## Abstract

This thesis is divided into two independent chapters which are, nevertheless, combined by the common subject of products and dual modules. The first one is on submodules of the Baer-Specker-module  $P = \prod_{i < \omega} Re_i$  which are also dual modules, while the second part provides a discussion on products, reduced products, and the commutativity of products with respect to the Chase radical within the category Z-Mod of abelian groups. Furthermore, in both parts we use combinatorial and set-theoretic ideas for the constructions and proofs. In the following we shall separately describe the contents of each chapter in more detail.

The first part is devoted to dual modules. Several authors considered abelian groups H, which can be represented as dual groups  $G^* = \operatorname{Hom}_{\mathbb{Z}}(G, \mathbb{Z})$ . The existence of such groups H is a non-trivial problem in abelian group theory. Here, we will concentrate on this problem in the context of R-modules (Ra countable domain containing a multiplicatively closed subset S suitable for defining a linear Hausdorff topology). In fact, we will search for dual modules H within the lattice of submodules of  $P = R^{\omega}$ . Recall, given an R-module G, its dual  $G^*$  is defined as  $G^* = \operatorname{Hom}_R(G, R)$ . Moreover, H is called a *dual module* if  $H \cong G^*$  for some G; we then also say that G is a *primal module* of H.

In the second chapter, we consider products and reduced products of abelian groups. In particular, we investigate the behavior of the Chase radical with respect to products. The Chase radical, defined by  $\nu G = \bigcap \{ \ker(\varphi) \mid \varphi : G \to X, X \aleph_1\text{-free} \}$ , is a famous example for radicals in abelian group theory. It provides a criterion for testing  $\aleph_1\text{-freeness}$  of groups.

As for any radical, it is natural to ask the following question:

"What is the minimal cardinal  $\kappa$  such that the Chase radical  $\nu$  does not commute with products with index set of size  $\kappa$ ?"

This means, we want to find the minimal  $\kappa$  for  $\nu$  such that  $\nu \prod_{\alpha < \kappa} G_{\alpha} \neq \prod_{\alpha < \kappa} \nu G_{\alpha}$  for some family  $\{G_{\alpha} : \alpha < \kappa\}$  of groups. Here we will prove, that the Chase radical does not commute with products over antichains of types of length  $\aleph_1$ . This finally proves that the exact bound equals  $\aleph_1$ . Moreover, our investigations also provide additional information on the  $\aleph_1$ -freeness of reduced products over rational groups. More precisely, we will show that a reduced product of rational groups is  $\aleph_1$ -free if and only if it is  $\mathbb{Z}$ -homogeneous; this property can also be characterized via conditions on the original product.