Robust Decentralized Control of Power Systems: A Matrix Inequalities Approach

Von der Fakultät für Ingenieurwissenschaften der
Universität Duisburg-Essen
zur Erlangung des akademischen Grades eines

Doktors der Ingenieurwissenschaften

genehmigte Dissertation

von

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Tag der mündlichen Prüfung: 31. Mai 2006

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Abstract

This dissertation presents an extension of robust decentralized control design techniques for power systems, with special emphasis on design problems that can be expressed as minimizing a linear objective function under linear matrix inequality (LMI) in tandem with nonlinear matrix inequality (NMI) constraints. These types of robust decentralized control design problems are generally nonconvex optimizations, and are proven to be computationally challenging. Therefore, this dissertation proposes alternative computational schemes using: i) bordered-block diagonal (BBD) decomposition algorithm for designing LMI based robust decentralized static output feedback controllers, ii) sequential LMI programming method for designing robust decentralized dynamic output feedback controllers, and, iii) generalized parameter continuation method involving matrix inequalities for designing reduced-order decentralized dynamic output feedback controllers.

First, this dissertation considers the problem of designing robust decentralized static output feedback controllers for power systems that guarantee connective stability despite the presence of uncertainties among the interconnected subsystems. The design problem is then solved using BBD decomposition algorithm that clusters the state, input and output structural information for the direct computation of the appropriate gain matrices. Moreover, the approach is flexible enough to allow the inclusion of additional design constraints such as the size of the gain matrices and the degree of robust stability while at the same time maximizing the tolerable upper bounds on the class of perturbations.

Second, this research considers the problem of designing a robust decentralized fixed-order dynamic output feedback controller for power systems that is formulated as a nonconvex optimization problem involving LMIs coupled through bilinear matrix equation. In the design, the robust connective stability of the overall system is guaranteed while the upper bounds of the uncertainties arising from the interconnection of the subsystems as well as nonlinearities within each subsystem are maximized. The (sub)-optimal robust decentralized dynamic output feedback control design problem is then solved using sequential LMI programming method. Moreover, the local convergence property of this algorithm has shown the effectiveness of the proposed approach for designing (sub)-optimal robust decentralized dynamic output feedback controllers for power systems.

Third, this dissertation considers the problem of designing a robust decentralized structure-constrained dynamic output feedback controller design for power systems using LMI-based optimization approach. The problem of designing a decentralized structure-constrained H_2/H_∞ controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting nonconvex optimization problem which involves bilinear matrix inequalities (BMIs) is then solved using the sequentially LMI programming method.

Finally, the research considers the problem of designing reduced-order decentralized H_∞ controllers for power systems. Initially a fictitious centralized H_∞ robust controller, which is typically high-order controller, is designed to guarantee the robust stability of the overall system against unstructured and norm bounded uncertainties. Then the problem of designing a reduced-order decentralized controller is reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized controller to the decentralized controller as the continuation parameter monotonically varies. The design problem, which guarantees the same robustness condition of the centralized controller, is solved using a two-stage iterative matrix inequality optimization algorithm. Moreover, the approach is flexible enough to allow designing different combinations of reduced-order controllers between the different input/output channels.

The effectiveness of these proposed approaches are demonstrated by designing realistic power system stabilizers (PSSs) for power system, notably so-called reduced-order robust PSSs that are linear and use minimum local-feedback information. Moreover, the nonlinear simulation results have confirmed the robustness of the system for all envisaged operating conditions and disturbances. The proposed approaches offer a practical tool for engineers, besides designing reduced-order PSSs, to re-tune PSS parameters for improving the dynamic performance of the overall system.

Acknowledgments

Although it is impossible to thank everyone who had helped me over the years to understand the field of Power and Control System Engineering, I feel that one person must be singled out. I am forever grateful to my instructor and supervisor Prof. Dr.-Ing. habil. István Erlich for his constant encouragement and invaluable suggestions. Prof. Dr.-Ing. habil. Jan Lunze who served in my dissertation committee also deserves special thanks for many discussions and his deep insights into the nature of large-scale complex systems.

I gratefully acknowledge all those members of the Institute of Electrical Power Engineering and Automation, University of Duisburg-Essen, for their invaluable assistance and kindness which made my study both worthwhile and rewarding.

This is a good opportunity to thank Prof. Girma Mullisa and Dr.-Ing. Fekadu Shewarega for their mentoring during my under and postgrad years at the Faculty of Technology, Addis Ababa University. I owe them my gratitude, not only for the professional relationship, but more importantly, for their friendship.

Finally, I would like to thank the Deutscher Akademischer Austausch Dienst (DAAD) for sponsoring my study at the Institute of Electrical Power Engineering and Automation, University of Duisburg - Essen, Germany.

My sincere thanks go to those who, either close at hand or at a distance were concerned about me and my studies. Last, but not least, thanks are also due to my wife Jemmy (Yemisrach M. Gebreselassie) and my father Kebede Befekadu Wolde-Mariam. Without their encouragements and continuous support this Ph.D. study could not have come to a satisfactory conclusion.

“Whatsoever thy hand findeth to do, do it with thy might....”

Eccles. 9:10

Contents

Pages

LIST OF FIGURES

Figure

LIST OF TABLES

Table

Chapter 1

Introduction

1.1 Problem Statement

The electric power systems such as those in Europe and North America have recently experienced unprecedented changes due to the emergence of deregulation in the sector and the development of competitive market for generations and energy services. These changes have caused a noticeable uncertainty in the load flow, and have pushed the networks further to their operational limits. Besides, the integration of offshore wind generation plants into the existing network is also expected to have a significant impact on the load flow of system as well as the dynamic behaviour of the network. On the other hand, the transmission grids have seen very little expansion due to environmental restrictions. As a result, available transmission and generation facilities are highly utilized with large amounts of power interchanges taking place through tie-lines and geographical regions. It is also expected that this trend will continue in the future and result in more stringent operational requirements to maintain reliable services and adequate system dynamic performances. Critical controls like excitation systems, power system stabilizers, static VAR compensators, and other new class of control devices based on modern power electronics (such as FACTS devices) will play increasingly key roles in maintaining adequate system dynamic performance. Moreover, proper design of these control systems that takes into account the continual changes in the structure of the network is imperative to ensure/guarantee robustness over wide operating conditions in the system.

With emphasis on the robustness and system performance, there is a need to analyze and design controllers in an integrated manner, taking into consideration the interaction between the various subsystems and controllers in the system. Currently, secure operations of power systems heavily rely on the controller schemes that are put in the system to manage disturbances or prevent the possible disastrous consequences. These control schemes are usually static in the sense that they do not adapt to changing network configurations and operating conditions. In addition, the design and parameter settings of these control schemes do not take into account the variations or changes in the system behaviour. Consequently, the system often tends to be unstable and is characterized by showing poor global behaviour.

In order to address the problems described above, new robust decentralized control mechanisms have been proposed that guarantee an accurate prediction of system response and system robustness to disturbances under various operation conditions. Decentralized control of large power systems, which are driven to the point of nonlinearity, presents new challenges due to the complex interactions among subsystems. Therefore, this dissertation deals with the various facets of the research performed including methodological, structural and computational issues pertaining to the formulation of robust decentralised controller design problems.

1.2 Proposed Approaches

This research mainly focuses on developing robust decentralized control techniques for power systems with special emphasis on problems that can be expressed in terms of minimizing a linear objective functional under linear matrix inequality (LMI) constraints in tandem with nonlinear matrix inequality (NMI) constraints. The NMI constraints including the bilinear matrix inequality (BMI) constraints render a computational challenge in designing decentralized controllers. Therefore, this dissertation proposes alternative computational schemes using: i) bordered-block diagonal (BBD) decomposition algorithm for designing LMI based robust decentralized static output feedback controllers, ii) sequential linear matrix inequality programming method for designing robust decentralized dynamic output feedback controllers, and, iii) generalized parameter continuation method involving matrix inequalities for designing reduced-order decentralized dynamic output feedback controllers. Moreover, these algorithms are computationally efficient and can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers. The local convergence properties of these algorithms for designing (sub)-optimal robust decentralized controllers have shown the effectiveness of these proposed approaches for designing power system stabilizers.

1.3 Outline of the Dissertation

The subsequent chapters of this dissertation are organized as follows: Chapter 2 addresses issues of power system stability, and identification of the different categories of power system stability problems. The chapter also includes summary of definitions and concepts of stability from mathematical and control theoretical point of view. Chapter 3 presents the modelling and analysis of large power system dynamics. This chapter also briefly discusses model reduction techniques that are necessary to simplify large interconnected power system models. Chapter 4 presents a brief review of convex optimization and LMI-based control optimization algorithms, which are essential to understanding the contributions of this dissertation. This is followed by the computational schemes used to solve the problems considered in this dissertation. Chapter 5 presents the main research contributions of this dissertation. First, a robust decentralized static output feedback controller design using convex optimization involving LMIs is presented along with the problem of connective stabilizability condition of interconnected power systems. Second, an extension of a decentralized dynamic output feedback controller design problem that guarantees a robust connective stability condition is presented. A sequential linear matrix inequality programming method is also discussed to solve such design problem (sub-) optimally. Third, a robust decentralized structure-constrained dynamic output feedback controller design for power systems is formulated using LMI-based optimization approach. The problem of designing a decentralized structure-constrained H_2/H_∞ controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting nonconvex optimization problem which involves bilinear matrix inequalities (BMIs) is then solved using the sequentially LMI programming method. Finally, decentralized H_∞ dynamic output feedback controller design for power systems is formulated using the general parameterized continuation optimization method involving matrix inequalities. Summary of the results obtained from solving these controller design problems for test systems is also presented in this chapter. Chapter 6 summarizes the work presented in this dissertation. The main contributions of this dissertation are highlighted, and a list of potential research directions for further study is given.

Chapter 2

Power System Stability

2.1 Introduction

Power system stability has been recognized as an important problem for secure system operation since the beginning of last century. Many major blackouts caused by power system instability have illustrated the importance of this phenomenon [1]. Historically, transient instability has been the dominant stability problem on most systems and also the focus of much of the power industry's attention concerning system stability. As power system has evolved considerably, different forms of power system instability have emerged owing to the continuing growth in interconnections, the use of new technologies and controls in the system. This has further created the need to provide proper classification of power system stability which is essential for satisfactory operation of power systems as well as developing system design criteria.

This chapter addresses issues of power system stability, and identification of the different categories of stability behaviours that are important in power system stability analysis. The organization of this chapter is as follows. Section 2.2 gives the formal definition of power system stability that conforms to theoretical system stability concepts. Section 2.3 identifies the different stability problems in power systems, and classifies them according to their physical nature and size of disturbances involved; and the time-span taken to assess their dynamic behaviour. Section 2.4 addresses the qualitative characterizations of power system stability that are of fundamental importance to power systems analysis. Section 2.5 and 2.6

provide analytical definitions of several types of stability including Lyapunov stability of a system, input/output stability concepts and stability of linear systems.

2.2 Definition of Power System Stability

The objective of this section is to provide a physically based definition of power system stability that conforms to the theoretical system stability concepts. This definition moreover will serve as a basis for classifying the different forms of power system stabilities.

DEFINITION 2.1: *Power system stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded so that practically the entire system remains intact* [2].

In general, power system is a highly nonlinear system that is subjected to a wide range of disturbances including small-disturbances in the form of load changes that occur frequently or severe nature such as a short-circuit on a transmission line or loss of a large generator. The latter types of disturbances are commonly referred to as large-disturbances which may lead to structural changes due to the isolation of the faulted elements. At an equilibrium condition, a power system may be stable for a given physical disturbance, and unstable for another. Thus, the stability of the system, when subjected to a disturbance, depends on the initial operating condition as well as the nature of the disturbance (i.e., power system stability is a property of the system motion around the initial operating condition). A stable equilibrium condition has a finite region of attraction; the larger the region, the more robust the system with respect to large-disturbances. Moreover, this region of attraction has a complex behaviour and changes with the operating condition of the power system.

2.3 Classification of Power System Stability

Power system stability is essentially a single problem; however, the various forms of instabilities, which a power system may experience, cannot be properly understood or effectively dealt with by treating it as such. Due to high dimensionality and complexity of power system stability problems, it is necessary to make suitable simplifying assumptions for analyzing specific types of problems using an appropriate degree of detail system representation and analytical techniques. Analysis of stability, including identifying key factors that contribute to instability and devising methods of improving stable operation, is greatly facilitated by classification of stability into appropriate categories [3]. Classification, therefore, is essential for practical analysis and resolution of power system stability problems.

The classification of power system stability presented here is based on the following considerations

- The physical nature of the resulting mode of instability as indicated by the main system variable in which instability can be observed.
- The size of the disturbance considered which influences the method of calculation and prediction of stability.
- The devices, processes, and the time span that must be taken into consideration in order to assess stability.

Figure 2.1 gives the overall picture of the power system stability problem, identifying its categories and subcategories. The following are descriptions of the corresponding forms of stability phenomena.

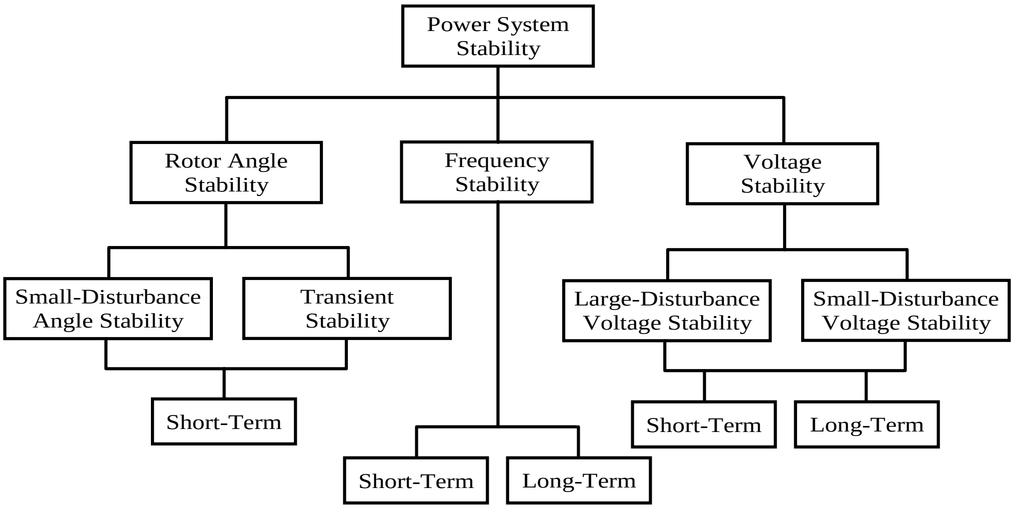


FIGURE 2.1 Classification of power system stability [2]

2.3.1 Rotor Angle Stability

Rotor angle stability refers to the ability of synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. It depends on the ability to maintain/restore equilibrium between electromagnetic torque and mechanical torque of each synchronous machine in the system. Instability that may result occurs in the form of increasing angular swings of some generators leading to their loss of synchronism with other generators.

The rotor angle stability problem involves the study of the electromechanical oscillations inherent in power systems. A fundamental factor in this problem is the way in which the power outputs of synchronous machines vary as their rotor angles change. Under steady-state conditions, there is equilibrium between the input mechanical torque and the output electromagnetic torque of each generator, and the speed remains constant. If the system is perturbed, this equilibrium is upset, resulting in acceleration or deceleration of the rotors of the machines according to the laws of motion of a rotating body. If one generator temporarily runs faster than another, the angular position of its rotor relative to that of the slower machine will advance. The resulting angular difference transfers part of the load from the slow machine to the fast machine, depending on the power-angle relationship. This tends to reduce the speed difference and hence the angular separation. The power-angle relationship is highly nonlinear and beyond a certain limit, an increase in angular separation is accompanied by a decrease in power transfer such that the angular separation is increased further. Therefore, instability occurs if the system cannot absorb the kinetic energy corresponding to these rotor speed differences. For any given situation, the stability of the system depends on whether or not the deviations in angular positions of the rotors result in sufficient restoring torques [3]. Loss of synchronism can occur between one machine and the rest of the system, or between groups of machines, with synchronism maintained within each group after separating from each other.

The change in electromagnetic torque of a synchronous machine following a perturbation can be resolved into two components:

- Synchronizing torque component, in phase with rotor angle deviation.
- Damping torque component, in phase with the speed deviation.

System stability depends on the existence of both components of torque for each of the synchronous machines, i.e., lack of sufficient synchronizing torque results in aperiodic or nonoscillatory instability, whereas lack of damping torque results in oscillatory instability. Depending on the nature of stability problem, it is useful to characterize rotor angle stability in terms of the following two subcategories:

Small-disturbance (or *small-signal rotor angle***) stability** is concerned with the ability of the power system to maintain synchronism under small disturbances. The disturbances are considered to be sufficiently small that linearization of system equations is permissible for purposes of analysis. Moreover, small-disturbance stability depends on the initial operating condition of the system. Instability that may result can be of two forms: *i*) increase in rotor angle through a nonoscillatory or aperiodic mode due to lack of synchronizing torque, or *ii*) rotor oscillations of increasing amplitude due to lack of sufficient damping torque.

Large-disturbance rotor angle (or *transient***) stability** is concerned with the ability of the power system to maintain synchronism when subjected to a severe disturbance, such as a short-circuit on a transmission line. The resulting system response involves large excursions of generator rotor angles and is influenced by the nonlinear power-angle relationship. Moreover, the stability of the system depends on both the initial operating state of the system as well as the severity of the disturbance. Instability is usually in the form of aperiodic angular separation due to insufficient synchronizing torque, manifesting as first swing instability, or could be a result of superposition of a slow interarea swing mode and a local-plant swing mode due to large excursion of rotor angle beyond the first swing [3].

2.3.2 Voltage Stability

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance from a given initial operating condition. It depends on the ability to maintain/restore equilibrium between load demand and load supply of the power system. Instability that may result occurs in the form of a progressive fall or rise of voltages at some buses. A possible outcome of voltage instability is loss of load in an area, or tripping of transmission lines and other elements by their protective systems leading to cascading outages. Loss of synchronism of some generators may result from these outages or from operating conditions that violate field current limit. Progressive drop in bus voltages can also be associated with rotor angle instability. For example, the loss of synchronism of machines as rotor angles between two groups of machines approach 180° causes rapid drop in voltages at intermediate points in the network close to the electrical centre. Normally, protective systems operate to separate the two groups of machines and the voltages recover to levels depending on the post-separation conditions. If, however, the system is not so separated, the voltages near the electrical centre rapidly oscillate between high and low values as a result of repeated pole slips between the two groups of machines.

The driving force for voltage instability is usually the loads; in response to a disturbance, power consumed by the loads tends to be restored by the action of motor slip adjustment, distribution voltage regulators, tap-changing transformers, and thermostats. Restored loads increase the stress on the high voltage network by increasing the reactive power consumption and causing further voltage reduction. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network and the connected generation [3] and [4].

A major factor contributing to voltage instability is the voltage drop that occurs when active and reactive power flow through inductive reactance of the transmission network; this limits the capability of the transmission network for power transfer and voltage support. The power transfer and voltage support are further limited when some of the generators hit their field or armature current time-overload capability limits. Voltage stability is threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources.

The system may experience over-voltage instability problem (in contrast to the most common form of progressive voltage drop instability) at some buses due to the capacitive behaviour of the network and under excitation limiters that preventing generators and synchronous compensators from absorbing excess reactive power in the system. In this case, the instability is associated with the inability of the combined generation and transmission system to operate below some load level. In their attempt to restore this load power, transformer tap changers cause long-term voltage instability.

Voltage stability problems may also be experienced at the terminals of HVDC links used for either long distance or back-to-back applications. They are usually associated with HVDC links connected to weak ac systems and which may occur at rectifier or inverter stations, and are associated with the unfavourable reactive power load characteristics of the converters. The HVDC link control strategies have a very significant influence on such problems, since the active and reactive powers at the ac/dc junction are determined by the controls. If the resulting loading on the ac transmission is beyond the transmission's capability, then voltage instability occurs. Such a phenomenon is relatively fast with the time frame of interest being in the order of one second or less. Voltage instability may also be associated with converter transformer tap-changer controls, which is a considerably slower phenomenon.

One form of voltage stability problem that results in uncontrolled over voltages is the self-excitation of synchronous machines. This can arise if the capacitive load of a synchronous machine is too large. Examples of excessive capacitive loads that can initiate self-excitation are open ended high voltage lines and shunt capacitors and filter banks from HVDC stations. The over-voltages that result when generator load changes to capacitive are characterized by an instantaneous rise at the instant of change followed by a more gradual rise that depends on the relation between the capacitive load component, machine reactance and the excitation system of the synchronous machine.

As in the case of rotor angle stability, it is useful to classify voltage stability into the following subcategories:

Small-disturbance voltage stability refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. This form of stability is influenced by the characteristics of loads, continuous controls, and discrete controls at a given instant of time. This concept is useful in determining, at any instant, how the system voltages will respond to small system changes. With appropriate assumptions, system equations can be linearized for analysis thereby allowing computation of valuable sensitivity information useful in identifying factors influencing stability. This linearization, however, cannot account for nonlinear effects such as tap changer controls (deadbands, discrete tap steps, and time delays). Thus, a combination of linear and nonlinear analyzes is used in a complementary manner.

Large-disturbance voltage stability refers to the system's ability to maintain steady voltages following large-disturbances such as system faults, loss of generation, or circuit contingencies. This ability is determined by the system load characteristics and the interactions of both continuous and discrete controls and protections. Determination of large-disturbance voltage stability requires the examination of the nonlinear response of the power system over a period of time sufficient to capture the performance and interactions of such devices as motors, under-load transformer tap changers, and generator field-current limiters. The time frame of interest for voltage stability problems may vary from a few seconds to tens of minutes. Therefore, voltage stability may be either a short-term or a long-term phenomenon as identified in Figure 2.1.

2.3.3 Frequency Stability

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe system upset resulting in a significant imbalance between generation and load. It depends on the ability to maintain/restore equilibrium between system generation and load, with minimum unintentional loss of load. Instability that may result occurs in the form of sustained frequency drop or increase leading to tripping of generating units and/or loads.

Severe system upsets generally result in large excursions of frequency, power flows, voltage, and other system variables, thereby invoking the actions of processes, controls, and protections that are not modelled in conventional transient stability or voltage stability studies. These processes may be very slow, such as boiler dynamics, or only triggered for extreme system conditions, such as volts/Hertz protection tripping generators. In large interconnected power systems, this type of situation is commonly associated with conditions following splitting of systems into islands. Stability in this case is a question of whether or not each island will reach a state of operating equilibrium with minimal unintentional loss of load. It is determined by the overall response of the island as evidenced by its mean frequency, rather than relative motion of machines. Generally, frequency stability problems are associated with inadequacies in equipment responses, poor coordination of control and protection equipment, or insufficient generation reserve. In isolated island systems, frequency stability could be of concern for any disturbance causing a relatively significant loss of load or generation.

During frequency excursions, the characteristic times of the processes and devices that are activated will range from fraction of seconds, corresponding to the response of devices such as under-frequency load shedding and generator controls and protections, to several minutes, corresponding to the response of devices such as prime mover energy supply systems and load voltage regulators. Therefore, as identified in Figure 2.1, frequency stability may be a short-term phenomenon or a long-term phenomenon. An example of short-term frequency instability is the formation of an under-generated island with insufficient under-frequency load shedding such that frequency decays rapidly causing blackout of the island within a few seconds. On the other hand, more complex situations in which frequency instability is caused by steam turbine over-speed controls or boiler/reactor protection and controls are longer -term phenomena with the time frame of interest ranging from tens of seconds to several minutes.

During frequency excursions, voltage magnitudes may change significantly, especially for islanding conditions with under-frequency load shedding that unloads the system. Voltage magnitude changes, which may be higher in percentage than frequency changes, affect the load-generation imbalance. High voltage may cause undesirable generator tripping by poorly designed or coordinated loss of excitation relays or volts/Hertz relays. In an overloaded system, low voltage may cause undesirable operation of impedance relays.

2.4 Qualitative Characterizations of Power System Stability

This section considers the qualitative characterizations of power system stability that are of fundamental importance to power systems analysis. The assumption here is the model of a power system is given in the form of explicit first-order differential equations (i.e., state-space description). While this is quite common in the theory of dynamical systems, it may not always be entirely natural for power systems due to the presence of a set of differential and algebraic equations (DAEs).

The outlined modelling problems are typically addressed in a power system analysis framework in the following way:

1. The problem of defining stability for general nonautonomous systems is very challenging even in the theoretical realm [5] and one possible approach is to require that a system to which the environment delivers square-integrable signals as inputs over a time interval is stable if variables of interest (such as outputs) are also square integrable. In a more general setup, one can consider signals truncated in time, and denote the system as well-posed if it maps square integrable truncated signals into signals with the same property. In a power system setting, it is assumed that the variables at the interface with the environment are known (or predictable) - e.g., that mechanical inputs to all generators are constant, or that they vary according to the known response of turbine regulators.
2. The disturbances of interest will fall into two broad categories-event-type (typically described by specific equipment outages) and norm-type (described by their size e.g., in terms of various norms of signals). One also observes that in cases when event-type (e.g., switching) disturbances occur repeatedly, a proper analysis framework is that of hybrid systems (see for a recent development in [6]). The focus here is on time horizons of seconds to minutes. On a longer time scale, the effects of market structures may also become prominent [7].
3. Given an emphasis on stability analysis, it is also assumed that the actions of all controllers are fully predictable in terms of known system quantities (states), or as functions of time [3].

A typical power system stability study thus consists of the following steps:

a) Make modelling assumptions and formulate a mathematical model appropriate for the time-scales and phenomena under study,
b) Select an appropriate stability definition,
c) Analyze and/or simulate to determine stability, typically using a scenario of events,
d) Review results in light of assumptions, compare with the engineering experience and repeat if necessary.

2.5 Power System Stability Analysis

Consider the system described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}) \tag{2.1}$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{f}(t,\mathbf{x})$ is sufficiently differentiable and its domain includes the origin. The system described above is said to be autonomous if $\mathbf{f}(t,\mathbf{x})$ is independent of t and it is said to be nonautonomous otherwise.

A typical scenario for power system stability analysis involves three distinct steps.

1. The system is initially operating in a pre-disturbance equilibrium set X_n; in that set, various driving terms affecting system variables are balanced (either instantaneously, or over a time interval).
2. Next, an event-type disturbance, that is characterized by a specific fault scenario (e.g., short circuit somewhere in the transmission network followed by a line disconnection including the duration of the event-fault clearing time) or norm-type disturbances (described by their size in terms of various norms of signals - e.g., load variations), acts on the system.
3. After an event-type disturbance, the system dynamics is studied with respect to a known post-disturbance equilibrium set X_p (which may be distinct from X_n). The system initial condition belongs to a starting set χ_p and one wants to characterize the system motion with respect to X_p, i.e., if the system trajectory will remain inside the technically viable set Ω_p (which includes X_p). Moreover, a detected instability (during which system motion crosses the boundary of the technically viable set $\partial\Omega_p$ e.g., causing line tripping or a partial load shedding) may lead to a new stability study for a new system with new starting and viable sets, and possibly with different modelling assumptions (or several such studies, if a system gets partitioned into several disconnected parts).

In general, the stability analysis of power systems is non-local as the various equilibrium sets may get involved. In the case of event-type disturbances, the perturbations of interest are specified deterministically, moreover, it is assumed that all X_p associated with the given X_n and disturbance are properly determined. In the case of norm-type perturbations, the uncertainty structure is different - the perturbation is characterized by size and the same equilibrium set typically characterizes the system before and after the disturbance.

Thus the following definition (formulation) of power system stability will serve from system theory to explore the salient features of general stability concepts.

DEFINITION 2.2: *An equilibrium set of a power system is stable if when the initial state is in the given starting set, the system motion converges to the equilibrium set, and operating constraints are satisfied for all relevant variables along the entire trajectory.*

The operating constraints are of inequality (and equality) type, and pertaining to individual variables and their collections. For example, system connectedness is a collective feature, as it implies that there exist paths (in graph-theoretic terms) from any given bus to all other buses in the network. Note also that some of the operating constraints (e.g., voltage levels) are inherently soft i.e., the power system analyst may be interested in stability characterization with and without these constraints. Note that the assumption of the model is accurate in the sense that there are no further system changes (e.g., relay-initiated line tripping) until the trajectory crosses the boundary a $\partial \Omega_p$.

2.6 Basic Theoretical Concepts for Dynamic System Stability

This section provides definitions of several types of stability including Lyapunov stability, input/output stability and stability of linear systems. Most of the concepts discussed in the following subsections are based mostly on the treatment in [8]–[11]. Of these various types, the Lyapunov stability definitions related to stability and asymptotic stability are the ones most applicable to power system nonlinear behaviour under large disturbances. The definition of stability related to linear systems finds wide use in small-signal stability analysis of power systems.

2.6.1 Lyapunov Stability

Consider again the nonautonomous system:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}) \tag{2.2}$$

where $\mathbf{x}(t)$ is the state vector, $\mathbf{f}(t, \mathbf{x})$ is a locally Lipschitz function and its domain includes the origin $\mathbf{x} = \mathbf{0}$. Suppose that the origin is the equilibrium point of the system (2.2), i.e., $\mathbf{f}(t, \mathbf{0}) = \mathbf{0}, \ \forall t \geq 0$. To characterize the qualitative behaviour of the equilibrium point of the system in the sense of Lyapunov, the following definitions are stated.

The equilibrium point $\mathbf{x} = \mathbf{0}$ of (2.2) is:

- *stable* if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon, t_0) > 0$ such that:

$$\|\mathbf{x}(t_0)\| < \delta \Rightarrow \|\mathbf{x}(t)\| < \varepsilon, \quad \forall\, t \ge t_0 \ge 0 \tag{2.3}$$

By choosing the initial conditions in a sufficiently small spherical neighbourhood of radius δ, one can force the trajectory of the system for all time $t > t_0$ to be entirely in a given cylinder of radius ε. This concept is pictorially depicted in Figure 2.2(a).

- *uniformly stable* if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$, independent of t_0, such that (2.3) is satisfied;
- *unstable* if not stable;
- *asymptotically stable* if it is stable and in addition there is $\mu(t_0) > 0$ such that:

$$\|\mathbf{x}(t_0)\| < \mu(t_0) \Rightarrow \|\mathbf{x}(t)\| \to 0 \text{ as } t \to \infty \tag{2.4}$$

It is important to note that the definition of asymptotic stability combines the aspect of stability as well as attractivity of the equilibrium. This is a stricter requirement of the system behaviour that eventually returned to the equilibrium point.

- *uniformly asymptotically stable* if it is uniformly stable and there is $\delta_0 > 0$, independent of t_0, such that for all $\|\mathbf{x}(t_0)\| < \delta_0$, $\mathbf{x}(t) \to 0$ as $t \to \infty$ uniformly in t and $\mathbf{x}(t_0)$; that is, for each $\varepsilon > 0$, there is $T = T(\varepsilon, \delta_0) > 0$ such that

$$\|\mathbf{x}(t_0)\| < \delta_0 \Rightarrow \|\mathbf{x}(t)\| < \varepsilon \quad \forall\, t \ge t_0 + T(\varepsilon, \delta_0) \tag{2.5}$$

By choosing the initial operating points in a sufficiently small spherical neighbourhood at $t = t_0$, it is possible to force the trajectory of the solution to lie inside a given cylinder for all $t > t_0 + T(\varepsilon, \delta_0)$. Figure 2.2(b) shows the behaviour of uniform asymptotic stability as $t \to \infty$.

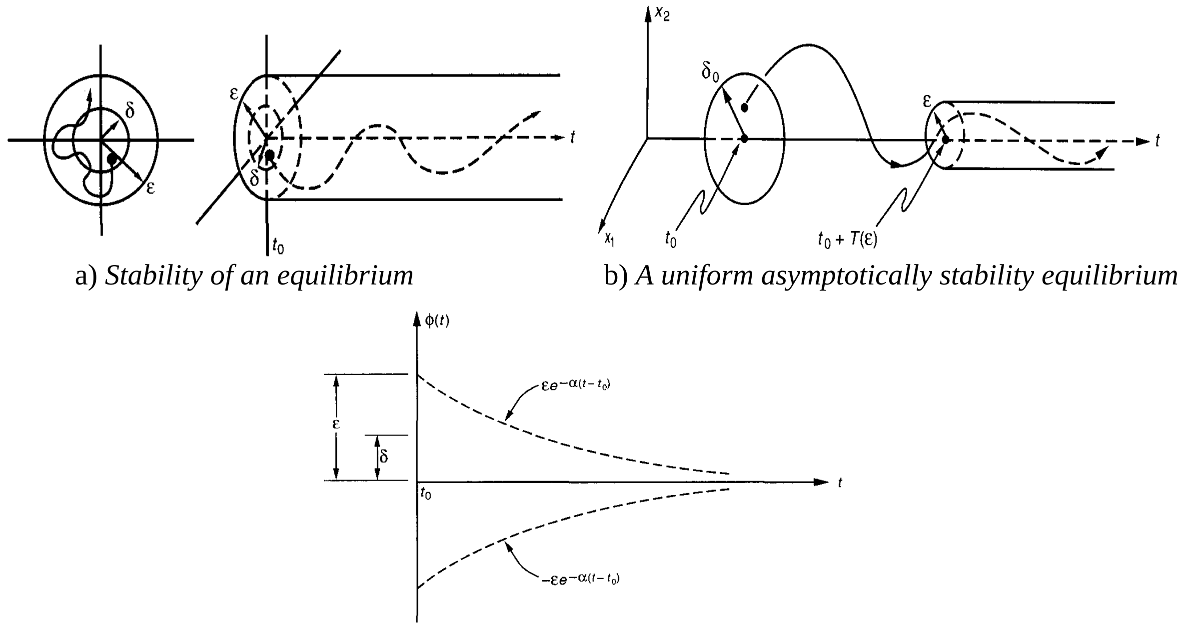
- *globally uniformly asymptotically stable* if it is uniformly stable, and, for each pair of positive numbers ε and δ_0 there is $T = T(\varepsilon, \delta_0) > 0$ such that:

$$\|\mathbf{x}(t_0)\| < \delta_0 \Rightarrow \|\mathbf{x}(t)\| < \varepsilon \quad \forall\, t \ge t_0 + T(\varepsilon, \delta_0) \tag{2.6}$$

- *exponentially stable* if there are $\delta > 0$, $\varepsilon > 0$, $\alpha > 0$ such that (see Figure 2.2(c)):

$$\|\mathbf{x}(t_0)\| < \delta \;\Rightarrow\; \|\mathbf{x}(t)\| < \varepsilon \|\mathbf{x}(t_0)\| e^{-\alpha(t - t_0)} \quad t \ge t_0 \tag{2.7}$$

- *globally exponentially stable* if the exponential stability condition is satisfied for any initial state.



a) *Stability of an equilibrium* b) *A uniform asymptotically stability equilibrium*

c) *An exponentially stability equilibrium*

FIGURE 2.2 Illustration of different types of stabilities

These definitions form the foundation of the Lyapunov approach to system stability, and can be checked for a specific system via so called Lyapunov functions. However, in power systems one is interested in the region of attraction $R(X_p)$ of a given equilibrium set X_p namely the set of points in the state space with the property that all trajectories initiated at the points will converge to the equilibrium set X_p. If the equilibrium set is a point that is asymptotically stable, then it can be shown that the region of attraction has nice analytical properties (i.e., it is an open and connected set, and its boundary is formed by system trajectories).

Owing to approximations and idealizations in large-scale power system modelling, the solution of the perturbed system may not necessarily approach the origin (equilibrium point) but ultimately bounded (i.e., $\|\mathbf{x}(t)\|$ is bounded by a fixed constant) for sufficiently large time t and given initial condition in a ball of a fixed radius. Characterization of stability in this case requires knowledge of the size of the perturbation term, and of a Lyapunov function for the nominal system. A related notion of practical stability is motivated by the idea that a system may be considered stable if the deviations of motions from the equilibrium remain within certain bounds determined by the physical situation, in case the initial values and the perturbation are bounded by suitable constants.

A power system is often modelled as an interconnection of lower-order subsystems, and one may be interested in a hierarchical (two-level) approach to determine the stability of the overall system. At the first step, analyzing the stability of each subsystem separately (i.e., by ignoring the interconnections among the subsystems). In the second step, combine the results of the first step with the interconnections information for analyzing further the stability of the overall system. In a Lyapunov framework, this results in the study of composite Lyapunov functions. An important qualitative result is that if the isolated subsystems are sufficiently stable, compared to the strength of the interconnections, then the overall system is uniformly asymptotically stable at the origin.

Another concept of interest in power systems is that of partial stability or set stability [12], where the stability behaviour does not concern all state variables (because of the properties of the norm used in (2.3) and elsewhere) but only a part of them or a function of state variables, i.e., a set of output variables. This formulation naturally leads to substantial simplifications and it has been used in the context of power system stability analysis.

2.6.2 Input/Output Stability

This approach considers the system description of the form:

$$\mathbf{y}(t) = \mathbf{g}(t, \mathbf{u}) \tag{2.8}$$

where $\mathbf{g}(t,\mathbf{u})$ is an operator that specifies the q - dimensional output vector $\mathbf{y}(t)$ in terms of the m - dimensional input vector $\mathbf{u}(t)$. The input belongs to a normed linear space of vector signals L_e^m - e.g., extended bounded or square integrable signals.

DEFINITION 2.3: A continuous function $\alpha:[0,a)\to[0,\infty)$ is said to belong to class K if it is strictly increasing and $\alpha(0)=0$. It is said to belong to class K_∞ if $a=\infty$ and $\alpha(r)\to\infty$ as $r\to\infty$.

DEFINITION 2.4: A continuous function $\beta:[0,a)\times[0,\infty)\to[0,\infty)$ is said to belong to class KL if, for each fixed s, the mapping $\beta(r,s)$ belongs to class K with respect to r and, for each fixed r, the mapping $\beta(r,s)$ is decreasing with respect to s and $\beta(r,s)\to 0$ as $s\to\infty$.

A mapping $\mathbf{g}:[0,\infty]\times L_e^m \to L_e^q$ is an L - stable mapping if there exists a class K function $\alpha(.)$ defined on $[0,\infty)$, and a nonnegative constant β such that:

$$\left\| (\mathbf{y}(t))_\tau \right\|_L \le \alpha\left(\left\| (\mathbf{u}(t))_\tau \right\|_L\right) + \beta \tag{2.9}$$

for all $\mathbf{u}\in L_e^m$ and $\tau\in[0,\infty)$.

It is finite gain L - stable if there exist nonnegative constants γ and β such that:

$$\|(\mathbf{y}(t))_\tau\|_L \le \gamma(\|(\mathbf{u}(t))_\tau\|_L) + \beta \tag{2.10}$$

for all $\mathbf{u} \in L_e^m$ and $\tau \in [0, \infty)$.

Consider a nonautonomous system with input:

$$\dot{\mathbf{x}} = \mathbf{f}(t, \mathbf{x}, \mathbf{u}) \tag{2.11}$$

A system in (2.11) is said to be locally input-to-state stable if there exists a class KL function $\beta(.,.)$, a class K function $\alpha(.)$ and positive constants k_1 and k_2 such that for any initial state $\mathbf{x}(t_0)$ with $\|\mathbf{x}(t_0)\| < k_1$ and any input $\mathbf{u}(t)$ with $\sup_{t > t_0} \|\mathbf{u}(t)\| < k_2$ the solution $\mathbf{x}(t)$ exists and satisfies:

$$\|\mathbf{x}(t)\| \le \beta(\|\mathbf{x}(t_0)\|, t - t_0) + \alpha\left(\sup_{t_0 \le \tau \le t} \|\mathbf{u}(\tau)\|\right) \tag{2.12}$$

for all $t \ge t_0 \ge 0$.

It is said to be input-to-state stable if the local input-to-state property holds for the entire input and state spaces, and inequality (2.12) is satisfied for any initial state $\mathbf{x}(t_0)$ and any bounded input $\mathbf{u}(t)$.

Next, consider the system (2.11) with the output $\mathbf{y}(t)$ determined from:

$$\mathbf{y}(t) = \mathbf{g}(t, \mathbf{x}, \mathbf{u}) \tag{2.13}$$

where $\mathbf{g}$ is again assumed smooth.

A system (2.11) is said to be locally input-to-output stable if there exists a class KL function $\beta(.,.)$, a class K function $\alpha(.)$ and positive constants k_1 and k_2 such that for any initial state $\mathbf{x}(t_0)$ with $\|\mathbf{x}(t_0)\| < k_1$ and any input $\mathbf{u}(t)$ with $\sup_{t > t_0} \|\mathbf{u}(t)\| < k_2$ the solution $\mathbf{y}(t)$ exists and satisfies:

$$\|\mathbf{y}(t)\| \le \beta(\|\mathbf{x}(t_0)\|, t - t_0) + \alpha\left(\sup_{t_0 \le \tau \le t} \|\mathbf{u}(\tau)\|\right) \tag{2.14}$$

for all $t \ge t_0 \ge 0$.

It is said to be input-to-output stable if the local input-to-output property holds for entire input and output spaces, and inequality (2.14) is satisfied for any initial state $\mathbf{x}(t_0)$ and any bounded input $\mathbf{u}(t)$.

2.6.3 Stability of Linear Systems

In this subsection, consider a system of the form:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\,\mathbf{x}(t) \tag{2.15}$$

which is the linearization of (2.2) around the equilibrium at the origin. General stability conditions for the nonautonomous case are given in terms of the state transition matrix $\mathbf{\Phi}(t,t_0)$.

$$\mathbf{x}(t) = \mathbf{\Phi}(t,t_0)\,\mathbf{x}(t_0) \tag{2.16}$$

In the general, such conditions are of little computational value, as it is impossible to derive a closed-form analytical expression for $\mathbf{\Phi}(t,t_0)$.

In the case of autonomous systems (i.e., $\mathbf{A}(t) = \mathbf{A}$): The origin of (2.15) is (globally) asymptotically (exponentially) stable if and only if all eigenvalues of $\mathbf{A}$ have negative real parts. The origin is stable if and only if all eigenvalues of $\mathbf{A}$ have nonpositive real parts, and in addition, every eigenvalue of $\mathbf{A}$ having a zero real part is a simple zero of the minimal characteristic polynomial of $\mathbf{A}$. In the autonomous case, an alternative to calculating eigenvalues of $\mathbf{A}$ is to solve a linear Lyapunov matrix equation for a positive definite matrix solution; if such solution exists, it corresponds to a quadratic Lyapunov function that establishes stability of the system.

2.7 Summary

This chapter is mainly focused on the issue of stability definition and classification in power systems from a fundamental as well as practical point of view. A comprehensive definition of power system stability that is precise enough to encompass all aspects of power system stability is also presented. The main focus of the chapter is to provide a systematic classification of power system stability, and the identification of different categories of stability behaviours that are important in power system stability analysis. The chapter also provides a brief treatment of definitions and concepts of stability from mathematical and control theoretical view point. Connections between different types of power system stabilities and the corresponding mathematical theory are also established.

Chapter 3

Power System Modelling

3.1 Introduction

This chapter briefly reviews the issue of power system modelling that is useful for analysis and control design in later chapters. Because of the intended use of these models for systematic power system analysis and control design, Power System Dynamics (PSD) software which is primarily used in this dissertation for simulation studies, is discussed in terms of sets of structural and functional subdivisions. In particular, these subdivisions precisely reveal the interrelations/interactions among the individual components as well as the computational structure for describing real large power systems. Further emphasis is laid on decomposing the linearized dynamic model as interconnected subsystems that could be used to design decentralized controllers for power system. Moreover, the problem of model reduction for large power systems is discussed by deriving a relatively low-order model which is necessary for applying controller design techniques developed in the following chapters.

Section 3.2 briefly explains modelling and simulation of power systems using a set of structural and functional subdivisions. Section 3.3 presents modelling of power system for small-signal analysis. Section 3.4 introduces a linear model description of power systems in which the interconnected subsystems and their interconnection terms are separated. Moreover, Section 3.5 provides a brief discussion of model reduction for large power systems. Finally, Section 3.6 presents a four machine two area test system which is used for all simulation studies in this dissertation.

3.2 Nonlinear Modelling and Simulation of Power Systems

Modern power systems are characterized by complex dynamic behaviours owing to their size and complexity. As the size of power systems increases, the dynamical processes are becoming more challenging for analysis as well as understanding the underlying physical phenomena. Power systems, even in their simplest form, exhibit nonlinear and time-varying behaviours. Moreover, there are numerous equipment found in today’s power systems, namely: (1) synchronous generators; (2) loads; (3) reactive-power control devices like capacitor banks and shunt reactors; (4) power electronically switched devices such as static Var Compensators (SVCs), and currently developed flexible AC transmission systems (FACTS) devices; (5) series capacitors and other equipments. Though these equipment found in today’s power systems are well-established and quite uniform in design, their precise modelling plays important role for analysis and simulation studies of the whole system.

To obtain a meaningful model of power systems, each equipment or component of the power system should be described by appropriate algebraic and/or differential equations. Combining the dynamic models of these individual components together with the associated algebraic constraints leads to the dynamic model of power systems. In general, the dynamic model of power systems can be formulated by the following nonlinear differential-algebraic equations

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{f}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}}) \\ \mathbf{0} &= \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}}) \end{aligned} \tag{3.1}$$

where $\tilde{\mathbf{x}}(t)$, $\tilde{\mathbf{y}}(t)$ and $\tilde{\mathbf{u}}(t)$ are the state, output and input variables of the power system, respectively. The parameters $\tilde{\mathbf{p}}(t)$ represent parameters and/or effects of control at particular time in the system.

In the following, modelling of power system will be further discussed based on the Power System Dynamics (PSD) software [13] that has been primarily used in this dissertation for analysis and simulation of power systems. Figure 3.1 shows the main structural components and their interrelations that are functionally implemented in the PSD software environment.

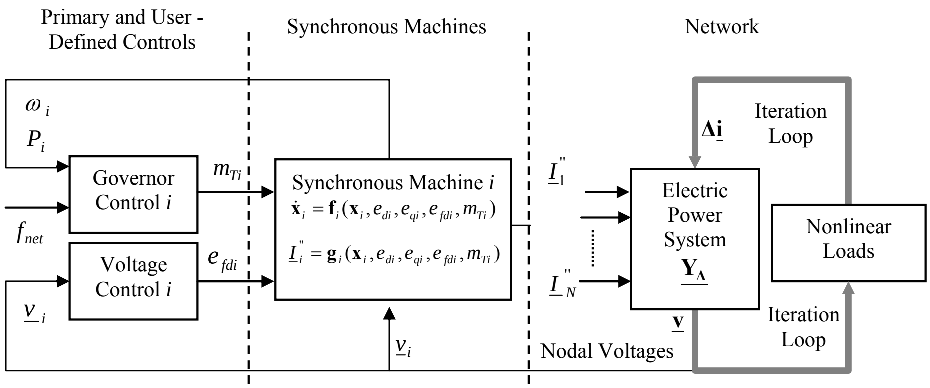


FIGURE 3.1 Nonlinear modelling and simulation of large power system.

In the following, a brief explanation of the PSD is given:

- The block in the middle of Figure 3.1 is used to describe the dynamics of synchronous machines. Their overall dynamics involve the full scale of energy-storing elements from mechanical masses to electric and magnetic fields, all driven by prime mover, normally turbines and under direct primary controls. Synchronous machines provide virtually all power generations in all today's power system. Moreover, synchronous machines have major influence on the overall dynamic performance of power systems due to their characteristics. A reduced 5th-order model, where stator transient dynamics are neglected [14], is used for all synchronous machines in this study. The model consists of a set of differential and a set of algebraic equations. Input variables to the models are the complex terminal voltage $\underline{v}_i$, the mechanical turbine torque m_{Ti} and the excitation voltage e_{fdi}. Moreover, the injected currents into the network which depend on the corresponding state variables of the synchronous machines are used as input to the algebraic network equations.

- The nodal voltages shown at the bottom of the right-side are computed by solving the algebraic network equations of the nodal admittance matrix. Moreover, nonlinear voltage dependent loads are incorporated in the system where the solutions for updating injection currents are carried out iteratively.

- The blocks in the left of Figure 3.1 represent the voltage and governor controllers. The governor control block contains, in addition to the direct primary control of the turbine torque (i.e., the governor mechanism), the mechanical dynamics of the equipment, such as the turbine or boiler that tie to the system dynamically through the governor control valve. Similarly, the voltage control block typically includes voltage regulators and exciters; and their dynamics depend on the nature of the feedback control arrangement and the nature of the source of DC voltage e_{fdi}. Moreover, user-defined controller structures can be easily incorporated either through voltage or governor controller sides and such options give greater flexibility in analysis and simulation studies.

The PSD performs nonlinear simulation for large power systems by using efficient numerical algorithms. Moreover, the PSD contains functional units for numerically linearizing the nonlinear differential-algebraic equations of the system, and then based on the modified Arnoldi's algorithm, it determines a set of eigenvalues and eigenvectors of the linearized system matrix near a given point on the complex plane.

3.3 Power Systems Modelling for Small-Signal Analysis

The starting model for small-signal analysis in power system is derived by linearizing the general nonlinear dynamic model of (3.1) around an operating (or equilibrium) point $(\tilde{\mathbf{x}}_0, \tilde{\mathbf{y}}_0, \tilde{\mathbf{u}}_0, \tilde{\mathbf{p}}_0)$ and given as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}_1\,\mathbf{u}(t) + \mathbf{B}_2\mathbf{p}(t) \tag{3.2}$$

where $\mathbf{x}(t) = \tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}_0$, $\mathbf{u}(t) = \tilde{\mathbf{u}}(t) - \tilde{\mathbf{u}}_0$ and $\mathbf{p}(t) = \tilde{\mathbf{p}}(t) - \tilde{\mathbf{p}}_0$. Here the tilde stands for the actual values of states $\tilde{\mathbf{x}}(t)$, outputs $\tilde{\mathbf{y}}(t)$, inputs $\tilde{\mathbf{u}}(t)$, and parameters $\tilde{\mathbf{p}}(t)$. Moreover, the matrices $\mathbf{A}$, $\mathbf{B}_1$ and $\mathbf{B}_2$ are evaluated at the operating point $(\tilde{\mathbf{x}}_0, \tilde{\mathbf{y}}_0, \tilde{\mathbf{u}}_0, \tilde{\mathbf{p}}_0)$ and given as follows:

$$\mathbf{A} = \left[\frac{\partial \mathbf{f}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{x}}} - \left(\frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{y}}}\right)^{-1} \frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{x}}}\right] \tag{3.3}$$

$$\mathbf{B}_1 = \left[\frac{\partial \mathbf{f}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{u}}} - \left(\frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{y}}}\right)^{-1} \frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{u}}}\right] \tag{3.4}$$

and

$$\mathbf{B}_2 = \left[\frac{\partial \mathbf{f}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{p}}} - \left(\frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{y}}}\right)^{-1} \frac{\partial \mathbf{g}(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}, \tilde{\mathbf{u}}, \tilde{\mathbf{p}})}{\partial \tilde{\mathbf{p}}}\right] \tag{3.5}$$

Depending on how detailed the model in (3.1) is used; the resulting linearized model (3.2) may or may not be applicable to study particular physical phenomena in power system. To start with, any disturbance affects all system states, and their exact changes are complex and can only be analyzed by using the full-order model. Despite this fact, much effort has been devoted to the reduction of power system dynamic models. As in modelling any dynamic system, this is typically done to avoid unnecessary complexity whenever possible. Moreover, model-reduction techniques have yet another conceptual importance in power systems. In a large power system consisting of weakly connected subsystems, it is possible to derive a relatively low-order model relevant for understanding the interactions among the subsystems (inter-area dynamics), as well as detailed models relevant for understanding the dynamics inside each subsystem (intra-area dynamics) [13], [15]–[17]. Once the models are introduced, the small-signal stability analysis of these models is straightforward. Basic analysis uses the elementary result that, given $\mathbf{u}(t)=\mathbf{0}$ and $\mathbf{p}(t)=\mathbf{0}$, the system of time-invariant linear differential equations (3.2) will have a stable response to initial conditions $\mathbf{x}(0)=\mathbf{0}$ when all eigenvalues of system matrix $\mathbf{A}$ are in the left-half plane. Moreover, the robustness of the system dynamics can be analyzed using the more involved sensitivity techniques with respect to parameter uncertainties [13], [15].

3.4 Interconnection Based Power System Modelling

In general, a large-scale interconnected power system $\mathbf{S}$ composed of N subsystems (or synchronous generators) $\mathbf{S}_i$, $i=1,2,\ldots,N$ can be described by the following equations:

$$\dot{\mathbf{x}}_i(t) = \underbrace{\mathbf{A}_i\mathbf{x}_i(t)+\mathbf{B}_i\mathbf{u}_i(t)}_{1} + \underbrace{\sum_{j=1}^{N}\left(\mathbf{M}_{ij}+\Delta\mathbf{M}_{ij}(t)\right)\mathbf{x}_j(t)}_{2} \tag{3.6}$$

where $\mathbf{x}_i(t)\in\Re^{n_i}$ is the state vector and $\mathbf{u}_i(t)\in\Re^{m_i}$ is the control variable of the subsystem $\mathbf{S}_i$. The matrices $\mathbf{A}_i$, $\mathbf{B}_i$ and $\mathbf{M}_{ij}$ are constant matrices of appropriate dimensions conformable to each $\mathbf{S}_i$. Furthermore, the matrix $\mathbf{M}_{ij}$ represents the interconnections and/or interactions among the subsystems. The term $\Delta\mathbf{M}_{ij}(t)$ is intentionally included to take into account the effect of any deviation from a given operating condition due to nonlinearities and structural changes in the system. Modelling of large power system in the form of (3.6) involves linearizing the nonlinear system equations of (3.1) for a particular operating condition and then decomposing the corresponding system equations into two parts: while the former describes the system as a hierarchical interconnection of N subsystems, the latter represents the interactions among the subsystems and nonlinearities with in the subsystems.

The interconnections and uncertainties terms in (3.6), which are used to characterize the interactions among the subsystems and the effects of nonlinearities within each subsystem, can be rewritten in the following form

$$\mathbf{h}_i(t,\mathbf{x}) = \sum_{j=1}^{N}\left(\mathbf{M}_{ij} + \Delta\mathbf{M}_{ij}(t)\right)\mathbf{x}_j(t) \tag{3.7}$$

With the assumption of no "overlapping" among $\mathbf{x}_i(t)$, the state variable $\mathbf{x}(t)\in\Re^n$ of the overall system can be denoted by $\mathbf{x}(t)=[\mathbf{x}_1^T(\cdot),\mathbf{x}_2^T(\cdot),\ldots,\mathbf{x}_N^T(\cdot)]^T$ and thus, the interconnected system $\mathbf{S}$ can then be written in a compact form as follows:

$$\dot{\mathbf{x}}(t)=\mathbf{A}_D\mathbf{x}(t)+\mathbf{B}_D\mathbf{u}(t)+\mathbf{h}(t,\mathbf{x}) \tag{3.8}$$

where $\mathbf{x}(t)\in\Re^n$ is the state and $\mathbf{u}(t)\in\Re^m$ is the input of the overall system $\mathbf{S}$, and all matrices are constant matrices of appropriate dimensions with $\mathbf{A}_D=\text{diag}\{\mathbf{A}_1,\mathbf{A}_2,\ldots,\mathbf{A}_N\}$ and $\mathbf{B}_D=\text{diag}\{\mathbf{B}_1,\mathbf{B}_2,\ldots,\mathbf{B}_N\}$.

Furthermore, the interconnection and uncertainty function in (3.8), i.e., $\mathbf{h}(t,\mathbf{x})$, is given by following expression

$$\mathbf{h}(t,\mathbf{x})=[\mathbf{h}_1^T(t,\mathbf{x}),\ \mathbf{h}_2^T(t,\mathbf{x}),\ \ldots,\ \mathbf{h}_N^T(t,\mathbf{x})]^T \tag{3.9}$$

The interconnection strength between the subsystems (i.e., generators/machines that are weakly and/or strongly coupled) considerably depends on the topology of the network as well as the loading conditions of the power system. The upper bound of $\mathbf{h}_i(t,\mathbf{x})^{\mathrm{T}}\mathbf{h}_i(t,\mathbf{x})$ can be considered as a qualitative bound on the interconnection term due to the influence of the other subsystems on the *i*th-subsystem as well as any deviation from the given operating conditions with in the *i*th-subsystem. Moreover, it can be used for accessing the connectively stability condition under light- and heavy-load operations of the whole interconnected system.

3.5 Model Reduction of Large Power Systems

Model reduction for large electric power systems is primarily carried-out for modelling one's own system in sufficient detail, while "equivalencing" the rest of the power system by a lower order model [13], [15], [17].

Consider the power system shown in Figure 3.2 in which the subsystem (i.e., the internal subsystem) indicated on the left-side is to be retained for further analysis and the remainder (i.e., the external subsystem) is to be reduced.

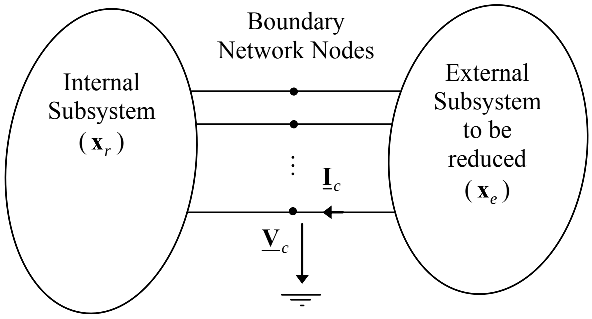


FIGURE 3.2 Large power systems for model reduction.

With minor abuse of notation, let the linearized set of the differential-algebraic equations at the given operating point for the internal subsystem (including the boundary network nodes) shown in Figure 3.2 be given by the following

$$\begin{aligned} \dot{\mathbf{x}}_r(t) &= \mathbf{A}_r \mathbf{x}_r(t) + \mathbf{B}_{r1} \underline{\mathbf{V}}_c + \mathbf{B}_{r2} \mathbf{u}(t) \\ \underline{\mathbf{I}}_c &= \mathbf{C}_r \mathbf{x}_r(t) + \mathbf{D}_r \underline{\mathbf{V}}_c \end{aligned} \tag{3.10}$$

where $\mathbf{x}_r(t)$ and $\mathbf{u}(t)$ are the state vector and the input vector of the internal subsystem, respectively. $\underline{\mathbf{V}}_c$ and $\underline{\mathbf{I}}_c$ are the coupling voltage and current measurements at the boundary network nodes, i.e., at the interconnection point.

Similarly, the external subsystem and coupling current measurements at the interface connection point be described by the following linearized set of differential-algebraic equations

$$\begin{aligned} \dot{\mathbf{x}}_e(t) &= \mathbf{A}_e \mathbf{x}_e(t) + \mathbf{B}_e \underline{\mathbf{V}}_c \\ \underline{\mathbf{I}}_c &= \mathbf{C}_e \mathbf{x}_e(t) + \mathbf{D}_e \underline{\mathbf{V}}_c \end{aligned} \tag{3.11}$$

where $\mathbf{x}_e(t)$, $\underline{\mathbf{V}}_c$, and $\underline{\mathbf{I}}_c$ are the state vector of the external subsystem, the coupling voltage and current measurements at the interconnection point, respectively. Notice also that there are no input variables for this external subsystem.

Thus, for the external subsystem given in (3.11) standard model reduction techniques such as balanced truncation [18]–[21], moment matching approximation [22], singular perturbation approximation [23] and optimal Hankel norm approximation [24] can be carried-out to reduce the order of the corresponding subsystem model. However, when performing model reduction for the external power subsystem, it is important to keep in mind that the reduced model of this subsystem should capture the system dynamics accurately in the frequency range under considerations and forcing inputs. The frequency range for the electromechanical dynamic studies of power systems usually lies between 0.1 Hz and 10.0 Hz.

Let the reduced order model for the external subsystem be given as follows:

$$\begin{aligned}\dot{\hat{\mathbf{x}}}_e(t) &= \hat{\mathbf{A}}_e\hat{\mathbf{x}}_e(t) + \hat{\mathbf{B}}_e\,\underline{\mathbf{V}}_c \\ \underline{\mathbf{I}}_c &= \hat{\mathbf{C}}_e\hat{\mathbf{x}}_e(t) + \hat{\mathbf{D}}_e\,\underline{\mathbf{V}}_c\end{aligned} \tag{3.12}$$

where $\hat{\mathbf{A}}_e$, $\hat{\mathbf{B}}_e$, $\hat{\mathbf{C}}_e$ and $\hat{\mathbf{D}}_e$ are the state matrices for the reduced external subsystem.

Using (3.10) and (3.12) the state space description for the whole system can then be written in a compact form as follows:

$$\begin{bmatrix}\dot{\mathbf{x}}_r(t)\\ \dot{\hat{\mathbf{x}}}_e(t)\end{bmatrix} = \begin{bmatrix}\mathbf{A}_r + \mathbf{B}_{r1}\hat{\mathbf{D}}_e^{-1}(\mathbf{E}_e - \mathbf{D}_r\hat{\mathbf{D}}_e^{-1})^{-1}\mathbf{C}_r & -\mathbf{B}_{r1}\hat{\mathbf{D}}_e^{-1}(\hat{\mathbf{C}}_e + (\mathbf{E}_e - \mathbf{D}_r\hat{\mathbf{D}}_e^{-1})^{-1}\mathbf{D}_r\hat{\mathbf{D}}_e^{-1}\hat{\mathbf{C}}_e) \\ \hat{\mathbf{B}}_e\hat{\mathbf{D}}_e^{-1}(\mathbf{E}_e - \mathbf{D}_r\hat{\mathbf{D}}_e^{-1})^{-1}\mathbf{C}_r & \hat{\mathbf{A}}_e - \hat{\mathbf{B}}_e\hat{\mathbf{D}}_e^{-1}(\hat{\mathbf{C}}_e + (\mathbf{E}_e - \mathbf{D}_r\hat{\mathbf{D}}_e^{-1})^{-1}\mathbf{D}_r\hat{\mathbf{D}}_e^{-1}\hat{\mathbf{C}}_e)\end{bmatrix}\begin{bmatrix}\mathbf{x}_r(t)\\ \hat{\mathbf{x}}_e(t)\end{bmatrix} + \begin{bmatrix}\mathbf{B}_{r2}\\ \mathbf{0}\end{bmatrix}\mathbf{u}(t) \tag{3.13}$$

where $\mathbf{E}_e$ is an identity matrix.

Representing large power system in the form (3.13), i.e. modelling a particular part of the system in sufficient detail, while "approximating" the rest of the power system by a very-low-order model, is necessary for both power system analysis and controller designs that are developed in the following chapters.

3.6 Four Machine Two-Area Test System

The four-machine two area test system, which is shown in Figure 3.3, has been specifically designed to study the fundamental behaviour of large interconnected power systems including inter-area oscillations in power systems [25]. The system, which is used for all simulation studies in this dissertation, has four generators. Each generator is equipped with IEEE standard exciter (IEEE Type DC1A Excitation System) and governor (Thermal Type Governor) controllers. The parameters for the standard exciter and governor controllers used in the simulation were taken from [3]. Moreover, the generators for all simulation studies in this research are represented by their 5th-order models. The following base loading condition was assumed: at node-1 a load of $[\mathrm{P_{L1}} = 1600\text{ MW},\ \mathrm{Q_{L1}} = 150\text{ Mvar}]$ and at node-2 a load of $[\mathrm{P_{L2}} = 2400\text{ MW},\ \mathrm{Q_{L2}} = 120\text{ Mvar}]$. Detail information about this test system including the controllers and their parameter values can be found in Appendix C. Furthermore, for all simulation studies as well as for the power system stabilizers (PSSs) designs, the general structure of the ith-generator together with an n_{ci}th-order PSS in a multimachine power system which is shown in Figure 3.4 is considered.

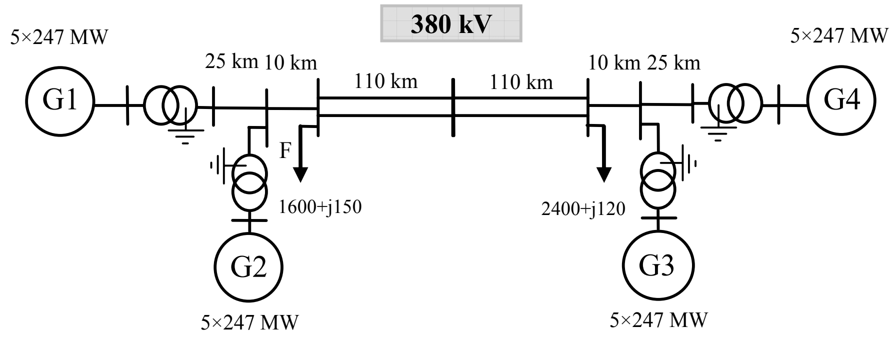


FIGURE 3.3 One-line diagram of four machine two area system.

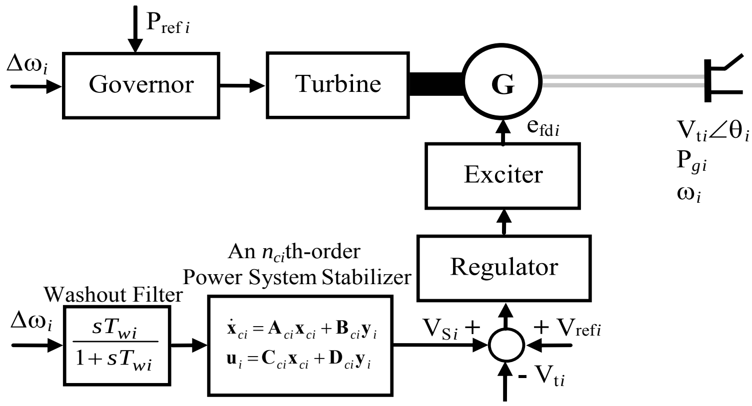


FIGURE 3.4 General structure of the *i*th-generator together with PSS in a multimachine system**.**

3.7 Summary

In this chapter, modelling and analysis of large power system dynamics are discussed. The chapter briefly discussed real large power systems representation with respect to the PSD software where the latter is primarily used for analysis and simulation studies in this dissertation. Moreover, the chapter presented a linear model description of linear power systems where the subsystems and their interconnection terms are separated. The chapter is also addressed the problem of model reduction for large power systems which is necessary for power system analysis and control designs. Finally, the chapter has briefly discussed a four machine two-area test system which is used to for all simulation studies in this dissertation.

Chapter 4

Background on LMI Based Robust Control Theory

4.1 Introduction

This chapter presents a brief review of convex optimization and linear matrix inequality (LMI) based controller synthesis theories, which are essential to understand the contribution of this dissertation. The materials presented in this chapter are intended to give a background to understand the fundamental facts about LMI based optimization problems such as H_∞ optimization problems of full-order controllers that can be reparameterized as convex optimization problems; therefore, the global optimum can be solved efficiently in practice using interior-point methods. On the other hand, constraints such as decentralized structure (i.e., controllers with “block diagonal” or other specified structure) and/or reduced-order controllers when included as design information in the problem; the nature of the problem cannot be parameterized as family of a convex optimization problem. Consequently, this type of design problems renders a computational challenge.

This chapter is organized as follows. Section 4.2 gives a brief review of convex optimization and semidefinite programming (SDP). Section 4.3 reviews H_∞ and optimization problems of state-feedback and full-order controllers together with associated SDP formulations. The proof of each formulation is presented to illustrate the limitations of LMI-based approaches in applying to more general problems. Section 4.4 treats the associated limitation of LMI based design formulation when the structure of the controller is either a block diagonal (i.e., decentralized) or other specified structure. Section 4.5 briefly discusses the problem of decentralized H_∞ controller for large scale system and the associated computation difficulty.

4.2 Convex Optimization and SDP Problems

4.2.1 Convex Optimization Problems

This section presents a brief overview of convex optimization. First, definitions of a convex set, a convex function, and a convex optimization problem are given.

DEFINITION 4.1: A set C is convex if the line segment between any two points in C lies in C, i.e., if for any $\mathbf{x}_1, \mathbf{x}_2 \in C$ any λ with $0 \le \lambda \le 1$, the following holds:

$$\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2 \in C \tag{4.1}$$

DEFINITION 4.2: A function $\mathbf{f} : \Re^n \rightarrow \Re$ is convex if $dom\,\mathbf{f}$ is a convex set and if

$$\mathbf{f}(\lambda \mathbf{x}_1 + (1-\lambda)\mathbf{x}_2) \le \lambda \mathbf{f}(\mathbf{x}_1) + (1-\lambda)\mathbf{f}(\mathbf{x}_2) \tag{4.2}$$

for any $\mathbf{x}_1,\ \mathbf{x}_2 \in dom\,\mathbf{f}$, and λ with $0 \le \lambda \le 1$.

Geometrically, this inequality means that the line segment between $(\mathbf{x}_1, \mathbf{f}(\mathbf{x}_1))$ and $(\mathbf{x}_2, \mathbf{f}(\mathbf{x}_2))$ lies above the graph of $\mathbf{f}(\mathbf{x})$. Similarly, a function $\mathbf{f}(\mathbf{x})$ is called concave if $-\mathbf{f}(\mathbf{x})$ is convex.

DEFINITION 4.3: The minimization problem of a convex function over the optimization variable $\mathbf{x} \in \Re^n$ subject to inequality constraints on convex functions of $\mathbf{x}$ and equality constraints on affine function of $\mathbf{x}$ is a ***convex optimization problem***, i.e.,

$$\begin{aligned} &\min_{\mathbf{x}\in\Re^n} \mathbf{c}(\mathbf{x}) \\ &\text{subject to } \mathbf{f}_i(\mathbf{x}) \le 0 \quad (i=1,\ldots,m) \\ &\qquad\qquad \mathbf{a}_j^T\mathbf{x} = \mathbf{b}_j \quad (j=1,\ldots,p) \end{aligned} \tag{4.3}$$

where $\mathbf{c}(\mathbf{x})$ and $\mathbf{f}_i(\mathbf{x})$, $i=1,\ \ldots,\ m$ are convex functions.

A fundamental property of convex optimization problems is that any local optimum is also globally optimal. The local and global optima for general optimization problems are defined as follows:

DEFINITION 4.4: Given a generalized optimization problem,

$$\begin{aligned} &\min_{\mathbf{x}\in\Re^n} \mathbf{c}(\mathbf{x}) \\ &\text{subject to } \mathbf{f}_i(\mathbf{x}) \le 0 \quad (i=1,\ldots,m) \\ &\qquad\qquad \mathbf{g}_j(\mathbf{x}) = 0 \quad (j=1,\ldots,p) \end{aligned} \tag{4.4}$$

$\mathbf{x}$ is locally optimal if $\mathbf{x}$ is feasible (i.e., $\mathbf{x}$ satisfies all the constraints) and

$$\mathbf{c}(\mathbf{x}) = \inf_{\mathbf{z}\in\Re^n} \left\{ \mathbf{c}(\mathbf{z}) | \mathbf{z} \text{ is feasible, } \|\mathbf{z}-\mathbf{x}\| \le R \right\} \tag{4.5}$$

for some $R>0$. If (4.5) holds for any feasible $\mathbf{z}$ (i.e., $R = \infty$), then $\mathbf{x}$ is globally optimal.

THEOREM 4.1: Suppose $\mathbf{x}$ is locally optimal for the convex optimization problem (4.3), then $\mathbf{x}$ is also globally optimal.

Consider an unconstrained minimization problem of a convex objective function, $\mathbf{c}(\mathbf{x})$, over a variable, $\mathbf{x} \in \Re^n$. When $\mathbf{c}(\mathbf{x})$ is differentiable, Theorem 4.1 assures that the global optimum, $\mathbf{x}^*$, can be found by simply computing the solution of $\nabla\mathbf{c}(\mathbf{x})=\mathbf{0}$, where $\nabla\mathbf{c}(\mathbf{x})$ denotes the gradient of $\mathbf{c}(\mathbf{x})$ at $\mathbf{x} \in \Re^n$, i.e. $\nabla\mathbf{c}(\mathbf{x}):=\{\partial\mathbf{c}(\mathbf{x})/\partial x_1, \partial\mathbf{c}(\mathbf{x})/\partial x_2, \cdots, \partial\mathbf{c}(\mathbf{x})/\partial x_n\}^T$. Such a problem can be solved in quite an efficient manner by iterative algorithms, which compute a sequence of points $\mathbf{x}^{(0)}, \mathbf{x}^{(1)}, \cdots$ with $\mathbf{c}(\mathbf{x}^{(k)})$ converging to the optimal point as $k \to \infty$.

THEOREM 4.2: Define the $\boldsymbol{\alpha}$ - sublevel set C_α of a function $\mathbf{f}: \Re^n \to \Re$ by

$$C_{\boldsymbol{\alpha}} = \left\{ \mathbf{x} \in dom\mathbf{f} \,|\, \mathbf{f}(\mathbf{x}) \le \boldsymbol{\alpha} \right\} \tag{4.6}$$

Sublevel sets of a convex function are convex.

THEOREM 4.3: The intersection of convex sets is also a convex. i.e., if sets $S_i\ (i=1,\cdots,n)$ are all convex, then their intersection $\bigcap_{i\in[1,n]} S_i$ is also convex.

4.2.2 Semidefinite Programming (SDP)

Consider the following problem of minimizing a linear objective function subject to a matrix inequality:

$$\begin{aligned} &\min_{\mathbf{x}\in\Re^m} \mathbf{c}^T\mathbf{x} \\ &\text{subject to } \mathbf{F}(\mathbf{x}) \preceq \mathbf{0} \end{aligned} \tag{4.7}$$

where

$$\mathbf{F}(\mathbf{x}) := \mathbf{F}_0 + \sum_{i=1}^{m} x_i \mathbf{F}_i \tag{4.8}$$

$\mathbf{c} \in \Re^m$ and the symmetric matrices $\mathbf{F}_i = \mathbf{F}_i^T \in \Re^{n\times n}$, $i = 0, 1, \cdots, m$ are given. The inequality symbol in (4.7) means that $\mathbf{F}(\mathbf{x})$ is negative semidefinite, i.e., $\mathbf{u}^T\mathbf{F}(\mathbf{x})\mathbf{u} \preceq 0$ for all nonzero $\mathbf{u} \in \Re^n$.

DEFINITION 4.4: The inequality $\mathbf{F}(\mathbf{x}) \preceq 0$ is a linear matrix inequality (LMI) and the problem (4.7) is a semidefinite programme (SDP).

4.3 Robust H_∞ Controller Design Using LMI Techniques

4.3.1 Problem Statement

The H_∞ optimal controller synthesis problem is formulated as follows. Suppose the closed-loop system is given as shown in Figure 4.1 where the plant model, $P(s)$, and the controller, $C(s)$, are assumed to be real, rational and proper continuous-time transfer function matrices. Suppose that the plant model, $P(s)$, can be written in the following continuous-time state space representation

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}_1\,\mathbf{w}(t) + \mathbf{B}_2\,\mathbf{u}(t)\\ \mathbf{z}(t) &= \mathbf{C}_1\mathbf{x}(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{12}\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}_y\mathbf{x}(t) + \mathbf{D}_{y1}\mathbf{w}(t) + \mathbf{D}_{y2}\mathbf{u}(t)\end{aligned} \tag{4.9}$$

The plant dimensions are summarized by $\mathbf{A} \in \Re^{n\times n}$, $\mathbf{D}_{11} \in \Re^{p_1\times m_1}$ and $\mathbf{D}_{y2} \in \Re^{p_2\times m_2}$.

The following assumptions are imposed on the plant parameters.

- $(\mathbf{A}, \mathbf{B}_2, \mathbf{C}_y)$ is stabilizable and detectable.
- $\mathbf{D}_{y2} = \mathbf{0}_{p_2\times m_2}$

Notice that neither of the assumption is restrictive in practice; the first assumption is necessary and sufficient to allow for plant stabilization by dynamic output feedback. The second assumption incurs no loss of generality while it considerably simplifies calculations for H_∞ optimization.

Suppose that the controller dynamics, $C(s)$, is given in the following state space representation:

$$\begin{aligned}\dot{\mathbf{x}}_c(t) &= \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{B}_c\mathbf{y}(t)\\ \mathbf{u}(t) &= \mathbf{C}_c\mathbf{x}_c(t) + \mathbf{D}_c\mathbf{y}(t)\end{aligned} \tag{4.10}$$

where $\mathbf{A}_c \in \Re^{n_c\times n_c}$, and the dimensions of $\mathbf{B}_c$, $\mathbf{C}_c$ and $\mathbf{D}_c$ are compatible with $P(s)$ given in (4.9).

The objective of the H_∞ - optimal controller synthesis problem is to find a controller $C(s)$ (i.e., $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$ and $\mathbf{D}_c$) such that: i) the closed-loop system is internally stable, and ii) the H_∞ norm of $F_L(P(s),C(s))$, i.e., the closed-loop transfer function from $\mathbf{w}$ to $\mathbf{z}$ in Figure 4.1. is minimized. It is often more convenient to look for a controller, $C(s)$, that achieves the closed-loop H_∞ norm $\|F_L(P(s),C(s))\|_\infty$ less than a given constant level $\gamma > 0$, rather than one that minimizes $\|F_L(P(s),C(s))\|_\infty$. The controller, $C(s)$, that i) internally stabilizes the closed-loop system, and ii) achieves $\|F_L(P(s),C(s))\|_\infty < \gamma$ for a given $\gamma > 0$, is called the H_∞ γ - suboptimal controller.

The conventional full-order H_∞ (sub)-optimization algorithm assumes that: i) the controller $C(s)$ is full-order, i.e., the order of the controller is the same as that of the plant model $P(s)$, i.e., $n_c = n$, and ii) every entry of ($\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, $\mathbf{D}_c$) is freely tunable.

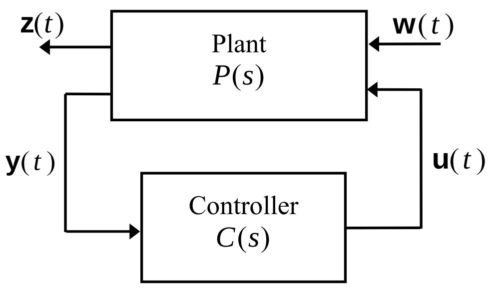


FIGURE 4.1 Feedback interconnection of the plant and the controller systems.

4.3.2 Robust H_∞ State Feedback Controller Design Using LMI Technique

This section presents a brief review of the LMI formulation of the H_∞ optimization problem of state feedback controllers. The materials presented in this section (and also in Section 4.3.3) are crucial to understanding the difficulties of applying LMI-based approaches to more general problems. Although the state feedback controller synthesis problem is not practical in most applications, its LMI formulation is much more straightforward to understand than that of the full-order controller synthesis case.

The problem is formulated as follows: all state variables of the plant, $P(s)$, given in (4.9) are assumed available, i.e. $\mathbf{C}_y = \mathbf{I}_n$ and $\mathbf{D}_{y1} = \mathbf{0}_{p_2 \times m_1}$. The problem is to find the H_∞ - optimal state feedback gain matrix, $\mathbf{K} \in \Re^{m_2 \times n}$, such that $\|F_L(P(s),\mathbf{K})\|_\infty$ is minimum.

The following lemma is crucial to interpret an H_∞ norm constraint as an LMI constraint.

LEMMA 4.1 (Bounded Real Lemma for Continuous-Time Systems) Consider a continuous-time transfer function $T(s)$ which is not necessarily minimal realization, $T(s)=\mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B}+\mathbf{D}$. Then, $\|T(s)\|<\gamma$ for $\gamma>0$ and $\mathbf{A}$ is asymptotically stable (i.e., $\mathrm{Re}(\lambda_i(\mathbf{A}))<0$) if and only if there exists a symmetric positive definite matrix, $\mathbf{P}$, that satisfies the following LMI:

$$\begin{bmatrix} \mathbf{PA}+\mathbf{A}^T\mathbf{P} & \mathbf{PB} & \mathbf{C}^T \\ \mathbf{B}^T\mathbf{P} & -\gamma\mathbf{I} & \mathbf{D}^T \\ \mathbf{C} & \mathbf{D} & -\gamma\mathbf{I} \end{bmatrix} \prec \mathbf{0} \tag{4.11}$$

By applying Lemma 4.1 to the closed-loop system $F_L(P(s),\mathbf{K})$, it can be easily observed that the H_∞ - optimal solution $\mathbf{K}^*$ can be computed by solving the following SDP problem.

LEMMA 4.2 (H_∞ - Optimal State Feedback Controller Synthesis) The continuous-time H_∞ - optimal state feedback gain matrix, $\mathbf{K}^*$, can be obtained by $\mathbf{K}^*=\mathbf{LQ}^{-1}$, where $(\mathbf{L},\mathbf{Q})$ is the optimal solution set of the following SDP problem:

$$\begin{aligned} &\min \gamma \\ &\text{over } \mathbf{Q}\in\Re^{n\times n},\ \mathbf{L}\in\Re^{m_2\times n} \text{ and } \gamma\in\Re \\ &\text{subject to } \begin{bmatrix} \mathbf{AQ}+\mathbf{QA}^T+\mathbf{B}_2\mathbf{L}+\mathbf{L}^T\mathbf{B}_2^T & \mathbf{B}_1 & (\mathbf{C}_1\mathbf{Q}+\mathbf{D}_{12}\mathbf{L})^T \\ \mathbf{B}_1^T & -\gamma\mathbf{I} & \mathbf{D}_{11}^T \\ \mathbf{C}_1\mathbf{Q}+\mathbf{D}_{12}\mathbf{L} & \mathbf{D}_{11} & -\gamma\mathbf{I} \end{bmatrix} \prec \mathbf{0} \\ &\qquad\qquad \mathbf{Q}\succ 0 \end{aligned} \tag{4.12}$$

Proof: By combining the plant model (4.9) and the state feedback law, $\mathbf{u}(t)=\mathbf{Kx}(t)$, the closed-loop system dynamics is given as follows:

$$\begin{aligned} \dot{\mathbf{x}}(t)&=(\mathbf{A}+\mathbf{B}_2\mathbf{K})\mathbf{x}(t)+\mathbf{B}_1\mathbf{w}(t) \\ \mathbf{z}(t)&=(\mathbf{C}_1+\mathbf{D}_{12}\mathbf{K})\mathbf{x}(t)+\mathbf{D}_{11}\mathbf{w}(t) \end{aligned} \tag{4.13}$$

Notice that the existence of $\mathbf{P}\succ 0$ satisfying (4.11) is equivalent to that of $\mathbf{Q}\succ 0$ satisfying:

$$\begin{bmatrix} \mathbf{AQ}+\mathbf{QA}^T & \mathbf{B} & \mathbf{QC}^T \\ \mathbf{B}^T & -\gamma\mathbf{I} & \mathbf{D}^T \\ \mathbf{CQ} & \mathbf{D} & -\gamma\mathbf{I} \end{bmatrix} \prec \mathbf{0} \tag{4.14}$$

Therefore, the H_∞ - norm of the closed-loop system (4.13) is less than γ if and only if there exists a positive definite symmetric matrix, $\mathbf{Q}\in\Re^{n\times n}$, such that

$$\begin{bmatrix} (\mathbf{A}+\mathbf{B}_2\mathbf{K})\mathbf{Q}+\mathbf{Q}(\mathbf{A}+\mathbf{B}_2\mathbf{K})^T & \mathbf{B}_1 & \mathbf{Q}(\mathbf{C}_1+\mathbf{D}_{12}\mathbf{K})^T \\ \mathbf{B}_1^T & -\gamma\mathbf{I} & \mathbf{D}_{11}^T \\ (\mathbf{C}_1+\mathbf{D}_{12}\mathbf{K})\mathbf{Q} & \mathbf{D}_{11} & -\gamma\mathbf{I} \end{bmatrix} \prec \mathbf{0} \tag{4.15}$$

Although there are bilinear terms of variables $\mathbf{K}$ and $\mathbf{Q}$ in the constraint (4.15), it can be equivalently transformed into the LMI constraint of (4.12) by simply introducing the new variable $\mathbf{L} = \mathbf{K}\,\mathbf{Q} \in \Re^{m_2 \times n}$. Since the problem (4.12) is an SDP problem, it has a unique optimal solution. Also notice that $\mathbf{Q}$ is always invertible, since it is restricted to be strictly positive definite. Therefore, the H_∞ - optimal state - feedback gain matrix, $\mathbf{K}^*$, can be uniquely obtained.

4.3.3 Robust H_∞ Full - Order Output Feedback Controller Design Using LMI Technique

This section considers the more general case where the controller is a dynamic output feedback controller whose state space representation is given in the form (4.10). The problem is to find the controller matrices $\mathbf{A}_c$, $\mathbf{B}_c$, $\mathbf{C}_c$, and $\mathbf{D}_c$ such that the H_∞ norm of the closed-loop system, $\|F_L(P(s),C(s))\|_\infty$, is minimized, where $P(s)$ is a given plant model in the form (4.9).

The standard LMI formulation for H_∞ full-order controller synthesis algorithm is based on the results in [26] and [27]:

THEOREM 4.2 (Solvability of Continuous - Time H_∞ Controller Design Problems) There exists a dynamic controller, $C(s)$, of order n_c such that $\|F_L(P(s),C(s))\|_\infty < \gamma$ if and only if there exist two symmetric matrices $\mathbf{X} \in \Re^{n \times n}$ and $\mathbf{Y} \in \Re^{n \times n}$ such that

$$\begin{gathered} \mathbf{N}_1^T \begin{bmatrix} \mathbf{AX}+\mathbf{XA}^T & \mathbf{XC}^T & \mathbf{B}_1 \\ \mathbf{C}_1\mathbf{X} & -\gamma\mathbf{I} & \mathbf{D}_{11} \\ \mathbf{B}_1^T & \mathbf{D}_{11}^T & -\gamma\mathbf{I} \end{bmatrix} \mathbf{N}_1 \prec \mathbf{0} \\ \mathbf{N}_2^T \begin{bmatrix} \mathbf{YA}+\mathbf{A}^T\mathbf{Y} & \mathbf{YB}_1 & \mathbf{C}^T \\ \mathbf{B}_1^T\mathbf{Y} & -\gamma\mathbf{I} & \mathbf{D}_{11}^T \\ \mathbf{C}_1 & \mathbf{D}_{11} & -\gamma\mathbf{I} \end{bmatrix} \mathbf{N}_2 \prec \mathbf{0} \\ \begin{bmatrix} \mathbf{X} & \mathbf{I} \\ \mathbf{I} & \mathbf{Y} \end{bmatrix} \succeq \mathbf{0} \\ Rank\,(\mathbf{XY}-\mathbf{I}) \le n_c \end{gathered} \tag{4.16}$$

where $\mathbf{N}_1 = \mathrm{diag}\{\mathbf{N}_{12}, \mathbf{I}\}$, $\mathbf{N}_2 = \mathrm{diag}\{\mathbf{N}_{21}, \mathbf{I}\}$. $\mathbf{N}_{12}$ and $\mathbf{N}_{21}$ are bases for the null space of $[\mathbf{B}_2^T, \mathbf{D}_{12}^T]$ and $[\mathbf{C}_2, \mathbf{D}_{21}]$, respectively.

First, notice that in the full-order controller synthesis case, i.e. $n_c > n$, the rank constraint in (4.16) is trivially satisfied for any $\mathbf{X}$ and $\mathbf{Y}$. Therefore, the problem for finding $\mathbf{X}$ and $\mathbf{Y}$ that satisfy the constraint (4.16) becomes an SDP problem.

The following two lemmas are required to proof the above theorem.

LEMMA 4.3: (Elimination Lemma) Given a symmetric matrix $\mathbf{\Psi} \in \Re^{m \times m}$ and two matrices $\mathbf{V}$, $\mathbf{U}$ of column dimension m, consider the problem of finding a matrix $\mathbf{\Theta}$ of compatible dimensions such that

$$\mathbf{\Psi} + \mathbf{V}^T \mathbf{\Theta}^T \mathbf{U} + \mathbf{U}^T \mathbf{\Theta} \mathbf{V} \prec \mathbf{0} \tag{4.17}$$

Denote by $\mathbf{W}_\mathbf{V}$ and $\mathbf{W}_\mathbf{U}$ any matrices whose columns form the bases of the null space of $\mathbf{V}$ and $\mathbf{U}$, respectively. Then, the problem in (4.17) is solvable for $\mathbf{\Theta}$ if and only if

$$\begin{cases} \mathbf{W}_\mathbf{V}^T \, \mathbf{\Psi} \, \mathbf{W}_\mathbf{V} \prec \mathbf{0} \\ \mathbf{W}_\mathbf{U}^T \, \mathbf{\Psi} \, \mathbf{W}_\mathbf{U} \prec \mathbf{0} \end{cases} \tag{4.18}$$

LEMMA 4.4: (Schur complement) The block matrix $\begin{bmatrix} \mathbf{P} & \mathbf{M} \\ \mathbf{M}^T & \mathbf{Q} \end{bmatrix}$ is negative definite if and only if

$$\begin{cases} \mathbf{Q} \prec \mathbf{0} \\ \mathbf{P} - \mathbf{M}\mathbf{Q}^{-1}\mathbf{M}^T \prec \mathbf{0} \end{cases} \tag{4.19}$$

$\mathbf{P} - \mathbf{M}\,\mathbf{Q}^{-1}\,\mathbf{M}^T$ is called the Schur complement of $\mathbf{Q}$.

Proof of THEOREM 4.2**:** From the plant model (4.9) and the controller (4.10), a state space representation of the closed-loop system (not necessarily minimal), $F_L(P(s), C(s))$, is given as follows:

$$\begin{aligned} \dot{\mathbf{x}}_{cl}(t) &= \mathbf{A}_{cl}\mathbf{x}_{cl}(t) + \mathbf{B}_{cl}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_{cl}\mathbf{x}_{cl}(t) + \mathbf{D}_{cl}\mathbf{w}(t) \end{aligned} \tag{4.20}$$

where $\mathbf{x}_{cl}(t) = [\mathbf{x}(t)^T \quad \mathbf{x}_c(t)^T]^T$ and

$$\mathbf{A}_{cl}=\begin{bmatrix}\mathbf{A}+\mathbf{B}_2\mathbf{D}_c\mathbf{C}_2 & \mathbf{B}_2\mathbf{C}_c\\ \mathbf{B}_c\mathbf{C}_2 & \mathbf{A}_c\end{bmatrix},\quad \mathbf{B}_{cl}=\begin{bmatrix}\mathbf{B}_1+\mathbf{B}_2\mathbf{D}_c\mathbf{D}_{21}\\ \mathbf{B}_c\mathbf{D}_{21}\end{bmatrix},$$
$$\mathbf{C}_{cl}=\begin{bmatrix}\mathbf{C}_1+\mathbf{D}_{12}\mathbf{D}_c\mathbf{C}_2\\ \mathbf{D}_{12}\mathbf{C}_c\end{bmatrix},\qquad \mathbf{D}_{cl}=[\mathbf{D}_{11}+\mathbf{D}_{12}\mathbf{D}_c\mathbf{D}_{21}] \tag{4.21}$$

The closed-loop system matrices can be rewritten as

$$\mathbf{A}_{cl}=\mathbf{A}_0+\mathcal{B}\mathbf{K}\mathcal{D}_{21},\ \ \mathbf{B}_{cl}=\mathbf{B}_0+\mathcal{B}\mathbf{K}\mathcal{C};\ \ \mathbf{C}_{cl}=\mathbf{C}_0+\mathcal{D}_{12}\mathbf{K}\mathcal{C};\ \ \mathbf{D}_{cl}=\mathbf{D}_{11}+\mathcal{D}_{12}\mathbf{K}\mathcal{D}_{21} \tag{4.22}$$

by using the following expressions:

$$\mathbf{A}_0:=\begin{bmatrix}\mathbf{A} & \mathbf{0}\\ \mathbf{0} & \mathbf{0}_{n_c}\end{bmatrix},\quad \mathbf{B}_{cl}:=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{0}\end{bmatrix},\qquad \mathbf{C}_0:=[\mathbf{C}_1\quad \mathbf{0}],$$
$$\mathcal{B}:=\begin{bmatrix}\mathbf{0} & \mathbf{B}_2\\ \mathbf{I}_{n_c} & \mathbf{0}\end{bmatrix},\quad \mathcal{C}:=\begin{bmatrix}\mathbf{0} & \mathbf{I}_{n_c}\\ \mathbf{C}_2 & \mathbf{0}\end{bmatrix},\quad \mathcal{D}_{12}:=[\mathbf{0}\quad \mathbf{D}_{12}],\qquad \mathcal{D}_{12}:=\begin{bmatrix}\mathbf{0}\\ \mathbf{D}_{12}\end{bmatrix} \tag{4.23}$$

and

$$\mathbf{K}=\begin{bmatrix}\mathbf{A}_c & \mathbf{B}_c\\ \mathbf{C}_c & \mathbf{D}_c\end{bmatrix} \tag{4.24}$$

From the Bounded Real Lemma, the controller (4.10) is an H_∞ γ - optimal controller if and only if there exists a symmetric positive definite matrix, $\mathbf{P}_{cl}\in\Re^{(n+n_c)\times(n+n_c)}$, that satisfies:

$$\begin{bmatrix}\mathbf{P}_{cl}\mathbf{A}_{cl}+\mathbf{A}_{cl}^T\mathbf{P}_{cl} & \mathbf{P}_{cl}\mathbf{B}_{cl} & \mathbf{C}_{cl}^T\\ \mathbf{B}_{cl}^T\mathbf{P}_{cl} & -\gamma\mathbf{I} & \mathbf{D}_{cl}^T\\ \mathbf{C}_{cl} & \mathbf{D}_{cl} & -\gamma\mathbf{I}\end{bmatrix}\prec\mathbf{0} \tag{4.25}$$

Using the expression (4.22), the above matrix inequality can be rewritten as:

$$\mathbf{\Psi}_{\mathbf{P}_{cl}}+\mathbf{V}_{\mathbf{P}_{cl}}^T\mathbf{K}\,\mathbf{U}+\mathbf{U}^T\mathbf{K}^T\mathbf{V}_{\mathbf{P}_{cl}}\prec\mathbf{0} \tag{4.26}$$

where

$$\mathbf{\Psi}_{\mathbf{P}_{cl}}:=\begin{bmatrix}\mathbf{P}_{cl}\mathbf{A}_{cl}+\mathbf{A}_{cl}^T\mathbf{P}_{cl} & \mathbf{P}_{cl}\mathbf{B}_{cl} & \mathbf{C}_{cl}^T\\ \mathbf{B}_{cl}^T\mathbf{P}_{cl} & -\gamma\mathbf{I} & \mathbf{D}_{cl}^T\\ \mathbf{C}_{cl} & \mathbf{D}_{cl} & -\gamma\mathbf{I}\end{bmatrix},$$
$$\mathbf{V}_{\mathbf{P}_{cl}}:=[\mathcal{B}^T\mathbf{P}_{cl}\quad \mathbf{0}\quad \mathcal{D}_{12}],\qquad \mathbf{U}:=[\mathcal{C}\quad \mathcal{D}_{21}\quad \mathbf{0}] \tag{4.27}$$

Therefore, the set of γ - suboptimal controllers of order n_c is nonempty if and only if (4.26) holds for some $\mathbf{K}\in\Re^{(n_c+m_2)\times(n_c+p_2)}$ and $\mathbf{P}_{cl}\succ\mathbf{0}$. Notice that there are bilinear terms in the constraint (4.26) with respect to $\mathbf{K}$ and $\mathbf{P}_{cl}$. The Elimination Lemma can now be invoked to equivalently transform the solvability condition (4.26) into a form depending only on $\mathbf{P}_{cl}$ and the plant parameters. Let $\mathbf{W}_{\mathbf{V}_{\mathbf{P}_{cl}}}$ and $\mathbf{W}_{\mathbf{U}}$ denote matrices whose columns form the bases of the null space of $\mathbf{V}_{\mathbf{P}_{cl}}$ and $\mathbf{U}$, respectively. Then, by Lemma 4.3, the constraint (4.26) holds for some $\mathbf{K}$ if and only if

$$\mathbf{W}_{\mathbf{V}_{\mathbf{P}_{cl}}}^T \mathbf{\Psi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{V}_{\mathbf{P}_{cl}}} \prec \mathbf{0}, \qquad \mathbf{W}_{\mathbf{U}}^T \mathbf{\Psi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{U}} \prec \mathbf{0} \tag{4.28}$$

Furthermore, it is straightforward to see that the existence of $\mathbf{P}_{cl}$ satisfying (4.28) is equivalent to that $\mathbf{P}_{cl}$ satisfying

$$\mathbf{W}_{\mathbf{V}}^T \mathbf{\Phi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{V}} \prec \mathbf{0}; \qquad \mathbf{W}_{\mathbf{U}}^T \mathbf{\Phi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{U}} \prec \mathbf{0} \tag{4.29}$$

where

$$\mathbf{\Phi}_{\mathbf{P}_{cl}} := \begin{bmatrix} \mathbf{A}_{cl}\mathbf{P}_{cl}^{-1} + \mathbf{P}_{cl}^{-1}\mathbf{A}_{cl}^T & \mathbf{B}_{cl} & \mathbf{P}_{cl}^{-1}\mathbf{C}_{cl}^T \\ \mathbf{B}_{cl}^T & -\gamma\mathbf{I} & \mathbf{D}_{cl}^T \\ \mathbf{C}_{cl}\mathbf{P}_{cl}^{-1} & \mathbf{D}_{cl} & -\gamma\mathbf{I} \end{bmatrix} \tag{4.30}$$

and $\mathbf{W}_{\mathbf{V}}$ is a matrix whose column form bases of the null space of $\mathbf{V} := [\mathbf{P}_{cl}^{-1}\mathcal{B}^T \quad \mathbf{0} \quad \mathcal{D}_{12}]$. The constraints in (4.29) are still not convex because they involve both $\mathbf{P}_{cl}$ and its inverse. Fortunately, they can be further reduced to pair of matrix inequality equations of lower dimension that are convex constraints. First, partition $\mathbf{P}_{cl}$ and $\mathbf{P}_{cl}^{-1}$ as follows:

$$\mathbf{P}_{cl} := \begin{bmatrix} \mathbf{Y} & \mathbf{N} \\ \mathbf{N}^T & * \end{bmatrix}, \qquad \mathbf{P}_{cl}^{-1} := \begin{bmatrix} \mathbf{X} & \mathbf{M} \\ \mathbf{M}^T & * \end{bmatrix} \tag{4.31}$$

where $\mathbf{X},\ \mathbf{Y} \in \Re^{n\times n}$ and $\mathbf{M},\ \mathbf{N} \in \Re^{n\times n_c}$.

By using this partition, the conditions $\mathbf{W}_{\mathbf{V}}^T \mathbf{\Phi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{V}} \prec \mathbf{0}$ and $\mathbf{W}_{\mathbf{U}}^T \mathbf{\Phi}_{\mathbf{P}_{cl}} \mathbf{W}_{\mathbf{U}} \prec \mathbf{0}$ are equivalently reduced to the constraint (4.16).

Theorem 4.2 only addresses the existence of a solution and does not include the computation of the optimal controller. The system matrices of the H_∞ controller, $\mathbf{K}$, can be obtained as follows. Once $\mathbf{X}$ and $\mathbf{Y}$ satisfying (4.16) are found, $\mathbf{P}_{cl}$ can be reconstructed from (4.31) by using the singular value decomposition (SVD). Then, $\mathbf{K}$ can be computed by solving (4.26). Notice that when $\mathbf{P}_{cl}$ is given, (4.26) becomes an LMI with respect to $\mathbf{K}$, and thus the problem of finding $\mathbf{K}$ is a convex optimization problem.

Theorem 4.2 states that: *i*) the controller that achieves the minimum $\|F_L(P(s), C(s))\|_\infty$ is, at most, the same order as the plant $P(s)$, and *ii*) the H_∞ - optimal controller of the same order as the plant, $P(s)$, can be computed by solving convex optimization problems. In the case where the order of the controller is restricted to less than the plant order, however, the problem becomes a nonconvex optimization problem due to the rank constraint (4.16).

Furthermore, the SDP formulation of the H_∞ optimization problem clarifies the difficulties of imposing an additional constraint on the controller system matrices, because the SDP formulation (4.16) no longer contains the set of controller system matrices.

This dissertation focuses on solving the problems that cannot be globally solved by convex optimization. Chapter 5 presents problem formulations and the associated algorithms for solving H_∞ and H_2/H_∞ optimization problems of reduced-order decentralized controller synthesis for large power system. Moreover, the nonlinear matrix inequality (NMI) formulation is also discussed to solve a more general class of H_∞ and H_2/H_∞ optimization problems that cannot be parameterized as SDP problems.

4.3.4 Limitations of the LMI – Based Approach

It remains true however that much difficulty remains in the robust synthesis of practical controllers. At least two very important classes of robust control design issues have not been found to be readily transformable in to LMI framework, namely, reduced-order (or fixed-order) controller synthesis and decentralized controller design (i.e., synthesis of controllers with “block diagonal” or other specified structure).

One of the main disadvantage to the modern control synthesis theories, such as H_∞, is that the resultant controllers are typically of high order - at least as high as the original plant and often much higher in very usual case where the plant must be augmented by dynamical scalings, multipliers or weights in order to achieve the desired performance.

Control system designs for very large-scale systems such as power systems must often be implemented in a decentralized fashion; that is, local loops are decoupled and closed separately with little or no direct communication among local controllers. The synthesis of optimally robust decentralized systems has obvious benefits and at the time presents new challenges. Thus, the objective of this dissertation (besides developing the algorithm) is to demonstrate that the NMI framework is sufficiently flexible to simultaneously accommodate all these of the foregoing types of control design specifications, in addition to handling all those which the LMI handles.

4.4 Decentralized H_∞ Controller Problem for Multi-Channel LTI Large Systems

Consider the following N - channel linear time-invariant large-system described by

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}_1\,\mathbf{w}(t) + \sum_{i=1}^{N}\mathbf{B}_{2i}\,\mathbf{u}_i(t) \\
\mathbf{z}(t) &= \mathbf{C}_1\,\mathbf{x}(t) + \mathbf{D}_{11}\,\mathbf{w}(t) + \sum_{i=1}^{N}\mathbf{D}_{12\,i}\,\mathbf{u}_i(t) \\
\mathbf{y}_i(t) &= \mathbf{C}_{y\,i}\,\mathbf{x}(t) + \mathbf{D}_{y1\,i}\mathbf{w}(t), \quad i = 1, 2, \cdots, N
\end{aligned} \tag{4.32}$$

where $\mathbf{x} \in \Re^n$ is the state variable, $\mathbf{w} \in \Re^r$ is exogenous signal, $\mathbf{z} \in \Re^p$ is the regulated variable, $\mathbf{u}_i \in \Re^{m_i}$ and $\mathbf{y}_i \in \Re^{q_i}$ are the control input and the measurement output signals of channel i $(i = 1, 2, \cdots, N)$. The matrices $\mathbf{A}, \mathbf{B}_1, \mathbf{B}_{2i}, \mathbf{C}_1, \mathbf{C}_{yi}, \mathbf{D}_{11}, \mathbf{D}_{12i}$ and $\mathbf{D}_{y1i}$ are all constant matrices with appropriate dimensions.

Consider the following decentralized dynamic output feedback controller for the system given in (4.32)

$$\begin{aligned}
\dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\,\mathbf{y}_i(t) \\
\mathbf{u}_i(t) &= \mathbf{C}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t)
\end{aligned} \tag{4.33}$$

where $\mathbf{x}_{ci} \in \Re^{n_{ci}}$ is the state of the ith-local controller, n_{ci} is a specified dimension, and $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$, $\mathbf{D}_{ci}$, $i = 1, 2, \cdots, N$ are constant matrices to be determined during the designing.

After augmenting the decentralized controller (4.33) in the system, the state space equation of the closed-loop system will have the following form

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= (\mathbf{A} + \sum_{i=1}^{N}\mathbf{B}_{2i}\,\mathbf{D}_{ci}\mathbf{C}_{yi})\,\mathbf{x}(t) + \sum_{i=1}^{N}\mathbf{B}_{2i}\mathbf{C}_{ci}\mathbf{x}_{ci}(t) \\
&\qquad + (\mathbf{B}_1 + \sum_{i=1}^{N}\mathbf{B}_{2i}\mathbf{D}_{ci}\mathbf{D}_{21\,i})\,\mathbf{w}(t) \\
\dot{\mathbf{x}}_{ci}(t) &= \mathbf{B}_{ci}\,\mathbf{C}_{y\,i}\,\mathbf{x}(t) + \mathbf{A}_{ci}\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\mathbf{D}_{21\,i}\mathbf{w}(t), \quad i = 1, 2, \cdots, N \\
\mathbf{z}(t) &= (\mathbf{C}_1 + \sum_{i=1}^{N}\mathbf{D}_{12i}\,\mathbf{D}_{ci}\mathbf{C}_{yi})\,\mathbf{x}(t) + \sum_{i=1}^{N}\mathbf{D}_{12i}\mathbf{C}_{ci}\mathbf{x}_{ci}(t) \\
&\qquad + (\mathbf{D}_{11} + \sum_{i=1}^{N}\mathbf{D}_{12i}\mathbf{D}_{ci}\mathbf{D}_{21\,i})\,\mathbf{w}(t)
\end{aligned} \tag{4.34}$$

Collecting together the controller states as $\mathbf{x}_c(t) = [\mathbf{x}_{c1}^T(t) \quad \mathbf{x}_{c2}^T(t) \quad \cdots \quad \mathbf{x}_{cN}^T(t)]^T$ and defining the decentralized controller matrix $\mathbf{K}_D$ as

$$\mathbf{K}_D = \begin{bmatrix} \widetilde{\mathbf{A}}_c & \widetilde{\mathbf{B}}_c \\ \widetilde{\mathbf{C}}_c & \widetilde{\mathbf{D}}_c \end{bmatrix} \tag{4.35}$$

where

$$\begin{aligned}
\widetilde{\mathbf{A}}_c &= \mathrm{diag}\{\mathbf{A}_{c1}, \mathbf{A}_{c2}, \ldots, \mathbf{A}_{cN}\}, \\
\widetilde{\mathbf{B}}_c &= \mathrm{diag}\{\mathbf{B}_{c1}, \mathbf{B}_{c2}, \ldots, \mathbf{B}_{cN}\}, \\
\widetilde{\mathbf{C}}_c &= \mathrm{diag}\{\mathbf{C}_{c1}, \mathbf{C}_{c2}, \ldots, \mathbf{C}_{cN}\}, \\
\widetilde{\mathbf{D}}_c &= \mathrm{diag}\{\mathbf{D}_{c1}, \mathbf{D}_{c2}, \ldots, \mathbf{D}_{cN}\},
\end{aligned}$$

The closed-loop system (4.34) can be further rewritten as

$$\begin{aligned}
\dot{\mathbf{x}}(t) &= (\mathbf{A} + \mathbf{B}_2 \widetilde{\mathbf{D}}_c \mathbf{C}_y)\,\mathbf{x}(t) + \mathbf{B}_2 \widetilde{\mathbf{C}}_c \mathbf{x}_c(t) + (\mathbf{B}_1 + \mathbf{B}_2 \widetilde{\mathbf{D}}_c \mathbf{D}_{21})\,\mathbf{w}(t), \\
\dot{\mathbf{x}}_c(t) &= \widetilde{\mathbf{B}}_c \mathbf{C}_y \mathbf{x}(t) + \widetilde{\mathbf{A}}_c \mathbf{x}_c(t) + \widetilde{\mathbf{B}}_c \mathbf{D}_{21}\,\mathbf{w}(t), \\
\mathbf{z}(t) &= (\mathbf{C}_1 + \mathbf{D}_{12} \widetilde{\mathbf{D}}_c \mathbf{C}_y)\,\mathbf{x}(t) + \mathbf{D}_{12} \widetilde{\mathbf{C}}_c \mathbf{x}_c(t) + (\mathbf{D}_{11} + \mathbf{D}_{12} \widetilde{\mathbf{D}}_c \mathbf{D}_{21})\,\mathbf{w}(t)
\end{aligned} \tag{4.36}$$

where

$$\begin{aligned}
\mathbf{B}_2 &= [\mathbf{B}_{21}, \mathbf{B}_{22}, \ldots, \mathbf{B}_{2N}], \\
\mathbf{C}_y &= [\mathbf{C}_{y1}^T, \mathbf{C}_{y1}^T, \ldots, \mathbf{C}_{y1}^T]^T, \\
\mathbf{D}_{12} &= [\mathbf{D}_{121}, \mathbf{D}_{122}, \ldots, \mathbf{D}_{12N}], \\
\mathbf{D}_{21} &= [\mathbf{D}_{211}^T, \mathbf{D}_{212}^T, \ldots, \mathbf{D}_{21N}^T]^T,
\end{aligned}$$

Moreover, the overall extended system equation for the system can be rewritten in one state-space equation form as

$$\begin{aligned}
\dot{\widetilde{\mathbf{x}}}(t) &= (\widetilde{\mathbf{A}} + \widetilde{\mathbf{B}}_2 \mathbf{K}_D \widetilde{\mathbf{C}}_y)\widetilde{\mathbf{x}}(t) + (\widetilde{\mathbf{B}}_1 + \widetilde{\mathbf{B}}_2 \mathbf{K}_D \widetilde{\mathbf{C}}_y)\mathbf{w}(t) \\
\mathbf{z}(t) &= (\widetilde{\mathbf{C}}_1 + \widetilde{\mathbf{D}}_{12} \mathbf{K}_D \widetilde{\mathbf{C}}_y)\widetilde{\mathbf{x}}(t) + (\widetilde{\mathbf{D}}_{11} + \widetilde{\mathbf{D}}_{12} \mathbf{K}_D \widetilde{\mathbf{D}}_{y1})\mathbf{w}(t)
\end{aligned} \tag{4.37}$$

where $\widetilde{\mathbf{x}}(t) = [\mathbf{x}^T(t) \quad \mathbf{x}_c^T(t)]^T$ is the augmented state variable for the closed-loop system and

$$\widetilde{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0}_{n\times n_c} \\ \mathbf{0}_{n_c\times n} & \mathbf{0}_{n_c\times n_c} \end{bmatrix}, \qquad \widetilde{\mathbf{B}}_1 = \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{0}_{n\times r} \end{bmatrix},$$

$$\widetilde{\mathbf{B}}_2 = \begin{bmatrix} \mathbf{0}_{n\times n_c} & \mathbf{B}_2 \\ \mathbf{I}_{n_c} & \mathbf{0}_{n_c\times m} \end{bmatrix}, \qquad \widetilde{\mathbf{C}}_1 = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0}_{p\times n_c} \end{bmatrix},$$

$$\widetilde{\mathbf{C}}_y = \begin{bmatrix} \mathbf{0}_{n_c\times n} & \mathbf{I}_{n_c\times n_c} \\ \mathbf{C}_y & \mathbf{0}_{q\times n_c} \end{bmatrix}, \qquad \widetilde{\mathbf{D}}_{11} = \mathbf{D}_{11},$$

$$\widetilde{\mathbf{D}}_{12} = \begin{bmatrix} \mathbf{0}_{p\times n_c} & \mathbf{D}_{12} \end{bmatrix}, \qquad \widetilde{\mathbf{D}}_{y1} = \begin{bmatrix} \mathbf{0}_{n_c\times r} \\ \mathbf{D}_{y1} \end{bmatrix},$$

$$n_c = \sum_{i=1}^{N} n_{ci}$$

Hence, the overall extended system can be rewritten in a compact form as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}_{cl}\,\tilde{\mathbf{x}}(t) + \mathbf{B}_{cl}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_{cl}\mathbf{x}(t) + \mathbf{D}_{cl}\mathbf{w}(t) \end{aligned} \tag{4.38}$$

where

$$\begin{aligned} \mathbf{A}_{cl} &= \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad & \mathbf{B}_{cl} &= \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{D}}_{y1}, \\ \mathbf{C}_{cl} &= \tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y, & \mathbf{D}_{cl} &= \tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1} \end{aligned}$$

Consider the following design approach where the controller strategy in (4.33) internally stabilizes the closed-loop of the transfer function $T_{\mathbf{zw}}(s)$ from $\mathbf{w}$ to $\mathbf{z}$ and moreover satisfies a certain prescribed disturbance attenuation level $\gamma > 0$, i.e., $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$.

Designing a decentralized H_∞ output feedback controller for the system is equivalent to that of finding the matrix $\mathbf{K}_D$ that satisfies an H_∞ norm bound condition on the closed loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from disturbance $\mathbf{w}$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}}(s)$ must be stable.

The following theorem is instrumental in establishing the existence of decentralized control strategy (4.33) that satisfies a certain prescribed disturbance attenuation level $\gamma > 0$ on the closed loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from disturbance $\mathbf{w}$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$.

THEOREM 4.3: The system (4.32) is stabilizable with the disturbance attenuation level $\gamma > 0$ via a decentralized controller (4.33) composed of N n_{ci}- dimensional local controllers if and only if there exist a matrix $\mathbf{K}_D$ and a positive-definite matrix $\tilde{\mathbf{P}}$ that satisfy the following matrix inequality

$$\begin{aligned} \mathbf{\Phi}(\mathbf{K}_D, \tilde{\mathbf{P}}) := & \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T\tilde{\mathbf{P}} & \tilde{\mathbf{P}}\tilde{\mathbf{B}}_1 & \tilde{\mathbf{C}}_1^T \\ \tilde{\mathbf{B}}_1^T\tilde{\mathbf{P}} & -\gamma\mathbf{I}_r & \tilde{\mathbf{D}}_{11}^T \\ \tilde{\mathbf{C}}_1 & \tilde{\mathbf{D}}_{11} & -\gamma\mathbf{I}_p \end{bmatrix} + \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} \\ & + \left\{ \begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} \right\}^T \prec \mathbf{0} \end{aligned} \tag{4.39}$$

The condition stated in the above theorem seems to be the same as that of the centralized H_∞ control cases [26], [27]. However, due to the specified structure on the controller (i.e., designing controllers with "block diagonal") the controller design problem using the above theorem is a nonconvex optimization problem.

4.5 Summary

This chapter has presented a brief review of convex optimization and LMI-based H_∞ controller synthesis algorithms, which are essential to understanding the contributions of this dissertation. The LMI formulation of the H_∞ optimization problem presented in [26] and [27] clarifies the following fundamental properties of the H_∞ optimization: i) H_∞ optimization problems of state feedback controllers and full-order controllers can be re-parameterized as convex optimization problems; therefore, the global optimum can be numerically computed in an efficient and reliable manner by using well-developed interior-point methods, and ii) when an additional constraint is imposed on the controller, the problem becomes nonconvex, and the LMI-based approach cannot be applied. It is simply because both formulations of the state feedback controller synthesis problem (Lemma 4.2) and the full-order controller synthesis problem (Theorem 4.2) no longer include the controller parameters, since they are eliminated in the process of transforming the problem into an SDP problem. This chapter has also briefly addressed the problem of designing controllers with block diagonal or other specified structure for large system where the associated design problem cannot be parameterized as a convex optimization problem.

Chapter 5

Robust Decentralized Control for Power Systems Using Matrix Inequalities Approach

5.1 Introduction

In the recent past, designing decentralized controller structure for interconnected large power systems that conforms to each subsystem was one of the predominant subjects [28]–[34], [3] and has also been intensively studied with most attention of guaranteeing the connective stability of the overall system despite the interconnection uncertainties among the different subsystems. A large number of results concerning robust decentralized stabilization of interconnected power systems based on the interconnection modelling approach - an approach which explicitly takes into accounts the interactions among the different subsystems - have been reported (to cite a few; [35]–[42]). In the work of [39], interesting decentralized turbine/governor controller scheme for power system has been presented. The salient feature of this approach is the use LMI optimization technique [43]–[46] to address the problem of robust stability in the presence of interconnection uncertainties among the subsystems. However, the local state feedback controllers designed through the approach need the corresponding state information of the subsystems and which may be either impossible or simply impractical to obtain measurements of the state information for all individual subsystems. An extension to robust static exciter output feedback controller scheme for power system, based on the approach outlined in [47], was presented in [40]. However, the additional information structure constraints that are used for decomposing the controller strategy might also lead to non-true decentralized controller schemes for large power systems. On the other hand, efforts have also been made to extend the application of robust control techniques to large power systems, such as L_∞ - optimization [48], [49], H_∞ - optimization [50], [51], structured singular value (SSV or μ) technique [52], [53]. Specially, the work presented in [53] relies on the sequential μ - synthesis technique where the design procedure is carried-out successively for each local input-output pairs in the system. Though the individual controllers are sequentially designed to guarantee the robust stability and performance of the whole system, the reliability of the decentralized controller depends on the order in which the design procedures of the individual controllers are carried-out. Moreover, a failure in the lower loop might affect the stability and/or the performance of the overall system. It is also clear from the nature of the problem that the order of the controller increases for each sequentially designed local controller. Another attempt is also made to apply a linear parameter varying (LPV) technique for designing decentralized power system stabilizers for a large power system [54]. The resulting controllers designed through this technique, however, are typically high order - at least as high as the system since the technique relies on H_∞ - optimization; and besides the problem formulation attempts to solve an infinite-dimensional LMI type optimization where the latter problem is computationally very demanding. Moreover, the approach did not consider the complete power system model with all interconnections in the design formulation.

In line with this perspective, this dissertation presents alternative approaches for designing robust decentralized controller to large power systems; whereby the interactions between subsystems (synchronous generators), changes in operating conditions as well as the effects of system nonlinearities can all be taken into account. The application of these approaches to a multimachine power system allows a coordinated tuning of controllers that incorporates robustness to changes in the system. Specifically, this chapter provides alternative approaches for designing robust decentralized controller design for power systems that is formulated as minimization problem of linear objective functional under linear matrix inequality (LMI) constraints coupled with nonlinear matrix inequality (NMI) constraints.

The rest of the chapter is organized in four sections. Each section is written to provide a self-contained treatment of the approaches used for designing decentralized controller for power systems. Moreover, each section provides PSS controllers design examples for the test system using actual industry data for generators, transformers, exciter and governor controllers and simulation results that show how the approaches can be applied to real large power. In Section 5.2, the robust decentralized static-output feedback controller design problem for an interconnected power system is formulated in the framework of convex optimization involving LMIs. Section 5.3 describes the extension of robust decentralized dynamic output feedback controller design for an interconnected power system. Section 5.4 focuses on designing a robust decentralized structure-constrained controller for power systems using LMI based H_2/H_∞ optimization technique. Finally, Section 5.5 presents a robust decentralized H_∞ controller design for power system based on a generalized parameter optimization method involving matrix inequality.

5.2 Decentralized Static-Output Feedback Controllers Design for Power Systems Using Interconnection Modelling Approach

This section presents a robust decentralized static-output feedback controller design for power systems using linear matrix inequality (LMI) technique. An interconnection based modelling approach, which reveals fundamental properties of the interconnected power system dynamics and offers a physical insight into the interactions between its constituent subsystems, is used in the design formulation. During the design procedure, the robust stability of the whole system is guaranteed by the decentralized controllers while at the same time the tolerable bounds in the uncertainties arising from the interconnection between subsystems, structural changes, nonlinearities and load variations, are maximized. This section also presents a decomposition procedure using the clustering technique of the states, inputs and outputs structure information to compute directly the appropriate diagonal structures of the output gain matrix for practical implementation. Moreover, the approach is demonstrated by estimating the optimal PSS gains for a test power system and simulation results together with performance indices are also presented.

5.2.1 Controller Design Problem Formulation

Consider again the model description of large power system with interconnection terms that has been discussed in Section 3.4

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}_D\mathbf{x}(t) + \mathbf{B}_D\mathbf{u} + \mathbf{h}(t,\mathbf{x}) \\ \mathbf{y}(t) &= \mathbf{C}_D\mathbf{x}(t)\end{aligned} \tag{5.1}$$

where $\mathbf{x}(t) \in \Re^n$ is the state, $\mathbf{u}(t) \in \Re^m$ is the input and $\mathbf{y}(t) \in \Re^q$ is the output of the overall system $\mathbf{S}$, and all matrices are constant matrices of appropriate dimensions with $\mathbf{A}_D = \text{diag}\{\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_N\}$, $\mathbf{B}_D = \text{diag}\{\mathbf{B}_1, \mathbf{B}_2, ..., \mathbf{B}_N\}$, and $\mathbf{C}_D = \text{diag}\{\mathbf{C}_1, \mathbf{C}_2, ..., \mathbf{C}_N\}$. Moreover, assume that there is no unstable fixed-mode in the system with respect to the system matrices [55].

Furthermore, the interconnection and uncertainty function in (5.1), i.e., $\mathbf{h}(t,\mathbf{x})=[\mathbf{h}_1^T(t,\mathbf{x}),\, \mathbf{h}_2^T(t,\mathbf{x}),\ldots,\, \mathbf{h}_N^T(t,\mathbf{x})]^T$, is also assumed to be bounded as follows

$$\mathbf{h}^T(t,\mathbf{x})\mathbf{h}(t,\mathbf{x}) \le \mathbf{x}(t)^T\left(\sum_{i=1}^{N}\xi_i^2\mathbf{H}_i^T\mathbf{H}_i\right)\mathbf{x}(t) \tag{5.2}$$

In the following, the feedback control law must satisfy decentralized information structure constraints conformable to the subsystems, so that each subsystem is controlled using only its locally available information [28]–[33]. This critical requirement implies that the *i*th-subsystem is controlled by local control law of the following form

$$\mathbf{u}_i(\mathbf{x}_i)=\mathbf{K}_i\,\mathbf{x}_i(t) \tag{5.3}$$

where $\mathbf{K}_i \in \Re^{m_i\times n_i}$ is constant matrix and thus, the control law for the overall system will have the following form

$$\mathbf{u}(\mathbf{x})=\mathbf{K}_D\,\mathbf{x}(t) \tag{5.4}$$

where $\mathbf{K}_D=\mathrm{diag}\{\mathbf{K}_1,\mathbf{K}_2,\ldots,\mathbf{K}_N\}$ is a constant block-diagonal matrix compatible with those of $\mathbf{A}_D$ and $\mathbf{B}_D$.

The instrumental theorem which is stated below is used in establishing the robust stability of the closed-loop interconnected system

$$\dot{\mathbf{x}}(t)=(\mathbf{A}_D+\mathbf{B}_D\mathbf{K}_D)\mathbf{x}(t)+\mathbf{h}(t,\mathbf{x}) \tag{5.5}$$

via a decentralized robust control strategy (5.4) under the constraints (5.2) on the function $\mathbf{h}(t,\mathbf{x})$.

THEOREM 5.1: The interconnected system (5.1) is robustly stabilized by the decentralized control strategy of (5.4) with uncertainty degree vector $\boldsymbol{\xi}=[\xi_1,\xi_2,\ldots,\xi_N]^T$ if there exist a symmetric positive definite matrix $\mathbf{Q}_D=\mathrm{diag}\{\mathbf{Q}_D^{(1)},\mathbf{Q}_D^{(2)},\ldots,\mathbf{Q}_D^{(N)}\}$ with blocks $\mathbf{Q}_D^{(i)}\in\Re^{n_i\times n_i}$ and a matrix $\mathbf{L}_D=\mathrm{diag}\{\mathbf{L}_D^{(1)},\mathbf{L}_D^{(2)},\ldots,\mathbf{L}_D^{(N)}\}$ with blocks $\mathbf{L}_D^{(i)}\in\Re^{m_i\times n_i}$ that satisfy the following matrix inequality:

$$\begin{aligned}&\mathbf{Q}_D\mathbf{A}_D^T+\mathbf{A}_D\mathbf{Q}_D+\mathbf{L}_D^T\mathbf{B}_D^T+\mathbf{B}_D\mathbf{L}_D\\&\quad+\textstyle\sum_{i=1}^{N}\xi_i^2\mathbf{Q}_D\mathbf{H}_i^T\mathbf{H}_i\mathbf{Q}_D+\mathbf{I}\prec\mathbf{0}\end{aligned} \tag{5.6}$$

Moreover, the controller matrix $\mathbf{K}_D$ is computed as

$$\mathbf{K}_D = \mathbf{L}_D \mathbf{Q}_D^{-1} \tag{5.7}$$

Proof: To establish the robust stability of the system with uncertainty vector of $\boldsymbol{\xi} = [\xi_1, \xi_2, ..., \xi_N]^T$, we use the following quadratic Lyapunov function

$$\mathbf{V}(\mathbf{x}) = \mathbf{x}^T(t)\mathbf{P}_D\mathbf{x}(t) \tag{5.8}$$

where $\mathbf{P}_D$ is a block diagonal symmetric positive definite matrix with blocks $\mathbf{P}_D^{(i)} \in \mathcal{R}^{n_i \times n_i}$. The robust stability of the system can then be established by ensuring the negative definiteness of the derivative $\mathbf{V}(\mathbf{x})$ for all trajectories of (5.5), i.e,

$$\begin{aligned} \left.\frac{d\mathbf{V}(\mathbf{x})}{dt}\right|_{(5.5)} &= \frac{d}{dt}\{\mathbf{x}^T(t)\mathbf{P}_D\mathbf{x}(t)\} \\ &= \nabla_{\mathbf{x}}\mathbf{V}(\mathbf{x})\{(\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D)\mathbf{x}(t) \\ &\qquad + \mathbf{h}(t,\mathbf{x})\} < -\boldsymbol{\varphi}(\|\mathbf{x}(t)\|) \end{aligned} \tag{5.9}$$

in the domains of continuity of $\mathbf{h}(t,\mathbf{x})$ and $\boldsymbol{\varphi} \in \text{class K}$ [8]–[10], [33] (see also Definition 2.3). Thus equation (5.9) reduced to the following form of

$$\begin{aligned} \mathbf{x}^T(\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D)^T\mathbf{P}_D\mathbf{x} + \mathbf{x}^T\mathbf{P}_D(\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D)\mathbf{x} \\ + \mathbf{h}^T\mathbf{P}_D\mathbf{x} + \mathbf{x}^T\mathbf{P}_D\mathbf{h} \prec \mathbf{0} \end{aligned} \tag{5.10}$$

or

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{h} \end{bmatrix}^T \begin{bmatrix} (\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D)^T\mathbf{P}_D + \mathbf{P}_D(\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D) & \mathbf{P}_D \\ \mathbf{P}_D & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{h} \end{bmatrix} \prec \mathbf{0} \tag{5.11}$$

Using S- Procedure [56], when (5.2) is satisfied, condition (5.11) (or condition (5.10)) is equivalent to the existence of a symmetric positive definite matrix $\mathbf{P}_D$ and a scalar number $\tau > 0$ such that the following matrix inequality holds

$$\begin{bmatrix} \mathbf{x} \\ \mathbf{h} \end{bmatrix}^{\mathbf{T}} \begin{bmatrix} (\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D)^T\mathbf{P}_D + \mathbf{P}_D(\mathbf{A}_D + \mathbf{B}_D\mathbf{K}_D) + \tau\sum_{i=1}^{N}\xi_i^2\mathbf{H}_i^T\mathbf{H}_i & \mathbf{P}_D \\ \mathbf{P}_D & -\tau\mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{h} \end{bmatrix} \prec \mathbf{0} \tag{5.12}$$

Thus by applying Schur complement formula (see Lemma 4.4) and multiplying it from both sides by τ, then equation (5.12) is equivalent to

$$(\mathbf{A}_D+\mathbf{B}_D\mathbf{K}_D)^T\tau\,\mathbf{P}_D+\tau\,\mathbf{P}_D(\mathbf{A}_D+\mathbf{B}_D\mathbf{K}_D)+\tau^2\sum_{i=1}^{N}\xi_i^2\mathbf{H}_i^T\mathbf{H}_i+\mathbf{P}_D\mathbf{P}_D\prec\mathbf{0} \tag{5.13}$$

Thus, with change of variables $\mathbf{Q}_D=\tau\mathbf{P}_D^{-1}$ and $\mathbf{L}_D=\mathbf{K}_D\mathbf{Q}_D$, equation (5.13) reduced to equation (5.6).

Remark 5.1: While proving the above theorem, it is possible to establish an α- degree of robust stability for the system by finding a quadratic Lyapunov function that ensures $d\,\mathbf{V}(\mathbf{x})/dt\le-2\alpha\,\mathbf{V}(\mathbf{x})$ for all trajectories of (5.5) and some positive α.

In most practical cases, it is not possible to get all the state variables $\mathbf{x}_i(t)$ of the system. However, if only linear combinations called output variables $\mathbf{y}_i(t)$ are used as feedback signals in the system, then the decentralized static-output feedback strategies will have the following form

$$\mathbf{u}_i(t)=\mathbf{F}_i\mathbf{y}_i(t)\equiv\mathbf{F}_i\mathbf{C}_i\mathbf{x}_i(t) \tag{5.14}$$

where $\mathbf{F}_i\in\Re^{m_i\times q_i}$ is a constant output controller matrix. One way of achieving the same effect with (5.14) as with the local state feedback controller of (5.3) is to require $\mathbf{B}_i\mathbf{K}_i=\mathbf{B}_i\mathbf{F}_i\mathbf{C}_i$ so that the closed-loop in (5.5) will be unaltered. By defining the structure of the matrices $\mathbf{Q}_D$ and $\mathbf{L}_D$ of the above theorem as

$$\mathbf{Q}_D=\mathbf{Q}_O+\mathbf{U}\mathbf{Q}_C\,\mathbf{U}^T \tag{5.15}$$

where $\mathbf{U}=\mathrm{diag}\{\mathbf{U}_1,\mathbf{U}_2,\ldots,\mathbf{U}_N\}$ is a fixed user defined block diagonal matrix with blocks $\mathbf{U}_i\in\Re^{n_i\times q_i}$, and $\mathbf{Q}_O=\mathrm{diag}\{\mathbf{Q}_O^{(1)},\mathbf{Q}_O^{(2)},\ldots,\mathbf{Q}_O^{(N)}\}$ and $\mathbf{Q}_C=\mathrm{diag}\{\mathbf{Q}_C^{(1)},\mathbf{Q}_C^{(2)},\ldots,\mathbf{Q}_C^{(N)}\}$ are symmetric block diagonal matrices with blocks $\mathbf{Q}_O^{(i)}\in\Re^{n_i\times n_i}$ and $\mathbf{Q}_C^{(i)}\in\Re^{q_i\times q_i}$, respectively.

Similarly, by defining the structure of the matrix $\mathbf{L}_D$ as

$$\mathbf{L}_D=\mathbf{L}_C\,\mathbf{U}^T \tag{5.16}$$

where $\mathbf{L}_C=\mathrm{diag}\{\mathbf{L}_C^{(1)},\mathbf{L}_C^{(2)},\ldots,\mathbf{L}_C^{(N)}\}$ is a block diagonal matrix with blocks of $\mathbf{L}_C^{(i)}\in\Re^{m_i\times q_i}$. With this, the matrix $\mathbf{K}_D$ can be computed by using matrix inversion lemma (see Appendix A) as follows:

$$\mathbf{K}_D=\mathbf{L}_C\mathbf{U}^T\mathbf{Q}_O^{-1}[\mathbf{I}-\mathbf{U}(\mathbf{Q}_C^{-1}+\mathbf{U}^T\mathbf{Q}_O^{-1}\mathbf{U})^{-1}]\mathbf{U}^T\mathbf{Q}_O^{-1} \tag{5.17}$$

Using relation (5.17) together with the general form of (5.14) (i.e., $\mathbf{F}_D$ with $\mathbf{F}_D = \text{diag}\{\mathbf{F}_1, \mathbf{F}_2, ..., \mathbf{F}_N\}$) for output-decentralized system and moreover by requiring $\mathbf{U}^T\mathbf{Q}_O^{-1} = \mathbf{C}_D$ as an additional constraint within the problem formulation, it is possible to compute directly the matrix $\mathbf{F}_D$ as

$$\mathbf{F}_D = \mathbf{L}_C\mathbf{U}^T\mathbf{Q}_O^{-1}[\mathbf{I} - \mathbf{U}(\mathbf{Q}_C^{-1} + \mathbf{U}^T\mathbf{Q}_O^{-1}\mathbf{U})^{-1}] \tag{5.18}$$

Remark 5.2: The decomposition steps involved in (5.15) and (5.16) together with the constraint $\mathbf{U}^T\mathbf{Q}_O^{-1} = \mathbf{C}_D$ clearly cluster the states, inputs and outputs structural information of the system into a block diagonal form. This clustering technique, which is used to compute directly the appropriate diagonal structures of the output gain matrix, can be exploited effectively to design decentralized static-output feedback controller for practical implementation.

5.2.2 Optimal Design Problem Using Convex Optimization Method Involving LMIs

Using (5.6) together with (5.15) and (5.16) of the preceding subsection, the decentralized controller design problem for the interconnected system, with uncertainties vector ξ, has been reduced to that of finding symmetric block diagonal matrices $\mathbf{Q}_O$, $\mathbf{Q}_C$ and a block diagonal matrix $\mathbf{L}_C$. The basic idea, motivated by the work of Siljak and others [57] on maximizing the class of perturbations that can be tolerated by the closed loop system, is as follows. By applying repeatedly the Schur complement to (5.6) and using the structure of matrices (5.15) and (5.16), the robust decentralized controller design can be formulated as convex optimization involving the LMIs. This formulation guarantees the numerical solvability of $\mathbf{Q}_O$, $\mathbf{Q}_C$ and $\mathbf{L}_C$ by maximizing at the same time the interconnection uncertainty bounds, and consequently, solving the robust output decentralized controller for the interconnected system. To make the problem more practical, the sum of $1/\xi_i^2$ which is related to the uncertainties in the system can be minimized while at the same time ensuring a prescribed upper uncertainty bounds on the individual interconnection terms. Furthermore, by limiting the norm of the individual gains of the controller the optimization problem can be formulated as follows:

$$
\begin{aligned}
&\text{Min} \quad \sum_{i=1}^{N}(\gamma_i + K_{\mathbf{Q}_i} + K_{\mathrm{L}_i}) \\
&\text{subject to} \quad \mathbf{Q}_D \succ \mathbf{0} \text{ and} \\
&\begin{bmatrix}
\mathbf{A}_D\mathbf{Q}_D + \mathbf{Q}_D\mathbf{A}_D^T + \mathbf{B}_D\mathbf{L}_D + \mathbf{L}_D^T\mathbf{B}_D^T & \mathbf{Q}_D\mathbf{H}_1^T & \cdots & \mathbf{Q}_D\mathbf{H}_N^T & \mathbf{I} \\
\mathbf{H}_1\mathbf{Q}_D & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbf{H}_N\mathbf{Q}_D & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\
\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I}
\end{bmatrix} \prec \mathbf{0}
\end{aligned}
\tag{5.19}
$$

and

$$
\gamma_i - \frac{1}{\bar{\xi}_i^2} < 0; \quad \begin{bmatrix} -K_{\mathrm{L}_i}\mathbf{I} & \mathbf{L}_D^{(i)T} \\ \mathbf{L}_D^{(i)} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0}; \quad \begin{bmatrix} \mathbf{Q}_D^{(i)} & \mathbf{I} \\ \mathbf{I} & K_{\mathbf{Q}_i}\mathbf{I} \end{bmatrix} \succ \mathbf{0}
\tag{5.20}
$$

where $\gamma_i = 1/\xi_i^2$; and $\mathbf{Q}_D$ and $\mathbf{L}_D$ are given according to (5.15) and (5.16), respectively. Moreover, $K_{\mathbf{Q}_i}$ and $K_{\mathbf{L}_i}$ are upper bound constraints on the magnitude of decentralized gains satisfying

$$
\mathbf{L}_D^{(i)T}\mathbf{L}_D^{(i)} \prec K_{\mathbf{L}_i}\mathbf{I}, \qquad (\mathbf{Q}_D^{(i)})^{-1} \prec K_{\mathbf{Q}_i}\mathbf{I}
\tag{5.21}
$$

Based on (5.18), (5.19) and (5.20), the algorithm for determining the robust decentralized output controller and the associated class of perturbations that can be tolerated by the interconnected system is given as follows:

ALGORITHM I: An LMI Based Convex Optimization Method

For a given $\bar{\xi}_i > 0$ (tolerable upper bound on the interconnection uncertainities) and $K_{\mathbf{L}_i}$ and K_{Q_i} (upper bounds on the controller gains).

Check the *feasiblity* of the LMI problem in (5.19) and (5.20).

If *feasible* **then**

(1) Solve the following optimization problem for $\mathbf{Q}_O$, $\mathbf{Q}_C$ and $\mathbf{L}_C$

$$
\text{Min} \quad \sum_{i=1}^{N}(\gamma_i + K_{\mathbf{Q}_i} + K_{\mathrm{L}_i})
$$

subject to $\mathbf{Q}_O \succ \mathbf{0}$, $\mathbf{Q}_C \succ \mathbf{0}$ and

$$
\begin{bmatrix}
\mathbf{\Psi} & (\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T)\mathbf{H}_1^T & \cdots & (\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T)\mathbf{H}_N^T & \mathbf{I} \\
\mathbf{H}_1(\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T) & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbf{H}_N(\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T) & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\
\mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I}
\end{bmatrix} \prec \mathbf{0}
$$

$$
\gamma_i - \frac{1}{\bar{\xi}_i^2} < 0, \quad \begin{bmatrix} -K_{\mathbf{L}_i}\mathbf{I} & \mathbf{U}^{(i)}(\mathbf{L}_C^{(i)})^T \\ \mathbf{L}_C^{(i)}(\mathbf{U}^{(i)})^T & -\mathbf{I} \end{bmatrix} \prec \mathbf{0}, \quad \begin{bmatrix} \mathbf{Q}_O^{(i)} + \mathbf{U}^{(i)}\mathbf{Q}_C^{(i)}(\mathbf{U}^{(i)})^T & \mathbf{I} \\ \mathbf{I} & -K_{\mathbf{Q}_i}\mathbf{I} \end{bmatrix} \prec \mathbf{0}
$$

(2) Compute the the output gain matrix using

$$
\mathbf{F}_D = \mathbf{L}_C\mathbf{U}^T\mathbf{Q}_D^{-1}(\mathbf{I} - \mathbf{U}(\mathbf{Q}_C^{-1} + \mathbf{U}^T\mathbf{Q}_O^{-1}\mathbf{U})^{-1})
$$

Else

Modify the design condition ($\bar{\xi}_i$, $K_{\mathbf{L}_i}$, K_{Q_i}) and repeat Steps (1) and (2)

End if

where $\mathbf{\Psi} = \mathbf{A}_D(\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T) + (\mathbf{Q}_O + \mathbf{U}\mathbf{Q}_C\mathbf{U}^T)\mathbf{A}_D^T + \mathbf{B}_D\mathbf{L}_C\mathbf{U}^T + (\mathbf{L}_C\mathbf{U}^T)^T\mathbf{B}_D^T$ and $\mathbf{U}^T\mathbf{Q}_O^{-1} = \mathbf{C}_D$.

Remark 5.3: The above algorithm, depending on the structure of the matrices $\mathbf{U}$, $\mathbf{Q}_O$ and $\mathbf{Q}_C$, can be used to cluster the states, inputs and outputs information of the system into diagonal (fully decoupled decentralized) or bordered-block diagonal (sparse decentralized) structures.

5.2.3 Simulation Results

The robust decentralized controller design approach presented in the previous subsection of this chapter is now applied on a test system that has been discussed in Section 3.6. In the design, speed signals from each generator are used for robust decentralized PSS through the excitation systems. Figure 5.1 shows the PSS block for the *i*th-machine including the values for T_{wi}, T_{i1} and T_{i2} that are used in the design. While the washout filter and the first order phase-lead block parameters are chosen according to conventional PSS design methods, the gain K_i were estimated based on the technique described in the previous section. After augmenting the washout filter and phase-lead block in the system, the design problem is formulated as a convex optimization problem involving LMIs so as to determine the optimal gain K_i for each controller. Moreover, issues such as upper bounds on the gains of the controllers that guarantee prescribed uncertainty bounds are included in the formulation while designing the optimal gain K_i for each PSS block.

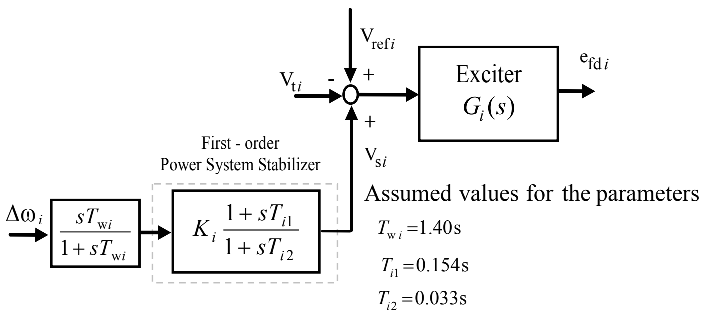


FIGURE 5.1 The PSS structure used in the design.

To demonstrate further the effectiveness of the proposed approach regarding the robustness, comparisons were made with simulation results obtained from a nonlinear based optimization for tuning power system controllers (a short description of the nonlinear based optimization technique can be found in Appendix B). A three-phase fault with different fault durations was applied at different locations to verify the performance of the proposed LMI based approach as well as that of nonlinear based optimization approach. The nonlinear based optimization approach was carried out for a fault duration of 150 ms applied to the bus near to generator G2 in Area-A for the base loading condition of [$P_{L1}=1600$ MW, $Q_{L1}=150$ Mvar] and [$P_{L2}=2400$ MW, $Q_{L2}=120$ Mvar]. For the LMI based controller design, the system was linearized for the same operating condition and the corresponding system equations were decomposed as a sum of two sets of equations. While the former describes the system as a hierarchical interconnection of N subsystems, the latter represents the interactions among the subsystems and uncertainties with each subsystem. For a short circuit of 150 ms duration at node F in Area-A, the transient response of generator G2 with and without PSSs in the system are shown in Figure 5.2.

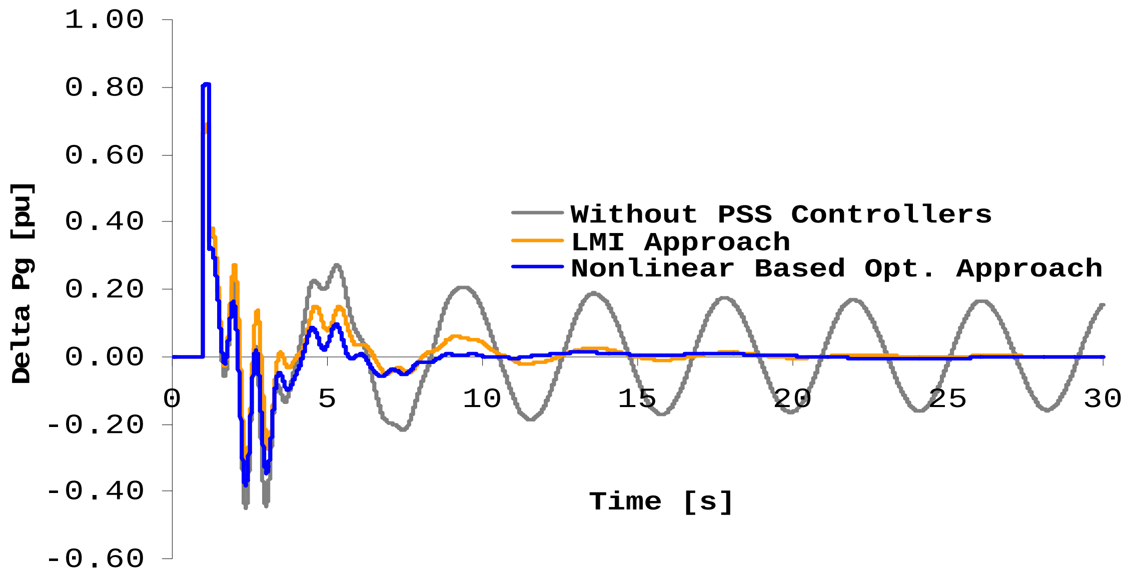


FIGURE 5.2 Transient responses of Generator G2 to a short circuit at node F in Area A.

The computed PSS gains are also given in Table 5.1. The PSSs designed through nonlinear based optimization approach were obtained by minimizing the quadratic deviation of the generator powers following a short circuit. Therefore, it is obvious that these PSSs provide slightly better damping for the considered operating condition. It is also observed that the damping achieved from the LMI controllers is quite good and acceptable.

TABLE 5.1 Gains computed for LMI and Nonlinear Based Optimization approaches.

Approaches	Gains	Gains Upper Bounds
LMI	$K_1 = 0.6544$ $K_2 = 4.8921$ $K_3 = 4.3837$ $K_4 = 0.2458$	$K_{Li} < 4$ $K_{Yi} < 5$ ($i = 1, 2, 3, 4$) $\lVert K_i \rVert^2 < K_{Li} K_{Yi}^2 = 100$
Nonlinear Based Optimization	$K_1 = 9.15$ $K_2 = 0.0$ $K_3 = 0.0$ $K_4 = 0.0$	$\lVert K_i \rVert^2 < \lVert K_{\max i} \rVert^2 = 100$

To further assess the effectiveness of the proposed approach regarding robustness, the transient performance indices were computed for different loading conditions at node 1 $[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$ while keeping constant total load in the system. The transient performance indices for excitation voltages e_{fdi}, generator powers P_{gi} and generator terminal voltages V_{ti} following a short circuit of 150 ms duration at node F in Area-A are computed using the following equations, respectively.

$$I^{E_{fd}} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| e_{fdi}(t) - e_{fdi}^{0} \right| dt \tag{5.22a}$$

$$I^{P_g} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| P_{gi}(t) - P_{gi}^{0} \right| dt \tag{5.22b}$$

$$I^{V_t} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| V_{ti}(t) - V_{ti}^{0} \right| dt \tag{5.22c}$$

These transient performance indices, which are used as a qualitative measure of the post-disturbance behaviour of the system for any fault and/or sudden load changes, are then compared for both approaches using the following ratio of performance index (IR):

$$IR = \frac{I_{\mathrm{LMI}}}{I_{\mathrm{NBO}}} \tag{5.23}$$

where I_{LMI} and I_{NBO} are the transient performance indices for the system computed using the LMI and nonlinear based optimization approaches, respectively. The ratios of the transient performance indices for excitation voltages e_{fdi}, generator powers P_{gi} and generator terminal voltages V_{ti} for different loading conditions are shown in Figure 5.3.

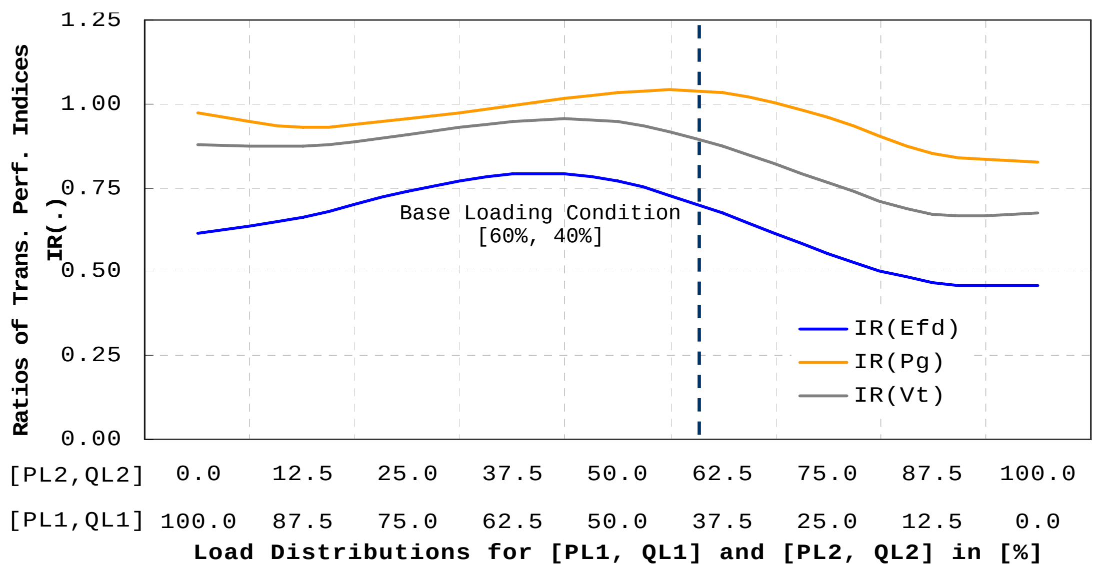


FIGURE 5.3 Plot of the ratio of the transient performances for LMI and Nonlinear Based Optimization approaches.

It can be seen from these results that, for the ratio of the transient performance index $\mathrm{IR}(P_g)$, the nonlinear based optimization approach performs better near or in the vicinity of the base loading condition, i.e. $[\mathrm{P_{L1}} = 1600\ \mathrm{MW}, \mathrm{Q_{L1}} = 150\,\mathrm{Mvar}]$ and $[\mathrm{P_{L2}} = 2400\ \mathrm{MW}, \mathrm{Q_{L2}} = 120\ \mathrm{Mvar}]$ corresponding to $[40\%, 60\%]$ in Figure 5.4. This is due to the fact that the nonlinear based optimization approach aimed only to improve the damping behaviour of the system by minimizing the transient responses of the generator output powers. However, as the loading conditions vary over wide ranges, the robustness of the controllers designed by the convex optimization involving LMI outperforms that of nonlinear based optimization approach as can be seen from the results shown in Figure 5.3. Though the approach proposed in this section seems computationally more involved and needs a relatively long computation time for large problems, its performances in all cases are adequate as compared with the nonlinear based optimization approach for power system stabilizers tuning.

Remark 5.4: The ratio of the transient performance index (IR) gives a qualitative measure of the behaviour of the system during any fault and/or sudden load changes. Value less than unity indicate that the designed PSSs using the LMI approach perform better as compared to those of the nonlinear based optimization approach.

Moreover, the approach has additional merits to incorporate design constraints like the size and the structure of the gain matrices, the degree of exponential stability and delays in the design formulation. While the nonlinear based optimization approach is straightforward for designing controllers for even large sized problems without encountering any computational difficulties, due to the nature of the problem, the optimal solutions depend on the initial values of the controller parameters, fault duration and fault locations used for determining the solution of the problem.

5.3 Decentralized Dynamic-Output Feedback Controller Design for Power Systems Using Interconnection Modelling Approach

In this section, the problem of designing a reduced-order robust decentralized output dynamic feedback controller for an interconnected power system is formulated as a nonconvex optimization problem involving LMI in tandem with NMI constraints. In the design, the robust connective stability of the overall system is guaranteed while the upper bounds of the uncertainties arising from the interconnected subsystems as well as the nonlinearities within each subsystem are maximized. The resulting optimization problem has a nonlinear matrix inequalities (NMIs) form which can be solved using sequential LMI programming method. The local convergence behaviour of the LMI sequential algorithm has shown the effectiveness of the approach for designing reduced-order power system stabilizers (PSSs) to a test system.

5.3.1 Controller Design Problem Formulation

In the following, consider designing a decentralized dynamic output feedback controller of an n_{ci}th-order for the system given in (5.1)

$$\begin{aligned} \dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\mathbf{y}_i(t) \\ \mathbf{u}_i(t) &= \mathbf{C}_{ci}\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t) \end{aligned} \tag{5.24}$$

where $\mathbf{x}_{ci}(t) \in \Re^{n_{ci}}$ is the state vector of the ith-local controller and $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$ and $\mathbf{D}_{ci}$ are constant matrices to be determined during the design.

After augmenting the ith-controller in the system, the state space equation of the ith-extended subsystem will have the following form

$$\begin{bmatrix} \dot{\mathbf{x}}_i(t) \\ \dot{\mathbf{x}}_{ci}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_i + \mathbf{B}_i\mathbf{D}_{ci}\mathbf{C}_i & \mathbf{B}_i\mathbf{C}_{ci} \\ \mathbf{B}_{ci}\mathbf{C}_i & \mathbf{A}_{ci} \end{bmatrix} \begin{bmatrix} \mathbf{x}_i(t) \\ \mathbf{x}_{ci}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{h}_i(t,\mathbf{x}) \\ \mathbf{0} \end{bmatrix} \tag{5.25}$$

which further can be written as follows:

$$\begin{bmatrix} \dot{\mathbf{x}}_i(t) \\ \dot{\mathbf{x}}_{ci}(t) \end{bmatrix} = \left\{ \begin{bmatrix} \mathbf{A}_i & \mathbf{0}_{n_i \times n_{ci}} \\ \mathbf{0}_{n_{ci} \times n_i} & \mathbf{0}_{n_{ci} \times n_{ci}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{B}_i \\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci} \times m_i} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{ci} & \mathbf{B}_{ci} \\ \mathbf{C}_{ci} & \mathbf{D}_{ci} \end{bmatrix} \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{I}_{n_{ci}} \\ \mathbf{C}_i & \mathbf{0}_{q_i \times n_{ci}} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{x}_i(t) \\ \mathbf{x}_{ci}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{h}_i(t,\mathbf{x}) \\ \mathbf{0} \end{bmatrix} \tag{5.26}$$

With minor abuse of notation, the above equation can be rewritten in a closed form as follows

$$\dot{\tilde{\mathbf{x}}}_i(t) = [\tilde{\mathbf{A}}_i + \tilde{\mathbf{B}}_i \mathbf{K}_i \tilde{\mathbf{C}}_i]\tilde{\mathbf{x}}_i(t) + \tilde{\mathbf{h}}_i(t, \tilde{\mathbf{x}}) \tag{5.27}$$

where $\tilde{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t), \quad \mathbf{x}_{ci}^T(t)]^T$, and the matrices $\tilde{\mathbf{A}}_i$, $\tilde{\mathbf{B}}_i$, $\tilde{\mathbf{C}}_i$ and $\mathbf{K}_i$ are given as follows

$$\begin{aligned}
\tilde{\mathbf{A}}_i &= \begin{bmatrix} \mathbf{A}_i & \mathbf{0}_{n_i \times n_{ci}} \\ \mathbf{0}_{n_{ci} \times n_i} & \mathbf{0}_{n_{ci} \times n_{ci}} \end{bmatrix}, \quad \tilde{\mathbf{B}}_i = \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{B}_i \\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci} \times m_i} \end{bmatrix}, \\
\tilde{\mathbf{C}}_i &= \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{I}_{n_{ci}} \\ \mathbf{C}_i & \mathbf{0}_{q_i \times n_{ci}} \end{bmatrix}, \quad \mathbf{K}_i = \begin{bmatrix} \mathbf{A}_{ci} & \mathbf{B}_{ci} \\ \mathbf{C}_{ci} & \mathbf{D}_{ci} \end{bmatrix}
\end{aligned} \tag{5.28}$$

Moreover, the function $\tilde{\mathbf{h}}_i(t, \tilde{\mathbf{x}})$ satisfies the following quadratic constraint

$$\tilde{\mathbf{h}}_i(t, \tilde{\mathbf{x}})^{\mathrm{T}} \tilde{\mathbf{h}}_i(t, \tilde{\mathbf{x}}) \leq \xi_i^2 \tilde{\mathbf{x}}^{\mathrm{T}}(t) \tilde{\mathbf{H}}_i^{\mathrm{T}} \tilde{\mathbf{H}}_i \tilde{\mathbf{x}}(t) \tag{5.29}$$

where the $\tilde{\mathbf{H}}_i \in \Re^{p_i \times (n + \sum_i^N n_{ci})}$ is partitioned according to the following

$$\tilde{\mathbf{H}}_i = [\mathbf{H}_{i1} \quad \mathbf{0}_{p_1 \times n_{c1}} \;\vdots\; \mathbf{H}_{i2} \quad \mathbf{0}_{p_2 \times n_{c2}} \;\vdots\; \cdots \;\vdots\; \mathbf{H}_{iN} \quad \mathbf{0}_{p_N \times n_{cN}}] \tag{5.30}$$

with $\mathbf{H}_{ij} \in \Re^{p_i \times n_i}$ for $j = 1, 2, \ldots, N$.

Thus, the overall interconnected system can then be rewritten in a compact form as follows:

$$\dot{\tilde{\mathbf{x}}}(t) = [\tilde{\mathbf{A}}_D + \tilde{\mathbf{B}}_D \mathbf{K}_D \tilde{\mathbf{C}}_D]\tilde{\mathbf{x}}(t) + \tilde{\mathbf{h}}(t, \tilde{\mathbf{x}}) \tag{5.31}$$

where $\tilde{\mathbf{x}}(t) = [\tilde{\mathbf{x}}_1^T(t), \tilde{\mathbf{x}}_2^T(t), \ldots, \tilde{\mathbf{x}}_N^T(t)]^T$ and all matrices are constant matrices of appropriate dimensions with $\tilde{\mathbf{A}}_D = \mathrm{diag}\{\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2, ..., \tilde{\mathbf{A}}_N\}$, $\tilde{\mathbf{B}}_D = \mathrm{diag}\{\tilde{\mathbf{B}}_1, \tilde{\mathbf{B}}_2, ..., \tilde{\mathbf{B}}_N\}$, $\tilde{\mathbf{C}}_D = \mathrm{diag}\{\tilde{\mathbf{C}}_1, \tilde{\mathbf{C}}_2, ..., \tilde{\mathbf{C}}_N\}$ and $\mathbf{K}_D = \mathrm{diag}\ \{\mathbf{K}_1, \mathbf{K}_2, ..., \mathbf{K}_N\}$.

Furthermore, the function $\tilde{\mathbf{h}}(t, \tilde{\mathbf{x}}) = [\tilde{\mathbf{h}}_1^T(t, \tilde{\mathbf{x}}), \tilde{\mathbf{h}}_2^T(t, \tilde{\mathbf{x}}), ..., \tilde{\mathbf{h}}_N^T(t, \tilde{\mathbf{x}})]^T$ is bounded as

$$\tilde{\mathbf{h}}^T(t, \tilde{\mathbf{x}}) \tilde{\mathbf{h}}(t, \tilde{\mathbf{x}}) \leq \tilde{\mathbf{x}}^T(t) \left[\sum_{i=1}^{N} \xi_i^2 \tilde{\mathbf{H}}_i^T \tilde{\mathbf{H}}_i \right] \tilde{\mathbf{x}}(t) \tag{5.32}$$

Thus, the above constraint is equivalent to the following matrix inequality

$$\begin{bmatrix} \tilde{\mathbf{x}} \\ \tilde{\mathbf{h}} \end{bmatrix}^T \begin{bmatrix} -\sum_{i=1}^{N} \xi_i^2 \tilde{\mathbf{H}}_i^T \tilde{\mathbf{H}}_i & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}} \\ \tilde{\mathbf{h}} \end{bmatrix} \preceq \mathbf{0} \tag{5.33}$$

The following two theorems are instrumental in establishing the robust stability of the closed-loop interconnected system (5.31) via a decentralized robust control strategy (5.24) under the constraints (5.33) on the function $\tilde{\mathbf{h}}(t,\tilde{\mathbf{x}})$.

THEOREM 5.2: The interconnected system (5.1) is robustly stabilized by the decentralized dynamic output control strategy of (5.24) with degree of uncertainty vector $\boldsymbol{\xi}=[\xi_1,\xi_2,\ldots,\xi_N]^T$ if there exist a symmetric positive definite matrix $\mathbf{Q}_D=\mathrm{diag}\{\mathbf{Q}^{(1)},\mathbf{Q}^{(2)},\ldots,\mathbf{Q}^{(N)}\}$ with $\mathbf{Q}^{(i)}\in\Re^{(n_i+n_{ci})\times(n_i+n_{ci})}$ and a number $\tau>0$ that satisfy the following matrix inequality:

$$\mathbf{Q}_D\succ 0,\quad \tau>0$$

$$\begin{bmatrix}\tilde{\mathbf{x}}\\ \tilde{\mathbf{h}}\end{bmatrix}^T\begin{bmatrix}\mathbf{Q}_D\tilde{\mathbf{A}}_D^T+\tilde{\mathbf{A}}_D\mathbf{Q}_D+\mathbf{Q}_D(\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D)^T+\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D\mathbf{Q}_D+\tau\sum_{i=1}^N\xi_i^2\mathbf{Q}_D\tilde{\mathbf{H}}_i^T\tilde{\mathbf{H}}_i\mathbf{Q}_D & \mathbf{I}\\ \mathbf{I} & -\tau\mathbf{I}\end{bmatrix}\begin{bmatrix}\tilde{\mathbf{x}}\\ \tilde{\mathbf{h}}\end{bmatrix}\prec\mathbf{0} \tag{5.34}$$

Proof: This theorem can be proved in the same way as Theorem 5.1 of Section 5.2.

With change of variables $\mathbf{Q}_D=\tau\mathbf{Q}_D$ and $\gamma_i=1/\xi_i^2$; and moreover by applying repeatedly the Schur complement, the bilinear matrix inequality (BMI) condition in (5.34) can be rewritten in the following form:

$$\mathbf{Q}_D\succ\mathbf{0}$$

$$\begin{bmatrix}\mathbf{Q}_D\tilde{\mathbf{A}}_D^T+\tilde{\mathbf{A}}_D\mathbf{Q}_D+\mathbf{Q}_D(\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D)^T+\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D\mathbf{Q}_D & \mathbf{Q}_D\tilde{\mathbf{H}}_1^T & \cdots & \mathbf{Q}_D\tilde{\mathbf{H}}_N^T & \mathbf{I}\\ \tilde{\mathbf{H}}_1\mathbf{Q}_D & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \tilde{\mathbf{H}}_N\mathbf{Q}_D & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0}\\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I}\end{bmatrix}\prec\mathbf{0} \tag{5.35}$$

Moreover, the above BMI condition in (5.35) can be rewritten in a closed form as

$$\overline{\mathbf{Q}}_D\overline{\mathbf{A}}_D^T+\overline{\mathbf{A}}_D\overline{\mathbf{Q}}_D+\overline{\mathbf{B}}_D\mathbf{K}_D\overline{\mathbf{C}}_D\overline{\mathbf{Q}}_D+\overline{\mathbf{Q}}_D\overline{\mathbf{C}}_D^T\mathbf{K}_D^T\overline{\mathbf{B}}_D^T\prec\mathbf{0} \tag{5.36}$$

where

$$\overline{\mathbf{Q}}_D=\begin{bmatrix}\mathbf{Q}_D & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \mathbf{0} & \mathbf{0} & \cdots & \gamma_N\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I}\end{bmatrix},\quad \overline{\mathbf{A}}_D=\begin{bmatrix}\tilde{\mathbf{A}}_D & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{I}\\ \tilde{\mathbf{H}}_1 & -\frac{1}{2}\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0}\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ \tilde{\mathbf{H}}_N & \mathbf{0} & \cdots & -\frac{1}{2}\mathbf{I} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & -\frac{1}{2}\mathbf{I}\end{bmatrix}, \tag{5.37}$$

$$\overline{\mathbf{B}}_D^T=\begin{bmatrix}\tilde{\mathbf{B}}_D^T & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\end{bmatrix},\qquad \overline{\mathbf{C}}_D=\begin{bmatrix}\tilde{\mathbf{C}}_D & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0}\end{bmatrix}$$

Thus, the problem of designing a decentralized dynamic output feedback controller strategy for the interconnected system of (5.1), which at the same time maximizing the tolerable upper bounds on the interconnection uncertainties, has the following form

$$\begin{array}{l} \text{Min} \quad Trace(\mathbf{\Gamma}) \\ \text{subject} \quad \text{to} \quad (5.35) \end{array} \tag{5.38}$$

where $\mathbf{\Gamma} = \text{diag}\ \{\gamma_1, \gamma_2, \ldots, \gamma_N\}$.

THEOREM 5.3: The interconnected system (5.1) is robustly stabilized by the decentralized controller strategy of (5.24) if there exist two symmetric positive definite matrices $\mathbf{X}_D = \text{diag}\{\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \ldots, \mathbf{X}^{(N)}\}$, $\mathbf{Y}_D = \text{diag}\{\mathbf{Y}^{(1)}, \mathbf{Y}^{(2)}, \ldots, \mathbf{Y}^{(N)}\}$ with blocks $\mathbf{X}^{(i)}$, $\mathbf{Y}^{(i)} \in \Re^{n_i \times n_i}$ and positive numbers γ_i for $i = 1, 2, \ldots, N$ that satisfy the following two LMI conditions

$$\begin{array}{l} \mathbf{Y}_D \succ \mathbf{0} \\ \begin{bmatrix} \mathbf{N}_{\mathbf{B}_D}^T(\mathbf{Y}_D\mathbf{A}_D^T + \mathbf{A}_D\mathbf{Y}_D)\mathbf{N}_{\mathbf{B}_D} & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_N^T & \mathbf{N}_{\mathbf{B}_D}^T \\ \mathbf{H}_1\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0} \end{array} \tag{5.39}$$

$$\begin{array}{l} \mathbf{X}_D \succ \mathbf{0} \\ \begin{bmatrix} \mathbf{N}_{\mathbf{C}_D}(\mathbf{A}_D^T\mathbf{X}_D + \mathbf{X}_D\mathbf{A}_D)\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_N^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{X}_D \\ \mathbf{H}_1\mathbf{N}_{\mathbf{C}_D}^T & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{X}_D\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0} \end{array} \tag{5.40}$$

where $\mathbf{N}_{\mathbf{B}_D}$ and $\mathbf{N}_{\mathbf{C}_D}$ denote arbitrary bases for the nullspaces of $\mathbf{B}_D$ and $\mathbf{C}_D$, respectively.

Moreover, the set of n_{ci}th-order decentralized controllers is nonempty if and only if the above two LMI conditions hold for some $\mathbf{X}_D$ and $\mathbf{Y}_D$ which further satisfy the following rank constraint

$$Rank(\mathbf{I} - \mathbf{Y}_D\mathbf{X}_D) = \textstyle\sum_{i=1}^{N} n_{ci} \le n = \sum_{i=1}^{N} n_i \tag{5.41}$$

Proof: Let us partition the matrix $\mathbf{Q}^{(i)}$ and its inverse $(\mathbf{Q}^{(i)})^{-1}$ conformable to the dimensions of $\mathbf{A}_i$ and $\mathbf{A}_{ci}$ as follows:

$$\mathbf{Q}^{(i)} = \begin{bmatrix} \mathbf{Y}^{(i)} & \mathbf{N}^{(i)} \\ (\mathbf{N}^{(i)})^T & * \end{bmatrix}, \qquad (\mathbf{Q}^{(i)})^{-1} = \begin{bmatrix} \mathbf{X}^{(i)} & \mathbf{M}^{(i)} \\ (\mathbf{M}^{(i)})^T & * \end{bmatrix} \tag{5.42}$$

where $\mathbf{X}^{(i)}, \mathbf{Y}^{(i)} \in \Re^{n_i \times n_i}$ and $\mathbf{N}^{(i)}, \mathbf{M}^{(i)} \in \Re^{n_i \times n_{ci}}$, moreover define the following block diagonal matrices

$$\mathbf{X}_D = \mathrm{diag}\{\mathbf{X}^{(1)}, \mathbf{X}^{(2)}, \ldots, \mathbf{X}^{(N)}\}, \quad \mathbf{Y}_D = \mathrm{diag}\{\mathbf{Y}^{(1)}, \mathbf{Y}^{(2)}, \ldots, \mathbf{Y}^{(N)}\} \tag{5.43}$$

Using the Elimination lemma (see Lemma 4.3), premultiplying (5.36) by the nullspace of $\overline{\mathbf{B}}_D$ and postmultiplying by its transpose the bilinear matrix inequality in (5.36), can be rewritten as

$$\mathbf{N}_{\overline{\mathbf{B}}_D}^T \begin{bmatrix} \mathbf{Q}_D \widetilde{\mathbf{A}}_{\mathrm{D}}^{\mathrm{T}} + \widetilde{\mathbf{A}}_{\mathrm{D}} \mathbf{Q}_D & \mathbf{Q}_D \widetilde{\mathbf{H}}_1^T & \cdots & \mathbf{Q}_D \widetilde{\mathbf{H}}_N^T & \mathbf{I} \\ \widetilde{\mathbf{H}}_1 \mathbf{Q}_D & -\gamma_1 \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \widetilde{\mathbf{H}}_N \mathbf{Q}_D & \mathbf{0} & \cdots & -\gamma_N \mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \mathbf{N}_{\overline{\mathbf{B}}_D} \prec \mathbf{0} \tag{5.44}$$

After explicitly computing the nullspace of $\overline{\mathbf{B}}_D$ and carrying out the block matrix products, the LMI equation in (5.44) can be equivalently expressed as

$$\begin{bmatrix} \mathbf{N}_{\mathbf{B}_D}^T (\mathbf{Y}_D \mathbf{A}_{\mathrm{D}}^{\mathrm{T}} + \mathbf{A}_{\mathrm{D}} \mathbf{Y}_D) \mathbf{N}_{\mathbf{B}_D} & \mathbf{N}_{\mathbf{B}_D}^T \mathbf{Y}_D \mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{B}_D}^T \mathbf{Y}_D \mathbf{H}_N^T & \mathbf{N}_{\mathbf{B}_D}^T \\ \mathbf{H}_1 \mathbf{Y}_D \mathbf{N}_{\mathbf{B}_D} & -\gamma_1 \mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N \mathbf{Y}_D \mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & -\gamma_N \mathbf{I} & \mathbf{0} \\ \mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0} \tag{5.45}$$

where $\mathbf{N}_{\mathbf{B}_D}$ and $\mathbf{N}_{\mathbf{C}_D}$ denote arbitrary bases of the nullspaces of $\mathbf{B}_D$ and $\mathbf{C}_D$, respectively.

Similarly, premultplying (5.36) by the nullspace of $\overline{\mathbf{C}}_D \overline{\mathbf{Q}}_D$ and postmultiplying by its transpose, the bilinear matrix inequality condition in (5.36) can be equivalently reduced to (5.40).

Thus, the existence of $\mathbf{K}_D$ is equivalent to the existence of $\mathbf{Y}_D$, $\mathbf{X}_D$ and γ_i for $i = 1, 2, \ldots, N$ that satisfy simultaneously the LMI conditions in (5.39) and (5.40) together with the rank constraint in (5.43).

Therefore, the problem of designing a decentralized control strategy given in (5.38) for the interconnected system (5.1) can be restated as a nonconvex optimization problem of the form

$$\begin{aligned} &\text{Min} \quad Trace(\boldsymbol{\Gamma}) \\ &\text{subject to} \quad (5.39), (5.40) \text{ and } (5.41) \end{aligned} \tag{5.46}$$

The coupling constraint in (5.41) can be further relaxed as an LMI condition as follows:

$$\begin{bmatrix} \mathbf{Y}_D & \mathbf{I} \\ \mathbf{I} & \mathbf{X}_D \end{bmatrix} \succeq \mathbf{0} \tag{5.47}$$

Furthermore, using the cone-complementarity approach [58], there exists a decentralized robust output stabilizing controller $\mathbf{K}_D$ if the global minimum of the following optimization problem

$$\begin{aligned} &\text{Min} \quad Trace(\boldsymbol{\Gamma}) + Trace(\mathbf{Y}_D\mathbf{X}_D) \\ &\text{subject to} \quad \mathbf{Y}_D \succ \mathbf{0},\ \mathbf{X}_D \succ \mathbf{0},\ \boldsymbol{\Gamma} \succ \mathbf{0}, \text{ and} \\ &\text{Equations (5.39), (5.40) and (5.47)} \end{aligned} \tag{5.48}$$

is $\alpha^* + \sum_{i=1}^{N} n_i$ where α^* is some positive number satisfying $Trace(\boldsymbol{\Gamma}) \leq \alpha^*$.

5.3.2 (Sub)-Optimal Design Problem Using Sequential LMI Programming Method

The optimization in (5.48) is a nonconvex optimization problem due to the bilinear matrix inequality term in the objective functional. To compute the (sub)-optimal solution of this problem, an algorithm based on a sequential LMI programming method is proposed. The idea behind this algorithm is to linearize the cost functional in (5.48) with respect to its variables and then to solve iteratively the resulting convex optimization problem subject to the constraints in (5.48). Moreover, the algorithm will set appropriately the direction of the feasible solution by solving a subclass problem of a Newton-type updating coefficient. Furthermore, the solution of the optimization problem is monotonically nonincreasing, i.e. the solution decreases for each iteration with the lower bound being $\sum_{i=1}^{N} n_i$ plus some positive number. The convergence behaviour of the whole optimization problem is ensured by checking the norm distances between the current and the previous solutions.

Thus, the sequential LMI programming method for finding the decentralized dynamic output feedback controllers has the following two-step optimization algorithms.

ALGORITHM II: Sequential LMI Programming Method

Solve the LMI feasibiliy problem of (5.48) for $\mathbf{Y}_D$, $\mathbf{X}_D$, and $\mathbf{\Gamma}$

Set the solutions $(\mathbf{Y}_D^0,\ \mathbf{X}_D^0):=(\mathbf{Y}_D,\ \mathbf{X}_D)$ and $\mu^0:=2Trace(\mathbf{Y}_D^0\mathbf{X}_D^0)/n$

Repeat unitl $\mu^k :\leq \varepsilon$ (for small number $\varepsilon > 0$)

(1). Solve the following optimization problem for $\mathbf{Y}_D$, $\mathbf{X}_D$, and $\mathbf{\Gamma}$

$$\text{Min } Trace(\mathbf{Y}_D\mathbf{X}_D^k + \mathbf{Y}_D^k\mathbf{X}_D) + Trace(\mathbf{\Gamma})$$

subject to $\mathbf{Y}_D \succ \mathbf{0}$, $\mathbf{X}_D \succ \mathbf{0}$, $\mathbf{\Gamma} \succ \mathbf{0}$ and

$$\begin{bmatrix} \mathbf{N}_{\mathbf{B}_D}^T(\mathbf{Y}_D\mathbf{A}_\mathrm{D}^\mathrm{T} + \mathbf{A}_\mathrm{D}\mathbf{Y}_D)\mathbf{N}_{\mathbf{B}_D} & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_N^T & \mathbf{N}_{\mathbf{B}_D}^T \\ \mathbf{H}_1\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0}$$

$$\begin{bmatrix} \mathbf{N}_{\mathbf{C}_D}(\mathbf{A}_\mathrm{D}^\mathrm{T}\mathbf{X}_D + \mathbf{X}_D\mathbf{A}_\mathrm{D})\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_N^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{X}_D \\ \mathbf{H}_1\mathbf{N}_{\mathbf{C}_D}^T & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{X}_D\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix} \prec \mathbf{0}$$

$$\begin{bmatrix} \mathbf{Y}_D & \mathbf{I} \\ \mathbf{I} & \mathbf{X}_D \end{bmatrix} \succeq \mathbf{0},$$

(2). Set the direction

$$(\Delta\mathbf{Y}_D^k, \Delta\mathbf{X}_D^k) := (\mathbf{Y}_D - \mathbf{Y}_D^k, \mathbf{X}_D - \mathbf{X}_D^k)$$

(3). Set $(\mathbf{Y}_D^{k+1}, \mathbf{X}_D^{k+1}) := (\mathbf{Y}_D^k, \mathbf{X}_D^k) + t_k(\Delta\mathbf{Y}_D^k, \Delta\mathbf{X}_D^k)$

(4). Set $\mu^{k+1} := Trace(\mathbf{Y}_D\mathbf{X}_D^k + \mathbf{Y}_D^k\mathbf{X}_D) - 2Trace(\mathbf{Y}_D^k\mathbf{X}_D^k)$ increment k by 1.

End do

Remark 5.5: The above algorithm has two parts: i) the initialization step which is an LMI feasibility problem used to obtain the initial values for the subsequent part of the algorithm, and ii) a sequential LMI programming problem with a linear objective functional under LMI constraints that computes iteratively the optimization problem.

Remark 5.6: The following subclass problem can be used to choose the appropriate value for t_k in Step (3):

$$\begin{aligned} &\text{Min } Trace(\mathbf{Y}_D^k + t_k(\mathbf{Y}_D - \mathbf{Y}_D^k))(\mathbf{X}_D^k + t_k(\mathbf{X}_D - \mathbf{X}_D^k)) \\ &\text{subject to } t_k \in [0, 1] \end{aligned} \tag{5.49}$$

To determine the decentralized controllers, it is first necessary to construct the matrix $\mathbf{Q}_D$ using the solution set of Algorithm II. Recall that the solution $\mathbf{X}_D$ and $\mathbf{Y}_D$ are related by

(5.42) to the solution of (5.38). Therefore, the block elements $\mathbf{Q}_D^{(i)}$ of $\mathbf{Q}_D$ can be computed using the following relations

$$\mathbf{N}^{(i)} = (\mathbf{Y}^{(i)} - (\mathbf{X}^{(i)})^{-1})^{1/2} \tag{5.50}$$

and

$$\mathbf{Q}_D^{(i)} = \begin{bmatrix} \mathbf{Y}^{(i)} & \mathbf{N}^{(i)} \\ (\mathbf{N}^{(i)})^T & \mathbf{I}_{n_{ci}} \end{bmatrix} \tag{5.51}$$

Hence, the following algorithm can be used to recover the decentralized controller $\mathbf{K}_D$.

ALGORITHM III: The LMI Problem to Determine $\mathbf{K}_D$

Solve the following LMI problem

$$\overline{\mathbf{Q}}_D^*\overline{\mathbf{A}}_\mathrm{D}^\mathrm{T} + \overline{\mathbf{A}}_\mathrm{D}\overline{\mathbf{Q}}_D^* + \overline{\mathbf{B}}_D\mathbf{K}_D\overline{\mathbf{C}}_D\overline{\mathbf{Q}}_D^* + \overline{\mathbf{Q}}_D^*\overline{\mathbf{C}}_D^T\mathbf{K}_D^T\overline{\mathbf{B}}_D^T \prec \mathbf{0}$$

where $\overline{\mathbf{Q}}_D^* = \mathrm{diag}\{\mathbf{Q}_D^*, \gamma_1^*\mathbf{I}, \ldots, \gamma_N^*\mathbf{I}, \mathbf{I}\}$ with $\mathbf{Q}_D^*$ and γ_i^* for $i = 1, 2, \ldots, N$ are the solutions of Algorithm II.

The above two algorithms, i.e. Algorithm II and Algorithm III, involve: i) minimizing a convex cost functional subject to LMI constraints at each iteration stage, i.e. the optimization problem in Algorithm II that gives the optimal values for $\mathbf{Y}_D^*$, $\mathbf{X}_D^*$, γ_i^* for $i = 1, 2, \ldots, N$, and ii) an LMI optimization problem in $\mathbf{K}_D$, i.e., the problem in Algorithm III, which gives the suboptimal robust decentralized output feedback controllers. These two algorithms can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers such as MATLAB LMI Toolbox [59].

5.3.3 Simulation Results

The robust decentralized dynamic output control design approach presented in the previous subsection is now applied to a test system that has been considered in Section 5.2.3. Moreover, some of the parameters of the regulator of the exciters are changed so as to create a weakly-damped inter-area oscillatory problem in the system. After augmenting the controller structure in each subsystem, the design problem was formulated as a nonconvex optimization problem involving linear matrix inequalities (LMIs) and coupling bilinear matrix inequality (BMI). Then using the proposed algorithms in the previous section, the (sub)-optimal robust decentralized second-order PSS for each subsystem was designed. The designed robust PSSs and the convergence behaviour of Algorithm II for a relative accuracy of $\varepsilon = 10^{-6}$ are given in Table 5.2 and Table 5.3, respectively. For a short circuit of 150 ms duration at node F in Area-A, the transient responses of generator G2 with and without PSSs in the system are also shown in Figure 5.4.

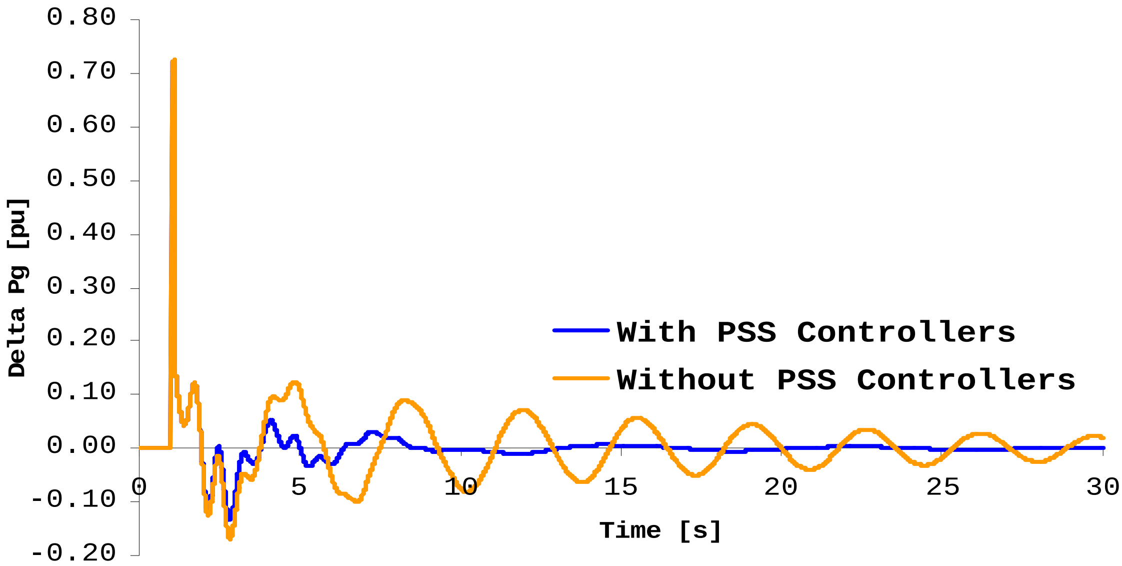


FIGURE 5.4 Transient responses of Generator G2 to a short circuit at node F in Area A.

TABLE 5.2 The robust decentralized controllers for the test system

Generator G_i	**Designed Controllers**
1	$\frac{1.83s^2+21.67s+64.11}{s^2+11.82s+34.96}$
2	$\frac{2.01s^2+24.08s+72.15}{s^2+11.92s+35.63}$
3	$\frac{1.98s^2+23.60s+70.32}{s^2+11.90s+35.44}$
4	$\frac{1.79s^2+21.24s+63.09}{s^2+11.87s+35.27}$

TABLE 5.3 The convergence properties of Algorithm II (for error $\varepsilon = 10^{-6}$)

Outer Iteration k	**Inner Iterations j**	**Trace(Y^kX^k)**	**J(Y^k, X^k, Γ^k)**
0	59	0.51523×10^7	0.19551×10^9
1	85	86.50271	0.10195×10^8
2	75	39.06272	133.01765
3	70	37.29666	75.72572
4	71	36.50326	73.59031
5	67	36.00285	72.16617
6	72	36.00002	72.09829
7	69	36.00000	72.06229

To further assess the effectiveness of the proposed approach regarding the robustness, the transient performance indices were computed for different loading conditions at node 1

$[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$ while keeping constant total load in the system. The transient performance indices for generator powers P_{gi}, generator terminal voltages V_{ti} and excitation voltages e_{fdi} following a short circuit of 150 ms duration at node F in Area-A are computed using (5.22). Moreover, for comparison purpose, these indices are normalized to the base operating condition for which the controllers have been designed:

$$I_N = \frac{I_{DLC}}{I_{BLC}} \tag{5.52}$$

where I_{DLC} is the transient performance index for different loading condition, I_{BLC} is the transient performance index for base loading condition.

The normalized transient performance indices for different loading conditions are shown in Figure 5.5. It can be seen from Figure 5.5 that the normalized transient performance indices for $I_N(P_g)$, $I_N(V_t)$ and $I_N(e_{fd})$ are either near unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behaviour exhibits robustness for all loading conditions.

Remark 5.7: The value of the normalized transient performance index $I_N(.)$ gives a qualitative measure of the behaviour of the system during any fault and/or sudden load changes. A value much greater than unity means that the system behaves poorly as compared to the base operating condition.

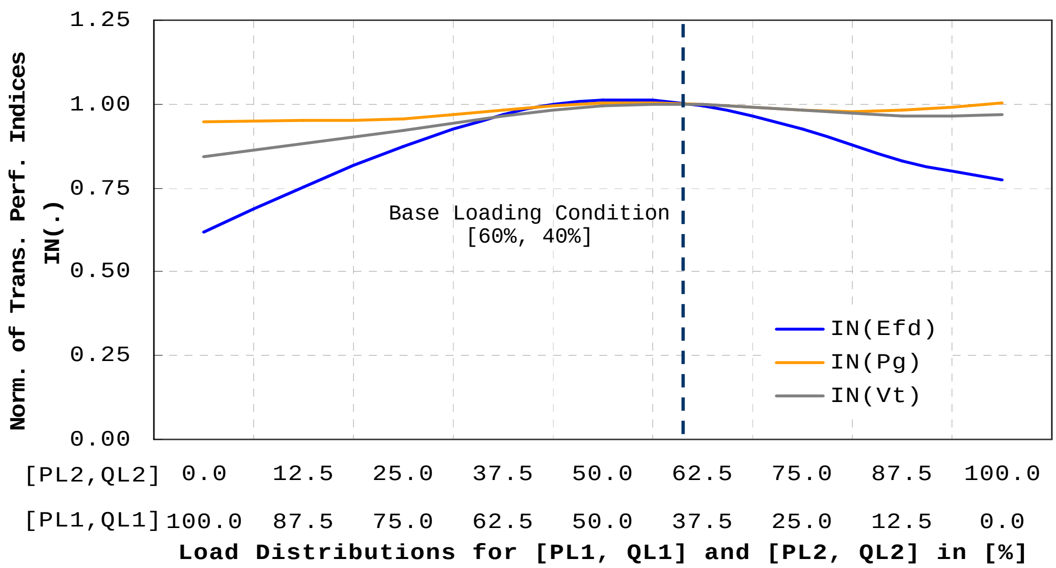


FIGURE 5.5 Plot of the normalized transient performance indices.

5.4 Robust Decentralized Structure-Constrained H_2/H_∞ Dynamic Output Feedback Controller Design for Power Systems

This section presents a linear matrix inequality (LMI) approach for designing a robust decentralized structure-constrained controller for power systems. The problem of designing a fixed-structure H_2/H_∞ dynamic output feedback controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting optimization problem has a general bilinear matrix inequalities (BMIs) form which can be solved using sequential LMIs programming method. The approach is demonstrated by designing fixed-structure power system stabilizers (PSSs) for a test power system. The approach has a number of practical relevance among which the following are singled out: i) the stability of the controller can be explicitly stated a priori in the fixed structure of the controller, ii) controller gains can be limited in order to avoid designing high gains that are often undesirable for practical implementation, and, iii) multi-objective optimization technique can easily be incorporated in the design by minimizing the H_2/H_∞ norms of the multiple transfer functions between different input/output channels.

5.4.1 Controller Design Problem Formulation

Consider the general structure of the *i*th-generator together with the PSS block in a multi-machine power system shown in Figure 3.4. The input of the *i*th-controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. To illustrate further the design procedure, consider the following first-order PSS:

$$K_i \left[\frac{1+sT_{i1}}{1+sT_{i2}}\right] \tag{5.53}$$

The PSS in (5.53) can be further rewritten in the following form

$$K_i \frac{1+sT_{i1}}{1+sT_{i2}} = \left[K_{i1} + K_{i2}\frac{1}{1+sT_{i2}}\right] \equiv \begin{bmatrix} K_{i1} & K_{i2} \end{bmatrix} \begin{bmatrix} 1 \\ 1/(1+sT_{i2}) \end{bmatrix} \tag{5.54}$$

where with K_{i1} and K_{i2} are easily identified as gain parameters that are to be determined during the design. Moreover, the gain parameters K_{i1} and K_{i2} together with T_{i2} (*a-priori* assumption made on the value of T_{i2}) determine the original parameters K_i and T_{i1}.

After augmenting the washout stage in the system, the *i*th-subsystem, within the framework of H_2/H_∞ design, is described by the following state space equation:

$$\begin{aligned}
\dot{\mathbf{x}}_i(t) &= \mathbf{A}_{ii}\,\mathbf{x}_i(t) + \sum_{j\neq i}\mathbf{A}_{ij}\,\mathbf{x}_j(t) + \mathbf{B}_{i0}\,\mathbf{w}_{i0}(t) + \mathbf{B}_{i1}\,\mathbf{w}_{i1}(t) + \mathbf{B}_{i2}\,\mathbf{u}_i(t)\\
\mathbf{z}_i(t) &= \mathbf{C}_{i1}\,\mathbf{x}_i(t) + \mathbf{D}_{i10}\,\mathbf{w}_{i0}(t) + \mathbf{D}_{i11}\,\mathbf{w}_{i1}(t) + \mathbf{D}_{i12}\,\mathbf{u}_i(t)\\
\mathbf{y}_i(t) &= \mathbf{C}_{iy}\,\mathbf{x}_i(t) + \mathbf{D}_{iy0}\,\mathbf{w}_{i0}(t) + \mathbf{D}_{iy1}\mathbf{w}_{i1}(t)
\end{aligned} \tag{5.55}$$

where $\mathbf{x}_i(t)\in\Re^{n_i}$ is the state variable, $\mathbf{u}_i(t)\in\Re^{m_i}$ is the control input, $\mathbf{y}_i(t)\in\Re^{q_i}$ is the measurement signal, $\mathbf{z}_i(t)\in\Re^{p_i}$ is the regulated variables, $\mathbf{w}_{i0}\in\Re^{r_{0i}}$ and $\mathbf{w}_{i1}\in\Re^{r_{1i}}$ are exogenous signals (assuming that $\mathbf{w}_{i1}$ is either independent of $\mathbf{w}_{i0}$ or dependent causally on $\mathbf{w}_{i0}$) for the *i*th-subsystem.

Now consider the following approach to design a decentralized robust optimal H_2/H_∞ controllers of the form (5.54) for the system given in (5.55), i.e. determining optimally the gains K_{i1} and K_{i2} within the framework of H_2/H_∞ optimization. This implies the incorporation of the dynamic part of the controller first in (5.55), namely

$$\left[1,\ \ 1/(1+sT_{i2})\right]^T \tag{5.56}$$

and then reformulating the problem as an extension of a static output feedback problem for the extended system. Hence, the state space equation for *i*th-subsystem becomes:

$$\begin{aligned}
\begin{bmatrix}\dot{\mathbf{x}}_i(t)\\ \dot{\mathbf{x}}_{ci}(t)\end{bmatrix} &= \begin{bmatrix}\mathbf{A}_{ii} & \mathbf{0}\\ \mathbf{B}_{ci}\mathbf{C}_{iy} & \mathbf{A}_{ci}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i(t)\\ \mathbf{x}_{ci}(t)\end{bmatrix} + \begin{bmatrix}\sum_{j\neq i}\mathbf{A}_{ij}\mathbf{x}_j\\ \mathbf{0}\end{bmatrix} + \begin{bmatrix}\mathbf{B}_{i0}\\ \mathbf{B}_{ci}\mathbf{D}_{iy0}\end{bmatrix}\mathbf{w}_{i0}(t) + \begin{bmatrix}\mathbf{B}_{i1}\\ \mathbf{B}_{ci}\mathbf{D}_{iy1}\end{bmatrix}\mathbf{w}_{i1}(t) + \begin{bmatrix}\mathbf{B}_{i2}\\ \mathbf{0}\end{bmatrix}\mathbf{u}_i(t)\\
\mathbf{z}_i(t) &= \begin{bmatrix}\mathbf{C}_{i1} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i(t)\\ \mathbf{x}_{ci}(t)\end{bmatrix} + \mathbf{D}_{i10}\,\mathbf{w}_{i0}(t) + \mathbf{D}_{i11}\,\mathbf{w}_{i1}(t) + \mathbf{D}_{i12}\,\mathbf{u}_i(t)\\
\tilde{\mathbf{y}}_i(t) &= \begin{bmatrix}\mathbf{D}_{ci}\mathbf{C}_{iy} & \mathbf{C}_{ci}\end{bmatrix}\begin{bmatrix}\mathbf{x}_i(t)\\ \mathbf{x}_{ci}(t)\end{bmatrix} + \mathbf{D}_{ci}\mathbf{D}_{iy0}\mathbf{w}_{i0}(t) + \mathbf{D}_{ci}\mathbf{D}_{iy1}\mathbf{w}_{i1}(t)
\end{aligned} \tag{5.57}$$

where $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$ and $\mathbf{D}_{ci}$ are the state space realization of (5.56) and are given by:

$$\mathbf{A}_{ci} = \left[-1/T_{i2}\right], \qquad \mathbf{B}_{ci} = \left[1/T_{i2}\right], \qquad \mathbf{C}_{ci} = \begin{bmatrix}0\\1\end{bmatrix}, \qquad \mathbf{D}_{ci} = \begin{bmatrix}1\\0\end{bmatrix} \tag{5.58}$$

Finally, the overall extended system equation for the system can be rewritten in one state-space model as follows

$$\begin{aligned}
\dot{\tilde{\mathbf{x}}}(t) &= \tilde{\mathbf{A}}\,\tilde{\mathbf{x}}(t) + \tilde{\mathbf{B}}_0\mathbf{w}_0(t) + \tilde{\mathbf{B}}_1\mathbf{w}_1(t) + \tilde{\mathbf{B}}_2\mathbf{u}(t) \\
\mathbf{z}(t) &= \tilde{\mathbf{C}}_1\mathbf{x}(t) + \tilde{\mathbf{D}}_{10}\mathbf{w}_0(t) + \tilde{\mathbf{D}}_{11}\mathbf{w}_1(t) + \tilde{\mathbf{D}}_{12}\mathbf{u}(t) \\
\tilde{\mathbf{y}}(t) &= \tilde{\mathbf{C}}_y\mathbf{x}(t) + \tilde{\mathbf{D}}_{y0}\mathbf{w}_0(t) + \tilde{\mathbf{D}}_{y1}\mathbf{w}_1(t) + \tilde{\mathbf{D}}_{y2}\mathbf{u}(t)
\end{aligned} \tag{5.59}$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix}
\mathbf{A}_{11} & \mathbf{0} & \mathbf{A}_{12} & \mathbf{0} & \mathbf{A}_{13} & \mathbf{0} & \dots & \mathbf{A}_{1N} & \mathbf{0} \\
\mathbf{B}_{c1}\mathbf{C}_{12} & \mathbf{A}_{c1} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\
\mathbf{A}_{21} & \mathbf{0} & \mathbf{A}_{22} & \mathbf{0} & \mathbf{A}_{23} & \mathbf{0} & \dots & \mathbf{A}_{2N} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{B}_{c2}\mathbf{C}_{22} & \mathbf{A}_{c2} & \mathbf{0} & \mathbf{0} & \dots & 0 & \mathbf{0} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\mathbf{A}_{N1} & \mathbf{0} & \mathbf{A}_{N2} & \mathbf{0} & \mathbf{A}_{N3} & \mathbf{0} & \dots & \mathbf{A}_{NN} & \mathbf{0} \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{B}_{cN}\mathbf{C}_{N2} & \mathbf{A}_{cN}
\end{bmatrix}$$

$$\tilde{\mathbf{B}}_0 = \mathrm{diag}\left\{\begin{bmatrix}\mathbf{B}_{10} \\ \mathbf{B}_{c1}\mathbf{D}_{1y0}\end{bmatrix}, \begin{bmatrix}\mathbf{B}_{20} \\ \mathbf{B}_{c2}\mathbf{D}_{2y0}\end{bmatrix}, \dots, \begin{bmatrix}\mathbf{B}_{N0} \\ \mathbf{B}_{cN}\mathbf{D}_{Ny0}\end{bmatrix}\right\}$$

$$\tilde{\mathbf{B}}_1 = \mathrm{diag}\left\{\begin{bmatrix}\mathbf{B}_{11} \\ \mathbf{B}_{c1}\mathbf{D}_{1y1}\end{bmatrix}, \begin{bmatrix}\mathbf{B}_{21} \\ \mathbf{B}_{c2}\mathbf{D}_{2y1}\end{bmatrix}, \dots, \begin{bmatrix}\mathbf{B}_{N1} \\ \mathbf{B}_{cN}\mathbf{D}_{Ny1}\end{bmatrix}\right\}$$

$$\tilde{\mathbf{B}}_2 = \mathrm{diag}\left\{\begin{bmatrix}\mathbf{B}_{12} \\ \mathbf{0}\end{bmatrix}, \begin{bmatrix}\mathbf{B}_{22} \\ \mathbf{0}\end{bmatrix}, \dots, \begin{bmatrix}\mathbf{B}_{N2} \\ \mathbf{0}\end{bmatrix}\right\}$$

$$\tilde{\mathbf{C}}_1 = \mathrm{diag}\{[\mathbf{C}_{11} \quad \mathbf{0}], [\mathbf{C}_{21} \quad \mathbf{0}], \dots [\mathbf{C}_{N1} \quad \mathbf{0}]\}$$

$$\tilde{\mathbf{C}}_2 = \mathrm{diag}\{[\mathbf{D}_{c1}\mathbf{C}_{1y} \quad \mathbf{C}_{c1}], [\mathbf{D}_{c2}\mathbf{C}_{2y} \quad \mathbf{C}_{c2}], \dots, [\mathbf{D}_{cN}\mathbf{C}_{Ny} \quad \mathbf{C}_{cN}]\}$$

$$\tilde{\mathbf{D}}_{10} = \mathrm{diag}\{\mathbf{D}_{110}, \mathbf{D}_{210}, \dots, \mathbf{D}_{N10}\}, \qquad \tilde{\mathbf{D}}_{11} = \mathrm{diag}\{\mathbf{D}_{111}, \mathbf{D}_{211}, \dots, \mathbf{D}_{N11}\}$$

$$\tilde{\mathbf{D}}_{12} = \mathrm{diag}\{\mathbf{D}_{112}, \mathbf{D}_{212}, \dots, \mathbf{D}_{N12}\}, \qquad \tilde{\mathbf{D}}_{y0} = \mathrm{diag}\{\mathbf{D}_{c1}\mathbf{D}_{1y0}, \mathbf{D}_{c2}\mathbf{D}_{2y0}, \dots, \mathbf{D}_{cN}\mathbf{D}_{Ny0}\}$$

$$\tilde{\mathbf{D}}_{y1} = \mathrm{diag}\{\mathbf{D}_{c1}\mathbf{D}_{1y1}, \mathbf{D}_{c2}\mathbf{D}_{2y1}, \dots, \mathbf{D}_{cN}\mathbf{D}_{Ny1}\}$$

Hence, the static output feedback controller for *i*th-subsystem is given as:

$$\mathbf{u}_i(t) = \tilde{\mathbf{K}}_i\,\tilde{\mathbf{y}}_i(t) \tag{5.60}$$

where $\tilde{\mathbf{K}}_i = [K_{i1} \quad K_{i2}]$. Moreover, the decentralized static output feedback controller for the whole system will then have the familiar block structure of the form

$$\mathbf{u}(t) = \tilde{\mathbf{K}}_D\,\tilde{\mathbf{y}}(t) \tag{5.61}$$

where $\tilde{\mathbf{K}}_D = \mathrm{diag}\{\tilde{\mathbf{K}}_1, \tilde{\mathbf{K}}_2, \dots, \tilde{\mathbf{K}}_N\}$.

Substituting the static output feedback strategy (5.61) into the system equation of (5.59), the closed-loop system can be rewritten as follows:

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}_{cl}\,\tilde{\mathbf{x}}(t) + \mathbf{B}_{cl0}\mathbf{w}_0(t) + \mathbf{B}_{cl1}\mathbf{w}_1(t) \\ \mathbf{z}(t) &= \mathbf{C}_{cl1}\mathbf{x}(t) + \mathbf{D}_{cl0}\mathbf{w}_0(t) + \mathbf{D}_{cl1}\mathbf{w}_1(t)\end{aligned} \tag{5.62}$$

where

$$\mathbf{A}_{cl} = \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\tilde{\mathbf{K}}_D\tilde{\mathbf{C}}_y, \quad \mathbf{B}_{cl0} = \tilde{\mathbf{B}}_0 + \tilde{\mathbf{B}}_2\tilde{\mathbf{K}}_D\tilde{\mathbf{D}}_{y0}, \quad \mathbf{B}_{cl1} = \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\tilde{\mathbf{K}}_D\tilde{\mathbf{D}}_{y1},$$
$$\mathbf{C}_{cl1} = \tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\tilde{\mathbf{K}}_D\tilde{\mathbf{C}}_y, \; \mathbf{D}_{cl0} = \tilde{\mathbf{D}}_{10} + \tilde{\mathbf{D}}_{12}\tilde{\mathbf{K}}_D\tilde{\mathbf{D}}_{y0}, \; \mathbf{D}_{cl1} = \tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\tilde{\mathbf{K}}_D\tilde{\mathbf{D}}$$

5.4.2 Optimal Design Problem Using Sequential LMI Programming Method

Designing an optimal static H_2/H_∞ output feedback controller for the extended plant is equivalent to that of finding the gain matrix $\tilde{\mathbf{K}}_D$ by minimizing the upper bound of the H_2 norm of the transfer function $T_{\mathbf{zw}_0}(s) = \mathbf{C}_{cl0}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl0} + \mathbf{D}_{cl0}$ from the disturbance $\mathbf{w}_0$ to the measured output $\mathbf{z}$ and which at the same time satisfies an H_∞ norm bound condition on the closed loop transfer function $T_{\mathbf{zw}_1}(s) = \mathbf{C}_{cl1}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl1} + \mathbf{D}_{cl1}$ from the disturbance $\mathbf{w}_1$ to the measured output $\mathbf{z}$, i.e. $\left\|T_{\mathbf{zw}_1}(s)\right\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}_1}(s)$ and $T_{\mathbf{zw}_0}(s)$ must be stable [26], [27].

Designing a static H_∞ output feedback controller for the extended system given in (5.59) is reduced to finding of a controller $\tilde{\mathbf{K}}_D$ and a positive definite matrix $\mathbf{P} \succ \mathbf{0}$ that satisfy the following matrix inequality

$$\begin{bmatrix} \mathbf{P}\mathbf{A}_{cl} + \mathbf{A}_{cl}^T\mathbf{P} & \mathbf{P}\mathbf{B}_{cl1} & \mathbf{C}_{cl1}^T \\ \mathbf{B}_{cl1}^T\mathbf{P} & -\gamma\mathbf{I}_{n_{\mathbf{w}1}} & \mathbf{D}_{cl1}^T \\ \mathbf{C}_{cl1} & \mathbf{D}_{cl1} & -\gamma\mathbf{I}_{n_{\mathbf{z}}} \end{bmatrix} \prec \mathbf{0} \tag{5.63}$$

Alternatively, using the Elimination lemma (see Lemma 4.3), the BMI form in (5.63) can be transformed into two LMIs equations coupled through a bilinear matrix equation.

$$\mathbf{N}_{\mathbf{U}}^T \begin{bmatrix} \tilde{\mathbf{A}}\mathbf{Q} + \mathbf{Q}\tilde{\mathbf{A}}^T + \gamma^{-1}\tilde{\mathbf{B}}_1\tilde{\mathbf{B}}_1^T & (\tilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^T)^T \\ \tilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{B}}_1^T & \gamma^{-1}\tilde{\mathbf{D}}_{11}\tilde{\mathbf{D}}_{11}^T - \gamma\mathbf{I} \end{bmatrix} \mathbf{N}_{\mathbf{U}} \prec \mathbf{0} \tag{5.64}$$

$$\mathbf{N}_{\mathbf{V}}^T \begin{bmatrix} \mathbf{P}\tilde{\mathbf{A}} + \tilde{\mathbf{A}}^T\mathbf{P} + \gamma^{-1}\tilde{\mathbf{C}}_1^T\tilde{\mathbf{C}}_1 & \mathbf{P}\tilde{\mathbf{B}}_1 + \gamma^{-1}\tilde{\mathbf{C}}_1^T\tilde{\mathbf{D}}_{11} \\ (\mathbf{P}\tilde{\mathbf{B}}_1 + \gamma^{-1}\tilde{\mathbf{C}}_1^T\tilde{\mathbf{D}}_{11})^T & \gamma^{-1}\tilde{\mathbf{D}}_{11}^T\tilde{\mathbf{D}}_{11} - \gamma\mathbf{I}_{n_{w1}} \end{bmatrix} \mathbf{N}_{\mathbf{V}} \prec \mathbf{0} \tag{5.65}$$

and

$$\mathbf{PQ} = \mathbf{I} \tag{5.66}$$

where $\mathbf{N}_{\mathbf{U}}$ and $\mathbf{N}_{\mathbf{V}}$ denotes arbitrary bases of the nullspaces of $\mathbf{U} = [\widetilde{\mathbf{C}} \quad \widetilde{\mathbf{D}}_{\mathbf{y}1}]$ and $\mathbf{V} = [\widetilde{\mathbf{B}}_2^T \quad \widetilde{\mathbf{D}}_{12}^T]$, respectively.

Remark 5.8: It is possible to find stabilizing controllers which at the same time ensure an α-degree of stability. This additional requirement will introduce terms $2\alpha\mathbf{Q}$ and $2\alpha\mathbf{P}$ for some positive value of α in (5.64) and (5.65), respectively.

The coupling nonlinear equality constraint in (5.66) can be rearranged as $\mathbf{P} - \mathbf{Q}^{-1} = \mathbf{0}$ and which further can be relaxed as an LMI expression as follows:

$$\begin{bmatrix} \mathbf{P} & \mathbf{I} \\ \mathbf{I} & \mathbf{Q} \end{bmatrix} \succeq \mathbf{0} \tag{5.67}$$

Moreover, using the cone-complementarity approach [58] there exists an H_∞ static output feedback controller $\widetilde{\mathbf{K}}_D$ if and only if the global minimum of the following optimization problem

$$\begin{aligned} &\text{Min} \quad Trace\ (\mathbf{PQ}) \\ &\text{subject} \quad \text{to (5.64), (5.65)} \quad \text{(5.66)} \end{aligned} \tag{5.68}$$

is $N + \sum_{i=1}^{N} n_i$.

The H_2 norm of $T_{\mathbf{zw}_0}(s)$ is finite if and only if $\mathbf{D}_{cl0} \equiv 0$ and which can be computed by the following equation

$$\left\| T_{\mathbf{zw}_0} \right\|_{H_2}^2 = Trace(\mathbf{B}_{cl0}^T \mathbf{L} \mathbf{B}_{cl0}) \tag{5.69}$$

where $\mathbf{L} \succ 0$ is the observability Gramian which satisfies the following Lyapunov equation

$$\mathbf{A}_{cl}^T \mathbf{L} + \mathbf{L}\mathbf{A}_{cl} + \mathbf{C}_{cl0}^T \mathbf{C}_{cl0} = \mathbf{0} \tag{5.70}$$

Whenever $\mathbf{P}$ satisfies the condition given (5.63), the H_2 norm of $T_{\mathbf{zw}_0}(s)$ satisfies the following upper bound condition [60], [61]

$$\left\|T_{\mathbf{zw}_0}\right\|_{H_2}^2 = Trace(\mathbf{B}_{cl0}^T \mathbf{L} \mathbf{B}_{cl0}) \le Trace(\mathbf{B}_{cl0}^T \mathbf{P} \mathbf{B}_{cl0}) \tag{5.71}$$

The above relation suggests minimizing of the upper bound given in (5.71) for (sub)-optimal static H_2/H_∞ output feedback problem instead of directly minimizing the H_2 norm of $T_{\mathbf{zw}_0}(s)$. Hence, minimizing the upper bound is equivalent to the following optimization problem

$$\begin{aligned} &\text{Min} \quad Trace(\mathbf{Y}) \\ &\text{subject to } \mathbf{P} \succ \mathbf{0},\ \mathbf{Y} \succ \mathbf{0} \text{ and} \\ &\begin{bmatrix} \mathbf{Y} & \mathbf{B}_{cl0}^T \mathbf{P} \\ \mathbf{P}\mathbf{B}_{cl0} & \mathbf{P} \end{bmatrix} \succeq \mathbf{0} \end{aligned} \tag{5.72}$$

The BMI expression in (5.72), i.e. $\mathbf{Y} \succeq \mathbf{B}_{cl0}^T \mathbf{P} \mathbf{B}_{cl0} \succeq \mathbf{0}$, can be further expanded as follows:

$$\begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \mathbf{P} \\ \widetilde{\mathbf{B}}_0 & \mathbf{P} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{P}\widetilde{\mathbf{B}}_2 \end{bmatrix} \widetilde{\mathbf{K}}_D \begin{bmatrix} \mathbf{D}_{y0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{y0}^T \\ \mathbf{0} \end{bmatrix} \widetilde{\mathbf{K}}_D^T \begin{bmatrix} \mathbf{0} & \widetilde{\mathbf{B}}_2^T \mathbf{P} \end{bmatrix} \succeq \mathbf{0} \tag{5.73}$$

Using the Elimination Lemma (see Lemma 4.3), the above matrix inequality can be equivalently rewritten as

$$\mathbf{N}_\mathbf{S}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \\ \widetilde{\mathbf{B}}_0 & \mathbf{Q} \end{bmatrix} \mathbf{N}_\mathbf{S} \succeq \mathbf{0},\quad \mathbf{N}_\mathbf{R}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \mathbf{P} \\ \mathbf{P}\widetilde{\mathbf{B}}_0 & \mathbf{P} \end{bmatrix} \mathbf{N}_\mathbf{R} \succeq \mathbf{0} \tag{5.74}$$

where $\mathbf{N}_\mathbf{R}$ and $\mathbf{N}_\mathbf{S}$ denotes arbitrary bases of the nullspaces of $\mathbf{R} = [\widetilde{\mathbf{D}}_{\mathbf{y}1} \quad \mathbf{0}]$ and $\mathbf{S} = [\mathbf{0} \quad \widetilde{\mathbf{B}}_2^T]$, respectively. Thus, the problem of designing a (sub)-optimal H_2/H_∞ stabilizing static output feedback controller will be reduced to solve simultaneously the optimization problems in (5.68) and (5.72) for the positive definite matrices $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$:

$$\begin{aligned} &\text{Min} \quad Trace(\mathbf{Y}) + Trace(\mathbf{PQ}) \\ &\text{subject to } \mathbf{P} \succ \mathbf{0}, \mathbf{Q} \succ \mathbf{0}, \mathbf{Y} \succ \mathbf{0} \text{ and} \\ &\begin{bmatrix} \mathbf{P} & \mathbf{I} \\ \mathbf{I} & \mathbf{Q} \end{bmatrix} \succeq \mathbf{0} \\ &\mathbf{N}_\mathbf{U}^T \begin{bmatrix} \widetilde{\mathbf{A}}\mathbf{Q} + \mathbf{Q}\widetilde{\mathbf{A}}^T + 2\alpha\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{B}}_1\widetilde{\mathbf{B}}_1^T & (\widetilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T)^T \\ \widetilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{D}}_{11}^T - \gamma\mathbf{I} \end{bmatrix} \mathbf{N}_\mathbf{U} \prec \mathbf{0} \\ &\mathbf{N}_\mathbf{V}^T \begin{bmatrix} \widetilde{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\widetilde{\mathbf{A}} + 2\alpha\mathbf{P} + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{C}}_1 & \mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11} \\ (\mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11})^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}^T\widetilde{\mathbf{D}}_{11} - \gamma\mathbf{I} \end{bmatrix} \mathbf{N}_\mathbf{V} \prec \mathbf{0} \\ &\mathbf{N}_\mathbf{S}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \\ \widetilde{\mathbf{B}}_0 & \mathbf{Q} \end{bmatrix} \mathbf{N}_\mathbf{S} \succeq \mathbf{0},\ \mathbf{N}_\mathbf{R}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \mathbf{P} \\ \mathbf{P}\widetilde{\mathbf{B}}_0 & \mathbf{P} \end{bmatrix} \mathbf{N}_\mathbf{R} \succeq \mathbf{0} \end{aligned} \tag{5.75}$$

The optimization in (5.75) is a nonconvex optimization problem due to the bilinear matrix term in the objective functional. To compute the (sub)-optimal solution of this problem, the sequential LMI programming method is invoked once more again. The idea behind this algorithm is to linearize the cost functional in (5.75) with respect to its variables and then to solve sequentially the resulting convex optimization problem involving only LMI optimization. Thus, the sequential LMI programming method for finding the stabilizing robust static output gain matrix has the following two-step optimization algorithms.

ALGORITHM IV: Sequential LMI Programming Method

For a given $\alpha > 0$, solve the LMI feasibility problems of (5.64), (5.65), (5.67) and (5.74) simultaneously for $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$.

Set the solutions $(\mathbf{P}^0, \mathbf{Q}^0, \mathbf{Y}^0) := (\mathbf{P}, \mathbf{Q}, \mathbf{Y})$

$\mu^0 = (2\,Trace(\mathbf{P}^0\mathbf{Q}^0) + Trace(\mathbf{Y}^0))/n.$

Repeat until $\mu^k \le \varepsilon$ (for small $\varepsilon > 0$), **do**

(1) Solve the following optimization problem for $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{Y}$

$$\text{Min } Trace\,(\mathbf{P}\mathbf{Q}^k + \mathbf{P}^k\mathbf{Q}) + Trace\,(\mathbf{Y})$$

subject to $\mathbf{P} \succ \mathbf{0}, \mathbf{Q} \succ \mathbf{0}, \mathbf{Y} \succ \mathbf{0}$ and

$$\begin{bmatrix} \mathbf{P} & \mathbf{I} \\ \mathbf{I} & \mathbf{Q} \end{bmatrix} \succeq \mathbf{0}$$

$$\mathbf{N}_{\mathrm{U}}^T \begin{bmatrix} \widetilde{\mathbf{A}}\mathbf{Q} + \mathbf{Q}\widetilde{\mathbf{A}}^T + 2\alpha\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{B}}_1\widetilde{\mathbf{B}}_1^T & (\widetilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T)^T \\ \widetilde{\mathbf{C}}_1\mathbf{Q} + \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{B}}_1^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}\widetilde{\mathbf{D}}_{11}^T - \gamma\mathbf{I} \end{bmatrix} \mathbf{N}_{\mathrm{U}} \prec \mathbf{0}$$

$$\mathbf{N}_{\mathrm{V}}^T \begin{bmatrix} \widetilde{\mathbf{A}}^T\mathbf{P} + \mathbf{P}\widetilde{\mathbf{A}} + 2\alpha\mathbf{P} + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{C}}_1 & \mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11} \\ (\mathbf{P}\widetilde{\mathbf{B}}_1 + \gamma^{-1}\widetilde{\mathbf{C}}_1^T\widetilde{\mathbf{D}}_{11})^T & \gamma^{-1}\widetilde{\mathbf{D}}_{11}^T\widetilde{\mathbf{D}}_{11} - \gamma\mathbf{I} \end{bmatrix} \mathbf{N}_{\mathrm{V}} \prec \mathbf{0}$$

$$\mathbf{N}_{\mathbf{S}}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T \\ \widetilde{\mathbf{B}}_0 & \mathbf{Q} \end{bmatrix} \mathbf{N}_{\mathbf{S}} \succeq \mathbf{0}, \quad \mathbf{N}_{\mathbf{R}}^T \begin{bmatrix} \mathbf{Y} & \widetilde{\mathbf{B}}_0^T\mathbf{P} \\ \mathbf{P}\widetilde{\mathbf{B}}_0 & \mathbf{P} \end{bmatrix} \mathbf{N}_{\mathbf{R}} \succeq \mathbf{0}$$

(2) Set the direction $(\Delta\mathbf{P}^k, \Delta\mathbf{Q}^k, \Delta\mathbf{Y}^k) := (\mathbf{P} - \mathbf{P}^k, \mathbf{Q} - \mathbf{Q}^k, \mathbf{Y} - \mathbf{Y}^k)$

(3) Set $(\mathbf{P}^{k+1}, \mathbf{Q}^{k+1}, \mathbf{Y}^{k+1}) := (\mathbf{P}^k, \mathbf{Q}^k, \mathbf{Y}^k) + t_k(\Delta\mathbf{P}^k, \Delta\mathbf{Q}^k, \Delta\mathbf{Y}^k)$ for $t_k \in [0, 1]$

(4) Set $\mu^k = Trace(\mathbf{P}\mathbf{Q}^k + \mathbf{P}^k\mathbf{Q}) + Trace(\mathbf{Y}) - (2\,Trace(\mathbf{P}^k\mathbf{Q}^k) + Trace(\mathbf{Y}^k))$ and increment k by 1

End do

Remark 5.9: The following sub-problem can be used to choose the appropriate value for t_k in Step (3):

$$\begin{aligned} &\text{Min } Trace(\mathbf{P}^k + t_k(\mathbf{P} - \mathbf{P}^k))(\mathbf{Q}^k + t_k(\mathbf{Q} - \mathbf{Q}^k)) + Trace(\mathbf{Y}^k + t_k(\mathbf{Y} - \mathbf{Y}^k)) \\ &\text{subject to } t_k \in [0, 1] \end{aligned} \qquad (5.76)$$

ALGORITHM V: The LMI Problem to Determine $\widetilde{\mathbf{K}}_D$.

Solve the following LMI optimization problem for $\widetilde{\mathbf{K}}_D$ that gives the least norm on the gains of the controller.

$$\begin{aligned}
&\text{Min} \quad \tau \\
&\text{subject to } \ \tau > 0, \text{ and} \\
&\mathbf{\Phi}_B \widetilde{\mathbf{K}}_D \mathbf{\Theta}_C + (\mathbf{\Phi}_B \widetilde{\mathbf{K}}_D \mathbf{\Theta}_C)^T + \overline{\mathbf{\Omega}} \prec \mathbf{0}, \\
&\begin{bmatrix} \mathbf{Y}^* & \widetilde{\mathbf{B}}_0^T \mathbf{P}^* \\ \widetilde{\mathbf{B}}_0 & \mathbf{P}^* \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{P}^* \widetilde{\mathbf{B}}_2 \end{bmatrix} \widetilde{\mathbf{K}}_D \begin{bmatrix} \mathbf{D}_{y0} & \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{y0}^T \\ 0 \end{bmatrix} \widetilde{\mathbf{K}}_D^T \begin{bmatrix} \mathbf{0} & \widetilde{\mathbf{B}}_2^T \mathbf{P}^* \end{bmatrix} \succeq \mathbf{0} \\
&\begin{bmatrix} -\tau \mathbf{I} & \widetilde{\mathbf{K}}_D^T \\ \widetilde{\mathbf{K}}_D & -\mathbf{I} \end{bmatrix} \succ \mathbf{0}
\end{aligned} \tag{5.77}$$

where $\mathbf{P}^*$ and $\mathbf{Y}^*$ are the solutions of Algorithm IV, and

$$\overline{\mathbf{\Omega}} = \begin{bmatrix} \mathbf{P}^* \widetilde{\mathbf{A}} + \widetilde{\mathbf{A}}^T \mathbf{P}^* + 2\alpha \mathbf{P}^* & \mathbf{P}^* \widetilde{\mathbf{B}}_1 & \widetilde{\mathbf{C}}_1^T \\ \widetilde{\mathbf{B}}_1^T \mathbf{P}^* & -\gamma \mathbf{I} & \widetilde{\mathbf{D}}_{11}^T \\ \widetilde{\mathbf{C}}_1 & \widetilde{\mathbf{D}}_{11} & -\gamma \mathbf{I} \end{bmatrix}, \ \mathbf{\Phi}_B = \begin{bmatrix} \mathbf{P}^* \mathbf{B}_2 \\ \mathbf{0} \\ \mathbf{D}_{12} \end{bmatrix}, \ \mathbf{\Theta}_C = \begin{bmatrix} \mathbf{C}_y & \mathbf{D}_{y1} & \mathbf{0} \end{bmatrix} \tag{5.78}$$

Remark 5.10: It is possible to include a different set of LMIs constraint in Algorithm II that could be used to limit the upper gain of the controller matrix $\widetilde{\mathbf{K}}_D$.

The above two algorithms involve the following: i) minimization problem of a convex cost functional subject to LMI constraints, i.e. the sequential LMI optimization problem in Algorithm IV, will give the optimal values for $\mathbf{P}^*$, $\mathbf{Q}^*$ and $\mathbf{Y}^*$, and ii) minimization problem of a least norm objective functional subject to LMI constraints, i.e. the optimization problem in Algorithm V, which gives the (sub)-optimal H_2/H_∞ stabilizing static output feedback controller $\widetilde{\mathbf{K}}_D$.

5.4.3 Simulation Results

The robust decentralized PSS design approach presented in the previous subsection is now applied to a test system that has been considered in Section 5.3.3. Here to demonstrate the applicability of the proposed approach a first order PSS with value for $T_{i2} = 0.29\,s$ is used, although it is possible to extend the method to any order and/or combinations of PSS blocks in the design procedure without any difficulty. After including the washout filter in the system, the design problem is reformulated as a nonconvex optimization problem involving linear matrix inequalities (LMIs) and bilinear matrix inequality (BMI) and which is solved using the sequential LMI programming method presented in the previous section.

The design procedure has been carried out for loading condition $[\mathrm{P_{L1}} = 1600\ \mathrm{MW}, \mathrm{Q_{L1}} = 150\,\mathrm{Mvar}]$ and $[\mathrm{P_{L2}} = 2400\ \mathrm{MW}, \mathrm{Q_{L2}} = 120\ \mathrm{Mvar}]$. Speed signals from each generator, the output of the PSSs together with the terminal voltage error signals, which are the input to the regulator of the exciter are used as regulated signals within the design framework. Moreover, the output of the washout block, i.e., measured output signal, is used as an input signal for the PSS in the system. For a short circuit of 150 ms duration at node F in Area-A, the transient responses of generator G2 with and without the PSSs in the system are shown in Figure 5.6. The calculated gains and parameters for each PSS are also given in Table 5.4.

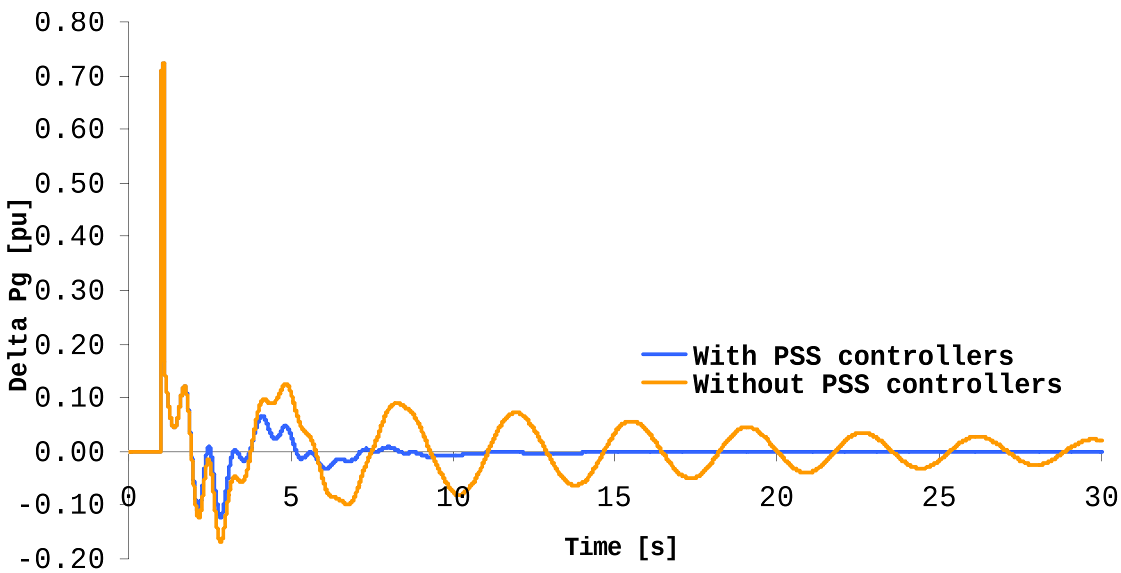


FIGURE 5.6 The transient responses of Generator G2 to a short circuit at node F in Area A.

TABLE 5.4 The computed robust PSS controllers gains and parameters corresponding to each generator.

Gains for the PSS	**Parameter T_{i1}**
$K_1 = 1.2565$	$T_{11} = 0.5357$
$K_2 = 1.0005$	$T_{21} = 0.5352$
$K_3 = 0.8778$	$T_{31} = 0.5338$
$K_4 = 1.1065$	$T_{41} = 0.5338$

To further assess the effectiveness of the proposed approach regarding robustness, the normalized transient performance indices for generator powers P_{gi}, generator terminal voltages V_{ti} and excitation voltages E_{fdi} were computed for different loading conditions at node 1 $[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$. These normalized transient performance indices for different loading conditions are shown in Figure 5.7. It can be seen from Figure 5.7 that the indices for $I_N(E_{fd})$, $I_N(P_g)$ and $I_N(V_t)$ are either near to unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behaviour exhibits robustness for all loading conditions (see Remark 5.7).

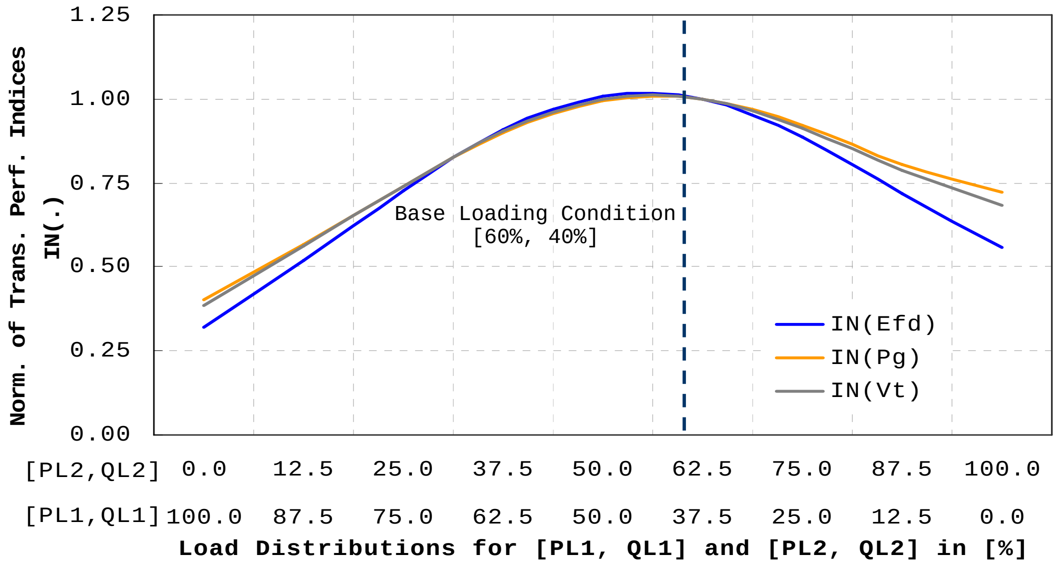


FIGURE 5.7 Plot of the normalized transient performance indices.

Similarly, the normalized transient performance indices after disconnecting one of the tie-lines from the system are shown in Figure 5.8 for different operating conditions. From these figures the normalized transient performance indices are also either near to unity or less than unity for wide operating conditions except for the condition when the total load distribution apparently concentrated to Area-B. This evidently shows the robustness of the proposed approach to structural change in the system.

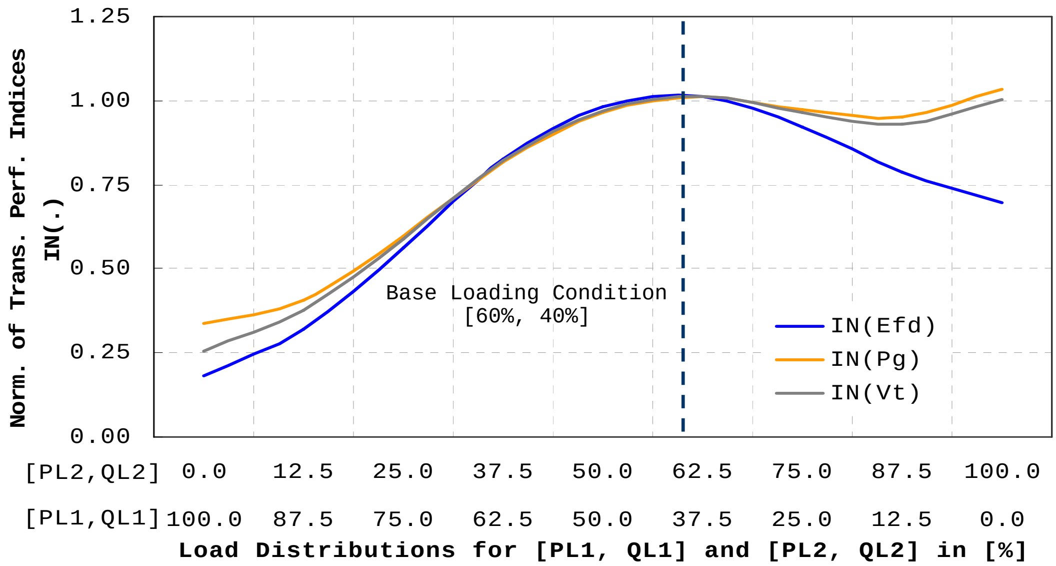


FIGURE 5.8 Plot of the normalized transient performance indices after disconnecting one of the tie-lines.

5.5 Robust Decentralized H_∞ Dynamic Output Feedback Controller Design for Power Systems Using Parameter Continuation Method

This section focuses on the extension of matrix inequalities based H_∞ optimization approach to problems of practical interest in power systems. The design problem considered here is the natural extension of the reduced-order decentralized H_∞ dynamic output feedback controller synthesis for power systems. In the design, initially a full-order centralized H_∞ robust controller, which guarantees the robust stability of the overall system against unstructured and norm bounded uncertainties, is designed. Then the problem of designing a decentralized controller for the system is reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized controller to the decentralized one as the continuation parameter monotonically varies. This section also proposes an algorithm based on two-stage iterative matrix inequality optimization method for designing reduced-order robust decentralized H_∞ controllers that have practical benefits since high-order controllers when implemented in real-time configurations can create undesirable effects such as time delays. Moreover, it is shown that the approach presented in this section has been demonstrated by designing realistic robust PSSs, notably so-called reduced-order robust PSSs design for a test power system.

5.5.1 Controller Design Problem Formulation

Consider the general structure of the *i*th-generator together with the PSS block in a multi-machine power system shown in Figure 3.4. The input of the *i*th-controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. After augmenting the washout stage in the system, the *i*th-subsystem, within the framework of H_∞ design, is described by the following state space equation:

$$\begin{aligned}
\dot{\mathbf{x}}_i(t) &= \mathbf{A}_{ii}\,\mathbf{x}_i(t) + \sum_{j\neq i}\mathbf{A}_{ij}\,\mathbf{x}_j(t) + \mathbf{B}_{1i}\,\mathbf{w}_i(t) + \mathbf{B}_{2i}\,\mathbf{u}_i(t) \\
\mathbf{z}_i(t) &= \mathbf{C}_{1\,i}\,\mathbf{x}_i(t) + \mathbf{D}_{11\,i}\,\mathbf{w}_i(t) + \mathbf{D}_{12\,i}\,\mathbf{u}_i(t) \\
\mathbf{y}_i(t) &= \mathbf{C}_{y\,i}\,\mathbf{x}_i(t) + \mathbf{D}_{y1\,i}\mathbf{w}_i(t)
\end{aligned} \tag{5.79}$$

where $\mathbf{x}_i(t)\in\Re^{n_i}$ is the state variable, $\mathbf{u}_i(t)\in\Re^{m_i}$ is the control input, $\mathbf{y}_i(t)\in\Re^{q_i}$ is the measurement signal, $\mathbf{z}_i(t)\in\Re^{p_i}$ is the regulated variables, and $\mathbf{w}_i\in\Re^{r_i}$ is exogenous signal for the *i*th-subsystem. The matrices $\mathbf{A}$, $\mathbf{B}_1$, $\mathbf{B}_{2i}$, $\mathbf{C}_1$, $\mathbf{C}_{yi}$, $\mathbf{D}_{11}$, $\mathbf{D}_{12i}$ and $\mathbf{D}_{y1i}$ are all constant matrices with appropriate dimensions. Moreover, assume that there is no unstable fixed mode [55] with respect to $\mathbf{C}_y = \mathrm{diag}\{\mathbf{C}_{y1}, \mathbf{C}_{y2}, \ldots, \mathbf{C}_{yN}\}$, , $[\mathbf{A}_{ij}]_{N\times N}$ and $\mathbf{B}_2 = \mathrm{diag}\{\mathbf{B}_{21}, \mathbf{B}_{22}, \ldots, \mathbf{B}_{2N}\}$.

Consider the following decentralized dynamic output feedback controller for the system given in (5.79):

$$\begin{aligned} \dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\,\mathbf{y}_i(t) \\ \mathbf{u}_i(t) &= \mathbf{C}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t) \end{aligned} \tag{5.80}$$

where $\mathbf{x}_{ci}(t) \in \Re^{n_{ci}}$ is the state of the ith-local controller, n_{ci} is a specified dimension, and $\mathbf{A}_{ci}, \mathbf{B}_{ci}, \mathbf{C}_{ci}, \mathbf{D}_{ci}$, $i = 1, 2, \cdots, N$ are constant matrices to be determined during the designing. In this section, the design procedure deals with nonzero $\mathbf{D}_{ci}$, however, it can be set to zero, i.e., $\mathbf{D}_{ci} = \mathbf{0}$, so that the ith-local controller is strictly proper controller.

After augmenting the decentralized controller (5.80) in the system of (5.79), the state space equation of the ith-extended subsystem will have the following form

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}_i(t) &= (\tilde{\mathbf{A}}_{ii} + \tilde{\mathbf{B}}_{2i}\mathbf{K}_i\tilde{\mathbf{C}}_{yi})\tilde{\mathbf{x}}_i(t) \\ &\quad + (\tilde{\mathbf{B}}_{1i} + \tilde{\mathbf{B}}_{2i}\mathbf{K}_i\tilde{\mathbf{C}}_{yi})\mathbf{w}_i(t) + \sum_{j\neq i}\tilde{\mathbf{A}}_{ij}\,\tilde{\mathbf{x}}_j(t) \\ \mathbf{z}_i(t) &= (\tilde{\mathbf{C}}_{i1} + \tilde{\mathbf{D}}_{12\,i}\mathbf{K}_i\tilde{\mathbf{C}}_{y\,i})\tilde{\mathbf{x}}_i(t) + (\tilde{\mathbf{D}}_{11\,i} + \tilde{\mathbf{D}}_{12\,i}\mathbf{K}_i\tilde{\mathbf{D}}_{y1\,i})\mathbf{w}_i(t) \end{aligned} \tag{5.81}$$

where $\tilde{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t), \quad \mathbf{x}_{ci}^T(t)]^T$ is the augmented state variable for the ith-subsystem and

$$\tilde{\mathbf{A}}_{ij} = \begin{bmatrix} \mathbf{A}_{ij} & \mathbf{0}_{n_i\times n_{ci}} \\ \mathbf{0}_{n_{ci}\times n_i} & \mathbf{0}_{n_{ci}\times n_{ci}} \end{bmatrix}, \quad \tilde{\mathbf{B}}_{1i} = \begin{bmatrix} \mathbf{B}_{1i} \\ \mathbf{0}_{n_i\times r_i} \end{bmatrix}, \quad \tilde{\mathbf{C}}_{yi} = \begin{bmatrix} \mathbf{0}_{n_{ci}\times n_i} & \mathbf{I}_{n_{ci}\times n_{ci}} \\ \mathbf{C}_{yi} & \mathbf{0}_{q_i\times n_{ci}} \end{bmatrix}, \ \tilde{\mathbf{D}}_{11\,i} = \mathbf{D}_{11\,i},$$

$$\tilde{\mathbf{B}}_{2i} = \begin{bmatrix} \mathbf{0}_{n_i\times n_{ci}} & \mathbf{B}_{2i} \\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci}\times m_i} \end{bmatrix}, \quad \tilde{\mathbf{C}}_{1\,i} = \begin{bmatrix} \mathbf{C}_{1\,i} & \mathbf{0}_{p_i\times n_{ci}} \end{bmatrix}, \quad \tilde{\mathbf{D}}_{12i} = \begin{bmatrix} \mathbf{0}_{p_i\times n_{ci}} & \mathbf{D}_{12i} \end{bmatrix}, \ \tilde{\mathbf{D}}_{y1\,i} = \begin{bmatrix} \mathbf{0}_{n_{ci}\times r_i} \\ \mathbf{D}_{y1\,i} \end{bmatrix},$$

$$\mathbf{K}_i = \begin{bmatrix} \mathbf{A}_{ci} & \mathbf{B}_{ci} \\ \mathbf{C}_{ci} & \mathbf{D}_{ci} \end{bmatrix}$$

Moreover, the overall extended system equation can be rewritten in one state-space equation form as follows

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= (\tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y)\mathbf{w}(t) \\ \mathbf{z}(t) &= (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1})\mathbf{w}(t) \end{aligned} \tag{5.82}$$

where

$$\tilde{\mathbf{A}} = [\tilde{\mathbf{A}}_{ij}]_{N\times N}, \qquad \tilde{\mathbf{D}}_{11} = \mathrm{diag}\{\tilde{\mathbf{D}}_{111}, \tilde{\mathbf{D}}_{112}, \ldots, \tilde{\mathbf{D}}_{11N}\},$$
$$\tilde{\mathbf{B}}_1 = \mathrm{diag}\{\tilde{\mathbf{B}}_{11}, \tilde{\mathbf{B}}_{12}, \ldots, \tilde{\mathbf{B}}_{1N}\}, \qquad \tilde{\mathbf{D}}_{12} = \mathrm{diag}\{\tilde{\mathbf{D}}_{121}, \tilde{\mathbf{D}}_{122}, \ldots, \tilde{\mathbf{D}}_{12N}\},$$
$$\tilde{\mathbf{B}}_2 = \mathrm{diag}\{\tilde{\mathbf{B}}_{21}, \ddot{\mathbf{B}}_{22}, \ldots, \tilde{\mathbf{B}}_{2N}\}, \qquad \tilde{\mathbf{C}}_y = \mathrm{diag}\{\tilde{\mathbf{C}}_{y1}, \tilde{\mathbf{C}}_{y2}, \ldots, \tilde{\mathbf{C}}_{yN}\},$$
$$\tilde{\mathbf{C}}_1 = \mathrm{diag}\{\tilde{\mathbf{C}}_{11}, \tilde{\mathbf{C}}_{12}, \ldots, \tilde{\mathbf{C}}_{1N}\}, \qquad \tilde{\mathbf{D}}_{y1} = \mathrm{diag}\{\tilde{\mathbf{D}}_{y11}, \tilde{\mathbf{D}}_{y12}, \ldots, \tilde{\mathbf{D}}_{y1N}\},$$
$$\mathbf{K}_D = \mathrm{diag}\{\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_N\}.$$

Hence, the overall extended system equation can be rewritten in a compact form as

$$\begin{aligned} \dot{\tilde{\mathbf{x}}}(t) &= \mathbf{A}_{cl}\,\tilde{\mathbf{x}}(t) + \mathbf{B}_{cl}\mathbf{w}(t) \\ \mathbf{z}(t) &= \mathbf{C}_{cl}\mathbf{x}(t) + \mathbf{D}_{cl}\mathbf{w}(t) \end{aligned} \tag{5.83}$$

where

$$\mathbf{A}_{cl} = \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad \mathbf{B}_{cl} = \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{D}}_{y1},$$
$$\mathbf{C}_{cl} = \tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y, \quad \mathbf{D}_{cl} = \tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1}$$

Consider the following design approach where the controller strategy in (5.80) internally stabilizes the closed-loop of the transfer function $T_{zw}(s)$ from $\mathbf{w}$ to $\mathbf{z}$ and moreover satisfies a certain prescribed disturbance attenuation level $\gamma > 0$, i.e., $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$.

In the following, the design procedure assumes that the system in (5.81) is stabilizable with the same prescribed disturbance attenuation level γ via a centralized H_∞ controller of dimension equal to or greater than $n_c := \sum_{i=1}^{N} n_{ci}$ in which each controller input $\mathbf{u}_i(t)$ is determined by the corresponding measured outputs $\mathbf{y}_j(t), 1 \le j \le N$. The significance of this assumption lies on the fact that the decentralized controllers cannot achieve better performances than that of centralized controllers. In the following subsections, the centralized H_∞ controller is used for the initial boundary value in the two-stage matrix inequality optimization method.

5.5.2 Design Problem Using Parameterized Continuation Method Involving Matrix Inequalities

Designing a decentralized H_∞ output feedback controller for the system is equivalent to that of finding the matrix $\mathbf{K}_D$ that satisfies an H_∞ norm bound condition on the closed loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from the disturbance $\mathbf{w}$ to the measured output

$\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}}(s)$ must be stable.

The following theorem is instrumental in establishing the existence of decentralized control strategy (5.80) that satisfies a certain prescribed disturbance attenuation level $\gamma > 0$ on the closed loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from the disturbance $\mathbf{w}$ to the measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}\|_\infty < \gamma$.

THEOREM 5.5. The system (5.79) is stabilizable with the disturbance attenuation level $\gamma > 0$ via a decentralized controller (5.80) composed of N n_{ci}th-order local controllers if and only if there exist a matrix $\mathbf{K}_D$ and a positive-definite matrix $\widetilde{\mathbf{P}}$ which satisfy the following matrix inequality:

$$\begin{aligned}\mathbf{\Phi}(\mathbf{K}_D, \widetilde{\mathbf{P}}) := {} & \begin{bmatrix} \widetilde{\mathbf{P}}\widetilde{\mathbf{A}} + \widetilde{\mathbf{A}}^T\widetilde{\mathbf{P}} & \widetilde{\mathbf{P}}\widetilde{\mathbf{B}}_1 & \widetilde{\mathbf{C}}_1^T \\ \widetilde{\mathbf{B}}_1^T\widetilde{\mathbf{P}} & -\gamma\mathbf{I}_r & \widetilde{\mathbf{D}}_{11}^T \\ \widetilde{\mathbf{C}}_1 & \widetilde{\mathbf{D}}_{11} & -\gamma\mathbf{I}_p \end{bmatrix} + \begin{bmatrix} \widetilde{\mathbf{P}}\widetilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \widetilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \widetilde{\mathbf{C}}_2 & \widetilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} \\ & + \left\{ \begin{bmatrix} \widetilde{\mathbf{P}}\widetilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \widetilde{\mathbf{D}}_{12} \end{bmatrix} \mathbf{K}_D \begin{bmatrix} \widetilde{\mathbf{C}}_2 & \widetilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p} \end{bmatrix} \right\}^T \prec \mathbf{0} \end{aligned} \tag{5.84}$$

The condition stated in the above theorem seems to be the same as that of the centralized H_∞ control case [26], [27]. However, due to the specified structure on the controller (i.e., designing controllers with "block diagonal") makes the problem a nonconvex optimization problem. To compute the optimal solution of this problem, the design problem is reformulated as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the continuation monotonically varies [62]. The parameterized family of the design problem in (5.84) is given as follows:

$$\widetilde{\mathbf{\Phi}}(\mathbf{K}_D, \widetilde{\mathbf{P}}, \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D,\ \widetilde{\mathbf{P}}) \prec \mathbf{0} \tag{5.85}$$

with $\lambda \in [0, 1]$ such that at $\lambda = 0$

$$\widetilde{\mathbf{\Phi}}(\mathbf{K}_F, \widetilde{\mathbf{P}}, 0) = \mathbf{\Phi}(\mathbf{K}_F, \widetilde{\mathbf{P}}) \tag{5.86}$$

and at $\lambda = 1$

$$\widetilde{\mathbf{\Phi}}(\mathbf{K}_D, \widetilde{\mathbf{P}}, 1) = \mathbf{\Phi}(\mathbf{K}_D, \widetilde{\mathbf{P}}) \tag{5.87}$$

where

$$\mathbf{K}_F = \begin{bmatrix} \mathbf{A}_F & \mathbf{B}_F \\ \mathbf{C}_F & \mathbf{D}_F \end{bmatrix} \tag{5.88}$$

is a constant matrix of the same size as $\mathbf{K}_D$ and composed of the coefficient matrices $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{C}_F$ and $\mathbf{D}_F$ of an n_c - dimensional centralized H_∞ for the disturbance attenuation level γ. The centralized controller $\mathbf{K}_F$ can be obtained via the existing method. Thus, the term $(1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D$ in (5.83) defines a homotopy interpolating centralized H_∞ controller and a desired decentralized H_∞ controller. Thus, the problem of finding a solution of (5.85) can be embedded in the family of problems as:

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, \lambda) \prec \mathbf{0}, \qquad \lambda \in [0,\ 1] \tag{5.89}$$

Thus, the algorithm based on parameter continuation method for finding the robust decentralized output feedback controller has the following two-stage iterative matrix inequalities optimization algorithm.

ALGORITHM VI: A Matrix Inequality Based Parameterized Continuation Method.

Compute the centralized controller $\mathbf{K}_F$ and $\tilde{\mathbf{P}}_0$ that guarantees
γ - disturbance attenuation level
Set $\lambda_0 = 0$, $k = 0$, $M = K$ (Large Number)
and $\mathbf{K}_{D0} = \mathbf{0}$ (Zero Matrix)
Repeat until $\lambda_k = 1$, **do**
$k \leftarrow k + 1$
$\lambda_k \leftarrow \lambda_{k-1} + 1/M$
(1) Compute $\mathbf{K}_{Dk}$ that satisfies
$\tilde{\mathbf{\Phi}}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk},\ \tilde{\mathbf{P}}_{k-1},\ \lambda_k) \prec \mathbf{0}$
(2) Compute $\tilde{\mathbf{P}}_k$ that satisfies
$\tilde{\mathbf{\Phi}}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk},\ \tilde{\mathbf{P}}_k,\ \lambda_k) \prec \mathbf{0}$
End do

Remark 5.10: If the problem in step-1 of Algorithm VI fails to be feasible, then the step-length should be changed to compute $\mathbf{K}_{Dk}$ that satisfies $\mathbf{\Phi}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_D,\ \tilde{\mathbf{P}}) \prec \mathbf{0}$ for the remaining interval.

The above two-stage iterative matrix inequality optimization method involves: i) computing a continuous family of decentralized feedback $\mathbf{K}_{Dk}$ starting from $\mathbf{K}_{D0} = \mathbf{0}$ at $\lambda = 0$ for each iteration, i.e., an LMI problem in step-1, and ii) computing the positive definite matrix $\tilde{\mathbf{P}}_k$ in step-2.

5.5.3 Reduced-Order Decentralized Controller Design

The algorithm proposed in the previous section can only be applied when the dimension of the decentralized H_∞ controller is equal to the order of the plant, i.e., $n_c = n$. However, it is possible to compute directly a reduced-order decentralized controller, i.e., $n_c < n$ by augmenting the matrix $\mathbf{K}_D$ as

$$\hat{\mathbf{K}}_D = \left[\begin{array}{cc|c} \hat{\mathbf{A}}_D & 0_{n_n \times (n-n_c)} & \hat{\mathbf{B}}_D \\ * & -\mathbf{I}_{(n-n_c)} & ** \\ \hline \hat{\mathbf{C}}_D & 0_{m \times (n-n_c)} & \hat{\mathbf{D}}_D \end{array}\right] \tag{5.90}$$

where the notation $*$, $**$ are any submatrices, and $\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, and $\hat{\mathbf{D}}_D$ are the reduced-order decentralized controller matrices. Note that the n- dimensional controller defined by $\hat{\mathbf{K}}_D$ of (5.90) is equivalent to the n_c- dimensional decentralized controller described by state-space representation of ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) if the controller and observable parts are extracted.

Next, define the matrix function $\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}, \lambda)$ (which is similar to (5.85)) as

$$\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}, \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda \hat{\mathbf{K}}_D,\ \tilde{\mathbf{P}}) \prec \mathbf{0} \tag{5.91}$$

Then, one can apply the algorithm proposed in the previous section with $\mathbf{K}_F$ of n-dimensional centralized H_∞ controller. In this case, at $\lambda = 0$ set the matrix $\hat{\mathbf{K}}_D$ to zero except $(2, 2)$ block $-\mathbf{I}_{(n-n_c)}$ and proceed with computing $\hat{\mathbf{K}}_{D_k}$ for each λ_k. If the algorithm succeeds, then the matrices ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) extracted from the obtained $\hat{\mathbf{K}}_D$ at $\lambda = 1$, comprise the desired decentralized H_∞ controller.

Remark 5.11: The approach outlined in the previous section considers the problem of designing decentralized controllers for the full-order system. It could also possible first to reduce the order of the system by applying standard model order reduction techniques, and then designing a decentralized controller for the reduced system. However, the controller designed for the reduced-order system may not perform well for the original system as expected.

5.5.4 Simulation Results

The test power system, which has been considered in Figure 5.3 (see also Figure 3.3 of Chapter 3), is considered again to demonstrate the applicability of the proposed approach. After including the washout filter in the system, the design problem is reformulated as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the continuation parameter monotonically varies. The speed of each generator and the voltage error signal which is the input to the regulator of the exciter are used as regulated signals within the framework of the design. Moreover, the output of the washout block, i.e., measured output signal, is used as an input signal for the PSS in the system. For a short circuit of 150ms duration at node F in Area-A, the transient responses of generator G2 with and without the PSSs in the system are shown in Figure 5.9. This generator which is the most disturbed generator in the system due to its relative nearness to the fault location shows a good damping behaviour after the PSS included in the system. The designed PSS for each generator are also given in Table 5.5.

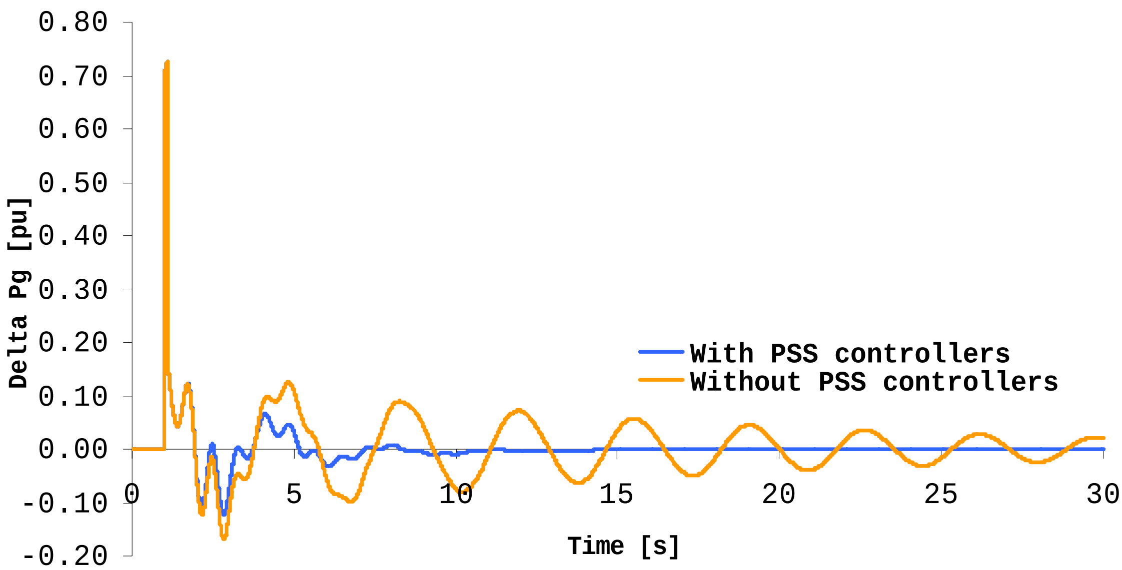


FIGURE 5.9 The transient responses of Generator G2 to a short circuit at node F.

TABLE 5.5 The robust decentralized controllers for the test system.

Generator G_i	Designed Controllers
1	$\frac{s^2+8.294s+20.122}{s^2+14.724s+29.431}$
2	$\frac{s^2+19.610s+21.010}{s^2+37.050s+22.344}$
3	$\frac{s^2+17.704s+22.120}{s^2+37.611s+17.862}$
4	$\frac{s^2+6.211s+6.508}{s^2+10.070s+11.832}$

To further assess the effectiveness of the proposed approach regarding robustness, the transient performance indices were computed for different loading conditions at node 1 [P_{L1}, Q_{L1}] and node 2 [P_{L2}, Q_{L2}] while keeping constant total load in the system. The normalized transient performance indices for different loading conditions are shown in Figure 5.9. It can be seen from Figure 5.10 that the indices for $I_N(E_{fd})$, $I_N(P_g)$ and $I_N(V_t)$ are either near to unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behaviour exhibits robustness for all loading conditions.

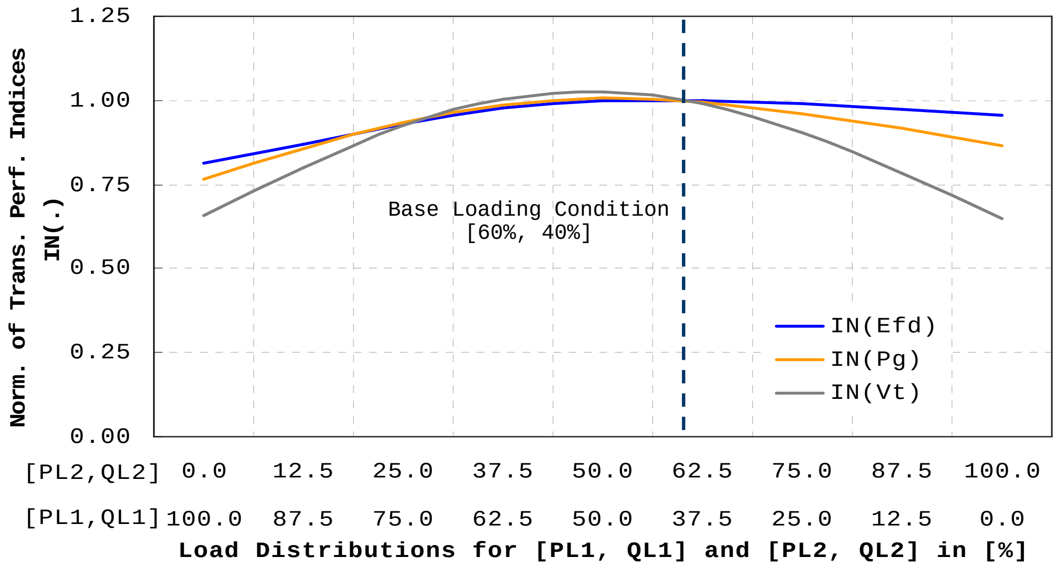


FIGURE 5.10 Plot of the normalized transient performance indices.

5.6 Summary

The central objective of this dissertation is to develop robust decentralized controllers design approaches for power systems with special emphasis on problems that can be expressed in terms of minimizing a linear objective function under LMI constraints in tandem with BMI constraints. Besides, developing the computational algorithms to solve such design problems, simulation results have also been carried-out to show the effectiveness of these proposed approaches for designing realistic robust power system stabilizers (PSSs). The followings are summaries of this chapter:

a) In section 5.2, a robust decentralized static-output feedback controller design based on LMI techniques for a power system that is described using the interconnection modelling concept, was presented. A decomposition procedure using the clustering technique of the states, inputs and outputs structure information has been incorporated in the design so as to compute directly (within the LMI optimization framework) the appropriate diagonal structures of the controller matrices. Issues such as the size and structure of the gain matrices and the robust stability degree have also been included in the design while at the same time guaranteeing the desired tolerable bounds in the uncertainties due to structural changes and nonlinearities in the system.

b) In Section 5.3, the problem of designing a reduced-order robust decentralized output dynamic feedback controller for a power system was reformulated as a nonconvex optimization problem involving LMIs and coupling BMI constraints. In the design, the robust connective stability of the overall system is guaranteed while the upper bounds of the uncertainties arising from the interconnected subsystems as well as the nonlinearities within each subsystem are maximized. A sequential LMI programming method was presented to solve such optimization problems for determining (sub)-optimal robust decentralized controllers for the test power system.

c) In Section 5.4, the H_2/H_∞ optimization algorithm for a structure-constrained linear controller was presented. The proposed approach consists of two steps: i) “Extraction" of controller parameters from the closed-loop system as a full constant block using system augmentation method, and ii) then synthesising the decentralized H_2/H_∞ static-output feedback controllers. Such decentralized H_2/H_∞ static-output feedback controller synthesis problem is generally a nonconvex optimization and solved using the sequential LMI programming method. This approach can be applied to almost any structure-constrained linear controllers provided that the controller parameters extracted properly as a full constant block.

d) In Section 5.5, the problem of designing a reduced-order decentralized H_∞ dynamic output feedback controller for power system was considered. In the design, initially a centralized H_∞ robust controller, which guarantees the robust stability of the overall system against unstructured and norm bounded uncertainties, is designed using the standard methods. Then the problem of designing a decentralized controller for the system was reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized controller to the desired decentralized controller as the continuation parameter monotonically varies. An algorithm using two-stage iterative matrix inequality optimization method is used to solve such design problem. Moreover, extending the approach to design reduced-order decentralized controllers, which have practical benefits since high-order controllers when implemented in real-time configurations can create undesirable effects such as time delays, was also treated properly.

Moreover, the nonlinear simulation results from these proposed approaches have confirmed the robustness of the system for all envisaged operating conditions and disturbances. The proposed approaches offer a practical tool for engineers, besides designing reduced-order PSS, to re-tune PSS parameters for improving the dynamic performance of the overall system.

Chapter 6

Summary and Future Work

6.1 Summary

This dissertation has studied an extension of robust decentralized control techniques for power systems, with special emphasis on design problems that can be expressed as minimizing a linear objective function under linear matrix inequality (LMI) constraints in tandem with nonlinear matrix inequality (NMI) constraints. The NMI constraints including the bilinear matrix inequality (BMI) constraints render a computational challenge in the designing decentralized and/or reduced-order controllers and should be properly taken into account so that the solutions can be numerically computed in a reliable and efficient manner. Alternative computational schemes, that can be used to solve the (sub)-optimal robust decentralized controller problems for power systems, have been proposed in this dissertation. These include: i) bordered-block diagonal (BBD) decomposition algorithm for designing LMI based robust decentralized static output feedback controllers, ii) sequential linear matrix inequality programming method for designing robust decentralized dynamic output feedback controllers, and, iii) generalized parameter continuation method involving matrix inequalities for designing reduced-order decentralized dynamic output feedback controllers. These algorithms are found to be computational efficient and can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers. The local convergence properties of these algorithms for designing (sub)-optimal robust decentralized controllers have shown the effectiveness of the proposed approaches. Moreover, the designed controllers when implemented in the test system have proven to work properly for all envisaged operation conditions and disturbances.

Specific contributions of this dissertation are on: 1) robust decentralized controller design for power systems using convex optimization involving LMIs, 2) robust decentralized dynamic output feedback controller design for power systems using an LMI approach, 3) robust decentralized structure-constrained controller design for power systems via LMI based approach, and 4) decentralized H_∞ controller design for power systems using general parameterized optimization method. The contributions are summarized below.

Decentralized Static-Output Feedback Controllers Design for Power Systems Using Interconnection Modelling Approach

This research deals with the application of robust decentralized controller design for power systems using linear matrix inequality (LMI) techniques. In the design, the desired stability of the system is guaranteed while at the same time the tolerable bounds in the uncertainties due to structural changes, nonlinearities and load variations, are maximized. The approach allows the inclusion of additional design constraints such as the size and structure of the gain matrices. The research also presents a decomposition algorithm using the clustering technique of the states, inputs and outputs structure information to compute directly the appropriate diagonal structures of the output gain matrix for practical implementation. Based on the proposed algorithm, this research demonstrates the effectiveness of the approach by designing power system stabilizers (PSSs) for a test system that is modelled as an interconnected power system.

Decentralized Dynamic-Output Feedback Controller Design for Power Systems Using Interconnection Modelling Approach

This research presents an LMI based robust decentralized dynamic output feedback controller design for interconnected power systems. The problem of designing fixed-order robust decentralized output dynamic feedback controllers, which act on the subsystem level with partial information of the state vector of the system, is formulated as a nonconvex optimization problem involving linear matrix inequalities (LMIs) coupled through bilinear matrix equation (BME). In the design, the robust connective stability of the overall system is guaranteed while the upper bounds of the uncertainties arising from the interconnected subsystems as well as the nonlinearities within each subsystem are maximized. The research also proposes a new approach to solve such optimization problems using a sequential LMI programming method to determine (sub)-optimally the decentralized output dynamic feedback controllers of the system. Moreover, the approach is flexible enough to allow the inclusion of additional design constraints such as the stability degree of the whole system and different dynamic orders of the controllers for each subsystem while at the same time maximizing the tolerable upper bounds on the class of perturbations. The approach is demonstrated by designing power system stabilizers (PSSs) for a test power system.

Robust Decentralized Structure-Constrained H_2/H_∞ Dynamic Output Feedback Controller Design for Power Systems

This research presents a linear matrix inequality (LMI)-based approach for designing a robust decentralized structure-constrained controller for power systems. The problem of designing a fixed-structure H_2/H_∞ dynamic output feedback controller is first reformulated as an extension of a static output feedback controller design problem for the extended system. The resulting optimization problem has bilinear matrix inequalities (BMIs) form which is solved using sequentially LMI programming method. The effectiveness of the proposed approach is demonstrated by designing (sub)-optimal fixed-structure power system stabilizers (PSSs) controllers for a test power system so as to determine the optimal parameters.

Robust Decentralized H_∞ Controller Design for Power Systems Using Parameter Continuation Method

This research presents a decentralized dynamic output feedback controllers design for power systems. The problem of designing decentralized robust dynamic output feedback controllers, which act on the subsystem level with partial information of the state vector of the system, is formulated as general parameterized optimization method involving a two-stage linear matrix inequalities (LMIs) optimization. In the design problem, initially a centralized H_∞ robust controller is designed for the system. This designed centralized controller is sufficient to guarantee the robust stability of the overall system against the unstructured and norm bounded uncertainties in the system. To design the decentralized controller for the system, we reformulate the problem as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the design parameter varies to its range space. The research also proposes a new approach to solve such optimization problems using a two-stage iterative LMI optimization method to determine optimally the decentralized output dynamic feedback controllers of the system. Moreover, the approach is flexible enough to allow the inclusion of additional design constraints such as the stability degree of the whole system and different and/or a combination of any order dynamic orders of the controllers for each subsystem while at the same time ensuring or meeting the robustness condition of the original centralized controller. The effectiveness of the approach is demonstrated by designing power system stabilizers (PSSs) for a test power system.

6.2 Future Work

Problem of robust decentralized controllers design for large power system has been an active research area for the last two or three decades. The ultimate goal of this subject is to develop decentralized controller design framework together with the necessary computational algorithm that will enable both power/control engineers to include and/or capture practical performance objectives and design constraints. There still remain unsolved problems in robust optimal decentralized controllers design in spite of intense research for decades. As discussed in this dissertation, additional constraints such as fixed-order controllers or controllers with block diagonal and/or other specified structure make the design problem a nonconvex optimization problem.

The design formulations presented in this dissertation can serve as a tool with prospect of extending them to other related control design problems in power systems. Although alternative efficient algorithms have been proposed to solve optimization problems involving matrix inequalities, none of these algorithms are sufficiently efficient to solve problems of practical large size. The importance of solving optimization problems involving LMI in tandem with NMI constraints is, however, clear for the extension of robust control theories for large system. The following are suggestions for future research directions:

1. ***Local search approaches.*** This dissertation proposed algorithms to solve locally problems involving linear objective functional under linear matrix inequality (LMI) constraints in tandem with nonlinear matrix inequality (NMI) constraints. Due to the NP-hardness of nonlinear matrix inequality (NMI) problems, local search approaches seem more promising for practical applications. The algorithm proposed in this dissertation is computationally intensive and might also be slow

when the problem size is large. More efficient and reliable algorithms must be developed to apply NMI - based approaches to practical large control problems in power systems.

2. ***Specific nature of NMI problems.*** This dissertation proposed an algorithm to solve a specific type of NMI problem arising form synthesis of H_2/H_∞ optimization decentralized structure-constrained controller. However, the proposed approach is restricted to this specific problem and could not be applied directly to general structured type controllers. Future research should focus on the nature of each specific problem to develop more efficient and reliable global/local search algorithms.

3. ***Application to large-scale problems.*** Numerical optimization approaches offers more advantages when applied to large-scale power system problems. However, the algorithms discussed in this dissertation are sufficient for moderate size problems. In particular, further research effort must be devoted to the development of efficient algorithms for large SDP problems arising in power systems. Efficient algorithms to solve sparse SDP problems are also of important.

Appendix A

Explanation of Symbols and Mathematical Relations

The purpose of this and the following appendices is to provide a list of mathematical definitions, notations, relations and results that are used in this dissertation.

Matrices and Vector Spaces

Let $\mathbf{A}$, $\mathbf{B}$ be real $n \times m$, and $\mathbf{M}$, $\mathbf{N}$ be real $n \times n$ matrices.

$\Re^{n\times n}$	: equipped with the usual standard norm.
$Rank(\mathbf{A})$	: The rank of $\mathbf{A}$.
$\mathbf{A}^T$	: The transpose of $\mathbf{A}$.
$\mathbf{S}$	: The set $\{\mathbf{S}\in\Re^{n\times n} \mid \mathbf{S}=\mathbf{S}^T\}$ of real symmetric $n\times n$ matrices.
$\mathbf{M} \succeq \mathbf{N}$	: $\mathbf{M}$, $\mathbf{N}$ symmetric matrices, $\mathbf{M} - \mathbf{N}$ positive semidefinite.
$\mathbf{M} \succ \mathbf{N}$	: $\mathbf{M}$, $\mathbf{N}$ symmetric matrices, $\mathbf{M} - \mathbf{N}$ positive definite.
$\mathbf{A}^+$	: The Moore-Penrose inverse of $\mathbf{A}$.
$\mathbf{I}_n$	: The $n\times n$ identity matrix.
$\mathbf{M}$ stable	: The real part of the eigenvalues of $\mathbf{M}$ are negative.

Recall the explicit formula for $\mathbf{A}^+$ via the singular value decomposition and the obvious consequences $\mathbf{A} \in S \Rightarrow \mathbf{A}^+\mathbf{A} = \mathbf{A}\mathbf{A}^+$ as well as $\mathbf{A} \succeq 0 \Rightarrow \mathbf{A}^+ \succeq 0$.

Partitions in matrices are only sometimes indicated and one should think of a matrix to carry a partition which is inherited e.g. to a product. Blocks of no interest are denoted by $*$.

If $\mathbf{A}_i$, $i = 1, 2, \cdots, N$ are square matrices, we denote the corresponding block diagonal matrix as $\mathrm{diag}\{\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_N\}$ with $\{\mathbf{A}_1, \mathbf{A}_2, \cdots, \mathbf{A}_N\}$ on the diagonal and zero blocks elsewhere.

LEMMA A.1: (Matrix Inversion lemma) Let $\mathbf{A}$ and $\mathbf{D}$ be square and non-singular matrices and $\mathbf{B}$ and $\mathbf{C}$ be matrices with appropriate dimensions. Then, if all the inverses below exist,

$$(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})^{-1} = \mathbf{A}^{-1} + \mathbf{A}^{-1}\mathbf{B}(\mathbf{D}-\mathbf{C}\mathbf{A}^{-1}\mathbf{B})^{-1}\mathbf{C}\mathbf{A}^{-1} \tag{A.1}$$

Remark: The expression in (A.1) can be verified by multiplying the right-hand side by $(\mathbf{A}-\mathbf{B}\mathbf{D}^{-1}\mathbf{C})$ and showing that the product is the identity matrix.

Acronyms

BME	: Bilinear Matrix Equations.
BMI	: Bilinear Matrix Inequality.
LMI	: Linear Matrix Inequality.
LPV	: Linear Parameter Varying.
LTI	: Linear Time Invariant.
MIMO	: Multi Input Multi Output.
NMI	: Nonlinear Matrix Inequality.
SDO	: Semidefinite Optimization
SDP	: Semidefinite Programming.
SSV	: Structured Singular Value.

Appendix B

Nonlinear Based Optimization for Tuning Power System Controllers

The problem of optimal controllers tuning could be interpreted as a class of nonlinear based optimization problem for power system dynamics analysis [63] and [64]. The objective of the nonlinear based optimization for tuning power system controllers is to force the system to have a post-disturbance stable operating point as well as a better damping behaviour as quickly as possible. Most of the problems can be reformulated as optimization problems of the form

$$\begin{aligned} &\min_{\boldsymbol{\theta}, t_f} J(\mathbf{x}(t,\boldsymbol{\theta}), t_f) \\ &\text{subject to} \\ &\quad \dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \boldsymbol{\theta}) \\ &\quad \mathbf{x}(t_0, \boldsymbol{\theta}) = \mathbf{x}_0(\boldsymbol{\theta}) \end{aligned} \tag{B.1}$$

where

$$J(\mathbf{x}(t,\boldsymbol{\theta}), t_f) = \phi(\mathbf{x}(t_f, \boldsymbol{\theta}), \boldsymbol{\theta}) + \int_{t_0}^{t_f} \ell(t, \mathbf{x}(t,\boldsymbol{\theta}), \boldsymbol{\theta}) dt \tag{B.2}$$

$\boldsymbol{\theta}$ are the decision variables (i.e., the design parameters of the controllers to be tuned), $\mathbf{x}(t)$ are state variables and t_f is the final time.

A typical conflicting requirement of improved damping behaviour without voltage degradation can be achieved minimizing the following objective function of the form:

$$\min_{\boldsymbol{\theta}} \sum_{i=1}^{N} \{ w_{1i} \int_{t_0}^{t_f} (P_{gi} - P_{gi}^0)^2 \, dt + w_{2i} \int_{t_0}^{t_f} (V_{ti} - V_{ti}^0)^2 dt + w_{3i} \int_{t_0}^{t_f} (e_{fdi} - e_{fdi}^0)^2 \, dt \} \tag{B.3}$$

where

w_{1i}, w_{2i} and w_{3i} are positive weighting factors for P_{gi}, V_{ti} and e_{fdi}, respectively.
P_{gi} the generator real power for the *i*th - generator.
V_{ti} the generator terminal voltage for the *i*th - generator.
e_{fdi} the exciter field voltage for the *i*th - generator.
$\boldsymbol{\theta}$ are parameters like gains and time constants for the controllers.

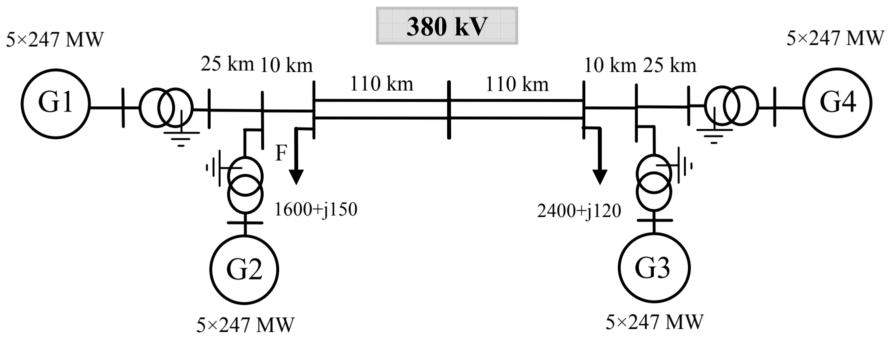
The optimization problem in (B.3) can be solved using gradient-based methods from IMSL Math/Library [65] or other standard optimization software. However, due to the nonlinear nature of the problem, the solutions of the optimization problem depend on the initial values of the controller parameters, fault locations and fault durations in the system.

Appendix C

Four Machine Two-Area Test System Data

The single line diagram, bus data, line, generator, excitation and governor data are given in this appendix.

C.1 Four Machine Two-Area Test System



C.2 Synchronous Machines

Parameters:

S_r/MVA	247	x'_q/p.u	-
U_r/kV	15.75	x'_q/p.u	-
T_m/s	7.0	x''_q/p.u	0.24
r_s/s	0.002	T'_d/s	0.93
$x_{\sigma s}$/p.u	0.19	T''_d/s	0.11
x_d/p.u	2.49	T'_q/s	-
x_q/p.u	2.49	T''_q/s	0.2
x'_d/p.u	0.36	$x_{\sigma fDd}$/p.u	-
x''_d/p.u	0.24		

C.3 Transmission Lines Data

380 kV Lines

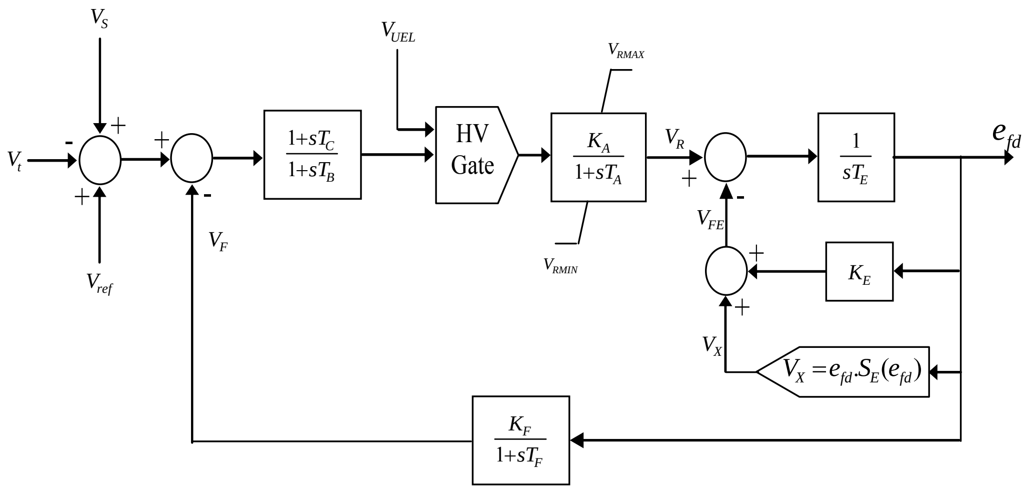
Single Lines:	$\underline{Z}_{11} = 0.0309 + j0.266\,\Omega/\text{km}$	$C_b = 0.0136\,\mu\text{F/km}$
Double Lines:	$\underline{Z}_{11} = 0.0155 + j0.1358\,\Omega/\text{km}$	$C_b = 0.0267\,\mu\text{F/km}$

C.4 Two Winding Transformers Data

Parameters

S_r/MVA	235
r_{ps} [%]	0.246
z_{ps} [%]	14.203

C.5 IEEE DC1A Type Exciter

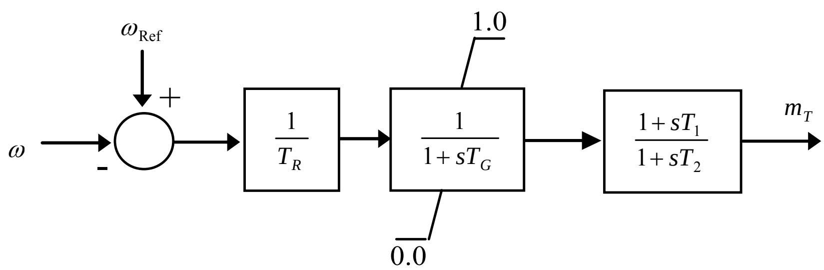


Parameters

T_C	0.173 s	B_{EX}	1.55
T_B	0.06 s	K_F	0.05
K_A	187	T_F	0.62
T_A	0.89 s	V_{RMAX}	1.7
T_E	1.15 s	V_{RMIN}	-1.7
A_{EX}	0.014		

K_E is computed so that initially $V_R = 0$.

C.6 Thermal Governor



Parameters

T_R	0.167 s	T_1	1.0 s
T_G	0.25 s	T_2	0.9 s

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List of Publications

[1] G. K. Befekadu and I. Erlich, “Robust Decentralized Controller Design for Power Systems Using Matrix Inequalities Approaches,” Presented to the 2006 IEEE Power Engineering Society General Meeting, Conference Proceedings, Montreal, QC, Canada, June 18-22, 2006.

[2] G. K. Befekadu and I. Erlich, “Robust Decentralized H_∞ Controller Design for Power Systems: A matrix Inequality Approach Using Parameter Continuation Method,” Presented to the IFAC Symposium on Power Plants and Power Systems Control 2006, Conference Proceedings, Kananaski, Canada, June 25-28, 2006.

[3] G. K. Befekadu, O. Govorun and I. Erlich, “Multi-Objective Optimization and Online Adaptation Methods for Robust Tuning of PSS Parameters,” Accepted to the 2006 Modern Electric Power Systems MEPS06, Wroclaw, Poland, September 6-8, 2006.

[4] G. K. Befekadu and I. Erlich, “Robust Decentralized Structure-Constrained Controller Design for Power Systems: An LMI Approach,” Presented at the PSCC 2005, CD-ROM Proceedings, Liege, Belgium, 2005.

[5] G. K. Befekadu and I. Erlich, “Robust Decentralized Controller Design for Power Systems using Convex Optimization involving LMIs,” Presented at the 16th IFAC World Congress, CD-ROM Proceedings, Prague, Czech Republic, 2005.

[6] G. K. Befekadu, “Robust Decentralized Controller Design for Power Systems: An LMI Approach,” Presented at the Dresden Kreis 2004, Dresden, Germany, March 28-29, 2004.

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Employment History:

2003 – To date	As a research co-worker at the Institute of Electrical Power Engineering, University of Duisburg - Essen, Germany.
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2003 - 2006	Ph. D. Degree in Electrical Engineering, University of Duisburg-Essen, Germany, Dissertation: “Robust Decentralized Control of Power Systems: A Matrix Inequalities Approach.”
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