Abstract

This thesis can be divided into two parts. The first part, containing Chapters 2 and 3, deals with balanced–exact sequences and the functor Bext. We will use this functor to define B–cotorsion pairs in analogy to cotorsion pairs which are defined using the functor Ext. The second part (Chapter 4) is devoted to the study of abelian groups A such that $\mathrm{Ext}(A, R) = 0$ for some rational group $R \subseteq \mathbb{Q}$. We will use combinatorial and set–theoretical methods to show that the characterization of these groups depends on the underlying model of set theory. In the following we will describe the content of the chapters in more detail.

The first chapter is on basic definitions and results which are elementary for the following chapters.

In Chapter 2 we shall prove the result of Eklof and Trlifaj, saying that for every module B over any ring there is a related module A such that $\mathrm{Ext}(B, A) = 0$, replacing Ext by Bext. We will use it to construct groups A such that $\mathrm{Bext}(A, A) = 0$ but $\mathrm{Ext}(A, A) \neq 0$. In particular, we will construct an infinite rank Butler group A such that $\mathrm{Bext}(A, A) = 0$ but $\mathrm{Ext}(A, A) \neq 0$.

Chapter 3 is devoted to cotorsion pairs for Bext, called B–cotorsion pairs. In analogy to Göbel, Shelah and Wallutis (2001) we will show that every power set can be embedded into the lattice of all B–cotorsion pairs. Hence there exist ascending and descending chains as well as anti–chains of arbitrary length in the lattice of all B–cotorsion pairs. In Section 3.2 we investigate the B–cotorsion pairs cogenerated by a rational group. Bican and Fuchs (1992) called a torsion–free group A an R–group if $\mathrm{Bext}(A, R) = 0$ for a rational group R. We will use their results on R–groups to show that all B–cotorsion pairs singly cogenerated by a rational group are incomparable.

In the final chapter we will characterize the groups A such that $\mathrm{Ext}(A, R) = 0$ for a rational group R. Since the case $R = \mathbb{Z}$ yields the Whitehead groups, we call such groups R–Whitehead groups. Assuming $V = L$ we will prove that every torsion–free R–Whitehead group A is R_0–free, i.e. the R_0–module $A \otimes R_0$ is free (here R_0 denotes the nucleus of R). In Section 4.4 we show that there is a non–R_0–free R–Whitehead group of cardinality $\aleph_1$ if and only if there is a ladder system on a stationary subset of ω_1 which satisfies 2–uniformization. This shows that in any model of set theory there is a non–free Whitehead group of cardinality $\aleph_1$ if and only if there is a non–R_0–free R–Whitehead group of cardinality $\aleph_1$.