

Teachers' Beliefs on Mathematics Teaching – comparing different self-estimation methods – a case study

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Abstract

Our primary concern is the quantitative and qualitative investigation of teachers' views of mathematics and its teaching. Hereby, Dionne and Ernest's characterization of beliefs on mathematics serves as a theoretical background; the dominant perspectives of views on mathematics can be described as toolbox aspect, system aspect and process aspect. Originally, our test subjects numbered a total of 13 experienced German teachers, whereby we restrict ourselves to 6 persons. In particular the experiences with the Dionne method are to be discussed here, whereby beliefs are interpreted as weight distributions for characteristic core representations, and are quantified by this method.

1. Introduction.

The literature contains controversial and abundant contributions¹ concerning teachers' beliefs and concepts as well as the process by which teachers change them in regard to their profession (see the survey articles of Thompson (1992) and Pehkonen & Törner (1996)). Marginal attention is dedicated, however, to the corresponding methodological questions concerning the research of these beliefs (Grigutsch 1994). According observations are to be reported here.

Originally, our test subjects amounted to a total of 13 experienced German teachers, five of whom teach at the *Gymnasium*, two at the *Realschule*, one at the *Hauptschule*, and five at the *Gesamtschule*. Here, we will limit ourselves to a study of six people.

1.1. Definition of Beliefs and their Role

The purpose of this subsection is to give a self-contained survey on the terminology used in this paper. For a survey on research conducted into teachers' beliefs, the reader is referred to the research synthesis of Thompson (1992) or Pehkonen (1994). However, there are hardly any no articles focusing on mathematical beliefs of German mathematics teachers.

An individual continuously receives perceptions from the world around him or her. According to his or her experiences and prior perceptions, s/he makes conclusions about different phenomena and their nature. The individual's personal knowledge, i.e. his or her beliefs, is a combination of these conclusions.

The concept of mathematical beliefs has many definitions in the literature. Here, we understand (mathematical) beliefs as one's subjective knowledge (which also includes affective loadings) of a certain object or concern for which indisputable grounds may not necessarily be found in objective considerations. One feature of such beliefs is that they can be held with varying degrees of conviction (Thompson, 1992). Beliefs cover personal convictions mixed with facts and external knowledge; thus the beliefs' subjective certitude range from truth-like facts to vague assumptions. Not in

¹ see <http://www.uni-duisburg.de/FB11/PROJECTS/MAVI.html>

all cases: the individual is however aware of the truth-degree of beliefs. In this case we are speaking of unconscious beliefs

The process of how and for what reasons a belief is adopted and defined by the individual him/herself is not well understood. The adoption of a belief may be based on some generally-known facts as well as beliefs, and on logical conclusions drawn from them. But in each case, the individual makes his or her own choice of the facts and beliefs to be used as reasons and his and her own evaluation of the acceptability of the belief in question; recently this question is partly discussed in the Schoenfeld's book (1998) *Toward a theory of teaching-in-context* (draft version²). Often for the individual there seems to be no objective distinction between facts and beliefs.

The individual compares his/her beliefs with new experiences and beliefs of other individuals and therefore these beliefs are subjected to a continuous evaluation and undergo change. When an individual adopts a new belief, it will automatically form a part of the larger structure of his/her personal knowledge and of the belief system, since beliefs never fully develop independently (Green, 1971).

It was pointed out by Green in 1971, beliefs come always in sets or groups, never in complete independence of one another. Thus we assume that an individual's beliefs form a structure. We will call this construct of beliefs a belief system or more generally his/her *views* of mathematics. This wide spectrum of beliefs around mathematics contains four main components which are also relevant for mathematics teaching: (1) beliefs about mathematics, (2) beliefs about oneself as a user of mathematics, (3) beliefs about teaching mathematics, and (4) beliefs about learning mathematics. These main groups of beliefs, in turn, can be split into smaller units. It is evident that these 'dimensions of beliefs' are interrelated. For more details on these ideas see Pehkonen (1995).

Törner & Grigutsch (1994), and also elaborated in a more recent paper by Grigutsch; Raatz & Törner (1998) attempted to investigate belief system structures using factorial analysis. They used the term "mathematical world view" which originally can be found in Schoenfeld's discussions (1985). This is why we adopt in this paper the abbreviated term "views".

1.2. Beliefs resp. Views on Mathematics and its Teaching

It is evident that views, in particular on mathematics, contribute an essential part to any belief system on mathematics. Thus there are numerous papers focusing this aspect of a belief system. For his research, Dionne (1984) used the following three perspectives of mathematics:

(T) Mathematics is seen as a set of skills (*traditional* perspective): Doing mathematics is doing calculations, using rules, procedures and formulas.

(S) Mathematics is seen as logic and rigor (*formalist* perspective): Doing mathematics is writing rigorous proofs, using a precise and rigorous language and using unifying concepts.

(P) Mathematics is seen as a constructive process (*constructivist* perspective): Doing mathematics is development thought processes, building rules and formulas from reality experience reality and finding relations between different notions.

It seems obvious that these perspectives reflect guiding aspects of mathematics; however different persons might evaluate these components differently and so Dionne let his test subjects assign weights to these 'dimensions'. In his book, Ernest (1991) describes three similar views of mathematics: instrumentist, Platonist and problem solving. These correspond more or less to the three perspectives Dionne (1984) mentioned above. Of course, there might be further relevant aspects

² <http://www-gse.berkeley.edu/faculty/aschoenfeld/TeachInContext/tic.html>

characterizing mathematics (see the book of quotations by Schmalz (1993)), e.g. mathematics as an art, however these seem to be the most leading ones in school mathematics.

In the following we define the basic view (T) as the toolbox view of mathematics, the formalistic perspective is interpreted as the system dimension (S) of mathematics, and the constructivist perspective is understood by us as the process aspect (P) of mathematics.

Finally, it is relevant to us to point out that in the present survey it appears problematic to distinguish between beliefs on mathematics and beliefs on the teaching of mathematics. In the university context of mathematics this may be easier: there is mathematics mediated to the students in lectures and seminars, there is applied mathematics, there is mathematics in research literature, etc. There are good arguments for assuming that mathematics in distinct contexts bears witness to various aspects of mathematics. In this view beliefs on the teaching of mathematics are not automatically linked to fundamental beliefs on mathematics.

The situation of mathematics in schools appears to us, however, to be a different one. In school, mathematics appears solely in the form of mathematics taught by teachers to pupils. Further aspects for both groups lie to a great degree beyond the scope of mathematics in schools. We therefore see no possibility of distinguishing between beliefs on mathematics and beliefs on the teaching of mathematics. Incidentally, the conditional inseparability is underlined by the well-known quotation³ of René Thom.

2. The Design of the Research

2.1. The General Setting

The subject of our survey, out of which we report some aspects here, is the survey of the view of mathematics of teachers from different school forms in North Rhine-Westphalia (Germany). Theme-centered, open interviews were conducted with 13 teachers who additionally filled out a questionnaire in our presence. Both situations were recorded on video. In a correspondence of letters following the interviews the teachers were requested to give a twofold self-assessment of their mathematics lessons, first by distributing 30 points to the factors (T), (S) and (P), and second by marking their own individual self-assessment on real mathematics lessons verses ideal mathematics lessons into an equilateral triangle with the three corner points representing (T), (S) and (P). This form of data representation, which develops the so-called Dionne method a step further, had previously not been described in the literature of the didactics of mathematics.

2.2. The Dionne Method

In a survey of Junior School teachers Dionne sees himself confronted with the problem of characterizing the beliefs of the test persons numerically. He reduces and quantifies the attitudes of the teachers by means of vectors with a standardized list of weightings by letting the teachers name their subjective numerical evaluations of three aspects of mathematics. More exactly: the teachers have to assign natural numbers to the three aspects (T), (S) and (P) (see above), whereby the sum of the three numbers has to amount to 30. For Dionne, this triple number characterizes the attitude of the teachers. Incidentally, by this method he views the field of beliefs as being two-dimensional. One should not forget that amidst all criticism of this admittedly rather crude method can be considered appropriate for a survey of mathematics at Junior level.

³ In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rest on a philosophy of mathematics. in Thom, R. 1973. Modern mathematics: does it exist? In A.G. Howson (Ed.), *Developments in Mathematical Education. Proceedings of the Second International Congress on Mathematical Education.* (pp. 194 - 209). Cambridge: Cambridge University Press.

Characteristic for this method is also that it is a self-assessment of the test persons, whereby it is relatively easy to determine the values.

The following table shows some median values from diverse surveys, also including the Dionne survey [1].

	T	S	P
Elementary Teachers (Dionne (84), N = 18) (Pretest)	10.8	6.3	13.7
Elementary Teachers (Dionne (84), N = 18) (Posttest)	9.5	7.5	13.4
Elementary Teachers (Dionne (84), N = 15) (Control group, Pre-test)	7.5	9.5	13.0
Elementary Teachers (Dionne (84), N = 15) (Control group, Post-test)	10.4	9.8	9.8
Teachers at Comprehensive Schools (Törner, 1995; N = 19)	10.8	11.2	8.0
Teachers at Gymnasium (Törner, 1994; N = 14)	12.8	10.1	7.1
Students at University (Törner, 1997; N = 15)	6.4	11.8	11.8

However one explains the individual differences, it is obvious that the self-assessments of the various groups on the aspects of mathematics clearly diverge.

3. Research Questions

At earlier stages authors have already phenomenologically employed the Dionne method. When giving self-assessments it quickly became apparent to us that the teachers like to distinguish between the reality of mathematics teaching on the one hand and ideal mathematics lessons on the other hand. We have taken this into consideration by taking into account **two** self-assessments for each individual test person, namely their view of their mathematics lessons in reality and also their view of ideal mathematics lessons.

After all, we are conscious of the fact that the weighting allocation to the components can only be conditionally objectified for the test persons. Beyond this, the Dionne method demands the nomination of three numbers of which the sum total underlies a marginal condition, namely it must amount to 30. Thus an in principal equivalent but in detail completely different graphic data allocation method appeared to us to be appropriate.

When one graphically represents the solution space of all possible answers by means of an equilateral triangle with the three corner points representing the dimensions (T), (S) and (P), then all the non-negative number triples (with the sum of 30 of the entities) can be converted into barycentric coordinates, which delivers a point inside the triangle. Inversely, all the points in the triangle can be interpreted as a triple; the corner points (T) resp.(S) resp. (P), then correspond to the points (30, 0, 0) resp. (0, 30, 0) resp. (0, 0, 30).

In view of the above described starting situation we intend to deal with the following research questions:

(1) *To what extent are the principally redundant information from both self-assessments of the test persons on mathematics lessons compatible to each other?*

By this approach we hope to indirectly conclude on the usefulness of the graphic method presented here. We also expect illumination on the processes leading to the points distribution within the self-assessment tasks.

4. Results

Before we discuss the results of the self-assessments of the test persons, the involved teachers are to be briefly introduced here in a gender-neutral fashion by employing quotations recorded during the interviews with the test persons. We limit ourselves to the discussion of 6 teachers from the group of altogether 13 teachers to keep the case study clear and concise. We furthermore consider the 6 persons to be typical. They are also representative for teacher profiles and school forms according to formal teacher certifications. For reasons of conciseness we do not offer an overview of the German school system here but refer instead to Robitaille (1997).

4.1. Quotations from the interviews.

Dylan (Dy) is a teacher in a Gesamtschule; she teaches mathematics in both the lower and the upper secondary classes. Dylan's view to mathematics and its teaching is revealed in the following quotation: *"I do not regard mathematics as dry, I find it fascinating. To me, mathematics is alive and I derive pleasure from it."* And furthermore: *"It is always of importance to me that the teacher makes the textbook understandable to the students ... That forms a visible line throughout my teaching, up to the maturity examination."* Whereas the next quotation shows that for him formalism is not important: *"Proof, just for the sake of proof I regard as arrogant abundance. Pillars could support mathematics just as well as solid walls."*

Harry (Ha) is a teacher at a Hauptschule; his pupils are aged 10-16. Harry's mathematical view is strongly process-oriented: *"I disapprove of any product-orientation, I regard the process as being too important"* a statement he mentioned several times. The core idea of his teaching is described by: *"We have to find a way to meet the demands of the labour market"*. Harry and his students have fun and derive pleasure from mathematics, *"It is a decisive factor that education should be enjoyed by my students as well as by me."*

Henry (He) has the same teacher's qualification as Dylan, i.e. he teaches in both the lower and the upper secondary classes. He also teaches at a Gesamtschule. Henry's view of mathematics is toolbox-centered: *"What works well, is giving formulas. When you give the formulas which are present in the classwork, you get the results."* His statements are reminiscent of a factory worker who manufactures products. Another notable point is his frequent use of the word "thing", when referring to mathematical contents. Furthermore, he himself offers another metaphor, namely that of a nursery school teacher who, *"... leads small children by the hand through a garden without leaving the path, in order not to confront unexpected things"*. This teacher made a tired, uninvolved impression.

Joseph (Jo) is the only teacher who teaches at a Realschule. Joseph's view on mathematics could only be indirectly considered as belonging to teaching mathematics. He understands his teaching as a continuous use of worksheets, which points to the toolbox aspect. His principle of teaching is reflected through the following quotation: *"The teacher-centered lesson has proven to be successful, because questions could be answered and problems cleared with regard to the whole class."* Joseph makes a tired, resigned impression, and is often preoccupied with his own thoughts.

The following mentioned teachers Ken (Ke) and Larry (La) have completed a full academic course on mathematics and are qualified to teach in all classes of the Gymnasium. They are also both employed as student teacher trainers.

Ken is always in the process of keeping his mathematical knowledge up-to-date on an elaborating level. His view can be considered as well-reflected, detailed and balanced. This interpretation is supported by the following quotation: *"In the beginning, I was strongly structurally-oriented; fractions were dealt with rather formalistically... and Freudenthal was not much of a help either because he, too, is very rule-oriented in this regard"*. He continues: *"My openness implies that I try to involve more visual elements than formalism, which compels..."* and further *"I came to realize that visuality stays in the memory longer, as well as association..."*. As a teacher, he is realistic,

pragmatic and at the same time lacking illusions: “*I suspect that my teaching was teacher-oriented in the beginning, and I suppose it still is today*”.

Larry regards mathematics as “*a colorful structure that allows the formal contents to be dealt with abstractly and systematically*”. He is still in contact to his former university and would like to obtain some new impulses from there. His teaching is moderately teacher-oriented, but his starting point, in general, is mathematics-oriented. His knowledge on mathematical topics is convincing. Furthermore, during the interview there was no mention of teaching in groups. In addition, he in fact expressed several objections against project-work, for example.

Further quotations are integrated in the discussion following in the next subsections. Finally, additional interview results can be found in the paper by Pehkonen & Törner (1998) focusing however the change aspect.

4.2. Self-Estimations

4.2.1 The Numerical Self-Estimation

In Table 4.1 we give the scores which teachers attribute to three components of the view of mathematics (see Section 1.2.), and which was asked for in the letter (see Appendix 1) sent to the teachers during the second round of the study.

Teacher	Tool	System	Process	Tool	System	Process
	REAL			IDEAL		
Dylan	15	5	10	5	5	20
Harry	9	1	20	4	1	25
Henry	14	8	8	6	12	12
Joseph	15	3	12	10	5	15
Ken	8	10	12	10	8	12
Larry	9	9	12	6	9	15

Table. 4.1. Scoring of the self-estimation; Tool = mathematics as a tool box, System = system aspect of mathematics, Process = process aspect of mathematics. The leading positions are marked in bold.

Each teacher, with the exception of Ken, wanted to change, more or less, towards “process“; also, Harry wanted to emphasize the process aspect even more.

At a first glance, the following is not surprising and became obvious through the table: None of the teachers chose an extreme position, neither in their real view nor in their ideal view. Thus Dionne’s polarization is in a particular way balancing.

Each teacher regarded the process aspect as the most important factor in his ideal view. Harry gave the process aspect the highest loading, which corresponded with his quotations in the interview, also regarding real teaching.

It is noteworthy that the estimations of Ken and Larry, though they had not met each other before, were about the same regarding their real teaching. The interviews depicted both teachers as highly qualified in mathematics (see Section 4.1). Also, Figure 4.1 showed the teachers as being close in their estimations of mathematics. They are more or less satisfied with the actual classroom situation. Larry points out that mathematics lessons are generally not very encouraging and motivating because of the subject. And therefore, Larry “... *is continuously looking for external stimulation*“. Larry hereby underlines his need for external stimulation for variation of lessons.

The interviews also supported the entries in Table 4.1, in which Dylan, Joseph and Henry share the highest loadings with respect to the toolbox aspect, namely “doing mathematics means working with figures, applying rules and procedures and using formulas“.

Of course, there is no canonique meaning and implication what toolbox aspect means, however the teachers' description in the interview is fitting to this line: Dylan points out the importance of a students' ability to handle school books, whereby Joseph and Henry believe that routine exercises are the only possible way to motivate the majority of the students within a class. Again, the three persons mentioned are precisely those teachers who are not quite satisfied with their own teaching of mathematics and would like to change their situation, however through quite different ways.

On the basis of the interview we may classify Dylan and Harry as the most innovative among these six teachers. The tendencies in their ideal view of mathematics are the same, however there are small differences concerning the role of systems and structures in mathematics (Dylan, System = 5; Harry, System = 1). Minor differences may originate in their different academic careers. However, on the basis of the figures for the real classroom lesson, the assessment of Dylan (Process = 10) is considerably more rational than Harry (Process = 20). Perhaps this discrepancy is explained by the fact that Dylan in contrast to Harry has completed a higher university degree, so Dylan’s mathematical horizon can be regarded as broader and so his estimation of what is happening in school is more modest.

Since there is no objective scale for the three mentioned aspects, the absolute numbers should not be overestimated. Moreover, it seems natural to us that primarily the weights set by the teachers, not the exact scoring, indicate their understanding of mathematics teaching, which leads to a linear ordering of the components. Thus, we derive Table 4.2:

<i>Teacher</i>	real	ideal
Dylan	$T > P > S$	$P > T = S$
Harry	$P > T > S$	$P > T > S$
Henry	$T > P = S$	$P = S > T$
Joseph	$T > P > S$	$P > T > S$
Ken	$P > S > T$	$P > T > S$
Larry	$P > T = S$	$P > S > T$

Table 4.2. The ranking of the components derived from Table 4.1 with T = tool, S = system and P = process aspect

Apparently, with the exception of Ken, all teachers placed the formalism resp. system aspect not dominant secondly in their real teaching. This can be understood through the interviews: In the past, Ken was extremely in favor of the formalism aspect; this conception may not have disappeared completely. It is remarkable that Larry has given formalism together with toolbox an equal ranking. This fact may probably be explained through their teaching career in a *Gymnasium* and its mathematics curriculum.

On the other hand, with respect to ideal teaching, the process aspect is ranked first by all of the teachers, and, in the case of Henry, on the same level as the system aspect. These observations are certified in the interview when he is favouring stronger dominance of formalistic aspects. Henry seems to be obliged to mathematics, which he has been taught at the university to be of structural importance. In the interview, the authors got the impression that he feels somewhat guilty since in his opinion his actual situation makes it impossible to present this subject in an adequate manner.

4.2.2. Triangular approach.

In Figure 4.3 we illustrate the marks within the equilateral triangle which were taken from the teachers' original responses (Appendix 1).

There are three features which come to mind at first glance: (1) the distribution of the respective positioning, (2) the tendencies of change which are represented as vector arrows and (3) the magnitude of assumed change.

First, the outer distribution underlines the observations in 4.2.1. Dylan, Joseph and Henry are found, with their implemented lessons, in the toolbox corner. We have listed some corresponding quotes in Section 4.1.

Secondly, the predominant tendency of change for the teachers underlines the importance of the process aspect. The two *Gymnasium* teachers Ken and Larry show some slight differences, in particular Larry. It should be noted that Larry does not estimate the necessity of change in his own classes as very high. Furthermore, the interview reveals that he does not believe in the possibility of fundamental change in the present system. Possibly the daily disturbances that take place in his classes lead only to marginal frustration, because for him (as is with Ken) mathematics exists outside of the classroom and, as a result, is a pure and philosophical discipline worthy of respect. Equally, he has fitted himself into the system.

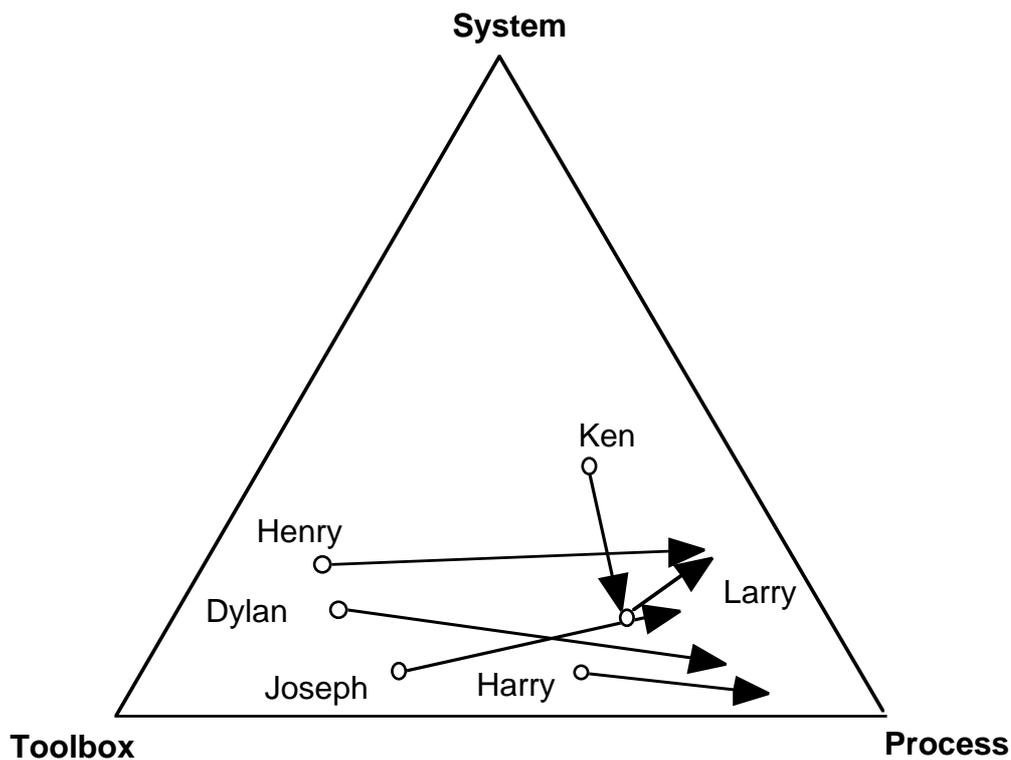


Figure 4.3. The self-estimation data in graphical form as given by the teachers. Arrows (real to ideal) indicating tendencies are drawn by the authors⁴.

⁴ The idea to show the tendency with arrows is due to Peter Berger (1995).

Thirdly, the diagram points on the whole to three classes in view of the arrow lengths: Dylan, Joseph and Henry; then Ken and Larry, whereby Harry takes on a middle position. These observations do not contradict the interview data when intervening feelings are included as a measure for change. In view of Ken and Larry we have already given explanations above. The numerically determined Euclidean distance on the basis of the table 4.1 is for Ken $\sqrt{8}$ and for Larry $\sqrt{18}$.

4.2.3. Comparison of Both Self-Estimations.

The question arises as to which data should be more seriously regarded: the graphical or the numerical data, or whether both 'messages' are equivalent. Of course, the representation modes are equivalent in a mathematical sense, each three-element-distribution can be calculated as a vector in the triangle and vice versa.

Next, we believe that the teachers are not trained to transform numerical data into barycentric coordinates. However, since the teachers are trained in mathematics they should be able to handle graphics. Furthermore, we don't believe that the teachers attempted to present direct translations of both data sources. However there was no chance to interview the teachers about that fact.

Our hypothesis is that both representations have their own messages and may cover partly different aspects. Many entries in the graph as well in the table could be supported by arguments and quotations from the interviews. Thus, the fact that corresponding data are not exactly coincident should not be overemphasized.

For example, Henry estimated his mathematics view by Toolbox = 14, System = 8, Process = 8, thus the aspects of System and Process are playing an equivalent but lower estimated role. In the ideal teaching his scores are Toolbox = 6, System = 12, Process = 12. This feature is not reflected in the graphical representation where the System aspect is remained unchanged. However, the length of the vector indicates his feeling that his real teaching differs greatly from ideal teaching.

Apparently there are some inconsistencies in the numerical and graphical data of Ken. His estimations of real and ideal teaching show some interchanging of the roles of Toolbox and System, which should be represented by a reflection of positions within the equilateral triangle. On the other hand, Ken's arrow in the graphical mode calls for some change, in particular, towards more Process aspect and less System.

Joseph's data in both surveys also deviate considerably; the Euclidean length of the alteration vector is $\sqrt{5^2+2^2+3^2}$, thus $\sqrt{37}$, whereas for Dyllan $\sqrt{200}$. However, the arrow length in the graphic representation are in the same length region. Under consideration of the presentations given in the interviews, these graphic informations reflect the situations more adequately.

Whereas it is easier to realize the tendencies and the direction of the changes in the graphical mode, the table may show some clues or patterns as to how the changes should take place. Note that all three, Dylan (System = 5), Harry (System = 1), and Larry (System = 9) would not be likely to change the absolute value of the factor System; they only prefer an exchange between the tool aspect in favor of the Process aspect. It must remain an open question whether or not it is an intentional exchange, or perhaps whether it is just merely a strategy to treat the data which is to follow the Dionnian categorization and to distribute 30 points twice among three entries. These arguments support again our hypothesis that the derived Table 3.6 is not less important than the absolute values.

4.3 Evaluation of the Information From The Different Sources

It is indisputable that Dionné's three-pole polarization, corresponding to the Ernestian categorization, must be seen only as a primitive model, in particular as through this approach itself the dimensionality of the views is limited to three. Nevertheless, in spite of its simplicity it still pos-

sesses a high degree of clarification power, especially when related to a first approach to the problem of becoming aware and identifying different views on mathematics. The detailed comparison of the partly different self-estimations on the part of the teachers, in view of the Dionnian components, makes clear that the numerical data is in need of commentary. They would have never been derived only from the interview data! The ranking table 4.3 derived from the information would also be too coarse to allow detailed conclusions, as it relates in our survey considerably different persons to the same ranking lists. In other words, determining views is of little evaluative value when one asks the interviewees for rankings.

It is decisive to highlight such examinations, whether or not they concern the real teaching of mathematics or are valued as the ideal teaching of mathematics. In turn, one should determine the weight distributions of both teaching situations and their loadings, using the Dionnian components. Since it is not evident how to choose 'objective' figures, the two respective point distributions stabilize themselves reciprocally.

Next, we highly recommend to use the tabular as well as the graphical mode of investigation in the sense of Dionne. However, the noticeable inconsistency should not be overly valued because both sources of data make allowances for varying emphasis.

Metrical aspects play an especially strengthened role in pictorial illustrations. The examinee can highlight his basic discrepancies, and, finally, a direction of change will become evident in relation to the three components represented by the three corners of the triangle. Whereas the graph informs one on the magnitude of a change, the table may show the kind of change.

In our experience it seems to be essential to address both assessments in a parallel fashion in order to give the examinee the possibility of internal self-comparison. The respective point distributions stabilize themselves reciprocally. Corresponding numerical data also possibly provide an additional indication of which changes the responder has in mind.

For these reasons we urge that when the relatively coarse Dionnian model of categorization is used attention is paid to the clarity between the actual and the ideal mathematics classroom situation. Both three-dimensional figures should be raised simultaneously, and the test person should be allowed the possibility of a graphic representation by the corresponding self-estimation.

5. Conclusion

Our predominant conclusion to research question through our observations is in a sense trivial: Related methods will mainly produce, partly redundant and additionally confirming information, thus consistent data. Furthermore, there were no *global* contradictions between the information of the teachers (obtained from different data sources) in particular with respect to the Dionne parameter (numerical as well graphical). This is of great importance, in particular when employing and mixing qualitative as well more quantitative methods. Individual inconsistencies, however, could be accounted for, i.e. made plausible by the statements made in the interviews.

Further, we can state that there is no best (indirect) method by which teachers' views can be investigated. Firstly, the open interview as well as the open discussion on the items of the questionnaire show once more that closed questionnaires are of limited use. Since we opened up our procedure, we had the possibility to correct some misunderstandings, e.g. we had to omit one item whose discussion showed the inadequate questioning.

Secondly, the interviews lead to some interesting central quotations describing main features of the teachers' views, however it is not easy to condense the verbal profile and to 'coordinatize' the teachers' positions in order to compare them objectively and quantitatively.

Thirdly, although the Dionne method appears to be a very rough quantitative tool ignoring some details and seeming not to be aware of other details, this method leads to some numbers which contain worthwhile information. Of course one cannot oversee that the Dionne approach (in both

versions employed here) projects a highly dimensioned world of attitudes into a 2- or 3-dimensional variety. Such a procedure leads inevitably to serious reductions. Individual conclusions on the persons, even when based on Dionee parameters, still contain many uncertainties, unless one couples these results to the statements in the interviews. The interview statements in this survey thus fulfill the central function of being explanatory and authoritative. As expressed before, one should clearly distinguish between real and ideal teaching. Then it is the pair of the two vectors which tells a story! The additional expenditure for the test subjects is of a marginal magnitude. Finally, we believe to have proven that additional collecting of graphic data is hardly more costly and is of important explanatory value. Intervening feelings and affects are revealed e.g. in the length of the arrows more clearly than in the numerical data.

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Appendix 1 The letter to the teachers

Starting point: a rough classification of mathematical views consist of the following three perspectives, which are part of every view of mathematics and the teaching of mathematics:

- T Mathematics is a large toolbox: Doing mathematics means working with figures, applying rules and procedures and using formulas.
- S Mathematics is a formal, rigorous system: Doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts.
- P Mathematics is a constructive process: Doing mathematics means learning to think, deriving formulas, applying reality to Mathematics and working with concrete problems.

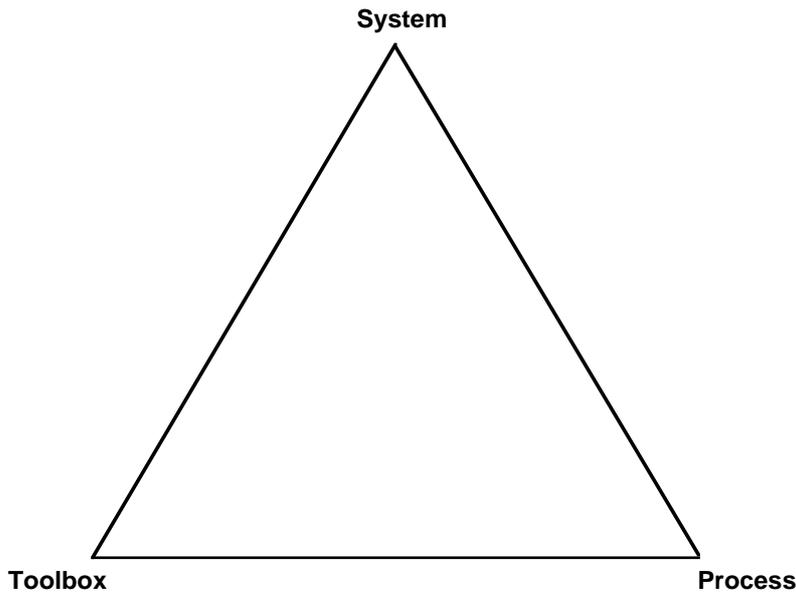
Question 1: Distribute a total of 30 points corresponding to your estimation of the factors, T, S, and P in which you value **your**

	T	S	P
real teaching of mathematics			
ideal teaching of mathematics			

For additional comments please use the reverse side of this page.

Question 2: Acknowledge your position on the three factors mentioned above by marking points within the equilateral triangle below.

- x** = real teaching of mathematics
- o** = ideal teaching of mathematics



For additional comments please use the reverse side of this page.

Thank you very much!