World views of mathematics held by university teachers of mathematics science

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The present empirical study deals with the question of the (world) view of mathematics (i.e. the image of mathematics) held by university mathematics teachers in countries of German as a first language. The basis of this study is a voluntary survey by means of a closed questionnaire of 119 persons during an annual meeting of mathematicians. This questionnaire was on the whole employed for two other studies on mathematics teachers (N= 300) and pupils (N=1650).

Four to five dimensions were defined by means of factor analysis and subsequently verified as relevant dimensions of the view of mathematics. These dimensions can be called the formalism aspect, the schema aspect, the process aspect, the application aspect and the Platonism aspect of mathematics. Attitudes towards these aspects differ on average, so that the "average" view of mathematics of university teachers is clearly accentuated in these five aspects. The process aspect acclaims the highest agreement, whereby the aspects formalism and application claim an average to above average assessment. In contrast, the Platonic aspect receives only weak to very weak agreement, and the schema aspect is on the whole rejected.

Furthermore the structure of the view of mathematics resulting out of the relations between the dimensions is investigated. The part of the view of mathematics that is considered here contains two different viewpoints in content, namely the static view of mathematics as a system and the dynamic view of mathematics as a process. In contrast to the observations in the other two populations the two viewpoints are not opposites. Mathematics is in the view of university teachers a complementary togetherness of both viewpoints. Their view of mathematics is insofar broader differentiated than the two other populations.

Which world view do mathematicians have of mathematics ?

"The working mathematician is a Platonist on weekdays, a formalist on weekends. On weekdays, when doing mathematics, he's a Platonist, convinced he's dealing with an objective reality whose properties he's trying to determine. On weekends, if challenged to give a philosophical account of the reality, it's easiest to pretend he doesn't believe it. He plays formalist, and pretends mathematics is a meaningless game." (Hersh, p. 39)

"The devastating effect of formalism on teaching has been described by others. [...] I haven't seen the effect of Platonism on teaching described in print." (Hersh, p. 238)

We do not know whether Reuben Hersh’s claim is right, that the mathematician is on week-days a Platonist and on the weekend a formalist. However, Hersh rightly points to the fact that mathematicians possess different "philosophies", i.e. subjective theories, belief and attitude systems on what mathematics is, and that one person can possess a number of "philosophies" alongside each other.

In his second statement Hersh points out that these "philosophies " are not neutral, but (can) have differing effects, e.g. on teaching mathematics, up to even "devestating" effects. Hersh thereby addresses one of the motives which is important to us, namely the question of research into subjective philosophies of what mathematics is to mathematicians:

1. Individual "philosophies", subjective theories, attitudes or opinions of university teachers on mathematics are possibly a significant influence factor on their research and teaching activities. Thus university teachers by virtue of their view of mathematicss possibly influence the views of mathematics students and thus later mathematics teachers as the multiplicators of mathematics and view of mathematics in schools.
Which view of mathematics are mathematics teachers familiarized with during their university course?

2. One can advocate the thesis that subjective theories originate in the process of adaptation of individuals towards their environment. Subjective theories of university teachers of mathematics insofar reflect their environment conditions at university, i.e. researching and teaching conditions.
Which conditions are mathematics teachers confronted with in research and teaching at universities and in science?

3. The view of mathematics of mathematics teachers at universities shows which view of "mathematics science" (i.e. of academic mathematics) is presently dominant amongst scientists, i.e. experts of mathematics. This view is an important orientation point in the discussion on the question of which view of mathematics should be imparted in schools.
Which view of mathematics does "science" posses?

These three motives reveal that interest in mathematics-related conceptions, beliefs, attitudes or subjective theories is related to their *origin* and above all to their *effect*:

Concerning the *effect*, individual attitudes towards mathematics (mathematics lessons, mathematical studies,...) constitute a relevant influence factor for using, learning, teaching and doing research in mathematics, in general for mathematics activities. These attitudes describe, even when they are unconscious, the context in which people view and do mathematics. They have an influence on the way one approaches mathematics (tasks, problems). Subjective attitudes towards mathematics influence

- how people use mathematics,
- how pupils and students learn mathematics,
- how teachers teach and learn mathematics,
- how university teachers teach mathematics and do research.

Furthermore, the *origin* of attitudes is revealing. The view of mathematics expressed in the attitudes of people is a precise reflection of their mathematical environment (i.e. their perception of the environment). Attitudes are acquired in learning processes increasingly influenced by environment conditions - and are equally adaptation instruments of individuals towards their environment. Attitudes, subjective theories, views, "Philosophies" can be viewed as strategies of individuals to optimally and economically come to terms with situational demands in school or lessons, in teaching or research.

Subjective attitudes towards mathematics can thus be seen as indicators for

- which mathematics lessons pupils and students experience,
- which teaching experiences teachers make,
- which conditions university mathematicians experience in research and teaching.

Concerning the relation between the influence of the mathematical environment on attitudes ("origin") and the influence of attitudes on doing mathematics ("effect"), we are dealing here only with hypotheses. Even though they are central for the research of mathematics-related attitudes, they have no yet been put to the test.

1. Theoretical framework and question formulation of the study

1.1. Theoretical framework of the "mathematical world view"

1. General framework of "attitude"

The basic theoretical term is "attitude". An attitude is a stable (i.e. temporally long-term sustaining) not necessarily conscious orientation and (latent) activity disposition (or intention) of an individual in relation to a social object (here: mathematics). "Orientation" denotes, in the contemporary cognitivist sense, a consistency in perception, cognitive representation and affective assessment. The feature of consistency is relevant to us. We are not so interested in spontaneous, unique or inconsistent cognitions, affections and behavior dispositions. We are more interested in patterns in cognitions, affections and behavior dispositions as, on the one hand, one can assume that such patterns have effects on learning behavior, teaching or doing mathematics, and on the other hand we can employ them for explaining learning and teaching behavior.

The answer on a questionnaire record initially and wholly only a belief (i.e. a cognitive component), which can be described as subjective knowledge of an individual. We believe that cognitions are closely related to affections, i.e. to affective meaning attributions, assessments, judgements and feelings. A number of empirical findings point to the close relation between cognitions and affections. We furthermore also believe that cognitions and affections possess a relation to activity dispositions (i.e. intentions), that is to activity schemata fitting to situations in which they have proven themselves best - in the way situations are perceived, categorized and assessed.

On the whole we advocate here an attitude concept close to the classical Three-components-approach. In this Three-components-approach of attitude theory attitudes are a system of cognition, affection and conation (i.e. willingness to act), assuming a principle tendency to agreement. We accept this Three-components-approach as an ideal-typical starting point of research, as it expresses that cognition and affection as a pair, as well as cognition, affection and conation are interwoven. We point out, however, that these relations do not always have to be 1:1 relations but can be principally only tendencies. The Three-components-approach and in particular the theorem of consistency between cognition, affection and conation is therefore to us not secured knowledge - it has moreover a heuristic function for considering relations between cognition, affection and conation schemata in research. For our purpose the terms "attitude" and "Three-components-approach" present an adequate theoretical framework; they take these three relations into consideration, even should they only be weak.

Conative components of attitudes are integrated into plans of activity. In the past a model of simple consistency between attitude and behavior has often been advocated. In this model it is assumed that behavior in a specific situation is exclusively determined by individually activated attitudes, that attitude and behavior are in agreement. We do not support this simple consistency model here as it does not model reality sufficiently enough. A number of empirical studies point out that the relation between attitude and behavior is at its best weak. We propagate the model of conditional consistency between attitude and behavior:

Situational behavior depends not only on one attitude (and its conative component), but on the interdependency of a number of factors:

- on a number of attitudes (and their structure properties),
- on factors related to the individual (e.g. self-awareness or self-control, individual properties of attitudes such as degree of centrality, awareness and generality of an attitude),
- on situation-related factors (resp. perception of situation) (e.g. persons of reference, routine adherence, social norms).

In this model of conditional consistency an attitude has, as do all factors, a direct influence on conation planning. It equally has an intervening effect on the influence of other variables, as these, in turn, can also have an intervening effect on an attitude-behavior relation. An attitude is conatively relevant under the condition that it corresponds to other attitudes (activated in the situation) and to the factors related to the person and the situation.

2. Attitude structure

We can assume that several attitudes (resp. attitude dimensions) possess relations amongst each other, so that an attitude structure develops. It is very important for our approach to research these attitude structures.

We believe that a single attitude does not reveal much about an individual; in particular an individual attitude may be less relevant for conation. In contrast, a complete "picture" (i.e. a number of attitudes and their structure) reveals more about the nature of this "picture", about its effects (on conation) and about the influencability of attitudes (comp. also the next section).

3. Mathematical world view

Mathematics as a world of experience and activity can be assumed to be a very complex field. Accordingly, one can assume that attitudes towards mathematics are very complex and multi-dimensional. There is certainly not only a positive and a negative attitude towards mathematics; one must assume that there are manifold differentiations.

We define here the system of all mathematics-related attitudes (i.e. single attitudes and their structure) as the "mathematical world view". This mathematical world view is an attitude structure encompassing a wide spectrum of cognitions, affections and conations towards mathematics. It can be roughly divided into four components: Attitude structures on

(1) mathematics ("view of mathematics"),
(2) learning mathematics,
(3) teaching mathematics,

(4) oneself (and others) as a learner and user of mathematics (self-concept)
(e.g. self-assessment of abilities, causal attribution on success - failure).

The questionnaire employed in this study contains almost only questions on the view of mathematics. This partial structure in turn comprises a wide spectrum of attitudes (or attitude structures) and contains at least the following components: attitudes or attitude structures on (1a) the nature of mathematics as such, as well as (1b) school mathematics, in particular university mathematics, (1c) on the nature of mathematical tasks and problems, (1d) on the origin of mathematical knowledge and (1e) on the relation between mathematics and empiricism (in particular on application value and practical use of mathematics).

Our research object is mathematical world views and not single attitudes (or beliefs), because
(1) there is a whole spectrum of attitudes towards mathematics (see above) which possibly influence each other, and because
(2) the structure of attitudes is possibly more relevant than single attitudes
- for nature: the nature of the mathematical world view or even single attitudes are possibly essentially defined by the relation of attitudes to each other.
- for conation relevance: attitude structures can possibly explain activities better than single attitudes.
- for change: ability to influence attitudes possibly depends essentially on how many and how strong relations are embedded into a network of attitudes.

1.2. Content approach of the study: antagonistic basic images of mathematics

For understanding the study approach and the concept of the questionnaire it is necessary to be familiar with our content approach. It definitely presents part of our own world view of mathematics.

The basic images of mathematics and thus the superordinated basic beliefs ("philosophies") for teaching mathematics in schools and universities are the two fundamental positions that mathematics can be expressed in the static view as a system or in the dynamic view as a process, i.e. as an activity. Both viewpoints cannot be simply separated from each other. The process of mathematical activity leads to products in the system, and the system originates from the process and is also the starting point for further mathematical activity. One therefore speaks of the two different sides of mathematics. In a paradigmatic view, however, both views are to be considered antagonistic.

In the static view mathematics is understood as an (abstract) system, i.e. a(n) (abstract) structure defined above all in the ideal case by academic mathematics. From this viewpoint mathematical theory consists of axioms, terms and relations between these terms. This "complete theory" consists on the one hand of accumulated knowledge, i.e. terms, rules, formulae and algorithms. On the other hand it consists of fixation rules of this theory, and thus of the (unspoken) agreement of exactness concerning definitions of concepts and language, of the strictly deductive method on an exact axiomatic basis, and of the strictness of proofs. We will simply call these two partial aspects the schema aspect and the formalism aspect.

Both at school as well as at university dealing with this ideal type of "complete" mathematics consists of learning, executing and applying definitions, mathematical facts and procedures, and on the basis of formal logical deduction, the verification or falsification of hypotheses and the systematization of attained knowledge. Therefore, in accordance to this view (which also has "recipe" character) in mathematics lessons (also at university) knowledge from the system is imparted, and formal-logical deduction as well as formalization and abstraction abilities of the pupils are more stressed as compared to intuition and content modes of thought.

This contrasts with the view that mathematics is an activity of contemplating about problems, acquiring realizations and creating knowledge: Mathematics activity starts with questions and problems. When doing mathematics (at least in exemplary cases) contents are understood and relations are realized, experiences are made and principles are discovered, which are classified on various levels into mathematical theorems. Mathematical activity consists of discovering or rediscovering mathematics as well as classifying experience fields and activity principles.

In its origin and development of new knowledge, mathematics is primarily a research-, realization- and theory-process. Mathematics therefore has process character because mathematical theories develop in a (at times dialectical) process of assumptions, proofs and falsifications, moreover under the influence of persons. Logical, deductive thought in a science understood as systematic and static is necessary for testing hypotheses, but it does not contribute foremost to winning hypotheses. From a cognitive view mathematics is last but not least an experimental, inductive science which to a considerable degree makes use of assumptions, intuitions as well as plausible and analogue inferences.

For mathematics activities and lessons the terms "process", in particular "contemplation process" and "development" are stressed when creating, developing, understanding and doing mathematics. Discovering, i.e. rediscovering mathematics, has preference over teaching "complete" mathematics. As a consequence, content-related contemplation and argumentation, assuming and trying out as well as intuition are predominantly preferred to formal-logical deduction as well as formalization and abstraction abilities.

"Good mathematics is not only characterized by a lack of mistakes, but by the quality of the [inherent] ideas. [...] Developing ideas is not less important than their critical testing and [formal-logical} verification of mathematical statements. […] Also in the world of professional mathematicians one can see that experts are slowly beginning to relativize the importance of Bourbacism and are increasingly turning to a [...] more informal, intuitive and content view of mathematics[…]." (Zimmermann 1991, p.40).

1.3. Integration and question formulation of the study

In research of mathematics-related attitude- and belief-systems the following four important global question formulations can be recognized:

1. Identification of attitude systems
   (attitudes and their structure)
2. Origin and development of attitude systems
   (including influence factors, e.g., cultural, national and social factors, learning biography (e.g. influence of school and university teachers, contents and methods of the school), everyday life circumstances and work relations (school form of the teacher, job of the person))
3. Effect of attitude systems ("conation relevance")

(in relation to learning, teaching, researching and doing mathematics)

4. Change of attitude systems
   (factors which initiate or encourage change, conditions of change)

There exist an enormous number of empirical studies on attitude systems of pupils and teachers, whereby most studies are related to the question formulations 1. and 2. (comp. overview article of Thompson 1992, Pehkonen and Törner 1996, or http://www.math.uni-duisburg.de/projects/mavi). On the other side, as far as we are informed, there exist only three studies on mathematics teachers at universities, and they are related to the first question formulation.

In his study Pehkonen (1997) researches the beliefs mathematics professors have on the view of mathematics of teachers. Our research interest in this study is however focused on beliefs that mathematics professors have of mathematics. Pehkonen also conducts research into this question in his extensive research project but has not yet published any results.

Mura (1993, 1995) asked university mathematicians and university mathematics educators at Canadian universities the open question "How would you define mathematics ?". By means of content analyis, she draws the answers of 106 mathematicians into 12 categories (Mura 1993) and the answers of 51 university mathematics educators (teacher trainers) into 14 categories (Mura 1995). These categories can be viewed as dimensions structuring the thought of university teachers towards mathematics and as dimensions, in which they hold attitudes towards mathematics. Finding such dimensions is a *first* step in order to describe the view of mathematics of university teachers (first question formulation).

To describe the picture the following questions arise:

1. Which attitudes do mathematics teachers possess towards mathematics in these dimensions, i.e. in some relevant dimensions ? (Which nature has thought on mathematics in these dimensions?)
2. In which relationship do these dimensions stand to each other, i.e. how is the "view of mathematics" structured ?

These are exactly the two questions that we will research in this survey, whereby we will restrict ourselves to a few dimensions which have crystallized as relevant during our own pre-studies and also in surveys in other populations (pupils, teachers). It is our objective to put their relevance to the test.

2. Method of the study

During the annual meeting of the German Mathematician Union (DMV) in Duisburg in Fall 1994 a questionnaire on the view of mathematics was handed out to the participants; 119 questionnaires were filled out and handed back.

We employed a closed questionnaire (see appendix 1) to enable us to research the view of mathematics of as many university mathematics teachers as possible to receive more representative and generalizable results. This questionnaire has also proven itself in samples with other populations (pupils in Grigutsch 1996, teachers in Grigutsch, Raatz and Törner 1998). What speaks for using this questionnaire is that firstly representative and generalizable results are our objective, secondly the questionnaire possesses a comprehensively developed instrumentarium for rough global recording of mathematics-related cognitions and attitudes, thirdly it allows comparison of the results of mathematics teachers at universities with other populations, and fourthly it demands from the interviewees a relatively low effort.

Against the application of a closed questionnaire speaks, amongst other reasons, that mathematics teachers at universities probably possess a highly elaborated and differentiated view of mathematics which can only be undifferentiatedly (at its best roughly) recorded by using a closed questionnaire. The depth and complexity of such view of mathematicss can be certainly better and more adequately recorded in extensive interviews. We have conducted such interviews with seven professors; the results will be published in another paper.

We have nonetheless employed this questionnaire to record generalizable results of a rough, global view in a larger sample and to be able to compare these results with other samples.

The representativeness of the sample was not systematically secured; the representativeness is hereby related to university teachers at universities in countries with German as a first language; we did not record further differentiations within the sample in the questionnaire. The participation at the annual meeting of the DMV and the voluntary participation in the survey is on the one hand a positive selection, but on the other hand it has, in our opinion, not influenced the representativeness to a considerable degree; nothing speaks for the possibility that the mathematicians who participated in the DMV meeting have particular attitudes towards mathematics in the surveyed aspects. We therefore assume that the sample does not deviate considerably from a representative sample.

We have employed factor analysis to group the statements of the questionnaire for the following reasons:

1. The conceptual argument

The concept "attitude" is, amongst other things, characterized by a consistency of reactions on a class of similar stimuli. Even though a survey is not a real-life activity situation, when assuming an attitude and to record it empirically one has to at least demand the following: A set of statements with a similar content have to be answered in a similar fashion. One can only assume the existence of an attitude under consistent answer behavior on a class of statements. (It is not sufficient to desire to record an attitude from a single observation from the response towards a single particular item.)

2. The validity argument

A particular problem of validity of closed questionnaires lies in whether the interviewees understand the statements of the questionnaire, i.e. whether they have the same understanding of the statements as the authors. Thus when the questionnaire statements are viewed by the authors as being semantically similar and are answered by the interviewees in a similar manner, one can then assume that they understand the statements the same way as the authors.

3. The measurement-theoretical argument

Attitudes towards mathematics can have different dimensions. The question posed here is which dimensions are relevant for the perception and cognitive representation of mathematics for the interviewed persons. Sets of statements answered similarly point to a dimension relevant for perception and cognitive representation of mathematics. These dimensions are not given for the survey *a priori*, but are derived from the answer behavior *a posteriori*. However the result (the factor analysis), namely the number (dimensionality) and the content meaning of the dimensions are not intersubjective but depend on the individual mode of application of the factor analysis (mode of sample, selection of items, mode of extraction and rotation). With the factor analysis we can only deliver (and not test) plausible arguments to what extent (i) presented dimensions are reproduced by the interviewed persons, thus being dimensions, and to what extent (ii) the recorded dimensions for the interviewed persons are the most relevant and important ones for the interviewed persons concerning attitudes towards mathematics in answering the questionnaire.

All three tasks lead to recording sets of homogenous items perceived by the interviewed persons as being similar in content, i.e. being similarly answered. Employing factor analysis enables sets of items to be generated which are similarly answered; it then remains the task to examine whether these sets are homogenous or not. (On the other hand it is <u>not</u> the task to of the factor analysis to validly determine the dimensionality and the dimensions of thought on mathematics in this questionnaire.)

All calculated factor analyses are based on 89 variables. They address the items 1 to 77 (without 22 and 23) of the view of mathematics according to the questionnaire also presented to the teachers and pupils (in a simplified language form), whereby the items M1 to M14 are quotations from famous mathematicians. The items were scaled as follows: 5 = "wholly correct", 4 = "on the whole correct", 3 ="undecided", 2 = "only partly correct", 1 = "not at all correct".

The sample scope in the factor analysis is n = 90 (due to missing values; "listwise deletion"), in the further statistic evaluations between 103 and 117 persons - the corresponding case number is given to each statistic.

First a principal component analysis was calculated to determine the eigenvalues and to conduct the scree test (comp. appendix 2). The scree plot shows that even under generous interpretation a maximum of approximately 8 - 10 factors can be extracted; for further factors the influence of coincidence (or item difficulty) is very large. We made the decision to initially extract 8 factors. All 8 factors have the eigenvalue greater than 1. This preliminary restriction to 8 factors will prove to be permissible in the following because further analysis shows that only 4 to 5 factors are relevant and meaningfully interpretable.

By a quasi-experimental procedure factor analyses were calculated with 4, 5, 6, 7 and 8 factors. A principle axes analysis and a Varimax-rotation were executed. In the orthogonal solutions for each factor those items were determined which had a loading above 0.4, and the loadings on the other factors above 0.3 (side-loadings) were noted.

Result of the quasi-experimental procedure:
How many and which factors are meaningful and important?

We have based the factor selection i.e. the determination of the number of factors (dimensions) on 4 criteria:

1. Statements within a factor have to be homogenous in content and meaningfully interpretable.
2. Statements within a factor have to be formally homogenous , i.e. Cronbach's Alpha has to be sufficiently high.

3. A factor has to be meaningful in content, i.e. present a meaningful structuring of the item amount in content. First a factor must be a superordinated term for a sufficient number of items, i.e. there must be a sufficient number of items loaded high on the factor. Second there is to be no other factor with similar content.
4. A factor has to be formally meaningful, i.e. it must clarify a sufficiently large variance contribution (on the total communality, or on the total variance).

Additionally, a factor should be stable over all considered 4- to 8- factor solutions, i.e. it should be represented by a defined set of items in all solutions.

First we shall view the solutions according to their content criteria, as they are of greater significance to us than their formal criteria.

There are 4 factors consistently present in all solutions (4- to 8- factor solutions): the factors formalism (factor 1), schema (factor 2), process (factor 3) and application (factor 4) are stable in all solutions and are contained in a stable core of at least 6 items. These items load consistently above 0.4 and as a rule have no side-loadings (above 0.3). A sufficient number of items (6 to 11) load above all of these 4 factors, and these factors present an independent content dimension. Consequently, these 4 factors are relevant in content. The items in the factors are also homogenous in content.

These 4 factors will be evaluated in this study. Appendix 3 and 4 contain the selected items with their factor loadings.[1]

There are a further 3 factors in the solutions which are homogenous in content but not so relevant in content as the first 4 factors.

The factor "rigid schema orientation" is homogenous in content. The items 4, 11, 16 and 20 describe that it is more important in a mathematics university course to learn complete and systematized results and facts (particularly for examinations) than solution ideas and the finding of further question formulations as independently as possible. This factor is not as significant as the first 4 factors in content, because only 4 items load high on it, one of them with a high side-loading. Furthermore, this factor is inconsistent as it does not occur either in the 6-factor solution or in the 10-factor solution. Under these restrictions one can view it as not so significant in content. Due to the content argument we will ignore this factor as it is related to the view of mathematics as a university course. Together with the considered factors this aspect is blended out in this study, as we will concentrate on the view of mathematics itself.

The factor "sole solution path" is homogenous in content. The items 52, 55, 58 and 62 describe that there is only one sole solution for solving a mathematical task or a mathematical

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1 We find it necessary to make three comments to the selection of the items.
1. As a rule all items are selected for operationalization which load sufficiently high in all solutions on the factor.
2. The items 3, 8, 12 and 17 load high on the factor "formalism" in all solutions. We have decided not to draw on these for operationalizing the factor for the following reason: These items describe university teachers' views on the mathematics university course for students, whereas the other items (as does the whole analysis in this study) are related to the view of mathematics itself. (The results are insignificantly altered by this).
3. A number of items load on the factor "schema" which do not fulfill the first condition: They do load on the schema factor, but not in all solutions or not high enough (>0.4) or with high side loadings. This refers to the items 42, 45, 55 and 62 as well as 2, 7, 18 and 20. These items were not selected for operationalization of the factor "schema". The items 2, 7, 18 and 20 deal, once again, with the view of mathematics university courses and not with the view of mathematics in general. (The results are insignificantly altered by this).

problem. We consider this factor not relevant in content. First only 4 items load over 0.4 on this factor - two of the items with equally high (!) loadings on the schema-factor. Second this factor incorporates only a very limited content expression; it does not unite a number of relatively different items with a similar or identical expression. Therefore, this factor does not present a relevant content structuring of the item set. This factor is not considered in the study.

On the factor "Platonism" load 4 items on the 7-factor solution without side-loading, namely M1, M3, M6 and M12. They are moderately homogenous in content but can nonetheless be interpreted. They roughly describe mathematics as distant from application but as secure and aesthetic "divine games" (- an exact description is undertaken in the next chapter 3.1.). With four items this factor is not very relevant. Under this restriction we nonetheless want to support this factor because (i) under the less significant 3 factors with 4 items without side-loadings it is the strongest one, because (ii) it categorizes different items into a relatively homogenous group, and above all (iii) it offers a new content aspect which brings in a new dimension to the analysis.

A look at the variance contributions of the factors as an indicator for their formal relevance offers a further formal argument for the content-based decision to limit the analysis to 4 or 5 factors.

	variance contribution to the total communality (= 28,2)		variance contribution to the total variance (= 89)	
factor		cumulated		cumulated
1	6,04 = 21,4 %	21,4 %	6,04 = 6,8 %	6,8 %
2	5,53 = 19,6 %	41,0 %	5,53 = 6,2 %	13,0 %
3	4,11 = 14,5 %	55,5 %	4,11 = 4,6 %	17,6 %
4	4,25 = 15,1 %	70,6 %	4,25 = 4,8 %	22,4 %
5	2,78 = 9,8 %	80,4 %	2,78 = 3,1 %	25,5 %
6	2,73 = 9,7 %	90,1 %	2,73 = 3,1 %	28,6 %
7	2,81 = 9,9 %	100,0 %	2,81 = 3,1 %	31,7 %

In the 7-factor solution[2] the first 4 factors clarify approx. 15% to 20 % of the total communality, whereby the three further factors clarify less than 10%. The variance quota of the total variance of all 89 items of the first 4 factors is above or slightly under 5%, whereas the other factors contribute to only 3% to variance clarification. This - relatively viewed - clear gap between the first four and the other factors is another formal argument which additionally justifies the limitation to 4 factors due to content: The 4 factors are also formally (relatively) significant, whereas further factors are too small in their formal explanation value.

It is confirmed in the 4-factor solution that these 4 factors with a proportion of 5% of the total variance is formally sufficiently relevant (comp. appendix 4). We will go into detail on the explanation value of the 4- and 7-factor model in the next section 3.1. when we interpret the results of the factor analysis.

2 We give the 7-factor -solution here and in appendix 3 because it is less complex to the reader than the 8-, 9- or 10-factor-solution.

The formal homogeneity of the 4 resp. 5 selected item groups (factors) was tested with the aid of Cronbach's Alpha. The results are for formalism α = .82 (10 items, n = 116), for schema α = .74 (6 items, n = 112), for process α = .71 (10 Items, n = 117), for application α = .76 (9 items, n = 109) and for Platonism α = .51 (4 items, n = 107). If one assumes the usual threshold value of α = .70 then the first 4 item groups (factors) homogenous in content are also formally homogenous. The factor "Platonism", only moderately homogenous in content, is also formally only moderately homogenous, but in comparison to the small item number (4 items) it is however formally surprisingly homogenous.

Conclusion of the factor analysis

We have made the decision to analyze the four factors formalism, schema, process and application. They are stable across all factor solutions and contain a solid core of items. They are significant in content because they structure a high number (6-11) of relatively different items into independent dimensions. These dimensions are homogenous and interpretable in content, formally homogenous and also formally the most relevant in the factor analysis.

The further 3 factors are homogenous in content, but formally and in content not so relevant. This may lie in the fact that the questionnaire is not designed to record these factors with a larger number of homogenous items reliably. Thus these factors point to further aspects and dimensions beyond these 4 dimensions, and they give a hint in which direction the hypotheses on the dimensions of the view of mathematics refinement can occur and the questionnaire be improved. The factors "rigid schema" and "only solution path" will not be further evaluated here.

The factor "Platonism", however, will be given further consideration here. It is in content and formally the most relevant factor under the less relevant ones. Above all it represents a completely new dimension we have not considered before; we are therefore curious and a particularly keen interest here. We view this factor under the limitation that it is represented by only 4 items and is not optimally operationalized.

3. Results of the survey

3.1. Dimensions in the view of mathematics of university teachers

In the factor analysis we made the decision to analyze five dimensions. Before we do this we will critically view the relevance of this result for the view of mathematics.

1. Each of the 5 dimensions describe a field of consistent answer behavior towards a homogenous item group. That is why they are on the one hand dimensions (and not only features) which structure perception, cognitive representation, affective assessment and conation towards mathematics. On the other hand attitudes are expressed in their consistencies. On the whole the 5 dimensions can be viewed as dimensions of attitudes towards mathematics.

2. One cannot claim that the view of mathematics of university teachers possesses 5 dimensions, because the questionnaire contains only a limited number of aspects (item selection). Also one cannot claim that "In the sample of university teachers the view of mathematics possesses in the section defined by the questionnaire the 5 dimensions formalism, schema, process, application and Platonism, because the result of a factor analysis - the number and the content relevance of the dimensions - depends on the method, in particular on the decision of factor

selection. Statements on the dimensionality and the dimensions can only be formulated on the basis of factor-analytical results at best as hypotheses, which cannot be objectively tested.

3. The results of the factor analysis demonstrate that there are further dimensions beyond the 5 selected dimensions. The 4 factors (their total communality) in the 4-factor solution only explain almost 23% of the total variance of the questionnaire, and with 7 factors only 32% is explained. This can be on the one hand due to the inexact measurement procedures of the Rating method and inexact procedures of recording attitudes, but on the other hand the cause can also (equally) be that there are more dimensions, and that an answer to one item can depend on further specific factors. Mathematics is thus perceived and structured by the interviewed persons as multifaceted and differentiated.

4. The 5 selected factors are relevant and the most important dimensions in the answer behavior in the questionnaire. This cannot be proven (objectively) by factor analysis because the result is method-dependent. However, this result is not subjectively and randomly determined - it presents the most possible optimal and clear structuring of answer behavior according to content and formal criteria. We hope that the above statement has been justified plausibly enough with sufficient arguments for the interpretation of the factor analysis.

5. One can claim that the 5 dimensions are generally relevant dimensions. This assumption is justified in that the questionnaire encompasses - compared to literature, other questionnaires and empirical results - a central (and not peripheral) area of thought on mathematics.

6. We advocate the belief that the 4 factors formalism, schema, process and application are not just relevant but are the most relevant global dimensions of the view of mathematics in general. This conviction evolved when developing the questionnaire and is supported in the literature and by comparison with other questionnaires and survey results. This hypothesis is not falsified by the results (and for logical and methodological reasons cannot be verified).

On the whole the "well-founded" hypotheses (which are hitherto not falsified) come together to a "small theory".

The view of mathematics of university teachers is multifaceted and differentiated; it entails many dimensions which individually cover only a very small field of stimuli, i.e. are valid only for a narrowly defined situation. Even in this narrowly defined situation these dimensions do not clarify behavior completely - here personal and situational intervening variables possibly interplay. Mathematics is thus perceived by mathematics teachers as being very structured and differentiated.

The aspects formalism, schema, process, application and Platonism only grasp a part of this multidimensional wholeness. However, they are dimensions in which mathematics teachers possess attitudes towards mathematics. Perception, cognitive representation and affective assessment of mathematics is guided and structured by mathematics teachers through the aspects formalism, schema , process, application and Platonism. These are also dimensions in which plans of activity are developed, whereby personal and situational intervening variables equally interplay.

These aspects are the most relevant dimensions in the answer behavior on the questionnaire. One can well-foundedly assume that these dimensions constitute generally relevant (not most relevant) dimensions of the view of mathematics. We are convinced, however, that to a considerable degree the 4 dimensions formalism, schema, process and application are the most relevant global dimensions in attitudinal thought on mathematics. They are constitutive elements in the differentiated and complex structure of the view of mathematics in the sense that to a considerable degree they define the basic orientation and characteristics of a complex and differentiated view of mathematics.

In the following we shall analyze the exact content relevance of these dimensions of the view of mathematics.

The formalism aspect

The following statements of the questionnaire operationalize the formalism aspect:

26 Mathematics is a logically uncontradicted thought building with clear, precisely defined terms and unequivocally provable statements.
28 Mathematics is characterized by strictness (rigor), namely a definitory strictness and a formal strictness of mathematical argumentation.
30 Very essential aspects of mathematics are its logical strictness and precision, i.e. "objective thinking".
32 Indispensable to mathematics is its conceptual strictness, i.e. an exact and precise mathematical terminology.
36 Mathematics especially requires formal-logical derivation and one´s capacity to abstract and formalize.
38 Mathematical thinking is determined by abstraction and logic.
40 Central aspects of mathematics are flawless formalism and formal logic.
48 The crucial fundamental elements of mathematics are its axiomatics and the strict, deductive method.
50 Characteristics of mathematics are clarity, exactness and unambiguity.
53 New mathematical theory first originates only if the (flawless) proof for a number of statements is present.
60 Development and logical safeguarding of mathematical theory belong together and are inseparable under correct mathematical contemplation, research and problem-solving.

In the formalism aspect mathematics is characterized by strictness on different levels. Firstly mathematics possesses strictness on the level of terminology and language with its exactly defined terms and precise expert language. Secondly mathematics is characterized by strictness in thought in the sense that it is characterized by "objective" thought. It is determined by logic, i.e. logical strictness and precision, flawlessness and abstraction. Thirdly mathematics possesses strictness on the level of argumentation, explanation and proof. Mathematical argumentation is conducted as flawless proof; it proceeds with formal strictness, i.e. with flawless formalism by means of formal logic and the strict deductive method. Fourthly, mathematical theory which develops by the means just mentioned possesses strictness in its systematic structure. Together with axiomatics, exactly defined terms, the strict deductive method and formal logic in argumentation leads to logical thought-building without contradiction development.

In comparison to mathematics teachers in schools, mathematics teachers at universities add a further partial aspect to this formalistic view concerning the origin of mathematics. (Such statements were not included in the questionnaires of the pupils.) When one characterizes mathematics according to this formalistic view it implies a particular "philosophy" on the origin of mathematics, and such statements akin to this view load indeed onto the formalism factor. Novel mathematical theory originates not when particular new ideas, contemplation and understandings are present, but when to a statement a (flawless) proof is achieved. The development of mathematical theory in the formalistic view is insofar inseparable from (flawless) logical safeguarding.

The schema aspect

The following statements of the questionnaire operationalize the schema aspect:

24 Mathematics is a collection of procedures and rules, which precisely determine how a task is solved.
29 Almost any mathematical problem can be solved through the direct application of familiar rules, formulas and procedures.
34 Doing mathematics demands a lot of practice in following and applying calculation routines and schemes.
39 Mathematics is the memorizing and application of definitions and formulas, mathematical facts and procedures.
44 Mathematics consists of learning, recalling and applying.
M2 Mathematics consists of elaborate tricks and ornate methods, and resembles on the whole very much a crossword puzzle.

The schema aspect focuses on schemata and algorithms in mathematics and describes mathematics as a "complete" set of knowledge: mathematics is a "toolbox" and a "formula package".

Mathematics is characterized as a set of procedures and rules exactly prescribing how the problem is to be solved. In this sense mathematics is "complete" as almost all mathematical problems can be solved with this "toolbox". The consequence for mathematical activity is that doing mathematics consists of recalling and applying (executing) definitions, rules, formulae, facts and procedures. Mathematics consists of learning and teaching (!), practicing, recalling and applying (in the sense of executing) routines and schemata.

Mathematics teachers at universities give the schema aspect a further particular aspect (- this statement was not included in the questionnaires of the pupils and the teachers at school): Schemata and algorithms can also take on the character of "skillful tricks" and "ornate methods" so that doing mathematics is not just simple application but (sometimes) more like solving a crossword puzzle.

The process aspect

The following statements of the questionnaire operationalize the process aspect:

25 Mathematics is an activity which is comprised of thinking about problems and gaining knowledge.
27 Mathematics consists of ideas, terms and connections.
31 Mathematics requires new and sudden ideas.
37 Doing mathematics means: understanding facts, realizing relationships and having ideas.

41 Above all, mathematics requires intuition as well as thinking and arguing, both relating to contents.
43 In mathematics one can find and try out many things for him/herself.
46 Central aspects of mathematics are contents, ideas and cognitive processes.
54 If one comes to grip with mathematical problems, he/she can often discover something new (connections, rules and terms).
56 When developing mathematical theory, it is only natural to make some mistakes; good ideas are what is important.
59 Flawlessness is required only for the logical proving of mathematical statements, not during their development.

Compared to the aspects "formalism" and "schema" of the static view, the process aspect describes the dynamic aspect of mathematics, namely of mathematics origination and development.

Mathematics is characterized as a process, as an activity of thinking about problems and achieving realizations. In this dynamic aspect of mathematics origination and development the content view is also stressed: Mathematics does not consist of axioms, definitions, laws and proofs (as in the formal view), but of ideas, terms, and relations (between terms and ideas).

Mathematics activity, the process of creatively developing mathematics, consists of two partial aspects. On the one hand mathematics is a discovery process: When one is concerned with mathematical problems one can try things out or find things out oneself, one can even discover something new (relations, rules, terms). The second partial aspect is related to this, namely the process of understanding: doing mathematics also means understanding content and setting up relations. Doing mathematics in this problem-related discovery and understanding process demands to a high degree content-related thought and argumentation, as well as inspiration, ideas, intuition and just simply trying things out.

In comparison to mathematics teachers in schools, mathematics teachers at universities add a further partial aspect to this process aspect (as was the case with the formalism aspect) concerning creative development of mathematics (- such statements were not included in the questionnaires of the pupils). In the process view of mathematics concerning theory building one distinguishes between development and safeguarding of mathematics. Development in mathematics can initially commence without logical safeguarding. The criterion is flawlessness: This flawlessness is first demanded at the later stage of logical safeguarding, whereas within the development process mistakes are allowed and good ideas are decisive.

The application aspect

The following statements of the questionnaire operationalize the application aspect:

65 The study of mathematics provides one with several abilities, which also continue to help in reality (for example, general, clear thinking in abstract and complex situations; concrete calculations).
66 Many parts of mathematics are either of practical use or are directly relevant to application.
68 Knowledge of mathematics is very important for the students later in life.
69 Mathematics is of general, fundamental use to society.

70 Only a few things learned from mathematics can be employed later in life.
71 Mathematics is of use to any profession.
72 Mathematics helps to solve daily tasks and problems.
74 With regard to application and its capacity to solve problems mathematics is of considerable relevance to society.
M5 All teachers remain united in that one must diligently do mathematics above all else, because this knowledge provides one with the greatest direct use in practical life.

The application aspect expresses that mathematics has a direct application relation or is of practical use. On the one hand mathematical knowledge is important for the student (in all cases) later on in life: either mathematics helps to solve everyday tasks and problems, or it is useful in every (!) job. On the other hand mathematics is of general, basic use for society.

Between mathematics teachers at universities and mathematics teachers in schools there are no considerable differences (in comparison to the items) concerning the content meaning of the application aspect. The partial aspect "general use for society" was not included in the questionnaires of the pupils.

The Platonism aspect

The following statements of the questionnaire operationalize the Platonism aspect:

M1 God is a child, and he did mathematics as he began to play. It is the godliest of games among mankind.
M3 It cannot be denied that a large part of elemantal mathematics is of considerable, practical use. However, these parts of mathematics appear rather boring when observed as a whole. These are those parts which possess the least asthetic value. "Real" mathematics from "real" mathematicians such as Fermat, Gauß, Abel and Riemann is almost totally "useless".
M6 The mathematicians, who are only mathematicians, are correct in their thinking, but only in the sense that all things can be explained to them using definitions and principles; otherwise their ability is limited and intolerable, because their thinking is only correct when it concerns only extremely clear principles.
M12 When the laws of mathematics are related to reality they are not secure, and when they are secure, they are not related to reality.

At a first glance the fifth dimension is less homogenous in content than the other 4 dimensions, and it demands greater effort to interpret a common meaning of the statements in the factor.

A first content agreement can be found between the statements M1 and M3. In M1 mathematics is "divine games". The term "game" expresses a range from application-distance up to complete uselessness. The term "divine" entails amongst other things the association "aesthetic". The statement M1 therefore entails that mathematics is aesthetic and low on application. This is confirmed by the statement M3: "Genuine" mathematics is "aesthetic" and "almost completely useless". On the whole, in this partial aspect mathematics is characterized as an application-distant, but aesthetic "divine games".

The second partial aspect in the fifth factor lies in the statements M6 and M12. M6 expresses that mathematicians only reason properly within the limits of definitions and principles, i.e. within the formal system. This formal system remotes them from the world ("otherwise they are limited and unbearable") but it secures them the "rightness" (validity) of contemplation. The statement M12 expresses the gap between the relation to reality and the security aspect: Either mathematics is reality-distant and secure, or it is close to reality and insecure. On the whole both expressions describe mathematics as reality-distant and secure.

Both partial aspects can not be immediately united together. (E.g.: What connects aesthetic mathematics to the security of the formal system?). Because both partial aspects are united by one factor this can mean that the interviewed persons see a relationship between the partial aspects. Mathematics can therefore be described here in this factor as reality-distant, but aesthetic and secure "divine games".

These partial aspects can however be indirectly brought together under the philosophy of Platonism. Today there exist a number of differentiated and modern forms of Platonist theory; we however refer to the original form, namely Plato´s ideas. This background theory does not only clarify the relation between the partial aspects "aesthetic - application-weak" and "application-distant - secure", yet it clarifies the relations in the partial aspects.

Plato distinguishes between the world of changeable images accessible to the senses (sense-experienced reality) and the world of fixed, permanent, and unchanging ideas which can only be recognized by reason. Both worlds are divided by a gap (*chorismos*), apparitions however are representations of ideas. The world of experience is, according to Plato, insecure knowledge (*doxa*), and the world of ideas is secure knowledge (*episteme*). The epistemological process of sense experience as the basis for contemplation on the nature of things is explained by Plato as "re-remembering" (*anamnesis*) because the ideas exist *a priori* (also without contemplation by humans) and the immortal soul has already viewed these ideas in a state of pre-existence. Mathematical objects fit, according to Plato, into an intermediate position between the sensual world and the world of ideas.

On the background of this theory the homogenity of the statements and the partial aspects appear to be self-evident: Mathematics exists as "divine games" in the world of ideas and is thus divided by a gap from the world of (sensual) apparitions i.e. of (sense-experienced) "reality". In this sense mathematics is aesthetic but application-distant. However, mathematics as part of the world of ideas is secure, and this security in the world of ideas corresponds to the gap to (sense-experienced) "reality" in the world of apparitions. Mathematics is thus reality-distant but secure contemplation from the world of unchanging principles.

The relation between both partial aspects can be presented wholly in the sense of Platonism by the statement M4: "Can we not approach the godlike by no other way but the symbolic, so we will best of all make use of mathematical symbols as these possess indestructible certainty" The statement M4 is related by the interviewed persons to the Platonism factor, but not so clearly as the above mentioned statements. (It loads exclusively on the Platonism factor but only with weak loadings.)

3.2. The attitudes in the five dimensions

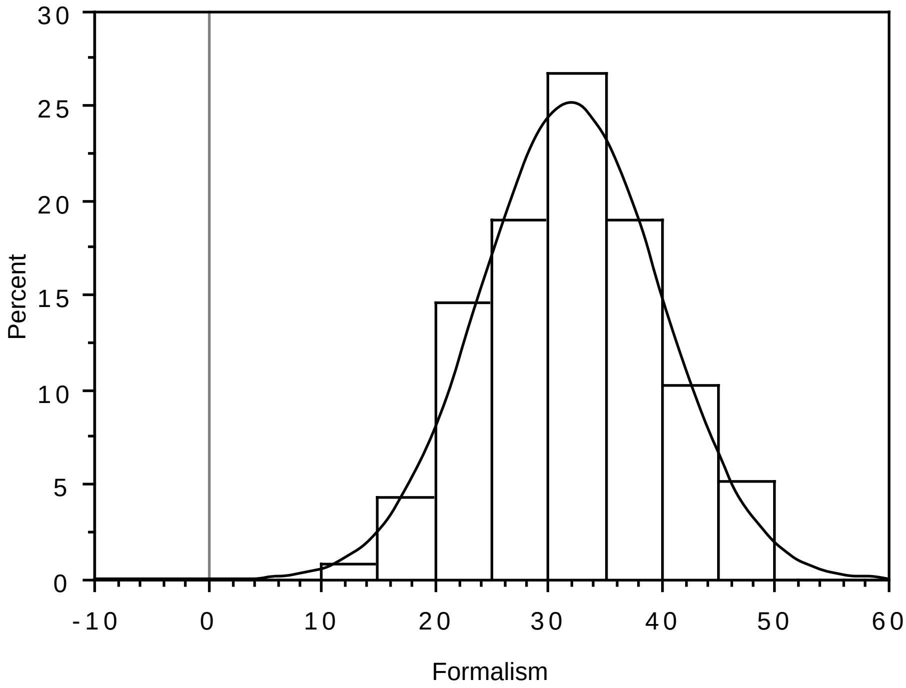
For the following survey, we set up a scale value for each of the 5 dimensions and for each of the interviewed persons. Each of the 5 dimensions were operationalized by 4 to 11 items (comp.

previous section 3.1). For each interviewed person the values in the statements of a dimension were counted together. Transformation and stretching lead to a scale of 0 - 50, whereby approx. 0 - 10 is the range of full rejection and 40 - 50 is the range of full agreement .

The reliability (Cronbach) on the formalism scale is 0.82 (10 items), for the schema scale 0.74 (6 items), for the process scale 0.71(10 items), for the application scale 0.76 (9 items) and for the Platonism scale 0.51 (4 items). The reliabilities for individual statements (on a single interviewed persons) lie under the limit of 0.9 demanded for psychological tests, but in this study we define group statements for which a reliability of 0.7 is demanded. The reliabilities are insofar good and acceptable for the first four scales, in view of the low item number and the usual measurement inaccuracies of the Rating process they are surprisingly high. One can thus assume reliability of the scale values in the first four dimensions. The scale in the dimension "Platonism" does not possess sufficient reliability - the reason for this probably being the low item number.

Formalism aspect

The frequencies in the formalism scale possess a "normal distribution"[3] (with good adaptation) an symmetrical distribution. The central tendency lies in the arithmetic average of 32.1 in the range of agreement (standard deviation = 7.92, median = 31.8, modus = 29.5).



3 This means that the observed distribution does not differ significantly in the Chisquare-adaption test (on the 5% level) from the normal distribution, which possesses the mean and the standard deviation as the observed distribution. (The zero hypothesis that the observed distribution is equal to the theoretical distribution could not be rejected on the 5% level.) The deviations between the empirical distribution and the normal distribution can thus be coincidental. The present empirical distribution can therefore come from a population which is normal distributed.

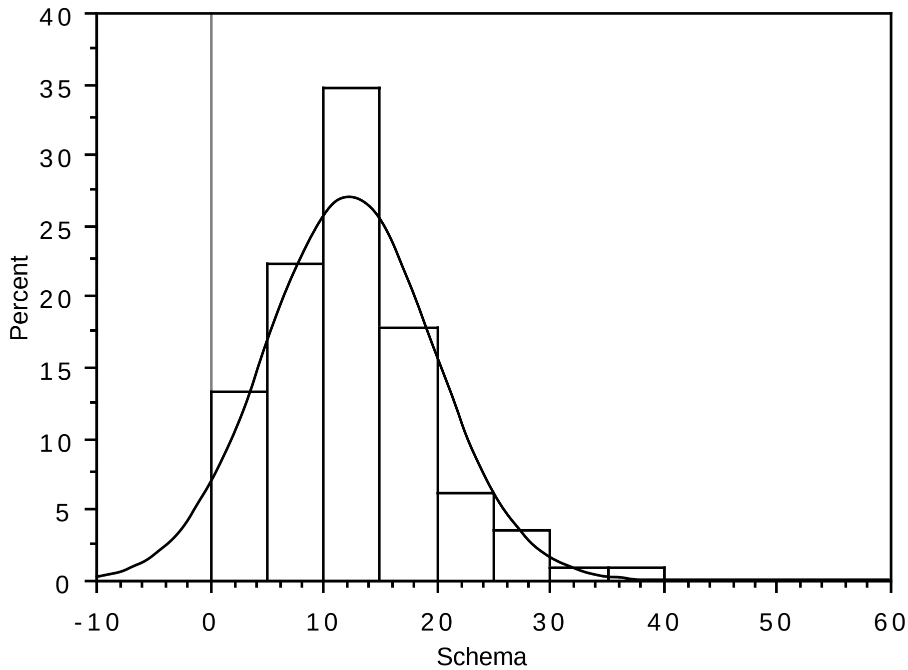
from (>)	to (≤)	count	percent	norm. count	norm percent	
0	10	0	0,0	0,3	0,3	don't agree
10	20	6	5,2	7,1	6,1	partly agree
20	30	39	33,6	38,6	33,3	undecided
30	40	53	45,7	51,6	44,5	mostly agree
40	50	18	15,5	17,0	14,7	fully agree
	Total	116	100,0	114,6	98,9	

More than 60 % of the university teachers clearly view in mathematics the formalism aspect and a high degree of formal strictness (45.7 % mostly , 15.5 % completely), and 36.6 % are undecided resp. represent a middle position. On the other side only 5.2 % of the interviewed persons think that mathematics is only partly characterized by a formal aspect, and no one completely rejects this view completely.

Mathematics thus posesses, in the view of most university teachers, a clear formalism aspect, whereas hardly any teacher views no formalism in mathematics. Formalism, i.e. formal strictness in language, contemplation, argumentation as well as in development and systematization of theory are a characteristic of mathematics akin to most of the university teachers on an average to clearly above average level.

Schema aspect

The distribution of frequencies in the schema scale is random-critically not distinguishable from a normal distribution (under weak adaptation in the Chisquare-test, comp. footnote 3), even though the estimations around the mean occur overproportionally. The location parameters also hint at a slight inclination to the right. The central position lies with an arithmetic mean of 12.4 resp. a median of 11.5 in the area of (clear) rejection of the schema aspect (standard deviation = 7.35, modus = 8.3).



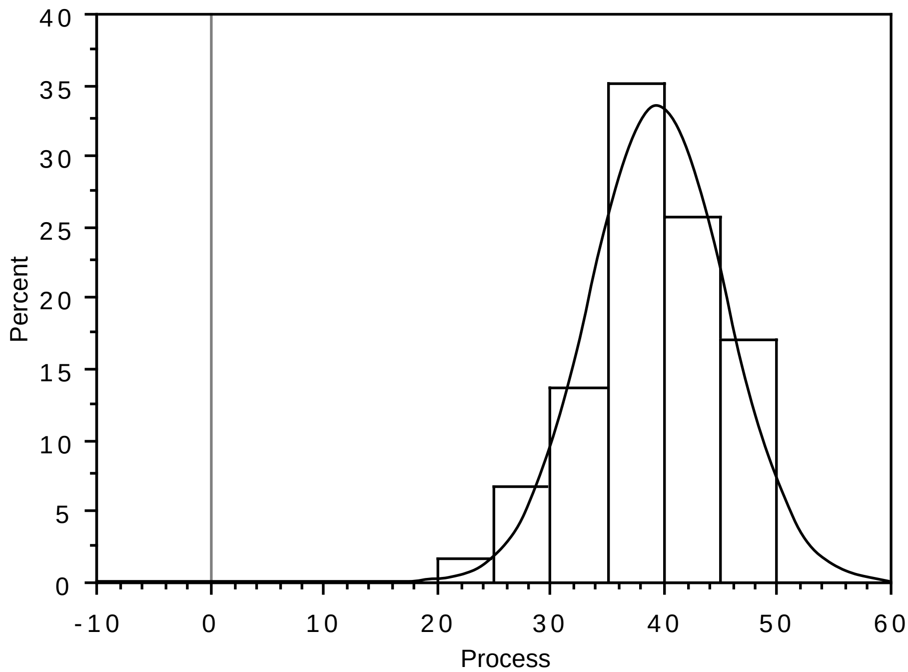
from (>)	to (≤)	count	percent	norm. count	norm percent	
0	10	40	35,7	36,5	32,6	don't agree
10	20	59	52,7	53,5	47,7	partly agree
20	30	11	9,8	16,0	14,2	undecided
30	40	2	1,8	0,9	0,8	mostly agree
40	50	0	0,0	0,0	0,0	fully agree
	Total	112	100,0	106,9	95,3	

Almost 90% of the university teachers (88.4%) reject an interpretation of mathematics as "toolbox" and "formula package", 35.7% completely and 52.7% mostly. On the one hand only 1.8% view a schema aspect in mathematics and only 9.8% are undecided.

In the image of almost all university teachers, mathematics is thus not a "complete" collection of algorithms, procedures and routines. The view of the schema aspect that mathematics is a "toolbox" and "formula package" or even consists of "skillful tricks" and "clever methods" is on the whole clearly rejected. This also means that for university teachers the use of mathematics does not consist of learning and teaching (!), retaining and applying complete knowledge.

Process aspect

The distribution of frequencies in the process scale is "normal" (under weak adaptation in the Chisquare-test, comp. footnote 3) and symmetrical. The mean values lie in the area of clear agreement, and on the border of this area even of unrestricted agreement. (standard deviation = 5.93, arithmetic mean = 39.5, median = 39.7, modus = 38.7).



from (>)	to (≤)	count	percent	norm. count	norm percent	
0	10	0	0,0	0,0	0,0	don't agree
10	20	0	0,0	0,0	0,0	partly agree
20	30	10	8,5	6,2	5,3	undecided
30	40	57	48,7	55,8	47,7	mostly agree
40	50	50	42,7	50,3	43,0	fully agree
	Total	117	99,9	112,3	96,0	

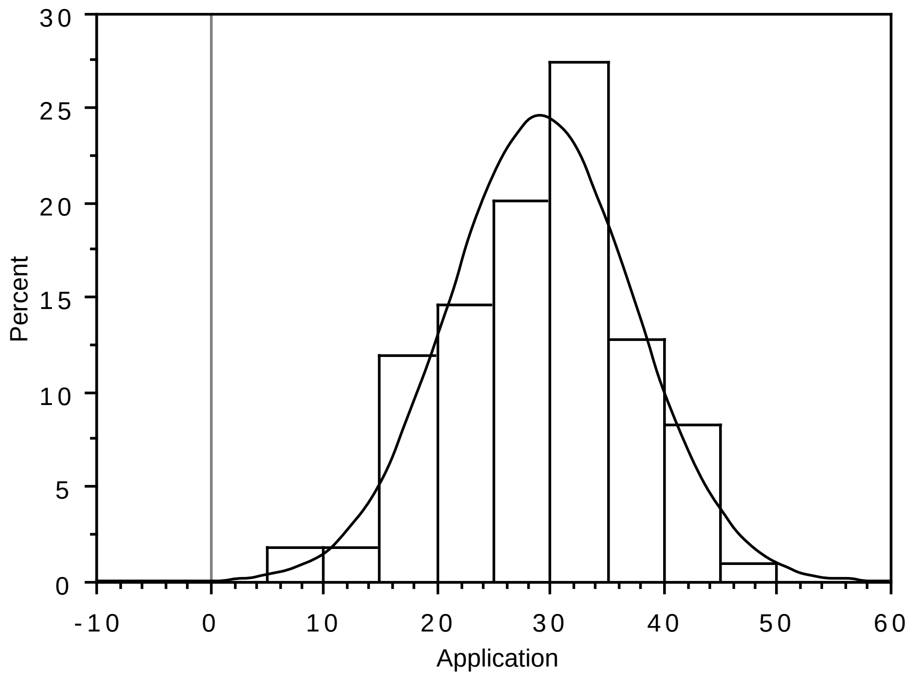
More than 90% of the university teachers agree to the view that mathematics is a process, whereby 48.7 % mostly and 42.7 % fully agree. None of the interviewed persons rejects to see this aspect in mathematics, and only 8.5 % are undecided.

In the view of mathematics of almost all university teachers the processual aspect of creating mathematics is a clear component; not one teacher does not view this aspect in mathematics.

The viewpoint of the process aspect to regard mathematics as a creative process of mathematics development and as a discovery and understanding process, is a widely spread and almost unlimitedly accepted view of university teachers.

Application aspect

With an arithmetic mean of 29.3 (median = 29.6, modus = 27.8, standard deviation = 8.10) the distribution of the application scale lies in the border range between undecided and agreement. This distribution is symmetrical and deviates random-critically not from a normal distribution (weak adaptation in the Chisquare-test, comp. footnote 3).



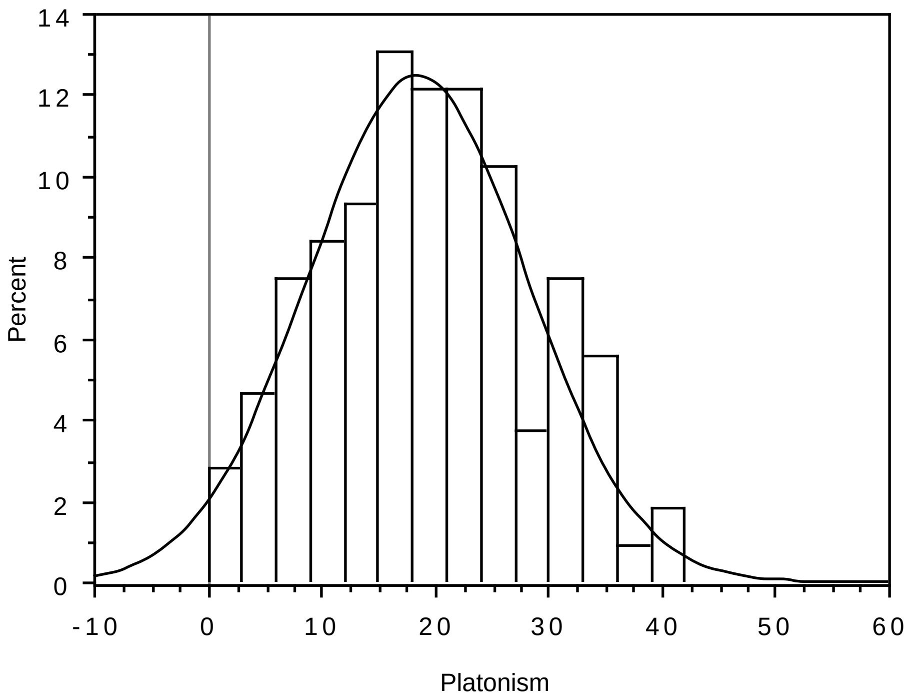
from (>)	to (≤)	count	percent	norm. count	norm percent	
0	10	2	1,8	0,9	0,8	don't agree
10	20	15	13,8	12,7	11,7	partly agree
20	30	38	34,9	44,5	40,9	undecided
30	40	44	40,4	40,6	37,2	mostly agree
40	50	10	9,1	9,6	8,8	fully agree
	Total	109	100,0	108,3	99,4	

Almost 50 % of the university teachers view mathematics as possessing an application aspect and as being useful, 40.4 % of them mostly and 9.1 % totally. Furthermore, more that one-third of the interviewed persons (34.9 %) fill in the undecided or middle position. In contrast, approx. 15 % deny an application aspect of mathematics (13.8 % on the whole, 1.8 % completely).

From the viewpoint of three quarters of the university teachers, mathematics has an average to clearly obove average (higher) application aspect and use in everyday life, in the professional occupation of mathematics students, and for society in general. Only a few, almost 15% of the interviewed persons, consider mathematics to be application-distant and useless.

Platonism aspect

The frequencies in the Platonism scale are "normal distributed" (with good adaptation in the Chisquare-test, comp. footnote 3) and on the whole symmetrical. The central position lies with the arithmetic mean of 18.5 in the border zone between rejection and undecidedness (standard deviation = 9.60, median = 18.3, modus = 15.6).



from (>)	to (≤)	count	percent	norm. count	norm percent	
0	10	25	23,3	17,1	16,0	don't agree
10	20	37	34,6	40,0	37,3	partly agree
20	30	28	26,2	34,6	32,3	undecided
30	40	15	14,0	11,1	10,4	mostly agree
40	50	2	1,9	1,3	1,2	fully agree
	Total	107	100,0	104,1	97,2	

Almost 58 % of the university teachers reject the view of a Platonism aspect in mathematics, 34.6 % on the whole and even 23.3 % completely. A further 26.2 % are undecided so that a majority of 60 % of the university teachers represent a position of moderate rejection to undecided, whereby rejection tips the scales in its favor. In contrast, there is only a small group of 15.9 % who on the whole allocate a Platonism aspect to mathematics.

Only a small group of university teachers (approx. 15 %) view a view of mathematics under the aspect of being application-free, but secure and aesthetic "divine games". The overall majority (almost 85 %) of the university teachers possess a view of mathematics with only a moderate or weak Platonism aspect, whereby some of the interviewed persons represent a middle position; others, however, demonstrate a stronger rejection of this aspect.

The interpretation of mathematics as being application-free, but secure and aesthetic "divine games" is therefore for the view of mathematics of most university teachers of moderate to small relevance.

Comparison of distributions

The "average" view of mathematics of mathematics teachers at universities is clearly accentuated in the 5 observed aspects. This means that these 5 aspects are not similarly relevant and describe mathematics similarly well, but some aspects are viewed more as a part of mathematics as forming part of the mathematics image, whereas other aspects are viewed as being less a part of mathematics and by this nonetheless also form part of the mathematics image.

In the "average" view of mathematics of university mathematics teachers the process aspect receives the highest agreement, whereas the aspects "formalism" and "application" receive an average or higher agreement. In comparison, the Platonism aspect receives only moderate or little agreement, and the schema aspect is (overtly) rejected.[4]

Mathematics is (according to the "average" view) to university teachers a discovery and understanding process, i.e. a process of mathematics origination, development and understanding. This characterization dominates in the surveyed 5 aspects; there are however possibly further aspects which characterize mathematics from the viewpoint of the interviewed persons with the same relevance.

4 The differences in the central tendency were tested non-parametrically by the Wilcoxon-test for pair differences. Hereby ties were considered. The measurement-pair-differences were symmetrically distributed around the median, and the number of zero differences was max. 3% the number of all differences.
The comparisons were made pairwise, and the levels of significance were Bonferroni-adjusted. The t-test for differences produced the same results:
All mean differences are under pair-wise testing highly significant, only the difference between the mean of the formalism and the application scale is significant.

Mathematics is from the viewpoint of the university teachers to a large degree characterized by formal strictness, i.e. mathematics is characterized by formal strictness of language, contemplation, argumentation, and systematic theory building. The university mathematicians also attested mathematics an average to considerable (but not high) relation to application and everyday usefulness, to the professional reality of mathematics students and for society in general.

Contrary to this, mathematics can be characterized as being less moderate or (at its best) moderate application-distant but secure and aesthetic "divine games". An understanding of mathematics as a collection of complete knowledge in the sense of "toolbox" or "formula package" is rejected.

With this interpretation of the mean differences between the individual features one must critically add the limitation that there can be two reasons for the differences: First they can be an expression of the differing attitudes of the interviewed persons towards different aspects (personal feature). Second these mean differences can be an expression of differing "difficulties" (or different scales in the various dimensions) of the items (item feature). For example, high agreement to the statements of the process aspect on the one hand can mean that the interviewees view a process aspect in mathematics. On the other hand agreement can (equally) be due to the fact that the statements were "too easy", i.e. that their formulation made agreement easy. If one assumes that the mean differences are exclusively determined by the degree of difficulty of the items, then statements of the kind "the process aspect is in the view of mathematics of greater relevance than the formalism aspect" are not permissible.

In our opinion the mean differences do not have so much to do with differing item difficulties but are grounded on real differences in the attitudes towards various aspects.[5] That is why we have also interpreted the mean differences but would like to compliment this under the reservation of possible differences in the item difficulties.

3.3. Relations between the dimensions: On the global structure of the view of mathematics

One approach to the view of mathematics is to describe attitudes in particular attitude dimensions. The 5 dimensions are independent attitude objects distinct from each other which initially are to be analyzed and evaluated individually. These attitudes are *one* feature of the view of mathematics.

A further approach, which is very important for our own approach, is to describe the structure which is generated by these dimensions. It is one of our basic assumptions (or convictions) that the view of mathematics (its nature) is not only formed by single attitudes but also by their structure. In particular, a single attitude can be less relevant for conation or even for conation

5 As first the mean differences are very big. The means lie in the clearly various zones of the scale "clear agreement" (P: 39.5), "agreement / undecided" (F: 32.1, A: 29.3), "undecided / disagreement" (Pl: 18.5) and "clear disagreement" (S: 12.4). These high differences are in our opinion not so much explainable by differing item difficulties, as the interviewed persons were clearly sorted out into the different sections of the scale (also on the questionnaire).
Second, the four scales formalism, schema, process and application clearly demonstrate other means in the sample of other populations (pupils in Grigutsch 1996, teachers in Grigutsch, Raatz, Törner 1998). This means that the items are not themselves "difficult" or "easy" but that they permit different ratings which are more than less marked by an attitude.

guidelines than the whole view of mathematics. If in contrast one views the total picture (i.e. the individual attitudes and the structure generated by them), then one receives more information on the nature of the view of mathematics. We believe that the structure of the view of mathematics considerably contributes to the nature of the view of mathematics, and to what degree an attitude has an influence on conation planning and to what degree an attitude can be modified or changed.

The structure of the view of mathematics can be regarded in relation to various criteria, e.g. quasi-logical networking, psychological relevance (centrality, intensity, extremity) or cluster properties (comp. Pehkonen 1994; also Törner, Pehkonen 1996). We can analyze clusters generated by relations between the dimensions; these relations in turn generate a structure, i.e. a global partial structure of the view of mathematics. Such attitude structures have already been described by us in other populations (pupils, teachers).

The five scales possess the following partial correlation coefficients (in the upper triangle matrix; significance levels in the lower half; n = 103):

	F	S	P	A	Pl
F	1,000	,235 *	-,072	,233 *	-,221 *
S	,018 *	1,000	-,052	-,169	,198 *
P	,477	,605	1,000	,234 *	,003
A	,020 *	,094	,019 *	1,000	,147
Pl	,027 *	,049 *	,978	,144	1,000

The schema aspect correlates significantly positive with the formalism aspect, this with the application aspect and this finally with the process aspect. The Platonism aspect stands in a significant positive relation to the schema aspect and in a significant negative relation to the formalism aspect. The correlation coefficient between the schema aspect and the application aspect is almost significant (10% > p > 5%), the correlation is negative.

These relations lead to the following partial structure of the view of mathematics (comp. diagram).

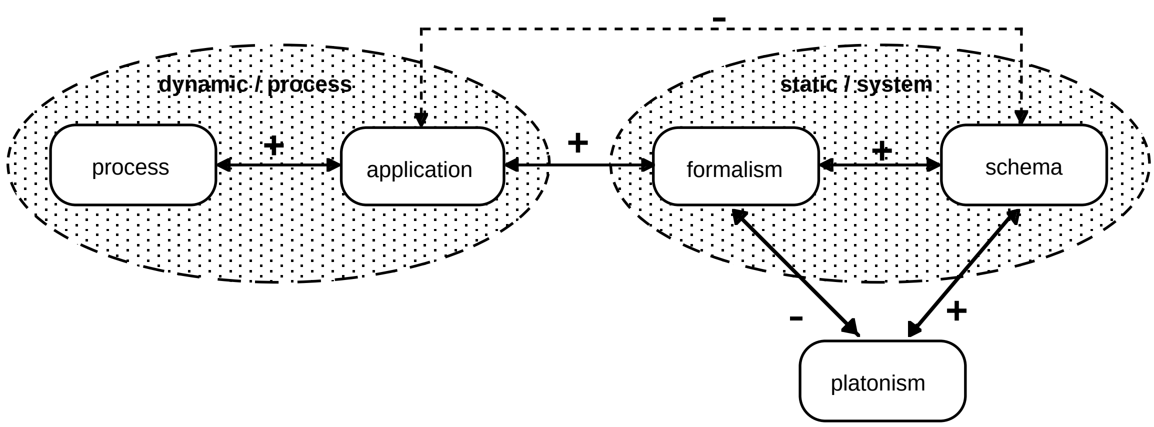


Diagram: Intercorrelative relations between the dimensions

The positive relation between the schema aspect and the formalism aspect agrees to the hypothesis from our content approach (pre-theory) namely that both these aspects represent the static view of mathematics as a system. The mathematics system consists on the one hand of the collection of complete knowledge (schema aspect), on the other hand of rules and conventions for building the system, i.e. rules for linguistic definition of knowledge, for the justification of its validity and for fitting the knowledge into the previous system. The correlation of these two aspects can mean that the interviewde university teachers view these relations similarly, i.e. in the view of the university teachers there is a static view of mathematics which consists of a schema aspect and a formalism aspect.

The positive relation between the process aspect and the application aspect can be plausibly interpreted in both directions. It can on the one hand mean that the process of mathematics origination and development is an activity related to application and reality (e.g. mathematics model building), in short: each mathematical discovery and understanding process is geared towards applicability and usefulness. This relation can inversely mean that reality and application-related tasks require a process, namely the process of problem-solving, of insight into relations, of achieving realizations, of creating and understanding mathematics. We draw the process aspect and the application aspect together to the dynamic view of mathematics as a problem-oriented and an epistemological process. The correlation of these two aspects can mean that also in the view of mathematics of university teachers there is a dynamic view of mathematics, in which the process is geared to applicability and that tasks of the reality require problem-solving, discovery and understanding processes.

In our theoretical prerequisites of the content approach we assumed that the static view of mathematics as a system and the dynamic view of mathematics as a process stand antagonistically to each other. This hypothesis could not be falsified in the teacher and pupil populations; here there were corresponding negative correlations between the aspects. In this sample of university mathematicians, however, the static and dynamic views do not stand antagonistically to each other. Instead both views stand next to each other and are even connected to each other by the positive relation "formalism"-"application". The view of mathematics of the professors is not characterized by two poles, by an "either-or", but by a "this-and-that". From the viewpoint of the university teachers, mathematics is - as one can roughly interpret the structure - not an antagonistic opposition but a complementary unity of two views.

A number of interpretations appear to us plausible for the positive relationship between the formalism aspect and the application aspect.

The relationship can on the one hand mean that the formalistic aspect of mathematics i.e. formally presented mathematics (from the viewpoint of the university teachers) is application-related and practically useful. Different formal elements of mathematics are important for solving tasks and problems of reality or for modeling reality: exact expert language serves precise formulation or modeling of situations, exact "objective", logical, rational and flawless contemplation supports correct thought and systematic construction of theory and therefore clear, well sorted, concise and uncontradictory presentation: It allows suitable theory segments to be found quickly.

Another interpretation is presented when one considers that the relation of the formalism aspect to the application aspect is the strongest and clearest relation to the statements 69 and 74 of the application aspect - the individual correlations, the side-loadings or the regression analyses demonstrate this. In both of these statements mathematics is attested a general, fundamental usefulness for society. This leads us to the following interpretation: University teachers view in mathematics a formal aspect, and in their work they are confronted with this aspect (literature, lectures, own presentations). They furthermore assume that this formal mathematics is useful - not for everyday applications or applications in particular professions, but for aspects of a more general and not further defined relation to usefulness.

Interestingly, the schema aspect does not positively correlate with the application aspect. According to this, mathematical rules and routines, facts, formulas and algorithms possess no application relation and usefulness. On the contrary: the partial correlation coefficient is almost significantly negative, i.e. the relation possesses more of a negative bias. One can therefore set up the interpretation and hypothesis that - in the view of the university teachers - complete knowledge in the form of rules, facts, formulas, algorithms and procedures (the "complete recipes") are even viewed as inhibiting for mastering mathematical problems in everyday life and in professional life.

The relationship of the Platonism aspect to the aspects "schema" (positive) and "formalism" (negative) are difficult for us to interpret as we have not yet found a convincing understanding for the interrelations.

The view describing mathematics as a collection of complete knowledge stands in a positive relationship to the interpretation of mathematics as Platonic "playing around". There are some approaches to the understanding of this relation. "Skillful tricks" and "clever methods" belong to the schema aspect, and "aesthetic playing around" belongs to the Platonic aspect. Knowledge and its documentation in the schema aspect is on the whole "complete" in the sense of pre-existent, given and unalterable. Finally, both aspects entail an application distance to mathematics. However, in these initial interpretation approaches we do not yet see a convincing understanding for this relationship.

The negative relation between the formalism aspect and the Platonic aspect evokes some questions. On the one hand they can be explained by the opposition of differing epistemological positions. Is knowledge from the viewpoint of formalism man-made and can be randomly set in definitions and axioms, so is knowledge from the Platonic viewpoint inalterable and preexistent and cannot be made by man but only be "recalled". However, this explanation of the negative relationship does not satisfy us completely.

On the other hand a positive relationship is also plausiblly explainable. The negative relation between the formalism aspect and the Platonic aspect is contradictory to the observation that formal mathematics is often characterized as "Platonic", whereby a positive relationship is implied. Strict logical thought and formal-logical deduction on the basis of axioms and precise definitions are necessary for mathematics to attain security (on the validity) of statements - to acquire unchangeable "Platonic truths". Equally, formal formulation and high degrees of abstraction in terms and statements leads to "Platonic" reality distance, or at least creates the impression of reality distance. And finally, (seemingly) arbitrariness of some definitions gives mathematics the reputation of "playing around with numbers".

Conclusion to the structure

In the section viewed by us the view of mathematics of university teachers contains in content two different viewpoints of mathematics. The static view of mathematics as a system consists of the schema aspect and the formalism aspect of mathematics. According to this view mathematics consists of a collection of "complete" knowledge and conventions for the integration of knowledge into this system. In the dynamic view the process aspect and the application aspect come together: Mathematics is described as a problem-related discovery and understanding process in which the process of mathematics origination and development is an activity related to reality and applications, and vice-versa, the solution of tasks from reality demands problem-solving, discovery and understanding processes.

The static view and the dynamic view of mathematics do not stand antagonistically to each other, mathematics is not "either system or process". Instead in the view of university teachers, mathematics is "as well system as process", e.g. a complementary unity of both viewpoints.

The relation between the two viewpoints is made through the relationship that formal mathematics is attested or assumed to have an application relation and usefulness. In contrast it seems that complete knowledge in the form of facts, formulas, rules and procedures (the "complete recipes") are not useful or are even inhibiting for the solution of mathematical problems from reality.

4. Discussion

We have previously presented the view of mathematics of university teachers (within the content framework of this study) descriptively. The question "What view of mathematics do university teachers possess ?" was the concrete question formulation of this study. Research into the view of mathematics of university teachers was however motivated by three global initial question formulations (comp. the introduction). Now we attempt to give a first answer to these global questions. Hereby the answer can only be hypotheses based on our results. (As other studies are not available, the basis for answering the general question formulation is still limited, and a comparison cannot be conducted).

4.1. What circumstances are university teachers confronted with in research and teaching, in universities and in science?

University teachers view mathematics as a discovery and understanding process, as a process of mathematics origination and development, whereby they are inclined to reject the view of mathematics as a collection of complete knowledge. The view of mathematics here is that of a science. The reality of mathematics teaching in practice is not identical with this view - lectures are the imparting of complete knowledge with a high degree of formal exactness. The view of mathematics of university teachers is thus strongly related to the research process of science - hardly to mathematical teaching practice. For the self-concept of university teachers of mathematics one can therefore conclude that they see themselves as scientists and researchers, lesser as teachers and mediators of mathematics. This hypothesis can reflect that mathematics teachers at universities - due to their professional socialization and the conditions of this socialization - should view themselves initially as scientists and researchers, and only in the second instance as teachers and mediators of mathematics. Their role seems to be defined or understood as "teaching researcher" and not as "researching teacher".

4.2. Which view of mathematics do mathematics teachers become familiar with or learn during their studies?

Mathematics teachers could learn a view of mathematics from university teachers that mathematics is essentially a process of mathematics origination and development, i.e. a discovery and understanding process, whereby it possesses only a moderate to weak Platonism aspect and cannot be understood as a collection of knowledge. They could learn that mathematics has a middle to high application relation and a higher degree of formal strictness. And they can learn that mathematics is not an opposition of the static system and the dynamic process but that both viewpoints stand next to each other, complement each other and are a prerequisite for each other.

Whether or inhowfar mathematics teachers learn this view of mathematics depends on a number of conditions:

i. University teachers have to translate their view of mathematics, their attitudes towards mathematics, into their activities and behavior.
ii. University teachers not only have to put into effect this view of mathematics in their behavior and activities in "scientific and research situations" but also in teaching situations. But everyday activities of teaching mathematics is not only influenced by the attitudes of mathematics as a science but also influenced by other attitudes: attitudes towards teaching mathematics, towards the learning behavior of students (learning modes, learning ability and willingness), towards the teaching forms "lecture" and "seminar", towards the institution "university" and optimal behavior in this institution, towards behavior in professional situations etc.
iii. Students have to perceive and interpret the view of mathematics presented to them in mathematics lectures. This takes place on the background of their own subjective attitudes (towards mathematics, learning, teaching, examinations etc.), which influence their perception and situation definitions.

On the whole it is doubtful whether students or teacher trainees of mathematics acquire a process-oriented view of mathematics from their academic mathematics teachers in lectures and seminars, in particular because lectures are often a form of mediating complete knowledge under a high degree of formal strictness. In other words: it appears doubtful whether university teachers (can) impart their own process-oriented view of mathematics to their students.

4.3. Is the academic view of mathematics a discussion contribution for the view of mathematics in schools ?

Academic mathematics is basically an important science for the didactics of mathematics so that the view of mathematics of academic mathematics is generally an important contribution. Moreover, Zimmermann assumes change in science:

"Also in the world of professional mathematicians one can see that experts are slowly beginning to relativize the importance of Bourbacism and are increasingly turning to a [...] more informal, intuitive and content view of mathematics […]." (Zimmermann 1991, p.40). If one assumes change (and this assumption can in the light of our results not be falsified and therefore is to be preliminary assumed as confirmed) then one has to discuss to what degree this change is being acknowledged for mathematics in schools.

In relation to the TIMSS studies, complaints were heard and deficits revealed that too many German pupils do not posses the ability to successfully tackle or even solve non-routine tasks demanding a higher degree of conceptual understanding and heuristic competence. The process-oriented academic view of mathematics can be a stimulating and fruitful contribution to the discussion on which mathematics - and this always means which view of mathematics - is to be imparted in schools. The view of mathematics of academic mathematics focusing more on the process of mathematics origination and development is insofar fruitful for the didactics of mathematics as it corresponds with well-known didactic demands which can be vividly described by such phrases as "learning process is research process", "genetic principle" and "constructivistic" learning and epistemology. This view stressing the process of mathematics origination and development in problem-oriented situations thus falls on fruitful ground in didactics and it can be fruitful for activities within the above mentioned didactic principles.

5. Personal statement and conclusions

Whereas the previous presentation of this study has primarily a descriptive nature, the question arises to the reader concerning the subjective evaluation of the results of the survey. We would like to present to the reader some of our own subjective evaluations.

Globally viewed, the view of mathematics presented in this study (comp. section 3) presents not an opposition, but a complementary side-by-side and togetherness of a static and dynamic viewpoint. This view is in our opinion extensive and more balanced in the sense of a comprehensive understanding of mathematics. Of interest appears to us the attitudes towards the individual aspects of mathematics. In the survey on university mathematicians it is hardly surprising that the formalism aspect received higher agreement.

Also, clear rejection of the schema aspect and the high level of agreement towards the description of mathematics as a process of creative mathematics development can be easily comprehended, as mathematics at universities is often experienced by the mediator in *statu nascendi*. Not using tools stands here in the foreground but the making and making available of tools. To what degree students participate in these activities remains an open question. From our point of view it would be regretful should university teachers not impart their process-oriented view of mathematics to their students, e.g. because their role is primarily understood as scientist and researcher and less defined and understood as mediator of mathematics.

From our point of view the interviewed persons assess the role of mathematics for application and everyday use more reservedly. The application aspect is designated a middle to high level of appreciation, but more than one-third of the interviewees are undecided, and formal mathematics is possibly considered to be of general practical use (comp. sections 3.2. and 3.3.). These observations underline the repeatedly claimed "ivory tower" nature of academic mathematics. That ("positive") changes are developing in the last years does not deny the introvert practice of mathematics in many places. Nevertheless, some mathematicians view mathematics as being increasingly application-oriented e.g. as a "key technology for the future" due to experience in "network projects between university and industry" (comp. Hoffmann et al.

1996). Assumedly, the interviewed persons have not yet had much experience with this perspective.

In particular attitudes towards the Platonism aspect deserve attention, as Hersh underlines their relevance:

> *Platonism, or realism as it's been called, is the most pervasive philosophy of mathematics. It has various variations. The standard version says mathematical entities exist outside space and time, outside thought and matter, in an abstract realm independent of any consciousness, individual or social.* (Hersh 1997, p. 9)

> *The working mathematician is a Platonist on weekdays, a formalist on weekends. On weekdays, when doing mathematics, he's a Platonist, convinced he's dealing with an objective reality whose properties he's trying to determine. On weekends, if challenged to give a philosophical account of the reality, it's easiest to pretend he doesn't believe it. He plays formalist, and pretends mathematics is a meaningless game.* (Hersh 1997, p. 39)

Hersh claims that university teachers are Platonists at least "on weekdays", i.e. in their work. In this survey the university teachers rate the Platonic aspect moderately or low. If we follow the view of Hersh (p.39) then it was to be expected that Platonists deny their philosophy. Our survey was, to remain in the Hersh argument, the "Sunday question", and the formalism aspect is rated higher, indeed.

Furthermore, Hersh offers the explanation that (naive) Platonism is only half-conscious and that one can only halfheartedly and with shame confess:

> *Yet most of this Platonism is half-hearted, shamefaced. We don't ask, How does this immaterial realm relate to material reality. How does it make contact with flesh and blood mathematicians. We refuse to face this embarrassment: Ideal entities independent of human consciousness violate the empiricism of modern science. For Plato the Ideals, including numbers, are visible or tangible in Heaven, which we had to leave in order to be born. For Leibniz and Berkeley abstractions like numbers are thoughts in the mind of God. That Divine Mind is still real for Somerville and Everett.* (Hersh 1997, p. 11)

The Platonism in our survey (in particular operationalized in item M1) is more naive Platonism (Davis 1994, p.133) or fundamentalistic Platonism. Again Hersh´s view is of interest:

> *Heaven and the Mind of God are no longer heard of in academic discourse. Yet most mathematicians and philosophers of mathematics continue to believe in an independent, immaterial abstract world - a remnant of Plato's Heaven, attentuated, purified, bleached, with all entities but the mathematical expelled. Platonism without God is like the grin on Lewis Carroll's Chescire cat. The cat had a grin. Gradually the cat disappeared, until all was gone - except the grin. The grin remained without the cat.* (Hersh 1997, p. 11)

Thus the rejection of a fundamentalistic, pseudo-religious Platonism aspect does not surprise one much, as a "Platonic heaven" is explicitly presumed here. This rejection does not however put into question Hersh´s thesis of a widely spread naive Platonism.

At present in many places the state of mathematics education and training is being critically reviewed, both at school and university levels. In particular TIMSS in the societies of the USA and in Germany has sharpened awareness towards mathematics lessons. The discussion has initially spread to the communities - when one takes as an evaluation basis the notices of the German Mathematician Union (DMV) (comp. e.g. Jäger, Törner 1997; Hefendehl 1997) as well as the notices of the American Mathematical Society (AMS) (Bass 1997; Garfunkel et al. 1998). The self-understanding of ones own academic discipline has now become topic for more intensive discussion. A philosophy of mathematics is not predominantly limited to the development of foundations within mathematics, but individual and group-specific ideologies of mathematics are becoming the object of debate itself. *It is remarkable how mathematicians (and not primarily didacticians of mathematics) are beginning to view the relevance and the effects of the view of mathematics of mathematics teachers for mathematical learning processes of learners as being important* (comp. e.g. Hersh 1997; Bass 1997).

When Davis (1994) points out to possible undesired developments in mathematics in this century, one must also point out that intensive discussion on possible misguided developments in the didactics of mathematics as well as in mathematics lessons is still at the beginning. Decisive responsibility in this respect lies in the hands of university mathematics teachers, and it is here that this study has its place. With the aid of the comments of Hersh it is becoming increasingly clear that we should pay more attention to the implicit world views of mathematicians. In this sense the following quote of Hersh is to be understood as a research project worth consideration:

> *The devastating effect of formalism on teaching has been described by others...* ***I haven't seen the effect of Platonism on teaching described in print.*** *But at a teachers' meeting I heard this "Teacher thinks she perceive other-wordly mathematics. Students is convinced teacher really does perceive other-wordly mathematics. No way does student believe he's about to perceive other-wordly mathematics."*

It therefore appears to us important on the one hand to take a closer, more detailed and precise look at the mathematics world view, and on the other hand to research into its effects on teaching, learning and doing mathematics. This however will demand more sophisticatedly developed methodological instruments than the questionnaire.

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Appendix 1: **Questionaire on Mathematics**

In order to shorten the length of the text, we do not transmit the whole questionnaire - we only transmit the statements. The categories for the answers were "wholly correct", "on the whole correct", "undecided", "only partly correct" and "not at all correct".

	***My* experiences with usual study of mathematics at university**
1.	I like to teach mathematics.
2.	Only those parts of mathematics which are tested in exams and written tests are - in my opinion - importand and worth knowing.
3.	In the studies of mathematics, the students have to think logically and precise.
4.	In the daily studies of mathematics it is often more important to learn facts and results than to find ideas and continuing questions by oneself.
5.	Knowledge from previous themes only plays a marginal role in the study of mathematics - one can forget it in most cases.
6.	In my opinion it is important and interesting if the study of mathematics shows up intermediate connections and relationships between single contents of mathematics.
7.	It is one of the main aims of the study of mathematics to teach solving tasks to the students.
8.	In the study of mathematics, the students have to explain everything very precisely.
9.	If the student can apply an algorithm correctly, then he often regards this to be "understanding".
10.	I don't like to give mathematics lectures.
11.	In the study of mathematics it is sufficient for the students to learn only those things which are asked for in exams and written tests.
12.	In the study of mathematics, the students have to prove everything very exactly.
13.	Sometimes a lecturer has to say to his students: "In this case you have to learn it by rote."
14.	Mathematical tasks should be done with those procedures which have been taught recently in the lecture.
15.	In the study of mathematics, a good cognitive ability and imaginativeness is often more important than a good ability to learn and remember.
16.	The learning of systemized and structured mathematical knowledge is more important than an active development of such knowledge.
17.	In the study of mathematics, the students have to use the mathematical terms correctly.
18.	The proof of a formula is - from my point of view - less important for the student. It is importand and descicive that he can apply formulas.
19.	Discovering and Re-discovering mathematics is more important than teaching resp. learning of "complete mathematics".
20.	In order to be successful in the study of mathematics, one has to learn many rules, terms and procedures.
21.	The descicive point in the study of mathematics is to obtain the right result.

	Mathematics as a science from *my* point of view.
24.	Mathematics is a collection of procedures and rules, which precisely determine how a task is solved.
25.	Mathematics is an activity which is comprised of thinking about problems and gaining knowledge.
26.	Mathematics is a logically uncontradicted thought building with clear, precisely defined terms and unequivocally provable statements.
27.	Mathematics consists of ideas, terms and connections.
28.	Mathematics is characterized by strictness (rigor), namely a definitory strictness and a formal strictness of mathematical argumentation.

29.	Almost any mathematical problem can be solved through the direct application of familiar rules, formulas and procedures.
30.	Very essential aspects of mathematics are its logical strictness and precision, i.e. "objective" thinking.
31.	Mathematics requires new and sudden ideas.
32.	Indispensable to mathematics is its conceptual strictness, i.e. an exact and precise mathematical terminology.
33.	Mathematical activity consists of ordering experiences and principles that have been gained in the work with examples.
34.	Doing mathematics demands a lot of practice in following and applying calculation routines and schemes.
35.	Mathematical activity consists of discovering and re-discovering of mathematics.
36.	Mathematics especially requires formal-logical derivation and one's capacity to abstract and formalize.
37.	Doing mathematics means: understanding facts, realizing relationships and having ideas.
38.	Mathematical thinking is determined by abstraction and logic.
39.	Mathematics is the memorizing and application of definitions, formulas, mathematical facts and procedures.
40.	Central aspects of mathematics are flawless formalism and formal logic.
41.	Above all, mathematics requires intuition as well as thinking and arguing, both related to contents.
42.	Doing mathematics demands a lot of practice in following rules and laws correctly.
43.	In mathematics one can find and try out many things for him/herself.
44.	Mathematics consists of learning, recalling and applying.
45.	Mathematics originates from setting axioms or definitions and from a formal-logical decuction of statements.
46.	Central aspects of mathematics are contents, ideas and cognitive processes.
47.	Good mathematics provides many strategies and procedures which can be successfully applied in numerous situations.
48.	The crucial fundamental elements of mathematics are its axiomatics and the strict, deductive method.
49.	If you want to understand mathematics then you have to create mathematics.
50.	Characteristics of mathematics are clarity, exactness, and unambiguity.

	On the origin of mathematics
51.	Mathematics is almost always discovered by especially creative human beings, and the others have to acquire their knowledge.
52.	Mathematical tasks and problems can be solved correctly on different ways.
53.	New mathematical theory first originates only if the (flawless) proof for a number of statements is present.
54.	If one comes to grip with mathematical problems, he/she can often discover something new (connections, rules and terms).
55.	If you want to solve a mathematical task, there is (in most cases) only one correct solution which you have to find.
56.	When developing mathematical theory, it is only natural to make some mistakes; good ideas are what is important.
57.	The average human being is mostly only the consumer and reproducer of the mathematics that other human beeings have created.
58.	There is usually more than one way to solve tasks and problems.
59.	Flawlessness is first demanded for the logical safeguarding of mathematical statements, not during their development.
60.	Development and logical safeguarding of mathematical theory belong together, they are inseparable for the right mathematical thinking, research and problem-solving.
61.	Every human being can discover or re-discover mathematics.

62.	If one has to solve a mathematical task, one has to know the only correct procedure - otherwise one is lost.
63.	Development and logical safeguarding of mathematical theory are different, separable processes.

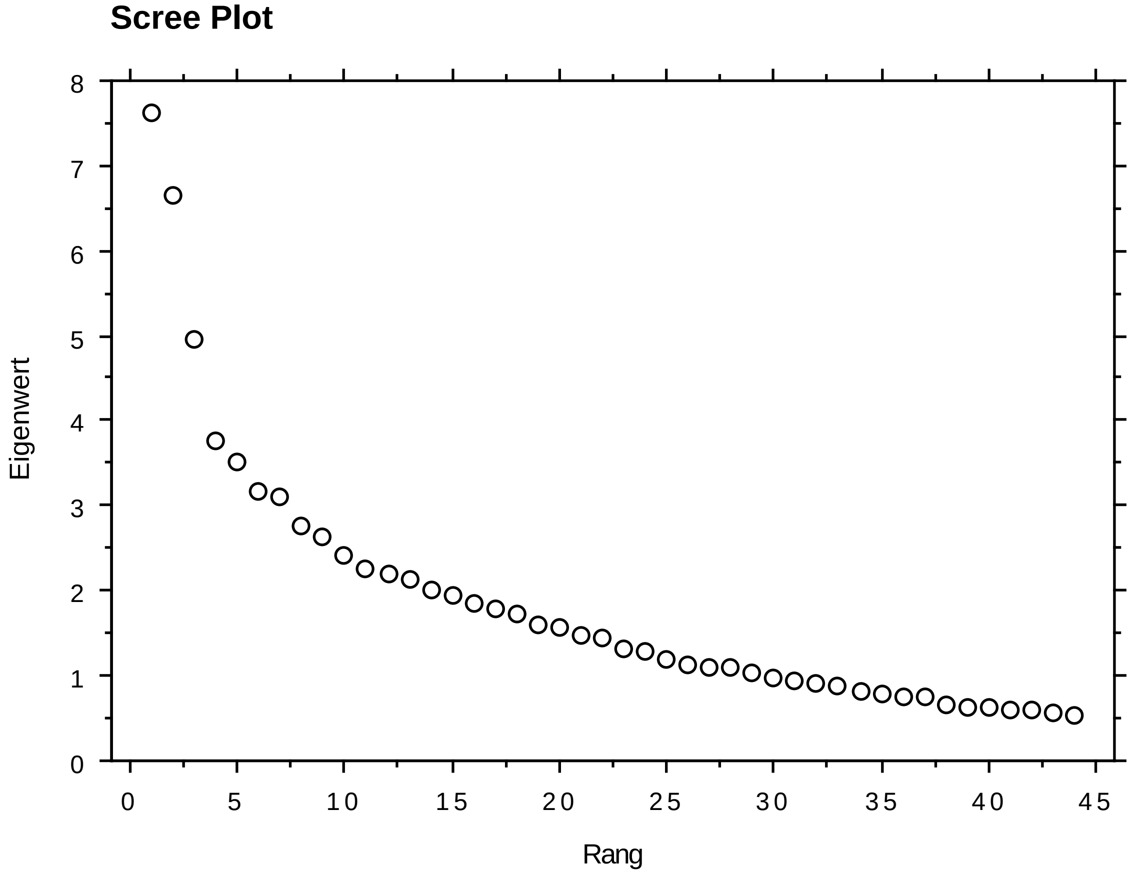
	Mathematics and reality
64.	The study of mathematics can give you a good, extensive knowledge for the reality - not more.
65.	The study of mathematics provides one with several abilities, which also continue to help in reality (for example, general clear thinking in abstract and complex situations, concrete calculations).
66.	Many parts of mathematics are either of practical use or are directly relevant to application.
67.	In the study of mathematics, one can - what ever will be taught - hardly learn something, which is useful in reality.
68.	Knowledge of mathematics is very important for the students later in life.
69.	Mathematics is of general, fundamental use to society.
70.	Only a few things learned from mathematics can be employed later in life.
71.	Mathematics is of use to any profession.
72.	Mathematics helps to solve daily tasks and problems.
73.	It would be worth enough if the study of mathematics delivers that knowledge which one needs in application, profession or life - anything more is waste of time.
74.	With regard to application and its capacity to solve problems mathematics is of considerable relevance to society.
75.	Mathematics is an aimless game, a occupation with objects without concrete relevance to reality.
76.	In the study of mathematics one is occupied with tasks which have a practical use.
77.	Some mathematical knowledge is important for some chosen professions.

	Mathematicians on mathematics
M1	God is a child, and he did mathematics as he began to play. It is the godliest of games among mankind.
M2	Mathematics consists of elaborate tricks and ornate methods, and resembles on the whole very much a crossword puzzle.
M3	It cannot be denied that a large part of elemental mathematics is of considerable, practical use. However, these parts of mathematics appear rather boring when observed as a whole. These are those parts which possess the least asthetic value. "Real" mathematics from "real" mathematicians such as Fermat, Gauß, Abel and Riemann is almost totally "useless".
M4	If we can approach to the godliest on no other way but by symbols, we will use the mathematical symbols because they possess undestroyable certainty.
M5	All pedagoges remain united in that one must diligently do mathematics above all else, because this knowledge provides one with the greatest direct use in practical life.
M6	The mathematicians, who are only mathematicians, are correct in their thinking, but only in the sense that all things can be explained to them using definitions and principles; otherwise their ability is limited and intolerable, because their thinking is only correct when it concerns only extremely clear principles.
M7	A pure mathematician has to leave the great task to reduce the suffer of mankind to luckier collegues.
M8	It happens that a mathematician has to judge a proof: "I see it, but I don't believe it."
M9	In mathematics education one can quickly obtain a high degree of formal education, because the memorizing abilities do not stand in the foreground. And consequently, pupils with a weak memory can find a way to formal education.

M10	And it seldom appears that the mathematicians are intuitive and the intuitives are mathematicians.
M11	Mathematics and nautural science are, where they had to restrict their high relevance for the vivid reality, for their weak representation reputed to be totally apart from normal human thinking.
M12	When the laws of mathematics are related to reality they are not secure, and when they are secure, they are not related to reality.
M13	It must always be possible to say "tables, chairs, beer-glasses" instead of "points, straight lines, planes".
M14	The knowledge of the godliest is unattainable for a mathematically totally uneducated person.

Appendix 2: Eigenvalues and Scree-Plot

Principal components analysis for 89 variables



Eigen-value no	value	variance contrib.*	Eigen-value no	value	variance contrib.*	Eigen-value no	value	variance contrib.*
1	7,636	8,6	16	1,850	2,1	31	0,939	1,1
2	6,660	7,5	17	1,798	2,0	32	0,924	1,0
3	4,963	5,6	18	1,730	1,9	33	0,868	1,0
4	3,751	4,2	19	1,586	1,8	34	0,831	0,9
5	3,510	3,9	20	1,560	1,8	35	0,793	0,9
6	3,162	3,6	21	1,467	1,6	36	0,756	0,8
7	3,097	3,5	22	1,429	1,6	37	0,743	0,8
8	2,768	3,1	23	1,324	1,5	38	0,664	0,7
9	2,637	3,0	24	1,290	1,4	39	0,643	0,7

10	2,416	2,7	25	1,190	1,3	40	0,625	0,7
11	2,268	2,5	26	1,129	1,3	41	0,605	0,7
12	2,196	2,5	27	1,110	1,2	42	0,582	0,7
13	2,118	2,4	28	1,088	1,2	43	0,563	0,6
14	2,007	2,3	29	1,020	1,1	44	0,518	0,6
15	1,932	2,2	30	0,982	1,1			

* Percentual contribution of the eigenvalue to the normalized variance of 89

Appendix 3: Factor loadings in the orthogonal 7-factor-solution

item-no.	factor 1	factor 2	factor 3	factor 4	factor 5	factor 6	factor 7	comm.[6]
1	,142	-,132	,212	,001	,208	,181	,079	,165
2	,007	,548	,101	,077	-,007	-,178	,264	,418
3	,570	-,126	-,034	,074	-,075	-,043	,067	,359
4	-,091	,032	,086	,021	,564	,043	-,004	,337
5	,025	,143	-,008	-,081	,330	,008	,002	,137
6	-,043	-,083	,236	,038	-,041	-,136	-,187	,121
7	,016	,376	,220	,083	,218	-,046	-,068	,251
8	,505	,041	-,093	,135	,281	-,110	-,230	,428
9	-,007	,077	,039	,282	,015	,049	,048	,092
10	-,341	,456	-,145	-,091	-,277	-,137	-,130	,466
11	-,048	,187	,055	,178	,492	-,049	-,039	,318
12	,392	,030	-,047	,201	,306	-,104	-,113	,314
13	,096	,231	-,107	,325	,071	,098	-,181	,227
14	,038	,122	,245	-,116	,094	,180	,029	,132
15	,064	,054	,353	-,014	-,156	-,087	,062	,168
16	,170	,051	-,103	-,152	,484	-,097	-,005	,309
17	,591	-,238	,074	,249	,216	-,032	,045	,523
18	-,195	,504	,062	-,096	,171	-,077	,140	,360
19	,014	,126	,287	,012	-,367	,124	,112	,261
20	,041	,339	,045	,046	,411	,137	,031	,309
21	-,004	,183	,111	,191	,202	-,267	,108	,206
24	,018	,686	-,002	,102	,057	-,164	-,066	# ,516
25	,098	,105	,473	,045	-,246	-,154	-,139	# ,350
26	,479	,188	-,048	-,083	-,026	-,087	,119	# ,296
27	,060	-,111	,444	,030	,131	-,329	-,029	# ,340
28	,473	-,023	,120	-,015	-,063	,012	-,085	# ,250
29	,102	,514	-,138	-,000	-,036	-,048	,047	# ,299
30	,669	-,024	-,117	,039	,011	,066	-,019	# ,468
31	,120	-,261	,476	,137	-,019	,057	,141	# ,351
32	,594	-,116	,188	,206	,140	-,086	-,160	# ,497
33	-,290	,305	,354	-,007	,164	,003	-,082	,336
34	,071	,413	,109	-,121	,231	,122	,060	# ,274
35	,110	,129	,330	,053	-,185	,102	,004	,185
36	,451	,173	,035	,029	,073	,258	,111	# ,320
37	,158	-,007	,451	,241	,048	,255	-,049	# ,356
38	,587	,150	,079	,070	-,039	,096	-,128	# ,405
39	,148	,628	-,108	,073	,148	,124	-,163	# ,497
40	,507	,319	,090	-,033	,086	,134	-,126	# ,409
41	-,019	,074	,592	,040	,048	-,061	-,090	# ,372
42	,282	,330	,067	-,006	,171	,246	,140	,302
43	-,031	-,188	,413	,091	-,101	,099	,284	# ,316
44	,072	,616	,177	-,042	-,076	,195	,171	# ,491
45	,399	,423	,022	-,050	-,044	,284	,045	,426

6 The column "comm." contains the communalities. Those items which have been chosen for the analysis in this text have been marked with a "#".

46	,048	-,022	,534	-,012	,180	-,048	-,197	# ,362
47	,081	,043	,366	,224	-,020	,145	,064	,218
48	,561	,233	-,121	-,094	-,048	,256	-,019	# ,461
49	,199	,010	,344	,314	-,016	-,247	,184	,352
50	,570	-,017	,217	,115	-,054	-,170	,027	# ,418

item-no.	factor 1	factor 2	factor 3	factor 4	factor 5	factor 6	factor 7	comm.
51	,327	-,142	,069	-,042	,211	-,204	,055	,223
52	,033	-,054	,062	,034	-,012	,628	-,083	,410
53	,518	,226	,070	,026	-,209	,042	,073	# ,376
54	,051	-,111	,404	,231	-,093	,178	-,108	# ,283
55	,022	,450	,021	-,081	,143	-,477	-,034	,459
56	-,269	,021	,443	-,071	,173	,003	,201	# ,344
57	,220	-,066	,145	-,090	-,017	-,001	-,095	,091
58	-,010	,015	,071	,105	,073	,506	,082	,284
59	-,101	-,163	,431	,061	,067	,203	,070	# ,277
60	,465	,058	-,185	,057	-,098	,028	,095	# ,276
61	-,140	,127	,245	,181	,229	,095	,094	,199
62	,109	,553	-,107	-,128	,097	-,495	,131	,617
63	-,154	,098	,321	-,120	,205	,028	-,006	,194
64	,074	,400	-,183	-,192	,007	-,245	,095	,305
65	,190	-,230	,191	,417	-,015	,175	,083	# ,337
66	-,036	-,099	,175	,489	-,134	-,162	,104	# ,336
67	-,092	,151	-,134	-,382	-,056	-,019	,168	,227
68	,118	-,157	,018	,567	-,084	,047	,151	# ,392
69	,428	-,267	,059	,470	,021	-,087	,073	# ,492
70	,027	,213	-,058	-,503	,173	,206	,151	# ,398
71	-,028	,291	-,063	,544	,133	-,004	,101	# ,413
72	-,012	-,008	,021	,716	,112	,093	-,003	# ,535
73	,008	,282	,036	-,184	-,061	-,037	,298	,209
74	,335	-,195	,068	,488	,027	-,155	-,002	# ,418
75	-,065	,055	-,077	-,378	,211	-,148	,342	,339
76	,119	,240	,129	,280	-,107	,021	,308	,274
77	,110	-,155	,220	-,095	-,032	,279	,025	,173
M1	,034	-,028	-,050	,221	-,057	-,023	,416	# ,230
M2	-,302	,474	-,121	-,016	-,086	,099	,132	# ,365
M3	-,226	,052	-,068	-,168	,030	,013	,529	# ,368
M4	,185	,069	-,070	,187	-,100	,073	,294	,181
M5	,250	,243	-,003	,462	-,189	,032	,246	# ,432
M6	-,024	,115	,043	,033	,031	-,016	,606	# ,385
M7	,109	-,058	,114	-,390	-,107	-,046	,255	,259
M8	-,041	-,040	,179	-,018	,272	,051	,337	,226
M9	,074	,096	,303	-,007	-,056	,156	,177	,165
M10	-,077	,225	,002	-,213	,091	,008	,287	,193
M11	-,167	-,084	-,139	,027	,277	,013	,231	,185
M12	-,283	-,018	,104	,045	,172	-,057	,405	# ,290
M13	-,012	-,036	,176	,175	,018	-,152	,154	,110
M14	,006	,218	,033	,131	-,185	-,194	,249	,200
sum of squares:	6,043	5,529	4,111	4,249	2,775	2,725	2,807	sum: 28,240

Appendix 4: Factor loadings of the orthogonal 4-factor-solution

item-no.	factor 1	factor 2	factor 3	factor 4	comm[7].	item-no.	factor 1	factor 2	factor 3	factor 4	comm.
1	,144	-,085	,306	-,042	,123	48	,593	,194	-,053	-,143	# ,413
2	-,033	,595	,052	,195	,396	49	,136	,050	,275	,415	,269
3	,537	-,115	-,035	,108	,314	50	,521	-,006	,146	,192	# ,330
4	-,066	,111	,174	-,026	,048	51	,275	-,059	,047	,022	,082
5	,037	,195	,042	-,096	,050	52	,117	-,128	,224	-,141	,100
6	-,044	-,119	,140	,054	,039	53	,501	,194	,047	,066	# ,295
7	,036	,373	,201	,088	,189	54	,079	-,181	,403	,168	# ,230
8	,519	,029	-,096	,111	,292	55	-,016	,480	-,128	,056	,250
9	,005	,056	,065	,264	,077	56	-,304	,107	,468	-,033	# ,324
10	-,312	,366	-,261	-,052	,302	57	,216	-,077	,117	-,088	,074
11	-,021	,228	,103	,144	,084	58	,048	-,009	,238	-,026	,060
12	,398	,043	-,028	,184	,195	59	-,099	-,148	,467	,018	# ,250
13	,159	,144	-,077	,243	,111	60	,453	,055	-,157	,071	# ,238
14	,053	,135	,302	-,152	,135	61	-,127	,156	,309	,152	,159
15	,029	,054	,275	,052	,082	62	,046	,615	-,239	,050	,440
16	,157	,146	-,050	-,137	,067	63	-,152	,136	,326	-,118	,162
17	,564	-,194	,124	,247	,432	64	,042	,443	-,257	-,086	,271
18	-,203	,559	,056	-,040	,358	65	,196	-,251	,269	,402	# ,335
19	,003	,075	,243	,039	,066	66	-,068	-,119	,124	,542	# ,328
20	,078	,373	,143	-,004	,166	67	-,120	,210	-,136	-,325	,183
21	-,038	,239	,069	,268	,135	68	,112	-,182	,065	,551	# ,353
24	,040	,646	-,080	,152	# ,448	69	,402	-,265	,071	,484	# ,471
25	,078	,040	,404	,112	# ,183	70	,029	,293	,033	-,512	# ,350
26	,443	,221	-,069	-,009	# ,250	71	-,001	,268	-,013	,518	# ,340
27	,005	-,056	,412	,127	# ,189	72	,034	-,067	,096	,619	# ,398
28	,470	-,048	,093	-,007	# ,232	73	-,039	,350	,039	-,094	,134
29	,112	,498	-,171	,043	# ,292	74	,316	-,210	,046	,509	# ,405
30	,674	-,033	-,082	,027	# ,463	75	-,136	,213	-,048	-,263	,135
31	,083	-,229	,493	,153	# ,326	76	,089	,258	,153	,331	,207
32	,589	-,131	,162	,205	# ,432	77	,124	-,162	,294	-,156	,152
33	-,263	,296	,323	-,013	,261	M1	-,014	,033	,001	,270	# ,074
34	,092	,446	,164	-,132	# ,252	M2	-,277	,453	-,096	-,015	# ,291
35	,112	,083	,300	,059	,113	M3	-,286	,188	,012	-,082	# ,124
36	,468	,178	,140	-,014	# ,271	M4	,128	,139	,022	,235	,091
37	,194	-,052	,423	,163	# ,246	M5	,236	,212	,019	,488	# ,339
38	,613	,094	,078	,045	# ,393	M6	-,100	,249	,119	,132	# ,104
39	,219	,552	-,081	,016	# ,359	M7	,036	,033	,091	-,282	,090
40	,544	,281	,106	-,064	# ,390	M8	-,081	,085	,273	,009	,088
41	-,027	,064	,508	,069	# ,268	M9	,060	,115	,343	,002	,134
42	,304	,353	,180	-,044	,251	M10	-,112	,316	,047	-,156	,139
43	-,076	-,138	,409	,120	# ,207	M11	-,181	,014	-,037	,023	,035
44	,089	,596	,207	-,029	# ,407	M12	-,324	,085	,147	,096	# ,143
45	,437	,425	,086	-,086	,386	M13	-,055	,007	,144	,243	,083
46	,055	-,027	,463	-,013	# ,218	M14	-,046	,242	-,030	,241	,120
47	,087	,022	,399	,203	,208						
						sum of squares:	6,058	5,849	4,247	4,148	sum: 20,303

7 The column "comm." contains the communalities. Those items which have been chosen for the analysis in this text have been marked with a "#".

	variance contribution to the total communality (= 20,3)		variance contribution to the total variance (= 89)	
factor		cumulated		cumulated
1	6,06 = 29,8 %	29,8 %	6,06 = 6,8 %	6,8 %
2	5,85 = 28,8 %	58,6 %	5,85 = 6,6 %	13,4 %
3	4,25 = 20,9 %	79,5 %	4,25 = 4,8 %	18,2 %
4	4,15 = 20,4 %	99,9 %	4,15 = 4,7 %	22,9 %