Optimization of Simultaneous Power Production and Trading by Stochastic Integer Programming

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Abstract

We develop a two-stage stochastic integer programming model for the simultaneous optimization of power production and day-ahead power trading. The model rests on mixed-integer linear formulations for the unit commitment problem and for the price clearing mechanism at the power exchange. Foreign bids enter as random components into the model. We solve the stochastic integer program by a decomposition method combining Lagrangian relaxation of nonanticipativity with branch-and-bound in the spirit of global optimization. Finally, we report some first computational experiences.

Key Words. Stochastic integer programming, power optimization, electricity trading.

AMS subject classifications. 90C15, 90C11, 90C90.

1 Introduction

Optimization of power production, mainly fuel cost minimization, is a traditional field of applied optimization in power engineering. In the course of the liberalization of electricity markets new trading instruments have emerged whose proper handling is of supreme importance for the efficient operation of electrical utilities as a whole. Among the new instruments there is day-ahead trading of power. In the present paper we address the simultaneous optimization of day-ahead trading and production of power. This problem is inherently uncertain, in that day-ahead trading involves bids that result in dispatch only after clearance of the market by an independent operator.

The need for simultaneous optimization of power production and trading arises if a utility has to cover with its installed generators a demand profile of power over time and, at the same time, has the possibility to offer into a day-ahead market. The demand profile may arise from physical bilateral contracts, for instance. The challenge then is to adjust production and trading in such a way, that "cheap" purchases replace "expensive" production and "cheap" production is used for "profitable" sales. Beside the mentioned uncertainty, time coupling of production decisions prevents an optimization hour by hour. Hence, a large-scale optimization problem arises.

The power system underlying our model is a hydro-thermal one as operated for instance by the German utility VEAG Vereinigte Energiewerke AG Berlin. It comprises pumped-storage hydro as well as coal and gas fired thermal power plants.

Day-ahead trading involves sealed selling and purchase bids for every individual hour of the day ahead. Each offer comprises volume and price. There is only one round of bidding, and the market price is cleared by an independent agent such that the total exchange is maximized. Selling offers strictly below and purchase offers strictly above market price are executed completely. Vice versa, sellings strictly above

as well as purchases strictly below do not become effective. Offers at market price in general are only partially executed, with specific splitting rules in case of several offers with identical price.
Decision making in the utility involves both production decisions for the generators and bids for the individual hours of the day ahead. Up to now, demand profiles and generator failures were typical sources of uncertainty in power optimization. The prime source of uncertainty in day-ahead trading are foreign bids, i.e., the bids of the other market participants.
We will formulate the problem of simultaneous optimization of power production and trading as a two-stage linear stochastic program with mixed-integer recourse. With a time horizon of one week, the model will provide non-anticipative production and bidding decisions for the first day such that the expected value of fuel costs minus trading income over time becomes minimal.
Stochastic integer programming is gaining increased interest in power optimization under uncertainty. For a recent contribution including a rich list of references we refer to [5].
Our paper is organized as follows. In the following two sections we develop submodels for power production and day-ahead trading. In Section 4 these are combined into a two-stage stochastic integer program with foreign offers as source of randomness. With a discrete probability distribution, the stochastic program turns into a large-scale mixed-integer linear program for which we outline a decomposition method in the second part of Section 4. Section 5 reports our computational experiences.

2 Power Production

Optimization of power production, often referred to as unit commitment, concerns the scheduling of start-ups and shut-downs and the finding of operation levels for power generation units such that the fuel costs over some time horizon are minimal, see [10] for a literature synopsis. In the following we will present a mixed-integer linear programming model for unit commitment. Given the algorithmic capabilities in mixed-integer *linear* optimization, we will either avoid nonlinearities or work with approximations involving mixed-integer linear expressions.
Consider a discretization of the planning horizon into $t = 1, ..., T$ equidistant, e.g. hourly, subintervals, and assume that there are I thermal and J pumped-storage hydro units. The variable $u_{it} \in \{0,1\},\ i = 1,\dots,I; t = 1,\dots,T$ indicates whether the thermal unit i is in operation at time t. Variables p_{it}, s_{jt}, w_{jt}, $i = 1,\dots,I$, $j = 1,\dots,J$; $t = 1,\dots,T$ are the output levels for the thermal units, the hydro units in generation and in pumping modes, respectively. The variables l_{jt} denote the fill (in terms of energy) of the upper dam of the hydro unit j at the end of interval t, $j = 1,\dots,J$; $t = 1,\dots,T$.
The power outputs of units and the fills of the upper dams have to be within the following bounds

$$\begin{array}{rcccllllll} p_i^{\min} u_{it} & \leq & p_{it} & \leq & p_i^{\max} u_{it}, & i & = & 1,\dots,I, & t & = 1,\dots,T \\ 0 & \leq & s_{jt} & \leq & s_j^{\max}, & j & = & 1,\dots,J, & t & = 1,\dots,T \\ 0 & \leq & w_{jt} & \leq & w_j^{\max}, & j & = & 1,\dots,J, & t & = 1,\dots,T \\ 0 & \leq & l_{jt} & \leq & l_j^{\max}, & j & = & 1,\dots,J, & t & = 1,\dots,T \end{array} \quad (1)$$

Here, $p_i^{\min}, p_i^{\max}, s_j^{\max}, w_j^{\max}$ denote minimal and maximal outputs, respectively, and $l_j^{\max}$ is the maximal fill of the upper dam. The hourly coverage of electrical load results in the constraints

$$\sum_{i=1}^{I} p_{it} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) \geq D_t, \quad t = 1,\dots,T, \quad (2)$$

where D_t denotes the demanded load at time t. In addition to load coverage, reserve management is a key issue in power production. Different reserve schemes are employed in practice. At least the following requirement involving a so-called spinning reserve R_t has to be met for the thermal units:

$$\sum_{i=1}^{I} (u_{it} p_i^{\max} - p_{it}) \geq R_t, \quad t = 1,\dots,T. \quad (3)$$

Reserve schemes involving pumped-storage hydro units are more elaborate but can still be handled using mixed-integer linear expressions, cf. [4].
In the pumped-storage plants the following water balances over time have to be maintained:

$$\left.\begin{array}{rcl} l_{jt} & = & l_{j(t-1)} - (s_{jt} - \eta_j w_{jt}), \\ l_{j0} & = & l_j^{in}, \qquad l_{jT} = l_j^{end}, \end{array}\right\} \quad j = 1,\dots,J,\ t = 1,\dots,T. \quad (4)$$

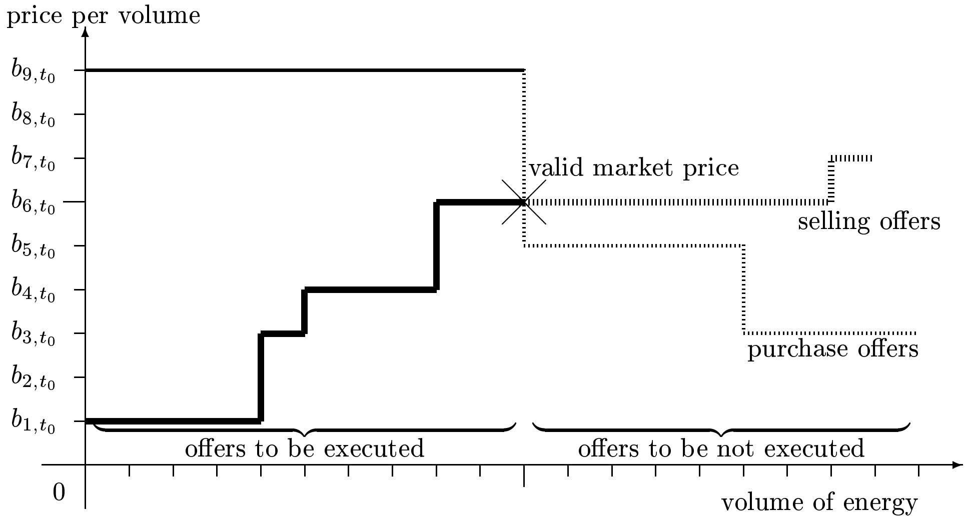

Figure 1: Market price determination

Here, l_j^{in}, l_j^{end} are the initial and final fills (in energy) of the upper dams, η_j denote the pumping efficiencies. The latter are known to be nonlinear functions of the fill in the upper dam. Our model will work with approximative constants that are tolerable due to test runs done with the real system of VEAG Berlin. Constraints avoiding simultaneous generation and pumping in the hydro plants are dispensable since it can be shown that such a deficiency can not occur in optimal points.
To avoid excessive thermal stresses in the coal fired blocks, they have to adhere to minimum up and down times σ_i and τ_i. These are modeled via

$$\begin{array}{rcllcl} u_{it} - u_{i(t-1)} & \leq & u_{i\sigma}, & \sigma & = & t+1, \ldots, \min\{t+\sigma_i - 1, T\}, \\ u_{i(t-1)} - u_{it} & \leq & 1 - u_{i\tau}, & \tau & = & t+1, \ldots, \min\{t+\tau_i - 1, T\}, \\ & & & i & = & 1, \ldots, I; t = 2, \ldots, T-1. \end{array} \tag{5}$$

The constraints (1) – (5) provide a mathematical model for basic features and basic interaction of the generating units of the VEAG system. A typical objective function to be minimized on the constraint set (1) – (5) is the fuel consumption for starting up and operating thermal units. It can be formalized by

$$\sum_{t=1}^{T} \sum_{i=1}^{I} C_i(p_{it}, u_{it}) + \sum_{t=1}^{T} \sum_{i=1}^{I} S_{it}(u_i), \tag{6}$$

where, with suitable constants a_{it}, b_{it}, and c_i, $C_i(p_{it}, u_{it}) = a_{it}p_{it} + b_{it}u_{it}$ denote the fuel costs and $S_{it}(u_{it}) = c_i \max\{u_{it} - u_{i,t-1}, 0\}$ the start-up costs for the thermal unit i. Being in a minimization framework, the max-term in the start-up costs can be transformed in a standard way into a linear objective term and additional linear constraints. In the above setting, start-up costs are independent of the preceding down time. Down time dependence can be modeled by suitable max-terms over linear expressions in Boolean variables as well, see [4] for details.

3 Day-Ahead Trading

The trading rules described in the Introduction result in the price formation mechanism depicted in Figure 1. Selling and purchase offers (or bids) are placed in (price-) ascending and descending orders, respectively, yielding two step curves where step length corresponds to volume and step height to price. The intersection of the curves determines both the market price and the total volume traded. In our model we make the basic assumption that executed offers are traded at market price, in contrast to trading at offer price.

In the following, we will model the pricing mechanism in mixed-integer linear terms. Since trading is carried out at any individual hour of a given time horizon, we again consider a time discretization

$t = 1, \dots, T$.
Bids are divided into own bids and foreign bids, the latter being those of the other market participants. Own bids will enter the model as indeterminates. Each bid consisting of volume and price, own bids create nonlinearities by the products of the two. To overcome this, a fixed equidistant discretization of the price range into $m = 1, \dots, M$ ascending levels with values $b_{mt} \geq 0$ will be used.
For all $t = 1, \dots, T$, $m = 1, \dots, M$ we introduce triplets $(v^s_{mt}, v^p_{mt}, v^c_{mt}) \in \{0, 1\}^3$ of indicators. In the end, we will have that $v^s_{mt} = 1$ ($v^p_{mt} = 1$) iff the selling (purchase) offer with price level m at time t is strictly below (above) market price and hence executed completely. Moreover, we will have that $v^c_{mt} = 1$ iff price level m coincides with the valid market price at time t.
In a first step, we introduce the following set of logical constraints. Beside the conditions that exactly one out of v^c_{mt} (t fixed and $m = 1, \dots, M$) and one out of $v^s_{mt}, v^p_{mt}, v^c_{mt}$ (m, t fixed) have to be one, the system assures that, once $v^s_{mt} = 1$, then also $v^s_{m't} = 1$ for all $m' > m$ and , once $v^p_{mt} = 0$, then also $v^p_{m't} = 0$ for all $m' < m$.

$$\begin{array}{rl} \sum_{m=1}^{M} v^c_{mt} = 1, & t = 1, \dots, T; \\ \left. \begin{array}{r} v^s_{mt} \geq v^s_{m+1,t}, \\ v^p_{mt} \leq v^p_{m+1,t}, \\ v^s_{mt} + v^c_{mt} + v^p_{mt} = 1, \end{array} \right\} & t = 1, \dots, T, \ m = 1, \dots, M. \end{array} \tag{7}$$

We distinguish variables for energy volumes of own bids of price level m in time period t (q^s_{mt}, $q^p_{mt} \geq 0$) and variables for volumes of own *executed* bids of price level m in time period t (p^s_{mt}, $p^p_{mt} \geq 0$). Executed bids cannot exceed bids:

$$0 \leq p^s_{mt} \leq q^s_{mt}, \ 0 \leq p^p_{mt} \leq q^p_{mt}; \quad t = 1, \dots, T, \ m = 1, \dots, m. \tag{8}$$

Own selling bids below and own purchase bids above market price are to be executed completely:

$$\left. \begin{array}{l} p^s_{mt} \geq q^s_{mt} - C_1(1 - v^s_{mt}), \\ p^p_{mt} \geq q^p_{mt} - C_1(1 - v^p_{mt}), \end{array} \right\} \quad t = 1, \dots, T, \ m = 1, \dots, m \tag{9}$$

with a sufficiently big constant C_1. Own selling (purchase) bids must not be executed, if they are above (below) the valid price:

$$\left. \begin{array}{l} p^s_{mt} \leq C_2(v^s_{mt} + v^c_{mt}), \\ p^p_{mt} \leq C_2(v^p_{mt} + v^c_{mt}), \end{array} \right\} \quad t = 1, \dots, T, \ m = 1, \dots, m \tag{10}$$

with a sufficiently big constant C_2.
The foreign selling and purchase bids of price level m for time t are denoted by $f^s_{mt}, f^p_{mt} \geq 0, m = 1, \dots, M, t = 1, \dots, T$, respectively. It is permitted that not the complete volume of a competitor's bid at market price is executed (as in our example, Figure 1). Then β^s_t, $\beta^p_t \geq 0$ denote the actually executed volumes of the competitors at valid market price:

$$0 \leq \beta^s_t \leq \sum_{m=1}^{M} v^c_{mt} f^s_{mt}, \ 0 \leq \beta^p_t \leq \sum_{m=1}^{M} v^c_{mt} f^p_{mt}, \quad t = 1, \dots, T. \tag{11}$$

Maximum exchange is reached at the equilibrium of supply and demand:

$$\sum_{m=1}^{M} (v^s_{mt} f^s_{mt} + p^s_{mt}) + \beta^s_t = \sum_{m=1}^{M} (v^p_{mt} f^p_{mt} + p^p_{mt}) + \beta^p_t, \ \ t = 1, \dots, T. \tag{12}$$

At market price, either a complete selling offer or a complete purchase offer or both a complete selling and a complete purchase offer have to be executed (cf. Fig. 2). Introducing another indicator $v^0_t \in \{0, 1\}$ attaining the value 1 iff at least all selling offers at market price are executed, the price formation model is completed by the following constraints:

$$\begin{array}{l} \beta^s_t \geq \sum_{m=1}^{M} v^c_{mt} f^s_{mt} - C_3(1 - v^0_t), \ \beta^p_t \geq \sum_{m=1}^{M} v^c_{mt} f^p_{mt} - C_3 v^0_t, \quad t = 1, \dots, T; \\ \left. \begin{array}{l} p^s_{mt} \geq q^s_{mt} - C_3(2 - v^c_{mt} - v^0_t), \\ p^p_{mt} \geq q^p_{mt} - C_3(1 - v^c_{mt} + v^0_t), \end{array} \right\} \quad t = 1, \dots, T, m = 1, \dots, M \end{array} \tag{13}$$

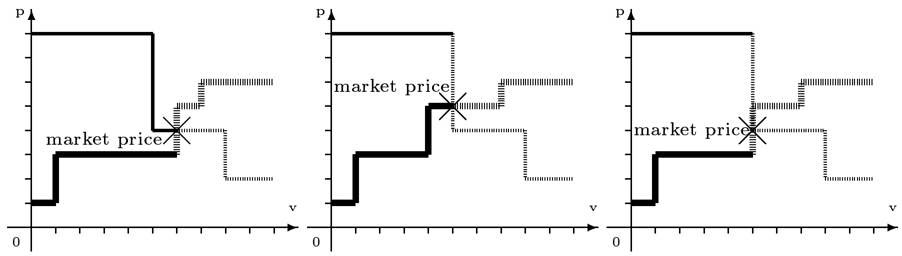

Figure 2: Cases at market price

with a sufficiently big constant C_3.
For selling and purchase offers at market price, the above relations, together with the final cost criterion, imply a splitting rule favouring the home utility. Indeed, equations (12) imply in particular that those shares p^s_{mt} and p^p_{mt} of the home utility's offers become committed that are optimal from the home utility's view alone, neglecting the preferences of the foreign utilities and leaving only the possibly remaining shares β^s_t and β^p_t to them.
By $g_{mt} \in I\!R$ we denote the net traded volume at price level m and time t. It has to fulfil

$$\begin{array}{c} \sum_{m=1}^{M} g_{mt} = \sum_{m=1}^{M} (p^p_{mt} - p^s_{mt}), \quad t = 1, \ldots, T, \\ -C_4 v^c_{mt} \leq g_{mt} \leq C_4 v^c_{mt}, \quad t = 1, \ldots, T,\ m = 1, \ldots, M \end{array} \tag{14}$$

with a sufficiently big constant C_4.

The above trading model is concatenated with the production model from Section 2 by modifying the load coverage constraints (2) into

$$\sum_{i=1}^{I} p_{it} + \sum_{j=1}^{J} (s_{jt} - w_{jt}) + \sum_{m=1}^{M} g_{mt} = D_t, \quad t = 1, \ldots, T \tag{15}$$

and adding the term

$$\sum_{t=1}^{T} \sum_{m=1}^{M} b_{mt} g_{mt}. \tag{16}$$

to the objective (6).

4 Stochastic Integer Programming Model

The trading model in Section 3 reflects the underlying mechanisms without considering availability of data information. In an operational setting this, clearly, is insufficent. In particular, the foreign bids $f^s_{mt}, f^p_{mt}, m = 1, \ldots, M, t = 1, \ldots, T$ are not known at the moment own bids have to be made. Bids for a day typically have to be handed in by early afternoon the previous day.
For the simultaneous optimization of power production and trading this suggests to consider all production and bidding decisions for the first day of the time horizon as here-and-now decisions, i.e., as decisions that have to be taken without precise knowledge of the missing data, the foreign bids. The latter enter the model in stochastic form through a finite number of scenarios with given probabilities. Decisions to be made at the second and at all further days of the time horizon then are stochastic as well, since they vary with the scenarios for the foreign offer.
In this way, one arrives at a decision problem that can be formulated as a two-stage stochastic program. The here-and-now decisions, also called non-anticipative, are collected into the first stage. The second stage assembles the remaining decisions which can be understood as recourse actions given the first-stage decisions and the realizations of the random data.
This can be formalized as follows. With stochastic foreign offers, the random optimization problem arising from concatenation of the models in Sections 2 and 3 adopts the form

$$\min_{x,y} \{c^T x + d^T y \ : \ Ax + W(\omega) y \ = \ h,\ x \in X,\ y \in Y\} \tag{17}$$

where x and y denote the first- and second-stage decisions, respectively. In the context of our power model, x comprises first-day production decisions, i.e., the variables $p_{it}, s_{jt}, w_{jt}, l_{jt}$ for suitable t and all i, j, and the utility's own offers for the second day, i.e., the variables q^s_{mt}, q^p_{mt} for suitable t and all m. We assume that the trading for the individual hours of the first day (based on bids from the day before) is known such that the corresponding variables are fixed. All the remaining variables are assembled into the vector y. The only place where the random foreign offers f^s_{mt}, f^p_{mt} occur is as coefficients in front of second-stage variables. This explains that in (17) the matrix W is the only random entity.

We assume that W lives on some probability space $(\Omega, \mathcal{A}, I\!\!P)$, that all the ingredients in (17) have conformal dimensions, and that $X \subseteq I\!\!R^{n_1}, Y \subseteq I\!\!R^{n_2}$ are polyhedra, possibly involving integer requirements to components of x and y.

The following two-stage linear stochastic program with mixed-integer recourse then aims at selecting a feasible non-anticipative decision x such that the expected value of the total random costs becomes minimal

$$\min_x \Big\{ c^T x + I\!\!E_\omega(\min_y \{d^T y \,:\, W(\omega) y = h - Ax,\ y \in Y\} \,:\, x \in X \Big\}. \tag{18}$$

As already mentioned, foreign offers, and thus the matrices $W(\omega)$, enter our model by a discrete probability distribution, with realizations $W_1, \ldots, W_N$ and probabilities $\pi_1, \ldots, \pi_n$, say. Problem (18) then can be equivalently rewritten as a large-scale block-angular mixed-integer linear program

$$\min_{x, y_j} \Big\{ c^T x + \sum_{j=1}^N \pi_j d^T y_j \;:\; Ax + W_j y_j \;=\; h,\ y_j \in Y,\ x \in X \Big\}. \tag{19}$$

Problem (19) is far too big to be handled directly by mixed-integer linear programming solvers. Instead we apply the following decomposition method from [2] for its solution.

Introduce copies $x_j, j = 1, \ldots, N$, according to the number of scenarios, and add the constraints $x_1 = \ldots = x_N$ (or an equivalent system), for which we use the notation $\sum_{j=1}^N H_j x_j = 0$ with proper $(l, n_1)-$matrices $H_j, j = 1, \ldots, N$. Problem (19) then becomes

$$\min \Big\{ \sum_{j=1}^N \pi_j (c^T x_j + d^T y_j) \,:\, A x_j + W_j y_j = h,\ x_j \in X,\ y_j \in Y,\ \sum_{j=1}^N H_j x_j = 0 \Big\}. \tag{20}$$

This model allows for Lagrangian relaxation of the constraints $\sum_{j=1}^N H_j x_j = 0$. For $\lambda \in I\!\!R^l$ we consider the functions

$$L_j(x_j, y_j, \lambda) \;:=\; \pi_j (c^T x_j + d^T y_j) \;+\; \lambda^T H_j x_j, \;\; j = 1, \ldots, N,$$

and form the Lagrangian

$$L(x, y, \lambda) \;:=\; \sum_{j=1}^N L_j(x_j, y_j, \lambda).$$

The Lagrangian dual of (20) then is the optimization problem

$$\max\{D(\lambda) \;:\; \lambda \in I\!\!R^l\} \tag{21}$$

where

$$D(\lambda) \;=\; \min \Big\{ \sum_{j=1}^N L_j(x_j, y_j, \lambda) \;:\; A x_j + W_j y_j \;=\; h,\ x_j \in X,\ y_j \in Y \Big\}. \tag{22}$$

For separability reasons we have

$$D(\lambda) \;=\; \sum_{j=1}^N D_j(\lambda) \tag{23}$$

where

$$D_j(\lambda) \;=\; \min\{L_j(x_j, y_j, \lambda) \;:\; A x_j + W_j y_j \;=\; h,\ x_j \in X,\ y_j \in Y\}. \tag{24}$$

$D_j(\lambda)$ is the pointwise minimum of affine functions in λ, and hence piecewise affine and concave. Therefore, (21) is equivalent to a non-smooth convex minimization problem that can be solved by bundle methods, see [6, 7] for background and details. At each iteration, bundle methods require the objective

value and one subgradient of D. Here, the separability in (23) is most beneficial. It is well known, cf. e.g. [9], that the optimal value φ_{LD} of (21) provides a lower bound to the optimal value φ of problem (19).
The total computational effort is distributed. The single-scenario problems in (24) have to be solved repeatedly to provide the input for (21). Powerful software is available for these tasks. In our computational experiments we resorted to CPLEX ([3]) for solving the single-scenario problems, and to the bundle method NOA 3.0 developed and implemented by K.C. Kiwiel ([7, 8]).
In Lagrangian relaxation, feasible points for the original problem are often obtained by suitable heuristics starting from the results of the dual optimization. Our relaxed constraints being very simple ($x_1 = \ldots = x_N$), ideas for such heuristics come up straightforwardly. For example, examine the x_j-components, $j = 1, \ldots, N$, of solutions to (24) for optimal or nearly optimal λ, and decide for the most frequent value arising or average and round if necessary.
If the heuristic results in a feasible solution to (20), then the objective value of the latter provides an upper bound $\bar{\varphi}$ for φ. Together with the lower bound φ_{LD} this yields a quality certificate (gap) $\bar{\varphi} - \varphi_{LD}$. If necessary, this certificate can be improved by embedding the procedure described so far into the following branch-and-bound scheme. Let $\mathcal{P}$ denote the list of current problems, $\varphi_{LD} = \varphi_{LD}(P)$ be the Lagrangian lower bound for $P \in \mathcal{P}$, and $Q(x) := I\!\!E_\omega(\min_y\{d^T y \,:\, W(\omega)y = h - Ax,\, y \in Y\})$.

Step 1 *Initialization:* Set $\bar{\varphi} = +\infty$ and let $\mathcal{P}$ consist of problem (20).

Step 2 *Termination:* If $\mathcal{P} = \emptyset$ then the solution $\hat{x}$ that yielded $\bar{\varphi} = c^T\hat{x} + Q(\hat{x})$ is optimal.

Step 3 *Node selection:* Select and delete a problem P from $\mathcal{P}$ and solve its Lagrangian dual. If the optimal value $\varphi_{LD}(P)$ hereof equals $+\infty$ (infeasibility of a subproblem) then go to Step 2.

Step 4 *Bounding:* If $\varphi_{LD}(P) \geq \bar{\varphi}$ go to Step 2 (this step can be carried out as soon as the value of the Lagrangian dual rises above $\bar{\varphi}$).

- **(i)** The scenario solutions x_j, $j = 1, \ldots, N$, are identical: If $c^T x_j + Q(x_j) < \bar{\varphi}$ then let $\bar{\varphi} = c^T x_j + Q(x_j)$ and delete from $\mathcal{P}$ all problems P' with $\varphi_{LD}(P') \geq \bar{\varphi}$. Go to Step 2.
- **(ii)** The scenario solutions x_j, $j = 1, \ldots, N$ differ: Compute the average $\bar{x} = \sum_{j=1}^{N} \pi_j x_j$ and round it by some heuristic to obtain $\bar{x}^R$. If $c^T\bar{x}^R + Q(\bar{x}^R) < \bar{\varphi}$ then let $\bar{\varphi} = c^T\bar{x}^R + Q(\bar{x}^R)$ and delete from $\mathcal{P}$ all problems P' with $\varphi_{LD}(P') \geq \bar{\varphi}$. Go to Step 5.

Step 5 *Branching:* Select a component $x_{(k)}$ of x and add two new problems to $\mathcal{P}$ obtained from P by adding the constraints $x_{(k)} \leq \lfloor \bar{x}_{(k)} \rfloor$ and $x_{(k)} \geq \lfloor \bar{x}_{(k)} \rfloor + 1$, respectively (if $x_{(k)}$ is an integer component), or $x_{(k)} \leq \bar{x}_{(k)} - \varepsilon$ and $x_{(k)} \geq \bar{x}_{(k)} + \varepsilon$, respectively, where $\varepsilon > 0$ is a tolerance parameter to have disjoint subdomains.

5 Computations

The model from Section 4 was validated with VEAG data for the power system and market prices of the Amsterdam Power Exchange (APX) [1] for day-ahead trading. The VEAG system comprises 17 coal and 8 gas fired thermal units as well as 7 pumped storage plants. Using APX market prices we constructed hourly foreign offers of different prices and volumes. These were assembled into 10 scenarios. To study the impact of trading we have also solved the pure production problem (Problem A) with trading volumes fixed to zero.
Problem sizes of the deterministic production problem A and the stochastic programs B-F (in the block-angular form of (19)) are displayed in Table 1. The sizes of B-F, clearly, suggest to apply our decomposition method rather than pursuing a direct approach with some general-purpose mixed-integer linear programming solver.
The test runs are reported in Table 2. They were carried out at a SUN E450 ultra SPARC with 300 MHz processor. The "Gap" column displays the relative size of the difference between the optimal value of the best feasible solution and the tightest lower bound found by the branch-and-bound method from the end of Section 4. The "Min. Saving" column contains the relative cost improvement over pure production achieved by the best solution found for the model of simultaneous production and trading. The "Gap" column then can be seen as an upper bound for possible additional saving provided one is able to solve the stochastic program to optimality. Given the very substantial amount fuel costs in power utilities

Prob.	power exchange	Constr.	Variables	Integers	Binaries
A	–	18 641	11 089	1 008	1 512
B	10 scenarios	563 959	332 848	8 784	47 736
C	20 scenarios	1 109 277	643 008	16 560	93 960
D	30 scenarios	1 654 595	963 168	24 336	140 184
E	40 scenarios	2 199 913	1 282 328	32 112	186 408
F	50 scenarios	2 745 231	1 603 488	39 888	232 632

Table 1: Problem characteristics and sizes

Prob.	Time h:min:sec	Best Solution	Lower Bound	Gap	Min. Saving
A	0:07:06	46 287 933	46 287 933	0.00 %	0.00 %
B	8:32:49	46 132 267	45 634 163	1.07 %	0.34 %
C	18:28:53	46 149 049	45 657 419	1.06 %	0.30 %
D	27:25:27	46 154 054	45 661 411	1.07 %	0.29 %
E	37:00:46	46 171 033	45 652 329	1.12 %	0.25 %
F	47:27:18	46 175 202	45 642 053	1.15 %	0.24 %

Table 2: Test runs

accumulate to, the savings in Table 2, although still below one percent, seem to be relevant, though. The tests confirm that problems with up to 50 scenarios are within the reach of computation. However, problem sizes then are really huge, and computation times (at our fairly slow serial machine) become quite excessive.

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References

[1] Amsterdam Power Exchange (APX). `http://www.apx.nl/vers200.htm`

[2] Carøe, C. C.; Schultz, R.: Dual decomposition in stochastic integer programming, Operations Research Letters 24 (1999), 37-45.

[3] Using the CPLEX Callable Library, CPLEX Optimization, Inc. 1999.

[4] Gollmer, R.; Nowak, M.P.; Römisch, W.; Schultz, R.: Unit commitment in power generation - a basic model and some extensions, Annals of Operations Research 96 (2000), 167-189.

[5] Gröwe-Kuska, N.; Kiwiel, K.C.; Nowak, M.P.; Römisch, W.; Wegner, I.: Power management in a hydro-thermal system under uncertainty by Lagrangian relaxation, in: Decision Making under Uncertainty: Energy and Power (C. Greengard, A. Ruszczyński eds.), IMA Volumes in Mathematics and its Applications Vol. 128, Springer-Verlag, New York 2001, 39-70.

[6] Hiriart-Urruty, J. B.; Lemaréchal, C.: Convex Analysis and Minimization Algorithms, Springer-Verlag, Berlin 1993.

[7] Kiwiel, K. C.: Proximity control in bundle methods for convex nondifferentiable optimization, Mathematical Programming 46 (1990), 105-122.

[8] Kiwiel, K. C.: User's Guide for NOA 2.0/3.0: A Fortran Package for Convex Nondifferentiable Optimization, Systems Research Institute, Polish Academy of Sciences, Warsaw, 1994.

[9] Nemhauser, G. L.; Wolsey, L. A.: Integer and Combinatorial Optimization, Wiley, New York, 1988.

[10] Sheble, G.B.; Fahd, G.N.: Unit commitment literature synopsis, IEEE Transactions on Power Systems 9 (1994), 128-135.