In this work we study Seshadri constants on ruled surfaces. In the case of $\mathbb{P}^1 \times \mathbb{P}^1$ we study Riemann-Roch expected curves in the context of the Nagata-Biran Conjecture. This conjecture predicts that for a sufficiently large number of points multiple point Seshadri constants of an ample line bundle on algebraic surface are maximal. Biran gives an effective lover bound N_0. We construct examples verifying to the effect that the assertions of the Nagata-Biran Conjecture can not hold for small number of points. We observe also that there is a strong connection between the Riemann-Roch expected curves on $\mathbb{P}^1 \times \mathbb{P}^1$ and the symplectic packing problem. Biran relates the packing problem to the existence of solutions of certain Diophantine equations. We construct such solutions for any ample line bundle on $\mathbb{P}^1 \times \mathbb{P}^1$ and a relatively small number of points. These solutions geometrically correspond to Riemann-Roch expected curves.

Finally we discuss in how far the Biran number N_0 is optimal in the case $\mathbb{P}^1 \times \mathbb{P}^1$. In fact we conjecture that it can be replaced by a lower number and we provide evidence justifying this conjecture.

Coming from the other end, motivated by Hwang-Keum, Szemberg-Tutaj-Gasińska we study impact of (low) Seshadri constants on the geometry of the underlying surface. First we study multiple point Seshadri constant and give a sharp upper bound on Seshadri constants resulting in detecting a fiber structure of the surface. Such a bound was given in the case of single point Seshadri constants by Szemberg and Tutaj-Gasińska. In that case we show that the only example satisfying their bound is a cubic surface in $\mathbb{P}^3$ and thus a better bound holds for all other surfaces.