In the first part of this thesis we study indecomposable vector bundles of degree zero over an elliptic curve. We show that every such bundle generates a ring and a Tannakian category, such that the ring and the group scheme associated to the Tannakian category are of the same dimension. Furthermore we show that this result does not extend to curves of genus 2.

In the second part we consider those flat bundles over a compact Kähler manifold, whose monodromy representation factors through the second power of the augmentation ideal. We show that they are extensions of trivial flat bundles, and give a condition on their de Rham cohomology group, such that the corresponding Higgs bundle via the Simpson correspondence has Higgs field equal to zero.