa to the problem of rabbits:

" A pair of adult rabbits produces a pair of baby rabbits once each month. Each pair of baby rabbits requires one month to grow to be adults and subsequently produces one pair of baby rabbits each month thereafter. Determine the number of pair of adults and baby rabbits alter some number of months. It is also assumed that rabbits are immortal."[84]

The problem, however, went beyond the rabbits.

"The reason that Fibonacci's name is so famous today is that the appearance of the Fibonacci sequence is far from being confined to the breeding of rabbits… We

[84] R.A. Dunlap, (1997). **The Golden Ratio and Fibonacci Numbers**. World Scientific Publishing. London. Page 35

shall encounter the Fibonacci sequence in an incredible variety of seemingly unrelated phenomena."[85]

In order to begin our own analysis, we will analyze the behavior of Fibonacci's succession versus the proportions among its elements.

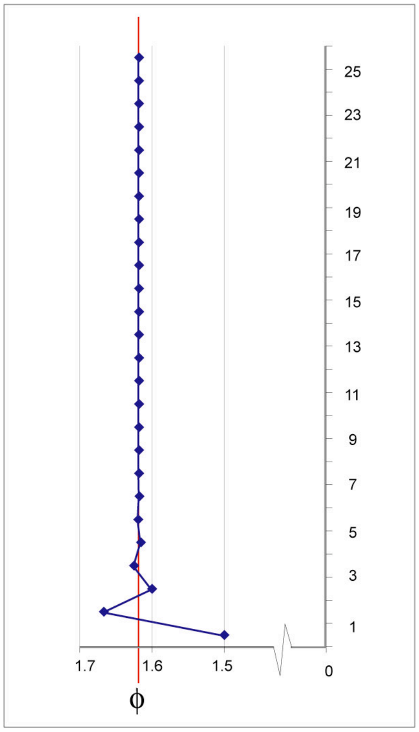

Figure 2.3. Graph that shows the relationship between each number of the Fibonacci series and its predecessors.

Position	Fibonacci	Proportion A_n / A_{n-1}
1	1	
2	1	1
3	2	1.5
4	3	1.666666667
5	5	1.6
6	8	1.625
7	13	1.615384615
8	21	1.619047619
9	34	1.617647059
10	55	1.618181818
11	89	1.617977528
12	144	1.618055556
13	233	1.618025751
14	377	1.618037135
15	610	1.618032787
16	987	1.618034448
17	1597	1.618033813
18	2584	1.618034056
19	4181	1.618033963
20	6765	1.618033999
21	10946	1.618033985
22	17711	1.61803399
23	28657	1.618033988
24	46368	1.618033989
25	75025	1.618033989
26	121393	1.618033989
27	196418	1.618033989
28	317811	1.618033989

Table 2.2: First 28 numbers of the Fibonacci series.

In the previous table, the first 28 numbers of the Fibonacci series are calculated; the first column indicates the position of the number within the series; the second column shows the Fibonacci succession and the third column shows the proportion between the previous number and the number in that line.

The graph is shown in Figure 2.3 display the left column in the above table, where one can observe the function

[85] M. Livio, (2002). **The Golden Ratio**. Broadway Books. New York. Page 98

that defines the behavior of the proportions between two successive elements of the Fibonacci progression. This graph is important because it shows how in the Fibonacci succession, the relation between consecutive numbers tends toward a specific ratio.

After a rough stumble during the first six or seven numbers, the proportions among them clearly approach the ratio 1.618033989. In this way, in the position 28 of the succession, the ratio is extremely close to 1.6180339887… or Φ .

"It is interesting to look at relationships between various Fibonacci numbers. Specifically the ratio of successive Fibonacci numbers is an interesting quantity… It seen that as n increases then the ratio F_n / F_{n-1} approaches the golden ratio. Figure 5.5 (likely our Figure 2.3) shows that this ratio oscillates around the value of τ (our Φ) as a function of n and asymptotically approaches this value. This may be expressed as

$$\lim_{n \to \infty} F_n / F_{n-1} = \tau$$

and is a fundamental property of the Fibonacci sequence and the golden ratio[86.]

Therefore, we can say that after a certain number (depending on our tolerance towards decimals), Fibonacci's succession becomes a geometrical progression where factor ***f*** is Φ.

This factor Φ, which is known as one of Euclid's' four irrationals, actually is:

$\Phi = \sqrt{5} + 1 / 2$ approximately 1.6180339887

This value was introduced in the first part of this study as one of the Greek's findings, which was mainly used to calculate the pentagon (base of the dodecahedron) in Euclid's' Elements[87] (Figure 2.4).

Likewise, this same value is frequently used as a base for the development of other important constructions concerning proportions, such as the logarithmic spiral, which we already mentioned.

"We note that √5+1 / 2 = 1.618 and √5-1 / 2 = 0. 618, where √5 = 2.236, is an important factor in the wave Principle and the logarithmic spiral"[88]

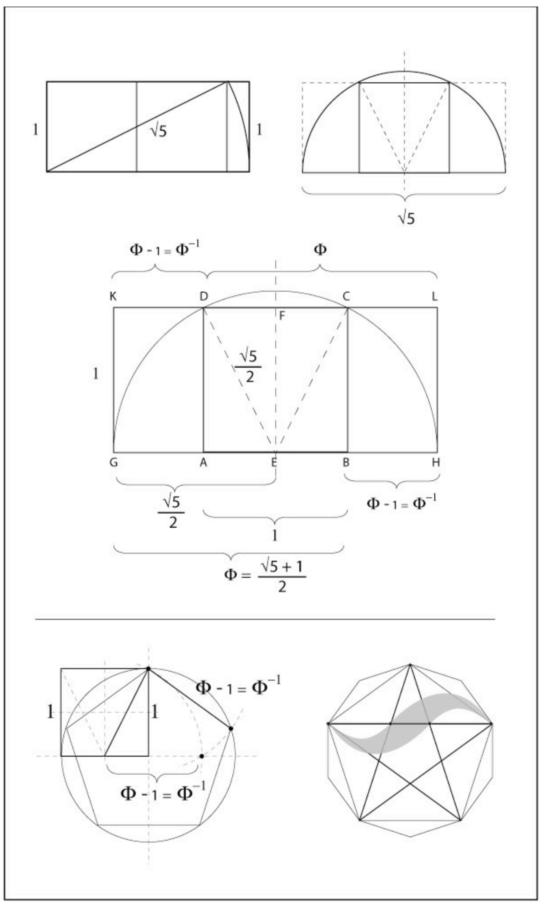

Figure 2.4. Construction on golden rectangles and the relationships in the pentagon, which were defined in Euclid's *Elements*.

[86] R.A. Dunlap, (1997). Page 43
[87] Euclid, (1956). **The Thirteen Books of the Elements**. Dove Publications. Vol II.
[88] E. D. Dobson, (1975). **Understanding Fibonacci Numbers**. Traders Press Inc. Page 3

In addition to the previous scope in this Euclidian construction, this factor has another important characteristic that is worth mentioning.

By solving the dilemma raised in relation to the subject of proportions between the mean and the unit, Φ is the only factor that the unit can be divided into, so that the proportions of the three sequential parts are the same.

That is, if we divide the unit in a cut that is equal to Φ, the part A will be to part B what part B is to part C — or the whole — (see Figure 1.1).

Therefore, if we have a succession of three parts A::B::C, we obtain a geometrical progression where the factor is ***f*** = Φ.

This is an important fact if we think that all the efforts invested in providing an artistic work would be wasted if the part provided is not proportional to the whole.

Some authors mark the difference between (1) the smaller and the medium and (2) the medium and the smaller; that is between A:B and B:A.

In the Fibonacci succession, it would be as saying that we observe the proportion either forward or backward; if it was forward, we would have to think what number should we multiply to obtain the following value in the series? The number would be Φ (in capital letter) almost 1.6180...; the other viewpoint would be which factor should I multiply backwards? That is, having a progression number, what factor should I multiply to obtain the previous number of such progression). This factor would be ϕ (in lower case letter), approximately 0.6180…

Due to this proportion's specific characteristics, the factor Φ and ϕ are closely related; for example, the reciprocal of Φ is ϕ:

That is, $1/\Phi = \phi$

Of course, besides $\Phi - 1 = \phi$

In the top of Figure 2.5, this relation is observed as a geometric shape. When we see a square where the diagonal's half is decreased (the diagonal of the square's half), the golden rectangle is generated; that is, a rectangle whose height is 1 and its base, Φ. This clear diagram graphically shows the way the unit is added to ϕ (0.618…) to total, together, Φ (1.618…).

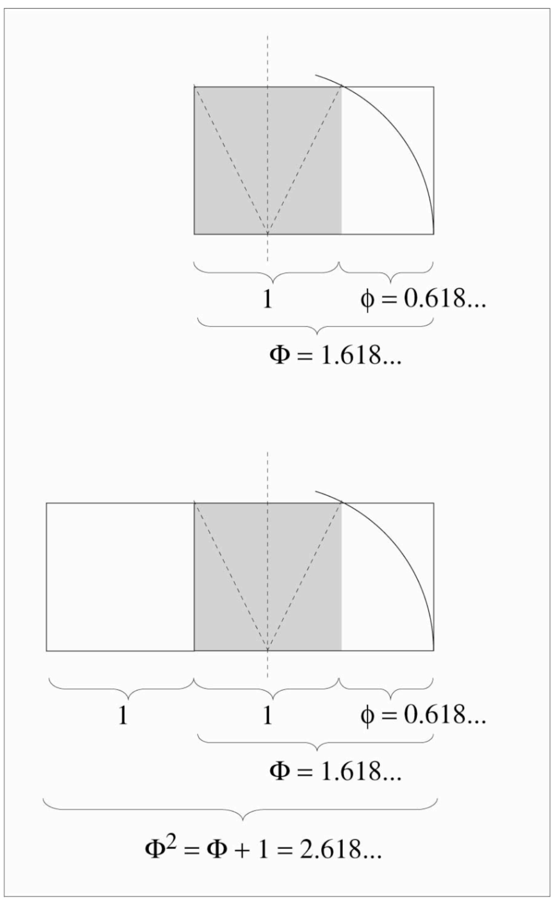

Figure 2.5. Classic construction of the golden rectangle and its relationship with the square of Φ

In this same Figure, it is evident that another square is added to the same construction; in this case, the resulting rectangle's proportion is:

Φ^2

which is the same as

$\Phi + 1 = 2.6180...$

In fact, as herein mentioned in the previous chapter, Cook reported in his classic "The curves of life", that:

"The most significant fact, for our purpose, about the Φ series is that it gives a double Fibonacci series, and in the Φ series the ratio of any two successive number is not merely approximate but exact, for the constant ratio is $\Phi = 1.618034$.
The double Fibonacci series in Φ is exhibited thus:

$\Phi^2 = 1 + \Phi$
$\Phi^3 = 1 + 2\,\Phi$
$\Phi^4 = 2 + 3\,\Phi$
$\Phi^5 = 3 + 5\,\Phi$
$\Phi^6 = 5 + 8\,\Phi$

and so on." [89]

Another interesting characteristic of this proportion and Fibonacci's succession is that the addition of two elements of the progression does generate another element (by definition), if we see it in a successive manner; that is, we partially solved the problem presented by the arithmetic progressions, where if we joined two specific dimensions in a design, the result would be unavoidably an out-of-system dimension.

This characteristic remains even if we begin with two other numbers different from 1,1, such as the classic example. Therefore, if we do this using 1,5,6, we would obtain, for example, the following:

1, 5, 6, 11, 17, 28, 45, 73, 118, 191, etc.

And, as it is easy to verify, ratios among numbers of the sequence again rapidly approach Φ. This characteristic, inherent to Fibonacci's progression, allows theoretics, such as Le Corbusier in the XX century, to take two or more numerical scales in this specific type of progression and interweave them to achieve a larger range of design possibilities.

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[89] T. Cook, (1914-1979). **The curve of life**. Dove Publications. New York. Page 420

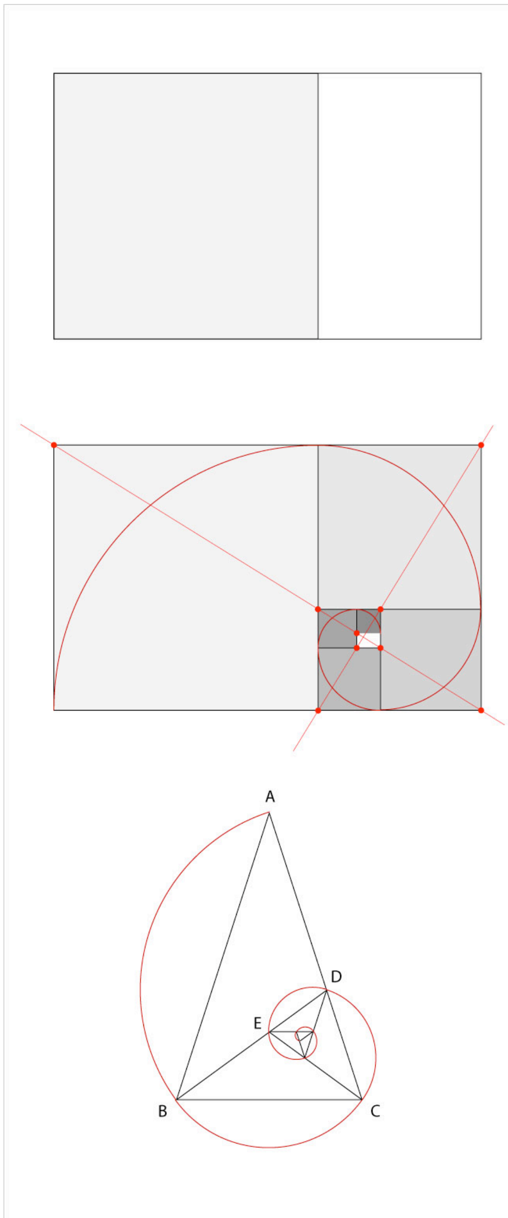

Figure 2.6. Development of the classical spirals beginning with rectangles and golden triangles.

As indicated before, (first part of the study) another practical problem faced by Le Corbusier and many other authors before him was the need to use whole numbers instead of irrational numbers. This is mainly due to the obvious need of designing real life things based on the measure of actual physical spaces and dimensions. Pure proportions such as the four Euclidian irrationals and ϕ are indeed incommensurable (they are never exact since decimals are infinite). On the contrary, Fibonacci's successions represent a wonderful alternative to the dilemma; since by being additive series, they always generate whole numbers.

"Whole-number additive progressions such as the Fibonacci series are at least as significant manifestations of the golden section and others theoretically incommensurable ratios as are the irrational limits towards which these progressions converge. And in architecture, just as in nature, whole numbers reflect the reality of building. Even when a building is not constructed of individuals unit like bricks, it is still necessary to measure it out in units: inches or centimeters…

In other words, in order to be architectonically expressive, number must be experienced in a very concrete way. It must be something like counting pebbles on a beach… The word ´calculate´ derives, incidentally, from Latin calculus, meaning a small stone"[90]

2.2.5.1 Fibonacci's spirals

It is now worthwhile to mention a very well-known type of construction with respect to proportions: spirals. Spirals are generally produced from the successions we have studied; let us analyze some examples:

A golden rectangle, as discussed earlier, is one with a base that is in golden proportion (Φ) to the height. This type of rectangle has the property of being divided into one square and one rectangle; however, the most important aspect of this division is that the resulting rectangle (beside the square) is also a golden rectangle. That is, its proportion is equal to the proportion of the original rectangle (see Figure 2.6).

This new rectangle could be, in turn, divided into a square and a new golden rectangle. If we repeated this process indefinitely and at the same time we draw arches of a quarter of the circumference circumscribed in the squares that we obtain, a golden spiral is obtained

[90] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Pages 47- 48

with a center that is in the intersection of the two diagonals. (See Figure 2.6)

In fact, this curve is not a logarithmic spiral because it is formed by circumference arches and not by a continuous shape, but it clearly illustrates the growth rate.

In the same way, a pseudo-logarithmic spiral can be produced from an Isosceles triangle with 36º-72º-72º angles.

In this case, if the relation among equal sides of the Isosceles triangle and their base is Φ (in Figure CB/AB = Φ), then it is a golden triangle.

In Figure 2.6, ABC is one of those triangles; if we bisect the angle in B, we will obtain two triangles: DAB and BCD. In DAB we have that AB/AD=Φ. The second one is similar to the original and, therefore, it is also a golden triangle. If in this triangle, we bisect the angle in C, we will obtain CDE, which is also similar to the other two. Continuing with this process, a golden-triangle spiral succession is obtained analogically to the process made with rectangles. These constructions are similar to the ad quadratum and ad triangulum constructions defined in Ancient Greece.

Thus, a line is obtained that describes a curve that is always similar to itself, in a factor of a different scale. It is a discrete approach to a logarithmic or equiangular spiral.

Concerning an actual logarithmic or equiangular spiral, the separation between them increases as the angle increases; that is, the vector ratio increases exponentially versus the turn angle. For this reason, it receives a third name: geometric spiral.

And its equation is:

$r = C^{k\theta}$

where r is the position ratio, C and k are constants and θ is the turn angle.

In relation to these spirals, the angle is also proportional to the ratio logarithm, which gave origin to its name: logarithmic spiral.

2.2.4 Polygonal and solid numbers

There are other important progressions used throughout the history of design as a dimension tool. The polygonal number is a good example.

The Pythagoreans developed these numbers; in those times, numbers were represented using small stones or beads (*calculus*), which were placed on the floor or a table. The word “number” was only used for whole numbers and ‘rational’ was used to name ratios (5/6, for example).

Thus, the “beads” were placed on the table for the corresponding counting; it is logical to assume that the ordering of these beads on the table was very important for the order of calculation and the resulting efficiency.

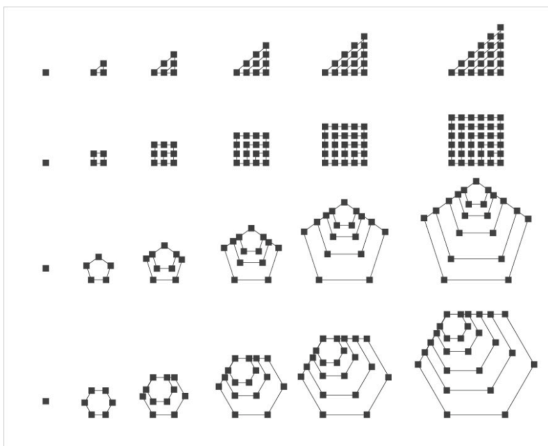

Figure 2.7. Some polygonal numbers and their classical construction.

Based on this idea, it was discovered that some numbers can form geometric figures, for example, three pebbles can form a triangle, four can form a square and five a pentagon, etc. (see Figure 2.7).

By increasing these figures proportionally, a succession of figures was obtained with pebbles which symbolized numeric progressions. The result of this process is a series of progressions named polygonal numbers; some of them are defined as follows:

Triangular numbers:
1, 3, 6, 10, 15, ...
are whole numbers of the type
$N = 1 + 2 + 3 + ... + n$

The square numbers:
1, 4, 9, 16, 25, ...
are whole numbers of the type
$N = 1 + 3 + 5 + 7 + ... + (2_{n-1})$

Pentagonal numbers:
1, 5, 12, 22, ...
are whole numbers of the type
$N = 1 + 4 + 7 + ... +(3_{n-2})$

Hexagonal numbers :
1, 6, 15, 28, ...
are whole numbers of the type
$N = 1 + 5 + 9 + ... + (4_{n-3})$

and so on.

In general, polygonal numbers are whole numbers of the following type:

$n + [\, n (n-1)\, \boldsymbol{b} / 2 \,]$

Where if ***b*** = 1 it is a triangular number; if ***b*** = 2, a square number and if ***b*** = 3 pentagonal number and so on.

If we take the first number of each polygonal number series (3, 4, 5, 6, ...), we obtain an arithmetic progression with a difference of 1.

With the second number of each series (6, 9, 12, 15, ...), we obtain an arithmetic progression with a difference of 3, and so on.

Arithmetic progressions are found in polygonal-number interweaved sequences.

This idea can also be extended to solid numbers and obtain tetrahedral, cubic numbers and then, *ad infinitum* (see Figure 2.8).

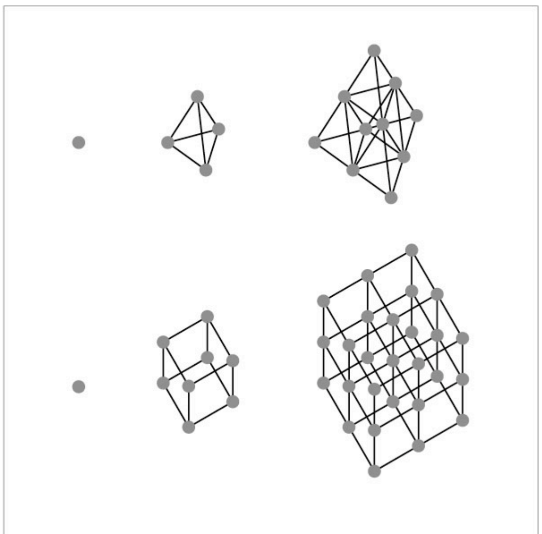

Figure 2. 8. Some solid numbers and the classic construction.

These numerical progressions are in accordance with the idea of a universe formed by indivisible units. At the beginning, Pythagoreans believed in the existence of a world based on discrete units.

“The unit was conceived as having a dimension, however small: it was a discrete quantity, a smallest indivisible whole. It followed from this those combinations of units formed characteristic space-filling shapes, depending on their arrangement. They could form series of plane figures or solid forms: triangular, square, oblong, cubic, etc. Matter was believed to consist of combination of such shapes.”[91]

This idea was improved in Pythagorean times with the appearance of irrationals, beginning with √2, as diagonal of the square. However, the successions formed were used, and are still used, as design tools since they can be applied to the actual world of design and architecture. As in Fibonacci’s Series, polygonal or solid progressions provide the designer with whole numbers, which can be used to measure different non-manageable irrationals.

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With the explanation of the polygonal number, we finish the review of the mathematical tools that have been mostly used throughout the history of our culture. As we explained before, although a significant number of measuring systems or proportion generating systems have been developed through out our material culture, it is also true that none of these systems absolutely satisfies the design’s basic expectation.

For example, it has not been possible to find a system where the addition of any two dimensions within the

[91] R. Padovan, (1999). Page 64

system results in another unmistakable amount within the system, except, of course, the arithmetic basic progression with difference ***d*** = 1.

This characteristic results in problems in the proportioning and dimensioning of any design work. For example, although we could provide a façade so that its height and width maintain a certain ratio (golden section for example) and define other internal golden ratios to place the doors, windows or locks, sooner or later, we will obtain a dimension from previous decisions and that is not in the proportional system.

This detail can only be resolved by using very small squares as the lesser dimension required and from that, define the remaining proportions. That is, we define a proportional system that (1) is only arithmetic (the most basic of all), (2) we use only whole units (intermediate measures are not allowed at a defined unit and (3) the unit is defined from the minimum dimensional need of the work.

However, this solution is the same as saying that it is possible to define the dimension or position of any element in any part of the work, because the grid is so fine that it allows anything. This, of course, is not a satisfactory solution to the "problem of the total integrated proportionality of the work," which is a paradigm searched by everybody, but with no solution in sight.

Other possibilities exist to understand the proportional processes. These are used constantly in the natural processes and offer us a different point of view for the solution of the problem.

Many of these strategies have already been studied in depth; in relation to their formal definition (often mathematical) the mechanisms behind these phenomenon have been disclosed. Until now, most of these strategies have not been used as design tools and are not mentioned in classical methodological art or design books. However, they are possibly the only actual strategies that form our perception of proportions and with them, our unconscious historical search for tools that are capable to recreate it.

2.3 Natural Patterns

This section looks back at the most important types of natural patterns; many of them are clearly defined forms and strategies with which the universe faces basic problems, such as packing, growth, equilibrium and multiplication. These strategies are defined and limited by the universal nature, by limits inherent to the mechanics of this four dimensional universe (three-space dimensions and a temporary dimension). Most of the times, these strategies alone represent the logical and unique options such phenomenon has to function with.

Some of these phenomena, due to their nature, define the proportions and behaviors that we have seen and pointed out as similar to those found in our material culture. This could give us a key to the "suspicious coincidence" of recognizing our own actions as influenced by our surrounding universe.

However, beyond our expectations in relation to the discovery of the mechanics behind these phenomena, this coincidence has only been useful to affirm the mystification of proportions themselves and with them, the historical instruments derived from them, such as the case of the "sacred geometry" previously mentioned.

The observation of patterns is a characteristic inherent to humanity. The inexhaustible search for identifiable forms is one of the autonomous functions of the perceptual human system.

"The complex programs for repetition of orders are unconsciously associated with cerebral libraries genetically programmed and recognized as nature. According to the paradigm of the human brain's peacefulness, the human brain involuntarily compares the perceptive context at all times. As a consequence of this analysis, it produces valuable judgments: "this is interesting", "this is not interesting", "this is beautiful", "this is dangerous", etc."[92]

"Looking for patterns is part of being human. People search for patterns –and usually find them. We seek familiar forms in clouds, in inkblots, in shadows. A drifting cloud could be a horse's head, or an elephant, or a fish…"[93]

This functional characteristic of the human perceptual system goes even beyond our comments; not only de we always try to find patterns in our immediate

[92] F. Hernández-Castro, (1995). **Estética Artificial**. Editorial MithOz. San José. Costa Rica. Page 89

[93] P. Murphy, (1993). **By Nature's Design**. Chronicle Books. San Francisco. CA U.S.A. Page 12

environment, but we also try to assign them with meaning. This characteristic, which is called the super-symbolic capacity of the superior primate brain, is extremely strong in human beings and it is probably the most powerful engine for progress. With these two qualities, our brain not only searches for patterns but associates them among them and assigns them a meaning.

“The World of human beings is a de facto World of signs, the thoughts they elicit, and their overall organization into a system of communal meaning that we call culture. If there is one trait that distinguishes the human species from all others, it is precisely the interplay of signs, thought, and culture in generating consciousness – the state of mind that provides humans with a means for making sense of who they are, of where they are in cosmic scheme of things, and of why they are here.”[94]

In this way, it is clearly explained why we identify a series of patterns that are repeated in nature and associate them to belief systems and we grant them pseudo-magic characteristics that are more superstition than scientific facts.

This section seeks to make a mechanist analysis of natural patterns underlining the coincidences with proportional historical instruments and the probable reasons of such coincidences. This scope is aimed at understanding the reasons of the occurrence of these proportions in these phenomenon and behaviors, as well as at demystifying these historical associations.

Several systems of classification of the natural patterns exist, varying depending on the author or objective of the classification. In our study, we will use the most common classification system, understanding that our objective is not the study of natural patterns themselves but the associations with our way of understanding the design and their instrumentation proportionally.

Therefore, we will divide, according to our purposes, the natural patterns in the following categories:

1. Spirals and helixes
2. Meanders and ribs
3. Branching
4. Fractals
5. Spheres and explosions
6. Packing

[94] M. Danesi, (2000). **Sign, Thoughy & Culture**. Canadian Scholars’ Press. Toronto, Canada. Page 7

2.3.1 Spirals and helixes

The first group of natural patterns that we could study is spirals and helixes. The spirals are well known in design and, as we said before, they have been used as reference in countless studies of proportion.

Certainly, in nature, spirals are very popular and particular, mainly logarithmic spirals. As mentioned in the previous section, the logarithmic spiral is the only type of spiral that keeps its shape when being rescaled.

This means that only this particular type of spiral forms similar curves (with exactly the same type of proportion among the parts) as it increases its size.

"The logarithmic spiral has the unique property that the curve is everywhere 'similar', differing in size but not in shape. In other words, as the curve rotates through a fixed angle, it grows uniformly in scale. This… helps us to see what the fundamental generating mechanisms of such forms are. Some things remain constant; for example, the angular speed of the curve's tip, and the shape of the curve, while other things, for example the linear (tangential) speed of the tip, change in a well-defined way."[95]

This fact explains the existence of many shapes in nature, which follow this standard.

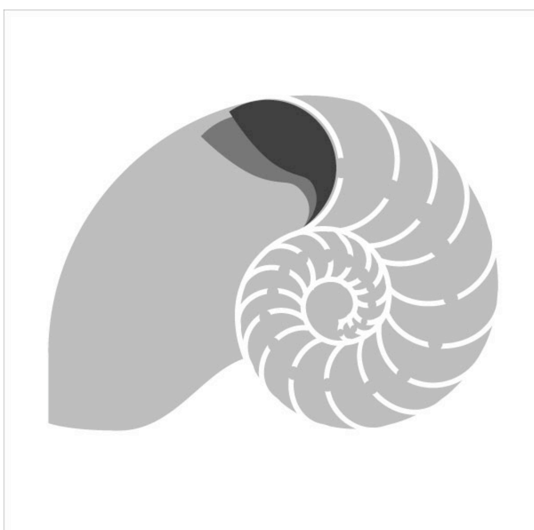

Figure 2.9. Graphical analysis of the Nautilus' transverse cut.

The diagram in Figure 2.9 shows the growth of the famous Nautilus in a transversal cut. The darkest sector represents the position of the animal in its current state of growth (without the appendixes). In the normal process of growth, the animal is always building a chamber, which is identical to the previous one, but bigger; this occurs in similar periods, so that periods of similar times correspond to angular movements (around the centroide), which are also similar, and correspond to an exponential growth rate (almost 6.3% each time, due to cellular growth).

These are exactly the generation characteristics of the logarithmic spirals; therefore, the obvious conclusion is that the shell grows with these characteristics. Indeed, it could be said that there is no other additional option because this is the only shape that meets such conditions.

Similar situations can be observed in the growth of many animals and vegetables; for example, the seeds of some flowers such as sunflowers, daisies, pineapples, the horns of animals, other shells, spider webs, etc.

[95] P. Ball, (1999). **The self-made Tapestry. Pattern formation in nature**. Oxford University Press. Page 12

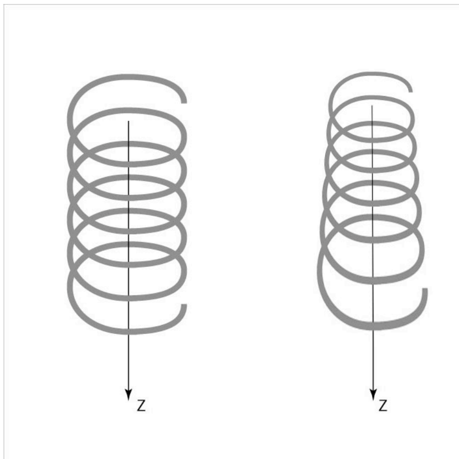

Figura 2.10. The growth of a classic helix with or without dilation.

Figure 2.11. Disposition of a helix in a Phyllotaxis stalk

“The florets, like the chambers of the nautilus, increase in size with their distance from the center. The oldest and largest florets are at the periphery; the new newest and smallest are at the center. The overall spiral shape is created by adding elements that have a shape identical to the existing elements, but a different size. You can see the same principle at work in the florets of a sunflower and the scales of a pinecone.”[96]

The construction of these spirals suggests the reason of their abundant presence as a form governing the growth of numerous living organisms. The two forces that trigger this growth are rotation and dilation; self-similarity additive-winding growth.

In this way, we can explain why many snail shells form logarithmic spirals; it would seem that there is no other possibility to growth than always being equal to themselves (as many of us).

The analysis that we made of spirals until now works in a two-space dimension; in fact, we talked about a transverse cut of the nautilus' shell. In relation to a three-dimensional space, there is another possibility of locating growth: instead of producing growth globally, such as in the case of the nautilus, growth can be produced longitudinally in the third dimension.

This form of growth is known as helix. In this sense, what we have is a circular growth, either constant or not, with a displacement in the third dimension, which is also known as the Z axis.

Figure 2.10 shows this type of movement with or without circular growth.

This type of arrangement is also present in the growth of leaves, circling the stems, for example.

“Phyllotaxis is the botanical term for a topic which includes the arrangement of leaves on the stems of plants. The arrangements are characteristic of the genera. Leaf “divergence” is technical term used to describe the angular separation of two successive leaf bases on the stem as measured by a helix drawn from the root of the plant upwards to its growing point. The leaf arrangement can be specified in term of divergence.

A helix is drawn to pass through each leaf base unit it reaches the first base which is vertically above the star point.”[97] (Figure 2.11)

[96] P. Murphy P, (1993). Page 23
[97] H.E. Huntley, (1970**). The divine Proportion. A study in mathematical beauty**. Dover Publications. New York. Page 161

This movement is also typical of organic growth:

“The general rule is clear. If the rates of growth or expansion of two surfaces are equal, the material lies straight. If the rates are unequal, the material curls so that the slower growing surface is inside the faster growing surface. We see manifestations of the rule everywhere we look.”[98]

This principle of several growth speeds works as a guideline in many forms of organic growth, such as in the case of a tendril. In these cases, the difference of growth rates is stimulated by any contact of the plant.

“If the outside of the tendril bumps into a twig or some other obstacle, the contact causes cells on the opposite side to grow faster. As a result, the tendril changes the direction of its twist to loop around the obstacle.”[99]

The typical helixes that we know are formed in this manner. The laws of growth, in these cases, keep the same behavior as the helixes herein described; in fact, they are the same type of form, but they developed in three dimensions.

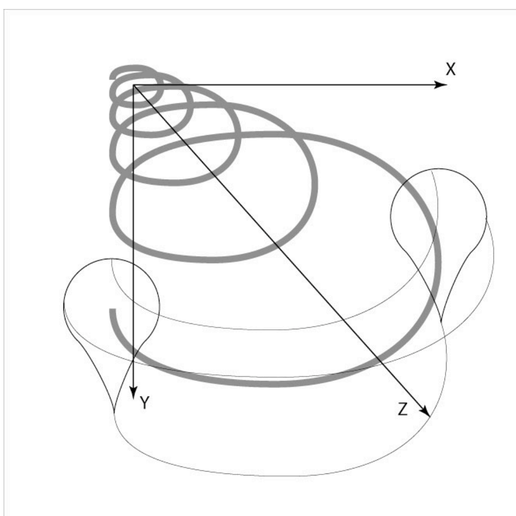

Figure 2.12. Three dimensional growth of the logarithmic helicoids.

Figure 2.12 shows the way in which both movements, the logarithmic helix of the Nautilus' transverse section and the helixes combine in order to produce the real organic growth forms.

“The surface of any shell may be generated by the revolution about a fixed axis of a closed curve, which, remaining always geometrically similar to itself, increases its dimension continually… The scale of the figure increases in geometric progression [exponential in time] while the angle of rotation increases arithmetic [at a constant rate].”[100]

Therefore, in relation to the analysis of the Nautilus' traverse section, the form growths (of living creatures) in geometric progression, combined with the arithmetic displacement, circling the Z axis and along itself generate the typical curve that we have described. Concerning this three-dimensional manifestation, spirals are known as logarithmic helicoids.

Based on the aforementioned paragraph, geometrical progressions maintain very particular proportional characteristics. Among them, there is the specific case of the growth factor $r = \Phi$, where the progression results in a self-similarity. This aforementioned property is the only possibility for two parts to remain proportioned so

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[98] P.S.Steven,(1974) **Patterns in Nature**. Penguin Books.Great Britain. Page 65
[99] P. Murphy, (1993). Page 26
[100] P. Ball, (1999). Page 12

that the sequence among them and the whole are in the same proportion.

On the other hand, it could also be said that this type of progression is the result of the organic structures' behavior when growing, because this is exactly what happens if we combine an exponential growth with an arithmetic displacement.

In other words, these organic growths don't have a choice: they must grow in golden proportion. From this angle, the proportion of logarithmic helicoids is an inevitable consequence of growth and the aforementioned displacement.

2.3.2 Meanders and ribs

Figure 2.13. Meander-type formations typical in organic growth, such as coralline formations.

The form defined by a river going down a hill, that is, that characteristic zigzag, is called meander. The name comes from the Maíandros River in Ancient Greece, which is a very sinuous river.

In fact, the meander is a form similar to the spirals we have previously discussed. Tendrils, for example, grow more on one side than the other, thus producing the spirals we know; if this movement takes place alternatively and periodically, a meander is produced. It is the same action that produces the contraction and dilation that a serpent makes when it moves.

“In the same way that the Archimedean spiral wraps around itself to fill space, the random meander twists and turns and doubles back on itself so that it too fills space.”[101]

The pattern is also followed by the growth of marine corals or the cerebral cortex. Figure 2.13.

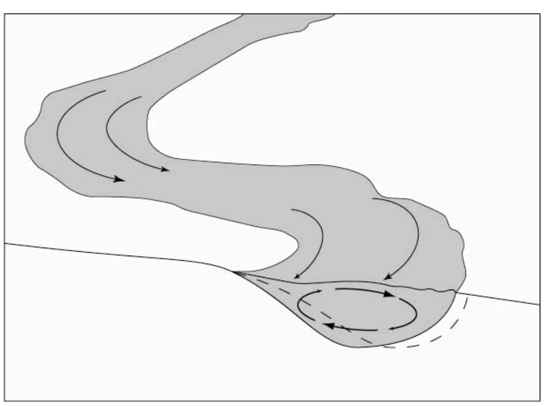

Figure 2.14. Turbulence generated in a meander due to the difference in speed between a bank and the curve.

Let’s analyze in further detail the case of rivers. In this situation, one would think that they should fall as straight as possible from a hill. This does not occur; instead, it forms meanders, a form that, as we saw, is also found in other natural formations.

Actually, the river’s first impulse is a natural and free fall as was mentioned earlier but right away it encounters the first obstacle. At that moment, the gyration to avoid the obstruction inevitably generates an internal turbulence. At first it is minimal, but as the speed and flow increases, so does this turbulence. As Figure 2.14 shows, this same curve generates a change of speed between the water on the internal side of the curve and its external side.

[101] P.S. Steven, (1974). Page 92

This difference in speed induces the circular turbulence (seen in the figure) and with it, erodes a part of the bank and deposits the sediment in another. This process results in the rivers' intermittent course, the small sand beaches on the internal side of the meanders as well as the sharp and vertical walls on their external side.

"From studies of over fifty rivers, the hydrologist Luna B. Leopold reports that no river, whatever its size, runs straight for more than ten times its width. Moreover, the radius of a bend is nearly always two to three times the width, and the wavelength –the distance between analogous points of analogous bend—is seven to ten times width. Thus, despite dramatic variations in size and bed conditions, rivers run a strikingly uniform course."[102]

Wind, like water, also presents the same type of behavior where erosion takes material from one side and deposits it in another. The formations of dunes in the desert or textures full of sand meanders in the beaches are an example of the same type of alternate erosion. In these cases, the repetitive meanders are called ribs.

"In many ways, flowing air acts much like flowing water. Like water, wind can carry bits of sand and silt for a distance and deposit this material somewhere new. Like water, wind picks up sand when it is flowing rapidly and drops it when it slows. Wind shapes both sand dunes and ripples on the surface of the dune."[103]

So, we find in meanders and ribs the same type of situation we had analyzed in other natural phenomena: the much talked about meanders are the inevitable result of the physics involved in the erosion process, caused either by air or water, in an hour or in thousands of years.

These forms have been used as examples to follow or as a parameter of beauty in many cultures. In almost every type of primitive ceramic we find decor that imitates the forms of meanders or ribs.

It is worth mentioning that this "copy" (conscious or unconscious) of these natural patterns clearly shows how much we appreciate these forms and proportions and how our inclination in this matter is clearly influenced by natural surroundings.

Further on in our study, we will delve into how and why natural patterns many times influence, with or without intention, our material culture. Suffice it to say for now

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[102] P.S. Steven, (1974). Page 94
[103] P. Murphy, (1993). Page 35

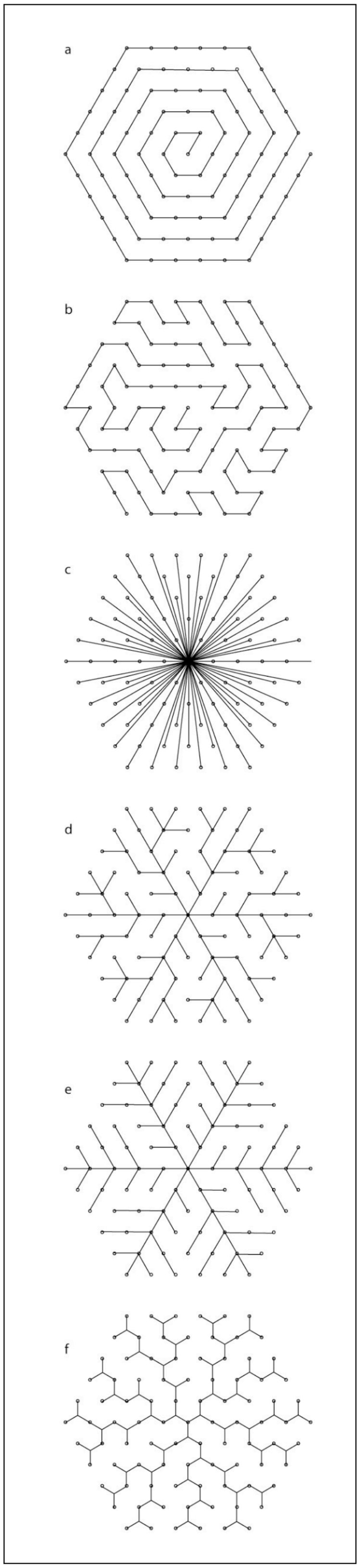

Figure 2.15. Different possibilities to connect a central point with a series of points in a hexagonal formation.

that the influence of these patterns, either explicitly (as in the case of the decorations in the shape of meanders) or implicitly (as in the desire to design with the same proportional canons) in our art and design, is practically omnipresent.

2.3.3 Branching and fractals

The next unavoidable topic in recognizable natural formations is branching. It is simply the division of a natural form, such as a plant's stem or a river bed. These phenomena often times refer to the equal distribution of some resource or flow from one point to an area; classic examples are tree and plant stems, the lung's blood vessels as well as the branching of a river on the plains.

These phenomena can be analyzed, as the former ones, in the light of the new axiom of minimum energy.

"With regard to how patterns and shapes come into being, we can readily accept the fundamental idea of the theory of evolution, that things evolve to their fittest form; we can accept the principle that things tend towards a configuration with least energy, that is to say, with the tightest fit, the lowest altitude, or the least motion; we can even accept the theory that exiting forms of nature are exactly those that are most likely to exist –taking into account all possible possibilities."[104]

With this idea in mind, let us take on the task of evaluating a configuration of dots (Figure 2.15). With a series of dots placed in a hexagonal reticulum, the task consists of connecting all of the dots to the central dot, directly or indirectly. Obviously, the exercise simplifies a typical natural situation; for example, the ramification of veins in a leaf or the radial growth of a plant.

There are, of course, several ways to reach our objective. We can begin with a spiral (Figure 2.15.a). This starts from the central point and begins to unravel until it reaches all dots.

Another way could be using a meander as observed in Figure 2.15.b.

To make an impartial comparison of the presented alternatives, one could imagine that the distance between two points is equal to one. With this in mind, one can count the total distances of the alternatives **a** and **b** (spiral and meander) and find that both are equal and occupy exactly 90 units.

[104] P.S. Steven, (1974). Page 37

Another important information we can evaluate from these configurations is the average longitude of the paths; therefore, evaluating each point and counting the distance of the necessary path between each of them and the center, we can tally all the necessary paths to reach all the points. Adding all of these paths, the total longitude is divided into the number of them and we obtain the average longitude of the evaluated configuration.

Once again, the average is identical in options **a** and **b**; exactly, 44.5 units.

The **c** configuration (Figure 2.15.c) shows a new strategy denominated "the explosion" where each path departs from the center and from there reaches all points. In this case, the longitudinal total is 233.1 units, much longer than in the former cases; however, this time the average path has barely 3.37, much smaller than the former configurations. In other words, in the explosion configuration, we have more longitude but much less distance in the individual paths; it makes the more direct connections.

Now, let us consider the branching strategy, where all paths divide. Figure 2.15.d shows a type of branching strategy where all paths divide. The longitudinal total of this focus is 90 units, equal to the spiral or meander strategy, but the average of paths is only 3.73 units, very close to the 3.37 of the explosion configuration. In the **e** configuration, we have another possible branching strategy with an even lower average – only 3.67 average units – but maintaining the longitudinal total of 90 units.

The branching strategy shown at the end of the figure (Figure 2.14.f) is nevertheless the shortest of all, with a total longitude of 77.9 units and below the spiral and the meanders. The average of paths between the dots and the center is 4.2 units this time, a little higher than in the better configurations.

The following table shows the results of our little study.

Strategy	Longitude	Average
a. spiral	90	44.5
b. meander	90	44.5
c. explosion	233.1	**3.37**
d. branching 1	90	3.73
e. branching 2	90	3.67
f. branching 3	**77.9**	4.2

Table 2.3: Longitude and average of natural typical strategies.

The configuration shown in figure 2.14.f is definitely the best with lesser absolute longitudinal total and a very low average. Another winner is the explosion with the lowest absolute average in all of the configurations.

Analyzing these results, we could consider branching as a good compromise between the longitude of the total system and the average reach to each point.

These conclusions explain which configurations are popular in nature and why. Its efficiency between the quantity of material used and achievement of objective is excellent, branching being an excellent medium point between the spiral-type strategies or meanders and the explosion-type strategies.

Once again, the exhaustive use of branching in nature is clear: its efficiency.

There is one more form of branching to study, which is very used and recognized at a natural and theoretic level – fractals.

2.3.4 Fractals

Actually, fractals are more than branching techniques but there are fractals, which work with this strategy, and it is this strategy that leads us to the topic of proportion.

" In the mid-1970s, Benoid B. Mandelbrot, a mathematician at IBM's T.J. Watson Research Center, developed a geometry that could analyze and quantify nature's crags, whorls, billows, and branchings. He called this new branch of mathematic fractal geometry, taking the name from the Latin adjective fractus, which means "fractured, fragmented, or broken…

Not every irregular shape is a fractal. To fit in this category, a shape must have what Mandelbrot called *self-similarity*, the details must look much like the large picture"[105]

We have already commented that the characteristic of the logarithmic spiral is the ability of being able to grow without changing its proportion (or reason among the parts). We mentioned that this unique characteristic makes it the resulting form in the growth of organisms such as the Nautilus. Using the terms of the Swiss mathematician, Jacob Bernouilli (1654-1705), the spiral logarithm, named by him, boasts the*: "Eadem mutata resurgo*" (to reoccur changed but equal).

[105] P. Murphy, (1993). **By Nature's Design**. Chronicle Books. San Francisco. CA. U.S.A. Page 92

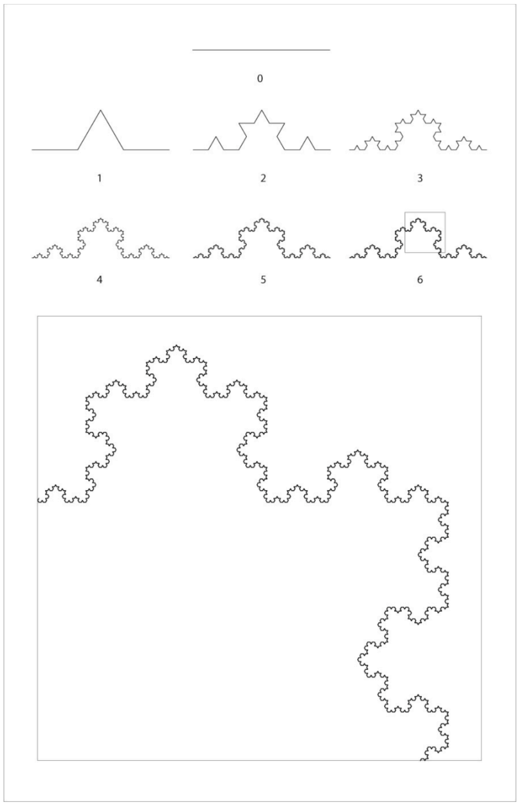

Figure 2.16. Koch Curve (Helge von Koch, 1870-1924). Detail of the methodology for the generation of fractals.

This interesting property of the logarithmic spiral is closely related with another that Bernouilli also discovered: self-similarity, which directly relates this spiral with fractal objects.

Simply defined, self-similarity refers to the property of some geometric constructions of having apparently the same form when they are observed from different scales. An easy example to follow is the clouds: they have different forms and shapes as one draws closer or further away but they always have a similar configuration. As a matter of fact, it is not possible to estimate the location of a cloud's distance. Our inability to do this is due to the fact that its self-similarity characteristic eliminates the visual clues that are typical of perspective (the further away an object, its details become smaller), clues that usually indicate how close or far away an object is located.

Figure 2.16 shows Koch's curve (Helge von Koch, 1870-1924)[106] which should assist us in considering these characteristics up close. In this Swiss mathematician's construction we can clearly observe the mechanics behind the self-similarity fractal.

Beginning with the 0 step, here we have the construction's departure position: a simple straight line.

This line has four operations:

1. Its original size is reduced by one third
2. The configuration is multiplied by four
3. One of the resulting constructions is rotated 60º and other -60º
4. They are assembled in a single horizontal line again

The result is the construction of Step 1 (figure 2.16). It is important to mention that this construction has the same horizontal longitude as the former, since strategically some of them were reduced by a third and rotated 60º. Note that the construction of the central peak corresponds to an equilateral triangle whose missing side has exactly the same longitude as the other two and therefore, equals the longitude of any of the horizontals in the construction's extremes. In other words, reduced to a third of its size and multiplied by three. So, its horizontal longitude is equal to the original line.

Here comes the fractal magic: you take the resulting construction (designated as 1) and repeat the four cited steps. This time, each one of the four sections of the new construction (designated as 2) takes on the shape of construction 1. This result in this previous stage

[106] G.W. Flake, (1998). **The Computational Beauty of Nature. Computer Explorations of fractals, chaos, complex system and adaptation**. Mit Press. Massachusetts. Page 64

begins to take the shape of the general configuration we know as Koch's curve.

The process is repeated once more. You take construction 2 and apply the four cited rules. As can be observed, the result is construction 3. You apply the rules to this last construction and obtain construction 4, and in this manner, obtain construction 5 and construction 6.

The resulting construction, designated as 6, has been enlarged in the frame in order to observe how it is constituted up close.

It is necessary to make several comments to understand the importance of these fractal constructions.

Let's see, for example, the concept we were studying: self-similarity. As one can observe from construction 4, construction 5 and construction 6 are practically the same. As observed in the construction, these are very different, a situation which is left clear in the frame.

This simple conclusion makes us come face to face, for the first time, with artificial constructions, which are not discreet as up to now, but rather are continuous and somehow irrational; definitely closer to the paradigm of the generation of natural forms than any other former construction.

The typical complaint of critics of the proportional systems was that the world cannot express itself in triangles, pentagons and squares and those theoretic proportions are irrational anyway (for example, $\sqrt{5}$); therefore, they could not be applied directly. In these fractal constructions, we have, for the first time, artificial systems that behave like natural ones.

"Euclidean geometry can be extended to account for objects with a fractional dimension. Such objects, knows as fractals, come very close to capturing the richness and variety of forms found in nature. Fractals possess structural self-similarity on multiple spatial scales, meaning that a piece of fractal will often look like the whole." [107]

By the way, this solves the cited problem of conventional proportional systems: the problem of the works' integral proportion. As commented, it has not been possible to find a system that, by adding two of the system's terms, unmistakably results in another term in the succession. In fractal phenomena, by definition, any part is completely equivalent to the whole and therefore,

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[107] G.W. Flake, (1998). **The Computational Beauty of Nature. Computer Explorations of fractals, chaos, complex system and adaptation**. Mit Press. Massachusetts. Page 59

proportional. This property hides another inherent characteristic of the system: its capacity to explicit itself in an extremely compressed manner.

“Related to this property is the fact that fractals are extremely compressible in the sense that an algorithm or recipe for the image is far simple to store than the image itself. This is due to the recursive and/or iterative nature of the fractal algorithm.”[108]

In this respect, it is worthy to mention that one of the most interesting mysteries which arose as a result of the discovery of the DNA structure, the famous double spiral, by James Watson and Francis Crick in 1953, was the speculation: if our bodies are all slightly different. How was it possible to store so much information in such a reduced space? The path of the blood vessels, for example, shows a special problem: there are no two people with the same distribution but its enormous extension insinuated that they could not be explicitly coded in the space foreseen by the DNA.

Fractals solved that problem leaving clear the possibility of coding structures that are infinitely complex in very small code fractions (like the four modification rules we saw).

In this manner, fractals represent the closest approximation we have had between an artificial construction and nature. We could include other information about fractals; for example, its dimensions that are defined between [1,2] (dimension 1.25 for example). However, this is not the objective of our study. For now, our interest lies in presenting a special type of fractals, which can shed light to our study of proportions: the L Systems.

[108] G.W. Flake, (1998). Page 59

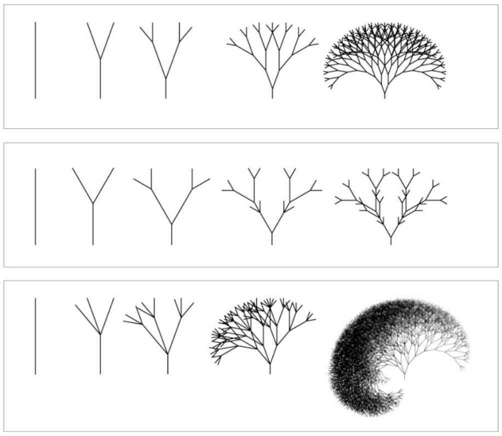

Figure 2.17. Samples of fractals using L-Systems. Departure positions and following steps.

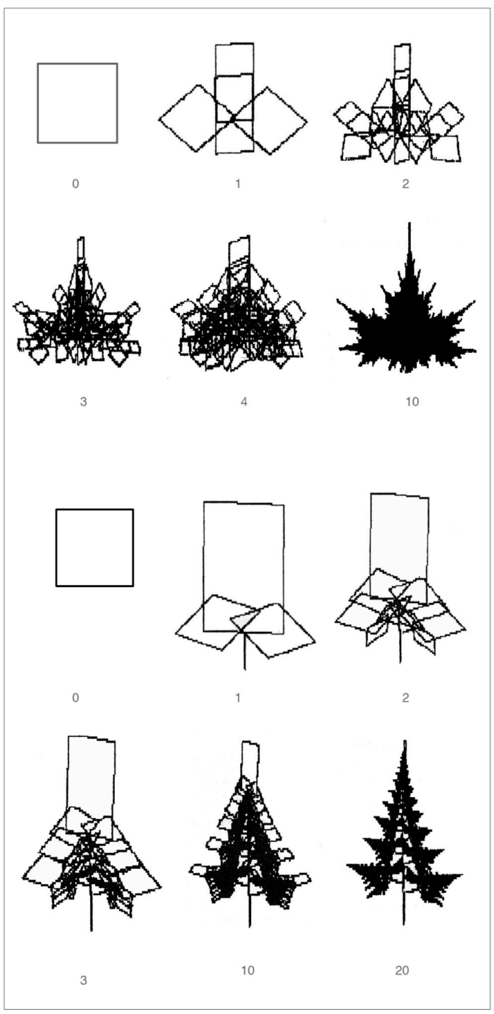

Figure 2.18. Generation of leaves by L-Systems.

2.3.4.1 The L-Systems

Developed by Aristid Lindenmayer (1925-1989), the L-Systems represent the best mathematical approximation ever developed to the model of growth of vegetable systems.

"The developmental processes (from plants) are captured using the formalism of L-systems. They are introduced by Lindenmayer as theoretical framework for studying the development of simple multicellular organisms, and subsequently applied to investigate higher plants and plant organs."[109]

To further explore this phenomenon, we turn back to the concept of sequence of constructions (figure 2.17). This figure describes three processes of modification very similar to the ones already exposed; in other words, a departure position is defined (also known as an axiom). A few rules to be followed are defined and finally, these rules are applied modifying the figure in cycles, where each result operates as an element to be modified in the next cycle. In the case of the L-systems, it is usually the modification of a line, an element that transforms itself, multiplies and rotates as convenient. In the figure, it is easy to follow the rules that were applied in the step by step modification.

These systems are generally coded in a text chain or "string" where each character is replaced by a few more characters.

"The main ingredient of the method is the algorithm for the string generation, of course. A first string consisting of only few characters must be given. It is called the axiom. Then each character of the axiom is replaced by a string take from a table of production rules. This substitution is repeated a prescribed number of times to produce the end result. In summary we have that the final picture is completely determined by:

1. The axiom
2. The production rules
3. The number of cycles"[110]

This summarized system exemplifies the extremely compact character of fractal coding. This codification seems to explain, at least theoretically, how certain design specifications are transferred from one plant to another.

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[109] P. Prusinkiewicz. A. Lindenmayer and others, (1990**). The Algorithmic beauty of Plants**. Spinger-Verlag. New York. Page VI
[110] H.O. Peitgen, D. Saupe and others,(1988). **The science of Fractal images**. Spinger-Verlag. New York. Page 275

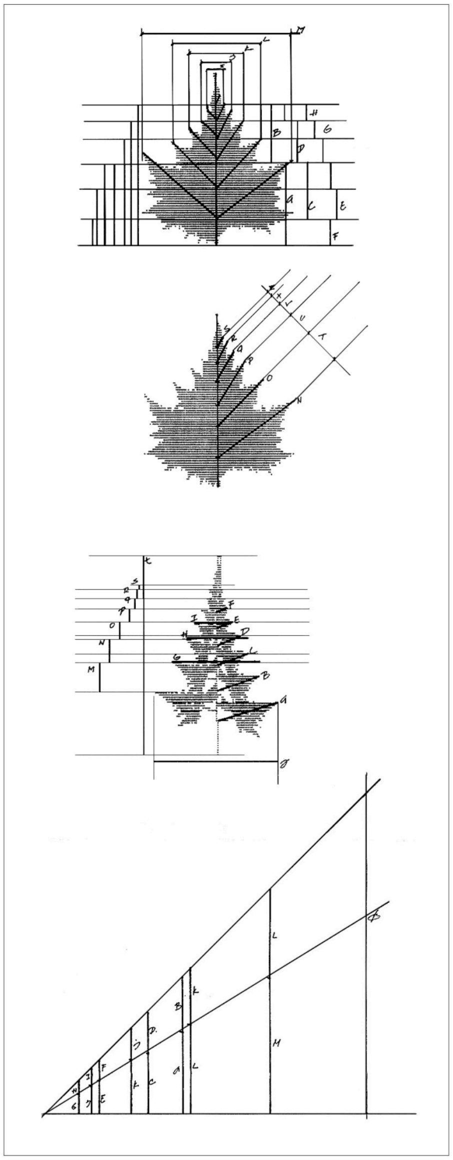

Figure 2.19. Analysis of the proportions in the structures obtained in the programming of the L-Systems.

As we mentioned earlier, the fractals' self-similarity is very easy to relate with the logarithmic growth and with it, the golden section. It is easy to imagine that, if snails use logarithmic spiral as base (since they don't have any other option), plants would be, due to their growth process, invariably subject to the same circumstances.

As part of a research work developed in the Hochschule für Gestaltung, Schwäbisch Gmünd (in 1989), in the German Federal Republic, we decided on our own to implement a calculation of fractal structures with the L-Systems. (Figure 2.18).

Figure 2.18 shows two different constructions with six different steps in each one, both trying to simulate somehow the natural growth in two different types of leaves: the composite leaf and the simple leaf.

In this experience, the idea was not to force the result to obtain a specific proportion, as usual; on the contrary, the desire was to follow a series of rules usually used in the L-Systems and later verify what type of proportion was shown, if any.

From a scientific perspective, the experience poses the requirement of not thinking about any of these factors when the experiment is being programmed. That is, why it was carried out, trying to simulate vegetable growth, by repetition, as has already been outlined.

The original program was written in 1989 in the "Basic" computer language and it is programmed only for repetition processes. The only thing that was programmed was the simulation of a fractal process of repetition of rules beginning with an axiom for a defined number of times.

After generating the artificial leaves, the following step was to verify the presence or absence of some type of proportional system. The result was amazing; using the classic methods of analysis for the golden proportions, the leaves comply with 95% of the generalized canons. Its different sections follow proportion measurement parameters as if they had been programmed for that purpose (Figure 2.19).

Furthermore, since the program generated leaves that were too perfect and unnatural, it was decided that random factors should be included. These were coefficients, which would vary by chance, acting on some repetitions but not on others, simulating natural conditions; for example, wind, rain, constant shadow to the side, etc. The inclusion of these factors resulted in more natural leaves, less perfect but more realistic. When analyzing factors such as proportion, the influence of these coefficients did not alter (as also happens in nature) the unavoidable appearance of proportions.

On the lower end of Figure 2.19, we can observe a comparison between the different measurements taken of the programmed leaves and the golden section, the middle line. From this scale, the differences are so small that they are hardly perceptible.

This interesting conclusion was, actually, very predictable. As was discussed earlier in the topic of logarithmic spiral, since it needs to grow keeping its proportion, it has no other choice but to grow in golden proportion. The L-Systems possess the same characteristic: they must grow and keep their self-similarity. This condition leads them, as in the case of the Nautilus, to the unavoidable conclusion of presenting the golden section in all its growth. In other words, even here the golden section is a collateral effect of the conditions of organic growth and not a norm imposed on purpose.

2.3.5 Spheres, explosions and packing

The following natural pattern in our study is the sphere. Since Ancient Greece, it is thought that the sphere, due to its perfection, is the divine counterpart of the circle. In nature, spheres possess the great advantage of its balance. As we have mentioned before, the law of minimum energy guides many natural processes. This can be interpreted in physical terms such as: the minimum surface, the minimum height, the minimum movement, etc.

The sphere, then, represents a good alternative for situations such as minimum surfaces.

"The soap bubble forms a sphere for the same reason, the least surface area for the most volume. The spherical soap bubble forms because the inward pull of elastic bubble film is opposed by the outward push of air inside the bubble. The balance between these forces creates a sphere, the shape that provides the most space for the air with the least stretching of the bubble film"[111]

In this manner, from planets to soap bubbles, form spheres as a result of the automatic search for balance and the least energy.

To understand this process, we must distance ourselves from our first intuitive impression of liquids. In general, we think of liquids as shapeless matter, which takes the

[111] P. Murphy, (1993). Page 48

form of the container it's in. It isn't easy to explain what the shape of water is; however, if we begin to think about small drops of rainwater in a crystal, we begin to change our perception about the form of water, even more if we think of a falling raindrop.

"Unlike crystals, in which the atoms are stacked into regular arrays like eggs in an egg box, liquids have no ordering of their constituent particles over long distances. The position of one molecule of water bears no relation to the position of another's few millionths of a millimeter away –every thing is a jumble. This means that the liquid looks the same in all directions –it is isotropic, and that is reflected in the 'perfect' spherical symmetry of a droplet."[112]

However, and as expected, there is another type of force that keeps liquids together. It could be described as forces that keep them from dispersing in a type of water air spray formed by individual molecules.

Actually, there are two types of forces in liquids: molecular forces which join atoms to one another in molecules keeping the oxygen atoms in contact with the hydrogen atoms, in the case of water. These forces are strong and are not easy to overcome (to separate the atoms) using conventional methods; for example, as a raindrop falls.

The second type of force is the force between molecules. This force, electric in nature, is much weaker and is produced by molecular asymmetry, which causes one part of the molecule to be more charged on the positive side (in the case of water, the hydrogen atom) and therefore, causes the other part of the molecule to be charged negatively (the oxygen atoms).

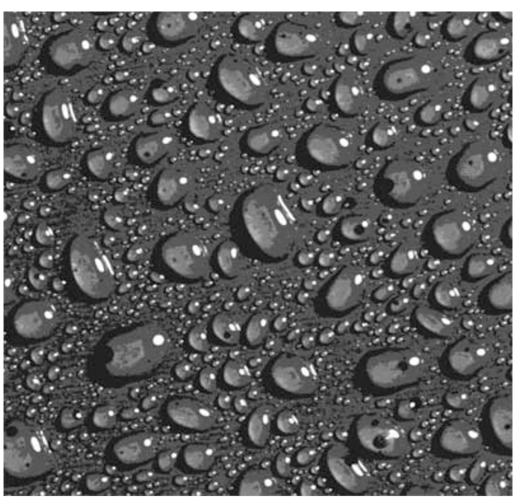

Figure 2.20. Typical formation of drops on a smooth surface, showing the effects of surface tension.

These two charges are jointly attracted between molecules and keep water as a relatively stable whole. This phenomenon is observed when we play with small drops of water on a smooth surface; the drops are generally shaped as flattened spheres (due to the force of gravity). If we draw two drops together, they attract and form one drop. This force is commonly known as surface tension (see Figure 2.20).

It is precisely surface tension that causes the spherical shape of drops and bubbles.

"As all physical systems like to reach their most energetically stable state (that is, their equilibrium state), they tend to minimize the areas of their surfaces. For a mass of a substance with certain volume, the shape that

[112] P. Ball, (1999). **The selft-made Tapestry. Pattern formation in nature**. Oxford University Press. Page 17

Figure 2.21. Vegetable formations in the form of explosions.

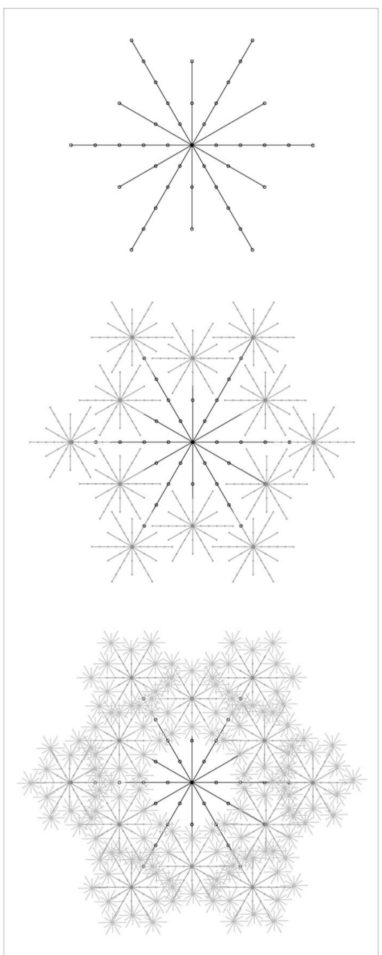

Figure 2.22. Fractal generation of an explosive vegetable formation.

has the smallest surface area is a sphere. So a droplet of water forms a sphere to minimize its surface excess energy. It is a statement of the same thing to say that surface tension pulls at the surface of a droplet equally from all directions, so that it acquires spherical symmetry."[113]

In this manner, we begin to notice a repetitive pattern in all of the examples of natural behavior we have seen. We are not talking about some specific form but clear evidence that a form in nature is not adopted by any organism or phenomenon, as has been commonly accepted, but it is rather a logical result of a process defined by the laws of physics of this universe.

Delving deeper into spherical forms, we could refer to explosions. When we talk of explosions, we are not only referring to the violent action of liberating a considerable amount of energy in a short time; it refers more to a natural growth phenomenon which adopts these same patterns in more relaxed time lapses. Many flowers, for example, demonstrate this type of construction.

As we had already discussed in the section on branching, specifically Figure 2.15, the explosion configuration has the least average of paths between the points to be reached and the center.

This means that if you wish to minimize the resources used to reach any point from the center, one must have an explosion-type configuration.

"The pattern of the explosion is really the first of our branching patterns. It represents a form of branching that is efficient in the sense that it is direct, but is inefficient in its failure to minimize the total length of network"[114]

Figure 2.21 shows some natural explosions in flowers and other plants. Some of these, as in the upper part of the figure, already show a fractal tendency. This phenomenon is better explained in Figure 2.22, where, following steps (as explained in the section on fractals), a fractal configuration is developed in the form of an explosion. As can be seen in the figures, these are widely used strategies in nature.

Finally, these two tendencies take us directly to the following type of natural patterns: packing.

[113] P. Ball, (1999). Page 18
[114] P.S. Steven, (1974). **Patterns in Nature**. Penguin Books. Great Britain. Page 98

2.3.6 Packing

The topic of packing is widely used in the theory of proportions. Since the time of Euclid, a differentiation was made between the possible forms in which a space can be filled with a recognizable and infinite order without leaving empty spaces.

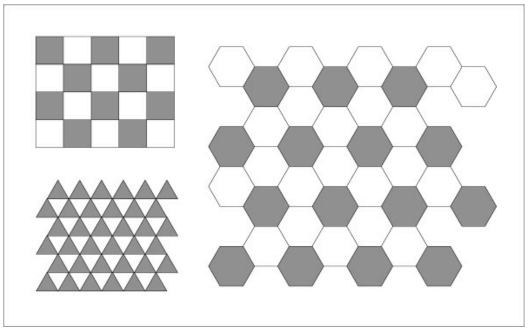

Figure 2.23. Tight-packing two-dimensional reticulum.

"Squares, equilateral triangles and hexagons are particularly easy to tile with, if one wants to cover the entire plane and achieve a pattern that repeats itself at regular intervals –know as periodic tilling…
The geometrical plane figure most directly related to the Golden ratio is, of course, the regular pentagon, which has a fivefold symmetry. Pentagons, however, cannot be used to fill the plane entirely and form a periodic tiling pattern… However, in 1974, Roger Penrose discovered two basic set of tiles that can fit together to fill the entire plane and exhibit the 'forbidden' five-fold rotation symmetry"[115]

The reference shows that not only is there a clear interest in recognizing the different orders that fill space (in this case, two dimensional) but in associating specific proportions such as the golden section with this type of reticulum.

Undeniably, there are specific periodic patterns in two and three-dimensional spaces, which are capable of filling space without leaving a vacuum. These configurations are called tight-packing regular reticulum.

As in the case of spirals, it is also possible here to carry out a two dimensional analysis in order to understand the dynamics behind this phenomenon.

Figure 2.23 shows a typical description of the base systems used to fill space in two dimensions. Truthfully, there are only two unique systems to comply with these conditions. Actually, the hexagonal system is a particular case of grouping of the triangular system; however, it is always drawn separately. One of the reasons for this is that the hexagonal system is one of the more used in nature.

In other words, all the systems or regular reticulum we observe (think of the work of e Dutch artist M.S. Escher (1898-1972)), are based on one of these two systems because in our universe and in particular, in our two-dimensional space, only these two regular tightly-packed reticulum are possible.

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[115] M. Livio, (2003). **The Golden Ratio. The store of phi**, Broadway Books. New York. Page 202

With respect to the natural systems, the prototype system par excellence is the honeycomb. We have all heard or seen the hexagonal geometry of the honeycomb. Of course, the question since ancient Greece to the present has been, why hexagons? or, why not squares or triangles?

The first one to contribute with an answer was R.A.F. de Réaumur (1673-1757), a French entomologist who discovered, in the XVIII century, that the relationship between the closed volume and the area of the walls in the construction was a very important factor.

"… the Frenchman R.A.F. de Reáumur proposed in the eighteenth century that it is the area of the walls, not the volume of the cavities, that matters. The total length of the cells walls for hexagonal cells filling a given area is less than that of square or triangular cells enclosing the same area. In other words, it takes less material to make a hexagonal wall."[116]

This is definitely a plausible explanation; however, it leaves us with one question: how do the bees know which one is the optimal solution?

The typical answer is that all the bees that made honeycombs with different configurations, including irregular ones, became extinct, leaving behind the surviving species, which makes hexagonal honeycombs. Sounds good, but not that convincing since the bees must have gone through a very trying period making the difference of invested energy in the construction of the honeycomb so decisive, that all others became extinct. One would think that those bees, which made extremely inefficient or irregular hives, became extinct, but of those, which made more square or triangular honeycombs, at least some of them should still be around. The survival crisis must have been very exact to reach the limit of extinction leaving behind only the bees, which use the more optimal system, and at that moment, stop so as to not extinguish those as well.

This sounds very unlikely; well, in science we say that the absence of proof does not prove its inexistence.

Luckily for our debate, D'Arcy Wentworth Thompson[117] (1860 - 1948) appeared. He was a XIX biologist who solved the problem in a more simple and convincing manner.

¨For everyone knows that a layer of bubbles packs together in just this hexagonal arrangement (like the honeycomb). If the wax of the comb is made soft enough

[116] P. Ball, (1999). Page 17
[117] D.W. Thompson, (1917). **On Growth and Form.** Cambridge University Press.

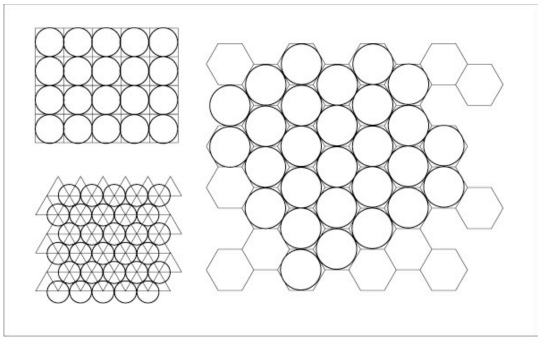

Figure 2.24. Analysis of the possible configurations of circles in a two-dimensional space.

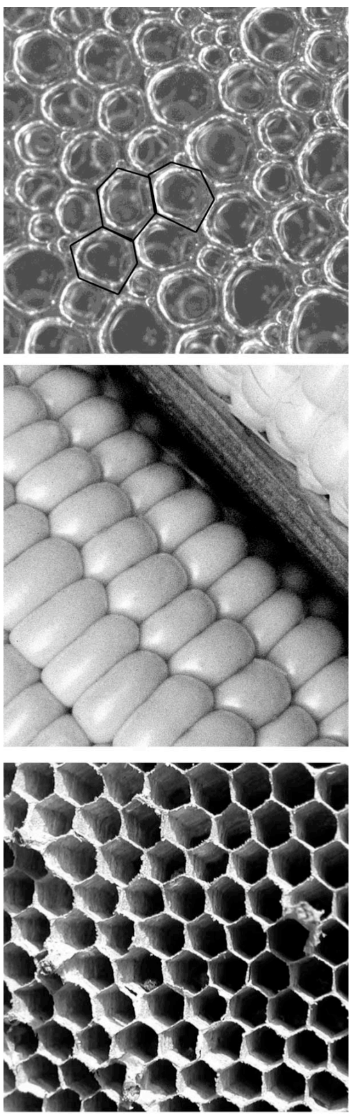

Figure 2.25. Examples of hexagonal formations in nature.

by the body heat of the bees, suggested Thompson, the it is reasonable to think of the compartments as bubbles surrounded by a sluggish fluid, and so they will be pulled into a perfectly hexagonal array by the same forces of surface tension that organize bubbles into hexagonally packed rafts. In other words, the pattern would form spontaneously, without any great skill on the part of the bees and without the guiding of natural selection.[118]

Now it seems that we can recognize the style of a typical natural construction. Bees do not choose from one design or another; they simply build their honeycomb and the force of surface tension makes it hexagonal.

This, of course, connects us to the aforementioned topic of soap bubbles. In reality, packing is no more than a collection of bubbles placed under specific conditions; for example, on a two dimensional surface.

If we go back to the regular reticulum described in Figure 2.20, but this time we think that an equal-sized circle is drawn in each center of the triangles or squares, we will have a typical regular shaped of two-dimensional packing (see Figure 2.24).

This type of packing is very used in nature. Figure 2.25 shows three very different examples where the bubbles, the corn and the honeycombs follow the same hexagonal pattern. There are certain common aspects, which clearly indicate what the mechanics of the process is.

“In all these situations, the surfaces, cracks, or partitions meet in three-way junctions at angles of 120 degrees.

Why 120 degrees? You can experiment to find out. Make four dots on a piece of paper and try connecting them with lines. Measure the length of line required to connect them all. The configuration where lines from dots meet in three-way junctions at 120-degree angles provides the shortest path connecting the dots.

In a froth of soapsuds, the bubbles invariably meet in three-way junctions; the angle between the soap films is about 120 degrees…

The wax walls of a honeycomb –like the soap films in a froth of bubbles—meet in groups of three, forming 120 degree angles…

In beehives and bubble foams, in some dried lakebeds and turtle shells, you can find three-way junctions at 120 degree angles…

[118] P. Ball, (1999). Page 17

In all these cases, the films or surfaces or cracks or cells take on the configuration that allows nature to accomplish the most with the least.[119]

In an ear of corn, where the kernels must grow neatly one next to the other, we face the same problem.

"On an ear of corn, the rows of kernels are staggered, so that the kernels on one row fit neatly into gaps between the kernels in the neighboring row, notice that the gaps between the kernels meet in three –like the soap bubbles and the wax walls of the honeycomb. Wherever you see tight packing of identical objects, look for this three-way junction." [120]

So, in three different examples we find the same pattern, a two dimensional packing is formed with hexagonal reticulum since this is the result of the equilibrium of forces in that space.

Analogically to what we commented with respect to the logarithmic spiral, the hexagonal regular reticulum is the two dimensional interpretation of a phenomenon that transcends its dimension to continue on to the three dimensions. If we think about crystal mineral formations, we immediately think about the mysterious organized constructions of crystal minerals.

Just like the exposed two-dimensional constructions, these also follow the defined laws of physics. We are, once again, talking about the interaction of the two forces we discussed at the beginning of the former topic.

As we said, a liquid is under the influence of two main forces: (1) the molecular force that maintains its two atoms forming molecules and (2) the electric force that attracts its molecules among them, also known as surface tension. However, if a liquid solidifies slowly, its molecules have the opportunity to align according to its minimum state of energy, where the second force is also in its minimum state. This configuration results in crystals with a defined form, which we know as crystal minerals.

" I might point out that surface tension can play a crucial role in determining the forms of solid objects too, in particular those of crystals. Crystals grow by adding atoms to those already packed into regular arrays; but there are several alternatives for where the newly added atoms might sit, and the positioning of these determines the shape of the facetted objects. It is better to add atoms onto the face of an existing layer, or add them on

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[119] P. Murphy, (1993). **By Nature's Design**. Chronicle Books. San Francisco. CA. Pages 74-75

[120] P. Murphy, (1993). Page 81

at the edges of the layer? In other words, which face of facetted crystal will grow fastest?

Whereas in a liquid droplet the surface tension is the same in all directions, the different faces of a crystal have different surface tensions (because the arrangement of atoms is different on each). The faces that grow faster will often be that with the greatest surface tension. These considerations determine whether, for example, a crystal like rock salt (sodium chloride) will grow as cubes or as octahedral. Either can be generated from the stacking arrangement of sodium and chlorine atoms, but the cubic shape is selected because of the way that certain facets grow faster than others.[121]

Crystals, therefore, create recognizable orderly forms. Quartz crystals, for example, form well-determined geometric shapes. They occur in prismatic forms with smooth surfaces. A common variety of quartz, for example, presents itself in a double-shaped pyramid (Figure 2.26.a); another variety acquires the shape of two hexagonal pyramids with a prism (Figure 2.26.b), whereas on other occasions, they present a distorted shape (Figure 2.26.c).

These forms continue to grow until they generate enormous structures that continue with recognizable forms at the geometric level.

The more popular formations of this growth are snowflakes. They are made up of small ice crystals formed from water vapor found in the highest clouds. The crystals, which float in the air, are attracted to each other and join to form snowflakes that begin to stick together as they descend.

These crystals have a hexagonal form (Figure 2.27). This shape results from the grouping of molecules according to its minimum energetic configuration. Here, as in the other cases, the configurations are just the logical and unavoidable result of the forces that formed them.

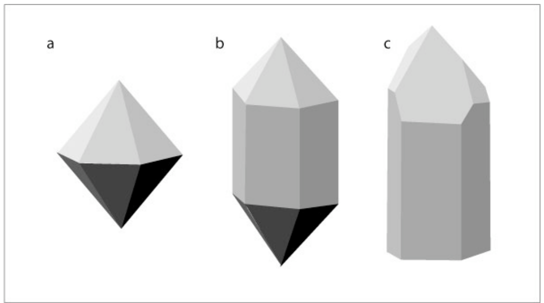

Figure 2.26. Typical geometric formations in quartz crystals.

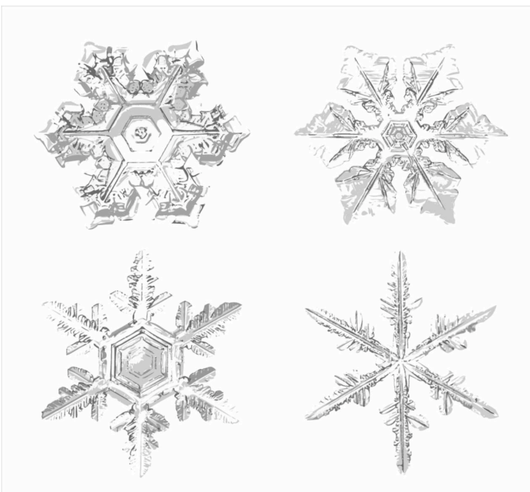

Figure 2.27. Snow crystals showing their typical hexagonal formation.

[121] P. Ball, (1999). Page 18

2.4 Conclusion to the second part

In this second part of our study, the theoretic nature of proportion is presented from two points of view: (1) the mathematic point of view and (2) the phenomenon of proportion in nature.

By analyzing the different tools used throughout history as proportion tools, we have encountered some irreconcilable incoherencies.

For example, in the more used system in history – the arithmetic system – the differences among proportions dictated by the method vary enormously from one side to the other of the scale. So, as we have seen, if we want to proportion small areas of our design, we must use one scale; and if we wish to proportion larger parts, we must use another. Despite this fact, Egyptians up to the Renaissance used arithmetic systems.

This problem was partially resolved with the *mis en scène* of the geometric systems but the problem of the works' integral proportionality arose. This new problem was defined as the need to have a system of proportions capable of proportioning the work's complete dimension.

The majority of the proportional systems, arithmetic, geometric or harmonious, are not additive; this means that it is not possible to add any two elements as proportional and find another element of the series. This problem is only partially resolved by the arithmetic systems, which use very small units as base. In this case, if we take the restriction of never doing anything that is not dimensional for a certain number of whole units, we could reach integral proportionality; however, this is almost like not having a proportional system.

Let's imagine we have an eight-floor building, some 25 meters high and define the width of the window frames as a minimum unit, some five centimeters. According to our arithmetic proportional system, we would have a progression with $\boldsymbol{d}$ = 5 cm. This means that we can dimension anything between 5, 10, 15 , 20, 25… etc. centimeters throughout the complete work; but, in the 25 meters in height and perhaps 50 cm wide, it is almost the same as saying that we can place any element anywhere.

A lot of work was put into trying to solve o trying to find a system that satisfied said requirements; however, the conclusion is still not found.

The geometric systems based on an $\boldsymbol{r} = \Phi$ factor work with a certain integral proportionality as long as we act sequentially. This means that if we use any measure and add it to its successor in the sequence, we can continue within the progression. However, if we take any measure and add it to any other measure, the majority of times we find a third measure out of the system.

The other obvious problem of the geometric systems is the use of rational or even worse, irrational, proportions. This means that if we use a geometric progression based on any non-whole factor the resulting numbers would also be non-whole numbers. This would not be important if it weren't for the fact that, to find a geometric progression that maintains the proportions among its successive parts, one must use $\boldsymbol{r} = \Phi$ as factor, an irrational.

In other words, if we partially solve a problem, we create another. The problem of whole numbers is mainly due to the need of using measuring wholes in the real works; as we said earlier:

" Whole-number additive progressions such as the Fibonacci series are at least as significant manifestations of the golden section and others theoretically incommensurable ratios as are the irrational limits towards which these progressions converge. And in architecture, just as in nature, whole numbers reflect the reality of building. Even when a building is not constructed of individuals unit like bricks, it is still necessary to measure it out in units: inches or centimeters…

In other words, in order to be architectonically expressive, number must be experienced in a very concrete way. It must be something like counting pebbles on a beach."[122]

In this manner, the typical mathematical progressions presented practical problems. The partial solution, once more, was the Fibonacci succession. This succession is capable of having whole units, which usually tend to maintain among them the same proportion and are sequential additives by definition. This solves many of the problems outlined except those which correspond to integral proportionality; even the Fibonacci succession is not capable of being additive in any combination.

Another advantage of the Fibonacci is its generalized use in nature, which rendered it almost official. This leads us directly to the second part of this chapter: natural patterns.

There is a series of natural patterns that constantly repeat themselves in our universe; they have been analyzed and described by many cultures. To a certain

[122] R. Padovan, (1999). **Proportion. Science, philosophy, architecture**. Spon Press. New York. Pages 47-48

extent, its obvious presence in many natural phenomena has earned it a status of sacred or divine.

Perhaps the most famous of all is the logarithmic or equiangular spiral used by the Nautilus to form its shell. The latter phrase reflects the prejudice we have toward the shape, and as we saw in detail: the Nautilus does not use the cited form to design its shell. The form is the obvious and unequivocal consequence of the animal's growing process.

This case is not exclusively of the Nautilus, but it can be observed in the great majority of the natural patterns. The riverbeds shaped like meanders, the crystalline configuration, the packing of corn, the snow crystals and the floral configurations, among others, are a result of its physical processes of formation.

All of these forms respond to the possibility of using the least level of energy possible. This law of economy is one of the more commonly used at the natural level: all systems tend towards its equilibrium.

"With regard to how patterns and shapes come into being, we can readily accept the fundamental idea of the theory of evolution, that things evolve to their fittest form; we can accept the principle that things tend towards a configuration with least energy, that is to say, with the tightest fit, the lowest altitude, or the least motion; we can even accept the theory that exiting forms of nature are exactly those that are most likely to exist –taking into account all possible possibilities."[123]

In this manner, a simple analysis of the efficiency of all possible solutions (as in Figure 2.15) reveals which are the more widely used solutions in nature for these problems and why.

While searching a little more for possible mathematical constructions to explain natural configurations, we meet up with fractals. These mathematical constructions proposed by B.B. Mandebrot in the mid-1970 give us the more exact theoretic approximation we have of the processes of universal growth, from galaxies to plants.

Within this type of theoretic focus, the L-Systems stand out, proposed by Aristid Lindenmayer (1925-1989). These systems represent the greatest approximation to the model of growth of vegetable systems.

"The developmental processes (from plants) are captured using the formalism of L-systems. They are introduced by Lindenmayer as theoretical framework for studying the development of simple multicellular

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[123] P.S. Steven, (1974). Page 37

organisms, and subsequently applied to investigate higher plants and plants organs."[124]

Furthermore, fractal systems represent the sole solution to the problem of integral proportionality. These systems, given their characteristics of self-similarity, are also self-proportional. From this point of view, each part of the fractal system is similar (at the mathematic level) to any other part.

Because of this same property, the systems with fractal growth present geometric proportions with an $\boldsymbol{r} = \Phi$ factor. This is due, once again, to its dynamic of growth.

This fact explains why many natural patterns, that use similar growing processes to fractals, also have these proportions.

Note that this factor is the only one that makes proportionality in growth possible and therefore, an unavoidable consequence of the process.

As was observed in the experiment in figures 2.18 and 2.19, with no intention whatsoever, the geometric proportionality based on the $\boldsymbol{r} = \Phi$ factor is a result of the processes of generation of forms.

Finally, spheres, explosions and packing, reveal the optimal solution to group elements in two and three dimensions. It is interesting to note that, despite the intention of theoretics to defend the pentagon as a fundamental piece in the construction of periodic reticulum with which it is possible to fill a space without leaving a vacuum, it is not possible to organize this type of reticulum. The reason why pentagons are sought after is because, as Euclid's pointed out, the pentagon encloses in its geometry the golden section and its characteristic of "divine" which has been proposed as a unifying characteristic.

The truth is that there is no unifying theory, neither in the matter of proportions nor in natural patterns.

The artificial systems of proportion do not respond in an integral manner to the needs outlined by every day practice. The natural patterns do not have a sole system, either.

Depending on each need, the physical conditions give one or another solution, from the packing of 120° angles to explosions connecting all the points to a center to minimize the time of access to them.

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[124] P. Prusinkiewicz. A. Lindenmayer and others, (1990**). The Algorithmic beauty of Plants**. Spinger-Verlag. New York. Page VI

The mythical patterns, like the Nautilus spiral, also have their explanation and space in a collection of the conclusions generated from the laws of physics that control every phenomenon in particular.

Third Part
3. Analysis of the proportional tools

3.1 Introduction to the third part.

In the previous sections of this study, we analyzed the tools that were used throughout history as proportional instruments in our western culture. We tried to define which tools were consciously used by the artists or architects and which ones were later associated to the works they did. We found that many of the uses we accept as true with respect to proportional instruments are, in the majority of the cases, well-intentioned assumptions.

In the more classical case, we discovered that there is no evidence that the golden proportion in architecture was used in Ancient Greece or that the Romans consciously used any proportional system beyond arithmetical proportions.

Further on, in the second part, we focused our attention on better understanding the possible focus that can be attributed to the problem of proportioning a work of art. Additionally, we saw how, despite the effort of many authors, it has not been possible to devise a proportional system that responds to such basic demands as the integral proportionality of the work.

We also studied the natural patterns more widely recognized as beautiful, and we also saw how these patterns are used as evidence by the theoretical ones to justify the use of the proportions they imply.

With the appropriate analysis, it was possible to prove; how the natural patterns we know, are repeated constantly (in different natural settings) since they represent the obvious consequences of the physical conditions of each natural phenomenon. Therefore, the natural solutions, from a group of soap bubbles to a corncob, are more a result than a decision and, therefore, are repeated constantly.

A decision presupposes the possibility to choose; nature, on the other hand, has no choice. The processes that result in the proportions of the Nautilus or the quartz's crystalline formations are physical phenomenon of cause and effect, whose inevitable conclusions are the patterns we observe and accept as a rule in nature.

In this part of our study (the third part), we want to critically analyze the methods that have been declared valid to proportion our material culture. We want to analyze the decisions we make, knowingly or unknowingly, when accepting a series of tools, which we have accumulated historically.

Why and on what basis have we accepted these tools? Do they have a solid foundation? Are they acceptable as a set of techniques to design? Are they successful as creators of more beautiful and acceptable designs?

If so, what reasons make one proportion more successful than another?

3.2 Reconsidered Analysis of the canon concept: from Polycletus to Le Corbusier

The basic idea of the canon was already instilled in Ancient Egypt. It was used as a tool to draw “correctly” according to the dispositions of the times.

The canon was particularly important; so much so that, as we have said, these conventions were established as a rule, where a minor deviation was considered a fault.

The importance of this canon was clearly observed in the fact that the rules were applied especially to the representation of gods and royalty; the lower the rank of the person represented, the greater the freedom allowed with the image.

This sole characteristic reflects the importance given to the rigorousness with which the canon was followed.

It is worth mentioning that although the canon assists in the faithful representation of reality, this is not its main objective. More than a search for purist realism, it seems we are in the presence of a specific aesthetic profile with which a culture, religion and cultural expression is identified.

In this case, the canon works very similar to how we understand the use of proportion; both are instruments used to reach a “more beautiful” representation.

This is a clear difference from the previous cultures, such as the Hellenic, where the accurate representation of reality is the canon’s purpose.

As a matter of fact, it was during the classic age where the explicit canons were developed. As we have said, the problem arose as a solution of Greek sculpture to the inexperience of artists. Until the canon was defined as a tool, Greek sculpture was not as perfect as we are accustomed to.

In mid-century V B.C., during the times of Pericles, Greek classicism takes shape thanks to the work of sculptors such as: Myron, Polycletus and Phidias.

Polycletus (450-420, B.C.) conducted a careful and detailed study of the proportions of the human body, a canon of the ideal male beauty based on strict mathematical proportions.

Polycletus’s renown “Canon” is the materialization of the theoretic principles formulated by this sculptor in his book, unfortunately lost, which has the same title: **Canon**.

It is worth mentioning that his focus had dozens of important followers during the following century.

Polycletus defined the head as the basic unit of his canon and from this measurement, resized the whole body. In this manner, the proportions of sculpture are defined, some related to each other, maintaining the human body's harmony.

As we mentioned in the first part, it is important to highlight that the canon's objective is to improve sculpture and serve as a tool to "copy" the model in a more faithful manner, in this case: the human body.

In the case of Greece, the canon's intentions are not to possess beauty in itself, as previously intended. Beauty is in the model and the canon makes it possible to reproduce this beauty in a more exact way than ever before. In other words, it is more a problem of technique than artistic creation.

For Greek sculptors, nature is beauty; therefore, a faithful copy of it results in something that is also beautiful. In other words, beauty is already in the model and the problem is reduced to making a reliable copy of it.

The majority of the canons shared a series of characteristics used from classic times to the present; among them we have:

- They choose a certain unit: (in the case of Polycletus: the head).
- This unit is defined as a sample and is repeated according to a factor or numerical sequence.
- Other dimensions are therefore produced which can be used in the parts of the work that require it.

Actually, there isn't additional information from what we have seen already on the conscious use of proportion or canons in the paintings, architecture or lesser art of Ancient Greece.

Vitruvius who, for the first time, explicitly implies using the beauty of the human proportions in buildings forward takes the following step. Aristotle had insinuated that proportions in the "natural order" possessed intrinsic beauty but Vitruvius expresses for the first time a clear intention to design art, design and architectural objects **according to natural orders.**

Vitruvius is, therefore, the first person who states that if natural things (in this specific case, the human body) are beautiful, then, somehow, this beauty is transported to art and design by the proportional system.

Here we make a conscious or unconscious decision to accept as fact that proportions are so important for beauty that if we use the same proportions possessed by a beautiful object (say, the human body), these will be so influential that it will make the object, upon which they are conferred, beautiful. In other words, it is like accepting that beauty resides in proportions and not in the color, form, size or any other of the object's characteristics. There is no historical evidence that gives us any idea as to the reason for such affirmation. We don't know if there was any evidence in this respect; what is true is that from Vitruvius onward, artists and architects tend to think that proportions are a fundamental piece in the intricate puzzle of art and design.

Vitruvius also describes all types of objects with their proportions: from war machines to temples and in this manner, leads his concept for the first time to the daily material culture from where it sets out up to modern industrial design.

Vitruvius' idea "fit like a glove" for renaissance masters whom, with their humanism, found it completely natural that the proportions of the human body embellished art and architectural works.

From this moment, it is accepted that canons or proportional systems are instruments used to embellish works and not instruments that guarantee a faithful copy of the model, as they are born.

As a matter of fact, the first proportional vocabulary appears with a selection of "well proportioned" forms, like a palette for the use of designers; an example of this type of vocabulary is the advice of Alberti and Palladio.

Beginning in the Renaissance, one can find a great number of books, articles and comments, *everywhere,* which promote the use of proportional systems as design tools. Many of these systems make reference to the thoughts of Vitruvius or Leonardo (as in the famous *Homo quadratus*), without questioning the reasons as to why these methodological instruments offered results.

The last, of these influential theoretical ones is, of course, Le Corbusier who makes the use of proportional systems in architecture obligatory toward the second half of the XX century.

Once would think that at this point the problem is very clear and that nobody would question that some proportions are more beautiful than others; therefore, these proportions must be used if one wishes to make "beautiful things". However, as we have seen, this was not the train of thought at least not for the Egyptian and Greek cultures, which founded the canons.

This pre-established idea by Vitruvius gained followers as time went by, many of them people with high reputation such as Dürer, Leonard da Vinci or Palladio. During the process, pseudo-evidence was gathered as to why proportions embellished works of art; for example, Palladio reanalyzed the classic, Greek and Roman buildings, finding the sought after proportions. Jointly with this process, there was also growing evidence as to how nature used these proportional systems and how, from the platonic solids to the Fibonacci series to Cook's[125] elaborate analysis, they gave faith to the fact that natural beauty was found in the proportions used.

As we have seen, proportions and natural patterns have their own causes and the uses of these proportions in the material culture do not seem to share these causes. Even when some proportions repeat themselves in nature, for one reason or another, this is not sufficient proof to declare that a design's success is significantly influenced by these proportions.

In conclusion, as a result of our historical analysis, we have found a logical fallacy that we have accepted as true: "natural proportions transport beauty to the material culture", thinking that by using these natural proportions, we can guarantee that natural beauty will influence the work in a positive manner.

This conclusion does not have a sound foundation; as a matter of fact, no influential theoretic asked the basic question: Is it true that natural proportions embellish artificial work?

The subsequent acceptance of this premise leads us to a false logic, which we will analyze in detail below.

[125] T. Cook, (1979**). The curve of life.** Dove Publications. New York.

3.3 The Scientific Method

In the practice of science, there are certain axioms of formal logic, which must be followed in detail to ensure that we are acting with truth.

In order to analyze the validity of our question (previously raised), we will try to use the classic instruments of logic discipline to prove the quality of the conclusions we have found, conclusions accepted as fact everyday in art and design schools.

The first of the basic axioms in science is reproduction; this means that before assuming a deduction is true, it is necessary that it can be reproduced by whoever wants to and inevitably obtain the foreseen result.

The second pillar is falsehood; this implies the possibility of designing tests by contradiction, which try to deny the outlined knowledge. If a sole example is found that bends the hypothesis, the implicit knowledge cannot be further accepted as true.

This experimental hypothetical deductive method for falsehood is known as *modus tollendo tollens* or test by contradiction (we will study this in more detail in section 3.4)

At this moment, let's delve into the first axiom in order to analyze our premise in the light of this tool.

Obviously, all science is based on the belief that natural phenomena can be explained based on a model. In our case, it means that we accept beauty in the material culture is possible through some kind of design strategy.

The basic model to analyze a stipulated proposal was well defined by Carl G. Hempel who summarizes the famous aphorisms in his book, *Aspect of Scientific Explanation*[126].

We will attempt to analyze our problem following these methodological steps in order to validate its logic using a general frame of reference (for example, the one used by Francisco Osorio G., social anthropologist. University of Chile)[127]:

Aphorism 1. To explain is to respond to the question, why? In other words, the first case is to ask oneself, why do proportions in nature transfer beauty over to artificial works?

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[126] C. Hempel, Carl. 1965 (1988). **La Explicación Científica**. Editorial Paidós. Barcelona.
[127] F. Osorio, (1988**). La Ciencia y el Método Científico**. Epistemología de las Ciencias Sociales. Universidad de Chile. <http://rehue.csociales.uchile.cl/antropologia/ciencia.htm>

This question outlines the issue in a different manner, by simply asking, why? the idea that proportions will carry beauty seems rather forced.

Aphorism 2. The explanation consists of two essential parts: one *explanandum* and one *explanans*.

Aphorism 3. The *explanandum* is a description of the phenomenon that will be dealt with. Let's see our *explanandum:*

There are certain dimensional characteristics in design, which are more relevant than others; for example, the total length in relation to the total height or the position of some internal elements like rhythm changes in horizontal or vertical dimensions such as a triangular porch in a facade. These proportions are, to a certain point, susceptible to being freely designed by the architect or designer (within a logical range, of course). These elements can be defined according to certain proportional systems, which we already studied in the previous section of this study.

Aphorism 4. The *explanans,* on the other hand, consists of two parts:

A. The statements that express previous conditions (C_1, C_2, ... C_k);

In other words, the precise description of the conditions, which must be present before the appearance of the phenomenon that will be explained.

In our hypothesis, we defined natural proportions, already mentioned in the previous section, as the conditions, which must be present in order to achieve our objective: a beautiful object. Because there are several possibilities, we will focus this short analysis on the golden proportion, because it is the most common in nature.

B. Statements that represent general laws with which we process previous conditions (L_1, L_2,... L_r).

In this case, we note that nature is beautiful in general and that it also possesses these proportions *everywhere.*

Aphorism 5. In summary, the phenomenon that will be studied will be explained showing that it was produced from general laws (B) and in the presence of previous conditions (A). Because nature is beautiful and it possesses golden proportions (B), therefore, if the design work possesses these proportions (A), it will also be beautiful.

In this manner, the question from the first step:
Why does the studied phenomenon occur?
Can now be translated into:
Under what previous conditions?
How many general laws occur to produce the phenomenon?

In other words, given the golden proportions in a work of design (given conditions), this work is beautiful because the golden proportions come from nature, which is also beautiful (general laws to create the phenomenon).

Aphorism 6. This explanation, of course, can come from other more general laws, which include the ones cited in this particular phenomenon.

Throughout history, we have many examples of these types of pseudo-laws; for example, the title "The Divine Proportion", gives the golden proportion a celestial status and if "God" uses it, then it most certainly must be beautiful. This is the kind of general law used as an argument to defend this type of proportion.

Up to know, this outline of the problem has been successful and as can be observed, it almost seems logical and possible to think that the deduction in question is true. Let us now analyze the necessary conditions that must exist to make a certain hypothesis true.

Aphorism 7. Necessary conditions for the explanation. Once the explanation is clear, it is necessary to show if it fulfills the minimum conditions conferred upon it by the law; for example:

A. Deduction. The *explanandum* must be logically deduced from the information contained in the *explanans*. (Logical character). Obviously, this is where the problems begin, is it logical to deduce that since nature is beautiful, that beauty will be transferred to the material culture through its proportions? As we have been commenting, the phenomenon we know as beauty could have been influenced by multiple factors such as form, color, contrast, etc. To subsequently think that proportions are especially important in this sense, more than in form, color, texture, etc., is a rather rash decision and groundless.

B. Laws. The *explanans* must contain general laws (previously proven) that adapt to the *explanandum*'s logical deductions. Our case would be more or less as follows: nature is beautiful, at least Aristotle thought so,[128] and with examples such as the human body (idealized by the Greeks), a sunset, a flower or a Rembrandt landscape, it seems that nobody questions this premise.

As a second step, we have nature using specific proportions and not just any proportion. The countless studies conducted by theoretical ones such as Adolf Zeising (1810-76)[129] and Sir Theodore Andrea Cook (1867-1928)[130] would have demonstrated the appearance (practically omnipresent) of the golden proportion in nature.

C. Empirical consequence. The *explanans* must be proven by experiment or observation (logical-empirical character). In this point we have another question: Is it possible to affirm that when natural proportions are used, the result is always beautiful? From this perspective, it is not as obvious as we accepted from the beginning. The logical analysis tools lead us to question the problem from a different point of view, demanding a not so obvious condition. It is not the same to affirm (1) that natural things are beautiful and possess golden proportions, or further, affirm that beautiful things possess a golden proportion than affirm (2) that the objects that possess a golden proportion are beautiful. In the first case (1), one can appeal to previously mentioned empirical evidence (such as Adolf Zeising (1810-76) and Sir Theodore Andrea Cook (1867-1928)), but in the second case (2), it is not possible to make an exhaustive inventory as empirical evidence. An empirical test from an exhaustive inventory supposes that we can analyze all objects that possess the golden proportion and verify if they are beautiful. As is expected, this is impossible so much so that the objects that would be designed in the future should also be analyzed.

Obviously, this isn't necessary either since finding a counterexample to the law would be enough to prove it is nonexistent. In other words, it is not possible to affirm that all objects that possess a golden section are beautiful without having analyzed all of them (an impossible task), but by finding just one example of an object with such characteristics that it is not beautiful, it is possible to affirm that the law is not fulfilled and that nothing can be affirmed about the next object to be analyzed since it could be another example belonging to the group of objects which do not fulfill the outlined law.

[128] Aristotle, (1960). **On the Parts of Animals**, quoted in F.M. Conford. Before and after Socrates. Cambridge University Press. Pages 93-94

[129] A. Zeising, (1854) **Neue Lehre von den Proportionen des menschlichen Körpers**, Leipzig, S. V.

[130] T. Cook, (1979). **The curve of life. Dove Publications**. New York. Page VII

The law, then, completely loses its use and is not applicable.

This is a typical problem of Aristotelian syllogism, which we will deeply develop in our next section.

D. Empirical requirement. The *explanans* statements must be confirmed by all relevant parameters before being considered true (empirical character). This is a good example of what could be happening with the problem of proportion as a tool to generate beauty. During the first stage of the process, the hypothesis was considered as valid and it was verified at that time with available experiences. However, a more thorough analysis of the phenomenon results in specific experiences where it is not possible to generate a relation between proportion and beauty. Therefore, the explanation was not, and has never been, correct.

Aphorism 8. Explanation and prediction. During this part of the method a formal declaration is made to the law that will be stated. In this case, the same previous formal analysis (including the four requirements) is applied to the **explanation** and the **prediction.** We will try to state these two definitions.

We have stated that if we use certain proportions found in nature (which is beautiful by axiom), the material work will also be beautiful (explanation).

Seen from the other extreme, this means that if a material work has the aforementioned characteristics (nature's proportions), then it is beautiful (prediction).

As mentioned, this prediction is not absolutely clear or proven; as a matter of fact, there is no empirical evidence that shows that if a work possesses golden proportions, it is at least a little more beautiful than if it didn't. This research is needed in the countless manifestos on proportion.

As we have pointed out in the first part of our study, Gustav Fechner (1834-87) was the only person who conducted work in this sense.

"Curios about the Golden Section… Fechner, late in the nineteenth century, researched the human response to the special aesthetic qualities of the golden section rectangle. Fechner's curiosity was due to the documental evidence of a cross-cultural archetypal aesthetic preference for the golden section proportion.

Fechner limited his experiment to the man-made world… He found that the average rectangle ratio was close to a ratio known as the golden section, 1: 1.1618… and that the majority of people prefer a rectangle whose proportions are close to the golden section."[131] (Figure 1.24)

As a result of Fechner's experiment, it is discovered that people prefer the golden section even in artificial forms; however, the experiment only deals with rectangles and not with objects of a different complexity.

These types of experiments have been conducted throughout time, comparing different proportioned rectangles or placing points in square surfaces[132], with the purpose of determining among those surveyed which are their favorite proportions or positions.

These kinds of experiences, despite being commendable, are not sufficient to affirm that golden proportions embellish objects; on the one hand, they don't always give the same results and on the other hand, many have argued that it is not the same to compare rectangles than it is to compare works of art or objects of design.

We developed an empirical research in Costa Rica's Technological Institute in order to try and obtain some information with respect to the influence of the golden section in more complex objects.

The objective of our study was to find at least one example of an object that had the golden proportions and was less beautiful than a similar one, which did not possess these characteristics; in other words, we sought the test by contradiction to Hempel's aphorism 7.

In order to determine if one object is more beautiful than another, we used a survey. Because we were researching daily preferences and not any other type of beauty (artistic quality, for example), we considered that the best way to gather an opinion was to consult a group of people of different ages, gender and social condition. The question had to be as easy as possible, where those surveyed would only have to respond to one question: which of the two objects presented was more agreeable?

Objects from industrial production and daily use were chosen; for example, photographic cameras, calculators, pens, cars, among others. The objects had to have a general sign or obvious part that could be identified with the golden section.

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[131] K. Elam, (2001). **Geometry of design, studies in proportion and composition**. Princeton Architectural Press. New York. Pages 6-7

[132] P.D. Stebbing, (2004**). A Universal Grammar for Visual Composition?** In Leonardo, Vol.37, No. 1, MIT Press. Pages 63-70

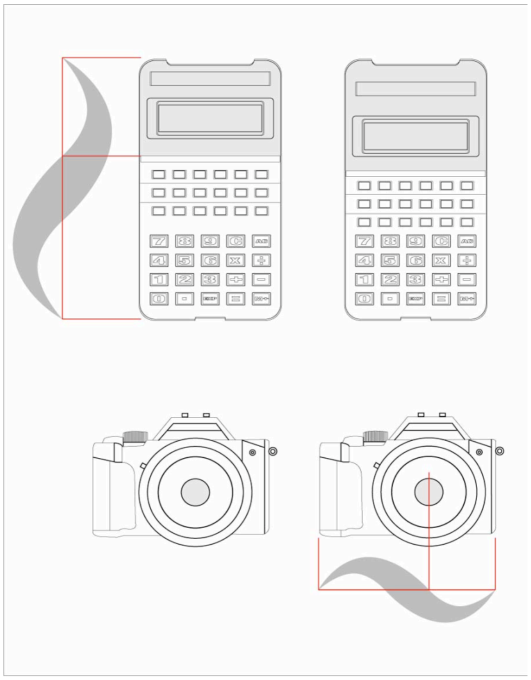

Figure 3.1 Examples of some of the objects used in the empirical research. Tec Costa Rica (2004). Students: Carlos Retana and Ely Marín, Silvia Venegas, César Chávez, Oscar Brenes. Professor: Franklin Hernandez-Castro.

Scientific calculators, for example, have an upper black area and the bottom is metallic. This difference was used to make two variations of the same object: one, where both areas were proportioned with a golden relation and another where there was no clear proportion among them.

The objects were presented in *outline* images to keep those surveyed from becoming distracted. In the case of cameras, the position of the lens was the part selected to transform, showing a version with golden proportion and another without. Both areas were slightly darkened to emphasize the area of interest even more (Figure 3.1).

Several of these images were consulted in pairs. Each consultation had two versions of the same object, one proportioned with golden sections and another avoiding these proportions. Those surveyed only had to choose which objects from each pair they found more agreeable.

In general, the results were predictable. Naturally, not all options with a golden section were selected as the most beautiful and neither were those, which did not possess this proportion. Actually, the results do not go beyond what is considered as statistical tendency; in other words, the percentage of objects considered more beautiful was 54% in favor of those, which possess a golden section, as, expected from a random selection.

Of those surveyed, 46% chose those objects which were prepared **not** to have the golden proportion. Figure 3.1 shows two of these examples where the object, which did not have said proportion, was chosen. The double golden curves have been added (not found in the original survey) to show where the proportions were placed.

Despite the difference in age, gender and social position, the results appear to be very similar. As was expected, it is not difficult to find a counterexample that disqualifies the hypothesis by contradiction.

There are studies that insinuate the existence of other factors that could, on some occasions, be more important than proportions; for example, symmetry.

During another research study conducted during my course “Perception of Beauty” in Costa Rica’s TEC in 2004, a group of children were surveyed to place an object in a rectangular space.

The students came up with the metaphor of a fish bowl to represent the rectangle. The advantage of the fish bowl is that it symbolizes a space with almost no gravity where all sides are almost equal.

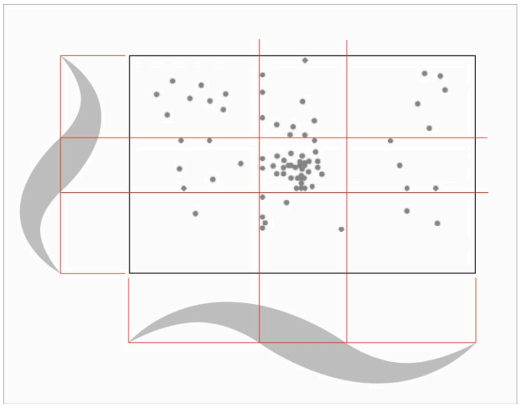

Figure 3.2 Data obtained after surveying children between the ages of 5 and 9. (The curves and lines were added later to help analyze the data). TEC Costa Rica (2004). Students: Evelyn Garita, Alina Leiva, Yorleny Mata. Professor: Franklin Hernandez-Castro.

The children were given a fish figure and were asked to place it where they wanted.

Because the children were surveyed individually, they were unable to influence the others; the summary of the preferred positions by the children is shown in figure 3.2. The lines and curves were added later to facilitate the data analysis.

As observed, the preferred location was the rectangle's center, favoring a symmetrical position above the lines represented by the golden proportions. It is worthy to mention that despite the aforementioned, the left vertical golden section also enjoyed some success.

The preference for symmetry is based on physiological reasons in human evolution which we will delve into later on; however, for now it suffices to say that the golden section is not always preferred by people either in objects to be selected (passive preferences, as in the case of the calculator) or in positions to be defined (active preferences, as in the case of children). In this manner, we can affirm that the more used and recommended proportions in all of western history are not much more successful than any other proportion.

It appears to be that although this golden zone is slightly favored, there are certain factors like symmetry (as in the case of the photographic camera and the fish bowl), which could have more influence upon selection.

Going back to our logical analysis, we can claim without a doubt, that we can find examples that contradict our hypothesis; and in these cases, where all the necessary conditions are not found to declare a hypothesis as law, the deductive logical method also classifies the result.

Aphorism 9. Pre-scientific explanation and incomplete explanation.

A. **The pre-scientific explanation** lacks prediction since the *explanans* does not provide explicit laws through which prediction can be realized. This is not our case because it is clear that what we supposedly have to do to achieve beauty in the material culture is use natural proportions.

B. **The incomplete explanation** can be the case where the previous conditions are fulfilled only during a percentage of the evaluations. This case does correspond to our analysis because there are cases in the material culture, without a doubt, that boast natural proportions and are beautiful at the same time.

To conclude, we would like to finish our logical analysis for the cases where the necessary conditions are found to declare a hypothesis as law. In these cases, the deductive logical method also qualifies the process.

Aphorism 10. Causal explanation
If “E” describes a fact, then one can declare that the previous circumstances described in C_1, C_2 ... C_n jointly "cause" that fact. In this manner, there are certain empirical regularities expressed by L_1, L_2... L_n, which imply that every time the conditions indicated by C_1, C_2 ... C_n occur, it will result in the fact described in “E”.

This means that:

A. A complete causal explanation becomes an adequate prediction.

B. The statements L_1, L_2... L_n, become causal laws when expressing general and ordinary connections.

C. The causal explanation is a type of deductive reasoning.

As expected, this is not the case of our hypothesis. As we have said, we cannot affirm that:

Every time that C_1, C_2 ... C_n conditions occur, a fact such as the one described in “E” will occur; in our case, as long as the golden proportions are available, the object that possesses them will be beautiful.

In the best of cases, our hypothesis is an incomplete explanation and not a proven law.

We have analyzed the facts that allow us to characterize a scientific explanation (according to Hempel’s[133] work), understood as reasoning for causal laws. In other words, the causal explanation is a type of deductive reasoning. Given this analysis, it has not been possible to declare our hypothesis as true indicating that natural proportions embellish material works.

It is important to note that this “incomplete explanation” was considered as a proven law for centuries and even today, it is still considered as such, in the majority of schools of art and design.

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[133] C. Hempel, Carl. 1965 (1988). **La Explicación Científica**. Editorial Paidós. Barcelona.

3.4 Propositional logic

"Although we may not be able to rationally justify our theories, and not be able to prove they are provable, we can criticize them in a rational and objective manner, seeking and eliminating mistakes in the search for truth, separating the better theories from the worst"

Karl Raimund Popper

There is another focus for the logical analysis of the problem we are dealing with: propositional logic.

This focus will delve into the deductive reasoning with respect to Aristotelian syllogisms either in its classical form or in the so-called symbolic logic, by inference rules.

In other words, we will try to approach our problem with syllogistics forms of reasoning, described first hand in Aristotle's *Organon*[134] (384 - 322 B.C.)

In the *Organon*, Aristotle develops logic and an epistemology that makes it possible to perfect and reach scientific knowledge independently of the proposition's content, creating a method that is also demonstrative and probatory.

For this reason, its logic guarantees a solid access to knowledge. With these ideas, Aristotle positions himself, as the creator of the "epistemological empirism", believing there is a reality that can be reached through empirical knowledge.

This focus is useful because, setting aside the content of the problem itself, it allows us to concentrate on the structure of thought behind it. Let's delve deeper into this topic:

Syllogism is a procedure in which, after establishing certain facts, other facts (different from the first) can be concluded only after the first. This method makes it possible to classify general deductive inferences, as our affirmation about proportions and beauty.

Syllogism is a special type of inference, where a process of deduction is established creating a cause-effect relation from the statements. This is exactly the problem we are dealing with because what we intend to analyze is an assumption of the cause-effect relation between the natural proportions and beauty of the artificial objects.

The inference rules are previously proven forms of reasoning which steer the use of the logical connectives and the step from the premise to the conclusion; in other words, they are possible logical and proven links between axioms or basic premises and the possible conclusions deduced from them.

[134] A.M. Olvera-González. **Comparaciones entre algunos modos del silogismo y las reglas de inferencia lógica.** Universidad Autónoma de Querétaro. México. <www.filosoficas.unam.mx/~Tdl/00-2/000831_olvera.htm>

The rules of inference are a fast and easy method to analyze a deduction. In the deductive processes, intuition is not enough; a more impartial verification of the relation between the propositions is needed.

There are more than twelve rules to work with this type of logic; however, we will only study those directly involved with our problem.

Let's begin by reviewing some formalism in traditional logic (syllogistic theory) as well as symbolic logic (or rules of inference), for example, if we say:

All fish live under water (A)

That is the same as saying:

If it is a fish, then it lives under water (P->Q)

If we wish to analyze the aforementioned affirmation with respect to inference rules, we can begin by representing the first proposition with the letter "A". This is the statement considered universal or affirmative; it is assumed that it has been proven and accepted as fact (of course, this same statement can also originate from another logical inference deduction).

In our case, one can reaffirm that "all natural things have certain proportions in common"; let us say that there is sufficient empirical evidence for such affirmation (like the works of Andrea Cook[135] or György Doczi[136], etc.).

Another axiom accepted as true in our case is "natural things are beautiful" based on the experience of the admiration of nature as the Greek canons and the Aristotelian confirmation:

"… we should approach the study of every form of life without disgust, knowing that in every one there is something of Nature and beauty. For it is in the works of Nature above all that design, in contras with random chance, is manifest; and the perfect form which anything born or made is designed to realize, holds the rank of beauty."[137]

The second phrase can be symbolized in a type P implied Q (P -> Q).

Note that one can work syllogism based on intuitive evidence (gathered information) and logical coherence

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[135] T. Cook, (1979**). The Curve of life.** Dove Publications. New York. Page VII

[136] D. György, (1981). **The Power of Limits**. Proportional harmonies in Nature, Art and Architecture. 5. Ed. Shambhala Publications Inc. Boston, U.S.A. Page 151

[137] Aristotle, (1960). On the Parts of Animals, quoted in F.M. Conford. Before and after Socrates. Cambridge University Press. Pages 93-94

(the way to make deductions) according to syllogistic figures and manners. By contrast, symbolic logic is proven applying the rules of inference.

We can, therefore, summarize the deduction we are dealing with in the following manner:

- Golden proportions (P) are in nature (R)
- Nature (R) is beautiful (Q)

This may seem rather rigorous because one proposition uses a relation of **location** and the other a **definition**; but this is precisely the problem we must pinpoint which has been understood for centuries.

It has been assumed that if P implies R and R implies Q, then P implies Q.

This rule is known as the rule of inference denominated hypothetical syllogism and is symbolized by:

P -> R
R -> Q
___
P -> Q [138]

In other words: if golden proportions are found in nature and nature is beautiful; then golden proportions are beautiful.

That's like saying that if 10 is greater than 6 and 6 is greater than 3; then, it is clear that 10 is greater than 3.

If 10 > 6 and 6 > 3, then 10 > 3

Here, of course, we have a serious problem. Generally, the rule of inference (or in mathematics, transitivity) is used when the two relations implied in the propositions are the same but this is not our case. Returning to our statement of inference:

"if golden proportions are in nature and nature is beautiful, then golden proportions are beautiful"

Taking into account that both relations are different, it would be like saying: if 10 is greater than 6 and 6 is twice 3, then 10 is the double of 3.

These terms denote the clear mistake of inference that has been made.

[138] In logical implications, it is generally used in ascending order. For a more normal reading, I have changed it to a descending order.

Even when we accept propositions as universally valid,

- Golden propositions (P) are in nature (R) (from the works of Andrea Cook[139] or György Doczi [140]).

- Nature (R) is beautiful (Q) (based on Greek canons and Aristotelian affirmation)

it is not possible to deduce an inference between two different relations.

However, it is exactly this conclusion, which has been accepted as true since Vitruvius made it so in the first century B.C., and it is also the conclusion reached and reaffirmed by Pacioli, Brunelesci, Alberti, Palladio, da Vinci, Caesarian, Dürer and Corbusier, among others.

But, there is more. Let's forget for now that it is not possible to infer from different relations and accept that if golden proportions are in nature and it is beautiful, then proportions will "transport" this beauty (by the transitivity of the relation) to the objects.

Even if we accepted this conclusion, we would have an additional problem. Let's analyze some hypothetical syllogisms.

The hypothetical syllogisms introduced by Crisip, in the II century B.C. recognize several kinds of implications; among them:

- If P implies Q, and P is true, then Q is true. (This syllogism was known in the Middle Ages as *modus ponens*)

- If P implies Q, and Q is false, then P is false. (This syllogism is known as *modus tollens*)

- If P implies Q, and Q is true, then P is also true.

This third implication is known in philosophical circles as the fallacy of affirming the consequence. It is an attempt to confirm that the information gathered in favor of a hypothesis can demonstrate that the hypothesis is valid.

This hypothetical syllogism is very significant in the philosophy of science and was very important in Sir Karl Popper's[141] falsacionism.

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[139] T. Cook, (1979**). The curve of life.** Dove Publications. New York. Page VII
[140] D. György, (1981). **The Power of Limits**. Proportional harmonies in Nature, Art and Architecture. 5. Ed. Shambhala Publications Inc. Boston, U.S.A., 1981. Page 151
[141] K.R. Popper**,** (1994). **The responsibility of living. Essays on politics, history and knowledge**. Ediciones Piados Ibérica, S.A. Buenos Aires.

During an inductive reasoning, as the one expressed in the syllogism we are dealing with, one begins with a premise and reaches a general conclusion and for this reason the conclusion is likely but not certain.

For example, if we observe three beautiful objects that possess golden proportions (singular statements), we can conclude that all objects that possess a golden section are beautiful (general statement), but this conclusion is not certain because we have been unable to observe all of the objects that possess a golden section.

If we observe one thousand beautiful objects that possess a golden section instead of three, the conclusion may be more certain, but its likelihood of success has not increased. This is known as the problem of induction; in other words, if it is justifiable or not to conclude a general rule from singular observations.

Falsacionism does not consider this inductive step as justifiable. If we follow our example and only find one object that possesses a golden section and is not beautiful (as in the example of empirical research in the former statement), we would have:

Our hypothesis:
"All objects that possess a golden proportion are beautiful",

And as a second premise:
"This object that possesses the golden section is not beautiful" (resulting from observation).

We can safely conclude, then, that it is false that all objects, which possess a golden proportion, are beautiful, with which we refute our theory. Popper calls this conception of the scientific method the theory of the deductive method of contrasting, also known as test by contradiction.

Here is where we limit what should be considered science and what should not. This is the criterion of refutability, which only those statements, which can be refuted, should be considered scientific (not that they refuted; in that case, they would no longer be science).

According to this criterion, a scientist's main interest should not consist on defending his or her theory but in trying to attack it, refute it by all means; in other words, a scientist must be on a permanent state of doubt.

But this isn't the only problem of this deductive syllogism; there is yet another one, which is so obvious and simple, given its characteristic, that it is generally not even, discussed. If P implies Q, this does not mean that Q implies P; in other words, the condition is not a biunivoc relationship.

Let's suppose again that we accept that all beautiful objects possess a golden section (note that this is the second conclusion that is obviously false, but we accept it as valid in order to follow the train of thought which has followed this topic historically).

If all beautiful objects possess a golden section and we (as we do everyday in art and design schools and academies) recommend the use of the golden section as a tool to generate beautiful objects, what we are affirming is that all objects that possess a golden section are beautiful, which is not the same.

In this manner, even if all beautiful objects had a golden section, this does not prove that all objects that possess a golden section are beautiful; if P implies Q, then it does not prove that Q implies P.

This is left clear with the following statement: if all fishes live in the water, not everything that lives in the water is a fish.

The problem is that this is exactly what Alberti, Palladio and Le Corbusier declared in their studies. All of them developed vocabulary of forms and proportions, which had to be used as tools in design to make the material work more successful. In other words, they take for granted that these proportions will embellish the design objects; therefore, Q implies P – all objects that possess nature's proportions are beautiful.

As we can see, there is a chain of incoherencies, which have been generated throughout history. The fact that nature's proportions in general and the golden section in particular, embellish objects is an unfortunate conclusion with no logical basis.

In the previous section of our study, we mentioned that the golden section and other known configurations are very recurrent only as a solution in nature. Furthermore, there is definitely empirical evidence that proves that the golden section is present in many objects considered beautiful throughout history, which generated a pan-proportionalism already mentioned in section 1.8. But none of it proves that:

1. That natural objects are beautiful because they possess the golden section.

2. That all beautiful objects possess the golden section.

3. That all objects that possess the golden section are beautiful.

In section 3.3, we stated that our hypothesis is an incomplete explanation and not a proven law; however, it is interesting to highlight that many objects considered beautiful possess the golden section.

In objects with some kind of functionality, from chairs to airplanes, as in the example in Figure 1.1 (cited by György Doczi[142]), one could grant that the physical natural processes could have defined part of these proportions. On the other hand, the human dimensions could have influenced the proportional definition of domestic objects, such as furniture or rooms. Nonetheless, none of this explains the recurrent appearance of the golden section in artistic works without a specific practical function.

The reason for this phenomenon could very well be the cause for this historical confusion we have commented and for this reason we will try to analyze it in the following section.

[142] D. György, (1981). **The Power of Limits**. **Proportional harmonies in Nature, Art and Architecture**. 5. Ed. Shambhala Publications Inc. Boston, U.S.A. Page 151

3.5 The nature of beauty

It is necessary to mention several aspects about the nature of beauty, although it is not the topic of this work, because it is precisely the search of beauty that has motivated the use and recommendation of one or another proportion throughout history.

The purpose of this section is also to clarify from what point of view the subjective classification we call beauty is being analyzed. Because it is such a controversial topic, it is necessary to indicate, with respect to the classification of objects:

What is beauty for the purpose of this study?
And, therefore, what is beautiful? or what is more beautiful?

There are many ways to define beauty and many points of view to consider. Our focus does not intend to be deep but rather concrete and specific.

The first thing we can say is that beauty is a rather vague, general and unspecific concept. Beauty is similar to “good” or “dear” in many aspects. All of these adjectives hold not one but several meanings.

“…it seems that what is beautiful is the same thing as what is good, and in fact in various historical periods there was a close link between the Beautiful and the Good.

But if we judge on the basis of our everyday experience, we tend to define as good not only want we like, but also what we should like to have for ourselves.”[143]

So, beginning to classify our specific concept of beauty, we can say that we wish to work with the phenomenon of beauty as a daily experience not as a philosophical, ethical or psychological phenomenon. We will deal with this topic more from the point of view of the daily shared and normal experience of human beings.

We had already introduced this topic in the first part of our study, making it clear that for our purpose, we would work this phenomenon of the perception of beauty more than the concept of aesthetics; these focuses are, of course, completely different.

The phenomenon of the perception of beauty, unlike the more general concepts of aesthetics or beauty refers, in an isolated or certain manner, to how the human brain perceives this phenomenon called beauty.

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[143] U. Eco, (2004). **History of Beauty**. Rizzoli International Publications, Inc. New York. Page 10

From a collective point of view, we will understand that:

"Beauty, as such, is a convention. Just like the length of a meter is a fanciful measurement defined historically... beauty is a convention defined by the evolutionary process of the interests of our species... beauty, like red or green, doesn't actually exist; it was just a translation of one or several physical conditions that help manipulate the universe to benefit our survival."[144]

The evolutionary process conditions our brain through more general mechanisms. We feel pain to avoid dangerous experiences; for example, contact with fire. Likewise, the things that were identified throughout time as beneficial for the survival of species are awarded with pleasure.

Beauty is one of those basic and instinctive pleasures; the same kind of pleasure experienced by a normal person during a sexual encounter or upon receiving a social award.

This type of natural pleasure, as it's intended here, is a mechanism inherent of every being and its function is to influence conduct in animals, in general, and in humans, particularly, toward a specific direction. For example, after going several hours without eating, satisfying our nutritional needs generates a great pleasure.

For a more chemical focus, one could talk about the generation of neurotransmitters or hormones, such as dopamine.

"Dopamine is desire personified; not just a sexual desire but the search for any pleasure. It is a well-known neurotransmitter due to its capacity of providing us with all kinds of pleasures. Without dopamine, we become emotionally down; we are not capable of feeling joy or pleasure, enthusiasm, excitement or exuberance. Dopamine is the common denominator of almost all, if not all, addictions, from cocaine to alcohol. It could also be the substance that makes us addicted to one another. Because its main function is to search for pleasure, it generally increases our sexual impulse. In the process, it intensifies the quality of our experience and reinforces our desire to repeat it. And more importantly: dopamine gets us moving, literally and figuratively. When we want something, it is the dopamine that gets us in the car – or in bed – to reach it, instead of just thinking about it."[145]

[144] F. Hernández-Castro, (1995). **Estética Artificial**. Editorial MithOz. San José. Costa Rica. Pages 36-37

[145] T. Crenshaw, (1997). **La alquimia del amo y el deseo**. Editorial Grijalbo, Barcelona. Page 35

These substances are chemical transmitters emitted by the brain in the blood flow. Its function is to award the body when the person has acted according to pre-programmed tendencies.

“These days, a biologist defines hormones in the following general terms: any cellular product capable of transporting itself, by any available means, to another cell and alter its function...It is more and more obvious that our control center is not just a physical place, but a process that implies the interrelation between the brain, the hormones, our biochemical constitutions and our environment.[146]”

Therefore, many of the addictions to drugs, sex, foods, among others, are due to the desire of feeling the pleasure we get after the dopamine dissolves in the blood.

This same mechanism is behind the perception of beauty; there is a natural pleasure (with the subsequent discharge of dopamine in the blood[147]), which is unchained when one observes something beautiful. As a matter of fact, this hormone and others such as vasopressin join forces when we define our territory, which many times translate into personal taste.

“Vasopressin could play an important role – not yet defined – in the formation of human social hierarchies, manifesting itself as an **aesthetic consciousness** which concerns itself with the clothes we wear, the fashion trends, jewelry, and perhaps, the toys we buy; in essence, the decoration of our flanks.”[148]

In this manner, the reasons for the phenomenon of the perception of beauty are aligned to any other type of pleasure mechanism in human nature. This whole mechanism is just our body trying to influence conscious behavior (in an unconscious manner) toward a specific action which has been an evolutionary advantage throughout million of years.

“Biologists World argues that at root the quest for beauty is driven by the genes pressing to be passed down and making their current habitat as inviting for visitors as possible.”[149]

From this point of view, the scientific concept surrounding the perception of beauty has changed radically in the last decades.

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[146] T. Crenshaw, (1997). **La alquimia del amo y el deseo**. Editorial Grijalbo, Barcelona. Page 21

[147] T. Crenshaw, (1997). Page 21

[148] T. Crenshaw, (1997). (bolds are from me), Page 140

[149] N. Etcoff, (1999). **Survival of the pretties, the science of beauty**. Doubleday Editions. New York. Page14

"Within the next there decades (since 1960) an explosion of research was to provide compelling evidence for a new view of human beauty. It suggested that the assumption that beauty is an arbitrary cultural convention might be simply not true.

The research comes at the time when scientists have begun to question a new many other assumptions about the relationships between human behavior cultures. As Leda Cosmides, John Tooby and Jerome Barkow point out: "Culture is not causeless and disembodied. It is generated in rich and intricate ways by information-processing mechanisms situated in human minds. These mechanisms are in turn the elaborately sculpted product of the evolutionary process". Clearly, culture cannot just spring forth from nowhere; it must be shaped by, and be responsive to, basic human instincts and innate preference. Until the 1960s, it was believed that languages could be arbitrary and without limit, but now there is a consensus among linguists that there is a universal grammar underlying this diversity. Similarly it was thought that facial expressions of emotion could arbitrarily vary across cultures until the psychologist Paul Ekman showed that many emotions are expressed by the same facial movements across cultures. Ekman made the important distinction between the facial expressions of emotion (smiles, frowns, scowls, and so on), which are universal, and the rules for when to display those emotions, which show cultural variation. Similarly, aspects of judgments of human beauty may be influence by culture and individual history, but the general geometric features of a face that give rise to perception of beauty may be universal…

The ability to perceive beauty and respond to it has been with us for a long as we have been men and women…

We will look at the argument for beauty as a biological adaptation. The argument is a simple one: the beauty is a universal part of human experience, and that it provokes pleasure, rivets attention, and impels actions that help ensure the survival of our genes. Our extreme sensitivity to beauty is hard-wired, that is, governed by circuits in the brain shaped by natural selection. We love to look at smooth skin, thick shiny hair, curved waists, and symmetrical bodies because in the course of the evolution the people who noticed these signals and desired their possessors had more reproductive success. We are their descendants."[150]

Obviously, there are other examples in nature of the use of visual attraction as an instrument of genetic manipulation.

[150] N. Etcoff, (1999). **Survival of the pretties, the science of beauty**. Doubleday Editions. New York. Pages 22-24

It is clear today whether the perception of beauty, so linked to concepts understood as completely and exclusively human, is no more than a more general reflex mechanism, a mechanism that we share with many species.

"The peacock's tail is the classic example of sexual selection through mate choice. It evolved because peahens preferred larger, more colorful tails. Peacocks would survive better with shorter, lighter, drabber tails. But the sexual choices of peahens have made peacocks evolve big, bright plumage that takes energy to grow and time to preen, and makes it harder to escape predators such as tigers. The peacock's tail evolved through mate choice. Its biological function is to attract peahens. The radial arrangement of its yard-long feathers, with their iridescent blue and bronze eyespots and their rattling movement, can be explained scientifically only if one understands that function. The tail makes no sense as an adaptation for survival, but it makes perfect sense as an adaptation for courtship." [151]

The nature of beauty, therefore, is associated to the evolutionary advantages that each phenomenon offers these species; in other words, mating, hierarchy or survival (how to select the right food, for example).

In the case of humanity, due to the super-symbolic capacity of the human mind, this phenomenon is not based on mere appearance (as in the case of animals) but it is transported to the material culture.

"The human mind's most impressive abilities are like the peacock's tail: they are courtship tools, evolved to attract and entertain sexual partners."[152]

In her book, *Homo Aestheticus*, anthropologist Ellen Dissanayake[153] gives us a clear explanation on how art can be a human adaptation, which was developed with evolutionary purposes.

His argument is based on three inarguable points:

1. Art is a completely intercultural activity. Throughout time and space, each culture has created its own cult to the generation of sculptures, clothing, decorations and improvement of the personal image of individuals.

2. Art is a source of pleasure for the artist as well as the observer, and as we have said, the evolutionary

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[151] G. Miller, (2000). **The mating mind. How Sexual Choice shaped the Evolution of Human Nature. Anchor Books.** New York. Pages 10-11
[152] G. Miller, (2000). Page 4
[153] E. Dissanayake, (1992). **Homo aestheticus: Where art comes from and Why**. Free Press. New York.

processes tend to convert evolutionary advantageous behaviors into something pleasurable; they arise as adaptations in time.

3. The artistic production is an investment in time and energy, and this type of investment always generates a greater interest than the investment from an evolutionary point of view.

Without a doubt, the omnipresence and the costs of the artistic activity (in our case, the production of beauty) indicate that under no circumstance this could be a biological accident.

Of course, going from art to the daily material culture must be handled in a symbolic manner because no one would doubt the influence of the artistic movements throughout history in the production of architecture, fashion and design of products in general. The brain's super symbolic capacity, creates an almost omnipresent diffusion of such concepts from the daily culture. That is how, for example, the importance of beauty as an instrument to select mating partners, adequate food or health climatic conditions is easily transferred to other cultural material elements.

"Gradually, throughout evolution, the image generated by external objects has been greatly substituted by more and more indirect representations of object or situations. This is the great advantage of assigning properties to objects, which are observed (or not) through the senses. Therefore, we drink from a glass, not just as a response to the perceived stimulus, but from our previous knowledge of what is a glass and what it may contain. On the other hand, a frog surrounded by dead flies is destined to die because although they are edible, the frog does not perceive them as such because they don't move."[154]

This capacity to infer concepts is probably the main cause of our progress. As Geoffrey Miller[155] declared, it is perfectly possible that all activities that generate a culture such as music or poetry have their origin in a mating instinct, but the mechanism goes beyond the selection rituals of mating partners.

We know, for example, from countless studies (like those conducted by Nancy Etcoff in Harvard [156] and our own here in the *Instituto Tecnológico de Costa Rica*[157]),

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[154] R.L. Gregory, (1998). **Occhio e Cervello. La psicología del vedere**. Raffaello Cortina Editore. Milano. Pages 12-13

[155] G. Miller, (2000). **The mating mind. How Sexual choice shaped the Evolution of Human Nature.** Anchor Books. New York.

[156] N. Etcoff, (1999). **Survival of the pretties, the science of beauty**. Doubleday Editions. New York

[157] F. Hernández-Castro, (1995). **Estética Artificial**. Editorial MithOz. San José, Costa Rica. Pages 67-69

that in the majority of cases, people often prefer more symmetrical faces than those less symmetrical.

"…we find symmetrical faces more attractive. This preference starts early, and may be inbuilt. Psychologist Judith Langlois collected slides of human faces and had them ranked for attractiveness by adults. When she showed the slides to infants three and six months old, she found that they stared longer at the faces the adults had rated most attractive."[158]

The majority of people also prefer more standard faces or prototypical to more particular or unique faces.

For example, in Sir Francis Galton's famous experiment[159] , conducted in the XIX century (revised many times), when several faces were combined, the result became more beautiful.

In the XIX century, Galton tried to identify the prototype characteristics in the faces of the criminals of that time.

With this objective in mind, he took a series of photographs (recently invented technology at that time) of criminals, and combined them in order to find similarities and in the process, tries to identify a "typical criminal".

His surprise was that the more faces he combined, the more beautiful the face became. The experiment was, of course, a failure: however, many existing investigations, such as Nancy Etcoff's in Harvard have recreated the experiment with modern technology but this time, seeking a prototypical beauty. The result was a sound success. In other words, the more prototypical the face, the more the average person finds it agreeable.

The reason this is so is perfectly compatible with what we have said until now. If, as we previously observed, beauty is a natural mechanism to select candidates, behavior or food, which are evolutionarily advantageous, it is very possible that our tendency to select prototypical or symmetrical features as the more beautiful is a kind of analysis of the state of health of the analyzed candidate.

"A possible explanation of the relation between beauty and the identification of the most minimum anomaly in a human face could be the convenience to mate with a healthy partner.

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[158] B. Bates and J. Cleesse, (2001). **The human Face**. Dorling Kindersley Publishing, Inc. New York. Page 147
[159] N. Etcoff, (1999). Page 55

It is possible to imagine then, that the evolutionary process fine tuned the evaluation of the state of health of a mating candidate in a rather unique manner."[160]

"Evolutionary theorists suggest that a high degree of symmetry may be an indication of particularly good genes, and perhaps resistance to the sorts of disease that can cause asymmetrical development. Highly symmetrical animals attracting other symmetrical mates would increase the hardiness of the species.[161]

3.5.1 The associative libraries.

Given our super-symbolic capacity, the phenomenon is not limited just to the selection of mating partners or in the selection of foods, but it's transferred to other elements of our daily and material culture.

These super-symbolic mechanisms are well known in our behavior and they draw us to an event or leads away from it if considered dangerous. In this manner, symbols, ideas, ideograms, legends and superstitions have been common throughout time, with the same symbol surviving time.

"Hitler's SS, Hell's Angels, the shamans, the pirates and even iodine bottlers, have used the symbol of the skull to generate fear. It is something completely and totally coherent. When I find myself in a room full of skulls, it is likely that somebody is around, perhaps a pack of hyenas, perhaps a gloomy and active decapitator whose occupation or hobby is to collect human skulls. These unsettling acquaintances need to be avoided or if possible, eliminated. The chill downs my spine, the racing of my pulse and heart, and the sticky sweat has been created by the evolutionary process to prepare me for the escape. Those who avoid decapitation leave behind a greater legacy; therefore, these sensations of fear constitute a clear advantage from an evolutionary perspective."[162]

It is common practice in our culture to transpose symbols as elements that offer positive or negative appraisals.

This mechanism acts as an associative library found in our brain, in an innate form as well as associations learned by experience. Our system has preprogrammed patterns that serve as a comparative base. When we perceive an object, a face or a circumstance, these are compared to internal patterns and an instinctive

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[160] F. Hernández-Castro, (1995). Page 66
[161] B. Bates and J. Cleesse, (2001). Page 149
[162] C. Sagan, (1981). **El cerebro de Broca. Reflexiones sobre el apasionante mundo de la ciencia**. Grupo Grijalbo-Mondatori. Barcelona. Page 25

response results from this comparison. One of these possible responses is the pleasure of observing something beautiful.

At a basic and innate level, we can recognize a good or bad smell by comparing it to these libraries.

“We consume in vast quantities fat, sugar, protein and salt in the form of burgers, shakes, French fries, and pizzas. Fast food chains are popular precisely because they serve theses elements in concentrated quantities. They revel the food preferences that evolved in a past environment of scarcity.”[163]

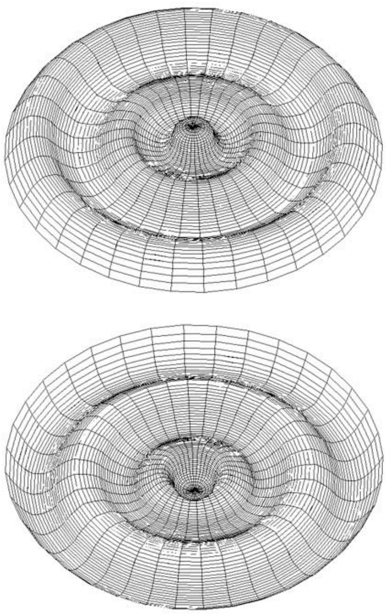

Figure 3.3 The same drawing with a 180° rotation generates a completely different interpretation.

It is important to note that the response to these stimuli not only depends on the perceived object (tasted, observed, etc.) but it also depends on the internal library with which it is associated.

Figure 3.3 shows the same drawing in two positions – one turned 180° with respect to the other (it is very a very interesting experience to turn the book and observe the figure again). The three dimensional interpretation of the figure is completely different; this kind of interpretation is one of those interpretations that results from the internal comparison within the associative libraries and not from the observed object.

In this innate level, the associative libraries have patterns of recognition that adapt as best they can to the visual stimulus, reach conclusions and directly transfer these already interpreted conclusions to the visual cortex.

As in the case of the skull, there are several evolutionary advantages in giving a specific form a different significance.

“ Why are parts so helpful to recognize? There are two main reasons. First, most objects are opaque. You can see the front of an object but no its back; you might not even see its entire front, if another object is in the way. Since you rarely see all of an object at once, you must recognize it from its visible parts…

Second, many objects are not rigid. Your body, for instance, has many movable parts –arms, legs, fingers, and toes. If they move, your body changes configuration. How shall you recognize a body despite such changes? Again parts come to the rescue. If you pick your parts prudently (and you do), then the parts won’t changes as the configuration does. This gives you a stable

[163] D.M. Buss, (2003**). The evolution of desire. Strategies of human Mating.** Basic Books. New York. Page14

description of objects and an efficient index into your memory shapes."[164]

Perceptions, then, are classified according to the patterns with which they are associated in a scale of importance, as well as the benefit or prejudice for the species. Beauty is one of these mechanisms; after all, for the sense of taste, beauty is a mechanism that savors good taste and for the sense of smell, it picks up a good aroma. The combination of both invites us to eat a ripe fruit instead of one that is in a state of decomposition.

"According to evolutionary biologists, these kinds of automatic reactions have remained imprinted in our nervous system because for a prolonged and crucial human prehistoric period, they marked the difference between survival and death...In terms of biological design for the basic neurological circuit of emotions, that with which we were born functioned better in the last 50,000 human generations, not in the last 500... and certainly not in the last five."[165]

Following the same relation, it is logical to suppose that visual beauty also follows the aforementioned type of processes.

3.5.2 Physiology of the perception of beauty

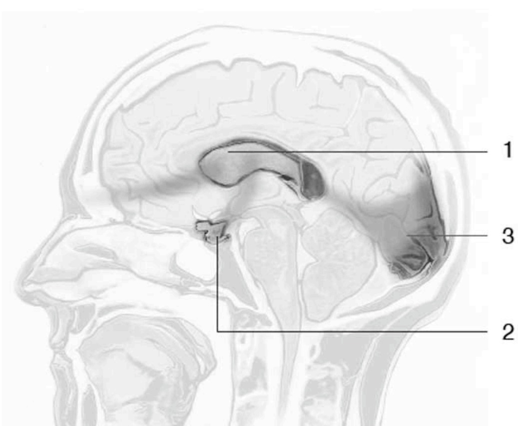

Figure 3.4 Basic circuit of vision on the human brain.

The visual perceptive system is specifically arranged so that the aesthetic prejudices, discussed earlier, are elaborated before they are communicated to the conscious part of the brain. In Figure 3.4, we can observe a transversal cut of the human brain where three areas are pointed out:

1. The hypothalamus
2. The tonsils
3. The visual cortex

It is important to note that the eyes are connected to the hypothalamus; this in turn, is connected to the tonsils and both are connected to the visual cortex. Due to the evolutionary timetable, the eyes send messages to the innermost brain complex in the Reptilian Complex (near the spinal cord) where previous decisions are made. Only after these decisions are made and carried out, does the message go to the visual cortex and it finally becomes conscious.

[164] D.D. Hoffman, (2000). **Visual Intelligence. How We Create What We See**. W.w. Norton & Company. London. Page 82
[165] D. Goleman, (1995). **La Inteligencia Emocional. Por qué es más importante que el cociente intelectual**. Javier Vergara Editor. México. Page 23

"In one of the most revealing discoveries of the last decade, LeDoux[166] 's work showed how the brain's structure gives the tonsils a privileged position as the emotional sentinel, capable of assaulting the brain. His research has shown that the sensorial signals from the eye and ear travel first through the brain to the thalamus and then – through a sole synapses – to the tonsils; a second signal from the thalamus goes to the neocortex, the thinking brain. This branching allows the tonsils to begin responding before the neocortex which elaborates information through different brain circuit levels before fully perceiving the information and finally giving its perfectly adapted response."[167]

This is the reason why sometimes we act instinctively (for example, when we avoid an object that is rapidly coming towards us) before even thinking about it.

"As soon as the message is released, the optical connections present in each one of the two hemispheres go through an intermediate station, the lateral nuclear bodies (numbers 1 and 2 in figure 3.4). It is interesting to observe how these organs receive more fibers downward (to order follow-up actions) from the internal cerebral centers than from the eyes (to perceive information). They constitute an anatomical support in which the internal centers modulate or increase the visual signals, giving the retinal images meaning."[168]

All visual perceptions, therefore, are reviewed and associated as much as possible with some type of judgment, and later, passed on to a phase of conscious thought. In other words, they are evaluated without giving our brain the opportunity to even perceive them and they are classified as good, bad, dangerous, adequate, beautiful or ugly.

"Some emotional reactions and emotional memories can be formed without the least conscious and cognitive participation. The tonsils can store memories and a series of responses we carry out without exactly knowing why because the path from the thalamus to the tonsil completely skips the neocortex. This detour makes the tonsils a warehouse of impressions and emotional memories of which we are never fully conscious. LeDoux proposes that it is the underground role of the tonsil in our memory, for example, which explains an amazing experiment where people preferred strangely formed geometric figures, which had been shown to them for an instant at such high speeds that they were not aware they had even seen them.

[166] J. LeDoux, (1993). **Emotional Memory System in the Brain**, Behavioral Brain Research. Page 58
[167] D. Goleman, (1995). Page 37
[168] R.L. Gregory, (1998). Page 110

Another research showed that in the first fragments of a second during which we perceive something, we not only unconsciously understand what it is, but we also decide whether **we like it or not**. The collective unconscious presents our consciousness not just the identity of what we see, **but an opinion about it.** Our emotions have a mind of their own; a mind that can hold different points of view with plenty of independence from our rational mind frame."[169]

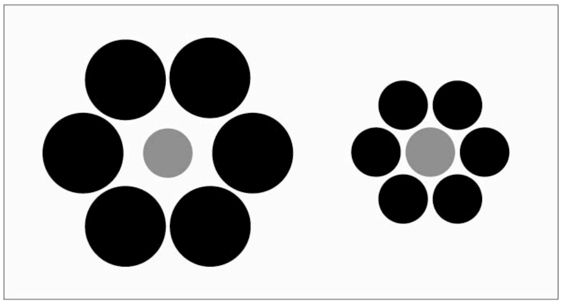

Figure 3.5 Typical optical illusion where the central circles, despite being of the same size, are perceived as being one smaller than the other.

This is likely the mechanism that is behind optical illusions, where the "wrong" conceptions are defined in the brain's internal neural centers and later transmitted to the conscious zone. So, when we see two circles (figure 3.5) which we know are the same size, our previous brain circuits have already determined they are **not**, so that the conscious centers like the visual cortex have no opportunity to change the previously defined judgments.

"Visual intelligence occupies almost half of your brain's cortex. Normally it is intimately connected to your emotional intelligence and your rational intelligence. It constructs the elaborate visual realities in which you live and move and interact. It forwards these constructions to your emotional and rational intelligence, which use them as raw materials in further constructions. The emotional world you inhabit is, like your visual world, a product of your own constructive genius."[170]

This phenomenon combines with another recent discovery: all perceptions converge in previous cerebral centers, located in the former brain, and are distributed from there to the different specialized zones which process sensory stimuli, such as the neocortex. In other words, not only are visual perceptions pre-elaborated in the brain's lateral sectors, but also they are evaluated and associated to other perceptions, to other senses, with which they could be related.

In this manner, when a perception communicates with the brain's conscious part, it is already a perception-goal, which includes not just previous judgments but several associations to other kinds of perceptual stimuli or sensorial channels.

In an experiment conducted in the University of Wisconsin-Madison[171] in USA as well as in the University of Strasbourg in France, an electrode matrix was developed connected to a video camera. These

[169] D. Goleman, (1995). Page 39
[170] D.D. Hoffman ,(2000). Page 202
[171] P.Weiss, (2001). **The Seeing Tongue**. Science News. Week of Sept. 1. Vol. 160, No 9. W, Washington.

electrodes translated into different electrical tension values the shades of grey perceived by the camera.

After discussing several other possibilities, it was decided to adapt the electrical tensions so the tongue of blind people could feel them. The electrode matrix became, therefore, a modest-sized square which could be introduced into the tongue and transmit certain visual information through the different values of electrical tension.

After a certain amount of training, the unthinkable occurred: the people in the experiment began to declare that they had perceived part of the visual information communicated through their tongue. Setting aside the obvious problems of resolution, the visual information was enough to perceive the light from a candle.

In the beginning, the tongue had not been considered as a possible receptor of these impulses.

“Earlier research had used the skin as route for images to reach the nervous system. That people can decode nerve pulses as visual information when they come from sources other than the eyes show how adaptable, or plastic, the brain is, says Wisconsin neuroscientist and physician Paul Bach-y-Rita, one of the device’s inventors.

‘You don’t see with the eyes. You see with the brain’ he contends. An image, once it reaches an eye’s retina, ‘becomes nerve pulses no different from those from the big toe’”[172]

“Not surprisingly, it is now common to refer to the eyes as an actual part of the brain. Both have direct, unimpeded neural connections to the visual cortex. Various components of eyes are actually made up of the same tissue from which brain matter originates. And of the approximated ten billon neurons in our heads, nearly half are dedicated to the processing and evaluation of visual information.”[173]

The idea of exchanging senses from tactile to visual is not new. Since the 1960’s, different devices have been used to carry out research. Obviously, the importance of this project not only lies in the possibility of restructuring the eyesight of the blind but in the revelation that nervous impulses are preprocessed and interpreted according to its nature.

“There’s plenty of evidence, he says, that even those brain regions devoted almost exclusively to a certain

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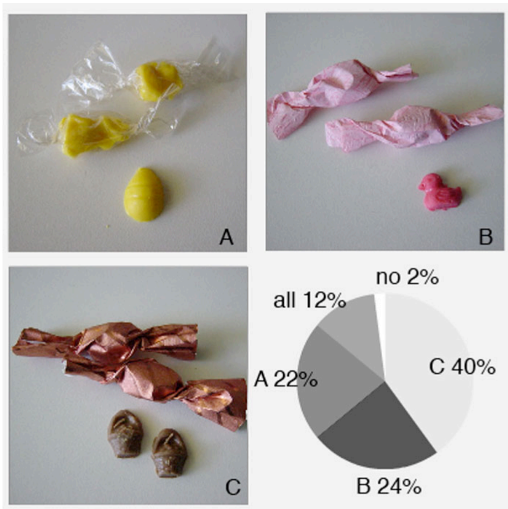
[172] P. Weiss, (2001)

[173] S. Aukstakalnis & D. Blatner, (1992). **Silicon Miracle. The Art and Science of Virtual Reality.** Peachpit Press. Berkeley, CA. Page 50



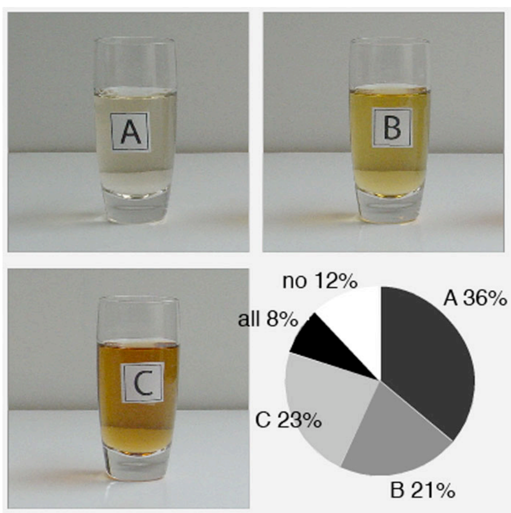


Figure 3.6. Empirical research developed by students Sandra Rojas, Carolina López, Cristina Mora and Hazle Brenes, TEC Costa Rica (2002-2003). Professor: Franklin Hernandez-Castro

sense actually receive a variety of sensory signals. 'We showed many years ago that even in the specialized eye region, auditory and tactile signals also arrive'.

Also, many studies over 40 years indicate that the brain is capable of massively reorganizing itself in response to loss or injury. When it comes to seeing via sense of touch, reorganization may involve switching portions of the visual cortex to the processing of touch sensations, Bach-y-Rita says."[174]

In this manner, and according to what we have been discussing about the tonsil's previous signal, it is possible that many of the sensory perceptions are preprocessed in this organ and are sent jointly to the neocortex in a compact and pre-evaluated form.

With this idea in mind, we designed in the *Instituto Tecnológico de Costa Rica*, an experiment with the objective of researching, in an empirical form, that perceptions comes in packages and are influenced and adjusted with each other before sending a value judgment to the neocortex.

The research consisted of producing a certain amount of chocolates with the same flavor but with different colors. Three different packages were also designed.

The first group was yellow and the chocolates were wrapped in a simple transparent plastic. The second group of chocolates was pink, wrapped in a waxy, opaque and pink paper; and the third group was chocolate-colored and was wrapped in metallic paper, which appeared more sophisticated or fancy.

Furthermore, three types of drinks were prepared which, although they were the same, had different colors.

With this material at hand, we proceeded to conduct a survey in the form of a taste test with one question, which do you like better and why?

As we had predicted, despite the fact that the flavor was the same in all of the products, an overwhelming majority of those surveyed (more than 300 people of all ages and social conditions), gave clear opinions with respect to the flavors. As a matter of fact, in the case of the chocolates, only 14% said they could not choose or that they liked all of them (meaning that they all tasted the same); however, 86% of those surveyed preferred one of the kinds of chocolates and gave their reasons why. They even assigned flavors such as banana for the yellow and strawberry for the pink (Figure 3.6).

[174] P. Weiss, (2001)

In the case of the drinks, 80% of those surveyed chose a favorite and had reasons for their chose. For example, “the darkest was too strong, too much flavor!” and only 20% either did not have an opinion or liked them all.

Despite the fact that the test was empirical, the results tend to insinuate that as we had guessed, the opinions surrounding sensory stimuli are not just the result of the object perceived, but also rather an elaborated function of perception. This function combines pre-conditioning with perceptions and creates previous value judgments, which are led to our conscious thought in a previously defined manner.

This phenomenon reminds us of the phenomenon of optical illusions, except that this time we would have to speak of **perceptual illusions.**

The results, then, seem to indicate that the perceptions from all senses combine before making a value judgment and lead them to the conscious part of the brain. And just like we can’t help and think that the circles in Figure 3.5 are a different size, we cannot isolate a flavor from its visual image or a smell from its physical origin. Perhaps that is the reason a meal will remind us of a song or a smell will bring up an image.

Perception is a complex phenomenon that originates in a stimulus (visual, for example) which will be evaluated, associated with internal patterns, combined with the simultaneous perceptions from other channels or senses and finally, perceived, properly speaking, in our conscious brain.

Although our anatomy is practically identical and this makes it possible for a great percentage of human beings to react the same to equal stimuli, it is very unlikely that this reaction only depends on an external stimulus. Rather, it seems our reaction is a “very human” interpretation of a universe that is very different from what we perceive.

3.6 Paradigm of tranquility

All phenomena described in this document can be analyzed based on the minimum energy axiom; in the second part of our study, we cited this axiom to explain some forms and proportions that are present in nature.

“With regard to how patterns and shapes come into being, we can readily accept the fundamental idea of the theory of evolution, that things evolve to their fittest form; we can accept the principle that things tend towards a configuration with least energy, that is to say, with the tightest fit, the lowest altitude, or the least motion; we can even accept the theory that exiting forms of nature are exactly those that are most likely to exist –taking into account all possible possibilities.”[175]

Indeed, this phenomenon is even more generalized to the extent that, in natural science, it is known as the principle of least energy, which establishes that all physical systems tend, in a natural manner, to find their state of minimum energy; that is, their more stable state.

When we talk about a ball that rolls inside of a half-concave sphere, we are talking about something evident: the ball will oscillate inside the sphere, losing strength due to its swinging movement — which reduces after each turn — until it reaches the state of minimum potential energy in the center of the half sphere.

In relation to gases, for example, the least energy state is related to the minimum temperature feasible and to the lowest pressures, so that their molecules are markedly separated among them as to avoid the presence of interactions.

We can say the same about the solidification of crystals; as we mentioned in the second part of this study, they accommodate to find the minimum energy state among their molecules, forming the geometrical forms all we know.

This behavior observed in systems not only applies to physics or chemistry, but it’s a universal law. Concerning evolutive processes, for instance, the efficiency of species to use energy may be the difference among extinction or survival; therefore, many species adapt to environmental conditions in order to obtain the maximum advantage from the available energy.

Some adaptations —such as a wider nose profile or a thinner body shape— are typical in some species, including human beings, because are required to release heat efficiently and adapt to warm weather.

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[175] P.S. Steven, (1974). Page 37

Perceptual systems are familiar with these phenomena; the use of energy is very expensive in relation to gathering, analyzing and defining the meaning and implications of visual impulses. For this reason, evolution has provided these interpretation means (such as tonsils) to respond efficiently and rapidly to situations that may eventually become a matter of life and death.

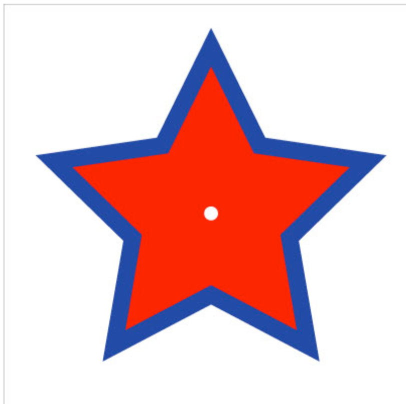

Figure 3.7 Classical figure to demonstrate the chromatic complementary effect caused by the observer's visual tiredness

A typical example of exhaustive use of energy in visual systems is the classical exercise of looking steadily at a geometrical shape, of a specific color, during a few seconds. Figure 3.7 provides an example of this exercise: if the observer fixes his or her sight on the white spot placed in the center of the figure for 30 seconds and then turns to look at a white surface, he or she will see what is known as a phantasmagoric image of perception.

This phantasmagoric perception is a star shape where the central area is in green and the contour is in lighter yellow. This phenomenon is due to the fact that color-perceiving cells operate generating electric tension between two polarities; therefore, in the presence of stimuli, cells charge with a specific electricity, let's say positive, and when the stimuli is ended, those cells are charged towards such end. The addition of this tension and the white image (neutral) — subsequently perceived — provides, as a result, an additive complementary color mixture.

This can be one of the reasons for our preference for balanced or complementary chromatic compositions where, according to Klee, the totality of color is present. Since these compositions have complementary colors or chromatically balanced colors — chromatic hue — (since they have color values that come from the entire chromatic circle), the state of electric excitement of photo-sensible cells is more balanced and, therefore, it has minimum tension; that is, less stress for the visual apparatus than what stress produced when observing a highly chromatically saturated figure, as shown in the previous figure (Figure 3.7).

As we can see, the system is also clearly influenced by the principle of least energy.

We have said that the most prototypical faces are the most beautiful and the most particular are less preferred. This fact, as we already explained, can be related with the capacity to have good genes or of avoiding serious diseases. However, it is also probably related to another advantage: reading speed.

We define reading speed as the speed required by the brain to analyze such perception. For most common observations, the speed required to interpret them is very important, since a person is exposed to a large volume of information, in a specific time. Such large volume of information cannot be analyzed deeply due to time and analysis capacities. For this reason, most information perceived through senses simply does not reach the visual cortex, or the corresponding processing centers (the conscious part of our perception); on the contrary, this information is analyzed by the peripheral centers and discarded as being non-important, before it can be "seen" consciously.

"We know that each actual problem (visual problems) contains both significant and irrelevant data upon which to base expedient behavior. For example, when we call our friend by name on the street it makes no difference to us whether we see him against the background of a grey or white house. Equally, it makes no difference whether we see him in profile or in full face. In all cases we call him, 'Hi John'. Our brain in this case transforms many situations into one action. It performs the degenerated reflection of a multiplicity of situations into a lower multiplicity of reactions."[176]

For this reason, reading speed is a critical factor involved in the identification of beauty. If something is evaluated as beautiful, it should be declared as interesting in the first place, and only after that, evaluated as positive and finally as beautiful. As a result of this process, the speed needed by an image to become clear in the brain is fundamental for the evaluation judgment it will receive.

"To be useful for perception, consciousness should be selected and be available in a fraction of a second; otherwise, the moment of action (or the essential instant for survival) vanishes. Therefore, the intelligence reserved for vision, acts with more speed than the intelligence used for the solution of other problems; this may explain the reason why visual perceptions are surprisingly different from the other conceptions, more abstract, in general, and in disagreement with them."[177]

We could add another concept to the importance of reading speed. As we have said, our conscious perception is indeed a mixture of those things that stimulate our eyes (the image itself) and the visual referendum we associate it with in our associative inner library.

"Certainly, vision (human) is not infallible; this is largely due to the fact that consciousness and the hypothesis we proposed, increase data in to the extent that it neither

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[176] M. Bongard, (1970). **Pattern Recognition**. Spartan Books. London. Page 5
[177] R.L. Gregory, (1998). **Occhio e Cervello. La psicología del vedere**. Raffaello Cortina Editore. Milano. Page 13

directly correlates with the images provided by the eyes nor is it limited by them, so that usually fictitious suppositions are produced."[178]

In other words, all we perceive is compared — in real time — with the brain's associative libraries in an attempt to find identifiable patterns; in this way, we recognize who is in front of us, what is behind, what is a figure and what is the background. This phenomenon is behind all optical illusions and is known since the introduction of Gestalt's psychology or form psychology.

This means that provided the identification is easier in relation to an image perceived, the faster is its reading speed and the more likely to enter into the perception's conscious phase. Obviously, this represents less energy to decode such image and, therefore, it is more pleasant to see it.

By extrapolating this concept and relating it to the phenomena of prototypical faces, we want to propose our own hypothesis. We named it:

3.6.1 Paradigm of tranquility (Definition)

Should a perception correspond to the expectation we have, it is immediately and successfully associated to the associative libraries and will have more chance to be assessed as beautiful.

This concept is based on the idea (as in the case of human being's faces) that if something is prototypical, that is, normal, natural or expected, it will be healthy and beneficial.

"In other words, everything is in its place, everything is correct and therefore, it is not dangerous but tranquil and beautiful…

The state of tranquility is produced when the analysis of the context derives in a conclusion of absence of danger; nothing is dangerous or suspicious; everything corresponds to the expected conventional values and to the associative libraries.

For example, in the country or forest, as long as everything is in its place and the breeze blows smoothly, and the sounds heard are the forest sounds and there is peace, then everything is "**beautiful**" and tranquil; on the contrary, if an opposing indicator is suggested, this value judgment may change.

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[178] R.L. Gregory, (1998). Page 10

Let’s say that suddenly, there is a total absence of noise, such incongruence with the normal conditions will make us suspicious: maybe a storm is about to come or a stalker is ready to attack; then, the place is no longer beautiful or riskless in relation to the judgment of beauty and that is what I define as the paradigm of tranquility. In other words, there is a certain implicit beauty in natural conditions and certain rejection (ugliness) in danger or abnormality of conditions, either consciously or unconsciously.

Note that this judgment is usually unconscious and immediate; the spectator feels uncomfortable and does not know how to define the fact that he/she wants to get out of the forest immediately.

The same occurs when the order observed in an abstract composition is not exact. Following our thesis of unconscious associations of observations, we could think that even abstract observations are analyzed and compared with our unconscious libraries in order to find a pre-programmed answer.

In this manner, a structure that correctly associates with our image of nature and tranquility could be recognized as beauty and, one that does not match, could be recognized as ugly.

That would mean that our unconsciousness compares them with ordering schemes and decides whether they are correct or incorrect and leave in our consciousness only the judgment of beauty or ugliness, as the conclusion derived from such analysis.

From this point of view, the orders of associative complexity with nature would be recognized as beauty and the “mistakes” in this order (non-associative to natural schemes) would be recognized as ugly.

The repetitive model of nature influences our inner comparison patterns and, therefore, our judgment on the esthetic values we assign to observations, in general, and to abstract observations, in particular.

Maybe this is the reason why all cultures throughout history have been doing art based on repetitive rhythms; some of them have made repetition the fundamental base of their artistic manifestations.

In other words, when an abstract composition does not follow an associative ordering (in relation to unconscious libraries), the spectator does not feel comfortable with respect to the composition and declares it ugly without knowing why. If not, then this is the reason why a highly structured or exact ordering is declared as beautiful, tranquil and suitable for “my interests” …

The repetitive character of nature is widely known (as we said in the second part of this study). The basic structure of a tree is repeated from the smallest leaf to the main trunk. However, something that has not been widely analyzed is that when observing a leaf, its internal veins keep the same structure and ramifications as those existing in the main trunk, even in relation to their proportions (since they are structured from the fractal repetition). In this manner, the same pattern changes from scale, growing, little by little, from the smallest size, medium and big branches to the main trunk, finally."[179]

Of course, the same can be said in relation to the golden section and natural patterns we studied in the second part of this work. That is, if many millions of years of evolution formed associative libraries with the purpose of influencing our decisions on those things we desire or not, this process necessarily provided some aspects of the natural proportions in our associative repertoire and therefore, they are more easily recognizable than others, and could clearly influence our associative judgment of beauty, based on the paradigm of tranquility.

This could be a plausible explanation for the diffused presence of the golden section of arts (either conscious or unconscious). In other words, if a talented artist is trying to add beauty to a work, it is possible that, in an associative manner and unconsciously, he/she makes use of his/her associative library, and if these are flowed with nature observations, it won't be weir that he/she used chromatic compositions and proportions, which are also present in nature.

Note that this doesn't oppose at all our logical analysis previously exposed in the first part of our study. That is, we have not either questioned that sometimes, nature has golden proportions, or that beautiful things also tend to have them. We questioned, and continue to do so, that even though many beautiful things have golden proportions, it does not mean, at all, that things which have golden proportions are beautiful; in other words, if many apples are red it does not imply that all red things are apples or that all apples are red.

Even though this seems insignificant, it could be the mistake made from Vitruvius to Corbusier. Besides, this could be the reason why we find golden proportions in beautiful things. Based on the paradigm of tranquility, it is expected that natural proportions, in general, and golden proportions, in particular, are found to be beautiful when they are in the material culture.

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[179] F. Hernández-Castro, (1995). Pages 89-91

In this manner, we have the hypothesis of why does the tendency exist to find them beautiful? (passive condition) and, Why do we find them usually in arts? (active condition).

That is, we propose a commitment with respect to the dilemma of our study, which matches facts with logics, a situation that has not been achieved today:

On the one hand, we must understand why are there so many golden and natural proportions, in the artistic works throughout history? And why do so many artists have been obsessed with the pan-proportionalism (as we called it)?

On the other hand, we found out that, logics does not let us apply the proportional system — such as that of the golden proportions— as a design tool. According to this scope, logics and facts are both compatible.

Indeed, all this explanation wouldn't be possible if we did not have human cerebral super-symbolic capacity, which allows us, as we said before, to transport one concept or pattern from one context to another.

Human beings handle complex symbolic systems that extend to all cultural manifestations not only visual but also all those which are converted into generalized and relational symbols, *everywhere*.

"These things are not only words, or images; they can also be forms of social behavior, political acts, and artificial landscapes. As Charles S. Pierce once said, 'a sign is something by knowing which we know something more' "[180]

In this manner, our capacity to "transport" meanings through symbolic systems from one side to another of our culture allows us to easily adapt (unconsciously) a concept, from one context to the other. The symmetric concept of human faces can be transported to other fields; this concept, for example, was born as the response to the need of identifying healthy mating subjects who had good genes; through the human super-symbolic capacity, it was adapted to other fields; in general, the preference of symmetric situations exists even in totally artificial situations (as in the investigation with kids figure 3.2).

This capacity of "communicating" by means of images or symbols has increased with the passing of centuries, enriching the unconscious codes we use to express and understand our culture.

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[180] U. Eco, (1986). **Travels in hyperreality**. Harvest Books. Orlando. USA. Page XI

"Humankind's ability to communicate visually (and to some extent our interest in communicating visually) has increased not steadily but exponentially. Arts was the primary method of expression for several thousand years and was then replaced by hand-written language, which, in turn, became obsolete after the fifteenth century when Gutenberg produced his first movable-type printing press (though movable type had already been used for hundreds of years in Asia). Over the past one hundred and fifty years, a plethora of new and efficient communication tools have been invented and quickly placed into ubiquitous use. Photography was invented in the mid- 1800s, and lithography near the close of the nineteenth century. The twentieth century has brought us motion pictures, teletypewriters, television, video, and finally computers.

Each of these forms of technology is primarily visual in its means of communications. And each uses particular signs an symbols to deliver its messages."[181]

Then, visual codes overlap and find their value in the new means, generating, in turn, new ways of interpreting old rules and generalizing situations that were created as evolution instruments to improve the species, in situations within real life, as well as describing customs, rites and objects of the post-modern material culture.

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[181] S. Aukstakalnis & D. Blatner, (1992). **Silicon Miracle. The Art and Science of Virtual Reality.** Peachpit Press.Berkeley, CA. Page 44

3.7 Conclusion to the third part

As we did with the previous sections of this study, we will summarize the most important aspects of this section.

Proportional systems were born in Egypt and Greece as instruments to faithfully copy a model. This model was usually a human body. Greeks cited, for the first time, beauty as the nature's inherent characteristic. Aristotle defined, in those early times, that natural orders had an intrinsic beauty.

Vitruvius, three hundred years later, recommended for the first time that architectural practices were used with human proportions. This was the first great step of the theory of proportion, since the prevailing canons did not aim to possess beauty itself, but copy beauty from a model, such as in Greece or an aesthetic profile, as in Egypt.

Later, this idea is considered as true; with some variations, it reached the Renaissance masters, such as Alberti and Palladio who took the next step by proposing formal vocabulary to design "correctly". Thus, a series of shapes and proportions were proposed, usually rectangular prisms, with specific measures that correspond to certain proportional systems. These shapes can be considered as a catalogue and an element to be used in architectural works that are being designed.

This is exactly the idea that Le Corbusier used again in the XX Century, proposing red and blue scales. This was also used as formal vocabulary wherefrom dimensions can be selected to specify an architectural work.

Besides this, in relation to the use of tools concerning proportional affairs, clear evidence was gathered throughout time that nature and the material work usually showed more one kind of proportions than others.

From the geometrics in Euclid's E*lements*, the trend of searching for some kind of proportions was evident (for example, the four irrationals) in many and highly dissimilar scenarios. Vitruvius, for example, was one of the first to analyze the Greek architectural works and founded golden proportions in them. Although we have said that it was not likely done on purpose by Greek architects, it is also true that it is relatively easy to find such proportions in their works. However, Vitruvius went beyond that and proposed all types of objects with certain proportional systems that included from catapults to chairs, beginning what we called the pan-proportionalism, which still prevails in our days.

Other theoretical ones like Luca Paciolli and Andreas Cook made the analysis of many natural patterns that also had these proportions. This discovery ended in the legitimization of the use of proportions for design purposes, since, “if God used them in His work”, how could they not be recommendable for man to use it?

Certainly, within this boom of discoveries and assumptions, a rational analysis wasn’t made in relation to those things and rapid specific conclusions were defined.

We may define the recommendation of ancient theoretical as follows:

“Since nature favors certain proportions, and since it is beautiful, if we design some objects with such proportions, they will be, in turn, more beautiful. ”

This statement prevails in our days, mainly in design and architectural schools, and it implies the theory that beauty moves in the “vehicle” of proportions. This assumption is not as evident as it seems to be, since it denies many other design factors.

By analyzing such statement more deeply, we find some intrinsic problems:

It is one thing if we affirm that nature is beautiful and has golden proportions

AND

another thing if we affirm that objects, which have golden proportion, are beautiful.

In other words, even though we have enough information, gathered empirically, to say that nature usually uses golden proportions and that nature is, by definition, beautiful, or that we affirm (without any decisive evidence) that all beautiful things have golden proportions, we cannot affirm the same in the opposite direction; that is, all things that have golden proportions are beautiful.

Finding one counterexample would be enough (as we have done, empirically in our study) to void the legitimacy of that law.

The problem with an incomplete explanation (as that discussed) is that it is useless in practice. If we cannot affirm that all objects which have the golden section are beautiful, then, the golden proportion is useless as a design tool; there is no way to figure out if what is being designed (as architect or designer) belongs to the group of things that have a golden section and are beautiful or to the group of things which has a golden section and are not beautiful.

In other words, this tool is **absolutely useless.** It is equivalent to say that a design may be successful only because it is higher than the average, but that it can also be a failure even though it is higher than the average. With these statements, nobody would recommend using the height of a building as a parameter to improve its beauty, and that is exactly what they did with proportion.

Therefore, based on our analysis, we can define some aspects, doing our best with what we have:

1. Definitively, due to physical reasons, some proportions more than others can be found in nature, such as the golden proportions (as we explained in the second section of our study).

2. The golden section is frequently found in some art objects and design works, in general.

3. It is easy to find examples of objects which do not have the golden section, but which are considered beautiful by many people.

Then, it seems that the explanation of the phenomenon — object of this study — is quite more complex than the simple idea of objects being beautiful because of the golden section.

To understand the things behind this phenomenon, we have to make a conscious analysis of the phenomenon of the perception of beauty.

Beauty is a natural mechanism that leads us to make decisions that have proven to be useful in our evolution. In general, this mechanism reward us with pleasure as many other natural mechanisms, and we share this with many other living beings.

Art, in this context, would constitute an adaptation of beauty-generating natural mechanisms in the super-symbolic field of human society.

As Geoffrey Miller[182] affirms, arts are humanity's peacock tail, either in the case of animals or humans, this mechanism generates pleasure for both the doer and the observer.

As a natural mechanism, the phenomenon of the perception of beauty leads us towards the selection of objects according to preferences and based on the

[182] G. Miller, (2000). **The mating mind. How Sexual Choice shaped the Evolution of Human Nature.** Anchor Books. New York. Pages 10-11

evolutionary viewpoint. A symmetric face, for instance, is found to be more beautiful than an asymmetric face. The explanation of this phenomenon is based on the statement that symmetric faces represent better genetic quality or resistance to diseases for the individual.

For this same reason, the most prototypical faces are considered more attractive than a face with more particular features.

If we apply these concepts to the material culture, taking advantage of the human brain’s symbolic capacity, it is easy to imagine the reason why, in many cases, symmetry is considered as beauty.

Thus, when we asked the children to place an object in a rectangular space, most of them chose the center (the place which favors symmetry), instead of any other place.

This phenomenon is combined with what we call the “paradigm of tranquility”. When a perception coincides with what it is expected from it (as in the case of the most prototypical faces), then, it is more likely to be found beautiful. It means that the decoding speed of a message by the brain is fundamental for a positive rating thereof at the aesthetic level. The sooner the perception is evaluated and the closer it gets to the idea we have (unconsciously) about it, the better will be the evaluation concerning beauty.

This is an excellent characteristic to clarify the reason why there are so many art objects with natural proportions. That is, if our perceptive system was programmed during millions of years with images coming from nature (that was our usual environment), it is evident that we find those things, which have natural structures and proportions as “normal”. And, as we said before, what we find “normal” is likely to be beautiful for us, as well; this explains why so many artists, designers and architects have used structures and proportions consciously and unconsciously.

Combining these two aspects:

1. Beauty as a natural mechanism for the selection of evolutionarily convenient options (as symmetry in faces);

2. The characteristic of considering beautiful, that which is natural (as prototypical faces),

we have a good explanation for the existence of other factors, as symmetry — even though in arts and design we usually find natural proportions (as the golden one) —, that may easily become more important than proportions in relation to esthetic evaluation.

This explanation answers our doubts; that is:

1. Why do many beautiful things have golden proportions as the diffused presence of this proportion in art through history? (Because they are the most common proportions in nature and due to the paradigm of tranquility, we find them beautiful).

2. Why do many beautiful things exist without a golden section? (Because there are many other factors, as symmetry, in many of our aesthetic judgments, which seem to be more important than proportion).

3. Not all objects having a golden section are beautiful (for the same reason).

4. Not all beautiful objects have golden sections (Because proportion is not the only factor involved in the convenient selection of objects).

In this way, we consolidate our theory that regroups facts without involving fallacies, as those we currently accept as true and *everywhere*.

Therefore, we may conclude by saying that although golden numbers are an important factor in all this process, it cannot be used as a tool for “a magic transportation of beauty” with its proportions.

As we said in this section:
If many apples are red, that doesn’t imply that all red things are apples or that all apples are red.

By demystifying the proportional systems as design tools, the remaining question is whether there is another project tool that can help us to focus the beauty perception problem in design.

During the past half-century, many scientists, from diverse fields, have studied the problems similar to those questioned in these pages. They represent a fresh and rational focus on an ancient and irrational problem of beauty. These viewpoints are the topic of our next and last section of our study: artificial esthetics.

Part Four
4. Proportion and virtuality, artificial aesthetic.

4.1 Introduction to part four.

As we near the final part of our study on proportion, that last thing we need to do is to analyze the possible future tendencies of the study of proportion.

As we have explained, it doesn't seem that the use of proportion makes any sense as an instrument to support design. For this reason, we believe that the theory of proportion (as we have understood it until now), will not be used beyond than what it has been already used for.

Due to its inefficiency as an instrument and to the past unsuccessful attempts, the reputation of the theory of proportion has gone downhill. Many architects and designers who enthusiastically embraced this theory became aware (in practice) of the method's inefficiency. To date, not many people consciously use this tool to face everyday problems.

There are different perspectives, however, developed in the last years, which can be applied to the problem of proportion; the majority of them from other fields that are not directly related (at least not obviously) to design and architecture. Because these approaches to the problem don't come from the field of design, they are not known by designers and of course, not used as a tool in the design process.

The majority of these new perspectives come from mathematics, computers and artificial intelligence, very different fields from what we are accustomed to seeing in the theory of design.

In this section of our study, we would like to make a modest review of some of these methodologies and techniques which have led scientists to the creation of real works of art, with a more controlled and recurring focus. We believe that the future of the proportional tools of design lies in the application of these techniques in the field of the theory of design, a future more aligned with the virtuality in which we live in this new century.

We will try to keep the argument and analysis of these topics at a less technical level than usual, trying not to lose sight of the designer's or architect's point of view, from his or her specialty and concerns; however, today's professional is (and must be) multidisciplinary. The architects and designers of the future will have to burrow in more formal topics if they want to keep pace with the times.

The wonderful tools of virtuality make it possible to focus on the problem of design with a whole new perspective. It will be useless, if not suicidal, to remain in the classic application of golden sections and proportional sequences.

4.2 Two perspectives for the same problem.

Other disciplines have studied the problems of proportion and the creation of artificial beauty (from the design and construction of the material culture) for thousands of years.

“The first step toward artificial intelligence was taken many years ago by Aristotle (384-322 B. C.), when he began to explain and encode certain deductive reasoning styles which he called syllogisms.”[183]

These syllogisms, which we had already analyzed in the third part of our study, allow us to consider a hypothesis and evaluate its validity independently of its content.

As we have suggested in the previous quote, the problem of the generation of beauty (through the proportional tools and others) is a typical problem of artificial intelligence. These kinds of problems are not easy to solve in a deterministic manner; in other words, they are not easily reduced to a group of steps that lead to a solution.

These problems, however, are not new. They have been studied by the discipline of artificial intelligence for more than fifty years, and although we don’t have any knowledge that any problems on the evaluation of beauty have been analyzed (only in exceptional cases which we will learn about further on), the nature of this problem is not wholly unknown.

As a matter of fact, these problems are not very different than those already analyzed in artificial intelligence. That is why, in this study as in another previously studied[184], we have intended to dub the term **artificial aesthetic** as the discipline that studies the topic of how to produce beauty artificially.

Artificial intelligence has two clearly distinctive perspectives: the symbolists and the sub-symbolists or connectionists.

The symbolists are the classics. They assume that all problems can be solved from a deterministic point of view; therefore, it is possible to develop a series of steps (or algorithms) to solve these types of problems.

As we advance on these topics, we will compare them with the classic theories of design in order to

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[183] N.J. Nilsson, (2001). **Inteligencia Artificial. Una nueva síntesis**. McGraw-Hill/Interamericana de España. Page 7

[184] F. Hernández-Castro, (1995). **Estética Artificial**. Editorial MithOz. San José. Costa Rica.

begin understanding the use of these topics in the present study.

In design, we can say that symbolists are the methodologists. As we know, there are many design theoretics, who believe that the operative methodology is a power instrument, capable of leading them to unmistakable solutions to almost all problems.

The connectionists or sub-symbolists, on the other hand, believe that knowledge is in the structure where information is stored and not fixed as in a recipe.

This last tendency deserves a little more attention. Copying models from the brain's function, the connectionists have managed to program structures of knowledge (in computers), which are capable of learning by themselves. As a matter of fact, these structures in general are not programmed; they are "trained" in order to recognize certain patterns.

"The neural networks are a well-known example of machines that come from the sub-symbolic school. These systems, inspired by biological models, are interesting basically because of their capacity to learn. There have also been interesting results obtained through processes that simulate certain aspects of the biological evolution: cross-mutation and reproduction of the better adapted organisms."[185]

In the theory of design, we would be thinking about the Academists, designers, who believe in "hands-on learning", a more renaissance style design. In this more traditional and classical perspective (in design), the students approach the "teacher" in order to learn how to design with him; in the process, they learn (almost by osmosis) and in the process, prepare to become "teachers themselves."

The majority of architectural schools today employ this method and represents, to a certain point, the legacy of the classic art academies.

Each one of these perspectives has its corresponding branch in the research of artificial intelligence and within them, the methodologies and perspectives that tend to deal with problems that are close to the generation of design. Further on, we will analyze each one in detail.

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[185] N.J. Nilsson, (2001). Page 7

4.3 Beauty, a formal approach.

Here we dare to make a formal approximation to beauty as a problem, exposing the process of creating a design object as a rational perspective. Note that the purpose of our subject of proportion, in reality, is to improve the perception of the objects and therefore, it becomes the problem of creating beautiful objects.

If the analysis in the previous section regarding the nature of beauty is correct, we can define that the answer to the question – is an object beautiful or not? – is influenced by a series of factors each one with a specific weight. Genetic, hierarchical and survival factors, as well as regional, commercial, social, group or even individual, combine with different degrees of importance, to define the observer's opinion (his or her aesthetic judgment) over an object (something observed).

In this manner, a great amount of factors are added and considered with different weight or relevance until a final value is obtained. This evaluation is summarized as a judgment of aesthetic value, either positive or negative.

Formally, it can be described in the following manner:

$$ß = \sum_{i=1}^{n} \omega_i \alpha_i \qquad (*1)$$

where:

ß is a real value that identifies an object's degree of beauty (negative for ugly)

α_i are the factors that affect decision

ω_i are the weight associated to these factors.

It is possible, of course, that the list of factors that affect the decision is long; after all, perception is a multivariable complex phenomenon.

This being the case, we could work the problem from different points of view to try and save this obstacle. An approximation of the problem, ordering the weights (ω_i) in a descending manner, would help us identify the more relevant steps.

In other words, if we took all the weights associated to the factors that play a role in the decision, in a descending manner, we would have the most important factors at the top of the list and the least important at the end. In other words, we try to analyze the different factors that influence the judgment of the studied value

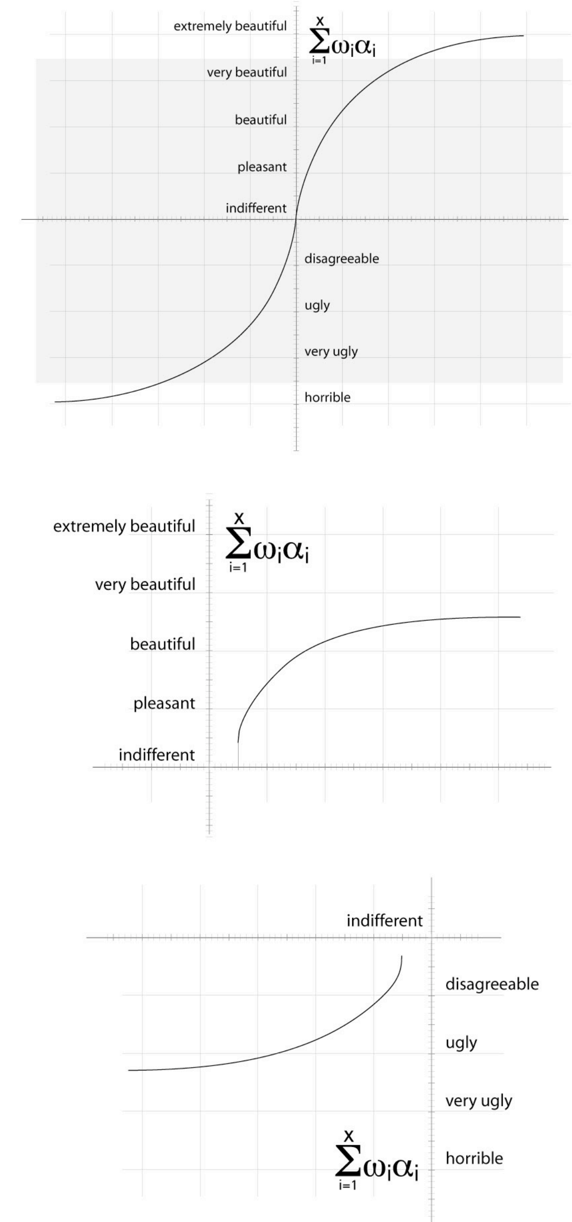

Figure 4.1 Graphic analysis of the possible behavior of a function that explains an aesthetic value judgment.

and we place them in order of importance, from greater to lesser.

Observing the sequence of ordered values, we would possibly face a very interesting situation. At the beginning of the sequence, the values ω_i (the importance of the factors) are relatively large; however, these will decrease rapidly until, after a few elements, the influence of the factors is not very significant.

Therefore, for example, if we analyzed why a certain fruit is more delicious than another, it would be easy to imagine that certain characteristics, such as its nutritional qualities, would probably play an important role in this decision. Later, in a scale of importance, other factors would come into play: it's easy to digest, its mineral ingredients and even color; finally, less importantly, regional factors such as cultural, family and personal influences.

Perhaps at first view, this may not appear as obvious because there is a generalized idea that tastes are very different. Actually, a more careful analysis reveals that although there is a large variety of tastes, say, for example, cooking recipes, it is also true that the great majority revolves around the same things; in other words, natural or nutritional products. None of them exceeds extremes: too acid, too bitter, too sweet, etc; in other words, there really aren't any great variations but small modifications of the same thing. No culture likes sand soup, probably because it is not nutritional.

All of this analysis, of course, is observed mainly in the case of evaluating the average statistical behavior, undermining the pathological cases. Note that the idea of this study is to have an approximation of how an aesthetic value judgment behaves for the majority of people and not for a particular individual. In individual cases, it could be that some personal or group values are important; nonetheless, when comparing this value with a large sample of people, these exceptional cases are not relevant for the whole of the population.

Following our ordered sequence of values, it would be interesting to create a graph where the increase or decrease of the ß value (depending on the aesthetic judgment: positive or negative) is ordered on the "Y" or vertical axis. The factors that are placed in descending order (taken into account at each moment) should be placed on the "X" or horizontal axis; in other words, the vertical graphs the value obtained by ß when values are added from the descending ordered sequence. (Figure 4.1)

In other words, in "Y" we have the ß value (how beautiful or ugly an object is) and in "X" we have the sum from 1 to *n* of the products $\alpha_i\ \omega_i$.

The result is a graph that exposes a logarithmic type of behavior. This type of curve is known as a *sigmoid* function curve mainly because of its "s" shape.

The curve studied by John Hopfield and David Tank exposes this behavior and defines this type of problem.

"In the mid-1980s, Hopfield and Tank (and many other researchers) applied this network to a number of combinatorial cost-optimization problems. Such problems are very easy to express but may have an optimal solution that is very hard to find." [186].

These characteristics fit in perfectly in the problem of beauty which, defined as it has been defined here, is precisely a problem of combination and optimization of costs. This is particularly obvious if we find which are the relevant factors and associated weights to make a clear definition of aesthetic judgment.

So, with the weights ordered in a descending manner, we could re-outline the formality of our initial expression (*1) as a function:

$$ß(x) = \sum_{i=1}^{x} \omega_i \alpha_i \qquad (*2)$$

where beauty becomes a function and it is possible to work it and manipulate it as such.

This function relates the number of parameters taken into consideration in the aesthetic value judgment on the horizontal axis and, on the vertical axis, the value gradually acquired by the judgment (in the beauty-ugly scale), as we increase the amount of values taken into account. We shall call this function an aesthetic recognition pattern PRE (for Patrón de Reconocimiento Estético), and it is a sole function between the observer and an object.

However, the functions between the same object and several spectators do not vary much among each other because the more important factors are common to the majority of human beings. By contrast, the more personal functions, which are found at the end of the function, affect the asintotic behavior of the function in a less dramatic way.

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[186] G.W. Flake, (1998). **The Computational Beauty of Nature**. The MIT Press, Cambridge Massachusetts. Page 320

The following figures are examples of these functions (Figure 4.1). They graph the possible behaviors of the function ß(x) for two specific PRE: one with a positive judgment and the other negative.

All of this analysis is based, of course, on the statistical behavior of beauty and not on an individual behavior. This means that if we take into account the value judgments possessed by several individuals with respect to an object (let's say, a rose), the majority of individuals will behave pretty much as the function predicts. This does not exclude exceptional cases that are completely different; however, because they are the minority, they are not relevant cases in this study.

This fact might appear to be irrelevant; however, if we superimpose a series of semiotic values in the "Y" vertical axis, the interpretation of this behavior reveals interesting facts.

Based on an interpretation of the semiotic differential concept[187], we could divide the ugly to beautiful scale into nine levels.

We would then have a semiotic differential as follows:

extremely beautiful—very beautiful—beautiful—pleasant—indifferent—disagreeable—ugly—very ugly—horrible

When we analyze the proposed graph with respect to the semiotic differential, it is obvious that it is possible to significantly reduce the number of values evaluated without losing the meaning in the final value judgment.

In other words, if one takes into account only the first values, one can define relatively extreme semiotic states as either very ugly or very pretty. An increase in the exactness of the function's calculation (increasing the number of values taken into account) will only lead us from a value such as very beautiful, for example, to extremely beautiful which is not very different, in a semiotic way.

The advantage of the aesthetic function behavior, practically speaking, is that it makes it possible to have, in the majority of cases, pretty exact semiotic values taking into consideration the few involved factors.

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[187] Concept of the classical Semiotic Differential used in the studies from the Semiotische Studien zur Kommunikation collection, specifically in the kinds of studies realized by Dr. M. Krampen in **Visuelle Kommunikation und/oder verbale Kommunikation?** Georg Olms Verlag Hildesheim.HdK, Berlin, 1983, pages 123-177. Another interesting reference in verbal classification is the work of Marcel Danesi in **Sign, thought & culture**, CanadianScholar's Press, Ontario, 1998; more specifically in its dimensions of representations.

Figure 4.2 Normal Statistical Curve. The shadowed area represents the majority of people, while the left and right sides represent the exceptional cases.

In real life, this means that the considered weight of the initial factors, such as the genetic, hierarchical and survival factors will be, on average, much more important than the individual or personal values, at least in a general statistical behavior. This behavior is typical of a normal statistical curve where the majority of people are placed in the center of the curve and only few exceptions on the side.

Figure 4.2 shows a typical representation of a Normal Statistical Curve. This behavior is very common in human characteristics; for example, if we graph the height of human beings or their visual capacity, we will very likely obtain a representation of this kind. In this case, the curve shows that the majority of human beings are found on similar average values and only a minority is found on the extremes, either lesser or greater. With respect to visual capacity, for example, this graph could be interpreted as follows: the majority of human beings (of a specific age) have a more or less average capacity, a small minority has very good visual capacity and another small minority has very bad eye sight.

Returning to our case of aesthetic function, the normal distribution (as it is known in statistics), tells us that the majority of human beings will have an average aesthetic value judgment (for example, with respect to a rose) and very few will find this flower horrible or extremely beautiful.

On the other hand, this also means that in a broader sense, it is feasible to treat the perception of beauty as a factor common to all humanity and that with due care, it is possible to avoid the changes in “tone” that could create cultural, social or historical differences.

4.3.1 Comments on complexity.

The theory of complexity[188] classifies problems according to its time of treatment, finding several orders or classifications according to the complexity of the problem.

The classification begins with the polynomial order problems (class P), those that can be solved in a time related in a polynomial manner with the size of the sample.

[188] The theory of complexity is a discipline that is one step ahead of algorithm and bases its work on the analysis of the complexity of problems more than in its capacity to solve them (the algorithm’s objective). To further learn about the subject: Garey, M.R., Johnson, D. **Computer and Intractability: A guide to the theory of NP-completeness**. W.H.Freeman and Company, 1979.

One climbs from this basic level in order of complexity until reaching a family of problems whose solution is not yet defined.

This other class of problems is called non-deterministic polynomial time problems (class NP[189]) and they are defined by the following characteristics:

1. If a problem will be defined as NP, it must be a decision problem; that is, have a yes or no answer.

2. Having a possible solution to the problem, it should be easy to verify if it is correct; specifically, the solution must be verified in polynomial time.

3. Finding a solution to a problem of this type is not trivial; generally, it is difficult to find in polynomial times.

Returning to our subject, it is possible to make an interpretation of our problem in order to classify it as a NP class problem.

First, we must define the problem in a decisional manner; for this we would say that given an object under observation, our question would be:

Is this object beautiful?

In this case, there are only two possible answers: yes or no (this helps the technical generation of solutions; let's think about a program that creates design solutions and an evaluation function which decides if they are acceptable or disposal).

Given the second characteristic of the NP problems, the answer could be interpreted in this manner: we have said that the capacity to recognize beauty is inherent in every human being; it is part of our genetic programming and an indispensable survival tool and evolution. Given a possible solution to the problem (as when an object is being considered beautiful or not), it will take an instant for a human to verify this affirmation. Remember we are working on the premise that there is a universal beauty and that the objects in our study are those whose positive aesthetic judgment is based mainly on genetic factors, common to all humans.

Lastly, (characteristic 3 of the NP problems), it is not a trivial matter to propose a candidate as a solution to our problem. In our example, this is interpreted as a beautiful solution for a specific case; create a combination of

[189] A very accessible treatment of this classification is found in Brassard, G., Bratley, P., (1997). **Fundamentos de Algoritmia**. Editorial Prentice Hall. Madrid, Pages 491-515

beautiful colors, for example. Experience tells us that it is not a trivial matter.

The NP problems are called that given their quality of being treated as polynomial time non-deterministic algorithm, and this is exactly the heart of the matter. From our point of view, some problems regarding beauty are dealt with this way.

This part of our study is based on this possibility, burrowing a little into the exploration of some problems related to beauty and its treatment in a non-deterministic algorithmic manner.

4.4 Copying nature, the sub-symbolic perspective

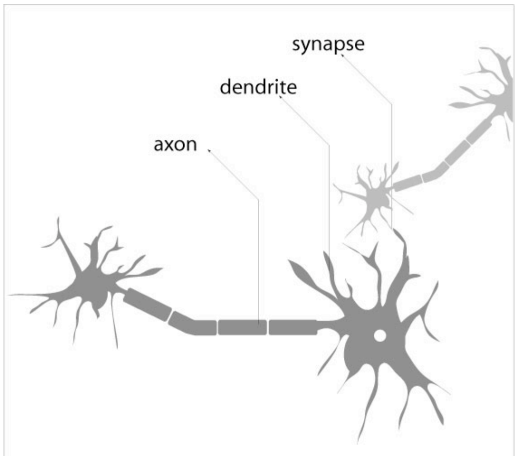

Figure 4.3 Basic diagram of a neuron.

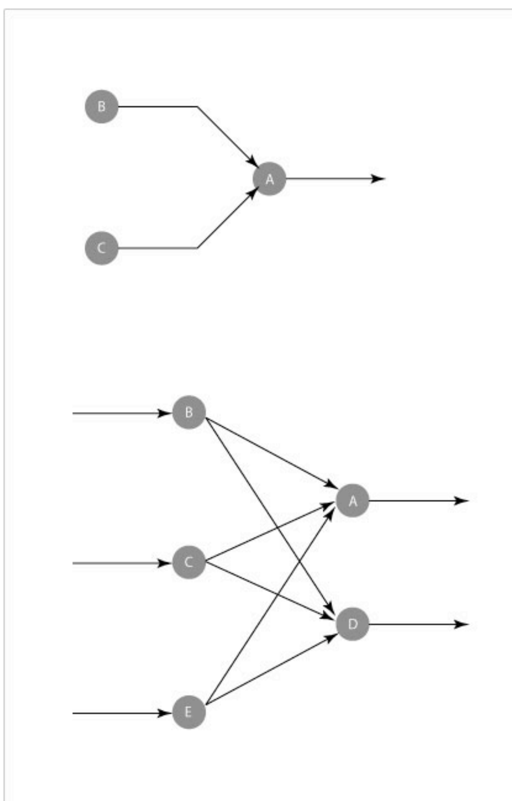

Figure 4.4 Diagrams of the architecture of artificial neurons.

Throughout history, we have tried to copy nature, consciously or unconsciously. In this first focus of the problem (the connectionist sub-symbolic focus), we have tried to copy the structure with which nature works these problems and not its strategy.

Scientists from other branches have studied some structures that could shed light on the possible direction new design tools will take. We will analyze them in some detail in the following sections.

4.4.1 Neural networks

The first theoretics who conceived the grounds of neuron computation were Warren McCulloch, a neurophysiologist, and Walter Pitts, a mathematician who, in 1943, presented their theory on how neurons work. They made a model of a simple neuron network by using electrical circuits.

The idea consisted in simulating the human cerebral structure through the simulation of neurons as basic units. Figure 4.3 shows how we can synthesize a neuron in three main parts:

1. Axon: body of the neuron, connects the head with rear;

2. Dendrites: are the “antennas” or appendages surrounding the nucleus and the rear of the neuron;

3. Synapses: although it isn’t really a part of the neuron, it is important to consider that the synapses connect two dendrites.

Based on these simple principles, virtual units were developed with the capacity of simulating and reproducing the natural structures that support cognitive tasks.

In Figure 4.4, the upper diagram shows the basic architecture of one of these virtual units. Units B and C in the figure are the sources of information for Unit A; the latter picks up the stimuli (from B and C) and decides if a new stimulus will pass through its frontal connection toward the rear units. All connections between units are weighed; this means that they all don’t weigh the same but can vary in weight. These weights define the importance of a connection and its influences on the units it connects to in a specific structure.

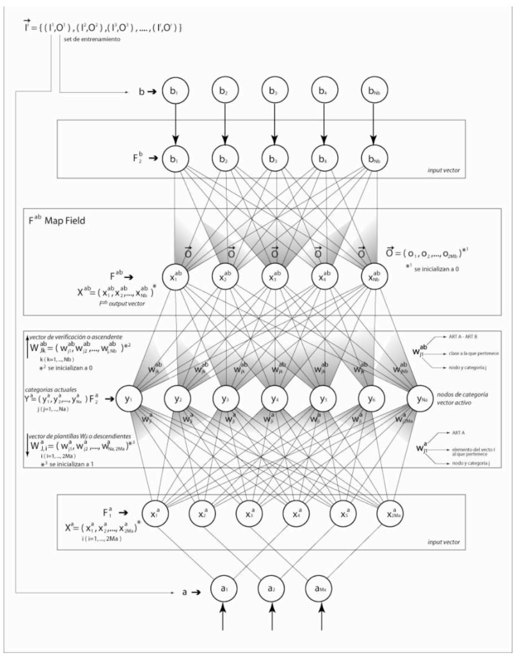

Figure 4.5 Architecture of an artificial neural network with a fuzzy adaptive resonance (Prof. Franklin Hernández-Castro).

In the lower part of the same figure, a small structure is observed with five units. This architecture possesses two layers of “neurons” and exemplifies how they interconnect among themselves.

When an artificial neural network is working, the first units receive the different stimuli which travel through its connections to the next layer of units; in turn, each one of these picks up all the entering connections, evaluates and combines them and decides if they need to send a stimulus to the neurons upfront.

Figure 4.5 shows an example of a Fuzzy Adaptive Resonance Theory neural network, which exemplifies the organizational complexity of these artificial networks. Nonetheless, the figure only shows the nature of these connections and not the amount; in a typical network, the number of neurons exceeds the hundreds even thousand of units.

There are several schemes or architectures to build artificial neuron networks; for example, the Bayesian Networks and Back-propagation Neural Networks, but this isn’t the topic of our study.

We would like to highlight, however, that these structures created from natural models are capable of learning; that is the reason why we have used them in this study.

The artificial neural network is basically programmed with an architectural format (like the ones we have shown here), and they are later “trained”. This training process consists in feeding the network with a specific stimulus and comparing it with the answer expected of it when it exits. During the training process, the connective weights are adjusted among all neurons so that the exit coincides with the entry stimulus. This process is repeated with entry examples several times and after a certain time, the network will be in capacity to give a correct answer to the specific stimulus.

An example can probably make the process easier to understand. The recognition of a face is one of the most common applications for neural networks. Generally, a fuzzy resonance network is programmed, such as the one indicated in Figure 4.5 with more than 10.000 entry neurons and some 100 exit neurons (in order to recognize 20 people; if it seems like too many, remember that a human brain has ten billion neurons and half of these are used for visual recognition). The architecture interconnects the neuron units and assigns them equal values.

At this point, the network is new and can be trained with any kind of information. Then, a set of photographs of 20 people is fed into the network, one a time, correcting the internal weights of the connections in order to correctly identify each photo with its corresponding individual. After having trained with the established group of information, the network can be supplied with a photograph of one of the individuals, not previously shown, and the network will be capable of recognizing the individual.

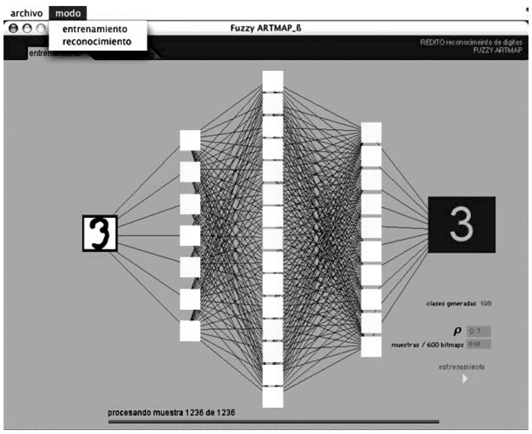

Figure 4.6 Example of a recognition network of handwritten characters. In the example, one can observe how the number 3, written on the left, is recognized by the network showing this same number on the right hand side. Programming: Franklin Hernández-Castro Jorge Monge-Fallas. Instituto Tecnológico de Costa Rica.

It is also possible to take this same network, “clean” the assigned weights and use it to recognize something else; for example, handwritten characters or atmospheric conditions. Figure 4.6 shows one of these networks trained to recognize handwritten characters.

It is this capacity of one same system to learn one thing or another that makes us believe that the knowledge of this kind of artificial intelligence is in the structure and not in the algorithm, although even the algorithm can learn several things. Once trained, a neuron network has the information it learned in a distributive manner; it is the sum of all the connection weights that were considered during the learning phase. It can’t be read or understood unless the network is used to ask what interests us.

The applications of this type of artificial intelligence are plenty; at present, they are very used in medicine to carry out diagnoses.[190]

Of course, and as the reader has probably predicted, this is the type of behavior we use to recognize beauty through our associative libraries. In other words, our brain is a neuron network with 10 billion interconnected neurons, trained to recognize countless things; among them, beauty and natural proportions.

There is sufficient evidence to think that part of our programming is already considered in our DNA; the rest is trained throughout our lives. Although our learning process in the recognition of patterns never ends, we do know it is much more intense in the first decade of our lives than in later years.

“During the first three or four years of life, a child’s brain grows approximately two thirds its final size and its complexity develops at a rate it will never reach again. During this period, the learning codes present themselves with greater effectiveness than in later years.”[191]

[190] J. Trujillano, J. March, M. Badia, A. Rodríguez, A. Sorbías, (2003). **Aplicación de las redes neuronales artificiales para la estratificación de riesgo de mortalidad hospitalaria**. Gac Sanit. Vol.17 # 6. Pages 504-511

[191] D.Goleman, (1995). **La Inteligencia Emocional**. Vergara Editor S.A. México. Page 230

In general, we can imagine how the mechanism that allows us to recognize beauty and golden proportions works; it is a pattern recognition mechanism by comparison in a neuron network structure.

This being the case, and having the tools to model the process naturally, it seems that the artificial neural network is the model to follow in order to recognize the beauty of proportions in a design work. In other words, we shouldn't be surprised if in the near future, the classical design tools (which we studied at the beginning of this work), make room for artificial intelligence tools based on neuron networks that help create successful designs in a more natural fashion. These tools would use their natural similarity with the recognition process itself of proportions and beauty in our brain to simulate and facilitate the process of design. It would be like having at hand a "cultured design curator", who could offer us an opinion.

In other words, more than having a methodology to measure, proportion or define a design (as we have done for centuries), we would have some kind of virtual assistant which would advice us on how our design could be more successful. Even an assistant with a specific style is easily imaginable because it is just as easy as training a network to recognize repetitive patterns in specific works, say a movement, like rationalism or postmodernism.

In this case, proportion, as a tool is only part of the analysis made on the requested recommendation. In this sense, proportion takes on the real importance it has in reality and not the disproportionate first place it has been given. It is only one more reference in a much more complex pattern such as the classification of an object within a fuzzy group[192].

Existing technology already makes it possible to talk about design tools in artificial intelligence that help us in the more critical part of the design process: the perceptual evaluation.

After all, programming an artificial neuron network is similar to what we do with students when they arrive in design school. Generally, we expose them to information related to design in magazines, books, history of design, touring important sights, exhibits, etc. and somehow, unconsciously, we try to train their neural networks to recognize and differentiate what their professors consider "good design".

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[192] This fuzzy group term is used in fuzzy mathematics and it is constantly applied in this type of connectionist intelligence. This type of mathematics works with percentages and not deterministic in groups.

The advantage of training a virtual design assistant lies, of course, in that once trained, it can be copied and distributed to the community of designers and architects and be always prepared to give an opinion.

This is the type of design tool which is in accordance with the virtuality of the XXI century and that would replace the existing deficient tools in a much more efficient manner.

The neuron networks are not the only possibility within the connectionist perspective to generate design tools. In following, we will analyze others.

4.4.2 Genetic algorithms

The other branch of sub-symbolic programming is evolutionary programming or genetic algorithms. This strategy tries to simulate the evolutionary process through generational virtual sequences.

A series of random solutions are produced for a specific problem. These solutions are evaluated and the best chosen among them. The selected solutions are then combined and a new generation of solutions is produced which are selected, in turn, to repeat the process from the beginning. This process repeats itself for a specific number of cycles, denominated, for obvious reasons, generations.

This procedure is very similar to the process followed by natural selection[193]. Let's define some terms a little better to systematize the strategy:

Population base: it is the group of solutions with which a selection process is begun. They are generally produced at random from the elements that could be a part of the solution.

Chromosomes: these are elements possessed by each individual in the population; in computer, binary elements are used (0 or 1); however, it can be points in another field, positions or simply elements of a possible solution.

Genes: the group of chromosomes in each individual.

Crossover: the process by which two individuals combine to create a new individual. This crossover consists of combining genes from each one of the parents and obtaining a new individual. There are

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[193] For a good introduction to the subject: Darrell Whitley, (1994). **A Genetic Algorithm Tutorial**. Statistics and Computing. Vol 4. Pages 65-85

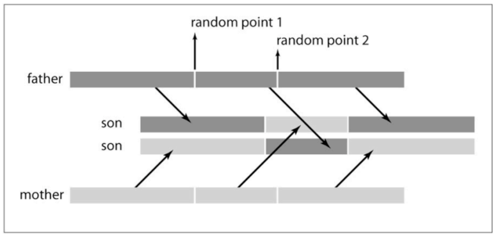

Figure 4.7 Two-point crossover: Intersection of two points used in genetic algorithms.

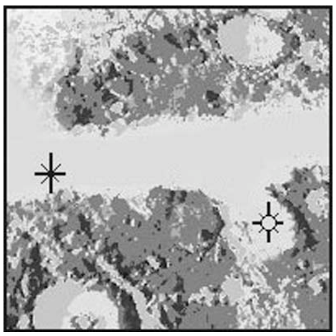

Figure 4.8 Search for a path between two points.

Figure 4.9 Population of 50 paths generated at random.

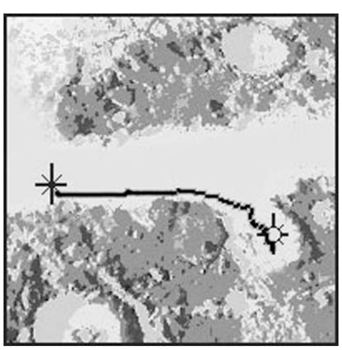

Figure 4.10 Selected as the best path in 50 generations.

several types of crossover techniques; figure 4.7 shows one of them.

Population: the number of valid individuals as a source of genes for new individuals.

Fitness function: it is the function analyzed by each individual awarding an evaluation with respect to the quality of his or her response to the outlined problem.

Generations: it is the number of times the gene mix and individual evaluation goes through the cycle.

Mutation: it is a modification of the chromosomes, at random, in order to increase the probability of finding better solutions to incorporate into the population base.

With these concepts cleared, we can concentrate on one example. Let's say we are evaluating a path with no obstacles between two points, as observed in Figure 4.8[194]. The idea is to find a path that connects both points marked on the map. This could appear to be easy for a human (the human being compares the map with his or her previous experience and decides what is high, what is low and which possibility is easier to follow), but it is not a trivial problem for an automated system.

The first step is to generate a population base (as in Figure 4.9); in this step, many paths are created between both points (the figure shows 50 of them). As observed, the paths can be created at random and many of them actually aren't very efficient; they go through high points and turn unnecessarily to connect the desired points.

These paths are examined for a function that evaluates them and gives them a quality grade. The paths with the worst grades are discarded and the ones that earned a higher grade are crossed again to begin the process anew.

[194] The illustrations are part of a system based on genetic algorithms to evaluate images. Programming: Franklin Hernández-Castro and Jorge Monge-Fallas. Instituto Tecnológico de Costa Rica.

The amounts that were used in the experiment of the figures are the following:

Population: 100
Generations: 50
Survival: 50
Chromosomes: 30

This means that you start with a population base of 100 possible paths that cross each other; the 50 worst individuals from each cycle are discarded and the remaining 50 are crossed again. The chromosomes identify the number of samples considered along its evaluation path; in other words, each path was evaluated in 30 points in order to determine its quality. Every so often, some generations introduced mutations at random among the surviving population, as a source of genetic variation.

Figure 4.10 shows the best path after 50 generations. As you can see, the path is very similar to the one a human would have chosen.

We have used this example because we consider it a simple way to understand the way genetic algorithms work. It is interesting to observe that although there is no programmed instruction to find the right path, the strategy finds a good solution. A typical characteristic of these strategies is that they find a good solution but often, it is not the optimal one; this is because there is no clear strategy in reference to the assigned task (finding the right path), but rather a trial and error perspective.

Once more, it is easy to see the similarities between the methodological strategy and the design process; in other words, the design process doesn't have a clear recipe that can be followed from beginning to end and the tools that can be used, as the use of proportion, are not blunt or deterministic and don't always have a valid solution. It seems more like the process becomes a generation of solutions (almost at random) and its evaluation, combination and improvement; all of these activities are similar to the shown algorithm.

It is for this reason that many scientists have tried to experiment with the genetic algorithm strategy in order to generate an automatic design. This has resulted in a new tendency of design called Generative Design.

"Evolution is now considered not only powerful enough to create biological entities as complex as humans and consciousness, but also useful in simulation to create algorithms and structures of higher levels of complexity as could easily be built by design. Genetic algorithms have shown to be a useful method of searching large spaces using simulated systems of variation and selection."[195]

More often, more sophisticated genetic algorithms have been used and have been implemented in the methodologies of design. These implementations, often times successful, are involved in great works that were designed, analyzed or consulted by generative design systems in computers.

"(We) investigate the possibility of encoding architectural design intentions into a generative design system, using as a test bed the School of Architecture at Oporto [Portugal], designed by Álvaro Siza. Based on language constraints derived from Siza's original design, the generative system [GS], consisting of a genetic algorithm and the DOE-2.1E building simulation program, created facade solutions resulting in lower annual energy consumption, while acting simultaneously as a diagnosis mechanism for problems occurring in the existing building."[196]

The systems are far from being perfect because they are in their initial stage, but they definitely represent one of the most solid promises as tools of the future.

The main problem with these systems lies, even today, in the inability to design a good fitness function capable of selecting and pre-selecting the great amount of design alternatives that can be created from a generative-genetic system (some advance has been made which we will analyze later).

Let's say that we are capable of elaborating a system that generates different architectural design options as solutions to a specific problem (this type of research is already being done quite successfully[197]). The result of our process is a group of proposals that represent the possible solutions to our problem; we could call it the Population Base. Because we are in a computerized system, our only restriction is the computer's speed because we could leave it running for a week to create (for example) a thousand solution proposals.

Obviously, the problem is barely beginning. We must now evaluate each one of the proposals and grade

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[195] K. Sims, (1991). **Artificial Evolution for computer graphics.** Computer Graphics, Vol. **25** #4. Pages 319-328. (ACM SIGGRAPH '91 Conference Proceedings, Las Vegas, Nevada)

[196] L. Caldas & L.Norford, (2001). **Architectural Constraints in a Generative design system: Interpreting energy consumption levels.** Massachusetts Institute of Technology in Seventh International IBPSA. Conference Rio de Janeiro, Brazil. Page 13-15

[197] J.S. Gero, S. Chase and M. Rosenman, (2001). **A Generative design system applied to Siza's School of Architecture at Oporto** CAADRIA, (2001). **Key Centre of Design Computing and Cognition**. University of Sydney. Pages 253-264

them. We could select the 100 best and cross them (with the previously explained techniques) to obtain a better generation. By repeating the process some 50 generations, we would be in capacity to obtain a relatively good solution, almost optimal, as in the example of the paths.

This isn't always like this. Today's systems generally only produce options that are mostly chaotic and evaluated by architects and designers in an inefficient interactive man-machine process. Once the user decides which are the best proposals, he or she lets the process run again and this cycle repeats itself until the user is satisfied with the computer's proposal. This process is slow and not necessarily the best.

"One particular way of melding computer technology with art holds a unique set of properties about it that sets it apart from everything else. By emulating the process of natural selection on a computer, artistic creations can be evolved. Instead of natural selection, aesthetic selection is used as the criteria for survival of these "artificial artisans.

At its heart, evolution is an algorithmic process, which leads to accumulations of design over time. Natural selection was Darwin's explanation of how things can be designed without a designer. With the natural world selecting for beneficial traits, any population of randomly variant self-replicators would tend to statistically favor the members that have better traits than others. Iterating this blind process yields non-teleological design. When humans interfere with nature (as we so often do) and make intentional choices over which things get to live and which things get to die and who gets to mate with whom, it is no longer termed Natural Selection but instead "Artificial Selection" or "Unnatural Selection".

The genetic algorithm, that which is key to the power of evolution, remains more or less unmodified. Only the criterion for selection has been changed. The process becomes goal oriented, or teleological. When the criterion behind artificial selection becomes aesthetic appeal, we will call it "Aesthetic Selection".[198]

Obviously, the optimal solution would be to have the computer evaluate the options, as in the case of the paths, and show us only the last ones (say, the last 10, for example) after a much more extensive process (let say 50 000). We could be sure then that the proposal is in reality a very careful selection of a long process.

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[198] A. Davidson, (2000). **Evolution of Art by Aesthetic Selection**. In http://spaz.ca/aaron/SCS/biography/selection.html

But the problem continues to be how to automatically discriminate artificially created options. This research is also in the process of being studied. These systems called Aesthetic Evolutionary Design (AED) allow non-expert users in design and architecture to explore previously developed and selected solutions by computers, which, lastly, play the role of architect or designer.

As deduced from the scenario presented, the evolutionary computer strategy applied to design and architecture is another of the great lines of research being developed at present with the purpose of providing new tools for design.

Within this strategy, the proportional tools strongly entered into the most critical area of the research, the fitness functions. It is expected that a more generalized function, which intends to grade design options, would have reference parameters of design's classical variables. Among these variables we will surely see symmetry, the golden proportion, speed of reading, rhythm, contrast and the anomaly, among others.

Weight, reading and specification such as the classification criteria of a design proposal is work that is just beginning within a new discipline "Aesthetic Selection"; however, there is no doubt that in the near future, this will be another of the common tools for designers and architects.

"Artificial evolution has been demonstrated to be a potentially powerful tool for the creation of procedurally generated structures, textures, and motions.

Reproduction with random variations and survival of the visually interesting can lead to useful results. Representations for genotypes, which are not limited to fixed spaces and can grow in complexity, have shown to be worthwhile.

Evolution is a method for creating and exploring complexity that does not require human understanding of the specific process involved. This process of artificial evolution could be considered as a system for helping the user with creative explorations, or it might be considered as a system, which attempts to 'learn' about human aesthetics from the user. In either case, it allows the user and computer to interactively work together in a new way to produce results that neither could easily produce alone."[199]

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[199] K. Sims, (1991). **Artificial Evolution for computer graphics.** Computer Graphics. Vol **25** #4. Pages 319-328. (ACM SIGGRAPH '91 Conference Proceedings, Las Vegas, Nevada)

4.4.3 Fractals as a tool of design

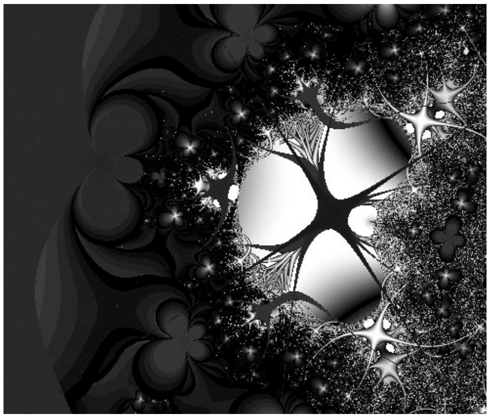

Figure 4.11 Example of a graphic art based on fractal calculations. Franklin Hernández-Castro, Instituto Tecnológico de Costa Rica (Made with EasyFractal 3.11).

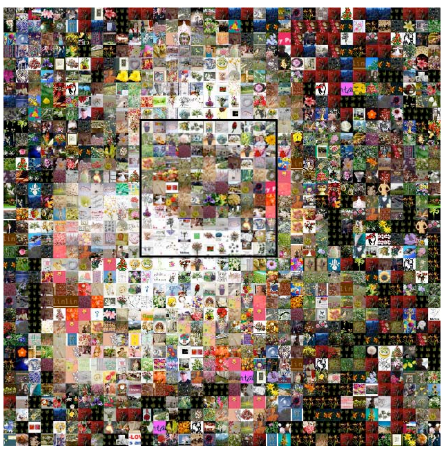

Figure 4.12 Photo mosaic of a face.

Figure 4.13 Area surrounding the eye in the photo mosaic above.

As part of the next strategy in the sub-symbolic perspective we would like to make reference to the least systematized use of some techniques applied to design. They consciously copy the structures and processes of nature and implement them to design and architecture.

Fractals, which we studied in the second section of our study, are usually the foundation of these procedures. Based on the fractal methodology of creating forms, designers and architects are creating design proposals. The first to use this possibility were graphic designers and artists (Figure 4.11). The fractal's natural form, based on its characteristic of self-similarity, gives these works a very natural depth and (as we had studied before) is the only way where a work can comply with the condition of integral proportionality. In other words, the works that are developed in a fractal manner are the only ones in which all of its parts are in the same proportional system, even the smallest ones.

This self-scaled natural complexity results in a very successful composition aesthetic wise; as a matter of fact, it is very coherent with what we have mentioned regarding the paradigm of tranquility. Because fractal compositions behave as many natural phenomena, it is not unusual that we regard them as beautiful; this is because we find in them proportions and compositions that agree with our previous programming in the associative libraries and are therefore, "correct and agreeable".

The famous photo mosaics were developed based on this concept. Robert Silvers[200], professor at the Massachusetts Institute of Technology (MIT) (a student during that time), developed a program capable of searching for images in a file of digital photos that fulfilled certain color requirements and luminosity and therefore, was capable of forming a larger photo. Figure 4.12 shows an example of this composition; the following figure shows the eye of the previous composition.

This type of composition appeals to self- similarity in its own way; each photo detail is, in turn, another photo. This is a very true characteristic of nature; like a forest where each unit is a tree and each part of the tree is a branch, which in turn, self-resembles the tree, as we have seen. This design focus also enjoys acceptance possibly for the already discussed reasons.

These same characteristics are being used in other branches of design especially in architecture.

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[200] M. Hawley, (1997). **Robert Silvers Fotomosaiken**. Wilhelm Heyne Verlag. Muenchen.

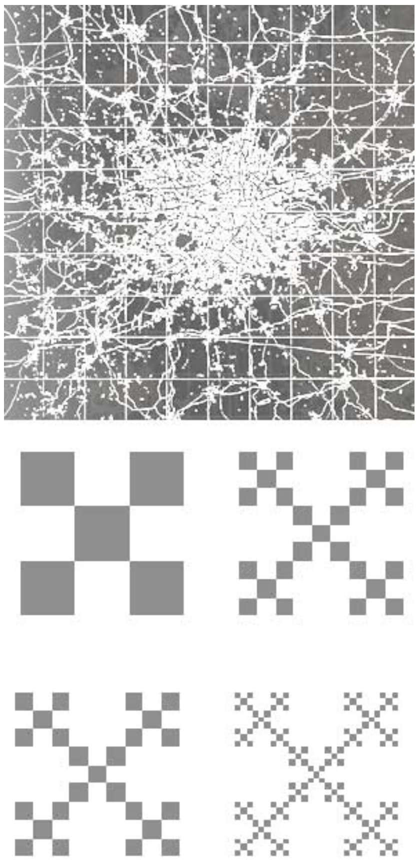

Figure 4.14. Comparative analysis of the a city and a classical fractal growth

"Architectural forms are handmade and thus very much based in Euclidean geometry, but we can find some fractals components in architecture, too.

We can divide the fractal analysis in architecture into two stages:

· **little scale analysis** (e.g., an analysis of a single building)

· **large scale analysis** (e.g., the urban growth).

The little scale analysis comprises:

· **the building's self-similarity** (e.g. a building's component which repeats itself in different scales)

· **the box-counting dimension** (to determine the fractal dimension of a building)"[201].

With this idea in mind, some researchers have begun to develop methodological approaches to the problem of architecture from a fractal point of view. These methodologies mainly focus on urban organization; because it is under constant growth, it adapts well to the fractal analysis.

"Urban planners cannot refer to the circular or linear city, and the cities are amorphous and irregular, without showing an inner organization. Studies about complex phenomena, such as non-linear phenomena, have shown the insufficiency of these traditional concepts. As an alternative, fractal geometry is the only approach with a pure geometric character."[202]

This behavior develops throughout history where cities grow similarly to what is predicted in the fractal calculation. Figure 4.14 shows a view of a city and a classic fractal growth; even in this basic level, the similarities are clear.

"The search for a unified theory of urban morphology has focused on the premise that cities can be conceptualized at several scales as fractals. At the regional scale, rank-order plots of city size follow a fractal distribution and population scales with city area as a power-law. More recently, it has been observed that the area distribution of satellite cities, towns and villages

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[201] N. Sala, (2000). **Fractal models in Architecture: A case of Study.** Proceedings International Conference on "Mathematics for Living". Pages 266-272

[202] M. Caglioni. & G.Rabino, (2003). **Contribution to fractal Analysis of cities: A Study of metropolitan Area of Milan**. Cybergeo: Revue européenne de géographie, N° 260. Theoquant. Milano

around large urban centers also obeys a power-law with exponent ≈ 2. ”[203]

This type of perspective has been used to analyze not only what has already occurred but to design future settlements.

4.4.4 Cellular Automata

A relatively simple technique to model natural behaviors is the cellular automata. This type of similarity makes it possible to reproduce the behavior of biological units from cells, virus or bacteria to animals such as ants or birds.

The Cellular Automata are virtual structures that make it possible to build digital models that emulate some of nature’s complex systems. It is possible, for example, to achieve simple digital models that faithfully represent some physics laws or biological behavior.

In other words, they are strategies to simulate natural behaviors in individuals, which were previously considered to simulate the behavior of cells or molecules.

The first thing that needs to be done is to represent a space in “cells”. Each cell behaves in a relatively simple manner and can change its state (in several defined states) based on the state of the cells that surround it. An important characteristic of these automata is that all cells change state at the same time.

The definition of cellular automata needs to define the following four points:

1. Group of cells. It is necessary to know how many elemental objects or cells will form the population. In general, there are no restrictions on the numbers. The space is formed by a finite group of cells distributed in a regular n-dimensional grid.

The automata can be one dimensional, for example; in these cases, the cells are distributed in a line. If it is two dimensional, the cells are distributed throughout a flat grid, which could be triangular, rectangular, square, hexagonal, etc. There are also three-dimensional or multi dimensional automata; in these cases, the grids are spatial or mathematical, respectively.

[203] R. Carvalho, S.Lida and A.Penn, (2003**). Scaling and universality in the micro-structure of urban space.** Proceedings: 4th International Space Syntax Symposium. London

On occasions, it is important to place them on a geographic region, identifying the cells with their respective coordinates.

2. The neighborhood. The adjacent cells form a cell's neighborhood and it is the same for all of the automata cells.

When the central cell changes, the neighborhood cells are affected; they are the field of action of each cell.

If the objects are associated with geographic coordinates, the criterion to build the neighborhood of an element is usually all those elements that are found at a short distance or radius from the corresponding cell. In this manner, one cell cannot exercise direct influence over the cells that are further away.

3. Group of states. Each cell can be in only one state during a specific period in time. State is understood as a change in the cell's characteristic; for example, color.

The more simple case corresponds to the elements that can only acquire one of two possible states; for example, white or black, alive or dead.

4. Guidelines for local evolution. The state of the cell changes from one instant to another according to a group of evolution guidelines common to all cells.

The evolution guidelines define the system's dynamic. They determine the following state of an element in a determined moment; the previous states are needed as information from the element under consideration and those in the neighborhood. The transition guidelines can be deterministic or statistical.

Once these concepts are defined, we can study an example to better understand how these systems function.

John Horton Conway (1937) developed one of the most popular cellular automata, an automata that he himself denominated the game of life[204].

The game of life is a two dimensional cellular automata on a grid with two possible states per cell. Each cell can be alive or dead (black or white).

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[204] D.C. Dennet, (1996). **Darwin's Dangerous Idea - Evolution and the Meanings of Life**. Tochstone Editions. New York. Page 163

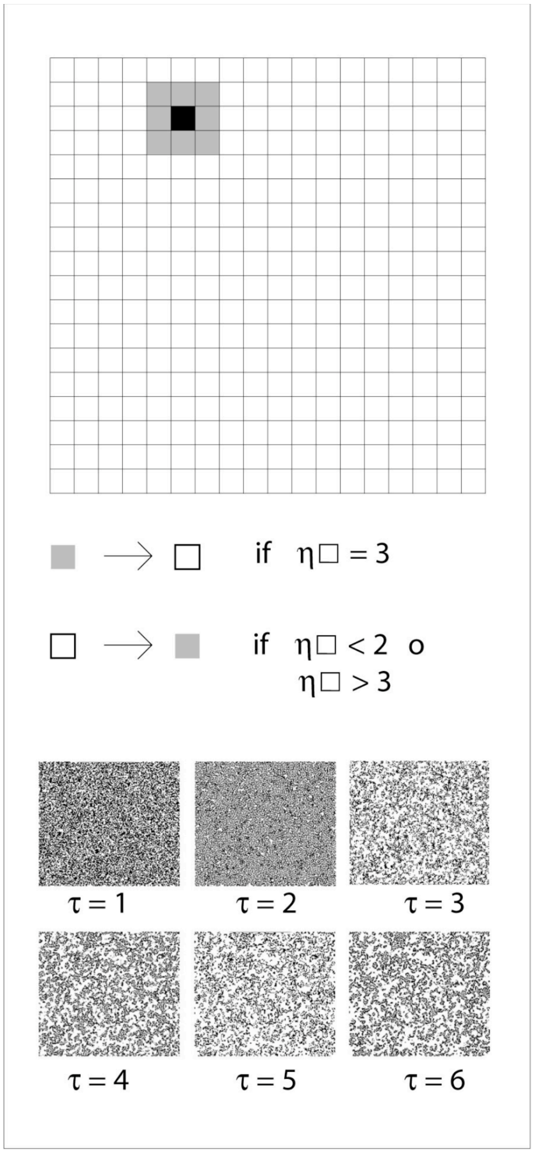

Figura 4.15. John Horton Conway's game of life. The upper part of the grid represents the total group of cells; a cell and its corresponding neighborhood; in the middle are the rules and in the lower part one can observe some periods of the automata's behavior.

An algorithm is applied to each generation that follows these three rules:

1.- Each live cell, with two or three neighboring live cells, survives to the following generation.

2.- Each live cell, with less than two or more than three live cells around it, is dead.

3.- Each dead cell, with three live neighboring cells, revives in the next generation.

Figure 4.15 illustrates this automata. The upper part of the grid shows the total group of cells, a cell and its corresponding neighbor; in the middle, the automata behavior rules and in the lower end, some periods were the automata's behavior can be observed.

The game of life presents final stable configurations, periodic or non-periodic. Some theoretics state that this automata also presents many properties we can find in live organisms, such as:

1. Catalysis (arbitrary construction actions).

2. Transportation (erasing structures and rebuilding them in another cellular space).

3. Structural (like static elements, barriers, etc.)

4. Regulation, defense or even informative.

These virtual automata present sufficient computational capacities to model functional behaviors that exhibit the micro-molecules in the molecular logic of life; in other words, the automata are comparable to the basic components of life, at least how we know it.

These kinds of automata are being used in several forms as tools of design. Like in the case of fractals, the automata are used to generate graphic art and decoration solutions; however, its more extensive use lies in the simulation of urban growth.

"Cellular Automata (CA) is a modeling technique defined on a raster space. Cell states usually represent land use or land cover, and the transition of a cell from one state to another depends on the states of the neighborhood cells. CA have proven to be useful for dynamic modeling… Most of this research emphasized urban simulation in order to understand the urban growth and its form."[205]

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[205] Hammam, Y., Moore, T., Whigham, P. and Freeman, C., (2003**). Towards the modelling of sustainable Cities with Multi-reactive agents**. D.Martin. Publisher: GeoComputation CD-ROM. Proceedings of the 7th International Conference on GeoComputation University of Southampton,United Kingdom.

This type of problem interpretation is very advantageous; this is one of the few existing perspectives of the urban problem that is capable of simulating the intrinsic complexities of this design subject.

“In order to make operational the general model proposed in 3.1 (an evolutionary urban model), cellular automata are specially suitable, for the following reasons:

- Cellular automata, as a tool for urban modeling, are gaining a big popularity, because they imply inputs and outputs in a qualitative and cartographic form, which is, for urban planners, more usual than numerical representations. In addition, this different form of representation allows an easy interface, which show an increasing diffusion.

- The working mechanism of cellular automata is relatively simple, because it consists mainly in the transition rules between one state and another of a cell, which depend on the state of the cell itself and of its neighboring (appropriately defined). This simplicity makes operatively easier the connection with the "learning system".[206]

These tools represent the possibility of using new technologies to support design. In these as in the previous ones, proportion is an intrinsic part of the process and not added artificially. Because these strategies simulate natural processes, proportion, as in nature, shows itself “**as a collateral process effect and not as a cause or initial parameter**”.

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[206] Colonna, A., Stephano, V., Lombardo, S., Papini, L., and Rabino, A., (1998). **Learning cellular automata: modelling urban modeling**. In Proceedings of the 3rd international conference on GeoComputation, University of Bristol, Bristol, UK.

4.5 The Algorithm of Nature - The Symbolic Approach

As we mentioned before — in the section of natural patterns — nature follows specific growth patterns. From a symbolic point of view, this fact facilitates the approach to the problem; that is, in some cases, it is possible to follow, step-by-step, the algorithm of nature and repeat it, accurately, to use its methods of design.

This is the symbolic approach. In this approach, a formal system of representations must be generated, which includes all the needs of the case under study. It is the same system used by a CPU (Central Process Unit) to play chess.

The chess champion was not a human being anymore when, in 1997, Garry Kasparov —who was the world champion at that moment— was defeated by Deep Blue (an IBM computer) in a six-game tournament[207]. This is only a way to say it, because these types of man-versus-machine tournaments do not occur every year.

However, the program used by Deep Blue was a program based on symbolic search; that is, a program that analyzes most of the alternatives existing in a given time (let us say, one play of the game). Then, the program assesses such alternatives and decides which is the best play to make in the next move.

As we see, there is nothing risky or structural here, which is opposed to the neuronal network or genetic algorithmic-based programs. This program is strictly for playing chess; it stores knowledge related to alternatives, good moves and evaluates the opponent's next possible reaction; that is, its acts are very similar to those carried out by human beings when playing chess.

It is not that human beings act only symbolically; in fact, most of our decisions are sub-symbolic, but all our abstract knowledge — as physics and mathematics— is indeed, symbolic knowledge.

We can change from one system to the other and learn by analyzing the data obtained among systems. A commonly known example is that of those disciplines whose execution is path-based.

A gymnast, a musician or a classic dancer begins with symbolic knowledge; that is, during the first years of training, many "recipes" are provided to them concerning

[207] N.J. Nilsson. (2001). **Inteligencia Artificial. Una nueva Síntesis.** MacGraw-Hill/Interamiericana de España. Page 9

the carry out this or that other technique in order to improve one movement. However, after many years, when he/she is prepared to perform, such training becomes repetitive and his/her neuronal networks automate such movements. So much so, that when he/she is interviewed, we hear them saying: “If I am performing and I think about the movements I have to make, I make a mistake".

In other words, he/she lets his/her knowledge act, which is accumulated in the structure of his/her networks, without taking into account the conscious movement sequence that he/she learned at the beginning of his/her training.

According to this example, we can divide knowledge into two great areas: the area where instructions are **sequences** to be followed (the symbolic knowledge) and the area where knowledge is stored in the neuronal networks (the relational or sub-symbolic knowledge).

In this section, we discussed the knowledge that is possible to store in a series of sequences; these “recipes” can be easily translated into algorithms that can be executed by a computer efficiently and rapidly, as in the case of Deep Blue and its chess game.

In design, both types of knowledge are used and recognized: one of them, and probably the most known, is the master who knows how to skillfully combine colors or forms but cannot explain with certainty the *know how* of it; the other is the methodologist, who by applying approach instructions to the problems, can find innovating solutions that can rarely be found outside the method. In spite of this and even though the design methodologies have been researched throughout many centuries, there are no infallible recipes that will give us a definite solution. There is still a lot of "analogical" (or sub-symbolic) hidden in the corners of the most sophisticated design methods.

We are certainly speaking about the design part wherein perception is important; that is, where “proportion” would be used as a design tool. Undoubtedly, an airplane can be designed without resorting to analogical randomness, as it was proposed by some pan-proportionalists, such as Doczi (György Doczi[208]).

Nevertheless, in relation to the perceptual part, it is a little more difficult to identify all factors influencing our decisions, as we mentioned in section 4.3.

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[208] G. Doczi, (1981). **The Power of limits**. **Proportional Harmonies in Nature, Art and Architecture.** 5. Ed. Shambhala Publications Inc. Boston, U.S.A.

Based on a formal point of view, the most important obstacle we face when dealing with design problems (in relation to perception) is the lack of common vocabulary.

There are few artistic activities that share a common language that allows experiencing, discussing, communicating and rejecting of ideas. Music is one of those few examples. Western music is based on a language that has been unified for several centuries and its roots are old. This allows music to be analyzed more objectively than all the other human arts.

The codification's quality is such that we are able to reproduce any work produced five hundred years ago, based on the original instructions of the author. This is the same as being able to paint again any of Leonardo da Vinci's works with the accuracy that only the original instructions would provide.

Probably, their temporary quality (music could not be recorded as such, until approximately one century ago) was the item that allowed for and forced musicians to develop these wonderful codification techniques.

"Concerning any knowledge representation scheme, it is necessary to define the corresponding vocabulary. In relation to semantic networks, efforts have been made to reduce these relations to a minimum number of non-coincident terms (primitive semantics)."[209]

In music, these "primitive semantics" are the notes divided in octaves and their sub-modules and time codification, such as black, round or quaver. Further on, we have the primitive syntax, such as triads or 4/4 or 4/2 times, etc.; that is, a linguistic structure to communicate, discuss and experience the inner side of arts."

A similar effort was made to understand the natural language, including several attempts made to describe all the world's aspects, in primitive terms, which are unique and non-ambiguous representations into which the statements of natural language can be converted into for a later translation into another language or other cognitive acts[210]."

In other arts, such as painting or architecture, any type of approach to the codification problem has not been reached. Nevertheless, if we paid attention to the different disciplines used as major input for these arts, a significant progress has already been made.

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[209] W.M. Gevarter, (1987). **Máquinas inteligentes. Una panorámica de la inteligencia artificial y de la robótica**. Ediciones Díaz de Santos S.A. Madrid. Page 41

[210] Idem

Let us use the theory of color as an example. Definitively, there is enough material to work with, not only with respect to color syntax, but also to its semantics. We know, with pristine clarity, the mechanics of color as light wavelength behavior, which we perceived, as well as the way they mix together and react to different means. We are acquainted with various theories that speak about combining possibilities and the mechanics behind nice or disagreeable chromatic composition.

“In color as well as music, harmony means an aesthetic arrangement of parts to form a pleasing whole; a musical composition, a painting, or a graphic design. All music from Mozart to Madonna consists of the same twelve notes, and all graphics design from Gutenberg to Glaser use the same palette of colors. If the science of color harmony knows which colors to use, the art is knowing what order to put the colors in, and what proportion of each.”[211]

We could say the same about some disciplines, such as the psychology of perception (Gestalt Psychology) or typesetting legibility, for example. In many of these disciplines, which are parallel to design, the way has been paved to theorize and formalize a language that allows us to experience, discuss, communicate and refute.

It is in these aspects that the new design tool-generating experiences take place, and allow us to produce options for specific problems, within "the almost mystical" process of producing new works of art, design or architecture.

[211] H. Chijiiwa, (1990). **Color Harmony. A Guide to creative color combinations**. Rockport Publishers. Rockport Massachusetts. Page 6

4.5.1 The chromatic proportion (ColorCalculator)

4.5.1.1 Proportion of Color

The proportion among colors is a part of proportion that is almost never considered in the conventional sense of the term. However, colors, as well as musical notes, must be carefully proportioned, if one wishes to obtain nice and successful color compositions.

As part of an investigation to burrow into the subject of "correct chromatic proportions", a digital tool was programmed with the aim of combining colors automatically.

Using the theory of color — that is very specific in chromatic ordering — and some general rules on correct chromatic combining considerations, a program was developed to "teach" the computer to develop “beautiful" chromatic compositions".

Before getting into the subject, we must standardize some terms. Generally, color is described based on the three following characteristic:

1. Hue: it is also called shade and it is the chromatic substance of a color, the difference between a violet and fuchsia or red and blue.

Strictly speaking, it is the position of each color in the wavelength spectrum, the primary vibration that makes it red, green, yellow, etc.

2. Luminosity: it is the amount of light in a color, how clear or dark it is. Having a fixed hue, such as red, it ranges from pink, going through pure red, to ocher or dark red. This clarity or darkness is called lightness.

3. Saturation: It is the degree of purity, the chromed substance’s quality and the amount of gray in a color. The more gray, the less saturated; the lesser the gray, the purer the color; in other words, the more saturated it is.

These characteristics define the difference or identity of one color with respect to another color. In some way, they are proportion measures because they define the differences of character, lightness and saturation between one color and the next one. They are also characteristics, which are enough to define a color completely, when speaking about colors in abstract. Nevertheless, if we spoke of chromatic compositions (colors which interact in groups), these characteristics are not enough.

In a chromatic composition, there are other variables that are very important; the **proportion** a color has with respect to another is a good example of these types of variables. The same happens if we speak about colors in design or architecture, where the material also offers connotations that nourish the color concept, since it is not the same to say gray or exposed concrete.

Thus, the most important variables are the following:

4. Mass: it is the amount of color existing in a composition with respect to its total area, and therefore, it is in proportional relation with respect to the other colors that appear in the composition; this item is measured based on the percentage of area covered versus the total area. This is the most evident application of the theory of proportions in chromatic compositions; however, it is almost never studied as such.

5. Chromatic proximity: it is the position of a color with respect to its neighboring colors. Based on a typical phenomenon of the theory of color, such as simultaneous contrast, each color acts in different ways if it is next to one color or other colors.

6. Texture: as the word indicates, it is the texture of the chromed surface, the association of color with a natural or artificial quality of the material, such as concrete, leather, metal, etc.

7. Reflection: it is the value of the light reflection on the chromed surface; it can be thought about in a scale from brilliant to opaque.

A color, in the strict sense of light, is defined based on the three first variables: character, lightness and saturation. However, the mass variable defines the amount of influence this color exerts on the composition.

"When you magnify them (the colors) a hundred times on your poster or layouts, you'll find that a seemingly innocuous color suddenly looks much bolder. Think of apple green, a lovely color in small doses, splashed over the walls of your apartment."[212]

In this way, the area a color occupies in a composition with respect to the area of the other colors is, in fact, the most important for the transmission of chromatic sensation. This variable is, of course, a variable of proportion.

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[212] H. Chijiiwa, (1990). **Color Harmony. A guide to creative color combinations**. Rockport Publishers. Rockport Massachusetts. Page 6

"Even though the amount and quality are often considered unrelated, in arts and music, they appear closely linked. We can even say that 'amount is quality', because, in this case, amount is not only understood as the proportion of a magnitude, such as weight or number, but also as a means to emphasize, to accentuate, and balance...

These studies of amount have taught us to think that, regardless of the rules of harmony, any color "matches" or "combines" with any other, supposing that their amounts are appropriate"[213]

In the same way, the chromatic proximity is a characteristic of the composition as well, and not of the color itself. From the abstract view, a color has the same "fingerprint" (its own, unique and invariable values of character, lightness and saturation); however, its effect on the observer will definitively vary, depending on its neighboring colors.

"Using a theatrical comparison: a group of four separate colors are ' the actors', and, altogether, they constitute 'the company' and, if they are presented or displayed in four different arrangements, they become the 'representations'.

Although hue and lightness are not altered as 'characters', and appear within an unaltered external frame, 'the scenario', will provide four different 'stages' or 'works' so different among them that a same group of colors will be perceived as four different groups presented/displayed by four different companies."[214]

Certainly, we are speaking about one of the classic seven contrasts defined by Johannes Itten in his book "*The art of Color*[215]", in the early 60's: the simultaneous contrast.

"It is about a phenomenon whereby our eyes, exposed to a specific color, simultaneously demands its other, that is that simultaneously demands the presence of its complementary color, and when it is not received, it presents it itself. This is the evidence that proves that chromatic harmony needs to respect the law of complementary colors. As final analysis, each color produces its own complementary simultaneously —

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[213] J. Albers, (1963-1971). **La interacción del color**. Alinza Editorial. Madrid. Pages 59-61
[214] J. Albers, (1963-1971). Page 61
[215] J. Itten, (1961-1994). **El arte del color**, Editorial Limusa, México

due to an analogous law to that static of action and reaction — with the purpose of obtaining balance."[216]

This principle is perfectly coherent with the Paradigm of Tranquility studied in section 3.6; it is practically the principle of minimum energy and (as we said), of the natural tendency of the eye to try to maintain its 'electric' balance to obtain its stability and reduce stress.

For this reason, the characteristic of chromatic proximity alters and identifies a chromatic composition, even though it only specifies the relative position of a color with respect to others.

The last two characteristics (texture and reflection) are more material, and they indicate the specific fact of the use of color in a defined work, such as a building or object. Therefore, we will delve deeply into this subject on another opportunity.

We could almost say that a chromatic composition is defined by the values of its colors in relation to the first five variables, as follows:

1. Hue
2. Luminosity
3. Saturation
4. Relative mass
5. Chromatic proximity

4.5.1.1 ColorCalculator implementation

With these values clear, a method was designed to code these variables in a single instrument.

This method is a preliminary approach to the idea of constituting a common language, which is able to transport or represent a design idea; in this specific case, it is for the chromatic composition codification.

By building a simple grid of 9 by 9-chromed elements approximately, we are speaking about identifying correctly the following:

1. Which colors are present in the (saturation, luminosity and hue) composition?

2. What is the proportion of these colors among themselves (relative mass)?

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[216] A. Marcolli, (1968-1978). **Teoría del campo. Curso de educación visual.** Xarait Ediciones y Alberto Corazón Editor. Madrid. Page 138

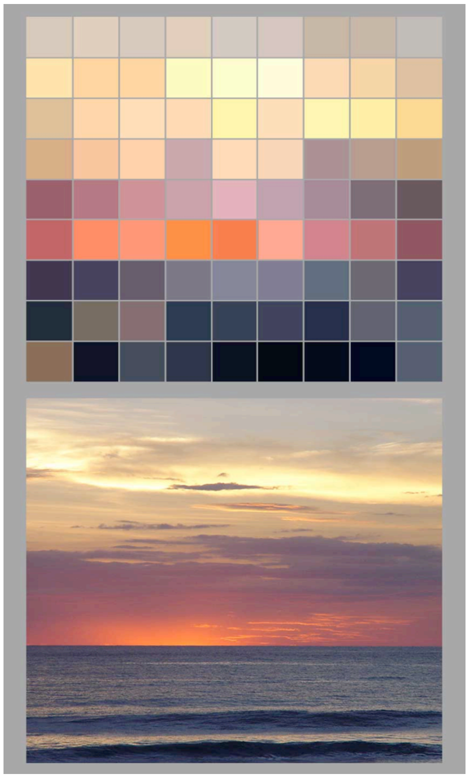

Figure 4.16. Codification of a chromatic composition in a matrix, conserving the hue, luminosity, saturation, mass and proximity of each color of the total composition.

3. Which is the proximity of each color in the composition?

With this practice, an abstract codification is obtained; it exposes and communicates (per each color) the 5 characteristics that we have defined as decisive in a chromatic composition, without making reference to the object that contains them.

In other words, this instrument codifies a chromatic sensation with an exact code, which is applied independently from the receiving substrate or object. Note that the goal is to reduce the chromatic touch sensation (as Harald Kueppers would say[217]) to a controllable and replicated codification, searching for a common language to use the topic of chromatic compositions and its related sensations.

Figure 4.16 shows one of these matrixes where the codification of chromatic sensation was attempted in the exposed photo. Evidently, the result is a chromatic sensation very similar to that, which gave origin to the matrix. This method codifies a chromatic composition with very exact variables. Its codification of five variables per color makes it possible for us to subsequently reproduce it and manipulate it without requiring the references of origin.

The application within our study is obvious; once a formal approach of a chromatic composition is defined, the method is useful to codify, transport, discuss and generate (in our case, automatically) the chromatic composition (the proportion between the colors), without losing its emotional chromatic message and without depending on any substrate of use.

With this problem solved, we could finally focus on the automatic generation of successful chromatic compositions, because without having an exact codification it was not possible to generate the candidates to evaluate.

Now the problem focuses on defining conditions that would build a nice chromatic composition automatically, or at least, one that is not disagreeable for many people. In other words, is it possible to define clear rules that automatically generate nice chromatic compositions?

By making a first approach to this question, we found out that, at least, there are three levels of factors involved in the origin of the sensation of beauty in a chromatic composition:

[217] H. Kueppers, (1980-2002). **Fundamentos de la teoría de los colores**. Ediciones Gustavo Pili, S.A. Barcelona.

1. The first level we will name the physiological level. Based on what we already discussed, the principle of minimum energy may also influence our chromatic sensations. In this way, our visual system (from the eye to the neo-cortex), would favor compositions that represent less stress for the system, as in the case of compositions that show the total chromatic.

2. The second level would be the genetically programmed system: the colors of a ripe mango are beautiful because the fruit is highly nutritive.

3. The third level would be more personal, such as social, historical, cultural and group factors.

If this approach is correct, our program could be involved, at least, in the first level, because the factors participating therein are clearly defined.

The second level is clearly related to the first one and both are based on natural laws. That is, in order to clearly identify a cherry amidst a landscape, the red color is useful since it facilitates hue contrast to the maximum extent, and therefore, it becomes more visible; thus, birds and animals in general, can find it more easily, eat it and transport its seeds far away from the tree that produced them, which is the ultimate mission of the fruit's existence.

"All these schemes are based upon the physical laws, relationships, and inherent structure of the color wheel. It is significant that these color formulation are the heart of the chromatic displays found in nature: red dessert sandstone glowing against a deep indigo sky; the variegated green perennial garden, accented with crimson blooms."[218]

In addition, the chromatic recognition scheme, which originates in the two first levels of the genetic programming, they that are both common to humanity and stable in time (at least, for the next 10 000 years, not so for the next 500 million years). This does not occur with the third level, which, due to its nature, is very complicated.

[218] R. Carter, (1997). **Color & type. Working with computer Type 3**. RotoVision Book. New York. Page 26

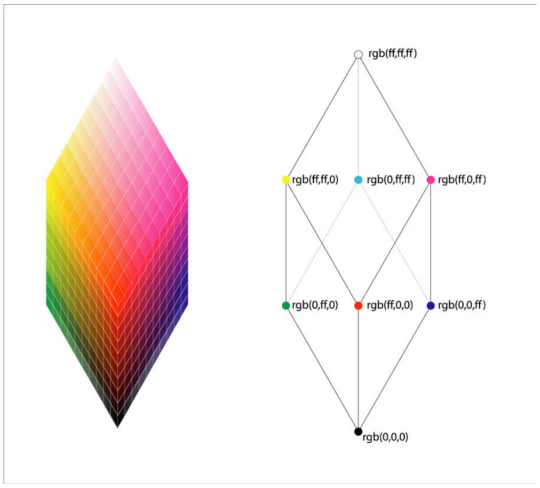

Figure 4.17. Kueppers' rhombohedra represents the organizational system of all visible colors.

Due to the eye's physiology (1st level of the analysis), cells that perceive colors get tired and react producing interesting effects, which are already documented in other studies:

1. When fixedly observing a color for a long time (about 60 seconds) and immediately turning to observe a white surface, the complementary color appears in the form of a "ghost color". (Figure 3,5)

2. A neutral color, let us say a gray square over a red surface, "contaminates" with the complementary color and it is dyed green. (Itten's simultaneous contrast).

These two examples suggest that a chromatic balance would be desirable, at least physiologically, and if such balance were good for the eye's "health". It would be reasonable to think that through the natural evolution, its effect would program a positive aesthetic judgment in relation to such compositions.

That is, if our hypothesis were correct, a given chromatic composition (from the point of view herein stated) would be considered as beautiful by most people.

In order to provide a clear definition of a chromatic composition, it was decided to work within Kueppers' rhombohedra framework (Rhombohedra Color Space) because this ordering adapts very well to the architecture whereupon the colors are used in computer science.

Harald Kueppers defined in his book, "Basic Law of Color Theory"[219], a chromatic space that organized all colors; this polyhedron is based on three vectors that start in a point specified as black and combines the three primary additive colors.

The three vectors — red, green and blue — are combined among them, reach the maximum point of saturation and continue to converge in only one upper point defined as a target. Figure 4.17 shows the three faces of Kueppers' rhombohedra and a scheme of its three-dimensional compositions, with the corresponding values in "rgb" in each vertex.

The "rgb" values (red, green and blue) are a codification of the additional colors and represent the three values of their primaries, specifying their intensities. This codification is especially useful when working with colors in the computer; this is the standard that is used to specify the colors in a conventional monitor. In the

[219] H. Kueppers, (1980-2002). **Fundamentos de la teoría de los colores**. Ediciones Gustavo Pili, S.A. Barcelona.

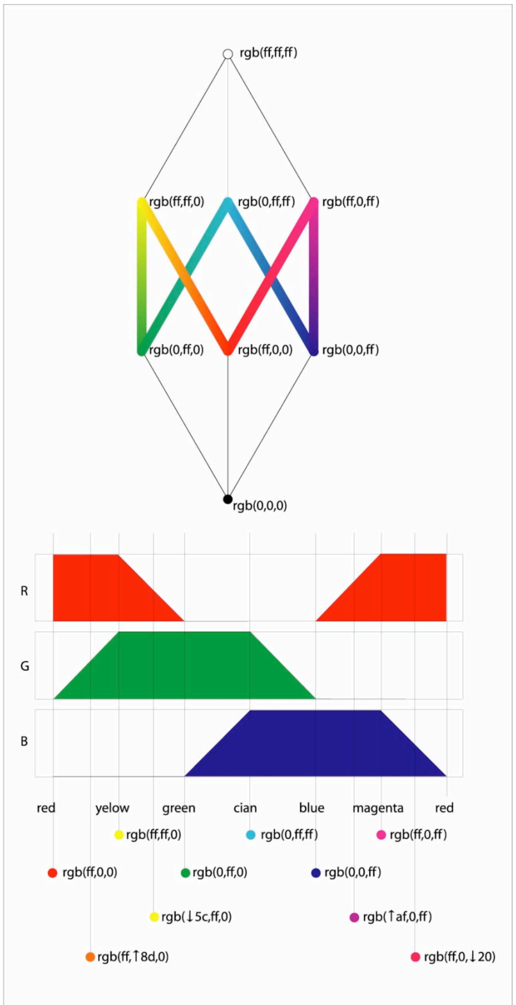

Figure 4.18. Equator of Kueppers' rhombohedra and the representation of the three primary color channels when going through this equatorial line.

figure, the values are represented in hexadecimal annotation with a scale of 0 to ff (which represents a scale from 0 255 in decimal notation). These scales are also the standard scales used to codify colors in virtual means.

Moving inside this polyhedron, it is easier to define the concept of the chromatic composition provided. Our concept of "proportion" for a composition of colors should be based on the five variables studied: character, saturation, luminosity, mass and proximity.

That is, to obtain proportionate chromatic composition, we should try to keep the balance in each variable. In other words, if a color is very luminous, it should be necessarily balanced with less luminous accompanying colors; if a color is highly saturated, it would be balanced with low-saturation colors; if there are many green colors, something red should appear or, if not, blue or yellow, together (character balance).

The chromatic rhombohedra equator represents the conventional chromatic circle wherein the colors are totally saturated; through this line, it is possible to move between the primary colors and secondary additive, thus maintaining its absolute purity.

Figure 4.18 shows this equatorial line and the graph below specifies the behavior of the three primary additive colors —when the solid's equator is crossed through. It is important to observe that in each point of this line, one of the primary colors always maintains the ff (its maximum value); other one of three primary colors always maintain 0 (its minimum value) and a third one varies gradually between 0 and ff. This is the mechanics of primary and secondary color combination with maximum saturation.

In order to meet this condition, no more than two colors must be present (for that reason, one value is always 0), since it would stand for gray, or — which is the same — a saturation value different from the maximum. Likewise, since they are additive colors, the other value, which is always the maximum, "is added" to the third color, which varies gradually.

Whenever luminosity or saturation is changed, color movements take place in the rhombohedra's upper or lower vertex.

Inside of this solid, plain shapes spinning around the inner rhombohedra would define a proportionate composition; each shape's vertex defines a color, which will be part of the composition. (Figure 4,19 above)

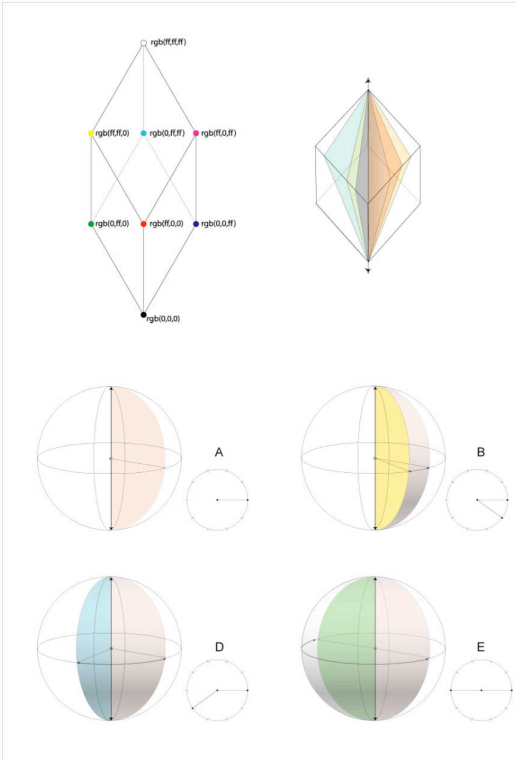

Figure 4.19. Basic models of chromatic combination used in the algorithm. Above: the polygons inside the rhombohedra and below, the spherical schemes (due to the clearness of the diagram) of chromatic distribution.

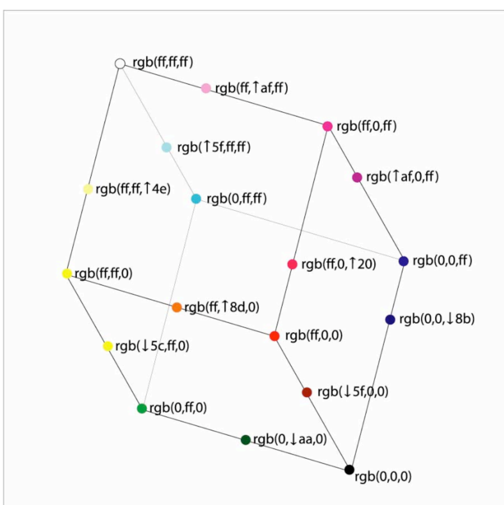

Figure 4.20. Typical "rgb" values of the most important routes taken by the algorithm (the arrows show the behavior of the third primary, with variable graduation, from the equator).

Having a clear idea of the chromatic ordering (based on Kueppers' rhombohedra), and feasible judgments on harmony with respect to color characters (because of the flat shapes in the inside of the solid), we must define which of these colors — and its proportion — must be displayed in our "suggested compositions".

After several analyses of proportion, considering the five variables described, we defined the following steps as a first color automatic combination algorithm:

1. Choose how many colors one wants to use. (1-10 in our example).

2. A "balanced" composition model is selected, either by the computer (randomly) or by personal preferences (interactive mode).

   Models were defined as follows: (Figure 4.19 below):
   A. Chromatic - achromatic
   B. Similar
   C. Contrasting
   E. Complementary

3. The computer selects the colors that will become part of the composition according to:

   • The previous selected model

   • The law of chromatic ambiguity: "no color should have similar luminosity or saturations".

4º The proportion among color areas is defined based on the following consideration:

   Accent Law: it should be an area with dominant hue and another area with abnormal or accent hue. The dominant area should have high mass percentage or a total chromed area; the accent area's mass should have a significantly smaller relative mass.

This means that a high percentage of involved colors must have a chromatic character (hue) similar to the dominant or to the neutral one; the remaining part of the composition should have another character or shade, whenever the model allows it (observe that the first and second model do not allow it).

5º The achromatic can be used freely, because they do not influence the composition's character balance and in our program, they are defined interactively.

With these steps, an algorithm was programmed to go through the rhombohedra by locating the "rgb" values,

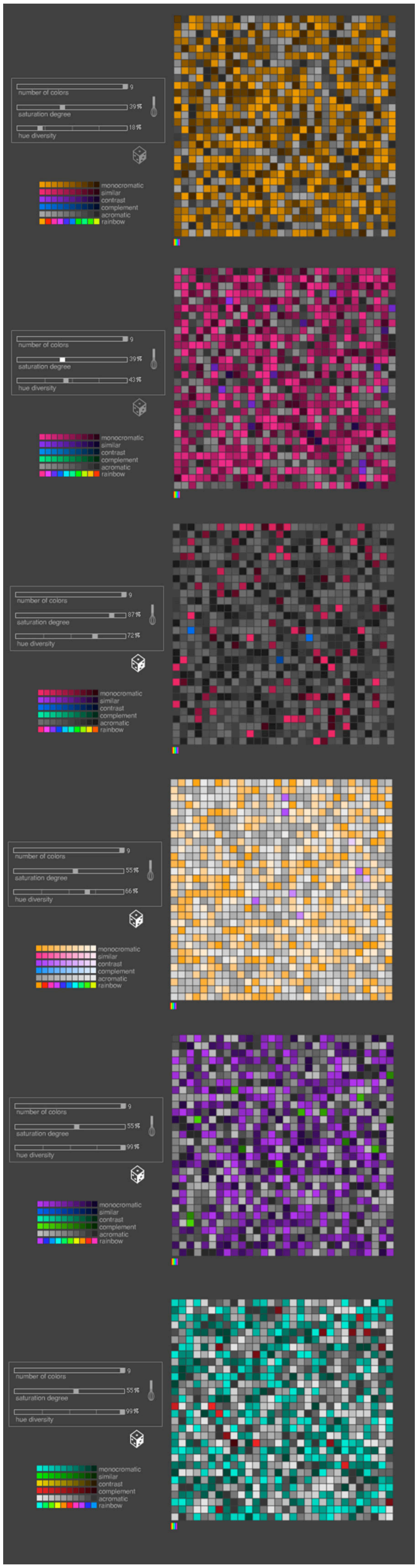

Figure 4.21. Chromatic compositions generated automatically by the “ColorCalculator”.

which were in the different compositions that were generated. (Figure 4.20)

The system we called "Color Calculator" can combine compositions, completely, randomly, or based on interactive parameters, and generate a composition that is considered “nice" by the system. In other words, a user can request the system to combine nice colors in general, or colors that “match" with a previous color selected by him/her.

This makes the system useful, because it gives the user the option to consult chromatic subjects of interest, such as a corporate color or a color that he/she wants as a dominating color in the composition for semiotic reasons.

Other parameters can be established interactively, such as the amount of colors to be combined, degree of global saturation in the composition, and degree of diversity in the hue.

The results seem to be encouraging and the compositions generated by the algorithm are largely well accepted. Figure 4.21 shows the selections for the combination that were completely generated by the computer.

In design, color is a very powerful tool to communicate concepts specially those of emotional nature. This was possibly generated throughout millions of years together with natural conditions — such as night and day — and weather conditions, such as a sunny or stormy day.

In this manner, the light conditions are immediately related to values such as safety, danger, fear or tranquility; therefore, when we see a chromatic composition (analogically, a face), it is almost impossible not to establish a relation with an internal library that will lead us — inevitably — to a value judgment (that is, a declaration in the scale of advisable/inconvenient, good/bad and ugly/beautiful).

This example clearly shows the manner whereby symbolic oriented computer tools can generate good foundations for the study of theory of design. This doesn’t mean that a ColorCalculator system can displace the capacity of the plastic artist masters to combine colors; however, it could provide the opportunity for designers to discuss the topic and give them the opportunity to concentrate on other stages of design.

It could be said that the ColorCalculator is some kind of generative design (as we discussed in the sub-symbolic section), that can provide the designer a series of options to choose from or analyze, with the difference

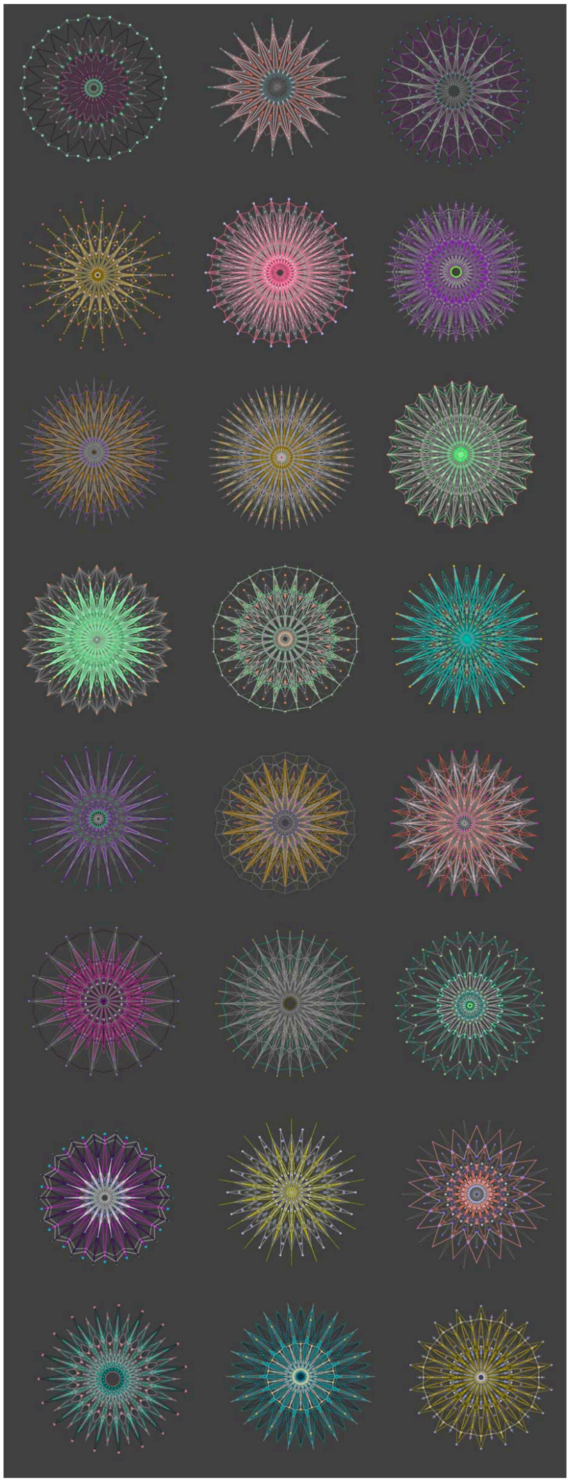

Figure 4.22. Alhambra Project: generation of random shapes with polar symmetry that are used as basis for the application of chromatic compositions, which were generated by ColorCalculator.

that such options are created from an algorithm that filters them, showing only those it considers successful. It is like a generative algorithm with a highly elaborated fitness function that is able to filter the options that are not considered successful; thus, it provides the designer only one careful selection of proposals.

Note that although this example does not tell us about proportion, in the conventional sense of the term, it directly states the problem of generating a selected design option. In addition, we see, one more time, that proportion is an intrinsic part of the solution and not a very important cause of the process; it is inherently mixed with the other parameters that influence the solution and it is almost impossible to isolate it.

Looking for a more specific application, a polar symmetry-based generator, was programmed to be used as basis for the chromatic compositions of the ColorCalculator. The "Alhambra Project" (as it was called due to the evident similarities with some of the compositions of this Islamic art masterpiece), uses the ColorCalculator technology in its interior to decide which chromatic compositions it will use in each one of its proposals.

It is interesting to emphasize that the Alhambra Project designs geometric compositions with polar symmetry; and as a secondary technique, "it chooses" the colors that it considers "suitable" to represent them, just as a human being would.

Figure 4.22 shows a vast range of results from the Alhambra Project; each of these 24 designs is especially interesting.

By carefully observing each unit, separately, we will be able to appreciate both the geometric beauty of the same construction, with its harmonious relations between lines and points and its chromatic selection, with elegant contrasts and harmonious chromatic compositions.

All this beauty was produced artificially —without human intervention— more than in the initial algorithm generation; therefore, it gives a wide vision of the infinite capacity of virtual tools in design.

In fact the amount and facility whereupon the Alhambra Project generated these small design works, inevitably involves an ephemeral character, proper of automatic generation works.

4.5.1 Simulated Annealing

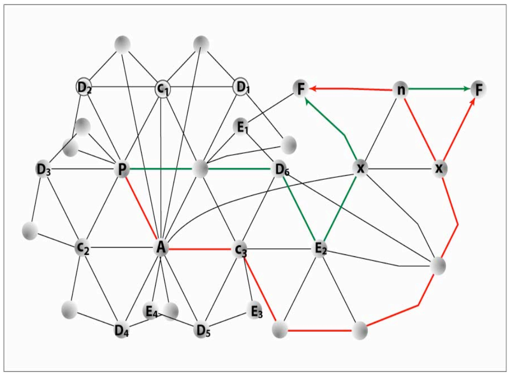

Figure 4.23. General diagram of a graph.

The simulated annealing is another interesting approach developed by computer science specialists to deal with the complex problems of optimization.

This approach has been used successfully to handle with one of the most common problems of computer sciences: graph agreeable drawings.

Graphs are diagrams that represent relations between objects, and in general, nodes and arcs that connect such nodes represent these relations. As an example, let us imagine the graphical representation of Internet servers. Figure 4.23 shows the diagram of a typical graph, which could represent too the different factors that affect a chemical process or a company's organizational chart.

As you may imagine, there are many possibilities of drawing these nodes and their relations; depending on the way it is done, the diagram becomes more legible and evident or less useful and illegible.

This problem has many similarities with the typical design problems; it is a problem with many, many possible solutions, which cannot all be tested since that would take too much time, and it is a problem where if we have a possible solution, it is easy to verify if it is a good or mediocre solution. We already talked about these kind of problems (in Section 4.3.1 Comments on Complexity) and we called them NP or problems with non-deterministic solutions. We said that the design problems fulfill these characteristics exactly, and therefore, they are classified as NPs.

The interesting aspect of designing graphs nicely is that, precisely, this computer problem strives for the "most beautiful" graph, and therefore, it was approached based on such a point of view[220]; in other words, we should use the adjective "most beautiful" as a parameter.

Another singular aspect of this approach is that, in order to solve the problem, it resorted to the simulation of chemical processes (specifically, metallurgical), which are based on the principle of minimum energy. Based on this other perspective, the problem is also coherent with the aspects discussed regarding natural proportion, beauty and processes. That is, the problem is focused trying to represent the graph as a chemical process and looking for the minimum energy state to solve it.

[220] R. Davison & D.Harel, (1996). **Drawing graphs Nicely Using Simulated Anneling**. ACM Transactions on Graphics, Vol. 15, No. 4. Pages 301-331

This approach to the problem of beauty parameter provides some evidence for one of our hypotheses: a solution that requires less energy is nicer than one that needs more energy.

The technique's origin is in the heuristic processes that attempted to simulate the behavior of a group of molecules, which were balanced from the beginning, at a specific temperature, and which were exposed to cooling. It is based on the crystallization phenomenon mentioned in the second part of our study.

As we said, if a material cools slowly, its molecules have the time to allow their "weak forces" to arrange in the position of less energy. A sudden cooling, however, would block the system in a value of greater global energy, which would also correspond to a state of descending order. In contrast, a cooling that is sufficiently slow or annealing derives in a more ordered state and less energy. The simulated annealing program is aimed at reproducing this phenomenon.

In fact, the annealing method is used in the metallurgical industry to obtain more resistant materials, with more ordered crystallizations to improve the material's qualities. Concerning industry, the process consists of "melting" the material by warming up it at very high temperature, whereby molecules acquire a random distribution within the material's structure and the system's energy rises at high levels. Then, the temperature is reduced slowly from one stage to the next stage, and so on, so that these molecules have time to reach their balance (that is, to allow the system to reach optimal configuration beyond that temperature). After several stages, molecules form a highly regular crystalline structure, and thus, the material reaches high resistance and global minimum energy.

This scheme was originally used by Metropolis[221] (the original algorithm of the 50's whereupon the simulated annealing was based) and which was later generalized to problems of combinatory optimization by Kirkpatrick [222] and almost simultaneously by Cerny [223].

"The foundation of this control is based on Metropolis's work (1953) in the field of statistical thermodynamics.

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[221] N. Metropolis, A. Rosenbluth, M. Rosenbluth, A. Teller, E. Teller, (1953). **"Equation of State Calculations by Fast Computing Machines"**, J. Chem. Phys.Vol.,21,# 6. Pages 1087-1092

[222] S. Kirkpatrick, C. D. Gelatt Jr., M. P. Vecchi, (1983). **"Optimization by Simulated Annealing**",Science, Vol. 220, #4598. Pages 671-680

[223] V. Cerny, (1985). **"Thermodynamical Approach to the Traveling Salesman Problem: An Efficient Simulation Algorithm"**, J. Opt. Theory Appl., Vol. 45, # 1. Pages 41-51

Basically, Metropolis modeled the process of annealing — hereinafter mentioned — simulating power changes in a particle's system, whereas temperature decreases, until it converges (frozen)"[224] in a stable state.

4.5.1.1 The algorithm

In order to use this idea to embellish graphs, it was necessary to make a representation of the graph's problem with the elements of the slow and orderly crystallization phenomenon of liquids.

In this case, random drawings are made of a graph maintaining all its structural characteristics that we want to represent; that is, we drew all the nodes, with their arcs connecting to the corresponding nodes, disorderly and carelessly.

The following step is to measure the system's total energy; in other words, the energy related to the random representation made. For this, we needed to assign it a score and evaluation in relation to the extent it approaches our concept of a nice graph (obviously, it is expected that the first drawing made at random will obtain a bad scoring).

Certainly, the problem is how to score it; at this point, we must define the parameters that we can say are characteristics of a nice graph. For example, Michael Coleman provides us with an interesting list of possible qualifiers:

"A list of layout aesthetics derived from the literature and common sense is as follows:

- Nodes should no be too close together
- Nodes should not be too far apart
- Edge lengths should be short
- Edge lengths should not be too short
- Edge lengths should be uniform
- Edge crossings should be avoided
- The angle between incident edges should not be too small
- Nodes should not be too close to edges
- Nodes should be within a bounding box
- Nodes should be near the center of a bounding box
- Nodes should be distributed evenly within a bounding box
- The width of the layout should be minimized…

[224] K.A. Dowslnad & B.A. Díaz, (2001). **Diseño de heurísticas y fundamentos del recocido simulado**. Revista Iberoamericana de Inteligencia Artificial No 20, 2001. Pages 34-35

- Inherent symmetry should be reflected by a layout."[225]

With these parameters in mind, we focus in generating a function that provides us with an evaluation of a specific graph. That is, we evaluated each parameter in that graph and we considered them somehow significant or important; we added them together and obtained the global qualification of the graph we are studying. This function is called aesthetic function and it describes a "candidate" we wish to evaluate.

"Given a vector of aesthetic functions

$f = (f_1, f_2, \ldots , f_k)$

Each of which measures undesirable features of graph layout, our problem is to find the layout $x \in X$ that finds minimal values for the vector,

$[f_1 (x), f_2(x), \ldots , f_k(x),]$

Unfortunately, the aesthetic functions cannot generally be simultaneously minimized. In the AGLO (Aesthetics Approach to Graph Layout) model, multiple aesthetics are combined to give composite aesthetics. Currently AGLO implements an adding composition:

$$f(x) = \sum_{i=1}^{k} \omega_i \, f_i(x) \qquad (*3)$$

where all ω_i are positive constants. Intuitively, in order for adding composition to make sense, we agree to a constant trade-off between our various aesthetics.
If we use

$f(x) = f_1 (x) + f_2(x)$

as our value function, for example, then we are willing to trade a decline of one unit in $f_1 (x)$ for an improvement of one unit in $f_2 (x)$ and vice versa"[226]

If this formula makes us remember our own deduction (*2) stated in section 4.3 (Beauty, a formal approach), it is not mere coincidence. Precisely, it is the formal interpretation of an optimization problem, which in this case is beauty. Based on this interpretation, our statement would be the following:

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[225] M.K. Coleman, (1996). **Aesthetics-based Graph Layout for Human Consumption.** Software-Practice and Experience, Vol. 26, No. 12, December. Pages 1415 - 1438
[226] Idem

$$\beta(x) = \sum_{i=1}^{k} \omega_i \beta_i(x) \qquad (*4)$$

Having cleared our evaluation function, we can go on with our algorithm. We said that:

1. A drawing of the graph under study is produced randomly.

2.The energy of this drawing is calculated (using the previous formula of beauty).

3. Immediately after, a random change is made in the drawing (the size of this change is generally defined by the temperature level existing at that moment).

4. This change generates a new representation of the graph and its energy is calculated again.

5. Now the two energies (or evaluations of beauty) are compared with the two configurations; if the new configuration's energy is less than the previous one (here, a step was made towards the solution), it is considered as the main one and the cycle starts again in point 3.

6. On the contrary, if the energy of the new configuration is greater than the previous configuration's energy, the difference is compared with a reference threshold that depends on the prevailing temperature (this strategy is used to avoid local minimums); if the difference is less than this value, the new configuration is considered as the main one and it goes back to step 3. If the difference is greater than the threshold, the new configuration is rejected and goes back to step 3
with the same configuration used to start the cycle.

7. This instruction cycle (3 to 6) repeats a defined number of steps (until the balance level is reached in this stage or temperature). Then, the temperature is reduced; in our case, this means to reduce the threshold that accepts the differences of energy (step 6) and the magnitude of the randomized changes in
step 3.8.

8. Another instruction cycle is repeated (3 to 6) using the new temperature and, again, it goes to step 7.

9. All the procedure is repeated according to a defined number of times or until an acceptable lower global energy of the system has been reached. In other words,

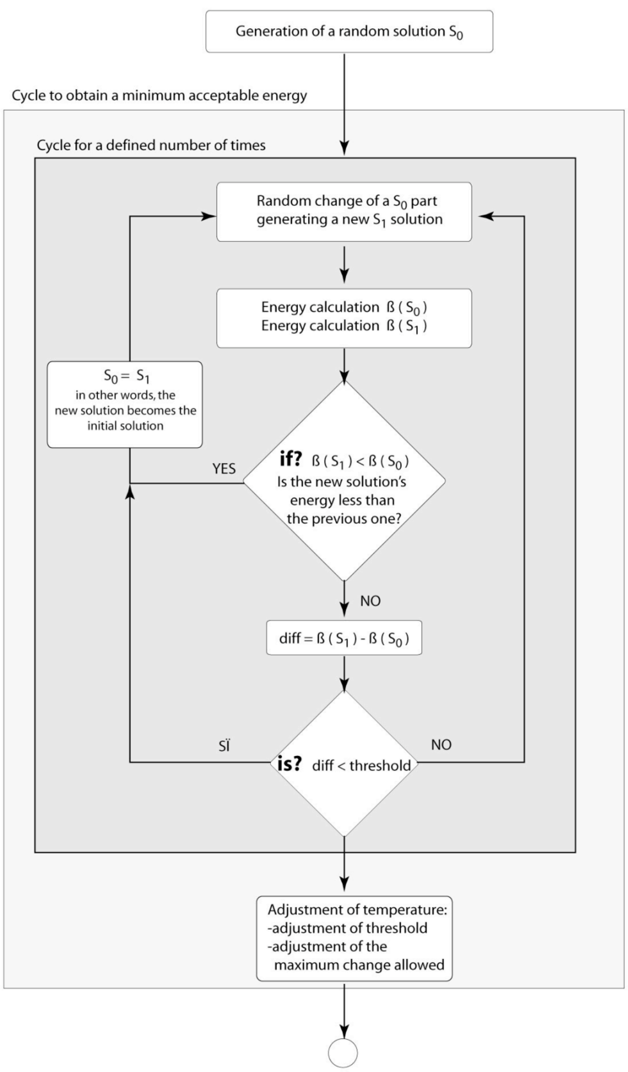

Figure 4.24. Basic flow chart of simulated annealing algorithm.

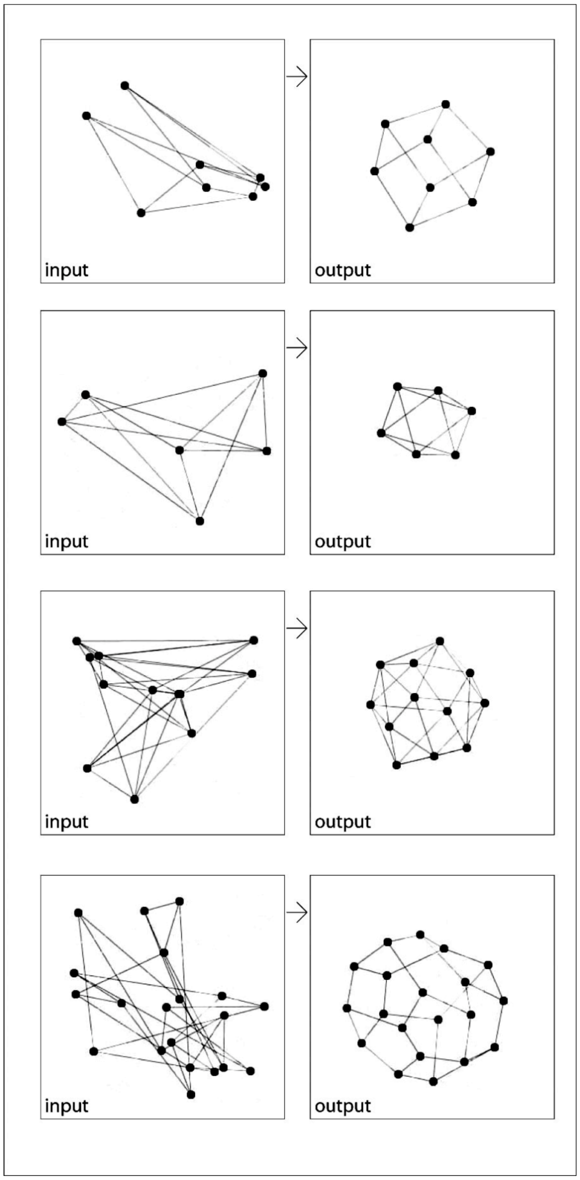

Figure 4.25. Examples of graphs redrew by the process described; the left boxes show the graphs provided as input to the system; the right boxes show the solution proposed by the system.

until the graph has reached acceptable levels of beauty (calculated by the function mentioned).

Figure 4.24 shows the flow chart of the algorithm; there are two gray boxes that show two cycles: an external cycle — wherein the temperature lowers and the size of the changes made is controlled — and an internal cycle wherein the small changes are made several times, until the energy is stable in relation to each temperature.

These cycles are the inheritance of the annealed industrial process since it reduces temperature; you must wait until molecules take their place in that new state of energy; then, temperature is lowered again and you must wait again. Actually, the process is repeated until the material is solidified.

Using this procedure, a system was programmed to redraw some common graphs; figure 4.25 shows four examples of graphs worked by the algorithm. The left boxes show the graphs provided as input to the system; the right boxes show the solution proposed by the system.

At first sight, more tension is observed in the left graphs than the graphs on the right side. Despite the fact that it is the same relations (or arcs) that connect with the same amount of nodes, the graphs on the right are much more legible, balanced and agreeable than those on the left.

In many aspects, this result affirms many thesis we used throughout the study:

1. The state of minimum energy in a composition; it is generally perceived as more beautiful.

2. Symmetry and balance are some characteristics that decide, to a greater extent, the judgment on aesthetic value than other specifics, such as proportion.

3. Beauty is susceptible to be formally expressed by a function of the following type:

$$\beta(x) = \sum_{i=1}^{k} \omega_i \beta_i(x)$$

4. It is possible to program algorithms that evaluate a design solution and improve it aesthetically, as evidenced in figure 4.25

5. The golden proportion, and other proportional systems are not more than a collateral effect of other parameters and processes. For example, in the last graph of figure 4.25, a series of pentagons were generated, forming a dodecahedron (according to the relations of the initial graph). This configuration exposes a series of golden proportions — as explained in the second part of our study — nevertheless, they become evident as a result of an energy reduction process and not as a previous parameter.

It is not difficult to identify a design problem as a graph worked in this subject. If we tried to systematize a problem in architecture, for example, we could consider the nodes as the rooms required for a certain project (bedrooms, baths, corridors, garage, etc.) and the arcs, as the obligatory connections between these rooms (bedroom -- bathroom, for example).

In this way, it is not difficult to think that the computer (by a process of simulated annealing) could propose a series of solutions in relation to the location of rooms, which could be useful for the architect. The proportions and arrangements among rooms would be made to obtain the lower tension among them; this would certainly result in more symmetrical, balanced, **proportionate** and harmonic organizations. These types of solutions are indeed hard to be found by using the trial-error system that we usually use in design.

We could think the same about the organization of displays or functions of a household electric device; it is possible that these and many other complex situations of design problems can further be represented in the form of relations between elements, and therefore, easily analyzable by a program of the type herein explained.

Going beyond this, we could think about defining a series of parameters to evaluate a design at the perceptual level, and then, to generate a solution randomly (as in the examples provided or in generative design) and let the computer "lower the energy of the system" until a more harmonious solution is proposed.

These are the kinds of tools “to proportion” that seem to be the future of design methodologies. These are not the typical tools used by the designer himself to optimize a solution by re-dimensioning, reorganizing or reproducing it, but smart tools by which the designer is allowed to define a design proposal or problem. Based on this material basis, the computer tool is able to work, improve and propose new solutions that are closer to the optimal solution than the original proposal. Thus, in a dialectic communion between computer tools and the designer, the design process could be much more efficient than those we have observed until now.

4.6 Conclusion to the fourth part

In this fourth and final part of our study, we focused on the future perspective of proportion tools. However, since we found out that proportion, as a tool, is not absolutely reliable, it was necessary to redirect the future perspective in order to deal with alternative tools for design methodology, in general, and for the use of proportion, in particular.

To classify the possible new approaches, we based ourselves on a classification generally used for artificial intelligence: the distinction between symbolic or algorithmic knowledge and sub-symbolic or structural knowledge. Indeed, although this classification derives from a discipline that is not related to the design, such classification is not strange in pedagogy or methodology of design.

As we said, most schools, academies and universities that teach design have two types of approaches that can be clearly distinguished in the programs (although they are not consciously classified in that manner):

1. The “hands-on” approach, which is very common in artistic academies, and

2. The methodological approach, which is more related to engineering and it is based on more rationalized methods.

Human knowledge is, in a certain way, divided into two types of knowledge. The structural knowledge, based on experience and the algorithmic knowledge based on “recipes” learned consciously.

Knowledge can move from one system to the other, either individually or collectively. A simple example is the knowledge related to learning how to drive a car; first, we receive all kinds of instructions regarding the car’s instruments and devices. In this first stage, it is not easy to think about everything: blinkers, gears, accelerator, clutch, etc. As time goes by, the student begins to automate everything he or she has learned to the point that the average driver, with more than two years of driving experience, never thinks about the gear changes or the sequence in which they must be carried out, only when one of them fails.

This is a good example to show how symbolic knowledge progresses to the automatization phase and becomes sub-symbolic. It is the approach of methodologists who after teaching the students of design subjects such as theory of structures, theory of proportion, color, materials, etc., expect that the experience and practice cause the student to be able to mix everything unconsciously in his/her future projects.

In the same way, there are some examples related to the manner whereby knowledge moves from unconscious or sub-symbolic phase to the conscious phase. For instance, a child cannot say a word during its first months of life and for a long time, only listens to people talking; after some months, about two years in general, it finally begins to say its own words, then phrases and finally, elaborated linguistic constructions. Years later, the child enters the school and discovers all the use of phonemes, syllables and phrases that he or she already knew unconsciously and uses them for reading and writing, learning, making this process a "prescription".

An example of the design pedagogy would be an academist student who after many years of carefully copying and observing teachers, begins to create works in his/her own style, which after many years can be defined as a set of identifiable characteristics.

Based on this classification of knowing and learning design, we concentrated on identifying some of the new tools useful for design as development instruments in the near future.

The neural networks, for example, are computer science systems able to learn patterns and recognize them. These patterns go from the recognition of faces in photographs and videos to the identification of medical cases.

This tool faithfully simulates the operation of the mechanism that allows us to recognize beauty and golden proportions; it is a mechanism of pattern recognition by comparison, in a structure of neuron network. Therefore, we could think that in the short term, the development of tools, based on neural networks, can provide value judgments about design proposals.

That is, more than having a methodology to measure, proportion or define a design (as we have done for centuries), we would have a type of virtual assistant that is able to advise us in relation to what could be more successful in a design.

In addition, a sub-symbolic approach is the one used by well-known tendencies of generative design. They use algorithms, based on strategies of evolutionary computer science or genetic algorithms, to generate multiple design options for a specific problem.

Even though these systems are in the experimental phase, they have already generated designs that have been implemented in areas as architecture and industrial design.

"Evolution is now considered not only powerful enough to bring about biological entities as complex as humans and consciousness, but also useful in simulation to create algorithms and structures of higher levels of complexity than could easily be built by design. Genetic algorithms have shown to be a useful method of searching large spaces using simulated systems of variation and selection"[227]

The most important problems of these systems are, even today, related to the lack of capacity to design a good fitness function that is capable of selecting or pre-selecting a large number of design alternatives generated from a generative-genetic system. However, based on other approaches of the problem already mentioned, the identification of beauty evaluation functions has also been studied with some level of success.

On the other side, the fractal analyses — such as cellular automata — already provided results in many design branches, especially in urban and architectural design. These represent powerful tools that adapt much more to the problem's reality than any other proportion system did in the past. Finally, thanks to these techniques, it is possible to analyze, simulate and project design problems from a theoretical framework, without losing consistency, a condition that had not been possible until now.

On the other hand, symbolic approaches have given some results already; two examples of this type of systems were submitted.

The ColorCalculator is a system (developed by the author), for the automatic combination of colors. We started by stating that it is not by chance we begin with the investigation of automatic color combination alternatives, because this is one of the disciplines of design theory, which is better afforded with language analysis level.

Based on physiological premises to evaluate the proportions in chromatic combinations, the system is able to combine colors at an intermediate successful level. Although we are not trying to say, with this

[227] K. Sims, (1991). **Artificial evolution for computer graphics.** Computer Graphics. Vol **25** #4. Pages 319-328. (ACM SIGGRAPH '91 Conference Proceedings, Las Vegas, Nevada)

example, that a system of this type is ready for the designer to depend on it completely, the system is acceptable to suppose, based on these first results, that the successful color combination may be handled by means of artificial intelligence and can become a tool of common use in a near future.

Indeed, there is information about the implementation of a small project able to generate geometric constructions successfully and use the technology of the ColoCalculator to decide the colors for these constructions. It is interesting to point out that the 'Alhambra Project' designs geometric compositions with polar symmetry; to the interior of this process and as secondary technique, "it chooses" the colors that it considers "suitable" to represent these geometric compositions, just as a human designer would do it.

Here, two types of the hypotheses we provided in our study are mixed:

1. First, the generation of successful geometric constructions, that are based on natural concepts such as symmetry, repetition and fractal techniques is able to generate, in a completely automatic manner, compositions that are largely perceived as beautiful

2. Second, the use of color automatic selection techniques to "color" such constructions.

This is a good example to illustrate the manner whereby these systems can, in a future, combine different technologies to obtain a more complex task in the field of CASD. Note that we are not longer speaking about computer aid design (CAD) but of computer-assisted design (CASD).

It is possible, just as it has happened in other branches of computer sciences, that the solution to the generation of design support tools is a smart combination of different techniques as those herein provided. Let us imagine a system that generates basic proposals with genetic algorithm techniques, which are soon improved with a simulated annealing, and presented/displayed with chromatic combinations that are derived from symbolic techniques. This would be closest to which it is made at the moment in relation to the development of design (where the designer mixes all his possibilities and generates the proposals), and therefore, the most logical route to follow.

Finally, we discussed the well-known technique of simulated annealing, which is used, among other things, to draw graphs nicely. The technique simulates the industrial cooling of materials that is made in a controlled manner to obtain a state of minimum energy in the final crystallization.

The interesting aspect of this technique is that it has many aspects in common with the problem "to embellish" or generate nice design options. For these reasons, this technique summarizes some of the approach strategies related to the problem of artificial design generation.

Among the most useful aspects of these similarities, we have the following:

1. The formalization of the aesthetic evaluation function for the valuation of intermediate solutions. This is indeed the function we said that required generative or genetic approaches and that could be useful for the pre-selection of the options generated.

2. The association between states of minimum energy in a system and "the beautiful" states, which is a hypothesis that we proposed in this study.

3. The production of solutions that show different types of classic design parameters such as symmetries, golden proportions, chromatic balance, formal coherence, etc. as a collateral product of the generation method and not as a previous condition of the system.

4. The clear possibility of automatically improving the design, from the formal point of view, with the appropriate approach at the level of artificial intelligence.

Certainly, nobody is proposing that, in a near future, computers will be capable of designing autonomously and independently. However, it is possible to conclude from these investigations, that the tools it will use to face the problem of design, in a near future, no longer will be the innocent proportional systems, so glorified since Vitruvius to Le Corbusier, nor the endless and inefficient trial and error processes to which we are used to, but rather a true computer-assisted design (CASD).

In this scenario, the designer will use the computer as his/her design assistant. This virtual assistant will be able to generate many valid alternatives for design, on which the designer can select or recommend one or another so that they can later be delved into; a scenario where the design process is really an interactive process between the designer and the computer.

In this interactive process, the tasks are divided in accordance with the capacities of each one, as in any good work team. Thus, the designer would take care more of selecting and discriminating works of proposals, and his or her virtual assistant focuses on the most tedious and repetitive processes, as the trial and error works, which are activities wherein the computer is, by nature, faster and more efficient.

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